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NUMERICAL SIMULATION OF MIXED CONVECTION FLOWS IN LID-DRIVEN SQUARE CAVITY

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ABSTRACT

Mixed convection heat transfer in two-dimensional lid-driven rectangular cavity filled with air (Pr =0.71) is studied numerically. A hybrid scheme with multiple relaxation time Lattice Boltzmann Method (MRT-LBM) is used to obtain the velocity field while the temperature field is deduced from energy balance equation by using the Finite Difference Method (FDM).

The main objective of this work is to investigate the model effectiveness for mixed convection flow simulation. Results are presented in terms of streamlines, isotherms and Nusselt numbers.

Excellent agreement is obtained between our results and previous works. This comparison demonstrates the robustness and the accuracy of our proposed approach.

INTRODUCTION

Nowadays, the Lattice Boltzmann Method (LBM) is considered as an alternative numerical method which has attracted much attention as a technique in fluid engineering [1]. This method based on a mesoscopic study of the macroscopic problem incorporates the basic conservation laws of the hydrodynamic variables such as density and velocity. This scheme is initially developed from its predecessor, the Lattice Gas Automata (LGA). The LBM has rapidly evolved into a self-standing research subject. Thereafter, it has to be an efficient tool for simulating problems of fluid mechanics and transport phenomena [2-7].

A literature survey shows that this method is also used for applications involving interfacial dynamics and complex boundaries such as multiphase flows [8,9], compressible flows [10,11] and porous media [12]. Moreover, LBM is well-suited for high-performance implementations on massively parallel processors such as, for example, graphics processing units.

Concerning the term of collision in the lattice Boltzmann equation, two types of collision operator are considered. One of the simplest and most widely used model, proposed by Bhatnagar et al. [13] and called BGK model, is based on a single relaxation time (SRT). It achieved considerable success due to its easy implementation and the ability to take complex geometries [15-17].

Despite the great advantages, this model, with single relaxation time, reveals deficiencies due to the numerical instabilities [14] and then difficulties to reach high Reynolds number flows. This deficiency can be easily treated by using the second type of collision operator called Multiple Relaxation Time (MRT) operator [18-20]. The MRT model presents numerous advantages compared to the BGK model. It leads to a stable solution for flows with higher Reynolds numbers.

NOMENCLATURE

c	$[m.s^{-1}]$	Lattice speed
dx	[m]	Lattice width
Gr	[-]	Grashof number
x	[m]	Horizontal coordinate
g	$[m.s^{-2}]$	Gravitation acceleration
y	[m]	Vertical coordinate
H	[m]	Cavity height
L	[m]	Cavity width
Nu	[-]	Average Nusselt number
Pr	[-]	Prandtl number
Ri	[-]	Richardson number
Re	[-]	Reynolds number
Ra	[-]	Rayleigh number
t	[s]	Time step
T	[K]	Temperature
u,v	$[m.s^{-1}]$	Velocity components
U,V	[-]	Dimensionless velocity components
U_0	$[m.s^{-1}]$	Lid-driven constant velocity
Greek symbols		
α	$[m^2.s^{-1}]$	Thermal diffusivity
β	$[K^{-1}]$	Thermal expansion coefficient
ν	$[m^2.s^{-1}]$	Kinematic viscosity
ρ	$[kg.m^{-3}]$	Density of fluid
θ	[-]	Dimensionless temperature
Subscripts		
h		hot
c		cold
f		fluid
w		wall

The main limitation of using LBM in engineering applications is the lack of satisfactory model for the thermal fluid flows problems. To remedy this problem, several approaches have been proposed which can be gathered into three categories: multispeed approach, double population approach and hybrid approach. The multispeed model [21,22] consists in extending the distribution function in order to obtain the macroscopic temperature. Previous workers [21-23] concluded that this model is not advantageous because it requires more computational resources and less stable than the other approaches described below. The double population approach, called passive scalar approach, has been proposed to use two independent distributions function [24,25]. The first approach is for the velocity field and the second is for the temperature field while using different relaxation time. In this approach the temperature is considered as a passive scalar transported without changing the velocity field. This model assumes that the viscous dissipation and compression work can be neglected for incompressible fluids. The evolution of the temperature is given by the advection-diffusion equation. However, this approach is considered ineffective, because it is not necessary to add a distribution function to simulate a passive scalar.

Concerning the hybrid model and according to Lallemand and Luo [26], the instability of previous models inherent the LB method is due to a coupling between the modes of collision operator. Authors show that the fault cannot be eliminated by increasing the number of speeds. Therefore, they argue that the best alternative to build a thermal model is to use a hybrid method in which the flow is determined by the LBM method and the energy equation is solved by another method. Thus, the present article aims to present a novel model based on the Hybrid Multiple Relaxation Time Lattice Boltzmann Method (MRT-LBM) for solving the mass and momentum conservation equations with D2Q9 lattice model, coupled with the Finite Difference Method (FDM) for computing the temperature. The method is validated thereafter for the classical MRT-lid-driven cavity. Finally, the model has been used to simulate the 2-D mixed convective flow in lid-driven square cavity.

This work is organized as follows. First we present the hybrid multi relaxation time Lattice Boltzmann Method with D2Q9 lattice model to simulate the fluid flow. In the second step we describe the Finite Difference Method (FDM) used to solve the energy equation. Thereafter, we present the MRT-lid-driven cavity for code validation and results for coupled (MRT-LBM) with (FDM) for mixed convection simulation in 2D-lid-driven square cavity. Finally, Effects of varying, both, Richardson and Reynolds numbers on the average Nusselt number are analyzed.

MULTIPLE RELAXATION TIME LATTICE BOLTZMANN METHOD (MRT-LBM)

The Lattice Boltzmann equation with the multiple relaxation time (MRT) operators can be expressed as:

$$f_i(\vec{x}+\vec{e_i}\Delta t, t+\Delta t) = f_i(\vec{x},t) - M^{-1}S_{ij}\left[m_j - m_{ij}^{eq}(\vec{x},t)\right] \quad (1)$$

where f_i is the discrete distribution function at computing node $\vec{x}$, at time t, moving with velocity $\vec{e_i}$. M is a transform matrix projecting the discrete distribution function f into moment space $|m\rangle = M.|f\rangle$, m_{ij}^{eq} is the equilibrium moment and S is the relaxation matrix.

Physically moments are given by:

$$|m\rangle = (\rho \;\; j_x \;\; j_y \;\; e \;\; \varepsilon \;\; j_x \;\; q_x \;\; q_x \;\; p_{xx} \;\; p_{xy})^T \quad (2)$$

where ρ is the intensity, j_x and j_y the x and y components of momentum, e is the energy, ε is defined as the kinetic energy, q_x and q_y correspond to the x and y components of the energy flux vector, p_{xx} and p_{xy} correspond to the diagonal and off-diagonal components of the viscous stress tensor, and T denotes the transpose operator.

The nine velocity square Lattice Boltzmann model D2Q9, as shown in Fig. 1, has been used in our work due to its widely and successfully simulation of the two-dimensional thermal flows.

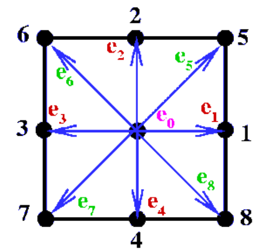

Fig. 1 Lattice structure for the (D2Q9) model.

For the D2Q9 lattices, the particle speeds $\vec{e_i}$ are defined as:

$$\begin{cases} \vec{e_i} = \vec{0} & i=0 \\ \vec{e_i} = (\cos\left[(i-1)\frac{\pi}{2}\right], \sin\left[(i-1)\frac{\pi}{2}\right])c & i=1,2,3,4 \\ \vec{e_i} = (\cos\left[(2i-9)\frac{\pi}{4}\right], \sin\left[(2i-9)\frac{\pi}{4}\right])c\sqrt{2} & i=5,6,7,8 \end{cases} \quad (3)$$

Where $c=\frac{dx}{dt}$ is the lattice speed, dx and dt are the lattice width and time step, respectively. It is chosen that dx=dt, thus c=1. The macroscopic variables such as density ρ velocity $\vec{u}$ are calculated as the moments of the distributions functions:

$$\rho = \sum_{i=0}^{i=8} f_i \quad \text{and} \quad \rho\vec{u} = \sum_{i=0}^{i=8} f_i \vec{e_i} \quad (4)$$

The equilibrium density distribution function which depends on the local velocity and density is given by:

$$f_i^{eq} = \omega_i \rho \left[1 + \frac{3\vec{e_i}\vec{u}}{c^2} + \frac{9(\vec{e_i}\vec{u})^2}{2c^4} - \frac{3\vec{u}\vec{u}}{2c^2}\right] \quad i = 0 \rightarrow 8 \quad (5)$$

Where ω_i is the weighting factor defined as:

$$\begin{cases} \omega_i = \frac{4}{9} & i = 0 \\ \omega_i = \frac{1}{9} & i = 1,2,3,4 \\ \omega_i = \frac{1}{36} & i = 5,6,7,8 \end{cases} \quad (6)$$

In the LBM the kinematic viscosity ν is related to the relaxation time by the relation:

$$\nu = (\tau - 0.5)c_s^2 \Delta t \quad (7)$$

where $c_s = \frac{c}{\sqrt{3}}$ is the speed of sound for the D_2Q_9 lattices. It is to be noted that the viscosity is positive which requires the choice of $\tau \succ 0.5$.

In the present work, the transformation matrix M, the velocity moment vector m and the equilibrium value of moments are defined in the same form as well as in Refs [24].

The diagonal relaxation matrix can be written as:

S=[S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8]

where $S_0=S_3=S_5=0$ for mass and momentum conservation before and after collision [15]. $S_7=S_8=1/\tau$ due to the fact that the viscosity formulation is the same as in SRT model [26].

In the present simulation S_1=1.64, S_2=1.2 and $S_4 = S_6 = 8\frac{(2-S_7)}{(8-S_7)}$.

FINITE DIFFERENCE METHOD FOR TEMPERATURE FIELD

The purpose of this study is to apply the lattice Boltzmann method (LBM) to simulate mixed convection in a lid-driven cavity with no source term inside. We assume that the viscous heating and compression work are neglected If we restrict our considerations to 2D cartesian coordinates, the energy balance equation is given by:

$$\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial^2 x} + \frac{\partial^2 T}{\partial^2 y}\right) \quad (8)$$

This equation is discretized by the Finite Difference Method (FDM) using the Taylor series expansion of the second order.

To improve the stability of the hybrid model used in this letter, Lallemand and Luo [26] suggest using a discretization in accordance with discretization speeds. While this choice is not unique, these authors proposed the following discretization for the derivatives:

$$\begin{cases} \frac{\partial T}{\partial x} = \frac{1}{\Delta x}\left(T_{i+1,j} - T_{i-1,j} - \frac{1}{4}\left[T_{i+1,j+1} - T_{i-1,j+1} + T_{i+1,j-1} + T_{i-1,j-1}\right]\right) \\ \frac{\partial T}{\partial y} = \frac{1}{\Delta y}\left(T_{i,j+1} - T_{i,j-1} - \frac{1}{4}\left[T_{i+1,j+1} - T_{i-1,j+1} + T_{i+1,j-1} + T_{i-1,j-1}\right]\right) \end{cases} \quad (9)$$

$$\nabla^2 T = \frac{1}{\Delta t}\left(T_{i+1,j} - T_{i-1,j} - \frac{1}{4}\left[T_{i+1,j+1} - T_{i-1,j+1} + T_{i+1,j-1} + T_{i-1,j-1}\right]\right) \quad (10)$$

For the time derivative, we use an explicit scheme:

$$\frac{\partial T}{\partial t} = \frac{1}{\Delta t}\left(T_{i,j}^{n+1} - T_{i,j}^{n}\right) \quad (11)$$

According to the above equations, the discrete form of the energy equation is written as:

$$\begin{aligned} T_{i,j}^{n+1} = {} & T_{i,j}^{n}(1-6\alpha) + T_{i+1,j}^{n}(-u+2\alpha) + T_{i-1,j}^{n}(u+2\alpha) + \\ & T_{i,+1}^{n}(-v+2\alpha) + T_{i,j-1}^{n}(v+2\alpha) + T_{i+1,j+1}^{n}\left(\frac{1}{4}u + \frac{1}{4}v + \frac{1}{2}\alpha\right) \\ & + T_{i-1,j+1}^{n}\left(-\frac{1}{4}u + \frac{1}{4}v + \frac{1}{2}\alpha\right) + T_{i+1,j-1}^{n}\left(\frac{1}{4}u - \frac{1}{4}v - \frac{1}{2}\alpha\right) \\ & + T_{i-1,j-1}^{n}\left(-\frac{1}{4}u - \frac{1}{4}v - \frac{1}{2}\alpha\right) \end{aligned} \quad (12)$$

The coefficient that accompanies ($T_{i,j}$) in the above equation plays an important role for explicit schemes. These schemes are conditionally stable and then lead to constraints on the time step and space step choices. One of the conditions of stability is that this coefficient is positive [26] implying that the thermal diffusivity α is limited: $\alpha = \frac{\nu}{Pr} \leq \frac{1}{6}$.

Concerning the Mach number, the assumption is that the flow remains within a Mach number limit (normally less than 0.3). Calculations have been done for incompressible fluid with a Mach number equal to 0.1 .

PROBLEM DESCRIPTION AND BOUNDARY CONDITIONS

Problem description

Simulations of mixed convection in a lid-driven cavity have carried out to demonstrate the applicability of this method. Fig 2 (case (b)) shows the 2D-lid-driven cavity when the vertical side walls are thermally insulated and the top wall moves at a constant velocity U_0=0.1.

The top and bottom walls are maintained isothermally at temperatures T_c and T_h, where $T_h > T_c$.

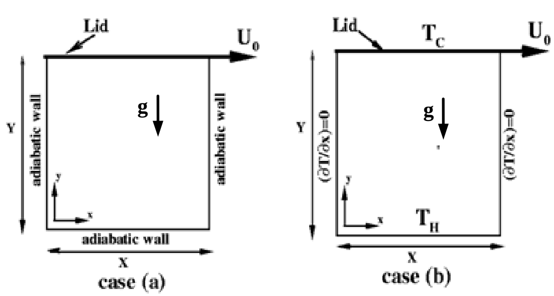

Fig. 2 Physical model and boundary conditions.

A multiple Relaxation Time (MRT) model is used to carry out LBM computations to obtain steady solutions for both geometric situations (Fig2.a and Fig2.b).

The dimensionless variables governing this problem are *U* the x-component velocity, *V* the y-component velocity. The Richardson number R_i , The Grashof number Gr, the Reynolds number Re and the Prandlt number Pr are defined as:

$$R_i = \frac{G_r}{Re^2},\ Gr = \frac{g\beta H^3 \Delta T}{\nu^2},\ Re = \frac{U_0 H}{\nu} \ \text{and}\ Pr = \frac{\nu}{\alpha}$$

The average Nusselt number, defined by temperature gradient at walls, is calculated via:

$$Nu = \frac{1}{T_h - T_c} \int_0^H \left(\frac{\partial T}{\partial y}\right)_{wall} dx.$$

The following dimensionless quantities are given by:

$$U^* = \frac{U}{U_0} \quad V^* = \frac{V}{U_0} \quad \theta = \frac{T - T_c}{T_h - T_c} \quad t^* = \frac{tU_0}{H}$$

Boundary conditions

Implementation of boundary conditions for simulation is very important. For the present work there are two types of boundary conditions applied to the computational domain.

The Dirichlet boundary conditions are given by:

for velocity: Top wall: $U=U_0=0.1$
$V=0.0$
Rest of walls: $U=V=0.0$
for temperature: Hot wall: $\theta = T_h = 1.$
Cold wall: $\theta = T_c = 0.0$
Adiabatic wall: $\left.\frac{\partial \theta}{\partial n}\right|_{wall} = 0$

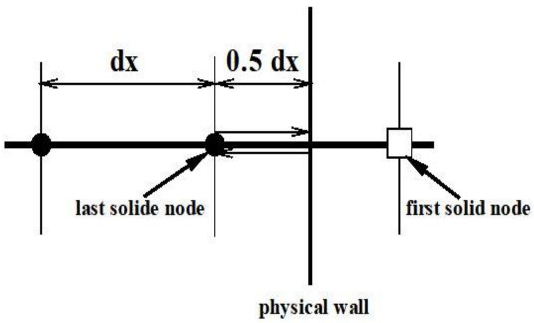

Fig. 3 Implementation of Bounce-back boundary condition

For the Lattice Boltzmann equation the difficulty is that the unknown distribution functions at the boundary nodes pointing to fluid zone must be specified. So, we apply the bounce-back condition [24] at the walls, as shown in Fig.3. The physical wall condition is located at the half grid spacing beyond the last fluid node (x_f). The particle moves from the last fluid node toward the physical wall (x_w) then it comes back to its place after being reflected by the stationary wall, so at the walls we have u=v=0, this is expressed as:

$$f_j(x_f, t+1) = f_i(x_f + e_i, t+1),$$

where f_j is the unknown distribution function of the velocity: $e_j \equiv e_i$.

As the scheme is built without a special treatment of the velocity-pressure coupling which arises in incompressible situations, LBM, in its standard form, works in the stability condition known from the explicit scheme for diffusion equation. The relation $\Delta t / \Delta x = 1$, is be frequently used [24].

RESULTS AND DISCUSSIONS

MRT-lid-driven cavity

Firstly, the present 2D-code is validated for the classical lid-driven square cavity (Fig.2 (case(a)) applying the hybrid MRT-LBM. In simplest of geometrical settings, the lid-driven cavity flow is of great scientific interests [28]. In terms of accuracy and numerical efficiency, the lid-driven cavity is considered as the classical test problem for the assessment of numerical methods and is relevant to many industrial applications and academic research [29-33]. That why it has attracted considerable attention and is probably one of the most studied fluid problem in computational fluid dynamics.

Figures from 4a to 4d show results of streamlines at different Reynolds numbers ranging from 400 to 5000, the number of lattice nodes in each coordinate direction is equal to 161 for $Re \le 1000$ and 321 when Re increased beyond 3200.

We should noted from these figures that, at low Reynolds numbers, a main vortex occupies most of the cavity volume and two secondary vortices at the bottom upstream and downstream corners.

When Reynolds number reaches 3200 another top left corner vortex starts to appear. In addition, small corner vortex emerged at the bottom right corner beyond Re=5000. Compared with earlier results in the literature [30,34-36] we noted that our computer streamlines are graphically comparable.

Table1 shows the numerical results of the locations of the primary vortex and two bottom secondary vortices at different Reynolds numbers. Numerical results given by many authors [30,34-37] are included for comparison.

Table 1: Comparison of the locations of the vortices at different Reynolds numbers Re, (case (a) Fig2).

	Primary vortex		Left secondary vortex		Right secondary vortex	
	x	y	x	y	x	y
Re = 400						
Our work	.5525	.6084	.0483	.0485	.8787	.1232
Ref [30]	.5547	.6055	.0508	.0469	.8906	.1205
Ref [34]	.5563	.6000	.0500	.0500	.8875	.1188
Ref [35]	.5532	.6055	.0528	.0439	.8908	.1384
Ref [36]	.5608	.6078	.0549	.0510	.8902	.1255
Ref [37]	.5571	.6071	.0500	.0428	.8857	.1142
Re = 1000						
Our work	.5294	.5677	.0817	.0783	.8670	.1143
Ref [30]	.5313	.5625	.0859	.0781	.8594	.1094
Ref [34]	.5438	.5625	.0750	.0813	.8625	.1063
Ref [35]	.5266	.5532	.0840	.0840	.8577	.1092
Ref [36]	.5333	.5647	.0902	.0784	.8667	.1137
Ref [37]	.5285	.5642	.0857	.0714	.8642	.1071
Re =3200						
Our work	.5129	.5439	.0813	.1164	.8293	.0892
Ref [30]	.5165	.5469	.0859	.1094	.8125	.0859
Re = 5000						
Our work	.5144	.5350	.0748	.1311	.8122	.0780
Ref [30]	.5117	.5352	.0703	.1367	.8086	.0742
Ref [34]	.5125	.5313	.0625	.1563	.8500	.0813
Ref [36]	.5176	.5373	.0784	.1373	.807	.0745

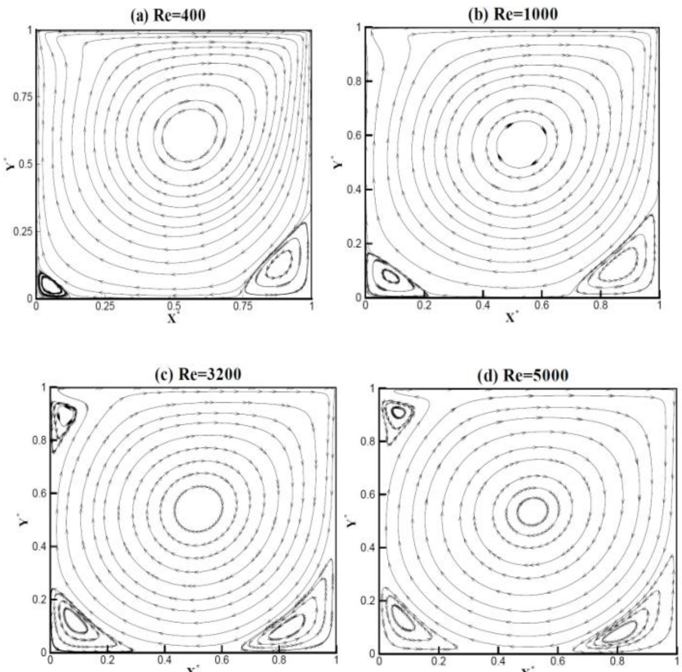

Fig 4 Streamline distributions at different Reynolds numbers, (case (a) Fig2).

These results include Ghia et al. [29] who employed the multigrid method for Navier-Stokes equations solution for the shear-driven flow in square cavity when Reynolds number is up to 10000. Vanka [34] used the multigrid technique for the coupled iterative solution for the momentum and continuity equations. In this work, asymmetrical Gauss-Siedel technique is proposed for the smoothing process. Schreiber and Keller [37] used an efficient and reliable numerical techniques of high-order accuracy for solving problem of driven cavity. The present comparison takes into account sstudy of Pandlt [35] who carried out the numerical simulations using a second order compact finite difference discretization of the fourth order stream function equation for rectangular geometry. Hou et al. [36] used the Lattice Boltzmann method with the BGK model to demonstrate the capabilities of this method for simulating the two-dimensional driven cavity flow.

From comparison of the locations of the vortices at different Reynolds numbers presented inTable1, we can conclude that the present MRT-Lattice Boltzmann method seems to be more accurate when Re increases because the difference between the present results and those of Ghia et al. [30] decreases. Thus, the present method is validated and can indeed be used for simulating mixed convective flows in lid-driven cavity.

Table2 Comparison of CPU times and number of steps with Ref [40] for different grid sizes, for Ra = 10^5

Grid size	Steps(LBM) / Steps(AFGM)	CPU(LBM) / CPU(AFGM)
32*32	13.5	0.73
128*128	64.2	0.278
256*256	115.1	0.606

Table2 summarize the comparison of CPU time ratio as well as number of steps ratio between LBM and finite volume multi-grid method used in Ref.[40] for the same geometrical and physical configurations.

In order to check the accuracy of LBM-MRT model, comparisons have been done between our results and those of Bruneau and Saad [33]. Table 3 shows velocity values (u and v) through the horizontal and vertical centerline of the cavity at Re=1000.

It seems from this table, that the present results are in good agreement with numerical results of Bruneau and Saad [33].

Table 3: Horizontal and vertical velocity, through the horizontal and vertical centerline of the cavity at Re= 1000 with Ref [33].

x	v		y	u	
	Ref [33]	Our work		Ref [33]	Our work
0.0000	0.0000	0.0000	1.0000	-1.0000	-1.0000
0.0547	-0.41018	-0.40836	0.9531	-0.47239	-0.46874
0.5000	0.02580	0.02574	0.5000	0.06205	0.06203
0.7734	0.33398	0.33217	0.2813	0.28040	0.28029
0.9297	0.29622	0.29592	0.0625	0.20227	0.20216
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Coupled hybrid-MRT Lattice Boltzmann Method with FDM for simulation of mixed convection

In this section we will study the association of the hybrid model of Lattice Boltzmann method with multiple relaxation time operators for collision term in order to solve the velocity field. Finite Difference Method is implemented for the energy equation resolution in order to explore the temperature field in the case of a lid-driven cavity shown in Fig.1b.

We present in Figs 5 a-c-e the temperature distributions and in Figs 5 b-d-f the velocity fields for different values of Richardson and Reynolds numbers corresponding to the 2D-lid-driven cavity when the vertical side walls are thermally insulated and the top wall moves at a fixed velocity $U_0=0.1ms^{-1}$.

In order to study the effect of Richardson number, as an important parameter providing a measure of the relative magnitude of natural convection effect compared to forced convection effect, calculations have been carried out for the three following cases: Ri>1, Ri=1 and Ri<1. The Grashof number is fixed at $Gr=10^6$, while Ri and Re are changed in the range of (10;316), (1;1000) and (0.1;3162), respectively.

For dominant natural convection (Ri=10, Re=316), Figs.5a-5b show the formation of two primary counter-recirculating vortices, one is upper driven by the moving top lid and the other is lower driven by buoyancy forces. This is due to the interaction between the shear forces and buoyancy due to unstable temperature gradient in a moving lid. It can noted that, in the lower portion of the cavity the recirculating vortex is larger than in the upper due to the effect of the buoyancy outweighs. The mixing hot and cold fluids between the counter-recirculating vortex and the impingement of cold and hot fluids on the top and bottom walls, simultaneously, result in steeper temperature gradients in these regions.

Streamlines and isotherms plots for (Ri=1, Re=1000) are shown in Figs.5c and 5d. The formation of two almost equal counter-recirculating vortices is due to the relatively balanced interaction of the buoyancy and shear effects. In addition, the apparition of the vortex in the lower half of the cavity is due to opposing action of the moving lid. This case is quantitavily in good agreement with Moallemi and Jang [38] (for Pr=1) who studied the effect of Prandtl number (.01<Pr<50) for laminar mixed convection using the control volume approach with the power low scheme. They used a non-uniform grids ranging from 22*22 to 52*52.

On the other hand, Figs.5e-5f show, respectively, the isotherms and streamlines predicted by the present hybrid Lattice Boltzmann-finite difference simulation for (Ri=.1, Re=3162).

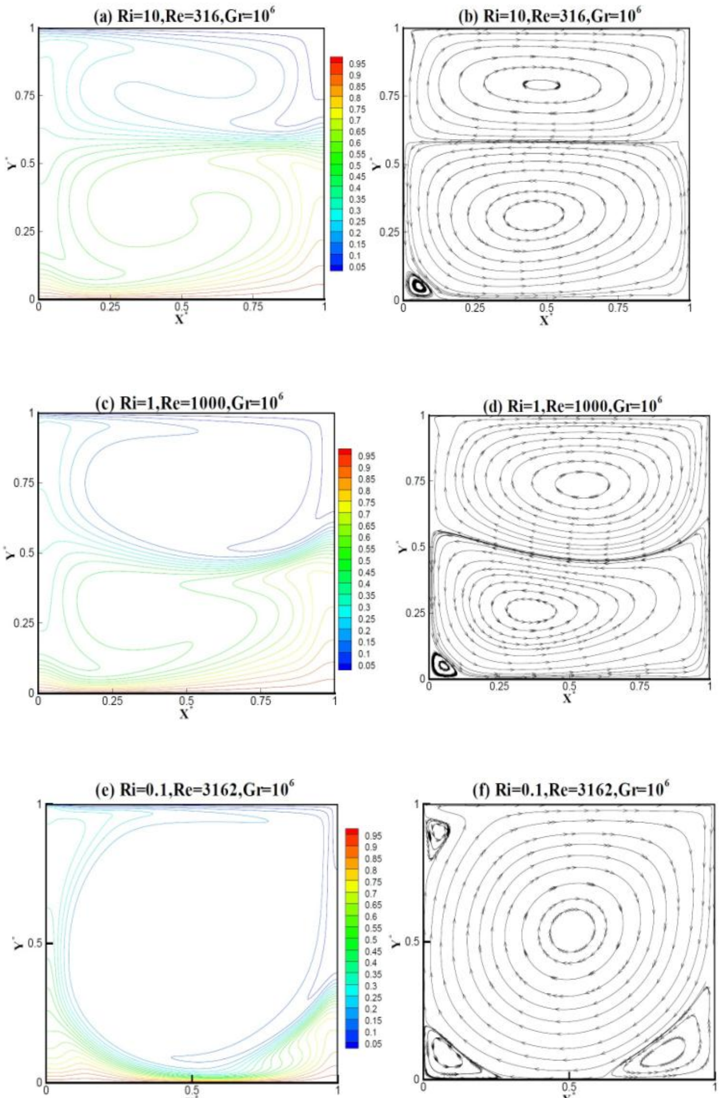

Fig 5 Richardson and Reynolds numbers effects on temperature (a-c-e) and velocity fields (b-d-f), (case (b) Fig2).

As shown from Fig.5f, the main circulation filled the entire cavity. Due to stagnation pressure and frictional losses three smaller secondary vortices are visible on the left vertical wall and near the bottom corners. All these results are similar to those of conventional mechanically-driven flow in absence of buoyancy for same Reynolds number (see Fig.4). Also, this case clearly shows that the effect of the mechanically-driven top wall (lid) dominates the entire cavity.

Figure 5e depicted the distribution of isotherms and indicates that the steep temperature gradients occur at locations near the top and bottom walls, and the cavity is filled with the well mixed cold fluids.

The effect of both Richardson and Reynolds numbers on the average Nusselt number are presented in Table 4. We observe that the heat transfer at the hot wall increases with decreasing the Richardson number Ri due to the increased buoyancy effect in the lower portion of the cavity. Comparison has been done with results of Cheng and Lin [39] who studied the effect of temperature gradient orientation on the fluid flow and heat transfer in lid-driven differentially heated square cavity using the higher-order compact scheme.

From these comparisons we can conclude that the use of MRT operators coupled with the FDM can capture fundamental behaviours in thermal flows of engineering interest and for simulating mixed convection in lid-driven square cavity.

Table 4 Comparison of average Nusselt number Nu with previous works for the lid-driven square cavity

Ri	Present work	Ref [39]	Error (%)
10	4.848	4.860	0.246%
1.	5.739	5.750	0.191%
0.1	12.138	12.161	0.189%

CONCLUSION

In this work, we have developed a hybrid Lattice Boltzmann Method with multiple relaxation time coupled with the Finite Difference Method to simulate mixed convection.

Firstly, the code has been validated for the classical lid-driven cavity. Second, we have simulated the effect of, both, Richardson and Reynolds numbers on mixed convection fluid flow inside a lid-driven square cavity.

The numerical results for a wide range of Reynolds number show that this method is adequate and numerically stable at high Reynolds number. The results predicted by the present model are in excellent agreement with other numerical results.

As a perspective of this work that this method will be tested for more general problems ultimately in three spatial directions and especially in the combined mode problems which are computationally very expensive. Work in this direction is underway.

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