

LAMINAR NATURAL CONVECTION HEAT TRANSFER FROM AN ISOTHERMAL VERTICAL RIBBED PLATE

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ABSTRACT

A finite-volume-based computational study of steady laminar natural convection from an isothermal vertical plate with roughness elements has been presented. Computational simulations of the flow and heat transfer have been performed in a two-dimensional array of rectangular elements. Heat transfer characteristics with roughness elements have been analysed by examining variations of the local and average Nusselt numbers in two-dimensional flow. It is found that significant heat transfer augmentation is obtained. A general correlation has been developed to predict the average Nusselt number as a function of different rib configurations. This general correlation will be very useful for industries who manufacture plate heat exchangers.

INTRODUCTION

Continuous efforts to improve the performance of heat exchange devices in all engineering applications have resulted in the selection of cooling technologies containing the thermal and flow characteristics of many investigated heat exchange devices. Among various modes of the cooling technologies for heat exchange devices, natural convective air cooling has been given much more attention because of its inherent high reliability. Heat exchange device performance is often limited by the air side because the heat transfer coefficients are inherently lower. The air side temperature distribution is intimately coupled to the velocity field, often taking the form of a thermal boundary layer. This temperature distribution is a manifestation of the air-side heat resistance, and it can be modified through roughness elements.

A large number of experimental works has been performed for the enhancement of air-side heat transfer; however, the flow profiles and the related heat transfer characteristics in the complex geometries are still needed to be verified and if possible new general correlations are to be brought forward so that the heat exchanger manufacturers can benefit from it.

Bhavnani and Bergles [1] conducted experiments on a vertical plate with repeated and stepped ribs. They found that the maximum increase of the local heat transfer coefficient is 23% when compared with equal projected area of a plain plate. They also found that the heat transfer performance of the ribbed surfaces with low thermal conductivity is slightly improved relative to high thermal conductive ribbed surfaces. The investigation was carried out through a Mach-Zehnder interferometer. Aydin [2] investigated the heat transfer in natural convection vertical ribbed plates by means of an infrared camera. The single rib and the two ribs configuration were analyzed for both high and low thermal conductive ribs. He found that the ribs have not got much influence on the local heat transfer coefficient, based on an equal area of the flat plate. The positions of minimum heat transfer and the reattachment point upstream of a rib were determined experimentally and were confirmed by the numerical predictions. Flow visualization as well as laser holographic interferometry and temperature measurement were presented for natural convection of air layers in vertical channels with asymmetrically discrete heated ribs in the paper of Lin and Hsieh [3]. They concluded that the existence of protruding 2D-ribs at the wall originates a flow separation/recirculation which is one of the major factors influencing the local temperature gradients.

Burak et al. [4] studied the case of a vertical heated plate in the presence of one or several rectangular steps by an interferometer. They concluded from flow visualization that a rotational flow between the ribs intensifies the process of heat transfer. On the contrary, considering an array of heated protrusions on a vertical surface in water, Joshi et al. [5] presented flow visualization with no evidence of vortex formation around the protrusions. Transient convective heat transfer and flow features in the vicinity of an array of transverse ribs along a vertical flat plate were experimentally studied with the aid of thermocouples in the paper of Polidori and Padet [6]. They concluded that during early transient an

important heat transfer enhancement occurs in the upstream part of the cavity between the ribs while increasing time reduces the heat transfer performance in the whole cavity due to complex eddy structures. An experimental study of natural convection heat transfer in smooth and ribbed uniform wall temperature and uniform heat flux vertical channels was carried out by Acharya and Mehrotra [7]. They indicated that a smooth surface yields a greater heat transfer rate when compared with ribbed geometries due to dead regions in the near- rib space. Tanda [8] performed an experimental study of natural convection inside vertical channels formed by heated ribbed surface and opposing unheated smooth surface. He found that the presence of ribs worsens heat transfer performance due to inactive regions just upstream and downstream of each protrusion. He also concluded that adding ribs for the purpose of heat transfer augmentation is useless for natural convection and that care must be taken in thermal design when large-scale roughness elements occur naturally. It was an experimental work carried out through schlieren optical technique.

Desrayaud and Fichera [9] studied numerically laminar natural convective flows in a vertical isothermal channel with two rectangular ribs. They specified in the case of channel that increasing the length of the rib has only a limited influence on the heat transfer while increasing its width decreases dramatically the heat transfer. Onbasioglu et al. [10] conducted experiments on a vertical adiabatic ribbed surface and also carried out numerical studies. They concluded that the ribs provide significant improvements in heat transfer. They also concluded that the heat transfer enhancement due to ribs depends on several geometric parameters and especially on the angle of inclination. A numerical investigation of the laminar natural convection on a periodically oscillating baffle plate was presented by Zhang et al. [11]. They found that three competing mechanisms are responsible for the heat transfer enhancement. The first mechanism of the enhancements is due to periodic redevelopment of the thermal boundary layer. The second is due to the thickness of the thermal boundary layer which becomes thinner under oscillating-wall conditions. In addition, a highly complex flow pattern including strong vortices produced by a simultaneous use of oscillating plate and baffles may contribute much to the heat transfer enhancements. They also reported that the enhancement ratio can be varied from 5.2 to 7.7, depending on the baffle spacing and baffle height.

More recently, an experimental study of natural convection heat transfer inside a vertical heated ribbed channel was investigated by Tanda [12]. He concluded that the enhancement of laminar, free convective heat transfer in vertical channels using transverse ribs with low-thermal conductivity occurs only locally, at a certain distance downstream of the ribs.

From the thorough literature survey summarised above, it reveals that the previous studies were concentrated mainly on experimental works. However, depending upon the thermo-geometric parameters and orientation of the plate and fin arrays may be vertical up ward, horizontal and vertical down ward. No attention has been given in the literature to determine thermal performance of heated vertical plate with non-conductive fins. In addition, researchers have rarely concentrated on optimization study of non-conductive fins under different

orientations. Thus, the analysis and optimum dimensions of a non-conductive fin for different orientations is essential to carry out the exact analysis of plate heat exchanger under natural convection. Therefore, a combined analysis for performance and optimization of non-convective pin fin is the prime motivation of the present the present study.

NOMENCLATURE

Gr	[-]	Grashof number based on L,
	$g\beta(T_w - T_\infty)L^3/\nu^2$	
H	[mm]	Height of the fin
L	[m]	Length of the vertical flat plate
N	[-]	Number of fins
Nu_y	[-]	Local Nusselt number
\overline{Nu}	[-]	Average Nusselt number
P	[m]	Pitch, center-to-center distance of two consecutive fins
Pr	[-]	Prandtl number, ν/α
Ra	[-]	Rayleigh number, Gr .Pr
T	[K]	Absolute temperature
H/t	[-]	Aspect ratio of the fin
P/L	[-]	Fin spacing
c_p	[J/kgK]	fluid specific heat
h	[W/m ² K]	Local heat transfer coefficient
\overline{h}	[W/m ² K]	Average heat transfer coefficient
k	[W/mK]	Thermal conductivity of fluid
q	[W]	Heat transfer rate
t	[mm]	Thickness of the fin
\vec{v}	[m/s]	Velocity vector
Y_1	[mm]	Location of fin in span-wise direction

Greek

α	[m ² /s]	Thermal diffusivity
β	[K ⁻¹]	Thermal coefficient of expansion
θ	[degree]	Angle of inclination
μ	[Pa.s]	Dynamic viscosity of fluid
ν	[m ² /s]	Kinematic viscosity of fluid
ρ	[kg/m ³]	Density

PHYSICAL DESCRIPTION OF THE PROBLEM

The schematic diagram of a heated vertical flat plate with non-conductive fins is shown in Fig.1. The co-ordinate system in which the plate is fixed and the computation would be carried out is also shown in the Fig 1. The plate is fixed with some non-conductive fins which have a height of H, thickness of t and fin separation pitch between two consecutive fins is P. The plate length is L and it has a constant temperature of T_w . Normally such type plate heat exchangers are used in paint shops of car manufacturing units and they are limited to a height of 0.65 m and at best a temperature difference of 25 °C exists between the plate and the surroundings where it is expected that the flow around the plate would be laminar ($Ra < 10^5$) and the heat transfer would be governed by laminar natural

convection. Even if the surface has ribs and there are vortices behind the rib the average flow velocity is very small in natural convection, particularly for the case considered. So the vortices will not produce turbulence because the viscous effects will suppress the turbulence production keeping the flow essentially to laminar. If a heat transfer augmentation factor for different rib configuration is known then ribs occasionally be used on the plates or even permanently without adding any extra cost to the device.

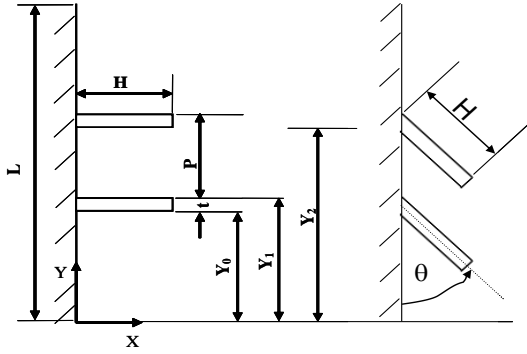


Fig.1 Schematic diagram of the heated plate with straight and inclined fins

MATHEMATICAL FORMULATIONS

In order to solve the flow and the temperature field around the finned plate we would use the Navier-Stokes equation with buoyancy driven terms in the y-momentum equation along with the energy equation which will solve for the temperature field in the domain . If the fin is non-conductive ($k = 0$) we will simply take out the non-conductive portion of the fin from the computational domain and attach a boundary condition on the surface of the non-conductive fin.

The assumptions in the mathematical formulation and the solution process are the following based on which the governing equations are written.

1. The flow field is steady, laminar and incompressible with the fluid stresses being Newtonian.

2. The properties of the fluid (air, its μ , k , c_p and ρ) are kept constant at free stream conditions in all the equations except for the density of air, ρ , in the buoyancy term of the momentum equation only where it is assumed to be a function of temperature so that the buoyancy force can be induced in the fluid. The Boussinesq approximation for the buoyancy is not assumed in the present study; rather density of air is taken to be a straight function of temperature through the ideal gas law and is provided as a table to the computing software. The temperature difference between the plate and the surroundings fluid is of the order of 25 °C, so Boussinesq approximation is not used. The computation of density from the ideal gas law is done by assuming the pressure to be atmospheric only since the equipments work in the atmospheric conditions.

3. Radiation from the plate and from the fin is also neglected because the plate temperature is hardly 25 °C more than the ambient temperature.

With the above assumptions the governing equations for the flow field and heat transfer around the vertical plate with fins would be the following in tensorial notations as:

Continuity Equation

$$\frac{\partial}{\partial x_i} (u_i) = 0 \quad (1)$$

Momentum Equation

$$\frac{D}{Dt}(\rho u_i) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + (\rho - \rho_\infty) g_i \quad (2)$$

p in Eqn. 2 is a modified pressure defined as $p = p_s + \rho_\infty g z$, where p_s is the static pressure in the fluid domain. p in the domain will vary since ρ is a function of temperature in the buoyancy term (else where it is constant).

Energy equation in fluid

$$\frac{D}{Dt}(\rho c_p T) = \frac{\partial}{\partial x_i} \left(k \frac{\partial T}{\partial x_i} \right) \quad (3)$$

4 Boundary Conditions

The boundary conditions for the solutions of Equations (1), (2) and (3) are shown pictorially in Fig. 2.

At the wall (Heated plate)

$$\text{All velocity components are zero and, } T = T_w \quad (4)$$

At the inlet

$$U = 0, \quad V = 0, \quad \text{and } T = T_\infty \quad (5)$$

At the pressure outlet boundary

Top pressure outlet

$P=0$, V determined from local pressure field and U found out to satisfy continuity equation,

$$\frac{\partial T}{\partial y} = 0, \quad (6)$$

On the pressure outlet boundary the flow can come in to the domain or can go out of the domain depending on the local pressure field. The velocity perpendicular to the boundary can be computed from pressure conditions and a realistic flow field can be established through this boundary condition. Other boundary conditions can not be applied to a boundary where the flow either goes out of the domain or comes in to the domain .

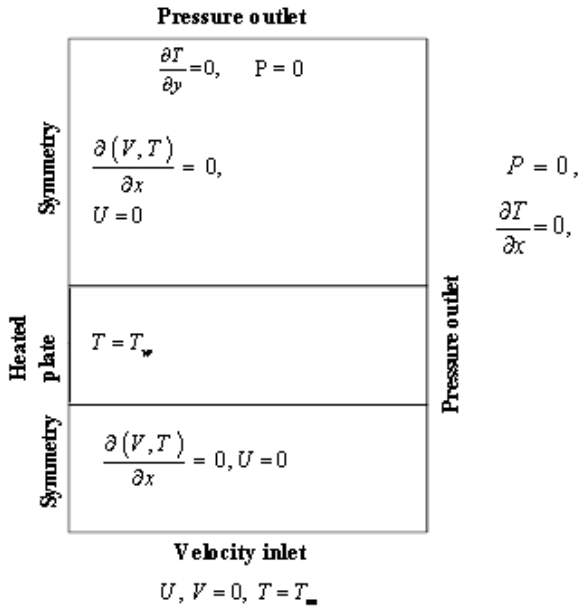


Fig. 2 Boundary conditions

Side pressure outlet

$P = 0$, U determined from local pressure field and V found out to satisfy continuity equation (Dash, 1996), either $T = T_\infty$

$$\text{or } \frac{\partial T}{\partial x} = 0 \text{ could be used.} \quad (7)$$

At the symmetry boundary

$$\frac{\partial(V, T)}{\partial x} = 0 \text{ and } U = 0 \quad (8)$$

NUMERICAL SOLUTION PROCEDURE

Semi-Implicit Method for the Pressure Linked Equations (SIMPLE) algorithm with a PREssure STaggering Option (PRESTO) scheme of Fluent 6.3 for the pressure interpolation (to find cell face pressure from cell center pressure) was used for the pressure correction equation. Under relaxation factors of 0.3 for pressure, 0.7 for momentum and 1 for energy were used for the convergence of all the variables. Rectangular cells were used for the entire computational domain for rectangular fins. The cells were also mostly rectangular for the inclined fins but they were paved at the plate because it was the only choice in such a geometry. There were few triangular cells near the tip of the inclined fins, but they did not pose any convergence problem. Convergence of the discretized equations were said to have been achieved when the whole field residual for all the variables fell below 10^{-3} for u , v , and p (since these are non-linear equations) whereas for energy the residual level was kept at 10^{-6} (energy being a linear equation).

Results and Discussion

Flow Field near the Plate with Different Ribs

Fig.3 shows the streamline patterns near the heated vertical flat plate with different ribs. Fig. 3 (a) shows a 6 mm high rib where the ambient fluid diverts a bit from its straight path and then again attaches to the vertical plate. Due to the reattachment the local Nusselt number on the plate rises a bit. When the rib height is 24 mm ($H/t = 8$) the stream of fluid diverts much away from the plate and reattaches to the plate at a higher length compared to the case of the rib having a height of only 6 mm. Due to a large deflection of the stream in to the ambient, the stream gets cold air from the ambient which reattaches to the plate thus increasing the local Nusselt number on the plate in the un-ribbed portion. Fig.3 (c) shows the streamline pattern of the fluid near the plate having ribs of 24 mm height but inclined at an angle of 45° . Here, the stream does not get deflected much in to the ambient but the reattachment length of the diverted stream is longer compared to the case of a straight rib shown in Fig.3 (b). Due to a relatively longer reattachment length (the wall shear stress on the heated plate increases from negative value to positive just after the rib. The stress increases to a maximum and then falls to zero again before the next rib and then again becomes negative. The length of the plate from zero stress to another zero stress is normally called the reattachment length [13] between two successive ribs.

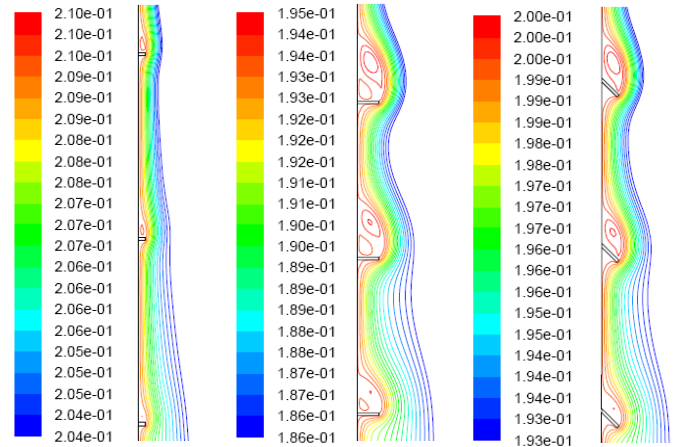


Fig. 3 Streamlines near the isothermal plate with different rib configurations (a) $H/t = 2$, (b) $H/t = 8$; $\theta = 90^\circ$, (c) $H/t = 8$; $\theta = 45^\circ$

The corresponding velocity field for $H/t = 8$ with different angle of inclination of ribs can be seen in Fig.4 which shows the secondary flow structure on the back side of the ribs, responsible for changing wall shear stress on the plate.) the local Nusselt number remains high for a longer length on the plate between two successive ribs. So it is probable that an inclined rib can produce higher average Nusselt number on a plate compared to a straight rib.

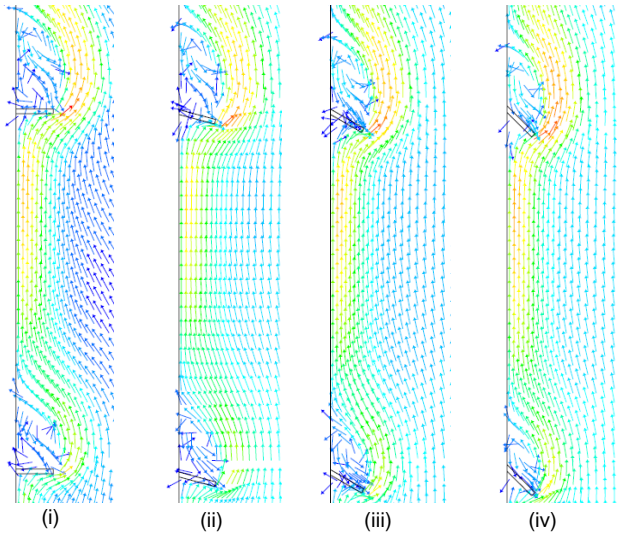
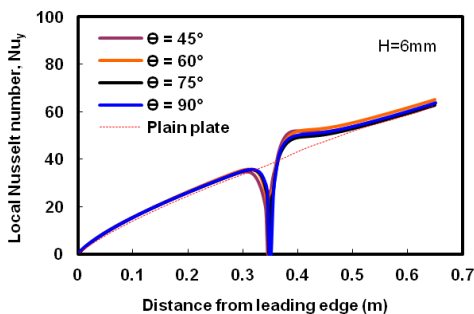


Fig. 4 Velocity field near the plate with fins ($H/t = 8$) having different inclination. (i) $\theta = 90^\circ$, (ii) $\theta = 75^\circ$, (iii) $\theta = 60^\circ$ and (iv) $\theta = 45^\circ$

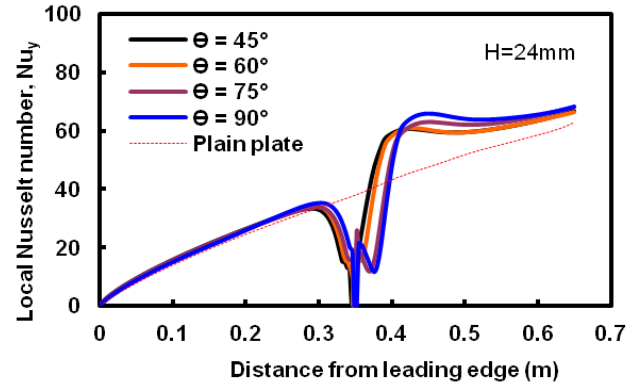
Variation of the Local Nusselt Number with Non-Conductive Fins

Vertical plate with one non-conductive fin

Local heat transfer coefficients from the vertical plate with non-conductive fin at different angles of inclination are simulated and compared with those obtained without non-conductive fin, as shown in Fig. 5. From this figure, in the region below the non-conductive fin, the protrusion surface has no influence on the local Nusselt number compared with the plain plate. Immediately below and above the non-conductive fin, local Nusselt number decreases. This is attributed to reduction in the flow velocity due to the stagnation and the resulting thickening of the boundary layer. Then, the local Nusselt numbers suddenly increase compared to the plain plate at the same elevation. This fact is due to the incoming of fresh air induced by the presence of the non-conductive fin. But a larger fin height, $H = 24$ mm in Fig. 5(b) the more fresh air is attached to the heated plate hence local Nusselt number is high.



(a)



(b)

Fig. 5. Local Nusselt number distribution along a vertical plate with one rib

Vertical plate with three non-conductive fins

Fig. 6 shows the distributions of local Nusselt number from a vertical flat plate with and without non-conductive fins. Along the leading region of the plate the Nusselt number distribution is coincident with that for the unribbed plate. A stagnation region appears to be responsible for the marked reduction in local Nusselt number values immediately upstream and downstream of the non-conductive fin. At the base of the non-conductive fins there is no heat transfer from the plate to the fins or the ambient since conductivity of the fins is assumed to be zero. So the local Nusselt number will be zero there which can be seen from Fig. 6 (a). But in the inter-fin space the local Nusselt number rises compared to the same location on a plain vertical plate due to the reattachment of the newly drawn ambient fluid. At last, significant enhancement of local Nusselt number due to high flow velocity past the finned region.

Fig. 6 (a) to 6 (d) shows the local Nusselt number distributions at different non-conductive rib heights. The variation trend of local Nusselt numbers is almost the same except for downstream of the non-conductive fins. Local Nusselt numbers on the downstream of the rib increase from the rib position. But at larger rib height, $H = 18$ mm and $H = 24$ mm in Fig 6. (c) to (d) , the local Nusselt numbers increase and again decrease due to recirculation flow on the downstream of the rib. As a result, the local Nusselt number maximum appears around the leading edge, which is not found in the case of $H = 6$ mm.

Development of a New Correlation for Average Nusselt Number

For the two dimensional simulation of the present numerical study, an empirical correlation for the average Nusselt number is developed for the vertical isothermal plate with nonconductive fins based on the present computation. For the case of non-conductive fins there are 96 computed data points. The objective is to bring out a correlation for the average Nusselt number which is a function of P/L , H/t and θ of the rib. The Prandtl number is constant at 0.7 and the Grashof number can be up to a maximum of 10^8 so that the value of \overline{Nu}_{pp} can be determined from some other correlation for a plain plate without ribs. Such a correlation will be useful for the actual industry. Eq. (9) shows the predicted Nusselt number for a plate with ribs as a function different rib configurations. \overline{Nu}_{pp} is the average Nusselt number for the plain plate without any ribs. The right hand side term next to \overline{Nu}_{pp} is the augmentation factor which is determined from this study.

$$\overline{Nu}_{pred} = \overline{Nu}_{pp} \left[\frac{(a + b\theta + c\theta^2) \left\{ d + e \left(\frac{P}{L} \right) + f \left(\frac{P}{L} \right)^2 \right\}}{g + \left(\frac{H}{t} \right)^h} \right] \quad (9)$$

Range of $\theta = \pi/4$ to $\pi/2$, $P/L = 0.5$ to 0.11 and $H/t = 2$ to 8 for which Eq. (9) can be used for laminar natural convection.

It can be seen from Eq. (9) that the dependence of the average Nusselt number is quadratic on θ and P/L and logarithmic on H/t . The constants a , b , c , d , e , f , g , and h are tabulated in the table 1.1. The present correlation gives a very good numerical match with the present computed CFD result. For the non-conductive fins Eq. (9) predicts the \overline{Nu} to an error limit of 5% for 92 data points, whereas for only 4 data points the error goes to a maximum of 9%. Eq. (9) has been developed from the Engineering Equation Solvers (EES) software with our present CFD result.

Table 1.1 The constants of Eq.(9)

Coefficients	value
a	1.877
b	0.154
c	-0.099
d	0.748

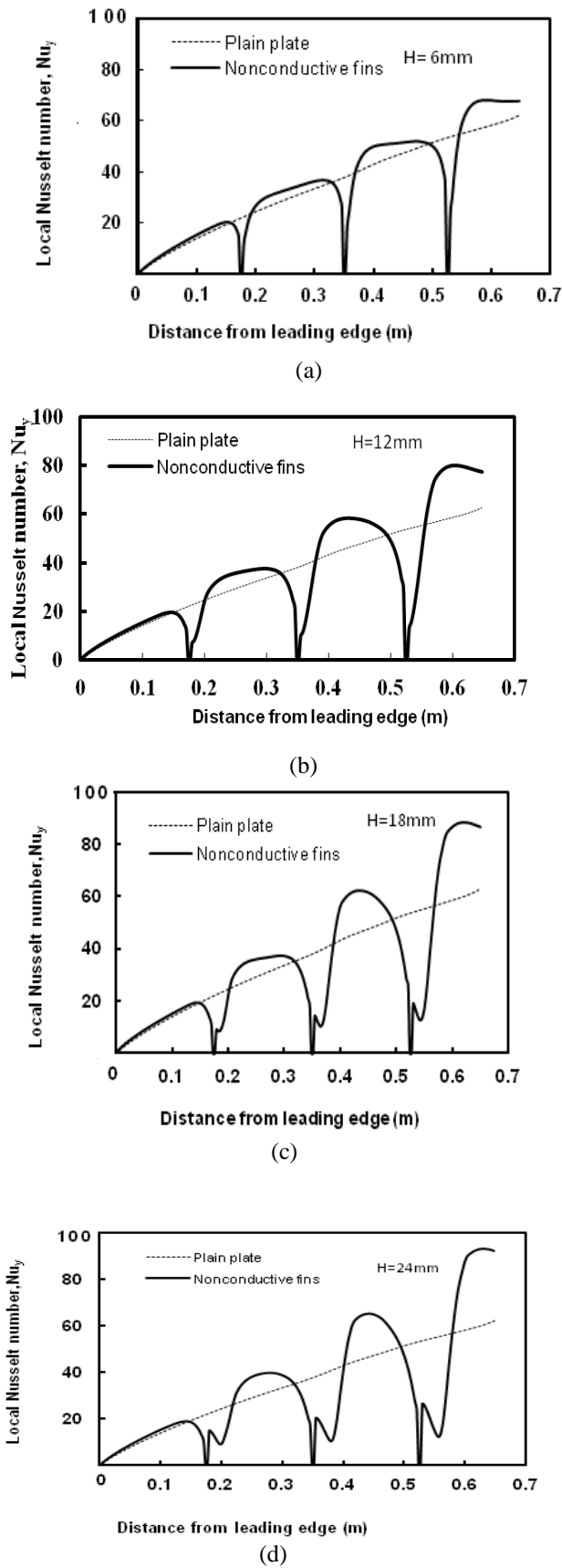


Fig. 6. Local Nusselt number distribution along a vertical plate with three ribs

e	1.880
f	-2.426
g	0.826
h	0.063

CONCLUSION

In the present study, the effects of rib geometry and the angle of inclination of the ribs on the natural convection heat transfer from a vertical flat plate with ribs have been investigated numerically. The most important results of the simulation are as follows.

1. Variations of the flow pattern in the boundary layer and their effect on the local Nusselt number are evaluated in presence of one rib at different angle of inclination or three ribs at angle of inclination of 90°
2. The maximum increase of the Nusselt number for a ribbed plate is observed to be around 10% as compared to a plain plate operating in similar conditions. This is obtained with a rib spacing of $P/L = 0.5$, fin height of 24 mm and angle of inclination of 45° .
3. The positions of minimum Nusselt number and the reattachment point upstream of a rib are obtained computationally.
4. Based on all computations, one correlation is proposed to predict the average Nusselt number of an array of non-conductive fins based on parameters \overline{Nu}_{pp} , θ , P/L , H/t .

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