NUMERICAL AND EXPERIMENTAL ANALYSIS
OF MIXED CONVECTION IN EXCHANGERS

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ABSTRACT
Fields of vector speed have been measured by the method of laser visualisation (PIV) in the case of a vertical heat exchanger in co current in mixed convection. The reversal zones have been given prominence for different numbers of Grashof and Reynolds. Where the internal and external flows are weak (in the region of 51/h) and the differences of internal and external temperature are in the region of 20°C, we see reversal zones appear in the two places simultaneously. These results show the importance of studying the thermal exchange of a heat exchanger in its totality and call into question the traditional methods of proportionality of a heat exchanger.

NOMENCLATURE
D Tube diameter
L Longer
Gr Grashof number
Pr Prandtl number
g Acceleration gravity
Re Reynolds number
V Axial velocity component
X, Y, Z Cartesian coordinates (Figure 1)
a Tube inclination

INTRODUCTION
The bi tube heat exchangers in co or contra currents, in a vertical or horizontal position are still very much in evidence today, notably in agribusiness (pasteurisation) or in the solar heating of swimming pools. The conditions of use of strong temperature gradients and weak loss, often resemble the study of mixed convection, that is to say when the conjugate effect of the forced convection and of natural convection due to the effects of the Archimedes principle is present.

A large number of numerical studies are concerned with heated pipes or isotherms in mixed convection. Significant results have been obtained by Hallman and al [1], Jackson and al [2], Wang and al [3], with horizontal and vertical tubes.

Reversal zones as well as instability have been highlighted by Zghal and al [4] and Chung and al [5].

The referenced experimental studies of Barrozi and al [6]; Zeldin and al [7]) describe the flow in a tube which has been uniformly heated, stressing temperature scales (most often in micro thermal couples) and axial velocities (pitot micro tubes essentially) and for certain points in specific plans.

This technique allows us to obtain really interesting results but not a total and continual vision of flow. Furthermore the presence of sensors in the flow of feeble loss causes disturbance.

Other experimental studies use a technique of visualisation, such as Lavine and al [8], Bernier and al [9] and Benhamou and al [10], who inject fluorescent particles into the flow before illuminating them with a diffuse light source. The first are interested in reversals where the unfavourable mixed convection for a vertical tube and the latter studied flow in an oscillating tube. The visualisations of hydrodynamic fields of flow in a heated or cooled tube in a vertical position do not appear to have been tackled.

Other numerical studies are concerned with the flow in the annular space of the heat exchanger bi tube such as Ianello and al [11] and Aung and al [12], but very few studies consider the context of a heat exchanger as a whole, taking into account the conduction of the inner wall. However our work Voicu and al [13] has shown important interactions due to the phenomena of axial diffusion and of reversal in mixed convection in a bi tube heat exchanger which disturbs thermal transfers. Another one of our studies [14] makes appear simplistic conditions which consist of considering the temperature or the constant flux of the inner wall, cause errors in the proportionality of the heat exchanger.

Studies on visualisation of the flow in these conditions do not seem actually to have been developed.

This text proposes to visualise the phenomena of reversal on the inner wall of a vertical bitube (or equivalent) heat exchanger.
The description of the theoretical study and the experimental operation follows, and then the validation of our numerical and experimental approach for a heated and cooled tube, the experimental study of hydrothermal fields in a vertically ascending co-current heat exchanger will be described using the technique PIV.

**EXPERIMENTAL STUDY**

We are interested in this study in the development of flow on the one hand in a vertical isothermal tube and on the other hand in a heat exchanger bi-tube (or equivalent) still in the vertical position. The schematic representation of the system studied is presented figure 1 using as reference the interior diameter of the inside tube D, the heat exchanger being a length of 100D. It is preceded by a vertical adiabatic zone of a length equivalent also to 100D allowing us to ensure a parabolic profile of Poiseuille at the insertions. The fluid is water and the flow is said to be stationary and laminar.

![Figure 1](image1.png)  
**Figure 1** Schematic representation

**Experimental set up**

The experimental plan is presented in figure 2. The designation of the elements is detailed in diagram 1. This setting allows the visualisation of internal and external flow of a fluid in a heat exchanger. The temperature control, of flows allows covering the Reynolds numbers from 0 to 1000, the Grashoff numbers from 0 to $10^5$; this is for a number of Prandlt data. The device is constituted from a tube of transparent glass with an inside diameter of 21 mm, 2 mm thick and 2 m long. This tube is placed in a squared section tube with an 80 mm side see figure 3, in Plexiglas covered with a styrodur type thermal thickness of 3 cm.

![Figure 2](image2.png)  
**Figure 2** Experimental set up

This area of visualisation limits the refraction of laser ray incident. The entrance of the tube and the annular space (between the internal and external tube) are each linked to a thermal reservoir with a constant level by a flexible cover of an insulated sheath 13 mm thick. These reservoirs have a capacity of 30 l and are situated at 5 m above the reference plane. The loss of the fluid entering into the tube as well as into the annular space is obtained by gravity and controlled by a miniature flow meter measuring 0.25 l/h. These losses are measured at the exit with the aid of a graduated vase with an absolute uncertainty of 5 ml.

The temperatures of the two fluids circulating respectively in the tube and in the annular space are controlled by type J thermocouples, measuring 0.3 °C , placed at the entrances and exits.

An opening 10 cm long and 5 mm thick has been made on the outer face and allows the passage of a light plane.

Everything is fixed on a soldered metal chassis, allowing on the one hand support of the effort generated by the weight of the ensemble (chamber + water + tube) and on the other hand to displace all the equipment camera / laser, thus avoiding the problems of vibration.

Finally the adiabatic zones in front of the heat exchanger are vertical and 2 m in length so as to ensure the obtaining of a developed entry flow. Condition, is shown in contrast to a possible bi-directional heat extraction boundary condition configuration.

**Visualisation technique**

The visualisation technique used is the PIV (Particle Image Velocimetry). It requires seeding, illumination by a lighting
plane obtained from a laser source, a camera, a system of acquisition and of image synchronisation.

This technique measures the displacement of a particle in a time interval and allows the obtaining of fields of velocities. The image of the camera is divided into interrogation zones and for each of these zones; we measure the displacement of the particles between two images taken in the interval of time between two laser beams. The two images taken by the same camera are transformed into pixels, then changed into the Fourier function (Fast Fourier Transformation), processed by an adaptive correlation and by a statistical study; then averaged into vector fields.

The lighting source is a YAG laser pulse emitting a laser beam with wave length 532 nm, with an energy of 0.120 Joules during 10ns. The pulse mode enables the sending of beams with 30ms spacing, and this every 66 ms. The 1.4 mm diameter beam obtained allows, by an optical system, to obtain a luminous plane 6 mm thick. A synchronisator enables the guiding of this pulsation and the synchronisator with an image taken by a camera. This camera is a Flow sensor camera M2/E 8 bits, and it has a shutter delay of 3 microseconds and an acquisition and of image synchronisation.

Governing equations. The flow is assumed to be laminar, steady and two dimensional (all the variables depend only on the radial and axial coordinates). Radial and axial conduction in the glass walls is taken into account; the conductivity of glass is considered uniform on the two sides. The profile is inherent to the form of the passage section and corresponds to that of a constant velocity.

Finally a statistical method average of all the picks obtained into vector fields. The density of the nylon is close to that of water to adapt. The density of nylon is close to that of water to simplify the computation. The results obtained [15] indicates an error relative to 0.5%on the scale of axial velocity.

Validation of experimental method

We have first of all validated our experimental setting by freeing the temperatures. The flow loss in the interior tube is 4.5 l/h and in the exterior tube 108 l/h, which corresponds respectively to the Reynolds numbers of 65 and 314.

The results, figure 3, show the evolution and the adimensional velocity (V/Vo) according to the beam for the different sections (L=2D, 3D, 4D, and 5D).

The disturbances upstream are due to a style of injection of fluid in this space. In fact the fluid arrives on one side only and this is only after several diameters that the flow can be considered uniform on the two sides. The profile is inherent to the form of the passage section and corresponds to that of a constant velocity.

These measures enable our experimental system to be validated (flow symmetry, parabolic profile at the entrance of the tube) as well as the technique of visualisation of flow and the precision of the measurement.

**STUDY OF ISOTHERMAL VERTICAL TUBE**

Numerical study

Governing equations. The flow is assumed to be laminar, steady and two dimensional (all the variables depend only on the radial and axial coordinates). Radial and axial conduction in the glass walls is taken into account; the conductivity of glass is assumed to be constant. Therefore, the thermo-hydrodynamic evolution for this conjugate heat transfer problem is modeled by the following equations.

For the fluid:

$$\frac{\partial (\rho v_r)}{\partial z} + \frac{\partial (\rho v_z)}{\partial r} + \frac{\rho v_r}{r} = 0 \quad (1)$$

$$\rho \left( v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_z}{\partial z} \right) = -\rho g - \frac{\partial p}{\partial z} + \frac{\mu}{3} \frac{\partial}{\partial z} \left( 2 \frac{\partial v_z}{\partial z} - \frac{\partial v_r}{\partial r} -\frac{v_r}{r} \right) \quad (2)$$

$$\frac{\partial}{\partial r} \left[ \frac{\mu}{r} \left( \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} \right) \right] + \frac{\mu}{r} \left( \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right) \quad (3)$$

$$k \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + k \frac{\partial}{\partial z} \left( \frac{\partial T}{\partial z} \right) + \Phi = \rho c_p \left( v_r \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z} \right) \quad (4)$$

Where
\[ \Phi = 2\mu \left[ \left( \frac{\partial N_r}{\partial r} \right)^2 + \left( \frac{v_r}{r} \right)^2 + \left( \frac{\partial N_z}{\partial z} \right)^2 \right] + \]
\[ \frac{1}{2} \left( \frac{\partial N_r}{\partial z} + \frac{\partial N_z}{\partial r} \right) - \frac{1}{3} (\nabla q)^2 \]  
(5)

and

\[ \nabla q = \frac{\partial N_r}{\partial t} + \frac{\partial N_z}{\partial z} + \frac{v_r}{r} \]  
(6)

For the solid, conservation of energy is expressed by

\[ k \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) - k \frac{\partial}{\partial z} \left( \frac{\partial T}{\partial z} \right) = 0 \]  
(7)

Boundary Conditions. The fluids are assumed to enter the system at \( z = -L' \) with uniform axial velocities and temperatures. At the outlet from the downstream adiabatic zone \( (z = L+L'') \) it is assumed that both flow fields are fully developed since this zone is very long. At the fluid-solid interfaces \( (r = 0.5D1) \) the constant temperature is applied.

Numerical solution and code validation. The system of coupled, non-linear, elliptic partial differential equations 1-7, subject to their boundary conditions, has been solved by finite volume approach with the aid of software FLUENT (2002). A second-order method has been employed throughout to compute the heat and momentum fluxes. The resulting discretised equations have been solved in a sequential manner, using the combination of the multiple-and-alternate-sweeping technique, the line-by-line technique and the TDMA (Three-Diagonal Matrix Algorithm). The pressure correction equation has been employed to calculate the pressure field and to correct the assumed velocity field in order to progressively satisfy all the discretised equations. Convergence criteria, grid independence tests as well as the validation of the model and the numerical procedure have been reported earlier [16] (Voicu et al, 2006). They are not repeated here for lack of space.

Comparison experimental numerical

Firstly we studied the situation in a vertical isothermal tube. For that one we chose a quite high flow (108 l/h) in the annular space and ensured that the exit temperature is close to the entry temperature. In these two trials presented, the temperature difference between entry and exit is inferior to 2°C with reference to the accuracy of the thermocouples validates the hypothesis of an isothermal tube.

Two situations were studied: study 1 with a heated tube and study 2 with a cooled tube. In both studies the flow of water entering the inside tube is 4.5 l/h which corresponds to a Reynolds number of 79 for the interior tube. The flow in the exterior space leads to a Reynolds number of 109. The results are presented in an adimensional form using: diameter \( D \) for the length, speed of entry \( v_{oc} \) for the velocities.

The results Figures 4 and 5 compare the evolution of the adimensional velocity \((V/V_{o})\) in the interior tube obtained by the PIV method with numerical results, according to the diameter and for the two sections \((L=3D\) and \(5D)\).

In the study 1 (figure 4), the temperature at the entrance of the inside tube is \( 23°C \) and the temperature of the exit is \( 38°C \). The external temperature is \( 43°C \), which corresponds to a Grashoff number of \( 3.9 \times 10^5 \).

In the study 2 (figure 28), the entrance temperature of the inside tube is \( 38°C \) and the temperature of the exit is \( 24.3°C \). The external temperature is \( 24°C \), which corresponds to a Grashoff number of \( 1.10^6 \).

Study 1: the velocity profiles are symmetrical everywhere; the maximum velocity is to be found near the walls. The behaviour of the axial velocity is due to the acceleration to which the fluid is subjected, which is heated up first near to the wall while the cooled fluid goes back down again affecting more the halfway centre of the tube, and this from the entrance of the tube \((L=3D \) to \( 5D)\). We note that the maximum velocities near the walls reach 2.6 to 3 times the initial speed and this symmetrically, whilst the velocity at the centre is near to zero, we observe negative velocities a the centre of the half way marks of the tube.

The lack of relative symmetry observed at the level of the velocities near to the walls, is due to the system of water injection in the annular space, a phenomenon described in the validation, and by the inaccuracies on the time for temperature stabilisation. The course of the water in the annular space explains the greater thickness of the positive velocity zones on the left of the tube in comparison with the simulation. The maximums of positive and negative velocities are weaker than in the simulation, on the one hand for the conservation of the
flow and on the other hand we observe delay in the standardisation of the temperature of the walls. These maximums increase with Z and confirm this observation. Thus it can be considered that the experiment allows an accurate observation of the phenomenon.

\[ -2 \leq R \leq 5 \]

\[ V \]

Figure 5 evolution of axial velocity, experimental/numerical, study 2

Study 2: the influence of the Archimedes principle which in this case opposes the flow is much more important. In fact, we observe a maximum of axial velocities six times greater than the average entrance velocity in the central part of the tube and negative velocities near the wall, near to \( v_{oc} \). The cooling on the walls, tends to make the fluid go back down again near the surfaces, that is towards \( L=5D \) that the effect on the value of the negative velocities and jointly on the maximum velocity at the centre is the most noticeable. We observe the same phenomenon of dissymmetry in relation to the simulation, due to the method of fluid injection in the annular space. The delaying effects are less pronounced. The uncertainties close to our experimental scales precisely describe these phenomena.

The fact that in study 1 and 2, we obtain negative velocities, even on the plane where heat exchange starts) is particularly important. It means that the cooled fluid is transported upstream into the adiabatic zone. This is due to the phenomena of axial diffusion. It is obvious that these reversal zones which are perceptible as well as the phenomena of axial diffusion in the adiabatic part, have an influence on the Nuusel numbers and therefore on the calculation of the proportionality of the heat exchangers.

STUDY OF VERTICAL EXCHANGER

The case of a comprehensive heat exchanger is studied experimentally in a vertical position and ascending water flows (flow in parallel currents). The wall not visible on the figure is in glass 2mm thick.

To study the influence of thermal transfer on velocity fields on the areas heat exchanger surfaces in mixed convection of two studies is presented. In study 1 of a heated tube and study 2 of a cooled tube. For each case the flow of the inside tube is 4.5 l/h, which makes a Reynolds number of 79 and a Grashof of \( 10^6 \) for study 1 and of \( 10^9 \) and \( 3.24 \times 10^5 \) for study 2.

The results Figures 6 and 7 show the evolution of the adimensional velocity (V/VO) in the half interior and exterior tube obtained by the PIV method, according to the diameter and for the sections (L = 3D, 4D et 10D).

Figure 6 evolution of axial velocity, experimental exchanger study 1

In study 1, figure 6, the flow outside the tube is 14.4 l/h which makes a Reynolds number of 66. Inlet temperature is
39.4°C and outlet 35°C. For the tube Inlet temperature is 23.3°C and outlet 33°C, which makes a Grashof number of 7.4 10^3.

In study 2, figure 7, the flow outside the tube is 6.4 l/h which makes a Reynolds number of 21.4. Inlet temperature is 23°C and outlet 27.3°C. For the tube Inlet temperature is 38.6°C and outlet 27.4°C, which makes a Grashof number of 2.13 10^3.

Which correspond to Richardson number of 1 inside and outside the tube.

As regards the profiles of velocities in the internal tube, we notice the same phenomena of acceleration near walls by a warmed tube and in the center for a tube cools. At the same time in the annular space the inverse phenomenon occurs because the warm fluid cools at the same time as the fluid of the tube warms itself and conversely.

On the other hand the effect is more pronounced and increases until the distance 10D. It seems that the zone of reversal appears on all the length of the tube.

These phenomena influence temperature of surface, and have an influence on the Nusselt numbers and therefore on the calculation of the efficient and sizing of the heat exchangers.

CONCLUSION

This study described an experimental method of visualization and obtaining of the fields of vector velocity in a vertical heat exchanger. This method was validated by a digital simulation in the case of a vertical isothermal tube.

The obtained results show zones of reversals flow pronounced in every case studied. These phenomena seem different when we consider a isothermal or an heat exchanger in general. This shows well that:

In mixed convection, the sizing of a heat exchanger is conditioned by the presence or not of zones of reversal flow.

The studies which consist in considering the constant temperature in the internal wall or an exponential variation, in the calculation of the thermal performances of a heat exchanger, seem approximate.

REFERENCES