# US inflation dynamics on long range data

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### Abstract

In this paper we evaluate inflation persistence in the U.S. using long range monthly and annual data. The importance of inflation persistence is crucial to policy authorities and market participants, since the level of inflation persistence provides an indication on the susceptibility of the economy to exogenous shocks. Departing from classic econometric approaches found in the relevant literature, we evaluate persistence through the nonparametric Hurst exponent within both a global and a rolling window framework. Moreover, we expand our analysis to detect the potential existence of chaos in the data generating process, in order to enhance the robustness of conclusions. Overall, we find that inflation persistence is high from 1775 to 2013 for the annual dataset and from February 1876 to May 2014 in monthly frequency, respectively. Especially from the monthly dataset, the rolling window approach allows us to derive that inflation persistence has reached to historically high levels in the post Bretton Woods period and remained there ever since.

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### 1. Introduction

Since the suggestion of Solow (1976) that "any time seems to be the right time for reflections on the Phillips curve", research on inflation dynamics and its impact to the economy has been a vigorous field of macroeconomics. The importance of inflation persistence stems from the fact that in its presence, the economy is susceptible to crisis contagion since exogenous shocks produce permanent effects on the series. Modern research builds on the Lucas (1976) critique and examines structural rather than autoregressive models in order to evaluate the relationship of inflation to the economy. For instance the FED exploits Dynamic Stochastic General Equilibrium (DSGE) models that model inflation according to the New Keynesian Phillips Curve (Chung et al., 2010). A rather appealing issue found in the literature is to identify the extent to which inflation persistence has shifted according to the evolution of world economic conditions and the inflation-targeting policies resumed by the FED in the post-1984 period. The high level of U.S. inflation persistence in the post WWII period is considered a stylized fact. Nevertheless, the existing literature is inconclusive regarding the exact level of persistence in the post-1984 period, since different methodologies reach to divergent conclusions.

In the existing literature there is no definition or consensus of what constitutes inflation "persistence". The earliest approaches use integration and unit root tests in order to detect whether the inflation series is stationary or not, or I(0) and I(1) in the Engle and Granger (1987) terminology. The detection of non-stationarity is an indication of persistence. Evans and Wachtel (1993), Kim (1993) and Culver and Pappell (1997) using different datasets find that inflation is stationary for the period after the WWII since the mid-1980's. From that point onwards inflation becomes an I(1) process. In contrast, Hassler and Walters (1995) and Baillie *et al.* (1996) among others, find that inflation is neither an I(1) nor an I(0) process. They suggest that such an arbitrary classification is rather restrictive and apply fractional integration techniques in order to identify the fractional level of integration for inflation. The empirical findings suggest that inflation exhibits long memory.

Building on the aforementioned studies, Kumar and Okimoto (2007) find that in the presence of fractional integration unit root tests could reach to diverging conclusions. They show that under different autoregressive (AR) model configurations of popular univariate unit root tests, inflation time series can be found both I(0) and I(1) according to the specification imposed in the number of lags included in the test. Moreover, the authors employ a rolling windows framework in evaluating the level of fractional integration in inflation series and find that its level changes through time from highly persistent to the post-WWII period to weak persistent after the inflation targeting policies of the FED in the post-1984 data. Overall, they conclude that inflation persistence increased after 1970 and declined significantly in the post-1984 period.

More recently, a significant number of researchers use univariate and structural AR models in order to evaluate inflation persistence measuring the Largest AutoRegressive Root (LARR) and the Sum of the AutoRegressive Coefficients (SARC). Taylor (2000) estimates the LARR and the SARC during the Volcker -Greenspan period (1979-1987 and 1987-2006, respectively) and finds that inflation persistence has been significantly lower than the previous two decades. Cogley and Sargent (2001) use monthly data of U.S. inflation, 3- year Treasury Bonds and civilian unemployment spanning the period January 1948 to April 2000 in developing a VAR model and find that as monetary authorities focus on inflation, the level of persistence is diminishing. Bihan and Matheron (2012) develop AR and VAR models using monthly U.S. sectoral data for January 1991 to June 2001 concluding that persistence rises with the aggregation level from sectoral prices to a CPI index. Kumar and Okimoto (2007) use AR fractionally integrated models on monthly 1960:4 - 2003:3 U.S. CPI data and show that inflation persistence is period dependent as it rises during the 1970's and falls during mid-80s. These findings are directly linked to the work of Dittmar et al. (2005) who argue that monetary authorities focusing on interest rates generate inflation persistence on the contrary to inflation targeting policies.

Contrary to the empirical findings of Cogley and Sargent (2001) and Kumar and Okimoto (2007), a large number of studies reject that inflation persistence has lowered in the post-1984 period. Pivetta and Reis (2007) use quarterly data for the period 1947Q2 to 2001Q3 and develop an AR(3) model testing for persistence with the half-life (the number of periods in which inflation remains above 0.5 following a unit shock), the LARR and the SARC measures on rolling windows. The authors show that inflation persistence rises to historically high levels during 1970 and remains there until 2001. Stock (2001) on a comment to Cogley and Sargent (2001) applies LARR on a rolling window and rejects the change in persistence detected by the latter in the post-1984 period. Noriega and Ramos-Francia (2009) also corroborate the findings of Pivetta and Reis (2007), accounting for structural breaks on the AR methodology employed by the latter. They find that inflation series exhibit an I(1) behavior with the exception of periods 1947-1950 and 1973-1983, where it behaves as an I(0) process. Finally, Benatti (2008) argues that a change in the level of persistence during mid-1980 and present can be detected only for the CPI and not for the GDP, GNP and PCE deflators. Overall, in the existing literature there is no clear consensus regarding U.S. inflation persistence for the post-WWII to the present period.

The innovation of this paper is threefold. We tackle the problem of inflation persistence from a completely different point of view. Departing from classic econometric methodologies, we empirically test inflation persistence using the nonparametric Hurst (Hurst, 1951) exponent. We estimate the Hurst exponent applying the Detrended Fluctuation Analysis (DFA) (Peng et al., 1994) and the Rescaled Range Analysis (R/S) methods in a rolling window framework that have never been used before in inflation persistence estimation. Secondly, instead of focusing solely on the level of persistence, we also extend our analysis to testing for the potential existence of chaos. The possible existence of a chaotic data generating process for the inflation will provide evidence in support of persistence. Even small shocks to the inflation rate will produce diverging trajectories in the time series if it is chaotic. In doing so, we estimate the maximum Lyapunov exponent as proposed in BenSaida and Litimi (2013). In this way, we attempt to fill an existing gap in literature, since in the presence of deterministic chaos the high complexity of the system renders every attempt to model the future evolution of the phenomenon practically impossible. The final innovation of this article is that we gather long range monthly and annual U.S. CPI data spanning from January 1876 to May, 2014 and 1774-2013 respectively. In this way, we test for inflation persistence within a very broad time framework and under different observation frequencies departing from period dependent studies. To the best of our knowledge no previous work has examined such an extended dataset.

# 2. Methodology and data

# 2.1 Methodology

2.1.1 Hurst Exponent

The Hurst exponent belongs to the broader category of nonparametric analysis methods and was first proposed by Hurst (1951) as a method for analyzing long-range dependence in hydrology series. The exponent H (Hurst exponent) takes values on [0, 1]. Values close to zero indicate an anti-persistent series: the series under examination is mean-reverting. Values close to 1 indicate that the series is persistent: the series never returns to equilibrium after an exogenous shock. An H = 0.5 indicates a Random Walk (RW). Hurst exponent analysis has been applied extensively in financial time series (e.g. equities, exchange rates, commodities, derivatives etc.<sup>1</sup>), but only sporadically in macroeconomic variables and never before in measuring inflation persistence.

The R/S is one of the oldest and best-known methods for calculating the selfsimilarity parameter H of a time series and was proposed by Mandelbrot and Wallis (1969). The R/S begins by using various evenly spaced partitions of the original series. The initial series X of length N is divided into q equally sized parts of length  $n = \frac{N}{q}$ . Each of the new segments  $m=1,2,3,\ldots,q$  is integrated by the cumulative sums:

$$Y_{i,m}^{(n)} = \sum_{j=1}^{i} X_{j,m}^{(n)} , \quad i = 1, 2, 3 \dots , q$$
(1)

where  $X_m$  is the *m*-th segment of the original series X. Next we find the range

$$R_m^{(n)} = \max\{Y_{1,m}^{(n)}, Y_{2,m}^{(n)}, \dots, Y_{n,m}^{(n)}\} - \min\{Y_{1,m}^{(n)}, Y_{2,m}^{(n)}, \dots, Y_{n,m}^{(n)}\}$$
(2)

and rescale it by the Standard Deviation  $S_m^{(n)}$  of each section *m* to take the mean of the rescaled range for all subseries of length *n* 

$$\binom{R}{S}_{n} = \frac{1}{d} \sum_{m=1}^{d} \frac{R_{m}^{(n)}}{S_{m}^{(n)}}$$
(3)

The  $\binom{R}{S}_n$  values are calculated for every partition and plotted against the partition segment size *n* in a log-log scale. The slope of the linear fit expresses the Hurst exponent *H*.

<sup>&</sup>lt;sup>1</sup> Due to space restriction the interested reader is referenced to Mulligan and Koppl (2011) and the papers cited therein.

Eke *et al.* (2002) show that in the presence of non-stationarity, the R/S method can yield inconsistent *H* exponents. In order to enhance the robustness of our results we also estimate the DFA method. The DFA method was proposed by Peng *et al.* (1994) for identifying long-dependence in DNA nucleoids series and can be summarized as follows. We again start by dividing our initial series into *q* equally partitioned subseries as in R/S method and integrate each segment by (1). We then estimate the OLS line for the points in each segment  $Y_{m,i}^{(n)} = a_m^{(n)}i + b_m^{(n)}$  and calculate the standard deviation residuals:

$$F_m^{(n)} = \sqrt{\frac{1}{n} \sum_{i=1}^n \left( Y_{m.i}^{(n)} - a_{m,j}^{(n)} - b_m^{(n)} \right)^2} \tag{4}$$

The average SD is calculated for all segements of length *n*:

$$\bar{F}(n) = \frac{1}{q} \sum_{m=1}^{q} F_m^{(n)}$$
(5)

The  $F_n$  values are calculated for every partition and plotted against the partition segment size n in a log-log scale. The slope of the linear fit expresses the Hurst exponent H.

# 2.1.2 Lyapunov Exponent

The use of the Lyapunov exponent in detecting deterministic chaos in economic time series has been applied extensively to financial market series, e.g. exchange rates (Serletis and Gogas, 1997), stocks (BenSaida and Litimi, 2013 and Hsieh, 1991), etc.

The basic idea behind the detection of chaos lies with the dependence of chaotic systems to initial conditions. More specifically, if we consider two points of the same series  $X_o$  and  $X_o + \Delta x_o$  and we generate a path for each one of them, then these two points will evolve through 2 different time paths. The difference in the trajectories of the two paths depends on the initial position  $X_o$  and the elapsed time, getting the form  $\Delta x(X_o, t)$ . If the system is stable this difference decreases asymptotically with time. In contrast, in a chaotic system the difference diverges exponentially. The Lyapunov exponent  $\lambda$  measures this difference  $\Delta x(X_o, t)$  between the two paths. In order to identify a system as chaotic, the corresponding Lyapunov exponent should be strictly positive and near unity. In this paper, we follow the procedure described in BenSaida and Litimi (2013) in order to estimate the maximum Lyapunov exponent. In mathematical notation, we consider a time series X with:

$$x_t = f(x_{t-L} + x_{t-2L} + \dots + x_{t-mL}) + \varepsilon_t$$
 (6)

Where L is the time delay, f is an unknown chaotic map, m is the embedding dimension of the system and  $\varepsilon_t$  represents the added noise. BenSaida and Litimi (2013) adopt the Jacobian based approach to compute  $\lambda$  since the direct approach is inefficient in the presence of noise in measurement. Briefly the exponent is given by:

$$\hat{\lambda} = \frac{1}{2M} \ln v_i \tag{7}$$

where *M* is an arbitrary selected number of observations often approximating the 2/3 of the total span and  $v_i$  is the largest eigenvalue of the matrix  $(T_M U_o)(T_M U_o)'$ , with

$$U_o = (1 \ 0 \ 0 \dots \dots 0)' \tag{9}$$

$$T_M = \prod_{t=1}^{M-1} J_{M-t}$$
 (10)

$$J_{t} = \begin{bmatrix} \frac{\partial f}{\partial x_{t-L}} & \frac{\partial f}{\partial x_{t-2L}} & \cdots & \frac{\partial f}{\partial x_{t-mL+L}} & \frac{\partial f}{\partial x_{t-mL}} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$
(11)

In the case of scalar time series the chaotic map f generating the series is usually unknown; as a result the Jacobean matrix in (11) cannot be estimated. Thus, we need to approximate the chaotic map with a data adapting function that can produce an exact approximation of the series. The authors choose to estimate the chaotic map based on a neural network with one hidden layer of neurons and one output layer. In mathematical notation the chaotic map f is approximated by the equation:

$$x_t \approx a_o + \sum_{j=1}^q a_j \tanh\left(\beta_{0,j} + \sum_{i=1}^m \beta_{i,j} x_{t-iL}\right) + \varepsilon_t \tag{12}$$

with q declaries the hidden layers of the neural network with a tangent activation function. The order of (L,m,q) defines the complexity of the system and is selected according to the triplet that provides the maximum value of the exponent  $\lambda$ .

A common problem in the identification of the maximum Lyapunov Exponent is the determination of noise in the system and misspecifications in the selection of the (L,m,q) values. As BenSaida and Litimi (2013) argue, when the noise frequency added to the system is sufficiently larger with respect to the output of the chaotic system, the chaotic map tends to be absorbed by noise and thus the system imitates a stochastic process, leading to small or negative values of the calculated exponent  $\lambda$ . The authors overcome these misspecification issues by proposing an auxiliary statistical test to the procedure of the evaluation of the maximum Lyapunov exponent, based on its asymptotic values. Assuming the existence of chaos as the null hypothesis of the test they attempt to reject it in favor of the non-existence of chaos in a one-sided statistical test<sup>2</sup>. In this way, a system is identified as chaotic when both assumptions are met: a) we find a positive Lyapunov exponent close to unity and b) we are unable to reject the null hypothesis on the existence of chaos.

# 2.1.2 The Data

We compiled long range data for the U.S. CPI on monthly and annual frequency in order to evaluate the persistence of inflation. More specifically, we gather monthly data spanning from January 1876 to May 2014, and annual CPI observations for the period 1774-2013, both with a base period of 1982-1984. The monthly data is obtained from the Global Financial database, while the annual data comes from the website of Professor Robert Sahr.<sup>3</sup> The annual frequency is also used in the empirical tests following the suggestion from Bihan and Matheron (2012) that persistence depends on the data frequency. In Table 1 we report descriptive statistics for both series.

Table 1:Data Descriptive Statistics							
<u>CPI annual</u> <u>CPI monthly</u>							
Number of Observations	240	1661					
Mean	35.546	54.014					
Median	12.600	18.500					
Max	233.800	237.900					
Min	7.400	6.762					
Standard Deviation	53.653	65.691					
Skewness	2.309	1.425					
Kurtosis	7.142	3.628					
Jarque-Bera ( <i>p</i> -value)	0.000***	0.000***					

Note: \*\*\* denotes rejection of the null hypothesis of normality at the 1% level of significance.

As we observe from Table 1, for both series the normality null hypothesis is strongly rejected. The use of first differences of the natural logarithms of the CPI

 $<sup>^{2}</sup>$  For more information on the derivation of the test, the interested reader is referred to BenSaida and Litimi (2013).

<sup>&</sup>lt;sup>3</sup> http://oregonstate.edu/cla/polisci/sahr/sahr.

yields us the U.S. inflation series.<sup>4</sup> Figures A1, A2, A3 and A4 respectively, plots the raw monthly and annual CPI series and the corresponding inflation rates in the Appendix of the paper.

#### 3. **Empirical results**

#### 3.1 Inflation persistence

We begin our analysis by estimating the Hurst exponent for a) the full span of the data and b) the rolling window approach. In both cases we report the Hurst exponent as it is estimated using both the DFA and the R/S methodology. In the rolling window approach we construct 3 alternative windows of different lengths in the effort to monitor the evolution of the Hurst exponent through time. These windows have a size of 40%, 50% and 60% of the total number of observations with a sliding step of one. In this way we estimate for the monthly data 1001, 831 and 661 Hurst exponents for the three window sizes respectively and 141, 121 and 101 for the annual series<sup>5</sup>. Hence, two consecutive Hurst exponents are estimated from data segments that differ in just one values. The rolling overlapping window procedure is interesting as: a) creates a smooth Hurst course over time and b) ensures that intertemporal Hurst exponent fluctuations are revealed. In Table 2 we report the estimated global Hurst exponents and the rolling window Hurst exponents' statistics for the DFA methodology.

Table 2: Hurst Exponent according to the DFA method						
		<u>40%</u>	<u>50%</u>	<u>60%</u>	<u>Global</u>	
		window	window	window		
	Observations	100	120	140	240	
Annual	Min	0.535	0.624	0.643		
	Mean	0.849	0.842	0.801	0.583	
	Max	1.000	1.000	0.994		
	SD	0.114	0.111	0.096		
	Observations	660	830	1000	1660	
Monthly	Min	0.786	0.798	0.823		
	Mean	0.916	0.901	0.926	0.869	
	Max	0.999	0.991	1.000		

<sup>&</sup>lt;sup>4</sup> Not surprisingly, unit root test results according to the ADF test (Dickey and Fuller, 1981), the Phillips – Perron test (Phillips and Perron, 1988) and the KPSS test (Kwiatkowski et al., 1992), suggested that both the monthly and the annual CPI series are I(1) in levels, but are stationary when converted to inflation rates. The details of these results are available upon request from the authors.

<sup>&</sup>lt;sup>5</sup> Numerical calculations are performed with the MATLAB code provided by Rafal Weron (2011).

SD 0.041 0.046 0.041
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As we observe from Table 2 the annual Hurst exponents fluctuate from 0.535 to 1.000 for 40%, 0.624 to 1.000 for the 50% and 0.643 to 0.994 for the 60% rolling window. According to the first window, inflation shows periods of both weak (almost RW) and strong persistence, while the 50% and 60% windows report that the series is consistently persistent. This change in the reported persistence can be attributed to the sample size of the windows. Delignieres *et al.* (2006) show that the power of the DFA method is low in small samples (less than 256 observations) and introduces significant bias to the estimated Hurst exponent towards a random walk (H = 0). As the sample increases DFA techniques can model more efficiently local trends in the subseries, yielding more consistent results. Under this limitation, the small number of observations in the case of the annual data included in all rolling windows may not allow for consistent exponent estimations. Even the globally estimated Hurst exponent results should be treated with caution.

Using the monthly frequency, the results provide strong evidence in support of inflation persistence on all rolling windows sizes and the global Hurst exponent. As the window size widens the fluctuation of the estimated Hurst exponents is decreased as it is evident from the calculated standard deviation. This produces a smoother series closer to the global Hurst exponent. In Figures 1 and 2 we depict the Hurst exponents evaluated with the DFA rolling window technique for the annual and the monthly frequency, respectively.



**Figure 1:** Rolling window Hurst exponents of different length for the annual data, estimated with the DFA methodology. Grey areas represent NBER recession periods.

It is evident in Figure 1 that there is, in general, an upward trend for all 3 rolling windows coupled with large fluctuations. Nevertheless, the small number of observations included in each rolling window and the significant deviation from the global value does not allow for robust conclusions.



**Figure 2**: Rolling windows Hurst exponents of different length for the monthly data, estimated with the DFA methodology. Grey areas represent NBER recession periods.

Figure 2 depicts the evolution of the rolling Hurst exponents for the 3 windows, estimated with the DFA methodology. We observe, in general, an upward trend of inflation persistence up to the early 1980's. The slope of this trend after a short-lived decline appears much steeper after the dissolution of the Bretton Woods system in the early 1970's. From the early 1970's and up to the early 2000 we have three decades where the Hurst exponents exhibit large fluctuations without any clear trend. Finally, after the early 2000's the Hurst exponent estimates display a downward trend.

Extending the findings of Pivetta and Rise (2007) in the pre-1965 period, we observe that inflation persistence is significantly high for the whole data span, on the contrary to Benatti (2008) who argues that inflation has only been persistent after WWII. More specifically, during the 1965-1984 period of high inflation in the U.S. persistence exhibits a clearly upward trend, meaning that the level of inflation could be restrained with inflation-targeting policies. In contrast we do not detect a significant drop in inflation persistence in the post-1984 period as reported by Kumar and Okimoto (2007) and Stock and Watson (2007). Focusing on the 40% window, we

observe a peak in the persistence series right after WWII (1946-1948), an erratic upward trend in the early 1960's and a sharp drop at the end of the Bretton Woods system and the Arab oil crisis (1971-1973) The 50% window exhibits erratic drops in the early 1950s and around 1965, near the initial point of high inflation in the U.S. The fact that peaks in the 40% window are identified in different dates from the other should be traced to the beginning of the rolling windows in the period of 1929-1933., The specific period is known as the Great Depression with the 1929 stock market crash and a steady decline in the development rate of the world since 1933. These effects are imprinted on inflation and produce erratic changes in its persistence.

Table 3: Hurst Exponent according to the R/S method					
		40%	50%	<u>60%</u>	Global
		window	window	window	
	Observations	100	120	140	
	Min	0.525	0.596	0.603	
Annual	Mean	0.712	0.713	0.686	0.216
	Max	0.826	0.805	0.797	
	SD	0.052	0.042	0.045	
	Observations	660	830	1000	
	Min	0.800	0.763	0.791	
Monthly	Mean	0.898	0.911	0.893	0.833
	Max	1.000	1.000	1.000	
	SD	0.047	0.060	0.052	

For comparison reasons we repeated the aforementioned procedure using the R/S methodology. The corresponding results are reported in Table 3.

As we observe from Table 3 the results on annual data are again inconclusive. The mean values of the Hurst exponents evaluated with rolling windows report inflation persistence, but the global exponent points to an anti-persistent behavior. On monthly frequency both global and rolling windows report strong inflation persistence. In Figures 3 and 4 we depict the annual and monthly rolling window Hurst exponent series with the R/S methodology.



**Figure 3:** Rolling windows Hurst exponents of different length for the annual data, estimated with the R/S methodology. Grey areas represent NBER recession periods.



**Figure 4:** Rolling windows Hurst exponents of different length for monthly data, estimated with the R/S methodology. Grey areas represent NBER recession periods.

The estimation of the rolling windows for the annual frequency results to diverging conclusions, as the global Hurst exponent reveals an anti-persistent behavior, but the rolling windows report persistence. Thus, again as in DFA the R/S tests on annual data should be treated with caution.

On the other hand on monthly data the results are qualitative the same with the ones from the DFA method (Table 4). As we observe from Figure 4 the rolling windows estimated with the R/S methodology depict an upward persistent behavior in the post-1973 period.

# 3.2 Tests for deterministic chaos

In the inflation persistence section we find that inflation persistence has been significantly high for the total span of the long range data that we use in this article, either in annual or monthly frequency. As a next step we test for the potential existence of chaos in the data generating process.

Initially we tested for the existence of non-linearities in the annual and monthly inflation series, using the MacLeod – Li (MacLeod and Li, 1983) test, the BDS (Brock *et al.*, 1996) test, the Cobivariate Hinich (Hinich and Patterson, 1985) test, the Engle (1982) test and the Tsay (1986) test<sup>6</sup>. In each test the null hypothesis is that the series are linear. All tests reject the null hypothesis of linearity in 1% level of significance for either the annual or the monthly frequency, evaluated globally and in rolling windows (Tables 4 and 5).

Table 4: P-values of linearity tests on annual data					
		<u>40%</u>	<u>50%</u>	<u>60%</u>	Global
		window	window	window	
	Observations	100	120	140	
Hinich	Min	1.000	1.000	1.000	
Bispectral	Mean	1.000	1.000	1.000	0.740
test	Max	1.000	1.000	1.000	
	SD	0.000	0.000	0.000	
	Min	0.000	0.000	0.000	
MacLeod Li	Mean	0.095*	0.035**	0.037**	0 000***
test	Max	1.000	1.000	1.000	0.000***
	SD	0.179	0.094	0.112	
BDS test	Min	0.000	0.000	0.000	0.000***

<sup>&</sup>lt;sup>6</sup> Due to space limitations non-linearity test results are not presented here and are available from the authors upon request. All calculations were performed with the "non-linear toolkit" software of Ashley and Patterson (2000).

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	Mean	0.015**	0.001***	$0.000^{***}$	
	Max	0.521	0.012	0.001	
	SD	0.062	0.001	0.000	
	Min	0.000	0.000	0.000	
Cobivariate	Mean	0.089*	0.027**	0.024**	0.000***
Hinich test	Max	0.962	0.235	0.219	0.000
	SD	0.163	0.031	0.026	
	Min	0.000	0.000	0.000	
En als tost	Mean	0.089*	0.027**	0.024**	0.002***
Engle test	Max	0.962	0.235	0.219	0.002***
	SD	0.163	0.031	0.026	
Tsay test	Min	0.031	0.000	0.000	
	Mean	0.456	0.024**	0.007***	0.000***
	Max	0.997	0.347	0.074	
	SD	0.300	0.053	0.014	

Note: \*, \*\* and \*\*\* report rejection of the null hypothesis at the 10%, 5% and 1% level of significance, respectively.

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Table 5: P-values of linearity tests on monthly data					
		<u>40%</u>	<u>50%</u>	<u>60%</u>	Global
		window	window	window	
	Observations	660	830	1000	
Hinich	Min	0.003	0.018	0.037	
Bispectral	Mean	0.087*	0.125	0.080*	0.272
test	Max	0.798	0.337	0.137	
	SD	0.081	0.059	0.018	
	Min	0.000	0.000	0.000	
MacLeod Li	Mean	0.004***	0.002***	0.001***	0 000***
test	Max	0.110	0.022	0.110	0.000
	SD	0.006	0.002	0.004	
	Min	0.000	0.000	0.000	
PDS test	Mean	0.000***	0.000***	0.000***	0.000***
BDS test	Max	0.000	0.000	0.000	
	SD	0.000	0.000	0.000	
	Min	0.000	0.000	0.000	
Cobivariate	Mean	0.004***	0.002***	0.001***	0 000***
Hinich test	Max	0.027	0.014	0.006	0.000
	SD	0.005	0.002	0.001	
	Min	0.000	0.000	0.000	
Engle test	Mean	0.004***	0.002***	0.001***	0 000***
Engle lest	Max	0.027	0.014	0.006	0.000
	SD	0.005	0.002	0.001	
	Min	0.000	0.000	0.000	
Tooy toot	Mean	0.018**	0.021**	0.008***	0.000***
i say test	Max	0.139	0.259	0.082	
	SD	0.020	0.040	0.013	

SE0.0200.0400.013Note: \*, \*\* and \*\*\* report rejection of the null hypothesis at the 10%, 5% and 1%<br/>level of significance, respectively.

Rejecting linearity, we then test whether the non-linearities detected in our data are the result of an actual stochastic process or deterministic chaos (that appears random). For this reason, we estimate the maximum Lyapunov exponent employing the methodology suggested by BenSaida and Litimi (2013). The maximum Lyapunov exponents for the global and rolling window data series and the p-values of the statistical tests of the null hypothesis accepting the existence of chaos ( $H_o: \lambda=1$ ) are reported in Table 6 for the annual data and in Table 7 for the monthly series.

Table 6: Lyapunov Exponents for the annual dataset					
		<u>40%</u>	<u>50%</u>	<u>60%</u>	Global
		window	window	window	
	Observations	100	120	140	
	Min	-0.144	-0.191	-0.216	
Exponent	Mean	0.062	-0.051	-0.071	-0.140
	Max	0.979	0.501	0.501	
	SD	0.173	0.119	0.128	
<i>p</i> -value	Min	0.011	0.000	0.000	
	Mean	0.553	0.224	0.198	0 000***
	Max	0.999	0.999	0.999	0.000***
	SD	0.318	0.297	0.296	

Note: \*\*\* denotes rejection of the null hypothesis regarding the existence of chaos in 1% level of significance.

Table 7: Lyapunov Exponents for the monthly dataset					
		<u>40%</u>	<u>50%</u>	<u>60%</u>	Global
		window	window	window	
	Observations	660	830	1000	
	Min	-0.160	-0.163	-0.179	
Exponent	Mean	-0.123	-0.120	-0.120	-0.144
	Max	-0.004	-0.024	-0.016	
	SD	0.027	0.027	0.026	
	Min	0.000	0.000	0.000	
<i>p</i> -value	Mean	0.000***	0.000***	0.000 * * *	0 000***
	Max	0.117	0.000	0.000	0.000
	SD	0.004	0.000	0.000	

Note: \*\*\* denote rejection of the null hypothesis regarding the existence of chaos in 1% level of significance.

As we observe in Table 6 the mean values of the maximum Lyapunov exponent for all 3 annual rolling windows is small and close to zero, while the value of the global exponent is negative. In Table 7 the mean values of all maximum Lyapunov exponents for the monthly rolling windows are negative, while the mean values of the p-values from the statistical tests in each rolling window reject the null hypothesis regarding the existence of chaos in the system in rolling windows and on global evaluation. Overall, we reject the existence of chaos for both data frequencies under both global and rolling windows setups. Thus, the identified inflation persistence should not be attributed to randomness.

# 4. Conclusions

In this paper we evaluate inflation persistence in the U.S. using long range monthly and annual data spanning the period from January 1876 to May 2014 and 1776 to 2013, respectively. Departing from classic econometric approaches found in the relevant literature, we evaluate persistence through the nonparametric Hurst exponent and test for the potential existence of chaos in the data generating process by estimating the maximum Lyapunov exponent. All estimations are performed within a global and rolling window (with alternative window sizes) framework. In contrast to Stock and Watson (2007) and Benati (2008), we find that inflation persistence remained high for the entire period under examination, exhibiting an upward trend up to the early 1980's. After a short-lived decline following the dissolution of the fixed exchange rates system in the 1970's, inflation persistence trends appear steeper up to the early 1980's. From that point onwards inflation persistence fluctuates erratically remaining at a high level but with no clear trend. Our empirical findings corroborate the ones of Pivetta and Reis (2007) as we do not detect any significant fall in inflation persistence in the post-1984 period, in contrast to the findings reported by Stock and Watson (2007) and Benati (2008). Both global and rolling window estimations reject the existence of deterministic chaos in either of the two series. Overall, we conclude that inflation exhibits a persistent behavior in the entire period under examination without significant shifts towards a RW process and has reached historically high levels in the post Bretton Woods period and remained there ever since.

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Figure A1: Plot of monthly CPI series (1876:1-2014:5)



Figure A2: Plot of annual CPI series (1774-2013)



Figure A3: Plot of monthly inflation rate series (1876:2-2014:5)



Figure A4: Plot of annual inflation rate series (1775-2013)