HEFAT2012
9th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
16 – 18 July 2012
Malta

ENTROPY GENERATION IN HEAT AND MASS TRANSFER IN POROUS CAVITY SUBJECTED TO A MAGNETIC FIELD

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ABSTRACT

Numerical investigation is carried out to predict the entropy generation for combined natural convection heat and mass transfer in a two dimensional porous cavity subjected to a magnetic field. The Darcy model is used in the mathematical formulation of the flow in porous media. The mathematical model is derived in dimensionless form. The governing parameters arise in the mathematical model are the Rayleigh number, Lewis number, buoyancy ratio and Hartmann number. The entropy generation is obtained as a function of velocity, temperature, concentration gradients and the physical properties of the fluid. The results are presented as average Nusselt number, Sherwood numbers and dimensionless form of local entropy generation rate for different values of the governing parameters.The numerical results show that increasing the magnetic field parameter (Hartmann number) leads to reduce the flow circulation strength in the cavity and this leads to a decrease in the rate of entropy generation.

INTRODUCTION

The natural convection in porous media has been studied and analysed widely in recent years. This interest was estimated due to many applications in, for example, packed sphere beds, high performance insulation for buildings, chemical catalytic reactors, to name of a few. Representative studies in this area may be found in the books by Kaviany [1], Nield and Bejan [2] and Ingham and Pop [3]. Double-diffusive convection in porous media concerns the processes of combined (simultaneous) heat and mass transfer which are driven by buoyancy forces. The buoyancy force not only affected by the difference of temperature, but also affected by the difference of concentration in the fluid. A detailed review of double-diffusive natural convection in porous media can be found in Mojtabi and Mojtabi [4]. Natural convection in a cavity saturated with porous media in the presence of magnetic a field is relatively a new topic and needs more investigation. The heat, mass and fluid flow can be described by means of the hydrodynamics, the convective heat and mass transfer mechanism and the electromagnetic field as they are linked together. In such cases, the fluid experiences a Lorentz force, which tends to oppose the fluid flow and hence reduce the flow velocities [5-6]. The irreversibility phenomena which are expressed by entropy generation are of important interest during the design of any thermodynamic system. Many studies concerning entropy generation in natural convection in porous media have been carried out [6-8]. However, the entropy generation during the double diffusive convection in enclosed cavities submitted to a magnetic field has not received much attention. The aim of this paper is to study numerically the problem of entropy generation in heat and mass transfer in square porous cavity filled with electrically conducting fluid and subjected to a magnetic field. A schematic diagram of the porous cavity and coordinate system is shown in Figure 1. Horizontal temperature and concentration differences are specified between the vertical walls and zero mass and heat fluxes are imposed at the horizontal walls.

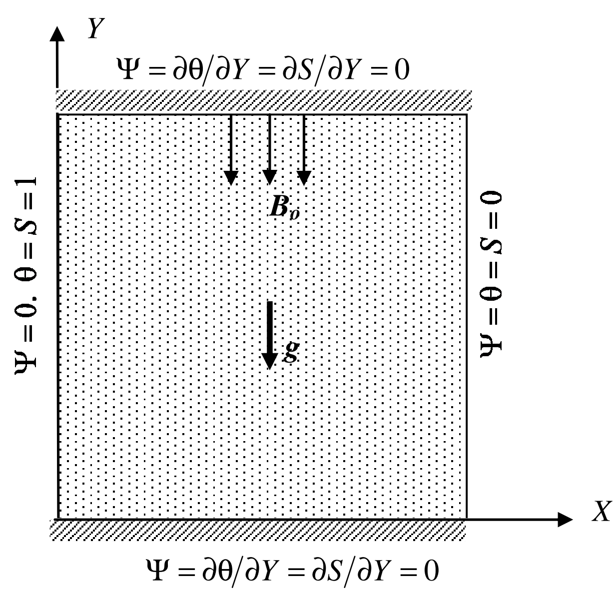


Figure 1 Schematic diagram of the physical model and coordinate system.

MATHEMATICAL FORMULATION

The mathematical model in the present problem is formulated based on the following assumption:

1. The convective fluid and the porous media are in local thermal equilibrium.
2. The properties of the fluid and the porous media are constants.
3. The mass flux produced by temperature gradients (Soret effect) and the heat flux produced by a concentration gradient (Dufour effect) are neglected.
4. The viscous drag and inertia terms of the momentum equations are negligible, which are valid assumptions for low Darcy and particle Reynolds numbers.
5. The Darcy law is applicable.

Under these assumptions, the conservation equations for steady flow can be written as:

$$\nabla.\vec{\mathbf{V}} = 0 \tag{1}$$

$$\vec{\mathbf{V}} = \frac{K}{\mu}\left(-\nabla p + \rho\vec{\mathbf{g}} + \vec{\mathbf{I}}\times\vec{\mathbf{B}}\right) \tag{2}$$

$$\nabla.\vec{\mathbf{I}} = 0; \quad \vec{\mathbf{I}} = \sigma\left(-\nabla\varphi + \vec{\mathbf{V}}\times\vec{\mathbf{B}}\right) \tag{3}$$

$$\nabla.(\rho\vec{\mathbf{V}}T - \alpha\nabla T) = 0 \tag{4}$$

$$\nabla.(\rho\vec{\mathbf{V}}C - D\nabla C) = 0 \tag{5}$$

where $\vec{\mathbf{V}}$ [ms^{-1}] is the velocity vector, K [m^2] is the permeability of the porous medium, μ [$kg.m^{-1}.s^{-1}$] is the dynamic viscosity, p [Nm^{-2}] is the pressure, ρ [$kg.m^{-3}$] is the density, $\vec{\mathbf{g}}$ [ms^{-2}] is the acceleration vector, $\vec{\mathbf{B}}$ [Wbm^{-2} or Tesla] is the external magnetic field vector, $\vec{\mathbf{I}}$ [A] is the electric current vector, σ [Ω^{-1} m^{-1}] is thefluid electrical conductivity, φ [V], is the electric potential, T [K] is the fluid temperature, C [$mol.m^{-3}$] is the concentration, α and D [m^2s^{-1}] are diffusivity of heat and constituent through the fluid saturated porous matrix respectively.

Garandet *et al.* [9] proposed an analytical solution of the equations of magnetohydrodynamics that can be used to model the effect of a transverse magnetic field on buoyancy driven convection in a two-dimensional cavity. According to Garandet et al. [9] equation (3) can be reduced to $\nabla^2\varphi = 0$ .The unique solution is $\nabla\varphi = 0$ since there is always an electrically insulating boundary around enclosure, where the gradients normal to the walls are zeros ($\partial\varphi/\partial n = 0$). It follows that the electric field vanishes everywhere as discussed by Alchaar et al. [10].

The solution that saturates the porous matrix is modelled as a Boussinesq incompressible fluid whose density variation can be expressed using the Oberbeck–Boussinesq approximation:

$$\rho \cong \rho_o\left\{1 - \beta_T\left(T - T_o\right) - \beta_c\left(C - C_o\right)\right\} \tag{6}$$

Where β_T and β_c are the thermal and concentration expansion coefficients. Subscript o stands for a reference state. For two-dimensional flow the pressure p in equations (2) can be eliminated by cross differentiation and a single momentum equation can be derived. The governing equations may be written in dimensionless form using the following non-dimensional variables:

$$X, Y = (x, y)/L; \quad U, V = (u, v)L/\alpha; \quad \theta = (T - T_c)/\Delta T;$$
$$S = (C - C_c)/\Delta C \tag{7}$$

where $\Delta T = (T_h - T_c)$ and $\Delta C = (C_h - C_c)$. The dimensionless forms of the governing equations (1) to (5) become:

$$\frac{\partial^2\Psi}{\partial X^2} + \left(1 - Ha^2\right)\frac{\partial^2\Psi}{\partial Y^2} = -Ra\left(\frac{\partial\theta}{\partial X} + N\frac{\partial S}{\partial X}\right) \tag{8}$$

$$\frac{\partial\Psi}{\partial Y}\frac{\partial\theta}{\partial X} - \frac{\partial\Psi}{\partial X}\frac{\partial\theta}{\partial Y} = \frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2} \tag{9}$$

$$\frac{\partial\Psi}{\partial Y}\frac{\partial S}{\partial X} - \frac{\partial\Psi}{\partial X}\frac{\partial S}{\partial Y} = \frac{1}{Le}\left(\frac{\partial^2 S}{\partial X^2} + \frac{\partial^2 S}{\partial Y^2}\right) \tag{10}$$

Where the stream function defined as $U = \partial\Psi/\partial Y$ and $V = -\partial\Psi/\partial X$, and the governing parameters are Rayleigh number, Buoyancy ratio, Lewis number and Hartmann number defined, respectively as:

$$Ra = \frac{\rho K g \beta_T \Delta T L}{\alpha\mu}; \; N = \frac{\beta_c}{\beta_T}\frac{\Delta C}{\Delta T}; \; Le = \frac{\alpha}{D}; \; Ha = B_0\sqrt{\sigma K/\mu} \tag{11}$$

where B_o is the magnitude of $\vec{\mathbf{B}}$. The governing equations in the present problem are subjected to the following boundary conditions:

$$\text{at } X = 0 \qquad \Psi = 0, \quad \theta = 1, \quad S = 1 \tag{12a}$$

$$\text{at } X = 1 \qquad \Psi = 0, \quad \theta = 0, \quad S = 0 \tag{12b}$$

$$\text{at } Y = 0 \qquad \Psi = 0, \quad \frac{\partial\theta}{\partial Y} = 0, \quad \frac{\partial S}{\partial Y} = 0 \tag{12c}$$

$$\text{at } Y = 1 \qquad \Psi = 0, \quad \frac{\partial\theta}{\partial Y} = 0, \quad \frac{\partial S}{\partial Y} = 0 \tag{12d}$$

The results will be presented in terms of the average Nusselt number $\overline{Nu}$ and the average Sherwood number $\overline{Sh}$ on the vertical walls, which are defined as follows:

$$\overline{Nu} = \int_0^1\left(\frac{\partial\theta}{\partial X}\right)_{X=0,1} dY; \; \overline{Sh} = \int_0^1\left(\frac{\partial S}{\partial X}\right)_{X=0,1} dY \tag{13}$$

ENTROPY GENERATION

In convection heat transfer problems, fluid friction and heat transfer in addition to magnetic field effects contribute to the rate of entropy generation. This entropy generation is due to the irreversible nature of heat transfer and viscosity effects, within the fluid and at the solid boundaries. For heat and fluid flow in porous media, which follows the Darcy model, the local rate of entropy generation can be calculated from the known temperature and velocity fields as [11]:

$$\dot{s}_{gen} = \frac{k}{T_o^2}(\nabla T)^2 + \frac{\mu}{KT_o}\left(u^2 + v^2\right) + \frac{\sigma B^2}{T_o}u^2 \tag{14}$$

Dimensionless form of equation (14) can be obtained by utilising the dimensionless variable defined in (7) as:

$$\dot{S}_{GEN} = \left(\frac{\partial\theta}{\partial X}\right)^2 + \left(\frac{\partial\theta}{\partial Y}\right)^2 + N_\mu\left\{\left(\frac{\partial\Psi}{\partial X}\right)^2 + \left(\frac{\partial\Psi}{\partial Y}\right)^2 + Ha^2\left(\frac{\partial\Psi}{\partial Y}\right)^2\right\} \tag{15}$$

where N_μ is the irreversibility distribution ratio related to velocity gradients, defined as:

$$N_\mu = \frac{\mu T_o}{k}\left\{\frac{\alpha^2}{K(\Delta T)^2}\right\} \tag{16}$$

The local entropy generation number would be integrated over the whole cavity to obtain the entropy generation number for cavity volume as:

$$N_S = \int_0^1\int_0^1 \dot{S}_{GEN}\, dXdY \tag{17}$$

NUMERICAL METHOD

The dimensionless governing equation (8)-(10) subjected to the boundary conditions (12) are integrated numerically using the finite volume method [12, 13]. The power law scheme [12] is used for the convection terms formulation of the energy and mass transfer equations. The resulting algebraic equations were solved by line-by-line using the Tri-Diagonal Matrix Algorithm iteration. The iteration process is terminated under the following condition:

$$\sum_{i,j}\left|\phi_{i,j}^{n} - \phi_{i,j}^{n-1}\right| \Big/ \sum_{i,j}\left|\phi_{i,j}^{n}\right| \le 10^{-7} \tag{18}$$

where ϕ is the general dependent variable which can stands for either θ, S or Ψ and n denotes the iteration step. The developed code is an extension of the code verified and validated in previous studies [14, 15]. The present numerical results are compared with the results obtained by Mahmud and Fraser [6] for the effect of magnetic field on the convective heat transfer in porous cavity. The results presented in Table 1 reflect the accuracy of the present results using uniform mesh of 40×40 cells.

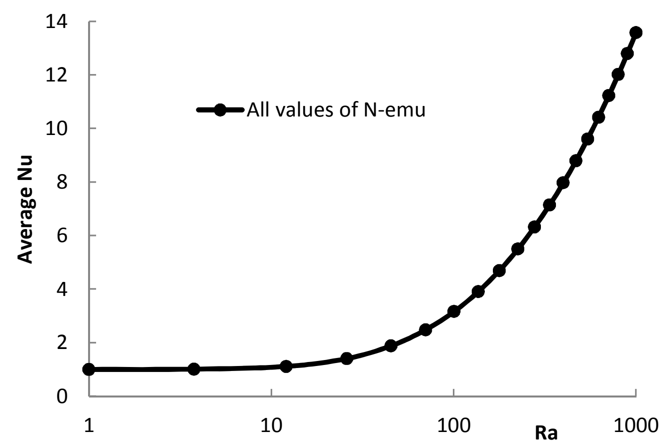
Table 1 Comparison of $\overline{Nu}$ with Mahmud and Fraser [6] results (*Ra*=250 and *N*=0)

Ha	0	2	4	6	10
Present results	5.883	3.195	1.516	1.140	1.021
Reference [6]	5.90	3.15	1.50	1.15	1.05

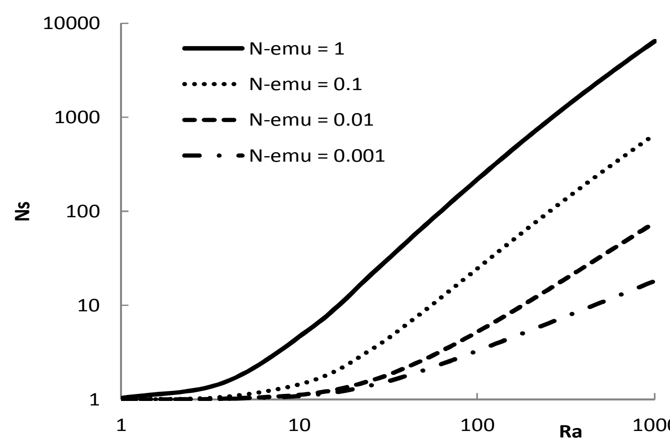
RESULTS AND DISCUSSION

The results are generated to show the effect of the following governing parameters $0 \le Ha \le 10$, $-5 \le N \le 5$, $0.001 \le N_\mu \le 1$ and $1 \le Ra \le 1000$ on the average Nusselt number, the average Sherwood number and the entropy generation number. In order to reduce the parameters, the Lewis number is kept constant at unity. It is important to note that the dimensionless temperature and dimensionless concentration distributions are identical for the case of Le =1, which leads to $\overline{Nu} = \overline{Sh}$.

The classical natural convection in porous cavity without mass transfer and zero magnetic field effects is considered first as a reference case. The variation of the average Nusselt number ($\overline{Nu}$) and the local entropy generation number (N_s) with Rayleigh number (Ra) are shown in Figure 2 with fixed values of $N = 0$ and $Ha = 0$. Figure 2a shows the classical variation of $\overline{Nu}$ with Ra. The effect of the irreversibility distribution ratio (N_μ) on the entropy generation number (N_s) with Rayleigh number (Ra) is shown in Figure 2b. The results show that increasing N_μ lead to higher entropy generation in the cavity. From the definition of N_μ, in order to reduce the rate of entropy generation, it is necessary to reduce the reference temperature and reduce the viscosity and the thermal diffusivity of the fluid. The rate of entropy generation can be minimized for the flow through a high permeable porous medium.



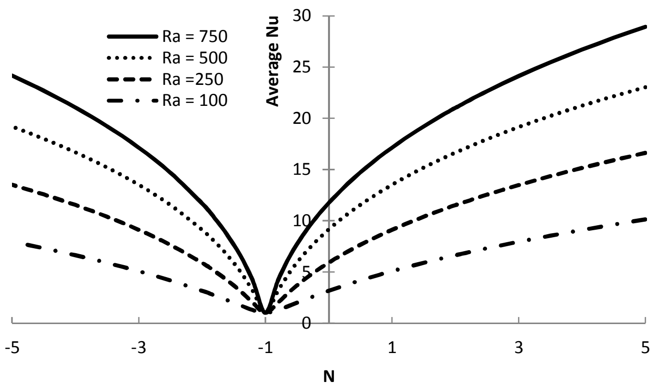
(a)



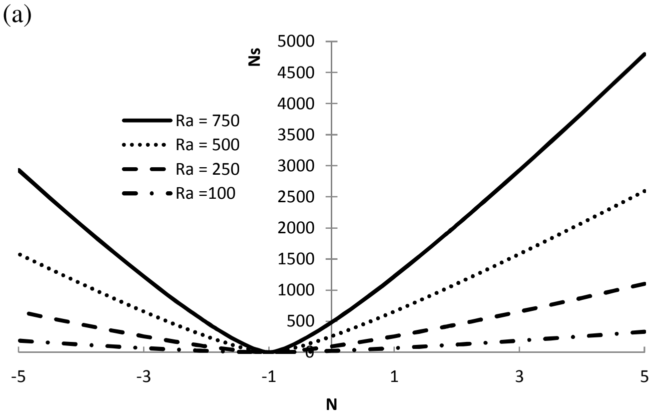
(b)

Figure 2 Variation of $\overline{Nu}$ and N_s with *Ra* with constant *N*=0 and *Ha*=0.

The variation $\overline{Nu}$ and N_s with buoyancy ratio (N) are shown in Figure 3 with fixed values of N_μ = 0.1 and Ha=0. The results depicted in Figure 3 show a minimum values of $\overline{Nu}$ and N_s are observed at N = -1 for all the values of Ra. Negative values of N means that the buoyancy forces generated due to temperature and concentration differences are in opposite directions. For Le =1, Ha = 0 and N = -1, equation (8) reduced to $\nabla^2\Psi = 0$ with $\Psi = 0$ at the importable walls, leads to a stagnate fluid with $\Psi = 0$ everywhere in the cavity. In this case the heat and mass transfer is a pure diffusion process, in which the values of $\overline{Nu}$ and N_s are the minimum. Increasing or decreasing the value of the buoyancy ratio parameter (N) leads to enhance the fluid circulation due to the increase of the resultant buoyancy force in the cavity. This leads to the increase the values of $\overline{Nu}$ and N_s as shown in Figure 3.



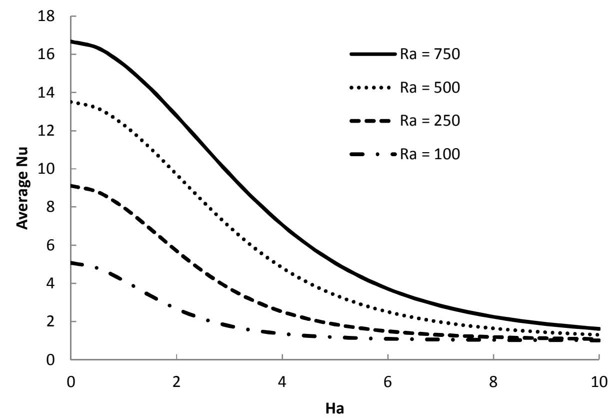
(a)



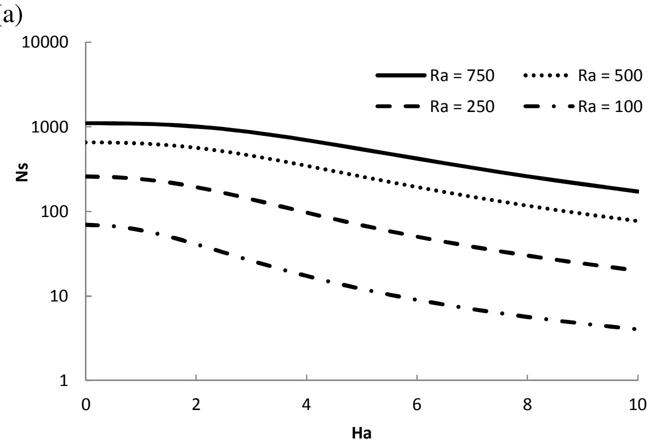
(b)

Figure 3 Variation of $\overline{Nu}$ and N_s with N at constant N_μ = 0.1 and Ha= 0.

The effect of the magnetic field on the variation of $\overline{Nu}$ and N_s is show in Figure 4 for different values of Ra and fixed values of N_μ = 0.1 and N= 1. Maximum rates of heat transfer as well as entropy generation are observed at Ha = 0. Increasing the magnetic field leads to increase the Lorentz forces and slowdown the fluid flow which leads to decrease $\overline{Nu}$ rapidly as shown in Figure 5a. For high values of Ha (Ha > 10), the effect of Ra on the heat and mass transfer process is vanished and the values of $\overline{Nu}$ approaching unity. The heat and mass transfer process is associated with the entropy generation and again the rate of entropy generation is maximum when the rate of heat and mass transfer is maximum as shown in Figure 4b.



(a)



(b)

Figure 4 Variation of $\overline{Nu}$ and N_s with Ha at constant N_μ = 0.1 and N= 1.

CONCLUSION

The entropy generation for the combined natural convection heat and mass transfer in a porous cavity subjected to a magnetic field is considered for investigation in the present study. In order to reduce the rate of entropy generation, it is necessary to reduce the reference temperature and reduce the viscosity and the thermal diffusivity of the working fluid. The rate of entropy generation can be minimized for the flow through a high permeable porous medium. The numerical results for Le =1, Ha = 0 and N = -1, show a stagnate fluid everywhere in the cavity. In this case, the heat and mass transfer is a pure diffusion process, in which the values of $\overline{Nu}$ and N_s are the minimum. The numerical results show that increasing the magnetic field parameter (Hartmann number) leads to reduce the flow circulation strength in the cavity and this leads to a decrease the rate of heat and mass transfer as well as the rate of entropy generation.

REFERENCES

[1] Kaviany, M., Principles of Heat Transfer in Porous Media, Springer, New York, 1999.

[2] Nield D.A., and Bejan A., Convection in Porous Media, third ed., Springer, New York, 2006.

[3] Pop I, and Ingham D.B., Convective Heat Transfer: Mathematical and Computational Modelling of Viscous Fluids and Porous Media, Pergamon,Oxford, 2001.

[4] Mojtabi, A. and Mojtabi, M.-C., Double-Diffusive Convection in Porous Media; in K. Vafai (Ed.), Handbook of Porous Media, 2nd edition, Taylor & Francis Group, New York, 2005.

[5] Bian, W., Vasseur, P., Bilgen, E., and Meng F., Effect of an electromagnetic field on natural convection in an inclined porous layer, *International Journal of Heat and Fluid Flow*, Vol. 17, 1996, pp. 36-44.

[6] Mahmud, S. and Fraser, R. A., Magnetohydrodynamic free convection and entropy generation in a square porous cavity, *International Journal of Heat and Mass Transfer*, Vol. 47, 2004, pp. 3245–3256.

[7] Baytas, A.C., Entropy generation for natural convection in an inclined porous cavity, *International Journal of Heat and Mass Transfer*, Vol. 43, 2000, pp. 2089–2099.

[8] Kaluri, R.S., and Basak, T., Role of entropy generation on thermal management during natural convection in porous square cavities with distributed heat sources, *Chemical Engineering Science*, Vol. 66, 2011, pp. 2124-2140.

[9] Garandet, J.P., Alboussiere, T., and Moreau, R., Buoyancy driven convection in a rectangular enclosure with a transverse magnetic field, *International Journal of Heat and Mass Transfer*, Vol. 35, 1992, pp. 741-748.

[10] Alchaar, S., Vasseur, P., and Bilgen, E., The effect of a magnetic field on natural convection in a shallow cavity heated from below, *Chemical Engineering Communications*, Vol. 134, 1995, 195–209.

[11] Woods, L.C., The Thermodynamics of Fluid Systems; Oxford University Press: Oxford, UK, 1975.

[12] Patankar, S.V., Numerical Heat Transfer and Fluid Flow, McGraw-Hill, New York, 1980.

[13] Versteeg, K. and Malalasekera, W., An Introduction To Computational Fluid Dynamics: The Finite Volume Method, H. Pearson Higher Education, 1995.

[14] Saeid, N.H. and Mohamad, A.A., Natural convection in a porous cavity with spatial sidewall temperature variation, *International Journal of Numerical Methods for Heat & Fluid Flow*, vol. 15, 2005, pp. 555-566.

[15] Saeid, N. H. and Pop, I., Transient free convection in a square cavity filled with a porous medium", *International Journal of Heat and Mass Transfer*, Vol. 47, 2004, pp. 1917-1924.