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SCATTERING OF THE TSUNAMI WAVE IMPINGING INTO A FOREST FOR DAMAGE PREVENTION

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ABSTRACT

A tsunami originated in a deep sea due to the tectonic activity such as earthquake is the transport phenomena of great scale energy, which makes much damage to sea-bounded area since its wave height becomes higher approaching the sea shore. Therefore, the tsunami wave is a kind of shallow water wave, and the characteristics similar with the shock wave in gas dynamics should be noted so remarkably that there exists a sharp discontinuity before and after the wave front. Actually there is a mathematical similarity between the equations of shallow water waves and gas dynamics. In this research, the existing code concerning the computation of unsteady shock waves is improved to simulate the propagation of tsunami waves. With this code, reflection, diffraction, and scattering of an incident shock wave are studied in the view point of physics, and the basic parameter analysis of stand density and the distance between trees is made for the application in the future.

INTRODUCTION

A tsunami, tidal wave originated from the earthquake, is produced from tectonic activity underwater in ocean[1]. In case that it is propagated to shallow water areas near to the coast line, the sudden high level tide often brings about serious damage, which appears well in the earthquake at Tohoku area, Japan, 2010. Through those damages, our society needs to prevent such natural phenomena.

The physics of tsunami in the shallow media has a mathematical analogy with gas dynamics equations. Many literatures have used this analogy, utilizing numerical schemes for nonlinear waves such as solitary or tsunami waves with solving Euler equations for the propagation of unsteady shock wave in the compressible gas media[2, 3]. This kind of analogy is used for the determination of the height of a sea wall[4], or extended to more realistic model considering the slope of coast terrain[5].

In this research, to prevent tsunami waves, the feasibility of a forest near at seashore for the protection is discussed, which seems possible in the viewpoint of intuition and experience. A tsunami is a kind of strong nonlinear wave, and it is the propagation of high-level natural energy. Therefore, like general waves such as shock waves, it will lost a part of the original energy when meeting a forest as an obstacle. This prediction will be proved with a simple numerical model concerning wave physics. Through the mathematical modelling, we will show reflection, diffraction, and scattering of an impinging tsunami wave as well as its shallow-water propagation.

NOMENCLATURE

g	[m/s^2]	*Gravitational acceleration*
h	[m]	*Height of wave, or local depth of water*
u	[m/s]	*Local flow velocity*
p	[Pa]	*Pressure*
t	[s]	*Time*
x	[m]	Spatial coordinate component
Fr	[-]	Froude number
Ma	[-]	Mach number

ρ	[kg/m^3]	Density

Subscripts

i, j	*Spatial coordinate index*
w	*Wave*
1	*Undisturbed region*
2	*Disturbed region*

METHODOLOGY

In this study, we attempt to solve the wave physics modelled under assumptions such as the following approximation for the simplification of the problem:

1) Deformation and destruction of trees are ignored in the process of propagation through.
2) Only stands of trees are modelled as two-dimensional cylinders except for crowns.
3) The tsunami wave is modelled as a planar discontinuous single wave, so this is a kind of Riemann problem mathematically.
4) The tsunami wave is assumed to propagate in the media of shallow water where the depth is almost constant.
5) The dissipation due to friction between fluid and forest is neglected, so the wave energy is almost conserved.

Especially, among the above assumptions, 2) and 5) might be treated most seriously because they can violate the real physics. However, as a preliminary research, we consider only a simplified model where additional energy loss like turbulent bore is discounted because the primary wave is so strong and straight- moving.

Therefore, the governing equations are written as follows:

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x_j}\left(u_j h\right)=0 \tag{1}$$

$$\frac{\partial}{\partial t}\left(u_i h\right)+\frac{\partial}{\partial x_j}\left(u_i u_j h\right)=-\frac{\partial}{\partial x_j}\left(\frac{1}{2} g h^2 \delta_{i,j}\right) \tag{2}$$

If we substitute h to density, and $gh^2/2$ to pressure, Eqs. (1)~(2) represents gas dynamics equations in compressible flow[6].

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_j}\left(\rho u_j\right)=0 \tag{3}$$

$$\frac{\partial}{\partial t}\left(\rho u_i\right)+\frac{\partial}{\partial x_j}\left(\rho u_i u_j\right)=-\frac{\partial}{\partial x_j}\left(p \delta_{i,j}\right) \tag{4}$$

Eq. (3) is the continuity equation for the conservation of mass while Eq. (4) is an equation for momentum conservation. This analogy allows us to reproduce the similar Riemann solver techniques used in the field of gas dynamics.

In the present research, we use Roe approximate Riemann solver and TVD(total variation diminishing) method based on van Leer flux limiter. MUSCL expansion is applied to extend to the FVM(finite volume method) numerical scheme as second-order accuracy in space[7]. The temporal accuracy is extended to second order with Runge-Kutta method.

With a very simple algebra, from Eqs. (1)~(2), we are able to show that the characteristic speed of wave is referred with the local value of $\sqrt{gh}$. Therefore, Froude number is defined as

$$Fr=\frac{u_s}{\sqrt{gh}_1} \tag{5}$$

If the height ratio between disturbed and undisturbed regions is 3:1, the Froude number in Eq. (5) should be 2.45. Therefore, we set this value as a baseline case.

In the same way, Mach number is defined from Eqs. (3)~(4),

$$Ma=\frac{u_s}{\sqrt{\gamma \frac{p_1}{\rho_1}}} \tag{6}$$

The specific heat ratio should be $\gamma=2$ for the similarity of Eqs. (5) and (6)[8].

Similarly with Rankin-Hugonite conditions in gas dynamics, with a coordinate transformation and from the weak solution of conservative integral equation before and after the discontinuity of tsunami wave: see Figure 1, and refer to Eqs. (1)~(2),

$$h_1 u_w = h_2\left(u_w - u_2\right) \tag{7}$$

$$\frac{1}{2} g h_1^2 + h_1 u_w^2 = \frac{1}{2} g h_2^2 + h_2\left(u_w - u_2\right)^2 \tag{8}$$

From Eq. (7),

$$u_w - u_2 = \frac{h_1}{h_2} u_w \tag{9}$$

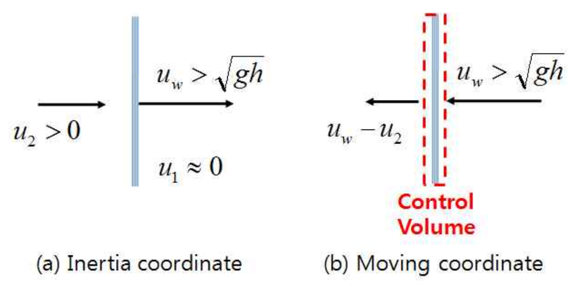


Figure 1 Transformation of the coordinate system.

From Eq. (5), the definition of wave Froude number,

$$u_w^2 = gh_1 Fr^2 \tag{10}$$

Substituting Eqs. (9)~(10) to Eq. (8),

$$\begin{aligned}&\left(\frac{h_2}{h_1}\right)^3 - \left(2Fr^2+1\right)\left(\frac{h_2}{h_1}\right) + 2Fr^2 \\ &= \left(\frac{h_2}{h_1} - 1\right)\left\{\left(\frac{h_2}{h_1}\right)^2 + \left(\frac{h_2}{h_1}\right) - 2Fr^2\right\} = 0\end{aligned} \tag{11}$$

The only physical solution of Eq. (11) is, if $h_1 \neq h_2$,

$$\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1+8Fr^2} - 1\right) > 1 \tag{12}$$

and solving Eqs. (9)~(10),

$$u_2 = \left(1 - \frac{h_1}{h_2}\right) Fr \sqrt{gh_1} \tag{13}$$

When the Froude number is given, we can easily set the initial conditions across the incident tsunami wave with Eqs. (12)~(13).

The shock wave Mach number in Rankin-Hugonite condition is a function of the pressure ratio before and after the wave:

$$Ma = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1} \tag{14}$$

Substituting $\gamma = 2$ to Eq. (14),

$$Ma = \sqrt{\frac{3}{4}\left(\frac{p_2}{p_1} - 1\right) + 1} = \sqrt{\frac{3}{4}\left\{\left(\frac{h_2}{h_1}\right)^2 - 1\right\} + 1} \tag{15}$$

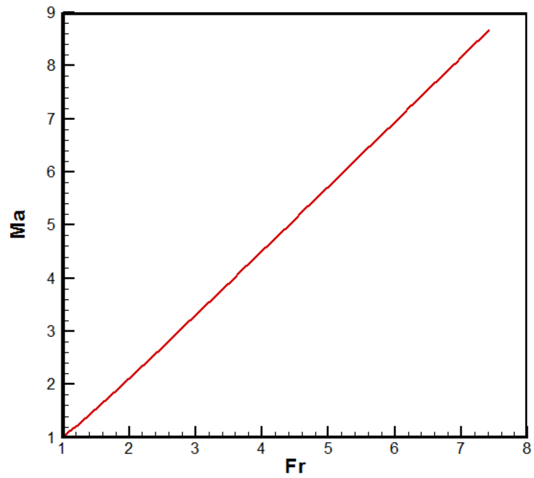
From Eq. (12),

$$Fr = \sqrt{\frac{1}{8}\left\{\left(2\frac{h_2}{h_1} + 1\right)^2 - 1\right\}} \tag{16}$$

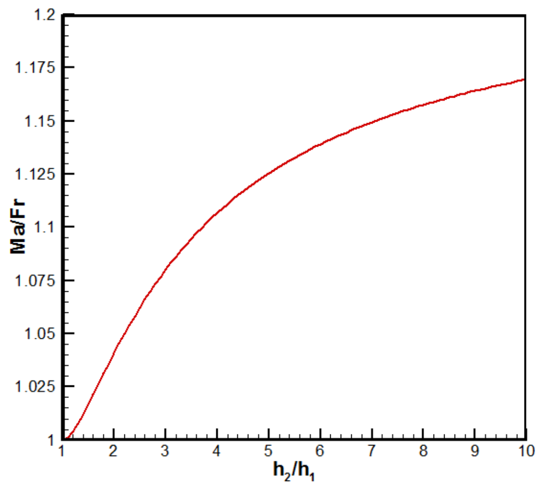
However, the initial conditions, Eqs. (15)~(16) cannot make sure to be identical if $h_1 \neq h_2$. The graphical relation of two parameters is visualized in Figure 2(a). If $h_2 / h_1 \to \infty$, for a large wave, the ratio is converged to

$$\frac{Ma}{Fr} \to \sqrt{\frac{3}{2}} = 1.22 \tag{17}$$

Eq. (17) is also checked in Figure 2(b). For the case of $Fr = 2.45$, $Ma = 2.65$ (about 9 % error), and this is due to the fact that the energy equation is an excessive one for the analogy, so we should delete out the energy equation in the gas dynamics code. In the real situation, energy is not conserved across the turbulent bore[8].



(a)



(b)

Figure 2 Relation between Mach number and Froude number.

Table 1 Four computational cases

Simulation case	Distance b/ trees (m)	Diameter of Trees (m)	Length of Forest (m)
I	7	0.3	100
II	2.2	0.3	100
III	2.2	0.15	100
IV	7	0.3	150

Table 2 Computational results

Simulation case	Reduction in flow velocity	Reduction in water depth
I	3.7 %	21.5 %
II	4.7 %	22.1 %
III	3.9 %	21.5 %
IV	3.1 %	21.5 %

COMPUTATIONAL CASES

The computational domain is specified with a slender rectangular shape whose length is 300 m, and width is varied from 2.2 to 7 m different in case by case. With application of periodic boundary condition at top and bottom of this rectangle domain, the computational load is significantly reduced. At the forest trees are arranged with constant spacing, and our area of interest is the wave field in the disturbed region after the tsunami has swept over: see Figure 3.

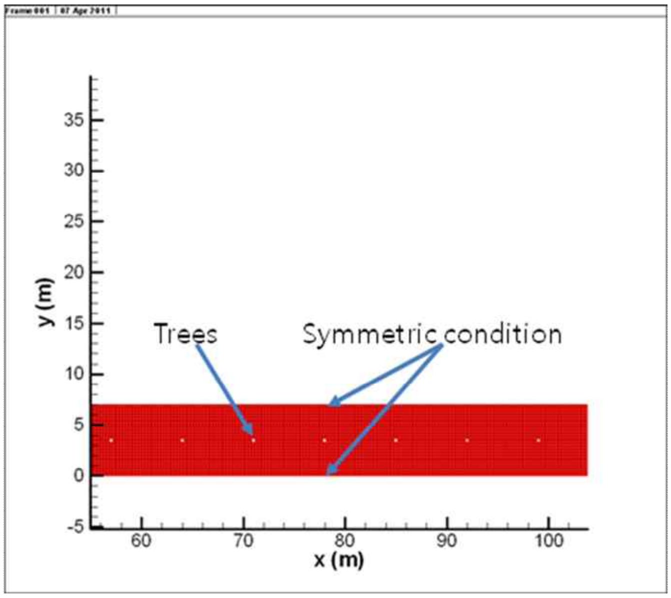


Figure 3 A sample of the computational domain

Four cases are computed, and conditions are listed like Table 1. The width 2.2 m is induced from the field data in a forest located in a southern coast of Korean peninsula where 220 trees per 1 ha. We changed this density parameter to ten times of this baseline case. The diameter of trees vary from 15 to 30 cm while the length of forest is from 100 to 150 m.

RESULT AND DISCUSSION

The tsunami wave starts from 40 m in the computational domain, and reaches the edge of forest at 50 m where the region of forest is 100 to 150 m.

At each unsteady time step, the instant scene such as Figure 4 can be obtained. Figure 3 is the field plot of water depth where the computational meshes are about 210 thousands. If we zoom in the plot along the symmetric line of the periodic surface, the difference of depth can be measured. The obstacles, or trees mitigates the strength of tsunami at the narrow wake region. Its effect is measured about 19 cm (6.3 % of initial tsunami).

At the final wave front, these effects are accumulated that the water depth is 21.5% reduced by forest. However, the reduction of water velocity is just 3.7% of the speed after the original incident wave.

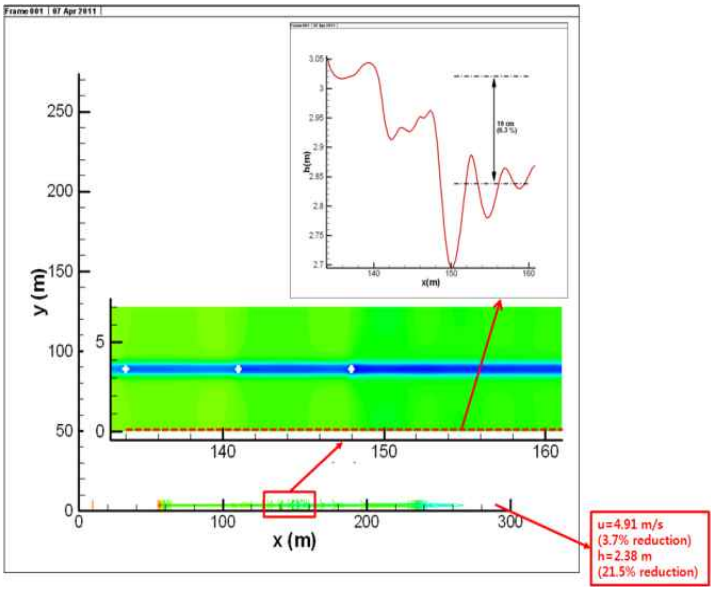


Figure 4 A sample of computational result (case I)

We summarized the parametric study of four cases in Table 2. The reductions in flow velocity and water depth are compared with each other.

From Table 2, we have arrived to the following provisional conclusion:

1) The dissipation effect to wave height or water depth is very similar for every cases. As we have assumed that the shock energy of wave is almost conserved, the reconstructed wave after the forest should have similar strength. Therefore, this effect should be distinguishable for the real-world physics.
2) The dissipation effect to wave velocity becomes to enhance when stand density gets greater (or the distance between trees gets nearer); and also the diameter of breast height gets greater.
3) However, the length of forest does not enhance the dissipation effect. Case IV simulation, for example, the flow speed is increased because the dense forest works like a nozzle, and effects on the increase of flow velocity: i.e. venturi effect.

Therefore, with trees of big diameters and a forest with high stand density, we can construct a forest of prevent at the area near coast effectively protective from tsunami waves in spite of small band of trees, comparatively.

However, the present model obviously has a limitation of application because the stand is just modelled as a cylinder for all cases. The real tree has its stems and branches, so this should be modelled precisely for more reasonable simulation, and the energy equation need to be fixed for the suitable loss model due to the wave dissipation such as porous media, reflective waves, effect of terrain, precursor undisturbed region(wet or dry conditions), turbulence, and cavitation, etc.

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