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SUSTAINABLE ENERGY SOURCE WITH TIME-VARYING TEMPERATURE

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ABSTRACT

Heat conductivity problem in rectangular region is studied numerically with algorithm describing in [1]. The region is divided into two equal parts. Boundary conditions have harmonic variation in time and are the same on the left and on the right sides. Heat conduction is assumed to behave as various linear functions of temperature on the left and on the right half of region. Heat conduction is an increasing function of temperature in the one part and is a decreasing function of temperature in the other part. It is found that heat transfer is not equal zero through the region for period. Similar effects for periodic natural convection were investigated in the works [2], [3]. It is also obtained that total heat flux for period through the region depends on its length.

INTRODUCTION

The temperature waves are periodic oscillations of temperature in some medium. They are usually considered as climatic oscillations of temperature [4]. Studies of the temperature oscillations in the ground were the first application of temperature wave problem for natural processes. The paper [5] was concerned with a solution of the problem in a semi-limited region. The propagation of temperature waves in rectangular area with harmonic oscillation of boundary temperature was considered. The thermophysical characteristics of a body were assumed to be constant. However temperature waves can be observed in the range of technological processes, specifically if there are the periodically repeating working cycles in engines where temperature of working medium and engine casing changes with some periodic law.

Analytical solutions for these problems are usually very complex if exist at all, since the heat conductivity is a function of temperature. Numerical methods are simple and effective way to find solution for this problem.

The temperature waves have a wide range of applications such as heating and cooling of the buildings, solar energy utilization, thermal energy storage, and cooling of electronic equipment. Gavriliev R.I. [6] found an analytical solution for temperature waves in a two-layer medium consisting of frozen ground and snow cover. The temperature waves that have both natural and technological character also can be used for getting source energy from cheap resources. As long ago as 1877 Baron Rayleigh [7] considered question about maintenance of mechanical oscillations using heat waves. Especially he noted problem about relation between phases of heat and mechanical oscillations. Kalabin E.V. et al. [2], [3] have investigated influence of temperature waves on natural convection in the inclined square cavity and got that there is a possibility of heat transferring from the time-average cold wall to the hot wall of the cavity.

In the present work we found a scheme for technological equipment with nonzero total heat transfer through the domain for a time period when instant temperatures of right and left bounds are equal.

NOMENCLATURE

a	[m^2/s]	Thermal diffusivity
c	[J/kgK]	Specific heat capacity
l	[m]	Length of cavity
l_0	[m]	Reference length
P	[s]	Period of boundary temperature oscillations
T	[K]	Temperature
T_a	[K]	Amplitude of boundary temperature oscillations [$(T_u$-$T_d)/2$]
T_d	[K]	Minimum value of boundary temperature
T_{mid}	[K]	Mean value of boundary temperature [$(T_u$+$T_d)/2$]
T_u	[K]	Maximum value of boundary temperature
t	[s]	Time
x	[m]	Cartesian coordinate
λ	[W/mK]	Thermal conductivity
λ_0	[W/mK]	Reference value of thermal conductivity for $T=T_{mid}$
ρ	[kg/m^3]	Density
ω	[rad/s]	Oscillation frequency

Dimensionless characteristics

a_I	[-]	Coefficient in discretization equation for θ_I
a_I^{old}	[-]	Coefficient in discretization equation for θ_I before θ_I^{old}
b_I	[-]	Coefficient in discretization equation for θ_I before θ_{I+1}
c_I	[-]	Coefficient in discretization equation for θ_I before θ_{I-1}
d_I	[-]	Free term in discretization equation for θ_I
L	[-]	Length of cavity [l/l_0]

q	[-]	Heat flux
Q	[-]	Total heat flux for period
X	[-]	Cartesian coordinate [x/l_0]
ΔX	[-]	Size of the typical control volume [$L/(N-2)$ for uniform grid]
$\Delta\tau$	[-]	Time step
δX	[-]	Distance between typical grid points
θ	[-]	Temperature [$(T-T_{mid})/T_a$]
θ_α	[-]	Temperature inside the domain when thermal conductivity is increasing function of temperature
$\theta_{-\alpha}$	[-]	Temperature inside the domain when thermal conductivity is decreasing function of temperature
$\tilde{\lambda}$	[-]	Thermal conductivity [λ/λ_0]
τ	[-]	Time [t/P]

Special characters

N	[-]	Number of grid points
α	[-]	Coefficient in dependence of heat conductivity on temperature

Subscripts

b	Boundary: for x=0, x=l or X=0, X=L
I	Number of the typical grid point
$I+1$	Number of the point on the right from I
$I-1$	Number of the point on the left from I
i	Number of the control volume limit between grid points I -1and I
$i+1$	Number of the control volume limit between grid points I and $I+1$
in	Initial: for t=0 or τ=0
old	Last time step

PROBLEM FORMULATION

The rectangular cavity of length l is considered. Boundary conditions have harmonic variation in time and are the same on the left and on the right (fig.1). The region is divided into two equal parts (fig. 2). Heat conduction is assumed with various linear functions of temperature on the left and on the right half of region. Heat conduction is an increasing function of temperature in the one part and is decreasing function of temperature in the other part.

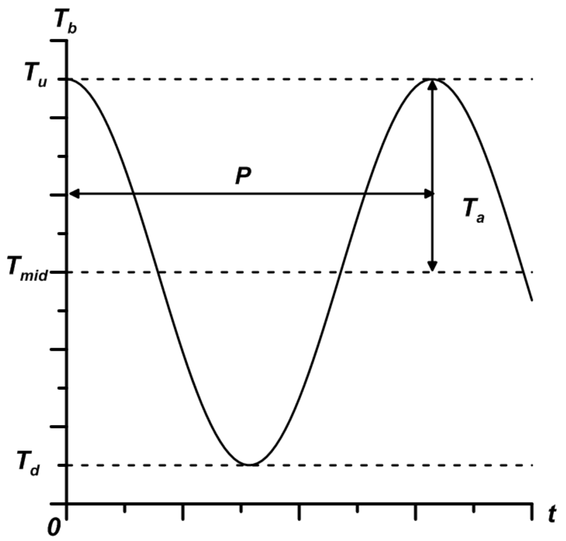

Figure 1 Dependence of boundary temperature on time

Mathematical model of the problem is one-dimensional nonstationary conduction equation in Cartesian coordinates with heat conductivity that depends on temperature:

$$\rho c\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right), \tag{1}$$

$$\lambda=\lambda_0\left(\alpha\frac{\left(T-T_{mid}\right)}{T_a}+1\right), \tag{2}$$

where

$$\begin{cases}\alpha>0,\ 0\le x<l/2\\ \alpha<0,\ l/2<x\le l\end{cases}. \tag{3}$$

Initial condition:

$$T\big|_{t=0}=T_{in}. \tag{4}$$

Boundary conditions:

$$T\big|_{x=0}=T\big|_{x=l}=T_b=T_{mid}+T_a\cos\left(\omega t\right), \tag{5}$$

$$T_{mid}=\frac{T_u+T_d}{2},\ T_a=\frac{T_u-T_d}{2}. \tag{6}$$

Introduce following dimensionless variables and parameters:

$$X=\frac{x}{l_0},\ \tau=\frac{t}{P},\ \theta=\frac{T-T_{mid}}{T_a},\ \tilde{\lambda}=\frac{\lambda}{\lambda_0},\ L=\frac{l}{l_0}, \tag{7}$$

where

$$P=\frac{2\pi}{\omega},\ l_0=\sqrt{aP},\ a=\frac{\lambda_0}{\rho c}. \tag{8}$$

After appropriate transformations original system has form:

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial X}\left(\tilde{\lambda}\frac{\partial\theta}{\partial X}\right), \tag{9}$$

$$\tilde{\lambda}=\alpha\theta+1, \tag{10}$$

where

$$\begin{cases}\alpha>0,\ 0\le X<L/2\\ \alpha<0,\ L/2<X\le L\end{cases}. \tag{11}$$

Initial condition:

$$\theta\big|_{\tau=0}=\theta_{in}. \tag{12}$$

Boundary conditions:

$$\theta\big|_{X=0}=\theta\big|_{X=L}=\theta_b=\cos\left(2\pi\tau\right). \tag{13}$$

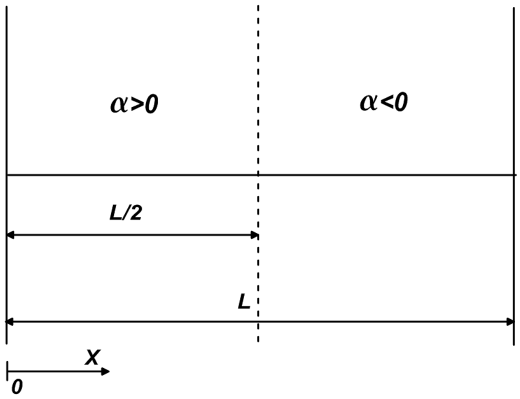

Figure 2 Geometry of the problem

The considered problem is the problem without initial conditions [5]. It means that in time moment that is quite distant from initial time moment, the initial conditions will not have influence on temperature distribution in the domain. There will be system state where temperature in every point of the domain will be changed with the same period that is on cavity boundaries.

NUMERICAL METHOD

The dimensionless differential equation (9), (10) with initial (12) and boundary conditions (13) are solved with transformation to algebraic equations. We use control volume method for discretization of the problem described in [1]. The typical control volume is shown on the fig. 3.

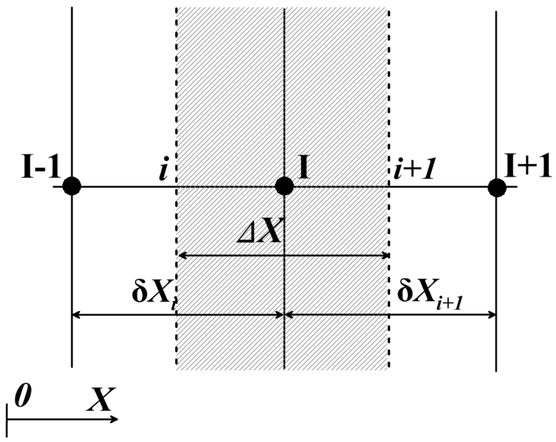

Figure 3 The typical control volume of the computational grid

Temperature θ is calculated in the grid points denoted with *I-1*, *I*, *I+1*. The small region between dashed lines *i* and *i+1* is the control volume for the grid point *I*. The size of control volume is ΔX.

Integating (9) over the control volume for point *I* and over the time interval with size $\Delta\tau$ we use implicit scheme on time and suppose that temperature θ has linear dependence on *X* between grid points:

$$\frac{\theta_I-\theta_I^{old}}{\Delta\tau}\Delta X=\tilde{\lambda}_{i+1}\frac{\theta_{I+1}-\theta_I}{\delta X_{i+1}}-\tilde{\lambda}_i\frac{\theta_I-\theta_{I-1}}{\delta X_i}, \quad (14)$$

where

$$\tilde{\lambda}_{i+1}=\frac{2\tilde{\lambda}_{I+1}\tilde{\lambda}_I}{\tilde{\lambda}_{I+1}+\tilde{\lambda}_I},\quad \tilde{\lambda}_i=\frac{2\tilde{\lambda}_I\tilde{\lambda}_{I-1}}{\tilde{\lambda}_I+\tilde{\lambda}_{I-1}}, \quad (15)$$

$$\tilde{\lambda}_{I-1}=\alpha\theta_{I-1}^{old}+1,\ \tilde{\lambda}_I=\alpha\theta_I^{old}+1,\ \tilde{\lambda}_{I+1}=\alpha\theta_{I+1}^{old}+1. \quad (16)$$

After that we obtain algebraic equations of the type (17) with respect on θ_I, θ_{I+1}, θ_{I-1}:

$$a_I\theta_I=b_I\theta_{I+1}+c_I\theta_{I-1}+a_I^{old}\theta_I^{old}+d_I,\ I=\overline{2,N-1}, \quad (17)$$

where a_I, b_I, c_I, a_I^{old} - known numerical functions which depend on temperature θ^{old} in the grid points taking from previous iteration and d_I=0 here. Adding initial condition

$$\theta_{\tau=0}^{old}=\theta_{in} \quad (18)$$

and boundary conditions

$$\theta_1=d_1,\ \theta_N=d_N \quad (19)$$

to the system (17) we can find the solution of the problem.

The structure of the algorithm

1. Set initial value of θ (18) and boundary conditions (19).
2. Calculate coefficients for the system (17). Find θ_I, *I=2,..,N-1*from the system (17) using TDMA (Three-diagonal matrix algorithm) [1].
3. Repeat step 2 until temperature in every point of the domain will be changed with the same period that is on cavity boundaries.

This algorithm is described in [1].

The parameters of calculation

Computations were carried on uniform grid which is consisted of *N*=502 nodes. Variation range of the parameter α defining dependence of heat conductivity on temperature was 0≤|α|<1. Dependences of dimensionless heat conductivity on temperature for all considered α are on the fig. 4. The dimensionless length *L* has been varied from 1 to 14.

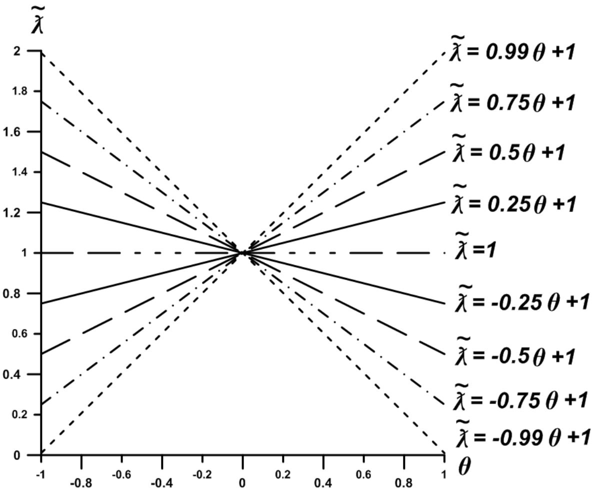

Figure 4 Dependences of dimensionless heat conductivity on temperature for different α.

ANALASYS OF OBTAINED RESULTS

If heat conductivity is a linear increasing function of temperature then temperature distribution will be almost constant and equal θ_{α} in some part of region that is far from the boundary. So θ_{α} will be higher than the mean value of temperature on the boundary of the domain: $\theta_{\alpha}>0$. If heat conductivity is a linear decreasing function of temperature then temperature distribution will be almost constant and equal $\theta_{-\alpha}$ in some part of region that is far from the boundary. Thus $\theta_{-\alpha}$ will be lower than mean value of temperature on the boundary of the domain: $\theta_{-\alpha}<0$. For the beginning we consider cavity with length L=14. Heat conductivity increases linearly in the left part of the domain and decreases linearly in the right part.

Let us consider temperature distributions in different time moments for different |α| which there are on fig. 5-8. Problem formulation with constant heat conductivity (α=0) has symmetry around the center point X=7 and therefore temperature in any time moment is the same for any points these have symmetry around center of the domain (fig. 5-8).

For time τ=0.25 (fig. 5) we observe displacement of local maximum in the left part of the domain to center in the temperature distribution and local maximum in the right part of the domain displaces to right boundary with increase |α|. For time τ=*0.75* we observe displacement of local minimum in the right part of the domain to the center and local minimum in the left part to the left boundary with increase |α| (fig.7).

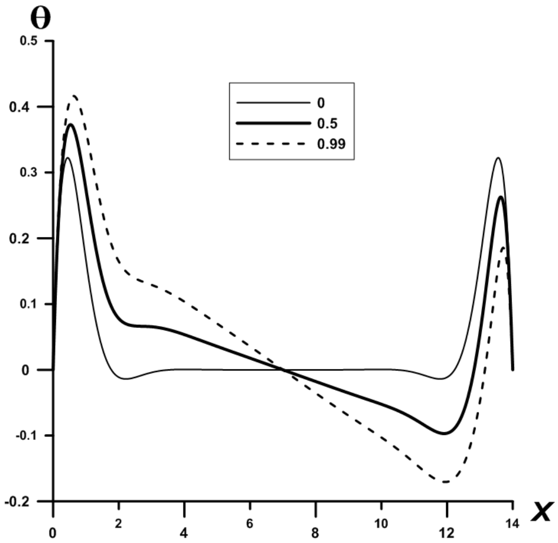

Figure 5 Dependences of dimensionless temperature θ on space coordinate X for time moment τ=*0.25,* |α|=0, 0.5, 0.99, L=14.

For α≠0 temperature inside the domain in any point of the left part is higher than temperature in the right part in symmetric point (fig. 5-8).

Thus as follows from fig. 5-8 for case α=0 (constant heat conductivity) temperature is almost constant in the significance part of the domain: $4<X<10$ and is equal $\theta=0$. For α≠0 and $4<X<10$ temperature is a decreasing function in all time moments. It means that heat flux has direction from left to right in this part of the domain. Consequently we get energy transfer through the domain with the same conditions of its boundaries.

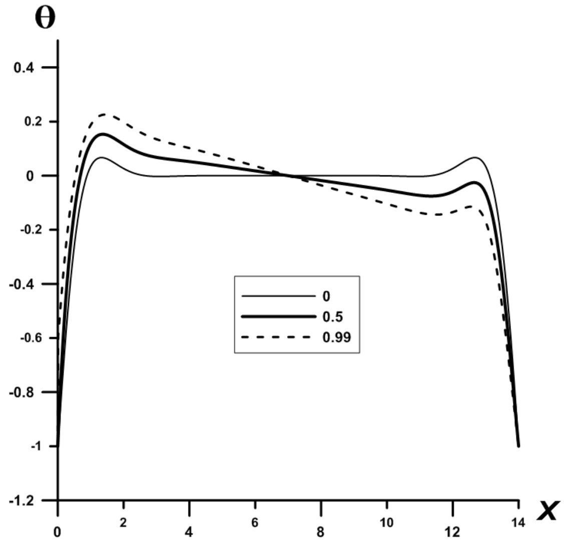

Figure 6 Dependences of dimensionless temperature θ on space coordinate X for time moment τ=*0.5,* |α|=0, 0.5, 0.99, L=14.

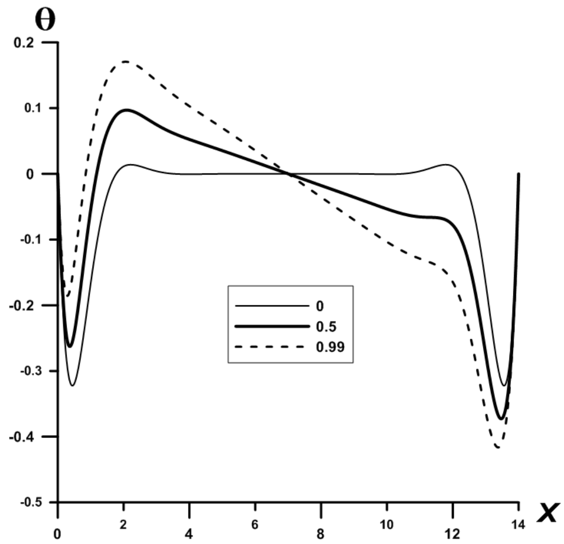

Figure 7 Dependences of dimensionless temperature θ on space coordinate X for time moment τ=*0.75,* |α|=0, 0.5, 0.99, *L=14*.

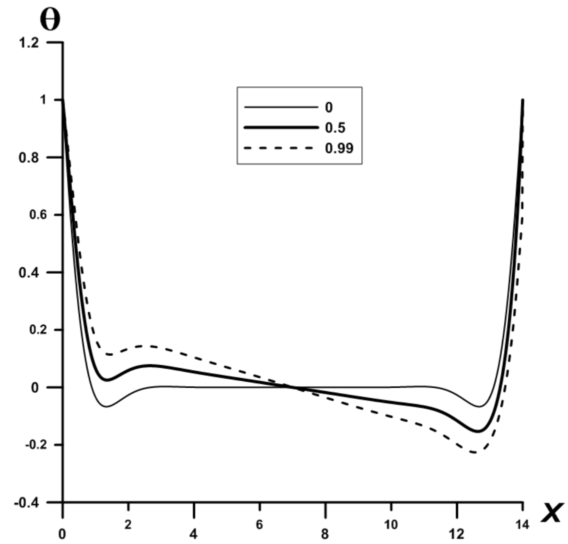

Figure 8 Dependences of dimensionless temperature θ on space coordinate X for time moment τ=1, |α|=0, 0.5, 0.99, L=14.

Consider dependences of dimensionless heat flux q on time for different values X:

$$q = -\tilde{\lambda}\frac{\partial \theta}{\partial X}. \quad (20)$$

Dependence of heat flux on time for $X=0$ is represented on fig. 9. The lag of maximum (minimum) value heat flux on the boundary from maximum (minimum) value of temperature on the boundary is equal π/4 for α=0. For |α|≠0 and X=0 heat flux minimum is achieved earlier and maximum is achieved later than for α=0. For X=0 the lag of heat flux peaks from peaks of boundary temperature is less than π/4 and the lag of corresponding minimum values on this boundary is more π/4.

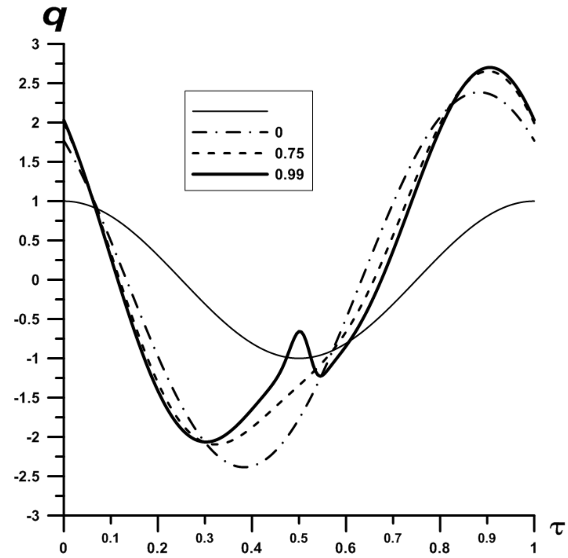

Figure 9 Dependences of dimensionless heat flux q on time for $X=0$, |α|=0, 0.75, 0.99 and dependence of boundary temperature on time, $L=14$.

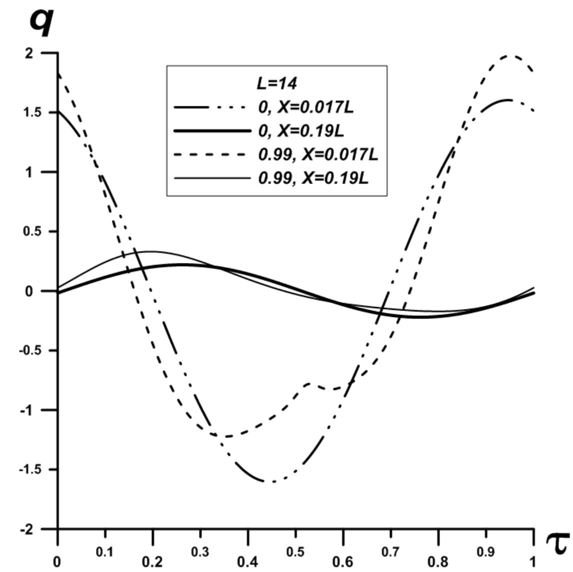

Figure 10 Dependences of dimensionless heat flux q on time for $X=0.017L$, $0.19L$, |α|=0, 0.99, L=14.

Choosing domain with length L=14 we consider heat flux on the left boundary (fig. 9) and heat fluxes in different points of the domain (fig. 10, 11). Comparing these figures we observe that amplitude of heat flux oscillation decreases exponentially with depth as amplitude of temperature oscillations. The length of cavity $L=14$ is chosen that heat flux in the center is almost constant (fig. 11). That is amplitude of it oscillation is so small that we can neglect it for considering of real physical process. Then we can get both constant heat flux in time placed it in the center of the domain and time-varying heat flux placed near boundaries. Thus amplitude of oscillations increases while approaching to boundaries of the domain.

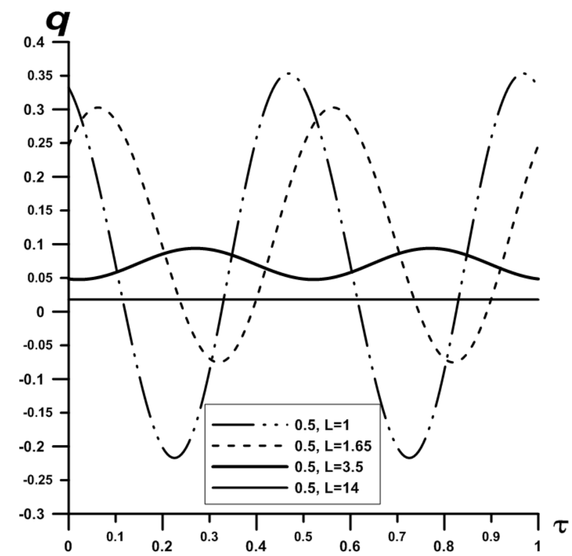

Figure 11 Dependences of dimensionless heat flux q on time for $X=L/2$, |α|=0.5, L=1, 1.65, 3.5, 14.

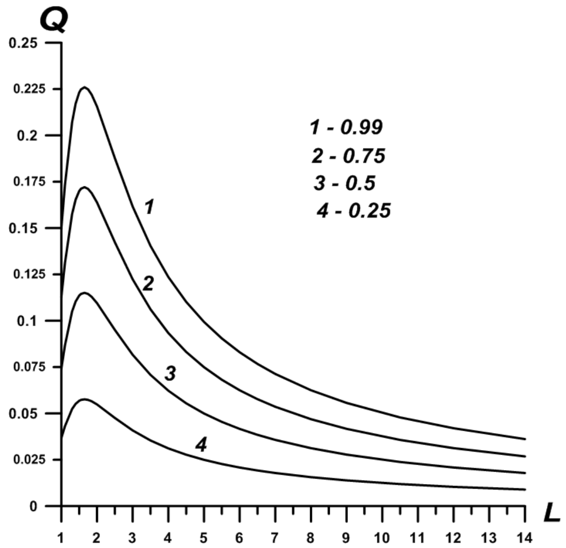

Figure 12 Dependences of dimensionless total heat flux Q for period through the region on length L of region for |α|=0.25, 0.5, 0.75, 0.99.

Varying length of cavity we get different values of dimensionless total heat flux for period for different L:

$$Q = \int_0^1 q d\tau. \qquad (21)$$

Looking at fig. 12 and tab.1 we see that dependence of Q on L is not monotonic and has peak for length L=1.65 for all |α|. It's clearly that value of Q also depends on |α|. The value of Q increases monotonically with growth of |α| (tab.1). Therefore dependences of Q on L for different values |α| are above each other on the fig. 12.

L	*|α|=0.25*	*|α|=0.5*	*|α|=0.75*	*|α|=0.99*
1	*0.0367*	*0.07414*	*0.11277*	*0.14983*
1.55	*0.05729*	*0.11447*	*0.17123*	*0.22482*
1.65	*0.05761*	*0.11508*	*0.17208*	*0.226*
1.75	*0.05731*	*0.11445*	*0.17112*	*0.22485*
2	*0.05485*	*0.10953*	*0.16383*	*0.21555*
3.5	*0.03541*	*0.07081*	*0.10617*	*0.14039*
7	*0.01785*	*0.03571*	*0.05358*	*0.07129*
10.5	*0.0119*	*0.02381*	*0.03571*	*0.04783*
14	*0.00892*	*0.01786*	*0.02682*	*0.03615*

Table 1 Values of dimensionless heat flux Q for period through the region.

We see on the fig. 11 that amplitude of oscillations of heat flux in the center of the domain decreases with growth of length L. Thus total heat flux for period has the maximal value for L=1.65 but it is varying in time (fig. 11, 13, 14).

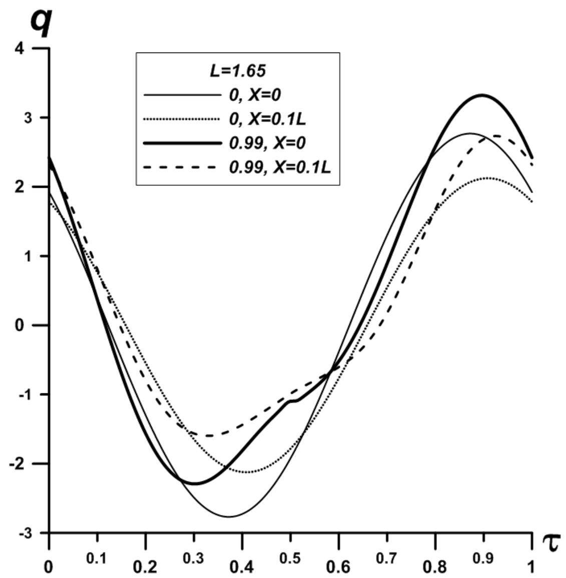

Figure 13 Dependences of dimensionless heat flux q on time for $X=0,\ 0.1\ L$, |α|=0, 0.99, L=1.65.

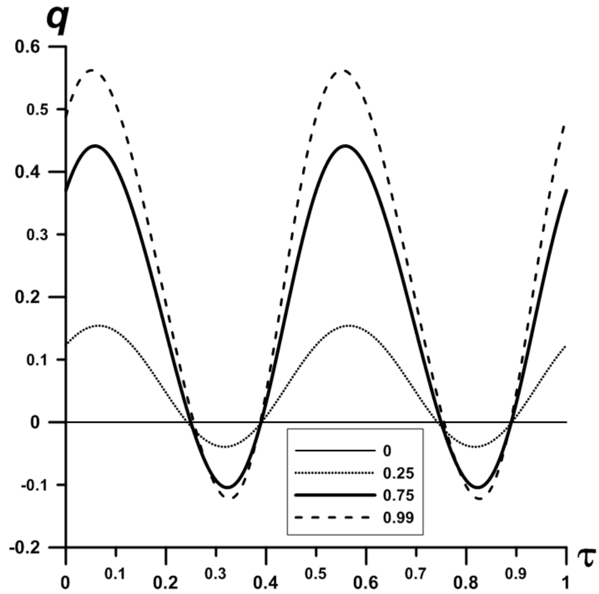

Figure 14 Dependences of dimensionless heat flux q on time for $X=L/2$, |α|=0, 0.25, 0.75, 0.99, L=1.65.

CONCLUSION

The one-dimension heat conductivity problem in limited rectangular area was solved. For present dependence of heat conductivity on temperature we received positive total heat flux for period through the domain with the same thermal boundary conditions on its boundaries.

The dependence of total heat flux on cavity length is not monotonic and has maximal value for length L=1.65 for all |α|. Total heat flux goes up monotonically with increasing |α| for any fixed L.

Using cavity with length L=14 we can get both constant heat flux in time taking it in the center of the domain and time-varying heat flux taking it near boundaries. The amplitude of heat flux oscillation increases while approaching to boundaries of the domain.

REFERENCES

[1] Patankar, S.V., Computation of Conduction and Duct Flow Heat Transfer: Innovative Research Inc., 1991, 349 p.

[2] Kalabin, E.V., Kanashina, M.V. , Zubkov, P.T., Natural-convective heat transfer in a square cavity with time-varying side-wall temperature, *Numerical Heat Transfer, Part* A, 47, 2005, pp. 621-631.

[3] Kalabin, E.V., Kanashina M.V., Zubkov, P.T., Heat transfer from the cold wall of a square cavity to the hot one by oscillatory natural convection, *Numerical Heat Transfer, Part* A, 47, 2005, pp. 609-619.

[4] Carslaw, H.S., Jaeger J.S. Conduction of heat in solids: Oxford, Clarendon Press, 1959, 510 p.

[5] Tikhonov, A.N., Samrski, A.A., Equations of Mathematical Physics: New York, Dover Publications, 1990, 800 p.

[6] Gavriliev, R.I., Analytical solution for the periodic temperature field in frozen ground with account for variation in mean daily air temperature, *Cold Region Science and Technology*, 83-84, 2012, pp. 110-114.

[7] Rayleigh, J. W. S., Baron, Theory of sound: Macmillan and co., 1877.