HEFAT2014
10th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
14 – 16 July 2014
Orlando, Florida

NUMERICAL OPTIMISATION OF SQUARE PIN-FINS FOR MINIMUM THERMAL RESISTANCE WITH NON-UNIFORM DESIGN DIMENSIONS

Olakoyejo O.T.[a,b,*], and Meyer, J.P. [b]
*Author for correspondence
[a]Department of Mechanical Engineering, University of Lagos, Lagos, Nigeria.
[b]Department of Mechanical and Aeronautical Engineering, University of Pretoria, Pretoria, Private Bag X20, Hatfield 0028, South Africa.
E-mail: olakoyejo@yahoo.com

ABSTRACT

In this paper we apply constructal theory and design to present a three-dimensional geometric optimisation of cooling square pin-fins in forced convection of solid base material subject to constant temperature at the bottom. The main objective was to optimise the configuration in such a way that the peak temperature was minimised from the solid to the fluid of the pin fin. The cross–sectional area of solid base was fixed and the thickness of the solid is allowed to change.

Three design cases are considered; the Case 1 has the pin-fins in a single row, Case 2 configuration has the pin-fins in two rows and Case 3 has configuration of the pin-fins in three rows arrangement. In all the three cases, the pin-fins geometric dimension are not uniform. The structure had four degrees of freedom as design variables: thickness of the solid base, hydraulic diameter of the pin, the height of the pin and pin spacing. The shape of the pin was not fixed but allowed to morph to determine the best configuration which gave the lowest thermal resistance.

The cooling fluid (air) was driven by forced convection to the pin-fin by the stream velocity. An optimisation algorithm called Dynamic-Q was applied in order to search for the best optimal geometric configuration which improved thermal performance by minimising thermal resistance for a wide range of Reynolds number. The effect of applied Reynolds number and constant wall temperature on the optimal geometry was reported. There was unique optimal design geometry for a given Reynolds number. Results obtained show that the effects of Reynolds number on minimised thermal resistance are consistent with those obtained in the open literature.

INTRODUCTION

Pin-fin heat sinks are commonly used in industries in the cooling systems of electronic components. They come in different shapes. Such as: cylindrical, square and rectangular and elliptical shapes. The most common used of them is the cylindrical type because of the ease at which it can be manufactured. The design of heat sinks has become critical to the global performance of electronic packages as heat flux densities increase with the miniaturisation of the product. The heat sinks must be designed and maintained without exceeding a satisfactory and allowable work temperature specified by the manufacturers [1].

Bejan and Morega [2] studied the geometry of an array of fins that minimise the thermal resistance between the substrate and the forced flow through the fins by modelling the array as the Darcy-flow porous medium and the local thermal conductance in dimensionless form. Peles *et al*. [3] conducted the convective heat transfer and fluid flow analysis across a pin-fin micro heat sink. They concluded that the cylindrical micro-pin-fin heat sink solution is superior to that of a microchannel heat sink.

Bejan [4] studied and extended the work of Jubran *et al*. [5] by providing the existence of an optimal spacing between the cylinders. Khan *et al*. [6] studied the optimal geometry of pin-fin heat sinks by an entropy generation minimisation for both in-line and staggered configurations. It was shown that In-line configurations gave lower entropy generation rates for both low and high thermal conductivity heat sink cases.

Bello-Ochende and Co-workers. [7-8] analytically and numerically optimised and determined the cylindrical configuration of two rows of pin–fins that maximised the total heat transfer rate. However, the thickness of the base of the heat sink was neglected and the volume of the pin fins was also, constraint. Obayopo *et al*. [9] also investigated the effect of pin-fins of small hydraulic diameter transversely arranged along the internal flow channel on the reactant gas distribution and pressure drop characteristics of the fuel cell performance. An optimal pin-fin clearance ratio offered minimum pumping power requirements and maximum fuel cell performance.

In this paper, we seek to apply constructal theory and design [10, 11] to search for the optimal geometry of square pin-fin heat sinks that minimises thermal resistance. Constructal theory and design [10, 11] has emerged as an evolutionary design philosophy for developing flow layouts that offer greater flow access and system performance. The application of this evolutionary design approach to the discovery of internal heat exchangers started with Bejan and Sciubba [12]. These researchers obtained the design rule for spacing an array of parallel plates to channels so that the heat transfer density of a volume filled with heat-generating components was maximum. The spacing was determined by using the method of asymptotes. This philosophy has been applied to all the facets of flow system design, from biology and physics, to engineering and social organisation [13-19]. The applications of this theory have been reviewed most recently by Bejan and Lorente [20]. The advantage of constructal law in the engineering field is that the flow architecture is not assumed in advance, but is the consequence of allowing the structure to morph [21].

This paper focuses on developing a three-dimensional geometric optimisation of a cooling square pin-fin heat sink in forced convection of solid base material subjected to heat at a constant wall temperature applied at the bottom of the pin-fins. It was an extension of our previous work where the pin-fins geometric dimensions were assumed to be uniform [22]. In this present work we assume that for Case 1, the pinfins are arranged in a row of pins; for Case 2, the pin-fins are arranged in two rows of pin-fins with different dimensions and for Case 3, the pin-fins are arranged in three rows with unequal sizes. Also the thickness of the heat sink base is considered as part design parameters. The thickness of the heat sink base is allowed to vary with the objective to enhance the removal of heat supplied at the bottom of the conductive material.

Our objective is to determine numerically the effect of pin-fin row arrangement on the flow resistance and forced convective heat transfer. This is done by building an elemental computational domain with which one can construct a larger construct body with several pin-fin arrangements. The heat transfer across the square pin-fin was by laminar forced convection of uniform isothermal free stream. The cross-sectional area of solid base was fixed and the thickness of the solid was allowed to change. The structure had four degrees of freedom as design variables: thickness of the solid base, hydraulic diameter of the pin, height of the pin and pin spacing. The geometry of the pin(s) was not fixed but allowed to morph to determine the best configuration, which gave the lowest thermal resistance. The cooling fluid (air) was driven by forced convection to the pin-fin by the constant velocity that corresponded to the Reynolds number. The optimisation process was carried out numerically under total fixed volume and manufacturing constraints.

NOMENCLATURE

AR	[-]	Aspect ratio
A_{el}	[m^2]	Cross-sectional area of the elemental domain
C_P	$[J/kgK]$	Specific heat at constant pressure
d_h	[mm]	Hydraulic diameter
f	[-]	Function
H	[m]	Structure height
h	[mm]	Elemental height
i	[-]	Mesh iteration index, number of rows
k	$[W/mK]$	Thermal conductivity
L	[mm]	Axial length
n	[-]	Normal
$\dot{q}$	$[W]$	Rate of heat transfer
R	[-]	Thermal resistance
Re	[-]	Reynolds number
s_1	[m]	Pin-fin spacing from the leading end
s_2	[m]	Pin-fin spacing between the first two pins, m
s_3	[m]	Pin-fin spacing between the last two pins
T	$[^0C]$	Temperature
T_{in}	$[^0C]$	Inlet temperature
$T_{\max}$	$[^0C]$	Peak temperature
$\vec{u}$	[m/s]	Velocity vector
V	[m^3]	Structure volume
V_p	[m^2]	Nuni volume of the pin-fin
v_{el}	[m^3]	Elemental volume
W	[m]	Structure width
w_{el}	[m]	Elemental width
t	[m]	Thickness of the base
x, y, z	[m]	Cartesian coordinates

Greek symbols

α	$[m^2/s]$	Thermal diffusivity
μ	$[kg.s/m]$	Viscosity
ν	$[m^2/s]$	Kinematics viscosity
ρ	$[kg/m^3]$	Density
∂	[-]	Differential
∞	[-]	far extreme end
ϕ	[-]	Porosity
Δ	[-]	Difference
∇		Differential operator
γ	[-]	Convergence criterion

Subscripts

s	Solid
f	Fluid
in	Free stream or inlet
max	Maximum
min	Minimum
opt	Optimum
w	Wall

COMPUTATIONAL MODEL

Figure 1 shows the physical configurations of the pin-fin heat sink studied in this paper. The system consists of a solid base body of fixed cross-sectional area *A*, which experiences uniform isothermal temperature T_w. Where the thickness of the sink base, *t*, the different height of the pin, and the different pin-fin diameter, are allowed to morph~~.~~

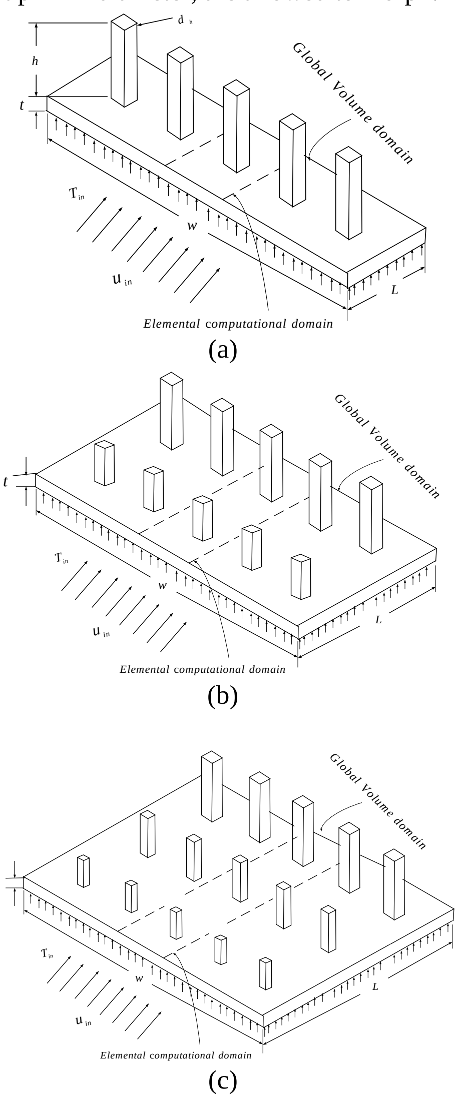


Figure 1: Three-dimensional (a) 1-row pin-finned (b) 2-row pin-finned and (c) 3-row pinned fins heat sink with constant wall temperature

Figure 2 shows the three-dimensional elemental computational domain of Case 1 to Case 3 of the pin-fin heat sinks. Case 1 consists of a single-finned row of the hydraulic diameters, d_h, height, h, spacing, s_1, from the leading edge of the heat sink and the thickness t of the heat sink base are allow to vary. Case 2, consists a two-finned row of different hydraulic diameters d_{h1}, and d_{h2} and different heights h_1 and h_2 , the spacing s_1 from the leading end of the heat sink to the first pin and spacing s_2 between the two cylinder pins. Finally, Case 3 consists of a three pin-finned row of different hydraulic diameters d_{h1}, d_{h2} and d_{h3} and different heights h_1, h_2 and h_3, the spacing, s_1 from the leading end of the heat sink to the first pin and spacing s_2 between the first two cylinder pins and spacing s_3 between the last two pins. For all the cases we assume that the pin diameters are of equal sizes and also the pin heights are all equal. The computational domain has fixed global cross-sectional area A_{el}.

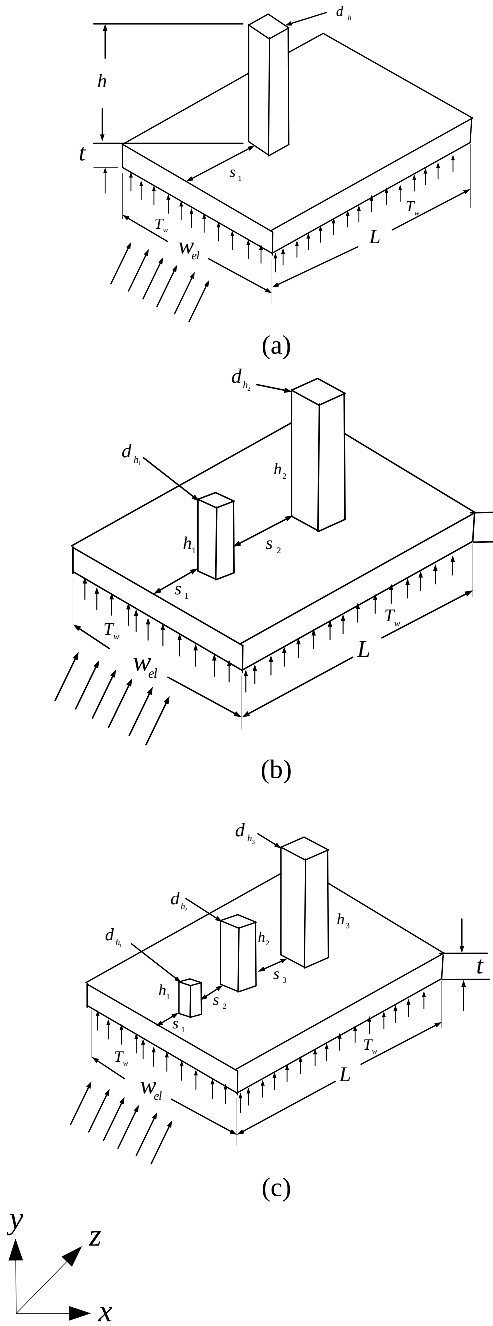


Figure 2. The three-dimensional elemental computational domain of (a) Case 1 (b) Case 2 and (c) Case 3 of pin-fin heat sink

The thickness of the heat sink base is allowed to vary with the objective to enhance the removal of heat supplied at the bottom of the conductive material. Also, the models are designed for micro-scale devices because of recent developments in largescale micro-electro mechanical systems (MEMS) with low-cost and small-space advantages, as well as high heat dissipation ability. The heat transfer in the elemental volume is a conjugate problem, which combines heat conduction in the solid and the convection in the working fluid

For a fixed elemental area computational domain,

$$A_{el} = w_{el}L \tag{1}$$

Therefore, volume of the each square pin-fin is not fixed but allowed to morph naturally on its own in order to select the best configuration. The volume of the unit pin-fin is defined as:

$$V_p = \frac{\pi\left(d_{hi}\right)^2}{4}h_i \neq \text{fixed} \tag{2}$$

where d_h is the hydraulic diameter of square pin-fin

Aspect ratio of the unit pin-fin is defined as:

$$AR_p = \frac{h_i}{d_{hi}} \tag{3}$$

Where i= number of rows, 1, 2, and 3

Also, aspect ratio of the elemental solid base to the pin-fin length is defined as:

$$AR_t = \frac{t}{L} \tag{4}$$

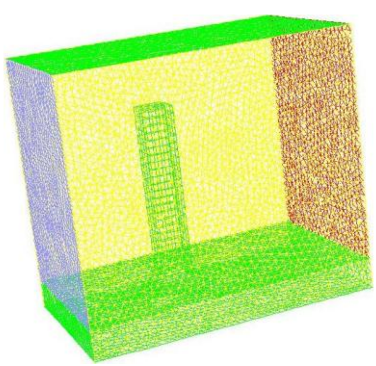
The numerical work begins by considering an elemental volume of a micro-pin-fin heat sink for different cases studied. The temperature distribution in the model was determined by solving the equation for the conservation of mass, momentum and energy numerically. The discretised three-dimensional computational domain of the configuration is shown in Figure 3. Air was considered the cooling fluid and forced by specified fluid velocity to flow. The fluid was assumed to be in single phase, steady and Newtonian with constant properties.

The governing differential equations used for the fluid flow and heat transfer analysis for cooling fluid within the heat sink are:

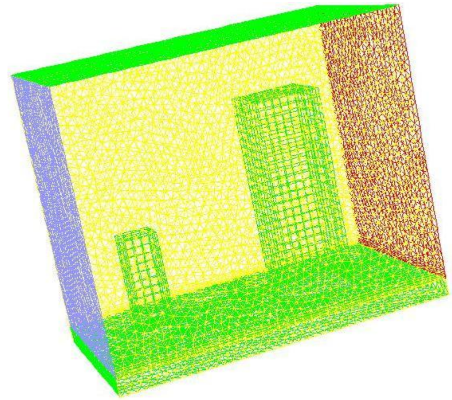
$$\nabla \cdot \vec{u} = 0 \tag{5}$$

$$\rho\left(\vec{u} \cdot \nabla\vec{u}\right) = -\nabla P + \mu\nabla^2\vec{u} \tag{6}$$

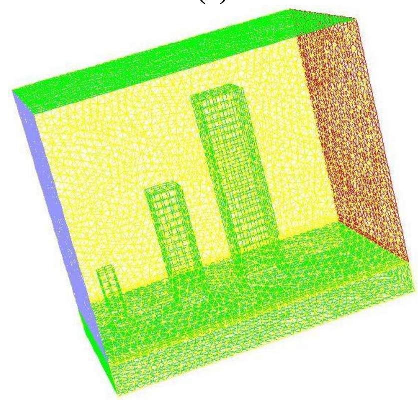
$$\rho_f C_{Pf}\left(\vec{u} \cdot \nabla T\right) = k_f \nabla^2 T \tag{7}$$



(a)



(b)



(c)

Figure 3 The discretised 3-D computational domains of (a) case 1, (b) Case 2 and (c) Case 3

The energy equation for the solid part of the elemental volume can be written as:

$$k_s \nabla^2 T = 0 \tag{8}$$

The continuity of the heat flux at the interface between the solid and the liquid is given as:

$$k_s \left.\frac{\partial T}{\partial n}\right|_w = k_f \left.\frac{\partial T}{\partial n}\right|_w \tag{9}$$

A no-slip boundary condition is specified for the fluid at the wall of the channel,

$$\vec{u} = 0 \tag{10}$$

At the inlet ($z = 0$),

$$u_x = u_y = 0, \quad T = T_{in}, \quad u = \frac{Re\mu}{\rho L} \tag{11}$$

where, Re is the Reynolds dimensionless number given as:

$$Re = \frac{\rho u L}{\mu} \tag{12}$$

At the outlet ($z = L$), the fluid is defined as outflow and the pressure is prescribed as zero normal stress.

$$\nabla \cdot \vec{u} = 0 \tag{13}$$

The thermal boundary condition imposed at the bottom side of the heat sink is the constant wall temperature, T_w and the uniform isothermal free stream (air) driven by specified Reynolds number is used as the working fluid. At the solid boundaries, the remaining outside walls and fluid and the plane of symmetry were modelled as adiabatic.

$$\nabla T = 0 \tag{14}$$

The measure of performance is the minimum global thermal resistance, which could be expressed in a dimensionless form as:

$$R_{\min} = \frac{k_f \left(T_w - T_{in}\right)}{\dot{q}/L} \tag{15}$$

and it is a function of the optimised design variables and the peak temperature.

$$R_{\min} = f\left(d_{h_{i,opt}},\ h_{i,opt},\ s_{i,opt},\ \mathrm{t}_{opt}\right) \tag{16}$$

$R_{\min}$ is the minimised thermal resistance for the optimised design variables. The inverse of $R_{\min}$ is the optimised overall global thermal conductance. q is the overall heat transfer rate, T_w and T_{in} are the wall and free-stream temperatures, respectively.

NUMERICAL PROCEDURE AND GRID ANALYSIS

The conjugate heat transfer problem is modelled with the thermal conductivity of the heat sink structure (aluminium) which is 202.4 W/m^{0}C , and a constant wall temperature T_w at the bottom wall of the heat sink was fixed at 100^{0}C. The thermophysical properties of uniform isothermal free stream [23] were taken at 300 K and the free stream air temperature was fixed at this temperature. The computional domain length of L = 1 mm and width w = 0.6 mm were fixed.

The numerical solution of the continuity, momentum and energy Eqs. (5) - (8) along with the boundary conditions (9) - (14) was obtained by using a three-dimensional commercial package FLUENT™ [24], which employs a finite volume method. The details of the method were explained by Patankar [25]. FLUENT™ was coupled with a geometry and mesh generation package GAMBIT [26] using MATLAB [27] to allow the automation and running of the simulation process. The computational domain was discretised using hexahedral/wedge elements. A second-order upwind scheme was used to discretise the combined convection and diffusion terms in the momentum and energy equations. The SIMPLE algorithm was then employed to solve the coupled pressure-velocity fields of the transport equations. After the simulation had converged, an output file was obtained containing all the necessary simulation data and results for the post-processing and analysis. The solution was assumed to have converged when the normalised residuals of the mass and momentum equations fell below 10^{-4} and while the residual convergence of energy equation was set to less than 10^{-8}. The number of grid cells used for the simulations varied for different computational domains used. However, grid independence tests for several mesh refinements were carried out to ensure the accuracy of the numerical results. The convergence criterion for the overall thermal resistance as the quantity monitored is:

$$\gamma = \frac{\left|\left(q_{\max}\right)_i - \left(q_{\max}\right)_{i-1}\right|}{\left|\left(q_{\max}\right)_i\right|} \le 0.01 \tag{17}$$

where i is the mesh iteration index. The mesh is more refined as i increases. The $i-1$ mesh is selected as a converged mesh when the criterion (17) is satisfied.

MATHEMATICAL OPTIMISATION

In this section, we introduce an optimisation algorithm that will search and identify the optimal design variables at which the system will perform at an optimum. A numerical algorithm, Dynamic-Q [28] was employed and incorporated into the finite volume solver and grid (geometry and mesh) generation package by using MATLAB. The Dynamic-Q is a multidimensional and robust gradient-based optimisation algorithm, which does not require an explicit line search. The technique involves the application of a dynamic trajectory LFOPC optimisation algorithm to successive quadratic approximations of the actual problem [29].

The algorithm is also specifically designed to handle constrained problems where the objective and constraint functions are expensive to evaluate. The details of the Dynamic-Q and applications can be found in Refs [28-30].

OPTIMISATION PROBLEM

The design variable constraint ranges for the optimisation are:

$$\begin{aligned}
&\mathrm{w_{el}} = 0.6\ \mathrm{mm},\quad \mathrm{L} = 1\ \mathrm{mm},\quad 0.05\ \mathrm{mm} \le d_{h_1} \le w,\\
&0.05\ \mathrm{mm} \le d_{h_3} \le w,\quad 0.05\ \mathrm{mm} \le d_{h_3} \le w,\\
&0.05\ \mathrm{mm} \le h_1 \le 0.7\ \mathrm{mm},\quad 0.05\ \mathrm{mm} \le h_2 \le 0.7\ \mathrm{mm},\\
&0.05\ \mathrm{mm} \le h_1 \le 0.7\ \mathrm{mm},\quad 0.05 \le \mathrm{t} \le 0.2,\\
&0.5 \le \mathrm{AR}_p \le 5,\quad 0.05 < \mathrm{s}_1 < \mathrm{L},\quad 0.05 < \mathrm{s}_2 < \mathrm{L},\\
&0.05 < \mathrm{s}_3 < \mathrm{L},
\end{aligned} \tag{29}$$

The pin-fin aspect ratios in the ranges specified in Equation (29) are the manufacturing and size constraint allowable for typical Pin-fins [31-32] and the inter-fin spacing is limited to 0.05 mm fabrication techniques consideration [33, 34]. The optimisation process was carried out for applied Reynolds number (*Re*) ranging from *Re = 25 to Re = 400*.

Figure 4 shows the minimised dimensionless global thermal resistance as a function of Reynolds number for design of the different cases of pin fin arrangements considered. The results show that the minimised dimensionless global thermal resistance monotonically decreases as the Reynolds number increases. It can be seen that the convective heat transfer is a strong function of the working fluid velocity. The results also show that the pin-fin row arrangements have a strong influence on the minimised thermal resistance. For a specified applied Reynolds number, as the number of pin-fin rows increases, the global thermal resistance decreases.

Also, Figures 5 to 8 show the optimal behaviours of the geometry as the applied Reynolds number increases for different cases of pin-fin arrangements. There is a unique optimal geometry for each of the pin-fin row arrangements.

Figure 5 shows that the optimal base thickness of the heat sink is fairly constant as the Reynolds number increases.

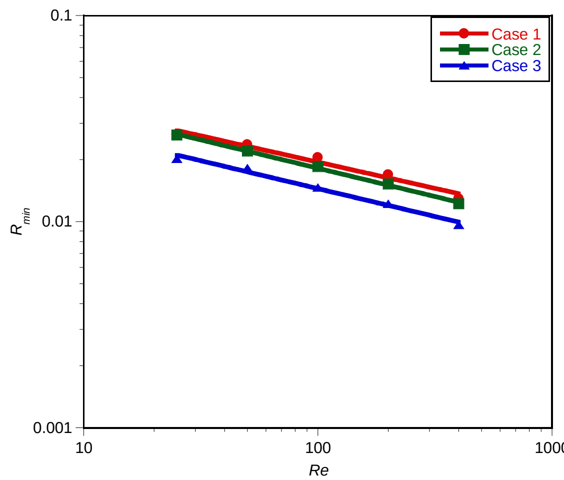


Figure 4. Effect of Reynolds number on the minimised dimensionless global thermal resistance

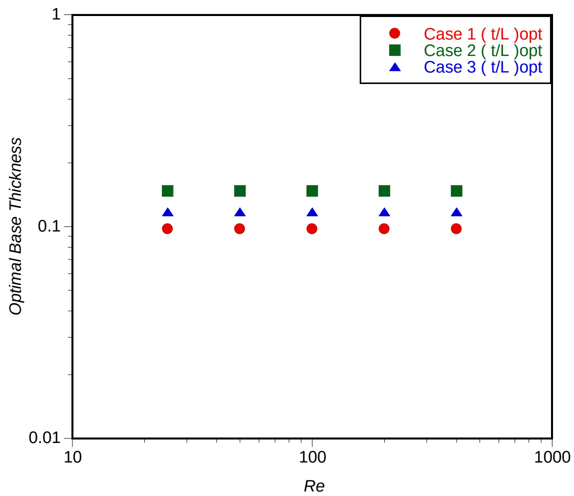


Figure 5. Effect of Reynolds number on heat sink base thickness.

Figure 6 shows the graph of optimal pin-fin aspect ratio against Reynolds number for different cases of pin-fin arrangements. This show optimal pin-fin aspect ratio is not sensitive to different applied Reynolds number.

Figure 7 shows that the optimal pin height-to-height ratio and optimal pin diameter-to-diameter ratio and fairly constant as the Reynolds number increases for different cases of pin-fin arrangements.

In all the three pin-fin row arrangements studied, the optimal aspect ratios, optimal pin height-to-height ratio, optimal pin hydraulic diameter-to-hydraulic diameter ratio and optimal spacing are not sensitive to the pin-fin row arrangements.

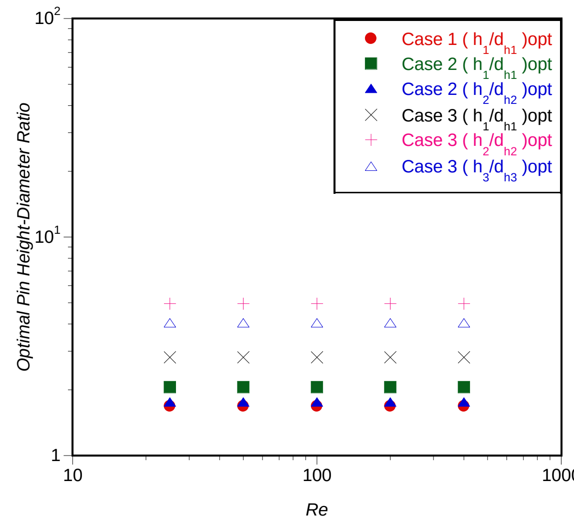


Figure 6. Effect of Reynolds number on optimal pin height/diameter ratio

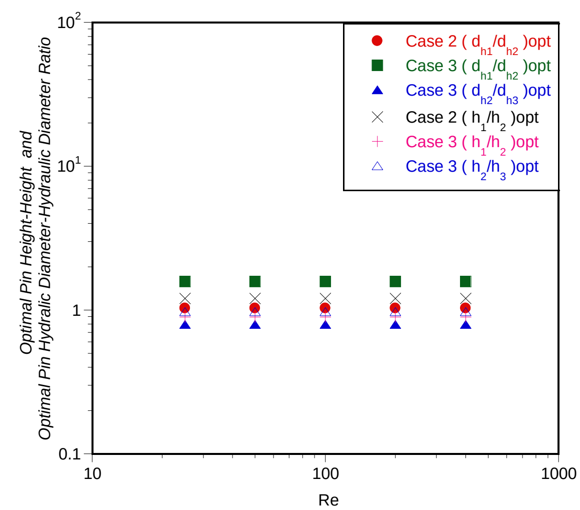


Figure 7. Effect of Reynolds number on optimal pin height/height ratio and diameter/diameter ratio

Figure 8 shows that the optimal spacing are also objectively constant as the Reynolds number increases. The optimal s_1 / L from the leading end and optimal s_1 / s_2 between the first two pin-fins decrease as the pin-fin row increases. The closer the first pin-fin is from the leading end, the better the performance of the heat sink pin-fin. Again, optimal s_2 / s_3 plays vital a role in optimal performance of the pin-fin heat sink. It shows that the closer the last two pin-fins the better the cooling is.

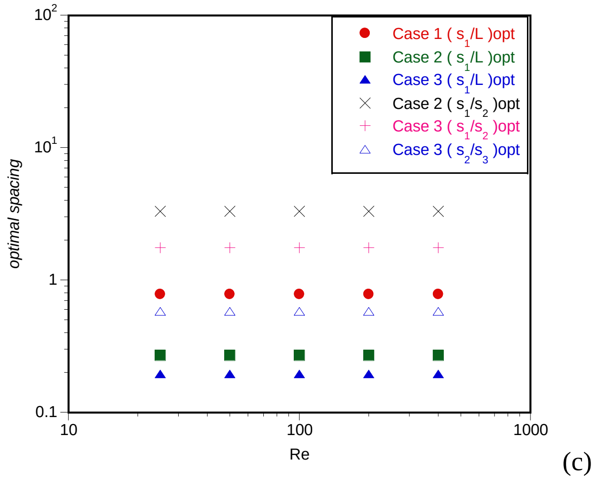


Figure 8. Effect of Reynolds number on optimal spacing

Figures 9a, 9b and 9c show the temperature contours of the elemental structure, with pin-fin and cooling fluid for different cases of pin-fin arrangements. The blue region indicates the region of low temperature and the red region indicates that of high temperature.

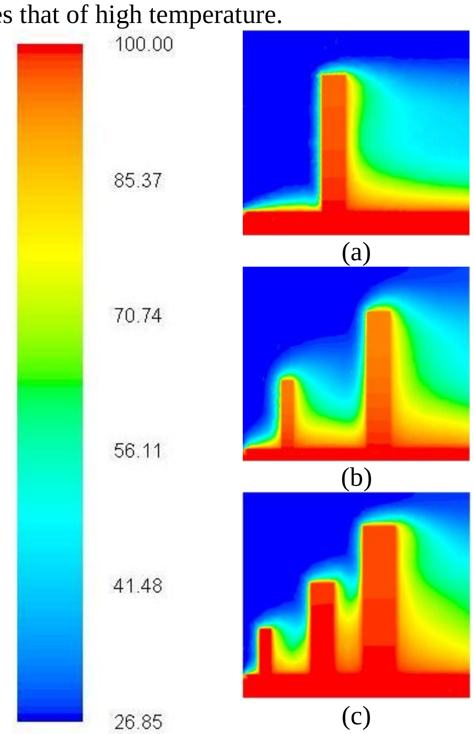


Figure 9. Temperature distributions on elemental structure with pin fin and cooling fluid for (a) Case 1 (b) Case 2 (c) Case 3

CONCLUSION

This paper studied a three-dimensional geometric optimisation of cooling square pin-fins in forced convection of solid base material subjected to a constant wall temperature applied at the bottom of the pin-fins based on constructal theory. Three pin-fin row arrangements were studied. The main objective was to optimise the configuration in such a way that the thermal resistance was minimised. The numerical solution shows that design parameters, in the various cases of pin-fin row arrangements, have strong influence on the minimised thermal resistance.

The numerical analysis also shows that the optimised geometry and minimised thermal resistance are functions of the Reynolds number. The results show that the minimised dimensionless global thermal resistance monotonically decreases as the Reynolds number increases. The optimal aspect ratios, optimal pin height-to-height ratio, optimal pin hydraulic diameter-to- hydraulic diameter ratio and optimal spacing are not sensitive to the pin-fin row arrangements. They are all constant as the Reynolds number increases. There is a unique optimal geometry for each pin-fin row arrangement for a given Reynolds number. The optimal spacing is also fairly constant as the Reynolds number increases. The optimal s_1 / L from the leading end and optimal s_1 / s_2 between the first two pin-fins decrease as the pin-fin row increases. The closer the first pin-fin from the leading end, and the closer the first two pin-fins are, the better the performance of the pin-fin heat sink is. Again, the optimal s_2 / s_3 plays a vital role in optimal performance of the pin-fin heat sink. It shows that the closer the last two pin-fins are, the better the cooling is. Future work will consider the cases where the pin fins arrangements are more than three rows.

ACKNOWLEDGEMENTS

The funding obtained from the NRF, TESP, Stellenbosch University / University of Pretoria, SANERI/SANEDI, CSIR, EEDSM Hub and NAC is acknowledged and duly appreciated.

REFERENCES

[1] Naphon, P., and Sookkasem, A., Investigation on heat transfer characteristics of tapered cylinder pin-fin heat sinks. Energy Conversion and Management, vol. 48, 2007, pp.2671–2679.

[2] Bejan A. and Morega A. M., Optimal arrays of pin fins and plate fins in laminar forced convection, ASME J. Heat Transfer, vol. 115, 1993, pp. 75–81.

[3] Peles, Y., Koşar, A., Mishra, C., Kuo, C., and Schneider B., Forced convective heat transfer across a pin fin micro heat sink, Int. J. Heat Mass Tran. 48, 2005, pp. 3615–3627.

[4] Bejan, A. The optimal spacing for cylinders in crossflow forced convection, ASME J. Heat Transfer, vol. 117, 1995, pp. 767–770.

[5] Jubran, B. A., Hamdan, M. A., and Abdullah, R. M., Enhanced heat transfer, missing pin, and optimization for cylindrical pin fin arrays, ASME J. Heat Transfer, vol. 115, 1993, pp. 576–583.

[6] Khan, T. W.A., Culham, J.R., and Yovanovich, M.M., Optimization of pin-fin heat sinks using entropy generation minimization, IEEE Trans. Compon. Packag. Technol. vol. 28, 2005, pp. 1-13.

[7] Bello-Ochende, T., Meyer, J. P., and Bejan, A., Constructal multi-scale pin fins, International Journal of Heat and Mass Transfer, vol. 53, 2010, pp. 2773-2779.

[8] Ighalo, F.U., Bello-Ochende, T., Meyer, J. P., Geometric Optimization of Multiple-Arrays of MicroPin-fins, Proceedings of the ASME/JSME 2011 8th Thermal Engineering Joint Conference, Honolulu, Hawaii, USA, 2011.
[9] Obayopo, S. O., Bello-Ochende, T., Meyer, J. P., Modelling and optimization of reactant gas transport in a PEM fuel cell with a transverse pin fin insert in channel flow, International journal of hydrogen energy, vol. 37, 2012, pp. 10286-10298.
[10] Bejan, A., Advanced Engineering Thermodynamics, 2nd ed., Wiley, New York, 1997.
[11] Bejan, A., Shape and Structure from Engineering to Nature, Cambridge University Press, Cambridge, UK, 2000.
[12] Bejan A., Sciubba E., The optimal spacing of parallel plates cooled by forced convection, *International Journal of Heat and Mass Transfer*, vol. 35, 1992, pp. 3259–3264.
[13] Bejan, A., Badescu, V., and De Vos, A., Constructal theory of economics, Applied Energy, vol. 67, 2000, pp. 37–60.
[14] Bejan, A., Optimal internal structure of volumes cooled by single phase forced and natural convection, Journal. Electron. Packaging vol. 125, 2003, pp. 200–207.
[15] Bejan, A., Two hierarchies in science: the free flow of ideas and the academy, International Journal of Design and Nature and Ecodynamics, vol. 4 , 2009, pp. 386–94.
[16] Rocha, L.O.A., Lorenzini., E., and Biserni, C., 2005, Geometric optimization of shapes on the basis of Bejan's Constructal theory, International Communication of Heat and Mass Transfer, vol. 32, pp. 1281–1288.
[17] Hajmohammadi, M. R. Poozesh, S., and Nourazar, S.S., 2012, Constructal design of multiple heat sources in a square-shaped fin, Journal of Process Mechanichal Engineering, vol. 226, pp. 324-336
[18] Salimpour, M.R., Sharifhasan, M., and Shirani, E., Constructal optimization of the geometry of an array of micro-channels, International Communication of Heat and Mass Transfer, vol. 38, 2010, pp. 93–99.
[19] Olakoyejo, O.T., Bello-Ochende, T., and Meyer, J.P., Mathematical optimisation of laminar forced convection heat transfer through a vascularised solid with square channels", International Journal of Heat and Mass Transfer, vol. 55, 2012, pp. 2402-2411.
[20] Bejan, A., and Lorente, S., 2011, The Constructal Law and the evolution of design in nature, Physics of Life Reviews, vol. 8, 2011, pp. 209–240.
[21] Reis, A.H. Constructal Theory – Complex flow structures in engineering and in Nature, in III Conferência Nacional em Mecânica de Fluidos, Termodinâmica e Energia (MEFTE - BRAGANÇA 09), 2009, pp. 1- 17.
[22] O.T and Meyer, J.P., Optimisation of square pin-fin for minimum thermal resistance, *Proceedings of the Constructal Law Conference 2013, 14 - 15 October 2013, Nanjing, China.*
[23] White F.M., *Viscous Fluid Flow*, 2nd Edition, McGraw-Hill International Editions, Singapore, 1991.
[24] Fluent Inc., *Fluent Version 6 Manuals*, Centerra Resource Park, 10 Cavendish Court, Lebanon, New Hampshire, USA, 2001 (www.fluent.com).
[25] Patankar, S. V., Numerical Heat Transfer and Fluid flow", Hemisphere, New York. 1980
[26] Fluent Inc., *Gambit Version 6 Manuals*, Centerra Resource Park, 10 Cavendish Court, Lebanon, New Hampshire, USA, 2001 (www.fluent.com).
[27] The MathWorks, Inc., *MATLAB & Simulink Release Notes for R2008a*, 3 Apple Hill Drive, Natick, MA, 2008 (www.mathworks.com).
[28] Snyman J.A., *Practical Mathematical Optimisation: An Introduction to Basic Optimisation Theory and Classical and New Gradient-Based Algorithms*, Springer, New York, 2005
[29] Snyman, J.A., Stander, N., Roux, W.J., dynamic penalty function method for the solution of structural optimization problems, Appl. Math. Model. vol 18, 1994, pp. 453–460.
[30] J.A. Visser, D.J. de Kock, Optimization of heat sink mass using the DYNAMIC-Q numerical optimization method, Commun. Numer. Meth. Engng vol.18, 2002, pp.721–727
[31] Achanta, V.S., An experimental study of end wall heat transfer enhancement for flow past staggered non-conducting pin fin arrays, PhD Thesis, Department of Mechanical Engineering, Texas A & M University, 2003.
[32] Lyall, M.E., 2006, "Heat transfer from low aspect ratio pin fins", PhD Thesis, Department of Mechanical Engineering, Virginia Polytechnic Institute and State University.
[33] Husain, A. and Kim, K., 2008, Shape optimisation of micro-channel heat sink for micro-electronic cooling", IEEE Trans. Compon. Packag. Technol. vol. 31, pp. 322-330.
[34] Li, J. and Peterson, G. P., 2006, Geometric optimisation of a micro heat sink with liquid flow, IEEE Transtraction Composition. Packaging. Technology, vol. 29, pp. 145-154.