

OSCILLATORY FLOW OF A CONDUCTING JEFFREY FLUID IN A COMPOSITE POROUS MEDIUM CHANNEL

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ABSTRACT

Unsteady flow of a conducting Jeffrey fluid in a horizontal composite porous medium channel is investigated. A uniform transverse magnetic field of strength B_0 is applied perpendicular to the composite channel. The flow in the channel is divided into two regions, namely porous and non-porous regions. The flow in the porous region is modeled using Darcy-Brinkman equation. The viscous and Darcian dissipation terms are also included in the energy equations governing the flow. The nonlinear governing equations are solved analytically using two-term harmonic and non-harmonic functions. The effects of the porous medium parameter, ratio of viscosities, oscillation amplitude, conductivity ratio, Prandtl number and Eckert number on the velocity and the temperature fields are studied in detail. It is found that the velocity decreases with the increase in the non-Newtonian Jeffrey parameter whereas the temperature shows same trend with the Jeffrey parameter. For a given ratio of viscosity m , the interface velocity decreases with increasing magnetic parameter M and porous medium parameter σ .

INTRODUCTION

Viscous flow through or past porous media is of fundamental importance in petroleum technology, powder metallurgy, industrial filtration, ceramic engineering, ground water hydrology and such other fields. In springs of the geothermal region, water is known to be an electrically conducting fluid. The abundant geofluids in the Earth's crust in the geothermal regions has to be brought up to augment fuel output. Earth's surface can be modeled as a natural permeable bed and hence the study on the flow through porous medium is necessitated.

The study of viscous conducting fluids plays a significant role, owing to its practical interest and abundant applications in astro-physical and geo-physical phenomena. The main impetus to the engineering approach to the electromagnetic fluid interaction studies has come from the concept of the hydrodynamics. The flow and heat transfer of

electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occurs in MHD generators, pumps, accelerators and flow meters and have applications in nuclear reactors, filtration, geothermal systems and others.

During the last few decades, interest in mathematical modelling and analysis of flows involving non-Newtonian fluids in various geometries has been increased. However, there is no model which can lonely predict the behaviour of all the non-Newtonian fluids. Among several non-Newtonian models proposed for physiological fluids, Jeffrey model is one of the simplest nonlinear non-Newtonian models governing the complex fluid behavior. It is significant because Newtonian fluid model can be deduced from this as a special case by taking the Jeffrey parameter $\lambda_1=0$. It has immense importance for their wide applications in engineering industries, for example in metal extrusion process, wire and blade coating, dyeing of papers and textiles etc. It is also recognized that many fluids commonly used in industry differ greatly from the Newtonian behaviour in their rheology.

Brinkman [1] proposed a non-Darcy law for the flow through highly permeable bed of spherical particles. Using the slip conditions Rudraiah and Wilfred [2] and Vajravelu et al. [3] analyzed the natural convection in an inclined layer bounded by porous material. Chamkha [4] presented analytical solutions for the flow of two-immiscible fluids in porous and non-porous parallel plates. Khan et al. [5] investigated for exact solutions for MHD flow of a generalized Oldroyd fluid with modified Darcy's law. Hayat and Ali [6] investigated the peristaltic motion of a Jeffrey fluid under the effect of a magnetic field. Kothandapani and Srinivas [7] Peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel.

Umavathi et al. [8] studied unsteady oscillatory flow and heat transfer in a horizontal composite porous medium channel, and they have investigated effects of porous medium and amplitude on the velocity and the temperature. Vajravelu et al. [9] examined the influence of heat transfer on peristaltic

transport of Jeffrey fluid in a vertical porous stratum. Vasudev [10] has examined MHD peristaltic flow of Newtonian fluid through a porous medium in an asymmetric vertical channel with heat transfer. Krishna Kumari et al [11] studied the effect of magnetic field on the peristaltic pumping in an inclined channel by considering Jeffrey fluid. Mahmoud et al. [12] studied the effect of porous medium and magnetic field on peristaltic transport of a Jeffrey fluid in an asymmetric channel.

Motivated by these studies, oscillatory flow of a conducting Jeffrey fluid in a composite porous medium channel is investigated. The velocity field, the temperature distribution and the volume flux are obtained in the porous and non-porous regions. The effects of various physical parameters on the velocity and temperature distributions are discussed.

MATHEMATICAL FORMULATION

Consider the flow of two electrically conducting, immiscible viscous fluids through an infinitely long composite channel, under the influence of uniform transverse magnetic field (Fig.1). The flow region between the plates is divided into two regions. The flow region between the lower plate $y = -h$ and the interface $y = 0$ is termed as Region 1 (porous matrix region) where as the flow region between the interface $y = 0$ and the upper plate $y = h$ is designated as Region 2 (clear viscous fluid region). The flow in Region 1 is governed by non-Darcy law and the flow in Region 2 is described by Navier-Stokes equations. The fluid velocities in the regions 1 and 2 are u_1 and u_2 . The fluid dynamic viscosities in the Regions 1 and 2 are μ_1 and μ_2 respectively. The magnetic field B_0 is applied perpendicular to the plates and the induced magnetic field is assumed to be negligible. The following assumptions are made in the analysis of the problem.

(a) The flow in both regions of the channel is assumed to be driven by a common Pressure gradient $\left(\frac{\partial p}{\partial x}\right)$ and the

temperature gradient $\Delta T = T_{\omega_1} - T_{\omega_2}$.

(b) The flow is unsteady and fully developed.

(c) The lower and upper plates are maintained at constant different temperature T_{w1} and T_{w2} where $(T_{\omega_1} < T_{\omega_2})$.

(d) The thermo-physical properties of the fluid and the effective properties of the porous medium are assumed to be constant.

With the assumptions mentioned above, the equations of motion and the equations of energy are

$$\rho_0 \left[\frac{\partial u_i}{\partial t} + v_i \frac{\partial u_i}{\partial y} \right] = \frac{\chi_\mu}{1 + \lambda_1} \frac{\partial^2 u_i}{\partial y^2} - \frac{\partial p}{\partial x} - \chi \frac{\chi_\mu}{k(1 + \lambda_1)} u_i - \sigma_e B_0^2 u_i \quad (i=1,2) \quad (1)$$

$$\rho_0 c_p \left[\frac{\partial T_i}{\partial t} + v_i \frac{\partial T_i}{\partial y} \right] = \chi_k \frac{\partial^2 T_i}{\partial y^2} - \chi_\mu \left[\frac{\partial u_i}{\partial x} \right]^2 + \chi \frac{\chi_\mu}{k(1 + \lambda_1)} u_i^2 \quad (i=1,2) \quad (2)$$

where $i=1,2$ gives equations for Regions 1 and 2 respectively, u is the x-component of fluid velocity, v_i is the y-

component of fluid velocity and T is temperature of the fluid. ρ_0 , μ_i , C_p and σ_e are the fluid density, dynamic viscosity and specific heat at constant pressure and electrical conductivity respectively. The parameter k is the permeability of the porous matrix. The other coefficients appearing in equations (1) and (2) are such that

$$\left. \begin{aligned} \chi_\mu &= \mu_{eff} \text{ for porous matrix region} \\ &= \mu \text{ for clear fluid region} \end{aligned} \right\}, \quad \left. \begin{aligned} \chi &= 0 \text{ for porous matrix region} \\ &= 1 \text{ for clear fluid region} \end{aligned} \right\}, \quad \left. \begin{aligned} \chi_k &= K_{eff} \text{ for porous matrix region} \\ &= K_0 \text{ for clear fluid region} \end{aligned} \right\}$$

where K_{eff} and K_0 are the thermal conductivities in porous and clear fluid regions respectively.

The boundary and interface conditions on velocity for the two fluids can then be written as

$$u_1(-h) = 0, \quad u_2(h) = 0, \quad u_1(0) = u_2(0), \quad \frac{\mu_{eff}}{1 + \lambda_1} \frac{\partial u_1}{\partial y} = \frac{\mu}{1 + \lambda_1} \frac{\partial u_2}{\partial y} \quad \text{at } y = 0 \quad (3)$$

The thermal boundary and interface conditions are given by

$$T_1(-h) = T_{\omega_1}, \quad T_2(h) = T_{\omega_2}, \quad T_1(0) = T_2(0), \quad K_{eff} \frac{\partial T_1}{\partial y} = K_0 \frac{\partial T_2}{\partial y} \quad \text{at } y = 0 \quad (4)$$

The continuity equations of both fluids (1) imply that v_1 and v_2 are independent of y . They can be utmost a function of time alone, we can write [assuming $v_1 = v_2 = v$]

$$v = v_0 \left(1 + \varepsilon A e^{i\omega t} \right) \quad (5)$$

where A is real positive constant, ω is frequency parameter and ε is small such that $\varepsilon A \leq 1$. Here it is assumed that the transverse velocity varies periodically with time about a non-zero constant mean v_0 .

Now, we introduce the following non-dimensional quantities.

$$\begin{aligned} u_i &= \bar{u}_i u_i^*, & y &= h y^*, & V &= \frac{v}{h} V^*, & V_0 &= \frac{v_0}{h}, & t &= \frac{h^2}{\nu} t^*, \\ \Delta T &= T_{w1} - T_{w2}, & P &= \frac{h^2}{\mu_i \bar{u}_i} \left(\frac{\partial p}{\partial x} \right), & v &= -v_0 \left(1 + \varepsilon A e^{i\omega t} \right) \\ \theta &= \frac{T_i - T_{\omega_2}}{T_{\omega_1} - T_{\omega_2}}, & Ec &= \frac{U_0^2}{c_p \Delta T} \text{ (Eckert number)}, & Pr &= \frac{\rho_0 \nu C_p}{K} \text{ (Prandtl} \\ & & & & & \text{number)}, & \nu &= \frac{\mu}{\rho_0}, & & \text{and } (v_0 = v_1 = v_2) \end{aligned} \quad (6)$$

In view of the above non-dimensional quantities, the basic equations (1) and (2) and the boundary conditions (3) and (4) can be expressed in non-dimensional form, dropping asterisks, as

$$\frac{\partial u_i}{\partial t} + v \frac{\partial u_i}{\partial y} = - \frac{A_i}{1 + \lambda_1} \frac{\partial^2 u_i}{\partial y^2} - \left(\chi \frac{\sigma^2}{1 + \lambda_1} + M_i^2 \right) u_i - P \quad (i=1,2) \quad (7)$$

$$\frac{\partial \theta_i}{\partial t} + v \frac{\partial \theta_i}{\partial y} = B_i \frac{\partial^2 \theta_i}{\partial y^2} - A_i Ec \left(\frac{\partial u_i}{\partial y} \right)^2 + \sigma^2 \frac{Ec}{1 + \lambda_1} (u_i)^2 \quad (i=1,2) \quad (8)$$

where $i=1, 2$ gives equations for Regions 1 and 2

$$A_1 = m, \quad A_2 = 1, \quad B_1 = \frac{n}{Pr}, \quad B_2 = \frac{1}{Pr}, \quad Pr = \frac{\rho_0 \nu C_p}{K_0}, \quad M^2 = \frac{\sigma_e B_0 h^2}{\mu}, \quad \sigma^2 = \frac{h^2}{\nu}$$

$$m = \frac{\mu_{eff}}{\mu} \text{ (ratio of viscosity) and } n = \frac{K_{eff}}{K_0} \text{ (ratio of thermal}$$

conductivity).

Region 1

$$\frac{\partial u_1}{\partial t} + v \frac{\partial u_1}{\partial y} = -\frac{m}{1+\lambda_1} \frac{\partial^2 u_1}{\partial y^2} - \left(\frac{\sigma^2}{1+\lambda_1} + M_1^2 \right) u_1 - P \quad (9)$$

$$\frac{\partial \theta_1}{\partial t} + v \frac{\partial \theta_1}{\partial y} = \frac{n}{Pr} \frac{\partial^2 \theta_1}{\partial y^2} - m Ec \left(\frac{\partial u_1}{\partial y} \right)^2 + \sigma^2 Ec (u_1)^2 \quad (10)$$

Region 2

$$\frac{\partial u_2}{\partial t} + v \frac{\partial u_2}{\partial y} = \frac{1}{1+\lambda_1} \frac{\partial^2 u_2}{\partial y^2} - M_1^2 u_2 - P \quad (11)$$

$$\frac{\partial \theta_2}{\partial t} + v \frac{\partial \theta_2}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta_2}{\partial y^2} - Ec \left(\frac{\partial u_2}{\partial y} \right)^2 \quad (12)$$

The non-dimensional form of the hydrodynamic and thermal boundary and interface conditions reduce to

$$u_1(-1) = 0, \quad u_2(1) = 0, \quad u_1(0) = u_2(0), \quad \frac{\mu_{eff}}{1+\lambda_1} \frac{\partial u_1}{\partial y} = \frac{\mu}{1+\lambda_1} \frac{\partial u_2}{\partial y} \text{ at } y = 0 \quad (13)$$

$$T_1(-1) = T_{\omega_1}, \quad T_2(1) = T_{\omega_2}, \quad T_1(0) = T_2(0), \quad K_{eff} \frac{\partial T_1}{\partial y} = K_0 \frac{\partial T_2}{\partial y} \text{ at } y = 0 \quad (14)$$

SOLUTION OF THE PROBLEM

The governing momentum and energy equations (9) to (14) are coupled partial differential equations that cannot be solved in closed form. However, they can be reduced to set of ordinary differential equations that can be solved analytically. This can be done by representing the velocity and temperature as $u_i(y, t) = u_{i0}(y) + \varepsilon e^{i\omega t} u_{i1}(y) + O(\varepsilon^2) + \dots$

$$i = 1, 2 \quad (15)$$

$$\theta_i(y, t) = \theta_{i0}(y) + \varepsilon e^{i\omega t} \theta_{i1}(y) + O(\varepsilon^2) + \dots$$

$$i = 1, 2 \quad (16)$$

This is a valid assumption because of the choice of v as defined in equation (5) that the amplitude $\varepsilon A \leq 1$ and ω is the frequency parameter.

By substituting equations (15) and (16) into (9) to (12) and equating the harmonic and non-harmonic terms and neglecting the higher order terms of $O(\varepsilon^2)$, one obtains the following pairs of equation for (u_{i0}, θ_{i0}) and (u_{i1}, θ_{i1}) where $i = 1, 2$.

Region 1

$$\frac{m}{1+\lambda_1} \frac{d^2 u_{10}}{dy^2} + \frac{du_{10}}{dy} - \left(\frac{\sigma^2}{1+\lambda_1} + M^2 \right) u_{10} = P \quad (17)$$

$$B_1 \frac{d^2 \theta_{10}}{dy^2} + \frac{d\theta_{10}}{dy} = A_1 Ec \left(\frac{du_{10}}{dy} \right)^2 - \sigma^2 Ec (u_{10})^2 \quad (18)$$

$$\frac{m}{1+\lambda_1} \frac{d^2 u_{11}}{dy^2} + \frac{du_{11}}{dy} - \left(\frac{\sigma^2}{1+\lambda_1} + M^2 + i\omega \right) u_{11} = -A \left(\frac{du_{10}}{dy} \right) \quad (19)$$

$$B_1 \frac{d^2 \theta_{11}}{dy^2} + \frac{d\theta_{11}}{dy} - i\omega \theta_{11} = A_1 \frac{d\theta_{10}}{dy} + 2A_1 Ec \frac{du_{10}}{dy} \frac{du_{11}}{dy} - 2\sigma^2 Ec u_{10} u_{11} \quad (20)$$

Region 2

$$\frac{1}{1+\lambda_1} \frac{d^2 u_{20}}{dy^2} + \frac{du_{20}}{dy} - M^2 u_{20} = P \quad (21)$$

$$B_2 \frac{d^2 \theta_{20}}{dy^2} + \frac{d\theta_{20}}{dy} = A_2 Ec \left(\frac{du_{20}}{dy} \right)^2 \quad (22)$$

$$\frac{1}{1+\lambda_1} \frac{d^2 u_{21}}{dy^2} + \frac{du_{21}}{dy} - (M^2 + i\omega) u_{21} = -A \left(\frac{du_{20}}{dy} \right) \quad (23)$$

$$B_2 \frac{d^2 \theta_{21}}{dy^2} + \frac{d\theta_{21}}{dy} - i\omega \theta_{21} = -A \frac{d\theta_{20}}{dy} + 2A_2 Ec \frac{du_{20}}{dy} \frac{du_{21}}{dy} \quad (24)$$

Using (15) and (16), the boundary and interface conditions may be written as

$$u_{10}(-1) = 0, \quad u_{20}(1) = 0, \quad u_{10}(0) = u_{20}(0), \quad m \frac{\partial u_{10}}{\partial y} = \frac{\partial u_{20}}{\partial y} \text{ at } y = 0 \quad (25)$$

$$u_{11}(-1) = 0, \quad u_{21}(1) = 0, \quad u_{11}(0) = u_{21}(0), \quad m \frac{\partial u_{11}}{\partial y} = \frac{\partial u_{21}}{\partial y} \text{ at } y = 0 \quad (26)$$

$$\theta_{10}(-1) = 1, \quad \theta_{20}(1) = 0, \quad \theta_{10}(0) = \theta_{20}(0), \quad n \frac{\partial \theta_{10}}{\partial y} = \frac{\partial \theta_{20}}{\partial y} \text{ at } y = 0 \quad (27)$$

$$\theta_{11}(-1) = 1, \quad \theta_{21}(1) = 0, \quad \theta_{11}(0) = \theta_{21}(0), \quad n \frac{\partial \theta_{11}}{\partial y} = \frac{\partial \theta_{21}}{\partial y} \text{ at } y = 0 \quad (28)$$

The solution of the equations (17) to (24) using the boundary and interface conditions (25) to (28) can be written as:

$$u_{10} = c_1 e^{a_1 y} + c_2 e^{a_2 y} - \frac{P}{M_{11}} \quad (29)$$

$$u_{20} = c_3 e^{a_3 y} + c_4 e^{a_4 y} - \frac{P}{M^2} \quad (30)$$

$$\theta_{10} = c_{11} + c_{12} e^{t_1 y} + t_7 y + t_8 e^{2a_1 y} + t_9 e^{2a_2 y} + t_{10} e^{m_1 y} + t_{11} e^{a_1 y} + t_{12} e^{a_2 y} \quad (31)$$

$$\theta_{20} = c_{13} + c_{14} e^{t_{13} y} + t_{14} e^{a_3 y} + t_{15} e^{a_4 y} \quad (32)$$

$$u_{11} = e^{\varepsilon_1 y} [R_1 \cos f_1 y + R_5 \sin f_1 y] + e_2 e^{\alpha_1 y} + e_4 e^{2\alpha_2 y} + i \left\{ e^{\varepsilon_1 y} [R_2 \cos f_1 y + R_6 \sin f_1 y] + e_3 e^{\alpha_1 y} + e_5 e^{2\alpha_2 y} \right\} \quad (33)$$

$$u_{21} = e^{\varepsilon_6 y} [R_3 \cos f_2 y + R_7 \sin f_2 y] + e_7 e^{\alpha_3 y} + e_9 e^{\alpha_4 y} + i \left\{ e^{\varepsilon_6 y} [R_4 \cos f_2 y + R_8 \sin f_2 y] + e_8 e^{\alpha_3 y} + e_{10} e^{\alpha_4 y} \right\} \quad (34)$$

$$\theta_{11} = e^{f_5 y} [R_{21} \cos f_5 y + R_{25} \sin f_5 y] + e^{2\alpha_1 y} [(r_{16} + r_{36}) \cos f_1 y + (r_{17} + r_{37}) \sin f_1 y] + e^{2\alpha_2 y} [(r_{26} + r_{46}) \cos f_1 y + (r_{27} + r_{47}) \sin f_1 y] + e^{\varepsilon_1 y} [r_{56} \cos f_1 y + r_{57} \sin f_1 y] + e^{\alpha_1 y} [g_5 + g_{16}] + e^{\alpha_2 y} [g_6 + g_{17}] + e^{2\alpha_1 y} [g_2 + g_{14} + g_{11} + g_8] + e^{2\alpha_2 y} [g_3 + g_9 + g_{12} + g_{15}] + g_1 e^{\alpha_1 y} + [g_4 + g_7 + g_{10} + g_{13}] e^{2\alpha_2 y} + i \left\{ \begin{aligned} & -e^{2\alpha_1 y} [(r_{18} + r_{38}) \cos f_1 y + (r_{19} + r_{39}) \sin f_1 y] - e^{\varepsilon_1 y} [r_{58} \cos f_1 y + r_{59} \sin f_1 y] \\ & -e^{2\alpha_2 y} [(r_{18} + r_{38}) \cos f_1 y + (r_{19} + r_{39}) \sin f_1 y] + e^{\alpha_1 y} [h_5 + h_{16}] + e^{2\alpha_2 y} [h_6 + h_{17}] \\ & + e^{2\alpha_1 y} [h_2 + h_{14} + h_{11} + h_8] + e^{2\alpha_2 y} [h_3 + h_{15} + h_{12} + h_9] + e^{\alpha_0 y} [h_4 + h_7 + h_{10} + h_{13}] \\ & + h_1 e^{\alpha_1 y} + \frac{A t_2}{\omega} + e^{f_5 y} [R_{22} \cos f_5 y + R_{26} \sin f_5 y] \end{aligned} \right\} \quad (35)$$

$$\theta_{21} = e^{f_6 y} [R_{23} \cos f_7 y + R_{27} \sin f_7 y] + e^{f_6 y} [l_7 \cos f_2 y + l_8 \sin f_2 y] + e^{f_6 y} [l_{17} \cos f_2 y + l_{18} \sin f_2 y] + g_{20} e^{\alpha_3 y} + g_{21} e^{\alpha_4 y} + g_{22} e^{\alpha_5 y} + g_{23} e^{2\alpha_3 y} + g_{24} e^{2\alpha_4 y} + g_{25} e^{f_6 y} + i \left\{ \begin{aligned} & e^{f_6 y} [R_{24} \cos f_7 y + R_{28} \sin f_7 y] + e^{f_6 y} [l_9 \cos f_2 y + l_{10} \sin f_2 y] \\ & + e^{f_6 y} [l_{19} \cos f_2 y + l_{20} \sin f_2 y] + h_{20} e^{\alpha_3 y} + h_{21} e^{\alpha_4 y} + h_{22} e^{\alpha_5 y} + h_{23} e^{2\alpha_3 y} + h_{24} e^{2\alpha_4 y} + h_{25} e^{f_6 y} \end{aligned} \right\} \quad (36)$$

The velocities and temperature distributions in the two regions are

$$u_1 = u_{10} + \varepsilon e^{i\omega t} u_{11} \quad (37)$$

$$u_2 = u_{20} + \varepsilon e^{i\omega t} u_{21} \quad (38)$$

$$\theta_1 = \theta_{10} + \varepsilon e^{i\omega t} \theta_{11} \quad (39)$$

$$\theta_2 = \theta_{20} + \varepsilon e^{i\omega t} \theta_{21} \quad (40)$$

RATE OF HEAT TRANSFER

The rate of heat transfer (Nusselt number) through the channel wall to the fluid is given by $Nu = \left[\frac{d\theta}{dy} \right]$ (41)

Based on the analytical solutions reported above the rate of heat transfer at the bottom wall is given by

$$Nu_1 = \left[\frac{d\theta_1}{dy} \right]_{y=-1} \quad (42)$$

$$Nu_1 = t_1 c_{12} e^{-\varepsilon_1} + t_7 + t_8 2a_1 e^{-2\alpha_1} + t_9 2a_2 + t_{10} e^{-\alpha_1} + t_{11} a_1 e^{-\alpha_1} + t_{12} a_2 e^{-\alpha_2} + \left\{ \begin{aligned} & f_6 e^{-f_6} [a_{11} \cos f_5 - a_{12} \sin f_5] + f_5 e^{-f_5} [a_{11} \sin f_5 + a_{12} \cos f_5] \\ & + a_{21} e^{-\alpha_1} [(r_{16} + r_{36}) \cos f_1 - (r_{17} + r_{37}) \sin f_1] + f_1 e^{-\alpha_1} [(r_{16} + r_{36}) \sin f_1 + (r_{17} + r_{37}) \cos f_1] \\ & + a_{23} e^{-\alpha_2} [(r_{26} + r_{46}) \cos f_1 - (r_{27} + r_{47}) \sin f_1] + f_1 e^{-\alpha_2} [(r_{26} + r_{46}) \sin f_1 + (r_{27} + r_{47}) \cos f_1] \\ & + e_1 e^{-\alpha_1} [r_{56} \cos f_1 - r_{57} \sin f_1] + e^{-\alpha_1} f_1 [r_{56} \sin f_1 + r_{57} \cos f_1] + a_1 e^{-\alpha_1} [g_5 + g_{16}] + a_2 e^{-\alpha_2} [g_6 + g_{17}] \\ & + 2a_1 e^{-2\alpha_1} [g_2 + g_8 + g_{11} + g_{14}] + 2a_2 e^{-2\alpha_2} [g_3 + g_9 + g_{12} + g_{15}] + t_1 g_1 e^{-\varepsilon_1} + a_{22} e^{-\alpha_2} [g_4 + g_7 + g_{10} + g_{13}] \\ & - a_2 e^{-\alpha_2} [(r_{18} + r_{38}) \cos f_1 - (r_{19} + r_{39}) \sin f_1] - f_1 e^{-\alpha_2} [(r_{18} + r_{38}) \sin f_1 + (r_{19} + r_{39}) \cos f_1] \\ & + a_{23} e^{-\alpha_2} [(r_{18} + r_{38}) \cos f_1 - (r_{19} + r_{39}) \sin f_1] - f_1 e^{-\alpha_2} [(r_{18} + r_{38}) \sin f_1 + (r_{19} + r_{39}) \cos f_1] \\ & - e_1 e^{-\alpha_1} [r_{58} \cos f_1 - r_{59} \sin f_1] - e^{-\alpha_1} f_1 [r_{58} \sin f_1 + r_{59} \cos f_1] + a_1 e^{-\alpha_1} [h_5 + h_{16}] + a_2 e^{-\alpha_2} [h_6 + h_{17}] \\ & + 2a_1 e^{-2\alpha_1} [h_2 + h_8 + h_{11} + h_{14}] + 2a_2 e^{-2\alpha_2} [h_3 + h_9 + h_{12} + h_{15}] + t_1 g_1 e^{-\varepsilon_1} + a_{22} e^{-\alpha_2} [h_4 + h_7 + h_{10} + h_{13}] \end{aligned} \right\}$$

$$\text{At the top wall, it is given by } Nu_2 = \left[\frac{d\theta_2}{dy} \right]_{y=1} \quad (44)$$

$$= t_{13} C_{14} e^{\alpha_1} + a_3 t_{14} e^{\alpha_1} + a_4 t_{15} e^{\alpha_1} + \varepsilon \cos \omega t \left\{ \begin{aligned} & f_6 e^{f_6} [a_{13} \cos f_7 + a_{14} \sin f_7] + f_5 e^{f_5} [a_{13} \sin f_7 + a_{14} \cos f_7] + f_6 e^{f_6} [l_7 \cos f_2 + l_8 \sin f_2] \\ & + f_7 e^{f_7} [-l_7 \sin f_2 + l_8 \cos f_2] + f_6 e^{f_6} [l_{17} \cos f_2 + l_{18} \sin f_2] + f_2 e^{f_6} [-l_{17} \sin f_2 + l_{18} \cos f_2] \\ & + g_{20} f_{13} e^{\alpha_3} + g_{21} a_3 e^{\alpha_4} + g_{22} a_4 e^{\alpha_5} + g_{23} 2a_3 e^{2\alpha_3} + g_{24} 2a_4 e^{2\alpha_4} + g_{25} f_{10} e^{f_6} \end{aligned} \right\} - \varepsilon \sin \omega t \left\{ \begin{aligned} & f_6 e^{f_6} [l_9 \cos f_2 + l_{10} \sin f_2] + f_2 e^{f_6} [-l_9 \sin f_2 + l_{10} \cos f_2] + f_6 e^{f_6} [l_{19} \cos f_2 + l_{20} \sin f_2] \\ & + f_2 e^{f_6} [-l_{19} \sin f_2 + l_{20} \cos f_2] + h_{20} f_{13} e^{\alpha_3} + h_{21} a_4 e^{\alpha_4} + h_{22} a_5 e^{\alpha_5} + h_{23} 2a_3 e^{2\alpha_3} + h_{24} 2a_4 e^{2\alpha_4} + h_{25} f_{10} e^{f_6} \end{aligned} \right\} \quad (45)$$

3.2. Mass flux

The dimensionless mass flow rate per unit width of the channel is $Q = F_1 + F_2$ (46)

$$\text{Where } F_1 = \int_{-1}^0 u_1(y, t) dy \quad (47)$$

$$F_1 = \frac{C_1}{a_1} [1 - e^{-\alpha_1}] + \frac{C_2}{a_2} [1 - e^{-\alpha_2}] - \frac{P}{\sigma^2 + M_1^2} + \varepsilon \left\{ \begin{aligned} & \frac{e_1}{e_1^2 + f_1^2} (R_1 \cos \omega t - R_2 \sin \omega t) + \frac{f_1}{e_1^2 + f_1^2} (R_6 \sin \omega t - R_5 \cos \omega t) + \frac{e_2 \cos \omega t - e_3 \sin \omega t}{a_1} \\ & + \frac{e_4 \cos \omega t - e_5 \sin \omega t}{a_2} - \frac{e^{-\varepsilon_1}}{e_1^2 + f_1^2} (e_1 \cos f_1 - f_1 \sin f_1) (R_1 \cos \omega t - R_2 \sin \omega t) \\ & + \frac{e^{-\varepsilon_1}}{e_1^2 + f_1^2} (e_1 \sin f_1 + f_1 \cos f_1) (R_5 \cos \omega t - R_6 \sin \omega t) \end{aligned} \right\} \quad (48)$$

And

$$F_2 = \int_0^1 u_2(y, t) dy \quad (49)$$

$$F_2 = \frac{C_3}{a_3} (e^{\alpha_3} - 1) + \frac{C_4}{a_4} (e^{\alpha_4} - 1) + d_2 + \varepsilon \left\{ \begin{aligned} & \frac{e^{\alpha_3}}{e_3^2 + f_2^2} [(e_3 \cos \omega t + f_2 \sin \omega t) (R_3 \cos \omega t - R_4 \sin \omega t)] + [(e_3 \sin f_2 - f_2 \cos f_2) (R_7 \cos \omega t - R_8 \sin \omega t)] \\ & + \frac{e^{\alpha_4}}{a_4} [e_4 \cos \omega t - e_5 \sin \omega t] + \cos \omega t \left[\frac{e_6 R_3}{e_6^2 + f_2^2} - \frac{f_2 R_4}{e_6^2 + f_2^2} + \frac{e_2}{a_3} + \frac{e_4}{a_4} \right] \\ & - \sin \omega t \left[\frac{e_6 R_3}{e_6^2 + f_2^2} - \frac{f_2 R_4}{e_6^2 + f_2^2} - \frac{e_2}{a_3} - \frac{e_4}{a_4} \right] \end{aligned} \right\} \quad (50)$$

Interface velocity

Taking $y = 0$ in the equation (37) or in equation (38) we get the interface velocity as

$$u_0 = C_1 + C_2 - \frac{P}{\sigma^2 + M_1^2} + \varepsilon \{ \cos \omega t [R_1 + e_2 + e_4] - \sin \omega t [R_2 + e_3 + e_5] \} \quad (51)$$

(or)

$$u_0 = C_3 + C_4 + d_2 + \varepsilon \{ \cos \omega t [R_3 + e_7 + e_9] - \sin \omega t [R_4 + e_8 + e_{10}] \} \quad (52)$$

GRAPHICAL RESULTS AND DISCUSSION

The effect of pressure gradient on the velocity profiles is discussed through Fig.2. It is observed that for a given pressure gradient the velocity profiles are parabolic. The velocity in the porous matrix region increases with the increasing y value and attains the maximum at the interface (i.e $y = 0$). After that the velocity decreases for increasing y in the clear viscous fluid region. And also it is noticed that the velocity increases with decreasing pressure gradient for any given y .

From Fig.3 it is noticed that for a given y , the velocity increases with decreasing magnetic parameter M . The velocity profiles are parabolic, in which it attains the maximum at the interface. Fig.4 is drawn to study the effect of Jeffrey parameter on the flow pattern.It is observed that the velocity increases with increasing Jeffrey parameter (λ_1). And also for any given λ_1 , the velocity profiles are parabolic. The velocity increases in the region 1 and decreases in the region 2. Similarly, from Fig 5.it is observed that the velocity decreases with increasing values of the ration of viscosity. The effect of porous medium on the velocity is discussed through Fig.6.For a given σ , the velocity profiles are parabolic. And also the velocity decreases with increasing σ .

The effect of various parameters on temperature is shown in Figs 7 – 13. Fig.7. is drawn to study the effect of magnetic parameter on the temperature. For a given magnetic parameter, the temperature decreases with increasing y in both the regions. And also it is noticed that temperature increases with decreasing values of Magnetic parameter. From Fig.8 the effect of Jeffrey parameter on temperature can be observed.The temperature decreases with increasing Jeffrey parameter.

It is observed from Fig 9,that the temperature increases with an increase in the ratio of viscosity. From Fig.10, it is noticed that the temperature decreases with decreasing ratio of thermal conductivity. The effect of Prandtl number on temperature is observed from Fig.11.it is noticed that temperature decreases with increasing Prandtl number. From Fig.12, it is observed that the temperature decreases with decreasing Eckert number. The variation of temperature in both the regions can be observed from Fig.12. The effect of porous medium parameter is observed from Fig 13. It can be seen that the temperature increases with increasing porous medium parameter σ .

The variation of interface velocity is calculated from equation (51 or 52) for different values of the ratio of Jeffrey parameter λ_1 with effect of viscosity ratio m , magnetic field parameter M , and porous medium parameter σ and is shown in the Table. 1. It is found that the interface velocity increases with the increment in Jeffrey parameter λ_1 , with the effect of the ratio of viscosity m , magnetic field parameter M , and porous medium parameter σ . For a given Jeffrey parameter λ_1 , the interface velocity decreases with increasing of ratio of viscosity m , magnetic field M and porous medium parameter σ .

CONCLUSION

In this paper, oscillatory flow of conducting Jeffrey fluid in a horizontal composite porous channel is investigated. The closed form solutions are reported for small ε such that oscillation amplitude $\varepsilon A \leq 1$. It following conclusions are drawn from this

1. Velocity increases with the decreasing of pressure gradient, Magnetic parameter, viscosity ratio, and porous medium parameter.
2. The velocity increases with the increasing of Jeffrey parameter.
3. Temperature increases with decreasing Magnetic parameter, Jeffrey parameter, Prandtl number.
4. Temperature increases with increasing viscosity ratio, Eckert number and porous medium parameter.

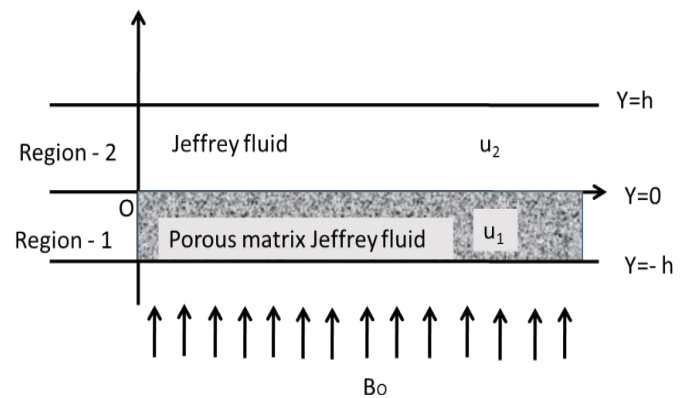


Fig. 1 Physical Model

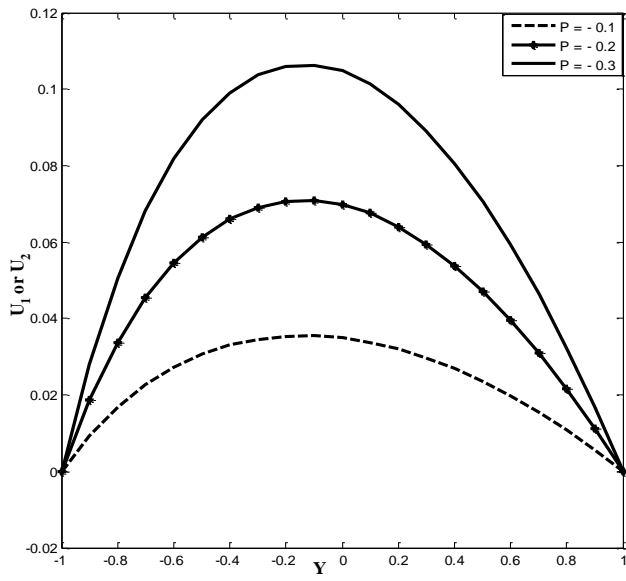


Fig. 2: Velocity profile for different values of Pressure gradient

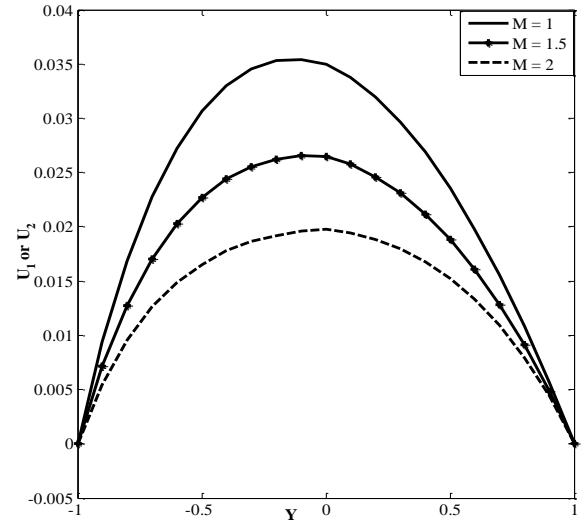


Fig. 3: Velocity profile for different values of Magnetic field parameter M.

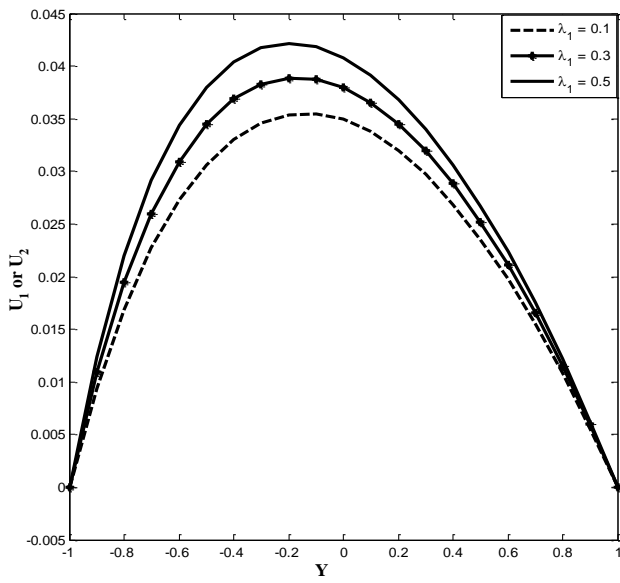


Fig. 4: Velocity profile for different values of Jeffrey parameter

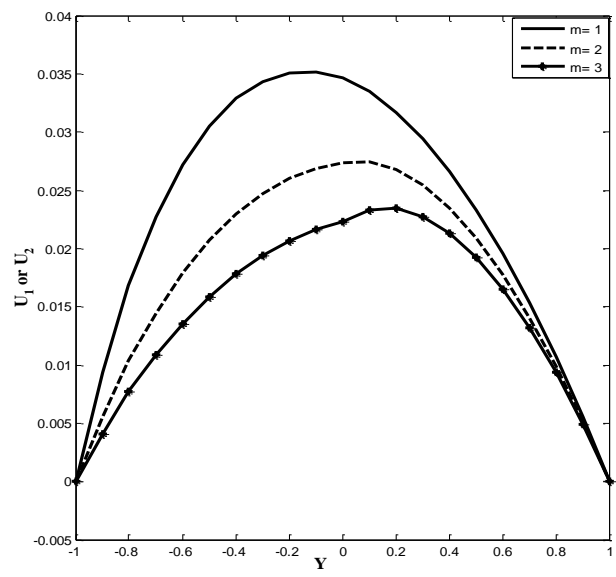


Fig.5: Velocity profile for different values of ratio of viscosity

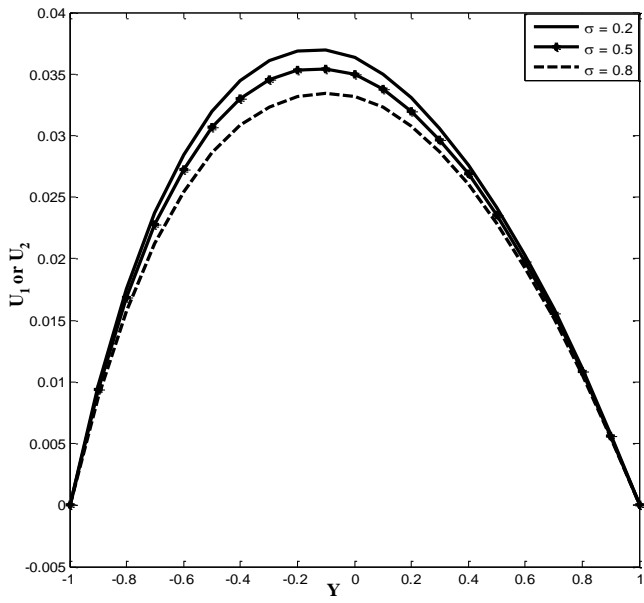


Fig.6: Velocity profile for different values of porous medium parameter σ

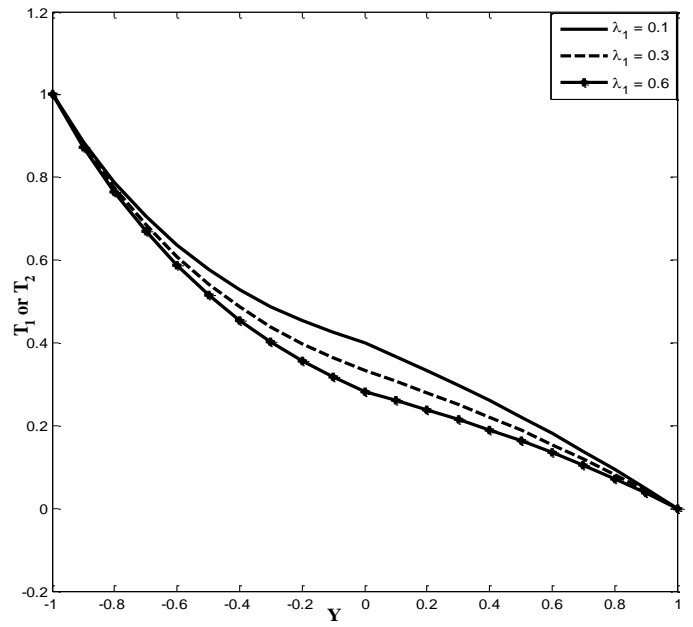


Fig. 8: Temperature profile for different values of Jeffrey parameter λ_1

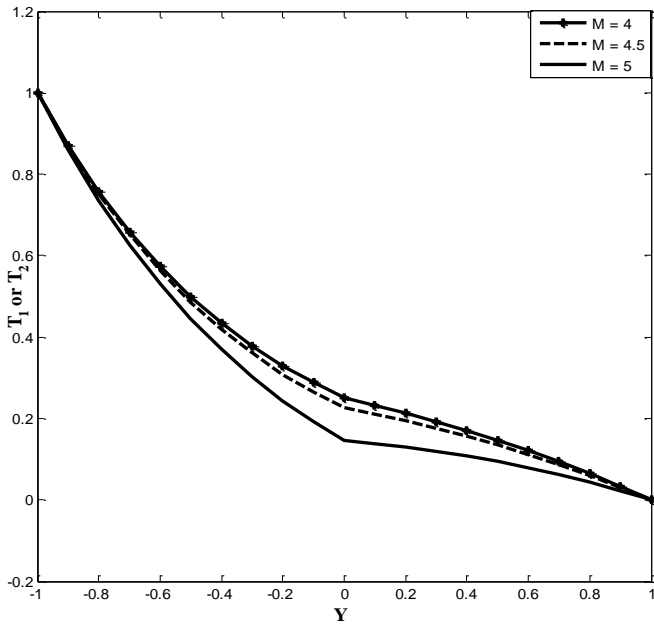


Fig. 7: Temperature profile for different values of Magnetic Field parameter

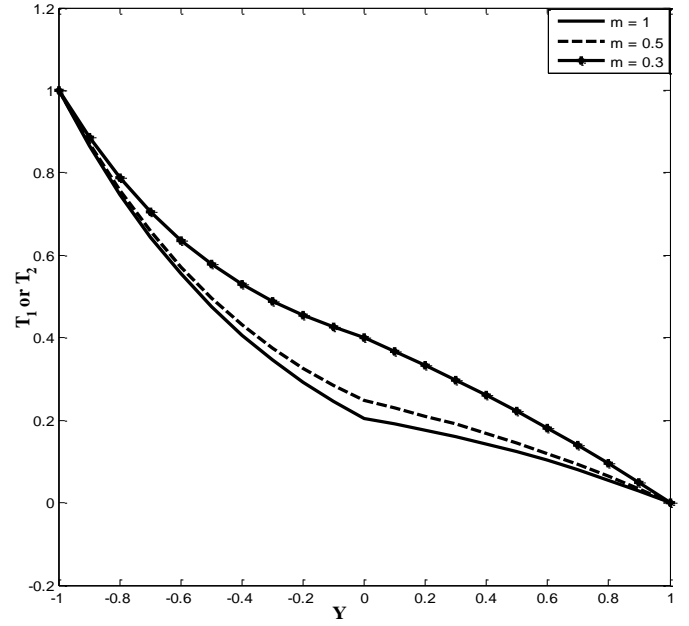


Fig.9: Temperature profile for different values of ratio of viscosity m .

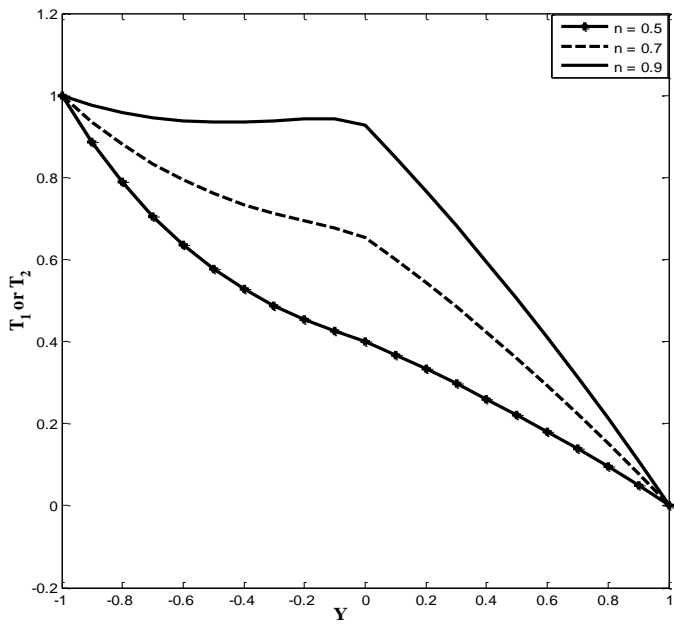


Fig.10: Temperature profile for different values of ratio of thermal conductivity n

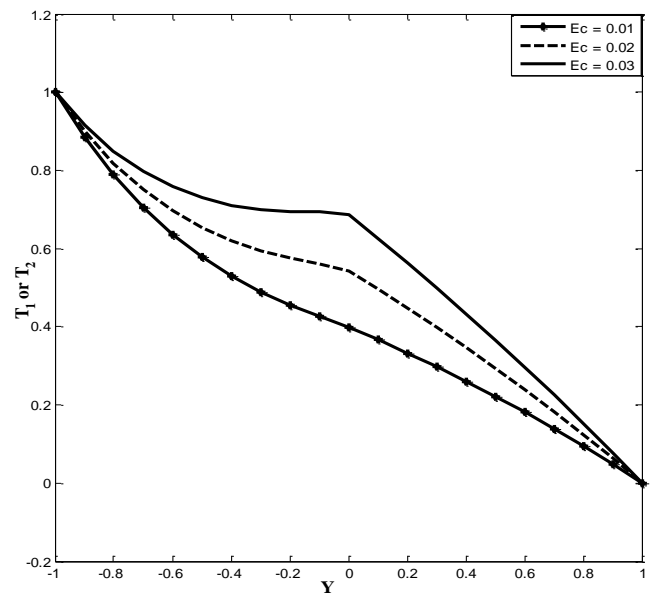


Fig.12: Temperature profile for different values of Eckert number Ec .

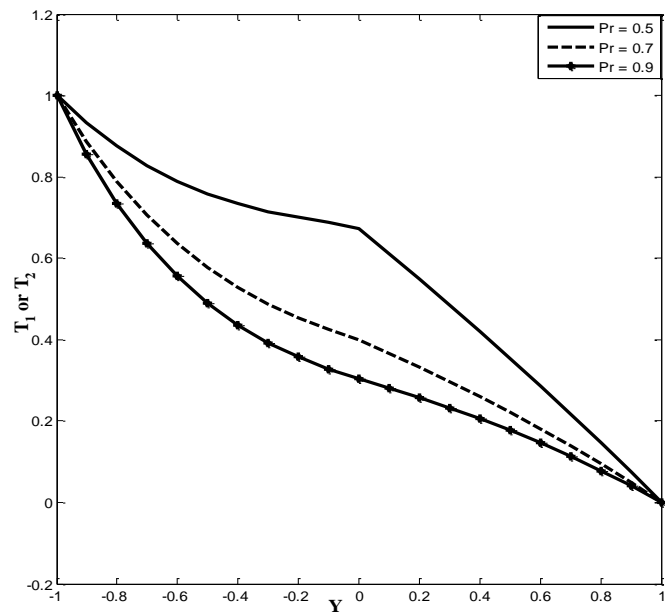


Fig.11: Temperature profile for different values of Prandtl number Pr .

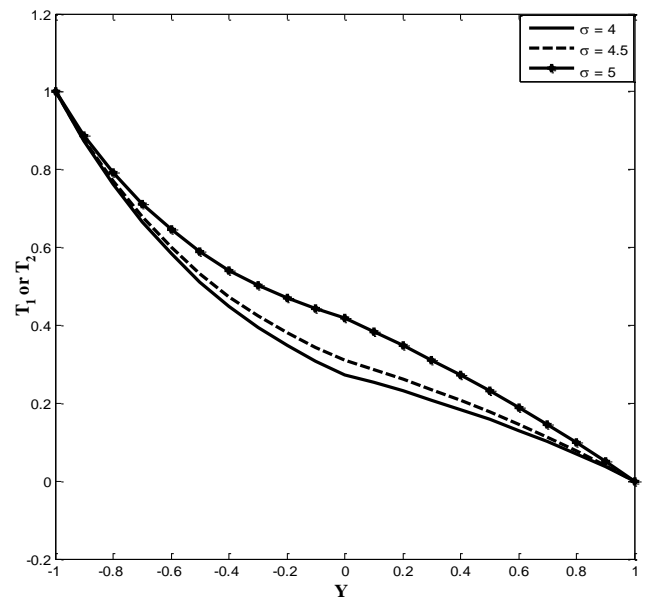


Fig.13: Temperature profile for different values of porous medium parameter σ

λ_1	$m = 1$	$m = 2$	$m = 3$	$M = 1$	$M = 1.5$	$M = 2$	$\sigma = 0.2$	$\sigma = 0.5$	$\sigma = 0.8$
0	0.0323	0.02541	0.0206	0.0323	0.0240	0.0175	0.033	0.0323	0.0308
0.2	0.0364	0.0291	0.0239	0.0364	0.0268	0.0196	0.0374	0.0364	0.0348
0.4	0.0395	0.0324	0.0269	0.0395	0.0282	0.02	0.0403	0.0395	0.0379
0.6	0.0421	0.0354	0.0297	0.0421	0.0294	0.0206	0.0428	0.0421	0.0405
0.8	0.0443	0.0380	0.0323	0.0443	0.0304	0.0210	0.0450	0.0443	0.0428
1	0.0462	0.0405	0.0347	0.0462	0.0313	0.0214	0.0468	0.0462	0.0448

Table 1: Interface velocity for different values of λ_1 varies as m , M , and σ .

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