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An Inverse Approach to Heat Transfer Coefficient Prediction Considering the Effect of Moving Heating Source in Machining Process

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Abstract

The present paper propound an analytical inverse method to calculate the heat transfer coefficient of cutting fluids in machining processes. This method starts by establishing an estimation of the heat sources in the transient heat conduction distribution in a rectangular domain with convective bounders in free convection. The non homogeneous partial differential equation (PDE) is solved by Integral Transform method. The test function for the heat generation term is obtained by modeling of chip geometry and thermomechanical cutting. Then the solution of PDE with heat generation term optimized is used to present an inverse problem to calculate the convective heat transfer coefficient.

keyword: Heat conduction, Integral transform, Heat source, Inverse problem

Nomenclature

(ξ, η)	Dimensionless coordinate system
(x, y)	Coordinate system
α	Thermal diffusivity of solid, $\frac{k}{\rho c_p}$, $[m^2/s]$
β_m	Eigenvalues, $m = 1 \cdots \infty$
Φ	Dimensionless heat-generation, $\frac{gL^2}{k(T_0 - T_\infty)}$
ρ	Density of solid or fluid, $[Kg/m^3]$
Θ	Dimensionless temperature, $\frac{T-T_\infty}{T_0-T_\infty}$
Bi	Biot number
c_p	Specific heat of solid or fluid, $[J/Kg \cdot K]$
F_0	Fourier number, $\frac{\alpha t}{L^2}$
g	Heat generation in the solid, $g(x, y, z)$, $[J \cdot m/s]$
h	Heat transfer coefficient at the boundary surface s_i
k	Thermal conductivity coefficient of the solid, $[W/m \cdot K]$
K_v	Eigenfunction, $v = \xi \quad or \quad \eta$
L	Characteristic dimension of solid, $\frac{\text{Volume}}{\text{Surface area}}$
q	Heat liberation rate of moving rectangular heat source $[J]$
s_i	Bounding surface of the solid, $i = 1, 2, 3, 4$
T	Temperature, T_∞ Ambient temperature, T_0 Initial temperature, [K]
t	Time, $[s]$

Introduction

In a typical metal cutting operation almost 100% of the total energy spent is converted into heat, which has to be dissipated by the tool cutting edge, workpiece, chip and also by the cutting fluid normally used. The cutting fluid is, in general, a liquid containing basically a mixture of water, oil and some other additives. The percentages of these, and other components, are carefully adjusted to satisfy the demands of a particular process. One of the main purposes of these cutting fluids is to cool down the chip formation region, which in turn, keeps the tool at acceptable temperatures delaying the wear and making the process more economically efficient. In addition, the dimensions are better kept if the heat is conducted away from the workpiece. Lately, with the tightening on legislation to dispose the cutting fluids, costs can be very high when using certain products and also there has been an increase in number of different formulations and basic substances. To be able to clearly test and distinguish among these new proposals a comprehensive study of the heat propagation and temperature distribution has become a basic requirement. Such study can, in the near future, set the grounds for a realistic and practical method to test and select the best products for metal cutting applications.

For such purpose simple mathematical formulations to deal with heat conduction problems is of prime interest in manufacturing processes optimization based on metal cutting. In this context the heat transfer coefficient is a important topic in the field of heat transfer technology. The heat transfer coefficient, regulates the heat transmission between the surface of a solid body and a neighboring fluid. In addition, The Biot number, the dimensionless form of the heat transfer coefficient, may physically be interpreted as the ratio of the internal and external conductances of a heat problem with convective boundary. In this paper, an method for estimating time-dependent heat transfer coefficient for linear inverse heat conduction problem is proposed. In [6] a method is proposed for the evaluation of the local convective heat transfer coefficient for an unidimensional steady heat conduction problem using Fourier transform. The same problem is studied in [5] where the time-dependent Biot number in a one-dimensional linear heat conduction problem is obtained from the solutions of the inverse heat conduction problems of determining boundary heat flux and boundary temperature. Non of the studies found in literature deals with the case of a inverse method to calculate the heat transfer coefficient for a two-dimensional transient heat conduction problem with heat generation subject to convective boundary in all body surface. To fill this gap, the present work studies the finite integral transform techniques to solve the two-dimensional, transient heat-conduction problem with general time-dependent heat sources and boundary conditions. The solutions is obtained, based on the work of [10]. This solution is formulated in terms of quasi-steady and transient terms and given in the form of infinite series. Using the Fourier theory to orthogonal function, an approximated solution is taken and an error is established. Then the heat transfer coefficient time dependent is formulated by direct inverse of this approximated solution. Finally, the inverse method is applied to the case of a plate with convective boundary conditions of third kind and moving heat generation.

Analytical Model of Temperature distribution

The process starts by modeling the heat distribution in a typical orthogonal cutting with fluid surrounding in all borders of a rectangular solid surface R. The dimensionless model of temperature distribution on a stationary, homogeneous, isotropic solid with constant thermal properties subject a heat generation near its boundary and convection dissipation too. The problem under consideration can be governed by the following equation, [11]:

$$\left(\frac{\partial^2\Theta}{\partial\xi^2}+\frac{\partial^2\Theta}{\partial\eta^2}\right)+\Phi=\frac{\partial\Theta}{\partial F_0} \quad \text{in } R, F_0>0 \tag{1}$$

where $\xi=\frac{x}{L}$ and $\eta=\frac{y}{L}$, with the following boundary conditions

$$\frac{\partial\Theta}{\partial N_i}\pm Bi\Theta=0 \qquad \text{on } s_i,\ F_0>0 \tag{2}$$

and initial condition

$$\Theta=1 \qquad \text{in } R, \qquad F_0=0 \tag{3}$$

where $R=[0,1]\times[0,1]$, s_i is a bounding surface of the solid, for $i=1,2,3,4$, and $\frac{\partial}{\partial N_i}\equiv$ differentiation along outward-drawn normal to the boundary surface s_i in the dimensionless coordinate system The dimensionless excess temperature Θ is defined by

$$\Theta=\frac{T-T_\infty}{T_0-T_\infty} \tag{4}$$

The dimensionless time variable F_0 , which can be called the Fourier number, is defined as follows:

$$F_0 = \frac{\alpha t}{L^2} = \frac{(\frac{k}{L})\cdot L^2}{(\rho c_p L^3/t)} = \frac{\text{rate of heat conducted across } L \text{ in the volume } L^3}{\text{rate of energy storage in reference to volume } L^3} \quad (5)$$

The dimensionless heat-generation variable Φ , is defined as follows:

$$\Phi = \frac{g(x,y,t)L^2}{k(T_0-T_\infty)} = \frac{g(x,y,t)L^3}{\frac{k}{L}L^2(T_0-T_\infty)} = \frac{\text{rate of heat generation in reference to volume } L^3}{\text{rate of heat conduction across } L \text{ in } L^3, \text{ with temperature difference } (T_0-T_\infty)} \quad (6)$$

The dimensionless heat-conduction parameter Bi, which is called the Biot number is interpreted as:

$$Bi = \frac{h}{k/L} = \frac{\text{heat transfer coefficient at the boundary surface}}{\text{unit conductance of the solid across thickness L}} \quad (7)$$

The equation (1) is a nonhomogeneous PDE. We can't use separation variables method to reduced the nonhomogeneous equation to a characteristic-values problem in each space variable involved. However, the integral transform method is convenient for nonhomogeneous problems due to the presence of heat generation term in the equation (equation (1)) or due to non-uniformity of boundary conditions, or both, see [12]. The dimensionless solution of the model governed by equation (1) is formulated by [10] as follows:

$$\Theta(\xi,\eta,F_0) = \sum_{m=1}^{\infty} \frac{1}{2\beta_m^2} K(\beta_m,\xi,\eta)\bar{\Phi}(\beta_m,F_0) + \sum_{m=1}^{\infty} e^{-2\beta_m^2 F_0} K(\beta_m,\xi,\eta) \left\{ \bar{F}(\beta_m) - \frac{1}{2\beta_m^2}\left[\bar{\Phi}(\beta_m,0)\right] - \frac{1}{2\beta_m^2}\int_0^{F_0} e^{-2\beta_m^2 \bar{F}_0}\left[\frac{\partial}{\partial \bar{F}_0}\bar{\Phi}(\beta_m,\bar{F}_0)\right] d\bar{F}_0 \right\} \quad (8)$$

where

$$\bar{F}(\beta_m) = \int_0^1 \int_0^1 K(\beta_m,\bar{\xi},\bar{\eta})d\bar{\xi}d\bar{\eta} \quad (9)$$

$$\bar{\Phi}(\beta_m,F_0) = \int_0^1 \int_0^1 K(\beta_m,\bar{\xi},\bar{\eta})\Phi(\bar{\xi},\bar{\eta},\bar{F}_0)d\bar{\xi}d\bar{\eta} \quad (10)$$

$$K(\beta_m,\xi,\eta) = K_\xi(\beta_m)K_\eta(\beta_m) \quad (11)$$

$$K_v(\beta_m) = \sqrt{2}\frac{\chi(v,\beta_m)}{\left[(\beta_m^2+B_i^2)\left(1+\frac{B_i}{\beta_m^2+B_i^2}\right)+B_i\right]^{1/2}}, \quad (12)$$

where $\chi(v,\beta_m) = \beta_m cos(\beta_m v) + B_i sen(\beta_m v)$ for $v=\xi$ or $v=\eta$.

Fourier theory of approximation

The uniform convergence of the infinite series in equation (8) is ensured by requirements that heat-generation variable Φ possesses continuous first and second order partial derivatives in the space variables, and possesses continuous first order partial derivatives with respect to time F_0, see [10, p. 310]. By the Cauchy criterion for series convergence one can approximate equation (8) by

$$\Theta(\xi,\eta,F_0) = \frac{1}{2\beta_1^2}K(\beta_1,\xi,\eta)\bar{\Phi}(\lambda_1,F_0) + e^{-2\beta_1^2 F_0}K(\beta_1,\xi,\eta)\left\{\bar{F}(\beta_1) - \frac{1}{2\beta_1^2}\left[\bar{\Phi}(\beta_1,0)\right] - \frac{1}{2\beta_1^2}\int_0^{F_0} e^{-2\beta_1^2\bar{F}_0}\left[\frac{\partial}{\partial\bar{F}_0}\bar{\Phi}(\beta_1,\bar{F}_0)\right]d\bar{F}_0\right\} \quad (13)$$

Substituting the equations (9) , (10) and (12) for $m = 1$ in the solution (13) and the heat generation with parabolic distribution, ie, $\Phi = q(aF_0^2 + bF_0 + c)$, we obtained the following equation:

$$
\Theta_{appr}(\xi,\eta,F_0) = \frac{1}{\left(\beta_1^2+Bi^2\right)\left(1+\frac{Bi}{\beta_1^2+Bi^2}\right)+Bi}\times
\left\{ \frac{2\Phi(\chi(\xi,\beta_1))(\chi(\eta,\beta_1))(-Bi-\sin(\beta_1)\beta_1+Bi\cos(\beta_1))^2}{\beta_1^4\left[\left(\beta^2+Bi^2+2Bi\right)\right]} + 2e^{-2\beta^2 F_0}(\chi(\xi,\beta_m))(\chi(\eta,\beta_1))\frac{(-Bi-\sin(\beta_1)\beta_1+Bi\cos(\beta_1))^2}{\left(\beta_1^2+Bi^2+2Bi\right)} \times\left(2-\frac{q}{\beta_1^2}+\frac{q(aF_0+b)(-1+e^{-2\beta_1^2 F_0})}{2\beta_1^4}\right)\right\} \tag{14}
$$

The approach given by the equations (14) is reasoned in the approximations theory of orthonormal functions in Fourier series in the sense of least squares, see [15]. From this theory one presents the following approximation error

$$
\begin{aligned}
E_{sq} &= \frac{\int_{area}[\Theta(\xi,\eta,F_0)-\Theta_{appr}(\xi,\eta,F_0)]^2 dA}{area} \\
&= \int_0^1\int_0^1[\Theta(\xi,\eta,F_0)-\Theta_{appr}(\xi,\eta,F_0)]^2 d\xi d\eta
\end{aligned} \tag{15}
$$

The error given by equation (15) will be used to evaluate the heat transfer coefficient error.

Heat Transfer Coefficient

The equation (7) indicates that simultaneous effects of k and h may be investigated in terms of a single dimensionless number, the Biot number.

The convective heat transfer though boundaries, h in equation (7), is important in the formulation and solution of conduction problems. The range of values of heat transfer coefficients h occurs under various conditions. It should be remembered that h, similar to but more strongly than k, depends on certain variables. These may include the space, time, geometry, flow conditions, and physical properties. The space wise averaged, steady values of commonly encountered heat transfer coefficients are given in Table 1.

Table 1: Range of heat transfer coefficients in several conditions

Conditions	Fluid	$h(W\cdot m^{-2}\cdot K^{-1})$
Free convection	Gases	5 - 30
	Water	100 - 900
Forced Convection	Gases	10 - 300
	Water	300 - 11.500
	Viscous oils	60 - 1300
	Liquid metals	5.700 - 114.000
Phase change	Boiling liquids	3000 - 57 .000
	Condensing vapors	5.700 - 114.000

Source:Adapted by [1]

The wide variation in the values of heat transfer coefficients suggests further investigations of the boundary condition under study for limiting values of h.

Method to calculate the Heat Transfer Coefficient

Taking the equation (14) as the approximate solution and knowing the temperature at some points for different times on the solid surface, one can find the values of β_1 and therefore the heat transfer coefficient h, by the Biot number Bi. Suppose that the temperatures at the point (ξ_0,η_0) for times F_0^0 and F_0^1 is known. Let $A_{0,1}=\frac{\Theta(\xi_0,\eta_0,F_0^0)}{\Theta(\xi_0,\eta_0,F_0^1)}$ and $\beta_1=\beta$ then

$$
A_{0,1} = \frac{\Phi(F_0^0)+e^{-2\beta^2F_0^0}\left\{2\beta^2-q\left[1-\frac{\left(aF_0^0+\frac{b}{2}\right)}{\beta^2}\left(e^{-2\beta^2F_0^0}-1\right)\right]\right\}}{\Phi(F_0^1)+e^{-2\beta^2F_0^1}\left\{2\beta^2-q\left[1-\frac{\left(aF_0^1+\frac{b}{2}\right)}{\beta^2}\left(e^{-2\beta^2F_0^1}-1\right)\right]\right\}} \tag{16}
$$

The equation (16) depends only on β_1, since Φ , F_0^0, F_0^1 and $A_{0,1}$ are input of problem.

Then to find the Biot number Bi one replaces the numerical value of β_1 calculated by equation (16) in the following equation

$$
\tan\beta = \frac{2Bi\beta}{\beta^2-Bi^2} \tag{17}
$$

It can be observed that equation (17) is a transcendental equation, therefore one can take only positive roots, see [11]. Moreover, in order to obtain the solution in real values, it is necessaries to apply an asymptotic analysis to choose the relation between F_0^0 and F_0^1.

The asymptotic analysis provide the relation $F_0^0 < F_0^1$ required to solve the equation in real values.

If the heat generation is the uniform distribution, ie, $\Phi = q$, then the equation (16) become

$$
A_{0,1} = \frac{q+e^{-2F_0^0\beta^2}\left(2\beta^2-q\right)}{q+e^{-2F_0^1\beta^2}\left(2\beta^2-q\right)} \tag{18}
$$

In particular case when $\Phi = 0$ this method according with calculus of Bi for homogeneous PDE, see [7] or [14].

$$
A_{0,1} = \frac{e^{-2F_0^0\beta^2}}{e^{-2F_0^1\beta^2}} \tag{19}
$$

Heat source modeling

The heat generation rate g can be modeled by assuming two different distributions: uniform and parabolic and various shapes. Komanduri and Hou [8] formulated a general equation for various plane heat sources (elliptical, circular, rectangular, and square) of different heat intensity distributions. The general equation for q_0 can be expressed as:

$$q_0 = \frac{q}{E \cdot A_{pl}} F \cdot G \tag{20}$$

The parameter E and F and G for rectangular (and square) heat intensity distributions are given in Table 2.

Table 2: Coefficients E, F, G and area A_{pl} for the for the general solution for uniform and parabolic heat intensity distribution

Distribution of intensity	E	F	G	A_{pl}
Uniform	1	1	1	$4a_0b_0$
Parabolic	$\frac{4}{9}$	$(1-(\frac{x_i}{a_0}))$	$(1-(\frac{y_i}{b_0}))$	or $(4a_0^2)$

Source: Adapted of [8]

Here, q_0 is the heat flow absorbed by the workpiece defined in equation (20), t is time and v_f is the source velocity. The heat generation is defined as

$$g(x,y,t) = \begin{cases} q_0 & 0 < x_i < v_f \cdot t \\ & -b_0 < y_i < b_0 \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

Calculus of the Heat Transfer Coefficient for a machining process

In order to investigate the applicability, the method will be applied to a machining process. The experiment selects the milling of the AISI H13 steel in orthogonal cutting being cooled by a fluid at $285K$. Table 4 shows the experimental thermomechanical data necessary for the Heat Transfer Coefficient calculation, extracted from [2]. Table 3 shows the thermal properties of steel AISI H13.

When the tool is cutting the contact area can be reasonably approximated by an area as shown in figure 1, being modeled as a moving heating source on the workpiece surface. An energy balance can be conducted on the workpiece area where the cutting energy is being generated. It is known that during cutting the energy is generated in the chip formation zone, CFZ. For orthogonal cutting, this volume can be

Table 3: Thermal properties of steel AISI H13

Specific heat c_p $[J/KgK]$	Thermal conductivity k $[W/mK]$	Density ρ $[Kg/m^3]$	Thermal diffusivity α $[m^2/s]$
475	38	7850	$1.18 \cdot 10^{-5}$

Source: Adapted from [3]

Table 4: Necessary parameters for reaching the Heat Transfer Coefficient

Rate feed $[m/s]$	Characteristic dimension $[mm]$	Inicial temperature $[K]$	Fluid temperature $[K]$
$v_f = 3.33$	$L = 50$	$T_0 = 297$	$T_\infty = 285$
shear angle	Heat intensity $[J]$	feed per tooth $[mm]$	depth cut $[mm]$
$\phi = 25°$	$q = 2.5$	$f = 0.1$	$a_p = 0.2$

associated to the shear plan introduced by Merchant's theory, [9]. For all practical purposes, the CFZ can be approximated with parallelepiped with sizes a, b, c, as shown figure 1. The length a was set equal to $\frac{b}{tan\phi}$, [13]. Analyzing the figure 1, one can observe that one of the faces of the CFZ is in contact with the workpiece surface. That can be modeled as the heat generation, the CFZ contact with the workpiece, by a heat source of rectangular shape dimension $c \times a$, see figure 1.

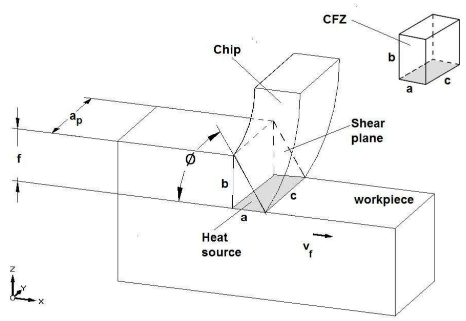

Figure 1: Heat source geometry

The heat generation propagation inside of workpiece can be model with uniform or parabolic distribution. Table 5 gives the heat generation, equation (6), where the function g is calculated by equation (20), considered $a_0 = a/2$ and $b_0 = c/2$.

The input data for Biot number calculus are $k = 38$, $F_0^0 = 0.41$ (for $t = 100s$), $F_0^1 = 1.22$ (for $t = 300s$), Φ

Table 5: Equation of heat generation propagation inside of workpiece

Distribution	$\Phi(\xi, \eta, F_0)$
Uniform	$\frac{q}{3.4\times10^{-5}}$
Parabolic	$\frac{q}{4.3\times10^{-8}}(-2.5\times10^{-3}F_0^2+1.5\times10^{-3}F_0)$

given by table 5 and

$$A_{0,1} = \frac{\Theta(0.5, 0.25, F_0^0)}{\Theta(0.5, 0.25, F_0^1)} = \frac{3/4}{25/12} \quad (22)$$

where $T(0.5, 0.25, F_0^0) = 294K$ and $T(0.5, 0.25, F_0^1) = 310K$.

The function Φ has continuous first and second order partial derivatives in the space variables, and has also continuous first order partial derivatives with respect to time F_0 for both distribution. Therefore one can use the approximation equation (14) to calculate the temperature Θ and find the Heat Transfer Coefficient of the fluid being tested. For this, firstly it is needed to find the values of β_1. The data obtained for a uniform distribution are in table 6. The table 7 shows the data calculated for a parabolic distribution.

Table 6: Heat Transfer Coefficient for uniform distribution: Data obtained versus expected data

q [J]	β_1 (Eq. (18))	h (Eq. (17))	h Table 1	Erro $\times10^{-8}$ (Eq. (15))
2.5	0.277	29.33	10 - 300	1.6
5	0.277	29.32	10 - 300	6.6
10	0.277	29.32	10 - 300	26
20	0.277	29.32	10 - 300	100

Table 7: Heat Transfer Coefficient for parabolic distribution: Data obtained versus expected data

q [J]	β_1 (Eq. (16))	h (Eq. (17))	h Table 1	Erro $\times10^{-10}$ (Eq. (15))
2.5	2.93	234.9	10 - 300	7.4
5	2.93	234.9	10 - 300	30
10	2.93	234.89	10 - 300	120
20	2.93	234.89	10 - 300	480

Conclusion

This work has examined the problem of transient heat conduction driven by convective cooling with a heat generation moving source. Using as a case-study the exact temperature solutions for two-dimensional model in a rectangular geometries, it can be concluded that:

1. The solution of the problem in a sense that Integral Transform given by equation (8) can be approximated by equation (13) since the heat-generation variable Φ has continuous first and second order partial derivatives in the space variables, and has continuous first order partial derivatives with respect to time F_0 and satisfy the hypothesis all boundary conditions is the same (unless the signal of) Bi.
2. The use of inverse analysis techniques permits the estimation of the heat transfer coefficient, from the knowledge of temperature and the thermomechanical of process.
3. The Heat Transfer Coefficient method calculate from equations (16) and (17) is a simple math tool to calculated the ratio of the internal and external conductances of a heat conduction, subject to convective boundary with a heat generation source. In addition, this method agrees with the particular case studies in literature, the steady heat conduction.
4. The simple analyses of this method admit the generalization for another simple geometry (cylinder and sphere). The generalization can be obtained replacing the model given by equation (1) by the respective model in appropriate coordinates (cylindrical and spherical) found in the following literature [4], [12] and [11]
5. The Heat Transfer Coefficient method formulated here applied in an experimental calculus are in good agreement with those predicted by empirical correlations cited in table 6 and 7.

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