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MIXED CONVECTION FLOW OF NON-NEWTONIAN CARREAU FLUID: EFFECT OF VISCOUS DISSIPATION

Amoura M.[1,*], Zeraibi N.[2], Benzaoui A.[1]
*Author for correspondence
[1]Université des Sciences et de la technologie Houari Boumediene, Faculté de Physique, Dépt. Energétique. B.P. 32 El-Alia, 16111 Bab-Ezzouar, Algiers, Algeria
[2]Université de Boumerdes, Faculté des hydrocarbures, Dépt. Transport et Equipements, Avenue de l'indépendance, 35000 Boumerdes, Algeria
Email: mlouniamoura@yahoo.fr / am_louni@yahoo.fr

ABSTRACT

In this paper, we present a numerical study of the flow characteristics and heat transfer mechanism of a non-Newtonian fluid in an annular space between two coaxial rotating cylinders taking into account the effect of viscous dissipation. The Carreau stress-strain relation was adopted to model the rheological fluid behavior. The problem is studied when the heated inner cylinder rotates around the common axis with constant and the cooled outer cylinder is at the rest. The horizontal endplates are assumed adiabatic.

A house code which is based on a Galerkin mixed finite element is developed to obtain numerical solutions of the complete governing equations and associated boundary conditions and is validated with the results reported in the literature.

It is found that five parameters can describe the problem under consideration, the Reynolds number (*Re*), the Grashof number (*Gr*) , the index of structure (*n*), Weissenberg number (*We*) and the Eckert number (*Ec*). The velocity, temperature and stream function distributions and the local Nusselt number variations are drawn for different dimensionless groups.

INTRODUCTION

The laminar flow and the heat transfer of a non-Newtonian fluid between rotating concentric annulus are encountered in a large number of industrial processes as the catalytic chemical reactors [1], the filtration devices [2], the blood plasmaphosresis devices [3], the plant cell bioreactors [4] and the liquid-liquid extractors [5].

The convective heat transfer mechanisms of the non-Newtonian fluids are the subject of considerable works and are well understood today. The mixed convection between two concentric horizontal cylinders is reported in references [6-8]. A survey of laminar flow of non-Newtonian fluids in a rotating concentric annulus has been reported by Batra and Eissa [9].

NOMENCLATURE

n	[---]	Flow index or index structure
r_i	[m]	Inner cylinder radius
r_e	[m]	Outer cylinder radius
u, v, w	[m/s]	Axial, radial and circumferentiel velocity components
We	[---]	Weissenberg number
Re	[---]	Reynolds number
Gr	[---]	Grashof number
Ec	[---]	Eckert number
Z	[m]	Axial coodinate

Special characters

$\dot{\gamma}$	[1/s]	Magnitude of the deformation rate tensor
μ	[Pa.s]	Apparent dynamic viscosity
μ_0	[Pa.s]	Zero-shear-rate dynamic viscosity
μ_∞	[Pa.s]	Infinite-shear-rate dynamic viscosity
λ	[s]	Time relaxation parameter
ρ	[Kg/m^3]	Density
Φ		Dissipation of energy

Subscripts

h	hot
c	cold
i	inner
e	extrenal

Flow of a Casson and Robertson-Stiff fluids between two rotating cylinder has been investigated by Batra and Eissa [10-11]. Kouitat *et al.* [12] investigated theoretically and numerically the laminar Couette at the start-up stage of the fluid motion within a coaxial cylinder viscosimeter. For a Carreau model, Khellaf and Lauriat [13] studied numerically the heat transfer between two rotating concentric vertical cylinders. A great deal of theoretical and numerical works dealing with flow and associated heat transfer characteristics of natural and mixed convection in annuli between two isothermal concentric cylinders are reported in the cited literature [13-18].

In this study, we present a numerical study of the flow characteristics and heat transfer mechanism of a non-Newtonian fluid in an annular space between two coaxial rotating cylinders taking into account the effect of viscous dissipation. The Carreau stress-strain relation was adopted to model the rheological fluid behavior. In comparaison with the Newtonian case, this model invoves four additional parameters, namely the zero-and infinite-shear rate viscosities ($\mu_0$and μ_∞, respectively), the relaxation time of the fluid, λ, which describes the transition to a constant viscosity in the limit of zero shear rate, and the index of structure, n, which is a measure of the degree of a non-Newtonian behavior. We consider here two cases, the case where the inner cylinder is rotated and the outer cylinder is at rest. The effects of viscous dissipation on heat transfer are examined. A computational code applied to the fluid mechanics and the heat transfer by using the finite elements method is developed. This code is validated by comparison with results reported in the literature. This computational code takes into account the non-Newtonian effects.

PROBLEM FORMULATION

The geometry under investigation is shown in Figure 1. We consider two coaxial cylinders with a finite length H. The inner cylinder, of radius r_i, is maintained at a hot uniform temperature T_h. The outer cylinder of radius r_e is at a cold temperature T_c.

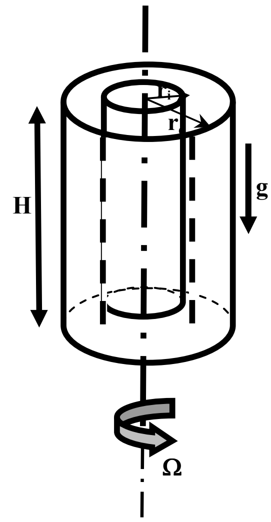


Figure 1 Geometry of the problem

The flow is assumed to be laminar, incompressible and axisymmetric. Non- Newtonian effects are considered for fluids obeying the Carreau constitutive relationship:

$$\eta = \frac{\mu - \mu_\infty}{\mu_0 - \mu_\infty} = \left(1 + \lambda^2 \dot{\gamma}^2\right)^{\frac{n-1}{2}} \tag{1}$$

where μ_0 is the viscosity at low shear rate, μ_∞ is the viscosity at high shear rate, λ is the time constant, n is the power law index, and $\dot{\gamma}$ is the shear rate. It is given by :

$$\dot{\gamma} = 2\left[\left(\frac{\partial v}{\partial r}\right)^2 + \left(\frac{v}{r}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right] + \left[r\frac{\partial}{\partial r}\left(\frac{w}{r}\right)\right]^2 + \left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial r} + \frac{\partial u}{\partial r}\right)^2 \tag{2}$$

In the following, it is assumed that the parameters of this constitutive equation do not vary with temperature. For many concentrated polymer solutions and melts, it can be assumed that $\mu_\infty << \mu_0$ [19]. So, μ_∞ is neglected here. η represents the dimensionless apparent viscosity. The fluid is Newtonian for $n = 1$, and the shear thinning behaviour becomes more significant where n becoming smaller.

The dimensionless form of the governing equations can be obtained by use of dimensionless variables defined as:

$$Z = \frac{z}{r_e - r_i},\; R = \frac{r}{r_e - r_i},\; U = \frac{u}{\Omega r_i},\; V = \frac{v}{\Omega r_i},\; W = \frac{w}{\Omega r_i},\; \theta = \frac{T - T_c}{T_h - T_c} \tag{3}$$

Variables *u, v* and w are the velocity components in the *z, r* direction and azimutal velocity. *T* is the temperature. The dimensionless deformation rate is the ratio $\frac{\Omega r_i}{r_e - r_i}$.

In dimensionless form, the relationship (1) is written as follow:

$$\eta = \left(1 + W_e^2 \dot{\gamma}\right)^{\frac{n-1}{2}} \tag{4}$$

where the flow index n and the Weissenberg number W_e, describe the rheological property of the fluid.

On the basis of the dimensionless variables defined in Eq.(3), the non-dimensional form of the conservation of mass, momentum and energy equations are:

$$\frac{\partial V}{\partial R}+\frac{V}{R}+\frac{\partial U}{\partial Z}=0 \tag{5}$$

$$\left(V\frac{\partial V}{\partial R}+U\frac{\partial V}{\partial Z}-\frac{W^2}{R}\right)=-\frac{\partial P}{\partial R}+\frac{1}{\mathrm{Re}}\left\{\frac{1}{R}\left[\frac{\partial\left(2R\eta\frac{\partial V}{\partial R}\right)}{\partial R}\right]-\frac{\eta V}{R^2}+\frac{\partial}{\partial Z}\left[\eta\left(\frac{\partial U}{\partial R}+\frac{\partial V}{\partial Z}\right)\right]\right\} \tag{6}$$

$$\left(V\frac{\partial U}{\partial R}+U\frac{\partial U}{\partial Z}\right)=-\frac{\partial P}{\partial Z}+\frac{1}{\mathrm{Re}}\left\{\frac{1}{R}\frac{\partial}{\partial R}\left[R\eta\left(\frac{\partial U}{\partial R}+\frac{\partial V}{\partial Z}\right)\right]+\frac{\partial}{\partial Z}\left(2\eta\frac{\partial U}{\partial Z}\right)\right\}+\frac{\mathrm{Gr}}{\mathrm{Re}^2}\theta \tag{7}$$

$$\left(V\frac{\partial W}{\partial R}+U\frac{\partial W}{\partial Z}+\frac{WV}{R}\right)=\frac{1}{\mathrm{Re}}\left\{\frac{1}{R}\frac{\partial}{\partial R}\left(R\eta\frac{\partial W}{\partial R}\right)+\frac{\partial}{\partial Z}\left(\eta\frac{\partial W}{\partial Z}\right)-\eta\frac{W}{R^2}\right\} \tag{8}$$

$$\left(V\frac{\partial\theta}{\partial R}+U\frac{\partial\theta}{\partial Z}\right)=\frac{1}{\mathrm{Re\,Pr}}\left\{\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)+\frac{\partial^2\theta}{\partial Z^2}\right\}+\frac{Ec}{\mathrm{Re}}\eta\,\Phi \tag{9}$$

The dimensionless boundary conditions in this case of this geometry are:

W = 1, U = V = 0, θ_1 at the inner cylinder

W = 0, U = V = 0, θ_2 at the outer cylinder

W = U = V = 0 and $\frac{\partial\theta}{\partial Z}=0$ at the horizontal endwalls.

The problem is characterized by the following parameters of similarity; Prandtl number $\mathrm{Pr}=\frac{\mu_0 c_p}{\lambda}$, Reynolds number $\mathrm{Re}=\frac{\Omega r_i\left(r_e-r_i\right)\rho}{\mu_0}$, Eckert number $Ec=\frac{(\Omega r_i)^2}{Cp\left(T_h-T_c\right)}$, Weisseneberg number $We=\frac{\Omega r_i}{r_e-r_i}\lambda$, Grashof number $Gr=\frac{g\,\beta\left(T_h-T_c\right)\left(r_e-r_i\right)^3}{\mu_0{}^2}$, (or Rayleigh number $Ra=Gr\,\mathrm{Pr}$)

NUMERICAL RESOLUTION

When simulating an incompressible fluid flow, we demand that divergence- free discrete velocities be attainable. Attempts to ensure that the fluid flow is everywhere and always incompressible have dominated the subject of computational fluid dynamics. We analyse equations (5-7) by using a mixed formulation rather than by using a segregated approach so that the mass and momentum conservations can be simultaneously coupled. The pressure in the incompressible Navier-Stokes equations serves as a Lagrangian multiplier. As a result, accurate predicted discrete solenoidal velocities may accompany a non-smooth pressure. Legitimate choice of finite element trial spaces for primitive variables is thus of importance because the mixed finite element method is subject to the LBB (Ladyzhenskaya- Babuska - Brezzi) stability condition. To retain a sufficiently smooth solution for the investigated elliptic system (5-7), we take an element free of the LBB stability constraint into consideration. By substituting the well-paired bilinear interpolation function for the pressure and the biquadratic interpolation function for the velocities into the weighted residual statement of (5-7), we can derive the following matrix equations along with bilinear test function for the mass conservation equation:

$$[C]\{U_n\}+[K]\{U_n\}=\{F\} \tag{10}$$

where

$$\{U_n\}=\begin{Bmatrix}U_j\\V_j\\P_j\end{Bmatrix},\quad \{F\}=\begin{Bmatrix}F_u\\F_v\\0\end{Bmatrix} \tag{11a, b}$$

and

$$F_u=-\iint N_i\left(\frac{W^2}{R}\right)RdRdZ\ ,\quad F_v=-\iint\frac{Gr}{\mathrm{Re}}N_i(\theta)RdRdZ \tag{12a,b}$$

The matrixes [C] and [K] are organized in the following way :

$$[C]=\begin{bmatrix}C(u)&0&0\\0&C(u)&0\\0&0&0\end{bmatrix},\quad [K]=\begin{bmatrix}\bar{K}_{11}&K_{12}&-Q_1\\K_{21}&\bar{K}_{22}&-Q_2\\-Q_1^T&-Q_2^T&0\end{bmatrix} \tag{13a, b}$$

where

$$C(u)=\iint N_i\left(U\frac{\partial N_j}{\partial Z}+V\frac{\partial N_j}{\partial R}\right)R\,dR\,dZ \tag{14}$$

$$\bar{K}_{11}=2K_{11}+K_{22}\ ,\quad \bar{K}_{22}=K_{11}+2K_{22} \tag{15a,b}$$

$$Q_1=\iint M_i\frac{\partial N_j}{\partial Z}RdRdZ\ ,\quad Q_2=\iint M_i\frac{\partial N_j}{\partial R}RdRdZ \tag{16a,b}$$

and

$$K_{11}=\iint\frac{1}{\mathrm{Re}}\left(\frac{\partial N_i}{\partial Z}\frac{\partial N_j}{\partial Z}\right)R\,dR\,dZ \tag{17}$$

$$K_{11}=\iint\frac{1}{\mathrm{Re}}\left(\frac{\partial N_i}{\partial R}\frac{\partial N_j}{\partial R}\right)R\,dR\,dZ \tag{18}$$

Again the finite-element technique and Galerkin's principle can be used for solving the rotating velocity and the energy equation. The following matrix equation (19 and 23) can be obtained by adopting the same approach as outlinned previously

$$N(W)\,W+K_wW=F \tag{19}$$

Where

W is the azimutal velocity

And

$$N(W)=\iint N_i\left(V\frac{\partial N_j}{\partial R}+U\frac{\partial N_j}{\partial Z}+\frac{VN_j}{R}\right)RdRdZ \tag{20}$$

$$K_W = \iint \frac{1}{\mathrm{Re}} \left(\frac{\partial N_i}{\partial R}\frac{\partial N_j}{\partial R} + \frac{N_i N_j}{R^2} + \frac{\partial N_i}{\partial Z}\frac{\partial N_j}{\partial Z} \right) R\, dR\, dZ \qquad (21)$$

$$F=0 \qquad (22)$$

$$N(\theta)\theta + K\theta = F \qquad (23)$$

where

$$N(\theta) = \iint N_i \left(V\frac{\partial N_j}{\partial R} + U\frac{\partial N_j}{\partial Z} \right) R\, dR\, dZ \qquad (24)$$

$$K = \frac{1}{\mathrm{Re}\,\mathrm{Pr}} \iint \left(\frac{\partial N_i}{\partial R}\frac{\partial N_j}{\partial R} + \frac{\partial N_i}{\partial Z}\frac{\partial N_j}{\partial Z} \right) R\, dR\, dZ \qquad (25)$$

and

N_i is the shape functions of the 9-node quadrilateral element and M_i is shape functions of the 4-node quadrilateral element

Calculation of the stream function and Nusselt number: The quantities of interest in the present problem are also the stream function and the Nusselt number. These can be calculated a posteriori once the solution for the velocity and temperature fields has been obtained.

The distribution of the stream function ψ can be obtained via the velocity field by solving separately the Poisson equation subject to the boundary condition $\psi = 0$ on all walls:

$$\begin{cases} U = \frac{1}{R}\frac{\partial \psi}{\partial R} \\ V = -\frac{1}{R}\frac{\partial \psi}{\partial Z} \end{cases} \quad ; \quad \frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R}\frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial Z^2} = -R\left(\frac{\partial V}{\partial Z} - \frac{\partial U}{\partial R} \right) \qquad (26a,b)$$

The mean Nusselt number is defined as the ratio of heat flux crossed through the total cylindrical surface of radius r to the conduction heat flux:

$$Nu = \frac{r}{AR} \ln\frac{r_e}{r_i} \int_0^K \left(-\frac{\partial \theta}{\partial R} + U\theta \right) dR \qquad (27)$$

where $AR = \frac{H}{r_e - r_i}$ *and* $K = \frac{r_e}{r_i}$

Although a Newton-Raphson iterative scheme would be recommended for the solution of nonlinear system of equations, here we have used a direct substitution scheme (Picard method), sometimes with underrelaxation to accelerate convergence. This avoids calculating the Jacobian which can be time consuming, and it also enjoys a wider convergence range. The solution process starts from the Newtonian field (n=1), which is used to obtain first approximation.

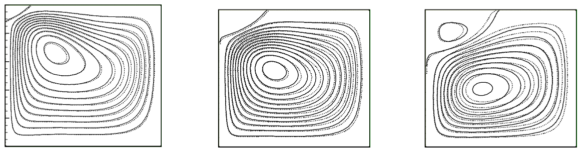
RESULTS AND DISCUSSION

Calculations were performed with two grids, one having 400 elements, 1681 nodes, and 3803 unknown degrees of freedom and another denser having 900 elements, 3721 nodes, and 8403 degrees of freedom. The results were virtually identical and thus mesh independent.

The convergence criterion used here is that the maximum relative change in dependant variables between successive Newton iterations is less than 10^{-5}. All the calculations have been conducted at We = 10 and for a fluid with Pr=5, corresponding to the dilute solutions of a polymer in water, the geometry of the annulus is fixed at AR=1 and K=2.

Furthermore, in order to validate the numerical code used for the present study, the steady-state solutions obtained as time-asymptotic solutions for an untilted square cavity with differentially heated sidewalls and adiabatic top and bottom walls, have been compared with the benchmark results by De Vahl Davis [20]. In particular, the average Nusselt numbers obtained at Rayleigh numbers in the range between 10^4 and 10^5, and the maximum horizontal and vertical velocity components on the vertical and horizontal midplanes of the enclosure, have been found to be within 1%-3% of the benchmark data.

The Figure 2 shows the streamlines for the mixed convection flows in two cases. Where we took into account the effect of viscous dissipation and where the energy dissipation is neglected. In such case of regime, the increase of the flow index leads to the appearance of a second cell flow occuping the high internal zone of the annular space. The intensity of this cell flow grows with the flow index.



n=0,6 n=0.8 n=1.0

Figure 2 Streamlines at *GR*=2000, *RE*=100 and *We*=10 (——Ec=0 - - - Ec= 0,5)

In this casev of regime, we show in Figures 3, 4 and 5 that the dissipation of temperature for n=1 is very affected by the dissipation of energy. The importance of this effect decreases with the flow index. We can explain this by the extent of the zone invaded by the flow generated by the conditions considered (show figure 2).

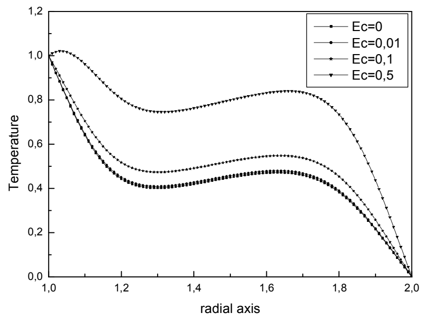


Figure 3 Variation of the temperature as a function of the radial coordinate and Ecker number for n=1

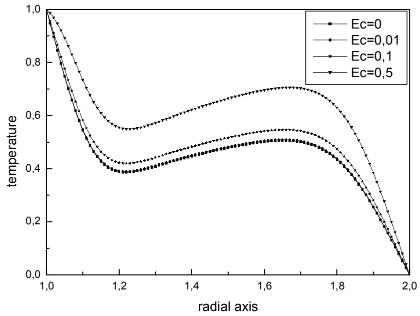


Figure 4 Variation of the temperature as a function of the radial coordinate and Ecker number for n=0.8

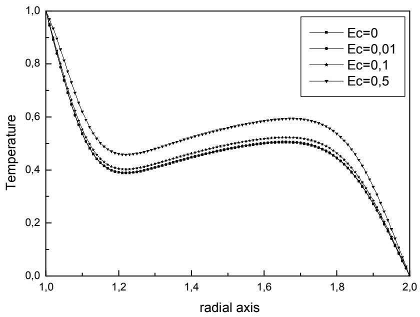


Figure 5 Variation of the temperature as a function of the radial coordinate and Ecker number for n=0.6

For Ec=0.5, we obseve an overheated of the zone near the wakll of the inner cylinder. What explains the evolution of the Nusselt number (Fig. 6).

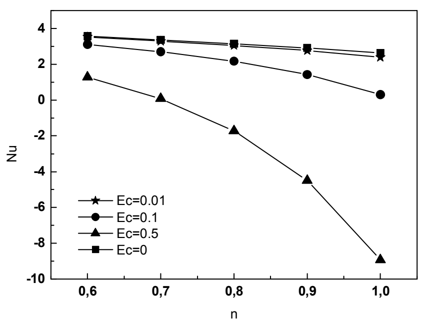


Figure 6 Variation Average Nusselt number as a function of the flow index and Ecker number (*Re*=100, *Ra*=10^4)

CONCLUSIONS

A numerical study of a fluid flow and a heat transfer is presented for non-Newtonian fluids confined in a differentially heated annular cylindrical space taking into account the effect of viscous dissipation. The shifted Carreau constitutive was adopted to model the rhological fluid caracteridtics. The flow regime is considered according to the speed of rotation of the inner cylinder: mixed convection. The results show that the non-Newtonian effects are important on the structure of the flow and on the heat transfer. The effects of dissipation of energy on heat transfer are examined. A parametric study of the effects of the flow index is presented to describe the flow behaviour.

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