HEFAT2007
5^{th} International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
Sun City, South Africa
Paper number: PJ4

A SIMPLE METHODS FOR PREDICTION OF TEXTILE FABRICS THERMAL CONDUCTIVITY

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ABSTRACT

The prediction of the thermal conductivity of fibrous structures is important for design purposes of new fabrics and prediction of their thermal comfort. It is well known that physiological comfort is strongly connected with the thermal comfort. There is a lot of fabrics' properties which influence the thermal comfort. Thermal insulation properties characterized by thermal resistance or thermal conductivity belong to the most important ones.

There exists a plenty of various models for prediction of thermal conductivity of multiphase materials which can be used for prediction of textile fabrics thermal conductivity.

Tai deduced mathematical expressions for the equivalent thermal conductivity of two and three-dimensional orthogonally fibre-reinforced composites in a one-dimensional heat flow model. Tai showed that whether a square slab model or a cylindrical fibre model is used makes little difference to the heat flux; while the fibre volume fraction matters. Transversal heat conductivity of fibrous composites is dependent on the yarn shape and fabric macroscopic porosity.

Krach and Advani investigated the effect of void volume and shape on the effective conductivity of a unidirectional sample of a 3-phase composite using a numerical approach consisting of a unit cell. Their findings clearly showed that the influence of porosity on thermal conductivity could not be described solely by the void volume. Militky used the plain weave cell model for prediction of cotton type fabrics thermal conductivity. Application of these models for systems in which in matrix phase replaced by air phase is complicated by fact that during measurement of thermal conductivity is fabric deformed, shape of yarns is not circular and therefore unit cell is then not precisely known. The simpler approach is to use estimated porosity and packing density as characteristics of fabrics porous structure.

The main aim of this paper is prediction of textile fabrics thermal conductivity as function of material (fibre type) and construction parameters (porosity or packing density). The relations between thermal conductivity and sound velocity or electrical conductivity are mentioned. Some approaches to predict thermal conductivity of multiphase systems with specific geometrical arrangements are shown. A measurement of the thermal conductivity is performed by the Alambeta apparatus. The set of cotton fabrics with plain weave and varying fineness of weft yarns is used as experimental material. The relations between total volume porosity and thermal conductivity of cotton weaves are predicted.

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INTRODUCTION

The prediction of the thermal conductivity of fibrous structures is important for design purposes of new fabrics and prediction of their thermal comfort. It is well known that physiological comfort is strongly connected with the thermal comfort (Fanger (1970)). There is a lot of fabrics' properties which influence the thermal comfort. Thermal insulation properties characterized by thermal resistance or thermal conductivity belong to the most important ones.

There exists a plenty of various models for prediction of thermal conductivity of multiphase materials which can be used for prediction of textile fabrics thermal conductivity (see e.g. Hashin and Shtrikman (1962) and Sulaiman et all (2006)).

Tai (1976) deduced mathematical expressions for the equivalent thermal conductivity of two and three-dimensional orthogonally fibre-reinforced composites in a one-dimensional heat flow model. Tai showed that whether a square slab model or a cylindrical fibre model is used makes little difference to the heat flux; while the fibre volume fraction matters. Transversal heat conductivity of fibrous composites is dependent on the yarn shape (see (Tai (1998)) and fabric macroscopic porosity.

Krach and Advani (1996) investigated the effect of void volume and shape on the effective conductivity of a unidirectional sample of a 3-phase composite using a numerical approach consisting of a unit cell. Their findings clearly showed that the influence of porosity on thermal conductivity could not be described solely by the void volume. Another predictions based on the models of fabric unit cell were presented by Stark and Fricke J (1993) or Ning and Chou (1995). Militky (2006) used the plain weave cell model for prediction of cotton type fabrics thermal conductivity. Application of these models for systems in which in matrix phase replaced by air phase is complicated by fact that during measurement of thermal conductivity is fabric deformed, shape of yarns is not circular and therefore unit cell is then not precisely known. The simpler approach is to use estimated porosity and packing density as characteristics of fabrics porous structure.

In this work, simple formulas are compared to predict the thermal conductivities of fabrics as function of effective thermal conductivity of moist fibres and the basic fabrics structural parameters. The fabric porosity is computed from various assumptions. The predictive ability of the resulting models is verified on the set of cotton fabrics with plain weave and varying fineness of weft yarns.

NOMENCLATURE

K	$[\mathrm{W\ m^{-1}K^{-1}}]$	Thermal conductivity
Q	[W]	Heat flow
u	$[\mathrm{ms^{-1}}]$	Sound velocity
R	[%]	Moisture regain
P_o	[-]	Porosity
D_C	$[\mathrm{m^{-1}}]$	Sett of weft
D_M	$[\mathrm{m^{-1}}]$	Sett of warpt
T_C	[m]	Weft yarn fineness
T_M	[m]	Warp yarn fineness
t_W	[m]	Fabric thickness

Special characters

ρ	$[\mathrm{kgm^{-3}}]$	Polymer density

Subscripts

C	Weft
M	Warp
D	Density
V	Volumes
s	Surface
P	Parallel
S	Serial

THERMAL CONDUCTIVITY

Thermal conductivity K [W m-1K-1] is defined as factor in the well known Fourier equation describing the steady state one directional transport of heat through body of cross sectional area A and length L due to thermal difference ΔT (see fig. 1).

$$Q = K\ A\ \frac{\Delta T}{L} \qquad (1)$$

where Q [W] is heat flow generated by the temperature gradient.

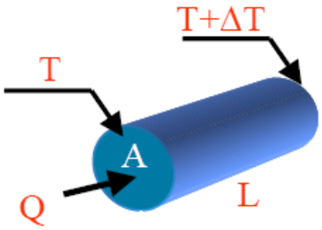

Fig. 1 Thermal transport though solid body

Thermal conductivity of solid particles is K about 1- 5 $[\mathrm{W\ m^{-1}K^{-1}}]$, for water is $K = 0.6$ $[\mathrm{W\ m^{-1}K^{-1}}]$, for ice is $K = 2.24$ $[\mathrm{W\ m^{-1}K^{-1}}]$ and for air is $K = 0.024$ $[\mathrm{W\ m^{-1}K^{-1}}]$.

No adequate theory exists which may be used to predict accurately the thermal transport in the polymeric systems. Simple phonon model of thermal conductivity is described by Van Krevelen (1992) In crystalline solids the thermal conductivity is enlarged by a concerted action of the molecules. Most of the semi empirical expressions for prediction of the thermal conductivity K are based on the Debye equation

$$K = C_P\ \rho\ u\ L$$

where ρ is the density, u is the velocity of the elastic waves (sound velocity), C_p is the specific heat capacity and L represents the average free path length (distance between molecules in adjacent layers).

Crystalline polymers show much higher thermal conductivity. For typical PET with 40 % crystallinity is $K = 0.272$ $[\mathrm{W\ m^{-1}K^{-1}}]$. Assuming that L is nearly constant it may be expected that a direct proportionality exists between K and sound velocity.

Thermal conductivity of textile fibres is generally dependent on theirs chemical composition, porosity and content of water. Haghi (2003) published thermal conductivity for some typical fibres. For practically nonporous polypropylene fibre he found $K = 0.518$ $[\mathrm{W\ m^{-1}K^{-1}}]$ and for porous acrylic fibre $K = 0.288$ $[\mathrm{W\ m^{-1}K^{-1}}]$. For hydrophilic fibres is thermal conductivity based on the moisture content characterized by regain R [%]. For wool fibres is dependence of K on R described by relation (Haghi (2003))

$$K = 10^{-3}\ (38.49 - 0.72\ (R/100) + \\ +0.113\ (R/100)^2 - 0.002\ (R/100)^3) \qquad (2)$$

Empirical relation between K and R for cotton fibre has the form (Haghi (2003))

$$K_y = 10^{-3}\ (44.1 + 63\ (R/100)) \qquad (3)$$

For expression of thermal conductivity of fabric is the simple to use two phase model consist from fibres (moist) having thermal conductivity K_y and air with thermal conductivity K_a in serial (lower limit) or parallel (upper limit) arrangements as is shown on the fig. 2. Relative portion of air phase is equal to porosity P_o and relative portion of fibrous phase is $1\text{-}P_o$.

In accordance with analogy between thermal and electrical conductivity is the thermal conductivity for parallel arrangement K_P (higher limit) equal to

$$K_P = P_O\ K_y + (1 - P_O)\ K_a$$

For serial arrangements is thermal conductivity K_S (lower limit) defined as

$$K_s = \frac{K_a\ K_y}{P_O\ K_a + (1-P_O)\ K_y}$$

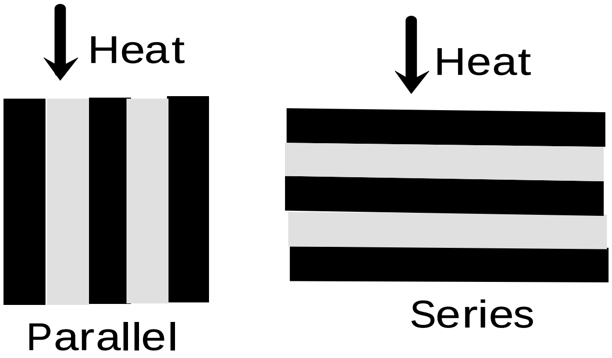

Fig. 2 Limit arrangement of yarns (black) and air (white) in conductivity model

Actual composition of a fibres and air phases can be presented by linear combination of parallel and series structures of its constituents' thermal resistance (Sulaiman (2006)). This might not give an accurate prediction of the fabrics thermal conductivity due to the specific orientations the fibres take within the yarns as well as the distribution, shape, and size of the pores. However, the parallel/series structure gives a first hand prediction and would give reasonable prediction accuracy for practical application due to its simplicity.

Different model combining thermal conductivities of air and fibres with account of fibres orientation has the form

$$K_b = K_y + \frac{K_a - K_y}{1 + \frac{1-P_O}{P_O}\left[1 + z\ \frac{K_a - K_y}{K_a + K_y}\right]}$$

with $z = 1$ when all fibres are perpendicular to the direction of heat flow, $z = 2/3$ for random fibre orientation and $z = 5/6$ for half of fibres being random and the other half being normal to the direction of heat flow (this value is used in our calculations here)

Hashin and Shtrikman (1962) developed lower K_{hd} and upper K_{hh} bounds of two phases mixture thermal conductivity (derived originally for spherical inclusions of one phase in continuous another phase)

$$K_{hl} = K_a + (1-P_O)/\left[1/(K_y - K_a) + P_O/(3\ K_a)\right]$$

$$K_{hh} = K_y + P_O/\left[1/(K_a - K_y) + (1-P_O)/(3\ K_y)\right]$$

In all of these relations can be thermal conductivity of fibrous phase replaced by K_y defined by equation [3]. Then the thermal conductivity is predicted as function of suitable definition of fabric porosity P_O.

FABRICS POROSITY

There exist a lot of models characterizing the idealized porosity P_O from some construction parameters of textile fabrics (Militký et all. (1998)). Classical parameters are sett (texture) of weft D_C [1/m], sett of warp D_M [1/m], fineness of weft yarn T_C [tex], fineness of warp yarn T_M [tex], planar weight of weave W_P [kg m^{-2}], density of fibres ρ_F [kg m^{-3}] and thickness of fabric t_W [m]. For the idealized arrangement of yarns in fabric is $t_I = d_C + d_M$, where d_C is diameter of weft yarn and d_M is diameter of warp yarn. In the case when $t_W \approx t_I$ the yarns in fabric are roughly circular. This type of arrangements is assumed in sequel.

For **idealized circular yarn** with the same packing density is simple to compute diameters from the relation

$$d_C = \frac{2\sqrt{T_C}}{\sqrt{10^6\,\pi\,\rho_C}} \tag{4}$$

$$d_M = \frac{2\sqrt{T_M}}{\sqrt{10^6\,\pi\,\rho_M}} \tag{5}$$

Here ρ_C and ρ_M are unknown densities of weft and warp yarns. These densities are combinations of densities of fibres ρ_F and air $\rho_A = 1\ 000$ [kg m^{-3}] according to the packing of fibres in yarns. For known packing density μ_M is $\rho_M = \mu_M\ \rho_F$ and the same relation is valid for a weft yarn. The values ρ_C and ρ_M are therefore function of twist and method used for yarn creation. For the moderate level of twist it has been empirically found that $\rho_C/\rho_F = \mu_C \approx 0.525$ and this correction can be imposed to the relations [4] and [5] for computation of d_C or d_M.

The „density" porosity of fabrics can be computed from relation

$$P_D = 1 - \rho_W / \rho_F \quad \text{where} \quad \rho_W = \frac{m_V}{v_V} = \frac{W_P}{t_W}$$

Here m_V [kg] is weight and v_V [m^3] is corresponding volume of fabrics having the surface of 1 m^2. From the measured planar weight W_P, fabric thickness t_W and known density of fibres is the simple to compute the „density" porosity

$$P_D = 1 - \frac{W_P}{\rho_F\ t_W}$$

Second possibility of porosity evaluation is based on the definition of hydraulic pore for the filtration purposes (Militký et all. (1998)). The „volume" porosity is defined as

$$P_V = 1 - \frac{volume\ \ covered\ \ by\ \ yarns}{whole\ \ accessible\ \ volume} =$$

$$= 1 - \frac{v_Y}{v_V} = 1 - \frac{v_Y}{t_W}$$

The v_Y is computed from equation derived by Militký et all. (1998). For the case of negligible yarns dimensional changes in fabrics can be porosity P_V expressed by the relation

$$P_V = 1 - \frac{\left[D_C T_C + D_M T_M\right]}{525\ 10^3\ \rho_F\ t_W}$$

More accurate determination of volume porosity is based on the idealized fabric surface structure projection shown on the Fig.3.

The unit cell (element of structure) shown on solid line contains a part of curved weft and warp yarns portions.

Volumes and lengths of these portions are computed from equation derived by Militký et all. (1998).

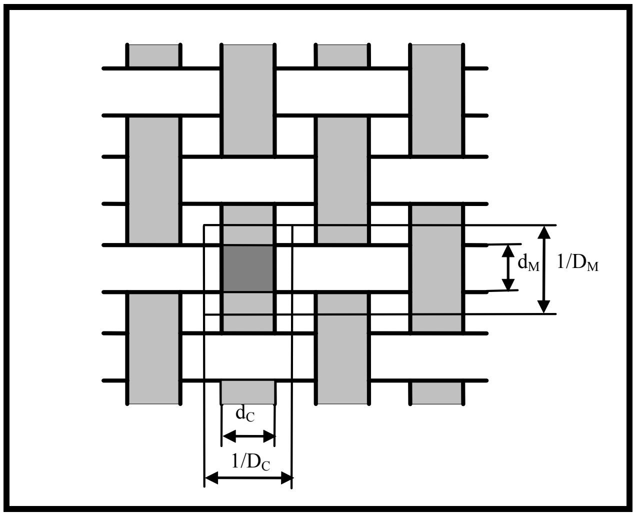

Fig. 3 Idealized surface of fabrics projection (solid lines bound the unit cell)

The unit cell (element of structure) shown on solid line contains a part of curved weft and warp yarns portions. Volumes and lengths of these portions are computed from equation derived by Militký et all. (1998). Corrected volume porosity is then defined as

$$P_V^* = 1 - \frac{\pi}{4\,(d_M + d_C)} \left[d_C^2\, D_C \sqrt{1.16\, d_C^2\, D_C^2 + 1} + d_M^2\, D_M \sqrt{1.16\, d_M^2\, D_M^2 + 1} \right]$$

From pure geometrical point of view can be defined the surface porosity

$$P_S = 1 - CF$$

The CF is the cover factor of fabric defined as

$$CF = D_C\, d_C + D_M\, d_M - d_C\, d_M\, D_C\, D_M\ .$$

EXPERIMENTAL PART

The 14 different kinds of the cotton fabrics for the summer clothing were investigated. Details about fabrics preparation and construction parameters are in the work of Matusiak (2005). There were the plain woven fabrics made on the basis of the same warp: cotton combed yarn of the linear density $T_M = 15$ tex. All fabrics were produced at the same nominal warp density $D_M = 2700$ [1/m] and at the same nominal weft density $D_C = 1450$ [1/m]. Differentiation of the fabric structure was achieved by the application of the different weft yarns: $T_C = 20$ tex, *25* tex, *30* tex, *40* tex, *50* tex and *60* tex. As characteristics of fabric structural parameters connected with transport properties the density porosity P_D, volume porosity P_V, corrected volume porosity P^*_V and surface porosity P_S were computed. The experimental thermal conductivity K_{ex} [W $m^{-1}K^{-1}$] was measured by means of the ALAMBETA device.

RESULTS AND DISCUSSION

For prediction of fabrics thermal conductivity the values of K_S, K_P, K_b, K_{hd} and K_{hh} were computed from equation defined in chap. 2. The K_y was computed from equation [3] for regain value $R = 10\%$, the thermal conductivity of air was $K_a = 0.024$. For investigation of influence of porosity P_O definition on the thermal conductivity the values of K_S, K_P, K_b, K_{hd} and K_{hh} for above mentioned surface P_S, volume P_V, **corrected volume** P^*_V and density P_D based porosities were computed. Based on the comparison of predicted and measured thermal conductivities the surface porosity was selected as optimal for prediction purposes (see fig.4). The scatter plot map for experimental and predicted thermal conductivities (porosity P_S) is given on the fig.4. The strong correlation between experimental values K_{ex} and predicted thermal conductivities are visible. The predicted thermal conductivities correlated strongly each other as well.

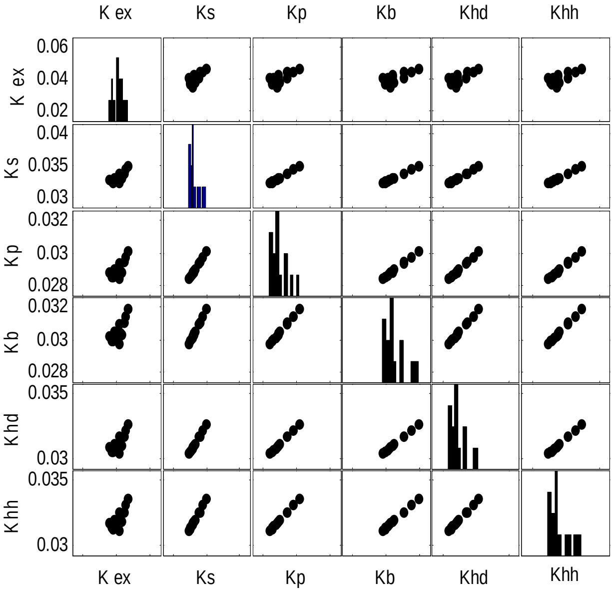

Fig. 4 Scatter plot map for thermal conductivities (case $P_O = P_S$)

The relation between experimental thermal conductivity, predicted conductivities (porosity P_S) and surface porosity P_S shown on fig. 5 demonstrates good prediction capability and the same trend. Apparent linearity between thermal conductivity and porosity is here due to relative small range of porosities.

The best prediction capability has upper limit K_P corresponding to the parallel arrangement or upper limit K_{hh}.

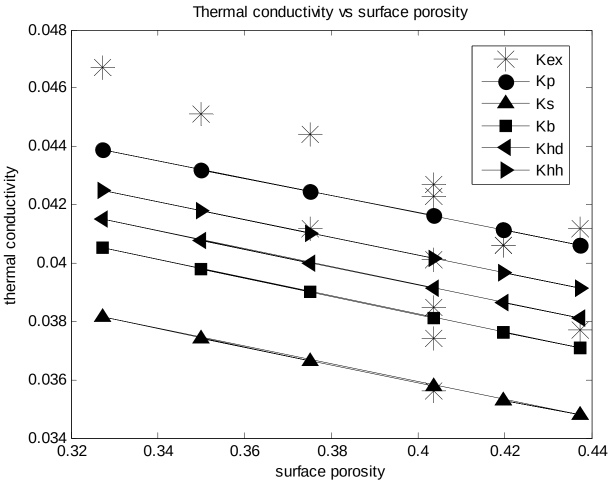

Fig. 5 Experimental and predicted thermal conductivities dependent on porosity P_S

CONCLUSION

On the basis of the carried out investigations the thermal conductivity of cotton type woven fabrics can be predicted from surface porosity P_S and model of parallel arrangements $K_{P.}$ Further improvements are in more precise definition of K_y and replacing the fibrous phase by yarn as composite of fibres and air. For rough prediction purposes is this approach quite sufficient and could be used for design purposes as well. This is the first step to shape the thermo – physiological features of fabrics by the appropriate designing of fabrics structure.

ACKNOWLEDGEMENT: This work was supported by the research project Textile Research Center 1M4674788501 of Czech Ministry of Education.

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