MEASUREMENT OF THE THERMAL DIFFUSIVITY OF BUILDING MATERIALS

Claude P Lewis and Alan Nurick
Department of Mechanical Engineering Science
University of Johannesburg
PO Box 524, Aucklandpark, 2006
South Africa
e-mail: alann@uj.ac.za

ABSTRACT
An apparatus was developed to measure the thermal diffusivity of solids with particular emphasis on building materials which do not necessarily have smooth surfaces. Time dependent temperature variations, which may be general, on the cold and hot surfaces of a specimen are used as boundary conditions for a numerical solution of the governing heat flow equation. A least average absolute temperature difference between measured and calculated mid-point temperatures is used to obtain the thermal diffusivity. The apparatus provides a base for the development of improved methods of measuring the thermal properties of materials.

NOMENCLATURE

- \( A \) [m\(^2\)] Area of specimen
- \( N \) Data point number
- \( t \) [s] Time
- \( T \) [°C] Temperature
- \( x \) [m] Distance from cold surface
- \( \alpha \) [m\(^2\)/s] Thermal diffusivity

Subscripts

- \( i \) \(^{\text{i}}\) th point in distance
- \( j \) \(^{\text{j}}\) th point in time
- \( S \) First data point
- \( F \) Last data point

INTRODUCTION
The thermal diffusivity of building materials is relevant during heating and cooling of buildings as it has a bearing on temperature gradients, and hence thermal stresses within structures as well as the rate at which temperature changes occur.
internationally accepted [1,5,6]. It should be noted that limitations in terms of specimen thickness and colour can result in errors.

Gustafsson [7,8] used a Hot Disk Technique in which a thin metallic foil combined heater/temperature probe was located between two pieces of test specimen and was used to generate a heat pulse. The resulting temperature response was analysed using the characteristic differential equation solved in terms of the boundary conditions. Vozár and Šrámková [9] used a step heating method in which a constant heat flux was applied to one side of a thermally insulated disc shaped sample and the temperature response on the other side of the disc was measured. The thermal diffusivity was obtained by fitting the measured temperature response to a series solution to the time dependent governing equation to determine the appropriate constants.

Stephenson [10] used a Guarded-Hot-Plate-Apparatus [11,12] to determine the thermal diffusivity of solid materials using two similar specimens with a temperature sensor placed between the specimens. The temperature of the heat sink plates located on either side of the specimens was varied at a constant rate and the variation of the temperature between the specimens with time was recorded. The thermal diffusivity was determined by matching the experimental data to a truncated series solution [1].

Waite et al [13] used a standard needle probe which simulates an infinite line source with a constant flow of heat. The thermal diffusivity was determined from a non-linear fit of the data obtained during the first few tenths of a second to a mathematical solution to the governing heat flow equation of Carslaw and Jaeger [1].

Typically, thermal diffusivity is determined by fitting a measured temperature response to an analytical solution to the governing heat flow equation. To meet the mathematical boundary conditions the time dependent heat source generally needs to conform to the mathematical boundary conditions. Thus the source could be of short duration, a constant temperature heat source applied to a material of different temperature, or it may be a constantly varying temperature. In effect, generally, experiments and measurements designed to measure thermal diffusivity are constrained by the need for specific boundary conditions which must be satisfied physically.

A simple method to measure the thermal diffusivity of building materials, which may be comprised of multi-components such as brick and mortar, and which may not have smooth contact surfaces is proposed. The manner in which heat is applied to the specimen need not be defined to satisfy specific mathematical boundary conditions, and may be of a general nature. Analysis of the results and the extraction of the thermal diffusivity are based on a simple application of a finite difference method which can be readily run on a spreadsheet.

**ANALYSIS**

The temperature in a homogeneous insulator, which varies with time over a single spatial dimension, is given by:

\[
\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\]  

(1)

A convenient formulation of (1), using central differences for the spatial dimension and backward differences for time is:

\[
T_{i,j} = \frac{F(T_{i,j} + T_{i-1,j}) + T_{i,j-1}}{2F + 1}
\]  

(2)

with:

\[
F = \alpha \frac{(\Delta x)^2}{\Delta t}
\]  

(3)

Central differences for the time dimension may also be used. The same value of \( \alpha \) is obtained but care needs to be taken in calculating the temperatures to ensure convergence.

Measured temperature variations with time on the upper and lower surfaces of the specimen, shown in figure 1, were used as boundary conditions. The thermal diffusivity \( \alpha \) was obtained by adjusting \( \alpha \) to minimise the average of the sum of absolute temperature differences between the measured and centreline temperatures over a selected time range of data points, given by \( \Phi \):

\[
\Phi = \min \left\{ \sum_{j=1}^{N_F} |T_{Calc,j} - T_{Meas,j}| \right\}
\]  

(4)

\[
\frac{N_F}{N_F - N_S + 1}
\]

**EXPERIMENTAL EQUIPMENT**

The test rig, shown in figure 1, was developed to measure the thermal properties of building materials which could include composites of mortar and brick and other material combinations which may not necessarily have flat surfaces conducive to the use of temperature controlled plates which should be in good thermal contact with the specimen to be measured [14]. As shown in figure 1, a constant temperature air pocket located between the steel heating plate, item (3), and specimen, item (1), was used to heat the hot side of the specimen, while the cold side was left open to the atmosphere. A duct which may be fitted to the upper side of the specimen was made to allow measurements of heat convection from the specimen to be made. Heat was supplied by means of a 2 kW halogen medium wave emitter. The temperature of the hot side of the specimen was controlled using a TCR-1 plug-in temperature controller with a set point accuracy of ± 2% of FS and hysteresis of 1°C at temperatures below 100 °C.
Specimen temperatures were measured using K-type thermocouples embedded 2 mm from its upper and lower surfaces. These time dependent temperatures provided the boundary conditions for the numerical analyses. The time dependent temperature variation at the centre of the specimen was measured for comparison with the calculated temperature variation. A Datataker DT500 was used to log the temperature readings. The power fed to the Halogen lamp was measured using a Zaptronix ZAP01DS Watt meter with a 10/80 Class 1 accuracy. All transducer outputs were collected on a PC and stored for later analysis.

Figure 1  Cross-Section of Experimental Equipment

EXPERIMENTAL PROCEDURE
To ensure that the heat supplied to the hot side of the specimen resulted in a uniform temperature on the specimen the temperature variation across the test rig was measured prior to the diffusivity tests. This was done using a board with 10 by 10 evenly spread locations for fixing thermocouples. The board was fixed at a distance of 250 mm from the Halogen light. To simplify the correlation of the results, rather than use a 2-dimensional distribution of the measured temperatures, the temperatures at various radial distances from the centre of the board are presented in figure 1.

Figure 2  Variation of the Mid-Point Temperature with Time (End time = 300 min)

Once the temperature stabilises $\alpha$ plays no role in the physical temperature distribution and it would be expected that the ensuing time independent temperatures would not materially affect the

RESULTS
The thermal diffusivity was obtained from the experimental data by adjusting $\alpha$ using equation (4), to give the least absolute average temperature difference between the temperatures measured at the mid-point and the calculated temperatures over a selected time period. The calculated temperature distribution was obtained using equation (2) and the measured temperatures at the hot and cold sides of the specimen were used as boundary conditions. For the numerical analysis, the thickness of the specimen was arbitrarily divided into 20 equal spatial distances. Temperature readings were recorded every 60s. Measurement of these temperatures at fixed time intervals simplified the numerical analysis. The variation of the measured and calculated temperatures presented in figure 2 was based on $\alpha$ obtained by minimising the average absolute error over the time period of 10 min to 300 min.

The average temperature of 48 points measured across the plate, shown in figure 1 was 100.4°C with a standard deviation of 2.6°C.
determination of $\alpha$. Also, too few readings could result in inaccurate measurement of $\alpha$ due to a limited temperature variation. To obtain an indication of the effects of ‘too few’ readings and the stable temperature readings on the calculated value of $\alpha$ the calculation was carried out for different end times. The results of these calculations are presented in Table 2. Similar results were obtained for other tests.

Table 2  Effects of End Time on Numerical Results

<table>
<thead>
<tr>
<th>Start Time (min)</th>
<th>End Time (min)</th>
<th>$\alpha \times 10^6$ (m²/s)</th>
<th>Difference from mean of $\alpha$ (%)</th>
<th>Average Temp Diff (°C)</th>
<th>Mid Point/Max Temp (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>200</td>
<td>1.16</td>
<td>6.42</td>
<td>0.49</td>
<td>89.5</td>
</tr>
<tr>
<td>10</td>
<td>300</td>
<td>1.10</td>
<td>0.92</td>
<td>0.63</td>
<td>95.5</td>
</tr>
<tr>
<td>10</td>
<td>400</td>
<td>1.08</td>
<td>-0.92</td>
<td>0.75</td>
<td>98.3</td>
</tr>
<tr>
<td>10</td>
<td>500</td>
<td>1.07</td>
<td>-1.83</td>
<td>-0.85</td>
<td>99.9</td>
</tr>
<tr>
<td>10</td>
<td>600</td>
<td>1.07</td>
<td>-1.83</td>
<td>0.93</td>
<td>99.8</td>
</tr>
<tr>
<td>10</td>
<td>700</td>
<td>1.06</td>
<td>-2.75</td>
<td>0.98</td>
<td>100.1</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.09</td>
</tr>
</tbody>
</table>

The analysis was started with data obtained after the experiment was running for 10 min to eliminate start-up errors. The values of $\alpha$ did not vary when measurements used spread over a large part of the time dependent period. From the measurements it appears that good results may be expected if the spread of data includes temperatures up to approximately 99% of the final stable temperature. It was found that approximately 1000 iterations are required on an Excel spreadsheet for convergence.

FURTHER DEVELOPMENT

The principle of operation of a simple test apparatus for measuring the thermal diffusivity of, in particular, building materials has been demonstrated. Further work will involve increasing the number of temperature measurements on the cold and hot surfaces of the specimens to allow an effective thermal diffusivity of non-homogenous building elements to be determined and improving the accuracy of the measurements. Also, the apparatus will be developed to allow the thermal conductivity to be measured directly with an accuracy similar to that typically obtained using a guarded hot plate [11,12], rather than combining the measured thermal diffusivity with the density and specific heat of the specimen to determine the thermal conductivity.

SUMMARY

A simple method for measuring the thermal diffusivity of building materials has been presented. Time dependent surface measurements are used as boundary conditions for a difference solution to the governing heat flow equation. The thermal diffusivity is obtained by minimising the average absolute temperature difference between calculated and measured mid point temperatures. The method is simple and temperature variations do not need to be designed to suit boundary conditions required when fitting experimental data to mathematical solutions to the governing heat flow equation.

REFERENCES
