HEFAT2012
9th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
16 – 18 July 2012
Malta

LONG WAVE INSTABILITY DEVELOPMENT IN THE LAMINAR BOUNDARY LAYER NEAR POROUS WALL. ASYMPTOTIC ANALYSIS

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ABSTRACT

As a result of laminar flows analysis for large Reynolds number asymptotic theory was developed to describe variety of flows with relatively large longitudinal gradients where classical Prandtl's boundary layer theory should be replaced by another theory. The most familiar example was associated with the theory of free interaction which allows to describe flows with small separated regions. This theory is applicable as well for many other flows including abrupt change in the boundary conditions, flows with reattachment etc.

Unsteady free interaction theory allowed to describe long wave instability processes in the laminar boundary layers. In fact linearized variant of this theory may be deduced from original Orr-Sommerfeld equation. At the same time asymptotic theory may be useful to describe nonlinear instability processes as well. It is important that boundary condition on the wall describing relation between pressure change and vertical velocity is linear and doesn't change uniformity of the problem. So it is possible to investigate as well linear stability problems incorporating early obtained results.

Presented are results of stability analysis describing long wave disturbances development.

These results may be useful to provide passive boundary layer flow control along with the buffet onset control.

NOMENCLATURE

u =tangent speed,

v =normal speed,

P =pressure disturbance,

γ =Darci coefficient,

α =wave nuber,

τ =time delay coefficient,

ω_i, ω_r =imaginary and real parts of field disturbances

Unless cpecifically stated, all of the variables are assumed to be

for the case of viscous wall layer.

INTRODUCTION

New materials development, in particular porous metal technology, lead to the opportunity to create new passive methods of the boundary layer flow control. It may be used to delay boundary layer separation as well as to delay laminar-turbulent transition.

Porous metal structure usually is associated with the surface flow suction (injection) due to pressure difference between external and internal surfaces of porous metal plate. In many cases it may be supposed that distributed mass transfer will exist which will obey Darcy law (or linear dependence between

vertical velocity distribution on the wall and pressure change distribution).
From mathematical point of view this condition allows to reconsider many early obtained classical results describing self-induced boundary layer separation in case of passive control. This model includes boundary layer equations with an additional relation determining induced pressure distribution.

PROBLEM FORMULATION

Porous wall structure supposes that pressure difference on the external and internal sides of porous surface may lead to the distributed suction (in the regions of relatively high pressure) or distributed injection. In many cases it may be supposed that mass transfer obeys the Darcy law (or linear dependence between vertical velocity on the wall and disturbed pressure distribution).
In fact this boundary condition allows us to reconsider early obtained results [1-3] describing self-induced boundary layer separation for the case of passive flow control.

Using results obtained in [1-4] mathematical problem for flows

near porous walls may be formulated as follows

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2}$$
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$y=0 \quad v=-\gamma p,\ u=0 \qquad y\to\infty \ u=y+A(x)$

$x\to-\infty \quad u=y$

$p=-\frac{\partial A}{\partial x}$ for supersonic flow

$p=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{A(x,t)}{(\xi-x)}d\xi$ for subsonic external flow

Equations presented are expressed in similarity variables [4].This problem differs from problems describing disturbed flow near impermeable wall due to condition for the vertical velocity on the wall [1-4]. In fact such conditions are well known in fluid mechanics [5]. We will use below generalized Darcy law with the inclusion so called time delay

$v(x,t)=-\gamma p(x,t-\tau)$

That is, normal velocity component changes later than pressure difference appears. This delay can be explained by a finite mass of the fluid considered inside a pore.

For small values of self-induced pressure next form of solution

may be considered

$$u=y+u_1,\ v=v_1,\ \ p=p_1 \qquad (2)$$

This form of solution gives next form of equations for the first approximation

$$\frac{\partial u_1}{\partial t}+y\frac{\partial u_1}{\partial x}+v_1+\frac{\partial p_1}{\partial x}=\frac{\partial^2 u_1}{\partial y^2} \qquad (3)$$
$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=0$$

As usual solution may be presented in the normal mode

approximation

$$(u_1,v_1,p_1,A)=e^{i\alpha(x-ct)}(U,V,P,B)$$
$$-i\alpha cU+iy\alpha U+V+i\alpha P=U''$$
$$i\alpha U+V'=0 \qquad (4)$$
$$U(\infty)=B$$
$$U(0)=0,\ V(0)=-\gamma P e^{-i\alpha c\tau}$$
$$y\alpha U'=U'''$$
$P=-i\alpha B$ for supersonic external flow
$P=\alpha B$ for subsonic external flow

$$i\alpha(y-c)F=F'' \quad F=V'$$
$$B=-\frac{1}{i\alpha}\int_0^{\infty}F\,dy,\ F(\infty)=0 \qquad (5)$$
$$F'(0)=i\alpha P(\gamma e^{-i\alpha\tau}-i\alpha)$$

After some transformations dispersion relation may be deduced. But for numerical analysis it may be possible to use aforementioned problem formulation.

RESULTS

We can present results as dependencies

$\omega_r+i\omega_i=f(\alpha)$, where

$\omega=i\alpha c$

Results have been obtained for different values of parameters $\gamma = 0 \div 0.4$, $\tau = 0 \div 1$.

Results obtained for impermeable wall ($\gamma = 0$) coincide with results obtained in [1].

At first, let's consider the results in case of supersonic regimes. On the figures 1 and 2 influence of coefficient γ is presented. It can be concluded that real part of parameter ω doesn't change significantly. At the same time velocity of long waves strongly depends on γ as does imaginary part of ω .

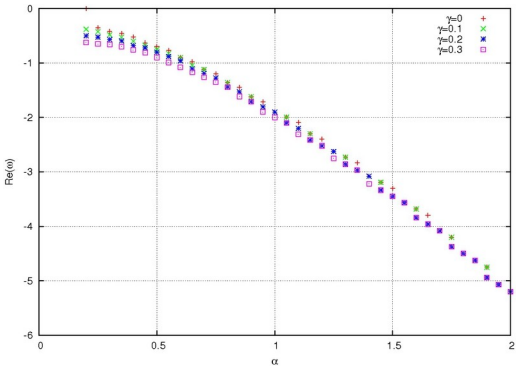

Figure 1: Relation of Re(ω) to α in case of subsonic regime without delay.

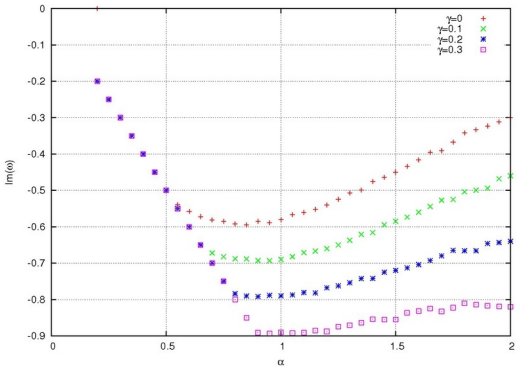

Figure 2: Relation of Im(ω) to α in case of subsonic regime. Penetrability dependance at $\tau = 0$.

On the next figures 3 and 4 influence of time delay parameter τ is presented. It may be concluded that mainly influenced is imaginary part of ω

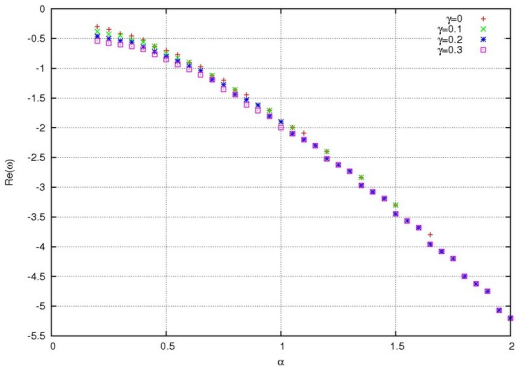

Figure 3: Relation of Re(ω) to α in case of supersonic regime.

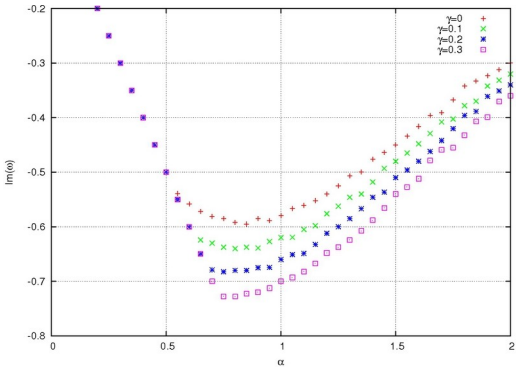

Figure 4: Relation of Im(ω) to α in case of subsonic regime. Penetrability dependance at $\tau = 0.4$.

Subsonic regimes were analyzed as well. On subsequent figures 5 and 6 influence of parameter γ is presented with time delay parameter equal to 0. It can be seen that in case of subsonic regimes the main parameter affected is imaginary part of ω while real part remains almost unaffected. That is, the amplification of small waves can be controlled. With small values of τ they tend to be amplified faster.

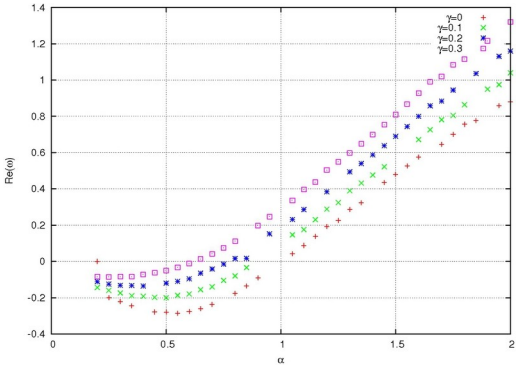

Figure 5: Relation of Re(ω) to α in case of supersonic regime without delay.

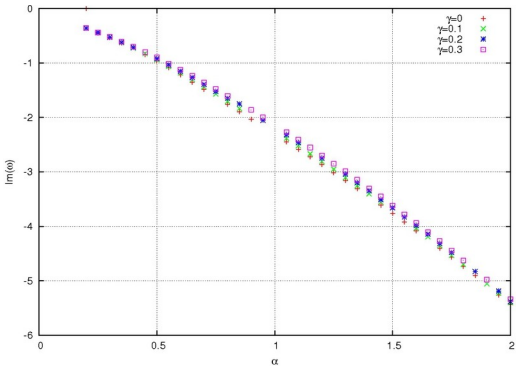

Figure 6: Relation of Im(ω) to α in case of supersonic regime without delay.

But if we take a look at dispersion relation with higher delay parameter, we may see a much more interesting picture(fig. 7 and fig. 8).

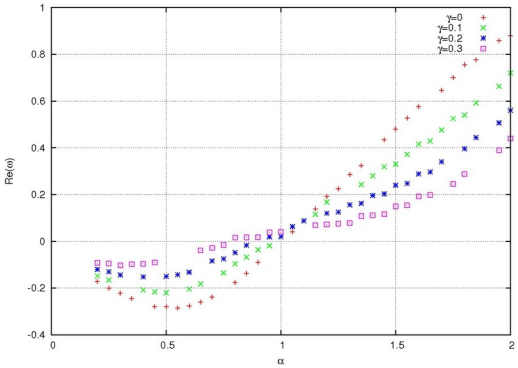

Figure 7: Relation of Re(ω) to α in case of supersonic regime. Delation parameter is equal to 0.8

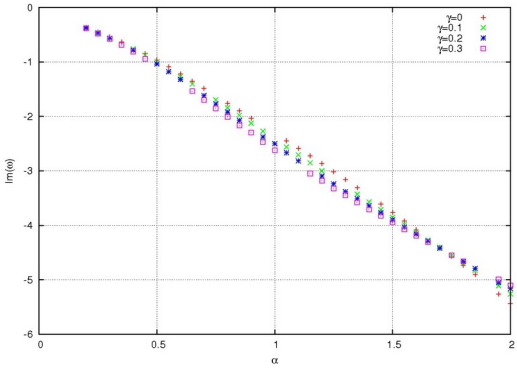

Figure 8: Relation of Im(ω) to α in case of supersonic regime. Delation parameter is equal to 0.8

It is seen that on figure 7 that the amplification level of relatively small waves becomes weaker, that is laminar-turbulent tranzition can be delayed!

Estimated values show that the problem considered can be applied in case of a real-scale aircraft at it's cruise speeds with a pored layer of reasonable thickness.

In general results presented show that permeable wall may be used to influence long waves parameters, providing influence on the laminar-turbulent transition and real-life applications as well

CONCLUSION

Generalized theory of long wave instability in the laminar boundary layer near porous wall is presented. This theory is based on the asymptotic analysis of Navier-Stokes equations leading to the so called triple deck disturbed flow structure. Linearized equations for small disturbances amplitudes may be reduced to the Airy equation and corresponding dispersion relation. This relation analysis shows that porous wall condition for vertical velocity significantly changes wave chracteristics like amplification factor and so on. In fact such method of passive boundary layer flow control may be as well used to prevent boundary layer separation. Corresponding method of passive boundary layer flow control may be useful as well to influence buffet onset.

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