HEFAT2012
9th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
16 – 18 July 2012
Malta

FLOW ORIENTATION IN CONJUGATE COOLING CHANNELS WITH INTERNAL HEAT GENERATION

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ABSTRACT

This work presents a three-dimensional geometric optimisation of conjugate cooling channels in forced convection with internal heat generation within the solid for an array of circular cooling channels with different flow orientations based on constructal theory. Three flow orientations were studied: Firstly, an array of channels with parallel flow; secondly, an array of channels in which flow of the every second row is in a counter direction to one another and thirdly, with the every flow in the array of channels in counter direction to one another.

The geometric configurations and the flow orientations were optimised in such a way that the peak temperature was minimised subject to the constraint of fixed global volume of solid material. The cooling fluid was driven through the channels by the pressure difference across the channel.

The system had hydraulic diameter and channel to channel spacing as degrees of freedom of the design variables. A gradient-based optimisation algorithm was applied to search for the best optimal geometric configurations that improve thermal performance by minimising thermal resistance for a wide range of dimensionless pressure differences. This optimiser adequately handles the numerical objective function obtained from numerical simulations.

The effect of porosities, applied pressure difference, flow orientation and heat generation rate on the optimal hydraulic diameter and channel to channel spacing were reported. Results obtained show that the effects of dimensionless pressure drop on minimum thermal resistance were consistent with those obtained in the open literature.

INTRODUCTION

Constructal theory and design [1, 2] have been adopted as an optimisation technique for the development of a procedure that is optimises a fixed global space constraint using a physical law (constructal law). The method seeks to optimise the flow architecture that predicts the flow and thermal fluid behaviour in a structure that is subjected to a global volume constraint. Bejan [1, 2] stated this law as: *For a finite-size system to persist in time (to live), it must evolve in such a way that it provides easier access to the imposed (global) currents that flow through it*.

The application of this theory started with Bejan and Sciubba [3], who obtained a dimensionless pressure difference number for optimal spacing of board to board of an array of parallel plate to channel length ratio and a maximum heat transfer density that can be fitted in a fixed volume in an electronic cooling application using the method of intersection asymptotes. This body of knowledge has been applied in all facet of lives; from humanity and nature to science and engineering [4-8].

In this paper our focus is on the original engineering application of Constructal theory, which is the geometric and shape optimisation especially in heat transfer and fluid flow analyses [9-11]. The advantage of constructal law in the engineering field is that flow architecture is not assumed in advance of the optimisation process, but is its consequence by allowing the structure to morph [12]. The applications of this theory have been reviewed most recently by the work of Bejan and Lorente [13], in which under certain global constraints, the best architecture of a flow system can be archived as the one that gives less global flow resistances, or allows high global flow access. In other words, the shapes of the channels and unit structure that is subject to global constraint are allowed to morph. The optimisation of heat exchangers and multiscale devices by constructal theory was also, recently reviewed and summarised by Fan and Luo Fan [14].

Yilmaz *et al*. [15] studied the optimum shape and dimensions for convective heat transfer of laminar flow at constant wall temperatures for ducts with parallel plate, circular, square and equilateral triangle geometries. Approximate equations were derived in the form of maximum dimensionless heat flux and optimum dimensionless hydraulic diameter in terms of the duct shape factors and the Prandlt number (*Pr*).

Da Silva et al. [16], optimised the space allocation on a wall occupied by discrete heat sources with a given heat generation rate by forced convection using the method of constructal theory in order to minimise the temperature of the hot spot on the wall.

Also, Bello-Ochende et al. [17] conducted a three-dimensional optimisation of heat sinks and cooling channels with heat flux using scale analysis and the intersection of asymptotes method based on constructal theory to investigate and predict the design and optimisation of the geometric configurations of the cooling channels. Rocha et al. [18]. Reis *et al*. [19] optimised the internal configurations of parallel plate and cylindrical channels using contructal theory to understand the morphology of particle agglomeration and the design of air-cleaning devices.

The recent comment by Meyer [20] on the latest review of constructal theory by Bejan and Lorente [21] shows that the constructal law's application in all fields of educational design is a wide road to future advances.

The above mentioned literatures focused on convective heat transfer analysis, but did not investigate the effects of flow orientations. However, Ma et al. [22] experimentally investigated the flow resistance and forced convective heat transfer influence for flow orientation in a packed channel that experience heating at the bottom. Wang et al.[23], carried out numerical investigation to study the effect of orientation of heat sink on the thermal performance of a PCM-based cooling system. Other research on the effect of orientation for different application can also be seen in open literature [24-27].

This paper focuses on the mathematical optimisation of laminar forced convection heat transfer in a heat generating solid volume with three types of flow orientations in circular channels. It examines the optimisation of a fixed and finite global volume of solid material with an array of circular cooling channels with a uniform internal heat generation. The objective is to numerically investigate the effects of three types of flow orientations on the flow resistance and forced convective heat transfer. This will be done building a smaller construct to form part of a larger construct body with different flow orientation of cooling channels that will lead to the minimisation of the global thermal thermal. The three types of flow orientations that will be investigated are: Firstly an array of channels with parallel flow. Secondly an array of channels in which flow of the every second row channel is in the counter direction to one another. Thirdly, the every flow in the array of channels is in counter direction to one another. From henceforth, shall be refered to as PF-1, CF-2 and CF-3 respectively.

NOMENCLATURE

A_c	[m^2]	Cross-sectional area of the channel
A_s	[m^2]	Cross-sectional area of the structure
Be	[-]	Dimensionless pressure drop number
P	[Pa]	Pressure
Re	[-]	Reynolds number
Pr	[-]	Prandtl number
q_s'''	$[W/m^3]$	Internal heat generation
$CF-2$	[-]	Counter - flow row
$CF-3$	[-]	Counter - flow channel
C_P	$[J/kgK]$	Specific heat at constant pressure
T	$[^0C]$	Temperature
T_{max}	$[^0C]$	Peak temperature
T_{in}	$[^0C]$	Inlet temperature
H	[m]	Structure height
PF-1		Parallel – flow
R	[-]	Thermal resistance
R_f	[-]	Flow resistance
V	[m^3]	Structure volume
W	[m]	Structure width
L	[mm]	Axial length
LFOPC	[-]	Leapfrog Optimisation Program for Constrained Problems
v_{el}	[m^3]	Elemental volume
v_c	[m^3]	Channel volume
w	[mm]	Elemental width
h	[mm]	Elemental height
d_h	[mm]	Hydraulic diameter
s	[mm]	Channel-to-channel spacing
N	[-]	Number of channels
x, y, z	[m]	Cartesian coordinates
n	[-]	Normal

Greek symbols

k	$[W/mK]$	Thermal conductivity
α	$[m^2/s]$	Thermal diffusivity
μ	$[kg.s/m]$	Viscosity
ν	$[m^2/s]$	Kinematics viscosity
ρ	$[kg/m^3]$	Density
∞		Far extreme end
ϕ	[-]	Porosity
Δ	[-]	Difference
i	[-]	Mesh iteration index
γ	[-]	Convergence criterion

Subscripts

0	Initial extreme end
f	Fluid
in	Inlet
max	Maximum
Min	Minimum
opt	Optimum
out	Outlet
s	Solid

COMPUTATIONAL MODEL

Figure 1 shows the schematic diagram of the physical configurations of the three different flow orientations and arrangements. The system consists of a solid body of fixed global volume, *V*, which is experience a uniform heat generation q_s''' within the solid. The body is cooled by forcing a single-phase cooling fluid (water) from the left side through the parallel cooling channels. The flow is driven along the length *L*, of the circular channel with a fixed pressure difference ΔP.

An elemental volume shown in Fig. 2 consisting of four (4) cooling channels and the surrounding solid was used for analysis because of the assumption of the symmetrical heat distribution and flow orientation process in the structure. The heat transfer in the elemental volume is a conjugate problem, which combines heat conduction in the solid and the convection in the working fluid.

Design variables

In Fig. 2, an elemental volume, v_{el}, constraint is considered to be composed of four(4) circular cooling channels of equal hydraulic diameter d_h. The surrounding solid of thickness s (spacing between channels) is defined as:

$$w = h \tag{1}$$

The elemental volume is

$$v_{el} = w^2 L \tag{2}$$

and the volume of a channel is:

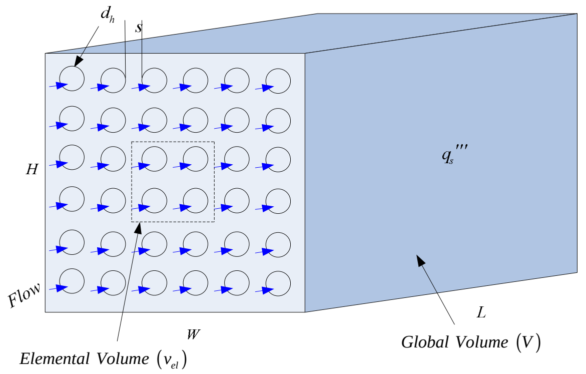
$$v_c = \frac{\pi}{4} d_h^2 L \tag{3}$$

For a fixed length of the channel, the cross-sectional area of the structure is

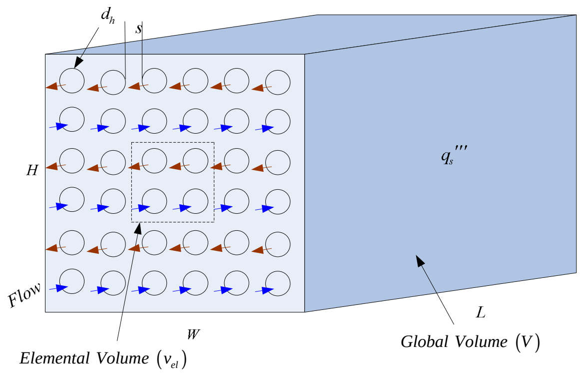
$$A_s = HW \tag{4}$$

Therefore, the number of channels in the structure arrangement can be defined as:

$$N = \frac{HW}{hw} \tag{5}$$



(a)



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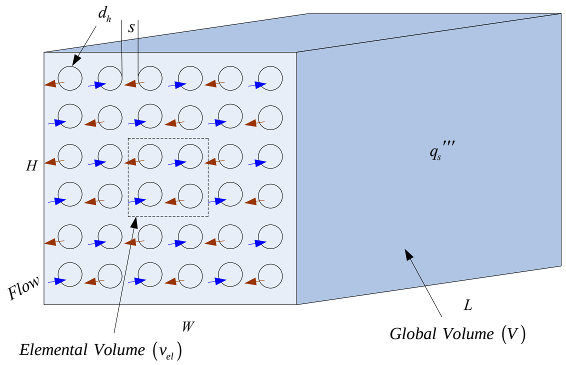

(c)

Figure 1 Three-dimensional parallel circular of (a) PF-1, (b) CF-2 and (c) CF-3 orietations.

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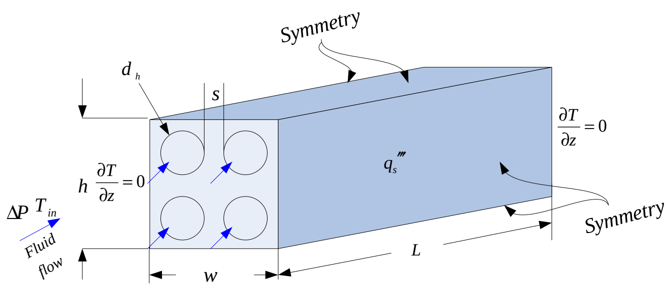

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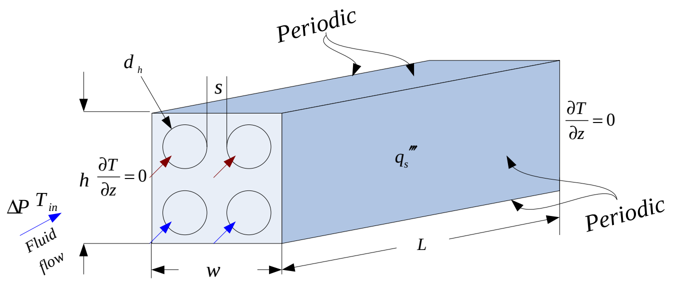

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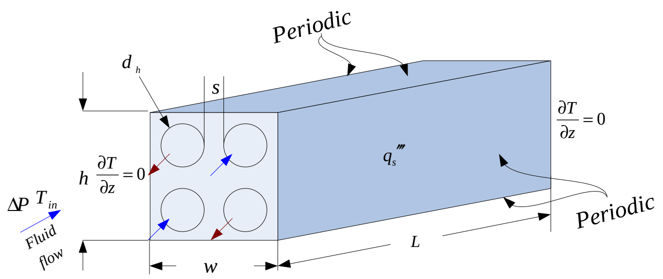

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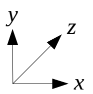

Figure 2 The boundary conditions of the three-dimensional computational domain of the elemental volume of (a) PF-1, (b) CF-2 and (c) CF-3 orietations

and the void fraction or porosity of the unit structure can be defined as:

$$\phi = 4\frac{v_c}{v_{el}} \tag{6}$$

The fundamental problem under consideration is the numerical optimisation of the channel hydraulic diameter, d_h, and the channel spacing, s, which corresponds to the minimum resistance of a fixed volume for a specified pressure drop for different flow orientations. The optimisation is evaluated from the analysis of the extreme limits of $0 \le d_h \le \infty$ and the extreme limits of $0 \le s \le \infty$. The optimal values of the design variables within the prescribed interval of the extreme limits exhibit the minimum thermal resistance.

The temperature distribution in the elemental volume was determined by solving the equations for the conservation of mass, momentum and energy numerically. A section of the discretised three-dimensional computational domain of the three flow orientation geometries is shown in Figure 3. The cooling fluid was water, which was forced through the cooling channels by a specified pressure difference, Δ*P*, across the axial length of the structure. The fluid is assumed to be in single phase, steady and Newtonian with constant properties. Water as fluid is more promising than air, because air-cooling techniques are not likely to meet the challenge of high heat dissipation in electronic packages [28, 29]. The governing differential equations used for the fluid flow and heat transfer analysis in the unit volume of the structure are:

$$\nabla \cdot \vec{u} = 0 \tag{7}$$

$$\rho(\vec{u} \cdot \nabla \vec{u}) = -\nabla P + \mu \nabla^2 \vec{u} \tag{8}$$

$$\rho_f C_{Pf} (\vec{u} \cdot \nabla T) = k_f \nabla^2 T \tag{9}$$

Energy equation for a solid given as:

$$k_s \nabla^2 T + q_s''' = 0 \tag{10}$$

The continuity of the heat flux at the interface between the solid and the liquid is given as:

$$k_s \frac{\partial T}{\partial n}\bigg| = k_f \frac{\partial T}{\partial n}\bigg| \tag{11}$$

A no slip boundary condition is specified at the wall of the channel, $\vec{u} = 0$, at the inlet (z = 0), $u_x = u_y = 0$, $T = T_{in}$ and

$$P = \frac{Be\alpha u}{L^2} + P_{out} \tag{12}$$

At the outlet (z = *L*), zero normal stress, $P_{out} = 1 \ atm$

At the solid boundaries,

$$\nabla T = 0 \tag{13}$$

The measure of performance is the minimum global thermal resistance, which could be expressed in a dimensionless form as:

$$R_{\min} = \frac{k_f (T_{\max} - T_{in})_{\min}}{q_s''' L^2} \tag{14}$$

And it is a function of the optimised design variables and the peak temperature.

$$R_{\min} = f\left(d_{h_{opt}},\ s_{opt},\ v_{el_{opt}},\ T_{\max_{\min}},\ \textit{flow orientation}\right) \tag{15}$$

R_{min} is the minimised thermal resistance for the optimised design variables. The inverse of R_{min} is the optimised overall global thermal conductance.

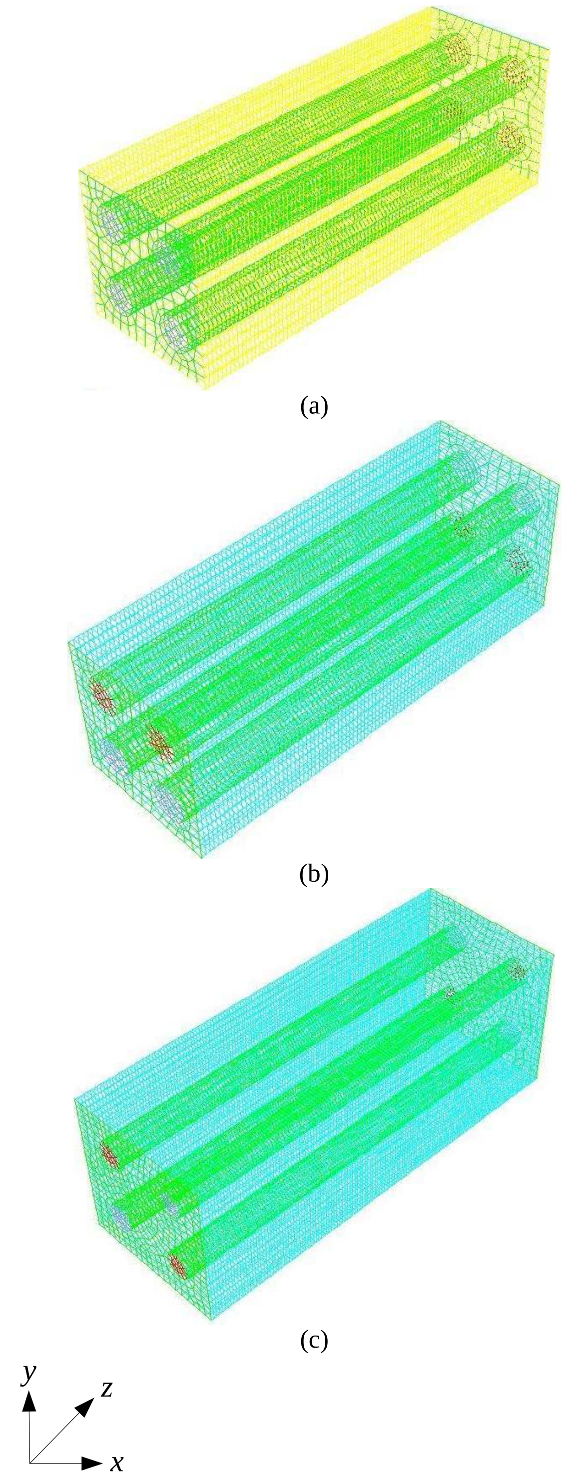

Figure 3 The discretised 3-D computational domains of (a) PF-1, (b) CF-2 and (c) CF-3 orietations

where, Be is the dimensionless pressure difference called Bejan number [30, 31].

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NUMERICAL PROCEDURE AND GRID ANALYSIS

The simulation work began by fixing the length of the channel, prescribed pressure difference, porosity, and heat generation and we kept varying values of elemental volume of the three flow orientation configurations in order to identify the best (optimal) internal configuration that minimised the peak temperature. The numerical solution of the continuity, momentum and energy Eqs. (7) - (10) along with the boundary conditions (11) - (13) was obtained by using a three-dimensional commercial package Fluent™ [32], which employs a finite volume method. The details of the method are explained by Patankar [33]. Fluent™ was coupled with the geometry and mesh generation package Gambit [34] using MATLAB [35] to allow the automation and running of the simulation process. After the simulation had converged, an output file was obtained containing all the necessary simulation data and results for the post-processing and analysis. The computational domain was discretised using hexahedral/wedge elements. A second-order upwind scheme was used to discretise the combined convection and diffusion terms in the momentum and energy equations. The SIMPLE algorithm was then employed to solve the coupled pressure-velocity fields of the transport equations. The solution is assumed to have converged when the normalised residuals of the mass and momentum equations fall below 10^{-6} and while the residual convergence of energy equation was set to less than 10^{-10}. The number of grid cells used for the simulations varied for different elemental volume and porosities. However, grid independence tests for several mesh refinements were carried out to ensure the accuracy of the numerical results. The convergence criterion for the overall thermal resistance as the quantity monitored was:

$$\gamma = \frac{\left|\left(T_{\max}\right)_i - \left(T_{\max}\right)_{i-1}\right|}{\left|\left(T_{\max}\right)_i\right|} \le 0.01 \tag{16}$$

where i is the mesh iteration index. The mesh is more refined as i increases. The $i-1$ mesh is selected as a converged mesh when the criterion (16) is satisfied.

NUMERICAL RESULTS

The elemental volume of the structure was in the range of 0.125 mm^3 to 20 mm^3 and the porosities ranged between $0.1 \le \phi \le 0.3$ and a fixed length of L = 10 mm and fixed applied pressure differences was specified as ΔP=50 kPa. . The thermal conductivity of the solid structure (silicon) is 148 W/m.K, and the internal heat generation within the solid was taken to be fixed as 100 W/cm^3. The thermo-physical properties of water [63] used in this study were based on water at 300 K and the inlet water temperature was fixed at this temperature.

Figures 5 and 6 show the existence of an optimum hydraulic diameter and elemental volume size in which the peak temperature is minimised at any point in the channel for the square configuration studied. Figure 4 shows the peak temperature as a function of the dimensionless channel hydraulic diameter. It shows that there exists an optimal channel hydraulic diameter, which lies in the range $0.005 \le d_h/L \le 0.025$ minimising the peak temperature. Also, the elemental volume of the structure has a strong effect on the peak temperature as shown in Figure 5. The minimum peak temperature is achieved when the optimal elemental volume is in the range 0.5 mm^3 $\le v_{el} \le$ 8 mm^3. This indicates that the global peak temperature decreases as the design variables (hydraulic diameter and elemental volume) increase or the global peak temperature decreases as the design variables decrease until it gets to the optimal design values.

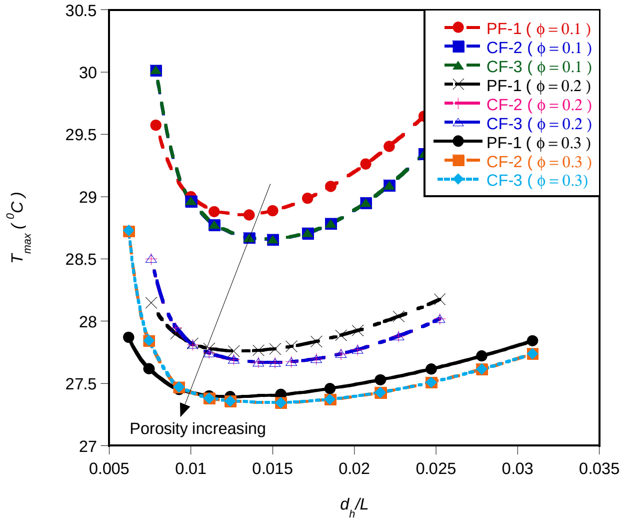

Figure 4 Effect of optimised dimensionless hydraulic diameter d_h on the peak temperature at ΔP = 50 kPa

Therefore, any increase or decrease in the design variable beyond the optimal values indicates that the working fluid is not properly engaged in the cooling process, which is detrimental to the global performance of the system. The results show that the optimal arrangement of the elemental volume for the entire structure at this fixed pressure difference should be very small in order to achieve a better cooling. The results also show that the flow orientation has a strong influence on the convective heat transfer as the peak temperature is lower in the two counter-flow arrangements compare to their parallel-flow counterpart and the two counter-flow arrangements show almost the same performance. However, the peak temperature of PF-2 orientation is slightly lower (if $d_h/L < 0.01$) than that of PF-3 orientation. Figures 4 and 5 also show that porosity has a significant effect on the peak temperature. The best cooling occurs at the highest porosity. That is, as the porosity increases, the peak temperature decreases.

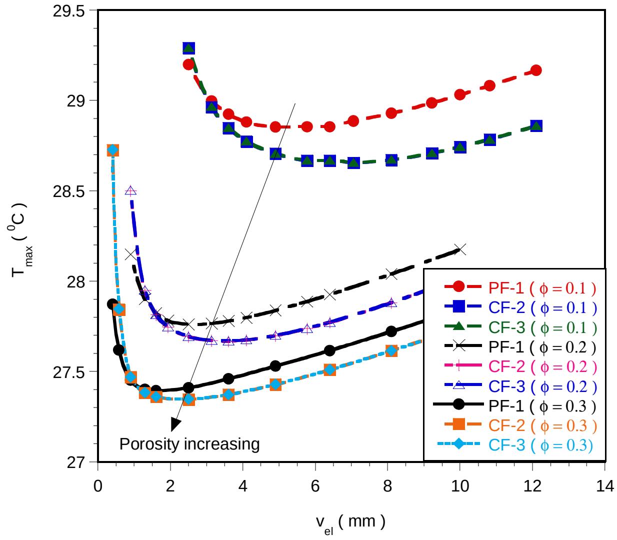

Figure 5 Effect of optimised elemental volume on the peak temperature at ΔP = 50 kPa

MATHEMATICAL OPTIMISATION

In this section, we introduce an optimisation algorithm that will search and identify the optimal design variables at which the system will perform best. A numerical algorithm, Dynamic-Q [37], is employed and incorporated into the finite volume solver and grid (geometry and mesh) generation package by using MATLAB for more efficient and better accuracy in determining the optimal performance.

The Dynamic-Q is a multidimensional and robust gradient-based optimisation algorithm, which does not require an explicit line search. The technique involves the application of a dynamic trajectory LFOPC optimisation algorithm to successive quadratic approximations of the actual problem [38]. The algorithm is also specifically designed to handle constrained problem where the objective and constraint functions are expensive to evaluate. The details of the Dynamic-Q and applications can be found in open literature [37-42].

OPTIMISATION PROBLEM

Design variable constraints

The constraint ranges for the optimisation are:

$$0.125\text{mm}^3 \le v_{el} \le 20\text{mm}^3, \quad 0.1 \le \phi \le 0.2, \qquad h = w, \quad 0 \le d_h \le w, \quad 0 \le s \le w \tag{17}$$

The design and optimisation technique involves the search for and identification of the best channel layout that minimises the peak temperature, T_{max} such that the minimum thermal resistance between the fixed volume and the cooling fluid is obtained with the desired objectives function. The hydraulic diameter and the channel spacing and elemental volume of the square configuration were considered as design variables. A number of numerical optimisations and calculations were carried out within the design constraint ranges given in (17) and the results are presented in the succeeding section in order to show the optimal behaviour of the entire system. The elemental volume of the structure was in the range of 0.125 mm^3 to 20 mm^3. The optimisation process was repeated for applied dimensionless pressure differences (Be) that correspond to $\Delta P = 5\ kPa$ to $\Delta P = 50\ kPa$.

Effect of applied pressure difference on optimised geometry and minimised thermal resistance

Figure 6 shows the effect of the minimised thermal resistance as a function of applied dimensionless pressure difference for the three flow orientation configurations. Minimised thermal resistance decreases as the applied dimensionless pressure difference and porosity increase. The results also show that the flow orientation has a strong influence on the convective heat transfer. For a specified applied dimensionless pressure difference and porosity, the PC-2 and PC-3 orientations have better performances than the PF1 orientation. The PC-2 and PC-3 orientations have almost the same performance. However, the performance of the PC-2 orientation is better than CF-3 orientation. The detail of the results can be seen in Table 1

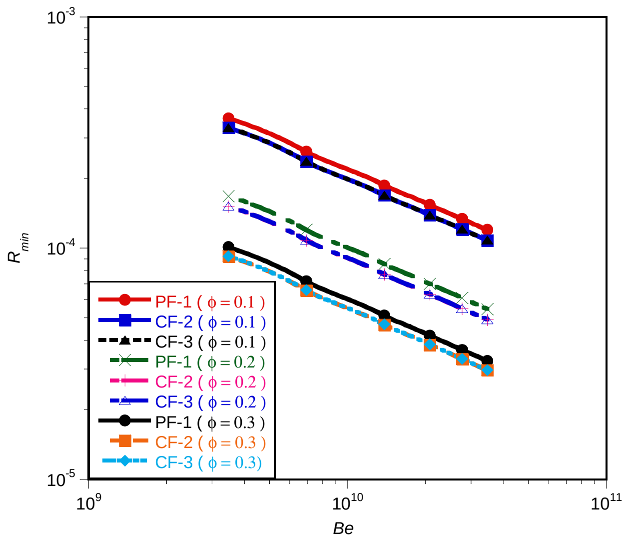

Figure 6 Effect of dimensionless pressure difference on the minimised dimensionless global thermal resistance

CONCLUSION

This paper studied the numerical optimisation of geometric structures of a conjugate cooling channels in forced convection with internal heat generation within the solid for an array of parallel cylindrical cooling channels configuration under the influency of three flow orientations based on constructal theory. The three flow configurations were, PF-1, CF-2 and CF-3.

Also Figures 7 and 8 show the optimal behaviours of the geometry with respect to the applied dimensionless pressure difference (or Bejan number) at different porosities for the three configurations. The Figure 8 show that the optimal hydraulic diameter $d_{h\,opt}$ decreases as the dimensionless pressure differences increase and there exists a unique optimal geometry for each of the applied dimensionless pressure differences for the three configurations.

In Figure 8, the optimal channel spacing s_{opt} is sensitive to the performance of the system. It decreases as the dimensionless pressure differences increase and there exists a unique optimal spacing for each of the applied dimensionless pressure differences for the configurations. It is also observed that the optimised spacing s_{opt} is directly proportional to the optimised hydraulic diameter $d_{h\,opt}$. This is also due to the fact that the elemental volume is not fixed, but it is allowed to morph for a fixed porosity. These results are also in agreement with past research work [35, 69].

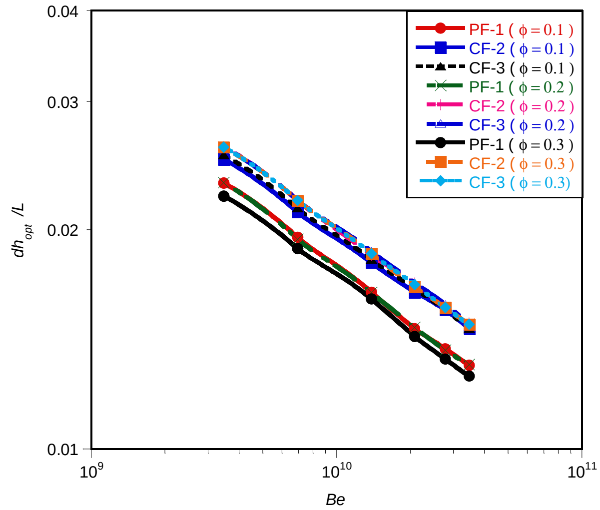

Figure 7 The effect of dimensionless pressure difference on the optimised hydraulic diameter

The geometric configurations and the flow orientations were optimised in such a way that the peak temperature was minimised subject to the constraint of fixed global volume.

The result shows that the optimsed geometry is a function of the dimensionless pressure difference number. The results also shows that there are unique optimal design variables (geometries) for a given applied dimensionless pressure number for fixed porosity that minimised dimensionless thermal resistance. Again the results show that the minimised dimensionless thermal resistance is sensitive to flow orientations. The results also show that the flow orientation has a strong influence on the convective heat transfer. For specified applied dimensionless pressure difference and porosity, CF-2 and CF-3 orientations perform better than the PF-1 orientation. The CF-2 and CF-3 orientations perform almost the same. However, the arrangement and performance of CF-2 orientation are better than CF-3 orientation.

The use of the optimisation algorithm coupled with a CFD package made the numerical results to be more robust with respect to the selection of optima geometries, flow orientations of the flow channels and dimensionless pressure difference.

Therefore, when designing the cooling structure of heat exchange equipment, the internal and external geometries of the structure, flow orientation and the pump power requirements are very important parameters to be considered in achieving efficient and optimal designs for the best performance.

ACKNOWLEDGEMENTS

The funding obtained from NRF, TESP, Stellenbosch University / University of Pretoria, SANERI/SANEDI, CSIR, EEDSM Hub and NAC is acknowledged and duly appreciated

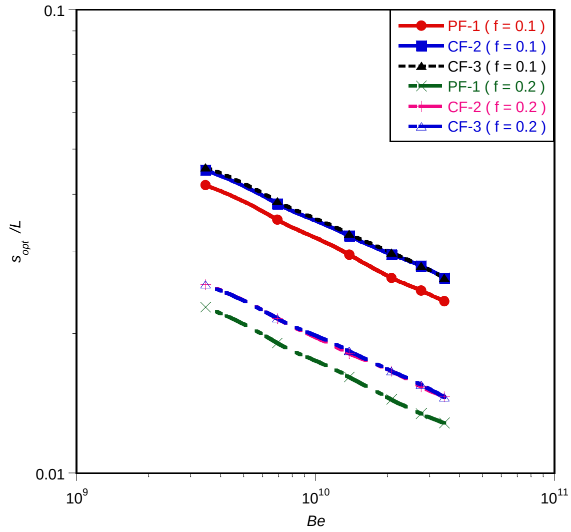

Figure 8 The effect of dimensionless pressure difference on the optimised spacing

Table 4: Global minimised thermal resistance R_{min} of the three configurations

$R_{\min} \times 10^3 =$

	$(\phi = 0.1)$			$\phi = 0.2)$			$(\phi = 0.3)$		
Be	PF-1	CF-2	CF-3	PF-1	CF-2	CF-3	PF-1	CF-2	CF-3
3.47E+09	0.3639	0.3315	0.3318	0.1678	0.1516	0.1518	0.1008	0.0922	0.0923
6.94E+09	0.2612	0.2369	0.2371	0.1199	0.1082	0.1083	0.0718	0.0657	0.0658
1.39E+10	0.1871	0.1692	0.1694	0.0855	0.0771	0.0771	0.0511	0.0467	0.0468
2.08E+10	0.1538	0.1389	0.1390	0.0701	0.0631	0.0632	0.0419	0.0382	0.0383
2.77E+10	0.1338	0.1207	0.1209	0.0609	0.0548	0.0548	0.0363	0.0332	0.0332
3.47E+10	0.1200	0.1082	0.1084	0.0546	0.0491	0.0491	0.0326	0.0297	0.0297

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