

A MILP Model for Energy Optimization in Multipurpose Batch Plants Using Heat Storage

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Abstract

The concept of heat integration in batch chemical plants has been in literature for more than a decade. Heat integration in batch plants can be effected in 2 ways, i.e. direct and indirect heat integration. Direct heat integration is encountered when both the source and the sink processes have to be active over a common time interval, assuming that the thermal driving forces allow. On the other hand, indirect heat integration allows heat integration of processes regardless of the time interval, as long as the source process takes place before the sink process so as to store energy or heat for later use. The thermal driving forces, nonetheless, must still be obeyed even in this type of heat integration. It is, therefore, evident from the foregoing statements that direct heat integration is more constrained than indirect heat integration. Presented in this paper is a mathematically rigorous technique for optimization of energy use through the exploitation of heat storage in heat integrated multipurpose batch plants. Storage of heat is effected through the use of a heat transfer fluid. The resultant mathematical formulation exhibits a mixed integer linear programming (MILP) structure, which yields a globally optimal solution for a predefined storage size.

1. Introduction

Until recently, heat integration has always been the privilege of continuous rather than batch chemical processes. This is mainly due to the fact that, in general, heat integration techniques assume steady-state behaviour, which is a feature of continuous processes. Moreover, batch operations tend to be less energy-intensive than their continuous counterparts. However, the increasing popularity of batch plants and the continuing global emphasis on emissions reduction is starting to warrant either the adaptation of the well-established heat integration techniques to or the development of novel techniques for batch processes. The increase in popularity is due to the flexibility and adaptability of batch plants, which is crucial in the current volatile market trends. It is also worthy of note that, although external utility requirement is a secondary economic issue in most batch facilities, e.g. agrochemicals and pharmaceuticals, it can be significant in others, e.g. dairy and brewing [1].

Early work on heat integration of batch plants was proposed by Vaselanak et al. [2]. These authors explored heat integration of batch vessels containing hot fluid that required cooling and cold fluid that required heating. Four cases were investigated. In the first case, the fluid from one vessel was allowed to return to the same vessel after exchanging heat with the fluid of another vessel via a common heat exchanger. In the second case, a heating or cooling medium was used to transfer heat between the hot fluid vessel and the cold fluid vessel, thereby maintaining the heat integrated fluids within the vessels throughout the heat exchange process. The third case entailed the transfer of fluids from their original vessels to receiving vessels while being heated or cooled. The fourth case was the combination of the above cases. Implicit in their analysis was the given schedule of the operations. A heuristic procedure was proposed for the cases where the final temperatures were not limiting and an MILP formulation for the cases where the final temperatures were limiting. Other methods that rely on a predefined schedule include the works of Kemp and Deakin [3] and Wang and Smith [4]. Although these authors considered opportunities for heat storage, the fact that they treated time as a parameter instead of a variable as it is actually the case in batch plants, is a significant limitation in their methods.

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The methodology presented in this paper is the extension of the methodology developed by Majozi [5] which was only aimed at direct heat integration of batch plants. The main advantages of this methodology are that the start and end times of processes need not be specified a priori and requires very few binary variables due to uneven discretization of the time horizon of interest. The extension pertains to the inclusion of heat storage as a possibility for saving more energy and allowing overall flexibility of the process. The mathematical model is linear which implies that the solution corresponding to a predefined size of storage is globally optimal.

2. Problem Statement

The problem addressed in this paper can be stated as follows.

Given:

- (i) production scheduling data, i.e. equipment capacities, task durations, time horizon of interest, recipe for each product as well as cost of raw materials and selling price of final products,
- (ii) hot and cold duties for tasks that require heating and cooling, respectively,
- (iii) cost of cooling water and steam,
- (iv) operating temperatures for the heat source and the heat sink operations,
- (v) allowed minimum temperature difference and
- (vi) available heat storage capacity,

determine the production schedule that results in minimum energy use or maximum profit. In the context of this paper, profit is defined as the difference between revenue and operating costs. The latter constitute raw material costs and external utility (cooling water and steam) costs. It is assumed that sufficient temperature driving forces exist between matched tasks for process–process heat transfer. Also, each task is allowed to operate either in an integrated or standalone mode. The integrated mode, in the context of this paper, is twofold, since a unit is allowed to be integrated with either heat storage or another operating unit. If heat integration cannot supply sufficient duty, external utility is supplied to complement the deficit. Whilst direct heat integration requires involved tasks to be active within a common time interval to effect direct heat transfer, they need not necessarily commence nor end at the same time. Moreover, the heat integrated tasks can either belong to the same process or distinct processes within reasonable proximity.

3. Mathematical Model

As aforementioned, the mathematical model proposed in this paper is an extension of the earlier work by the same author. It entails the following sets, variables and parameters.

Sets

$$U = \{u \mid u \text{ is a heat storage unit}\}$$

$$J = \{j \mid j \text{ is a processing unit}\}$$

$$J_c = \{j_c \mid j_c \text{ is a processing unit that requires cooling}\} \subset J$$

$$J_h = \{j_h \mid j_h \text{ is a processing unit that requires heating}\} \subset J$$

$$P = \{p \mid p \text{ is a time point}\}$$

$$S_{in,j} = \{s_{in,j} \mid s_{in,j} \text{ is an input stream to a processing unit}\}$$

$$S_{out,j} = \{s_{out,j} \mid s_{out,j} \text{ is an output stream from a processing unit}\}$$

Variables

$$t_p(s_{out,j}, p) = \text{time at which the stream is produced from unit } j$$

$$t_u(s_{in,j}, p) = \text{time at which the stream enters the processing unit } j$$

$$T_0(u, p) = \text{initial temperature in the storage vessel at time point } p$$

$$\begin{aligned}
T_f(u, p) &= \text{final temperature in the storage vessel at time point } p \\
CW(j, p) &= \text{amount of external cooling required by operation } j \text{ at time point } p \\
ST(j, p) &= \text{amount of external heating required by operation } j \text{ at time point } p \\
Q(j, u, p) &= \text{amount of heat exchanged with storage at time point } p \\
y(j, u, p) &= \begin{cases} 1 \leftarrow \text{if unit } j \text{ is exchanging heat with storage unit } u \\ 0 \leftarrow \text{otherwise} \end{cases} \\
x(j, j', p) &= \begin{cases} 1 \leftarrow \text{if unit } j \text{ is exchanging heat with another unit } j' \\ 0 \leftarrow \text{otherwise} \end{cases} \\
y(s_{in, j}, p) &= \begin{cases} 1 \leftarrow \text{if unit } j \text{ is active at time point } p \\ 0 \leftarrow \text{otherwise} \end{cases}
\end{aligned}$$

Parameters

$$\begin{aligned}
T(j) &= \text{operating temperature for processing unit } j \\
\tau(j) &= \text{duration of operation } j \text{ in standalone mode} \\
\tau'(j, j') &= \text{duration of operation } j \text{ when directly heat integrated} \\
\tau''(j, u) &= \text{duration of operation } j \text{ when integrated with storage} \\
\Delta T^{\min} &= \text{minimum temperature difference} \\
Q(j) &= \text{amount of heat required by or removed from the operating unit } j \\
M(u) &= \text{capacity of heat storage } u
\end{aligned}$$

Constraints

The mathematical model is based on the superstructure shown below. The heat transfer fluid in heat storage remains in the storage vessel during heat transfer with only the process fluid pumped around. The superstructure also shows that each unit is capable of receiving external heating or cooling in addition to direct and indirect heat integration

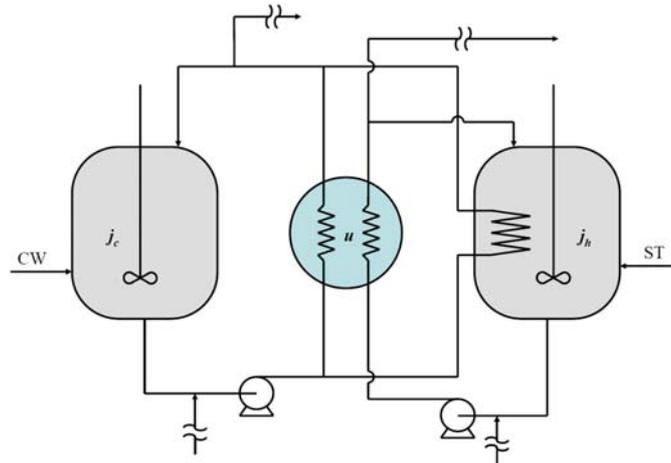


Figure 1 Superstructure for the mathematical model

In addition to scheduling constraints that have been presented in detail in another publication [4], the following constraints are necessary to cater for heat storage. Constraints (1) and (2) ensure that direct heat integration involves exactly one pair of units so as to simplify process operability. In essence, these constraints state that if 2

units are heat integrated at any given point in time, then these units must also be active at that point in time. However, if a unit is active at a given time point it is not necessary that it be heat integrated with another unit.

$$\sum_{j' \in J_c} x(j, j', p) \leq y(s_{in,j}, p), \quad \forall p \in P, j \in J_h, s_{in,j} \in S_{in,j} \quad (1)$$

$$\sum_{j \in J_h} x(j, j', p) \leq y(s_{in,j'}, p), \quad \forall p \in P, j \in J_c, s_{in,j'} \in S_{in,j'} \quad (2)$$

Constraints (3), (4) and (5) quantify the amount of heat transferred and received from storage unit, respectively. They ensure that if there is no heat integration between a processing unit and storage, then the amount of heat related to storage is not disturbed. T_{start} is the temperature of heat storage at the beginning of the time horizon.

$$Q(j, u, p) = M(u)c_p(T_f(u, p) - T_{start})y(j, u, p), \quad \forall j \in J_c \subset J, u \in U \quad (3)$$

$$Q(j, u, p-1) = M(u)c_p(T_f(u, p) - T_0(u, p-1))y(j, u, p-1), \quad \forall j \in J_c \subset J, p \in P, p > p_0, u \in U \quad (4)$$

$$Q(j', u, p-1) = M(u)c_p(T_0(u, p-1) - T_f(u, p))y(j', u, p-1), \quad \forall j' \in J_h \subset J, p \in P, p > p_0, u \in U \quad (5)$$

Constraint (6) ensures that only one unit is heat integrated with storage at any given point in time. Constraints (7) and (8) ensure that the temperature of the storage unit is not changed if there is no heat integration with any unit. These constraints carry the same meaning as constraints (3) - (5). Nonetheless, they are necessary since they pertain to temperature whilst the latter pertain to the amount of heat. Overall, constraints (3) - (8) govern the relationship between heat and temperature of storage.

$$\sum_{j \in J_c} y(j, u, p) + \sum_{j' \in J_h} y(j', u, p) \leq 1, \quad \forall p \in P, u \in U \quad (6)$$

$$T_0(u, p-1) \leq T_f(u, p) + \max_j \{T(j)\} \left(\sum_{j \in J_c} y(j, u, p-1) + \sum_{j' \in J_h} y(j', u, p-1) \right), \quad \forall p \in P, p > p_0, u \in U \quad (7)$$

$$T_0(u, p-1) \geq T_f(u, p) - \max_j \{T(j)\} \left(\sum_{j \in J_c} y(j, u, p-1) + \sum_{j' \in J_h} y(j', u, p-1) \right), \quad \forall p \in P, p > p_0, u \in U \quad (8)$$

Constraint (9) ensures that the initial temperature in heat storage at any given point in time is the same as the final temperature at the last time point. This condition is always true, regardless of the heat integration status in the previous time point.

$$T_0(u, p) = T_f(u, p-1), \quad \forall p \in P, u \in U \quad (9)$$

Constraints (10) and (11) ensure that if there is heat integration between any unit and heat storage, then the stipulated minimum driving force should be obeyed. Constraint (10) applies if heat storage is integrated with the heat source, whilst constraint (11) applies if heat storage is integrated with the heat sink.

$$T(j) - T_f(u, p) \geq \Delta T^{\min} - \max_j \{T(j)\} (1 - y(j, u, p - 1)), \quad (10)$$

$$\forall p \in P, p > p_0, j \in J_c \subset J, u \in U$$

$$T_f(u, p) - T(j) \geq \Delta T^{\min} - \max_j \{T(j)\} (1 - y(j, u, p - 1)), \quad (11)$$

$$\forall p \in P, p > p_0, j \in J_h \subset J, u \in U$$

Constraint (12) states that cooling in any heat source will be accomplished either by direct heat integration, external cooling or heat integration with storage. Constraint (13) is similar to constraint (12) but applies to a heat sink.

$$Q(j)y(s_{in,j}, p) = Q(j, u, p) + CW(j, p) + \sum_{j' \in J_h} \min_{j, j'} \{Q(j), Q(j')\} x(j, j', p) \quad (12)$$

$$\forall j \in J_c \subset J, p \in P, u \in U$$

$$Q(j)y(s_{in,j}, p) = Q(j, u, p) + ST(j, p) + \sum_{j' \in J_c} \min_{j, j'} \{Q(j), Q(j')\} x(j, j', p) \quad (13)$$

$$\forall j \in J_h \subset J, p \in P, u \in U$$

Constraints (14) and (15) state that if a unit is directly heat integrated with another unit, then it cannot be simultaneously integrated with heat storage. This is also a condition imposed solely to simplify operability of the overall process.

$$\sum_{j' \in J_h} x(j, j', p) + y(j, u, p) \leq 1, \quad \forall j \in J_c \subset J, p \in P, u \in U \quad (14)$$

$$\sum_{j \in J_c} x(j, j', p) + y(j', u, p) \leq 1, \quad \forall j' \in J_h \subset J, p \in P, u \in U \quad (15)$$

Constraint (16) is a feasibility constraint which ensures that if a unit is not integrated with storage, then the associated duty should not exist.

$$\delta y(j, u, p) \leq Q(i, u, p) \leq \max_{j \in J} \{Q(j), Q(j')\} y(j, u, p) \quad (16)$$

Constraint (17) shows how the variation in duration due to the heat integration mode is accounted for in the mathematical model. It is very likely that the duration times will be affected by the mode of operation and this should not be ignored in the formulation.

$$t_p(s_{out,j}, p) = t_u(s_{in,j}, p - 1) + \tau(j)(1 - y(j, j', p) - y(j, u, p)) \quad (17)$$

$$+ \tau''(j, u)y(j, u, p) + \tau'(j, j')y(j, j', p),$$

$$\forall j \in J, p \in P, u \in U, s_{in,j}, s_{out,j} \in S$$

The foregoing constraints constitute the full heat storage model. With the exception of constraints (3) - (5), all the constraints are linear. Constraints (3) - (5) entail nonconvex bilinear terms which render the overall model a nonconvex MINLP. However, the type of bilinearity exhibited by these constraints can be readily removed without compromising the accuracy of the model using the so called Glover transformation.

4. Literature example

Figure 2 is the representation of the case study that was used to demonstrate the performance of the proposed model it is taken from literature [6]. Whilst direct heat integration resulted in 36% improvement in terms of external heat load, use of heat storage showed more than 82% improvement.

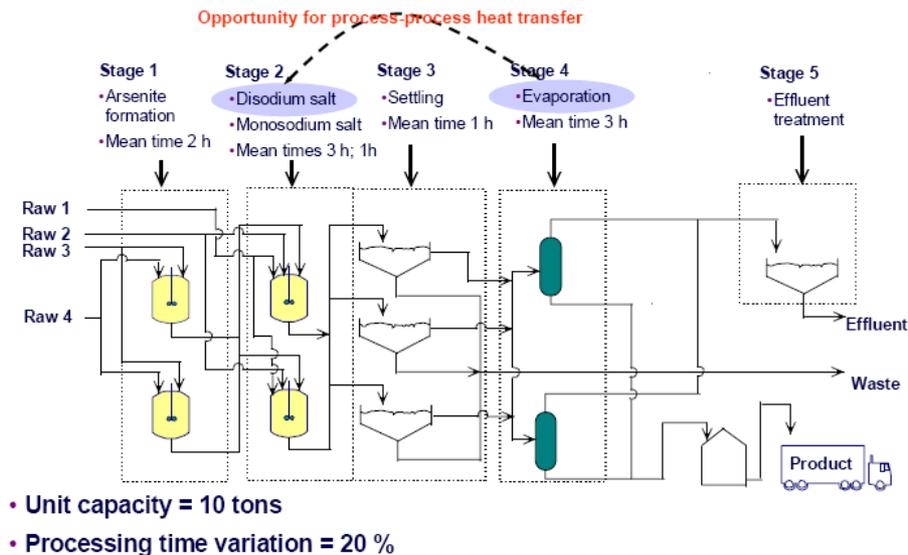


Figure 2 Process flowsheet for the case study

5. Conclusions

A mathematical approach for optimization of energy use in heat integrated multipurpose batch plants has been presented and tested in a literature example. The results have shown that heat storage certainly results in more energy savings than direct heat integration.

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