IMPROVEMENTS TO LONGITUDINAL CLEAN DEVELOPMENT MECHANISM SAMPLING DESIGNS FOR LIGHTING RETROFIT PROJECTS

by

Herman Carstens

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SUMMARY

IMPROVEMENTS TO LONGITUDINAL CLEAN DEVELOPMENT MECHANISM SAMPLING DESIGNS FOR LIGHTING RETROFIT PROJECTS

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Herman Carstens

Supervisor(s):	Prof. Xiaohua Xia
Department:	Electrical, Electronic and Computer Engineering
University:	University of Pretoria
Degree:	Master of Engineering (Electrical Engineering)

Keywords:	Measurement and Verification, Polish Efficient Lighting Project, longitudinal sampling, metering, Compact Fluorescent Lamp, Clean Development Mechanism, retrofit, performance tracking, energy efficiency, demand side management, population survival, reliability

An improved model for reducing the cost of long-term monitoring in Clean Development Mechanism (CDM) lighting retrofit projects is proposed. Cost-effective longitudinal sampling designs use the minimum number of meters required to report yearly savings at the 90% confidence and 10% relative precision level for duration of the project (up to 10 years) as stipulated by the CDM. Improvements to the existing model include a new non-linear Compact Fluorescent Lamp population decay model based on the results of the Polish Efficient Lighting Project, and a cumulative sampling function modified to weight samples exponentially by recency. An economic model altering the cost function to a nett present value calculation is also incorporated.

The search space for such sampling models are investigated and found to be discontinuous and stepped, requiring a heuristic for optimisation; in this case the Genetic Algorithm was used. Assuming an exponential smoothing rate of 0.25, an inflation rate of 6.44%, and an interest rate of 10%, results show that sampling should be more evenly distributed over the study duration than is currently considered optimal, and that the proposed improvements in model accuracy increase expected project costs in nett present value terms by approximately 20%. A sensitivity analysis reveals that the expected project cost is most sensitive to the reporting precision level, coefficient of variance, and reporting

confidence level, and that large interaction effects between these factors exist.

The study provides a more realistic model for implementation in real-world projects, and demonstrates the impact that various factors have on expected project cost. This will provide guidance for engineers designing long-term sampling plans.

OPSOMMING

VERBETERINGE AAN SKOON ONTWIKKELINGSMEGANISME LONGITUDINALE MONSTERONTWERPE VIR BELIGTING-VERVANGINGSPROJEKTE

deur

Herman Carstens

Studieleier(s):	Prof. Xiaohua Xia
Departement:	Elektriese, Elektroniese en Rekenaar-Ingenieurswese
Universiteit:	Universiteit van Pretoria
Graad:	Magister in Ingenieurswese (Elektriese Ingenieurswese)

Sleutelwoorde:	Meting en verifiëring, Poolse energiedoeltreffendheidsbeligtingprojek, spaarlamp, longitudinale monstering, Skoon Ontwikkelingsmeganisme, retrofit, verrigtingsanalise, energiedoeltreffendheid, aanvraagsbestuur, bevolkingsoorlewing, betroubaarheid

'n Verbeterde model vir die kosteminimering van langtermyn Skoonontwikkelingsmeganisme (SOM) moniteringsprojekte word voorgestel. Koste-effektiewe longitudinale monsterontwerpe gebruik die minimum aantal meters wat benodig word om die 90% vertrouensinterval met 'n 10% presiesheidsvereiste te haal vir die duur van die projek, wat deur die SOM bepaal is om tot 10 jaar te wees. Verbeterings aan die bestaande model sluit 'n nuwe nie-linieêre spaarlamp bevolking-oorlewingsmodel in wat op die Poolse energiedoeltreffendheids-beligtingsprojekdata gebasseer is, asook 'n kumulatiewe monsterfunksie wat gewysig is om meer onlangse monsters eksponensieël gewigtiger te reken. 'n Ekonomiese model wat die kostefunksie wysig om netto huidige waarde in ag te neem, word ook ingesluit.

Die soekruimte vir sulke bemonsteringsprojekte word ondersoek en daar word gevind dat dit diskontinu en trapsgewys is, wat die gebruik van 'n genetiese algoritme noodsaak. 'n Eksponensiële vervaltempo van 0.25 word aanvaar, tesame met 'n inflasiekoers van 6.44% en 'n rentekoers van 10%. Resultate toon dat die monsters meer eweredig versprei behoort te word oor die duur van die studie as wat tans as optimaal geag word, en dat die voorgestelde verbeteringe in die modelakkur-

raatheid kostes met meer as 20% verhoog in huidige waarde terme. 'n Sensitiwiteitsanalise toon dat die verwagte projekkoste sensitief is vir die verslagdoenings-presisheidsvereiste, die variasiekoëffisiënt, en die verslagdoenings-vertrouensvlak. Daar is ook noemenswaardige interaksie tussen hierdie faktore.

Die studie verskaf 'n meer realistiese model vir gebruik in regte-wêreld projekte, en wys op die impak wat oorwegings soos die ongelyke weging van monsters in verskillende jare, asook inflasie, kan hê op die monsterverspreiding van langtermynprojekte. As sulks verskaf die studie riglyne aan ingenieurs van langtermyn longitudinale monsterprojekte ontwerp.

PUBLICATIONS

The following articles have been published as a result of this research:

H. Carstens, X. Xia, X. Ye, J. Zhang, Characterising compact fluorescent lamp population decay, in: IEEE Africon Conference, Pointe-Aux-Piments, Mauritius, 2013.

H. Carstens, X. Xia, X. Ye, Improvements to longitudinal Clean Development Mechanism sampling designs for lighting retrofit projects. *Applied Energy*, vol. 126, pp. 256-265, 2014

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NOMENCLATURE

Symbols

A	population difference equation matrix
B	number of backup meters
E	daily energy use
$\bar{X}$	sample mean;
	in this study a random variable in distribution
	$\mathrm{N}(\mu_i, \sigma_i^2/n_i)$
P	cumulative relative precision
H	average annual lamp operating hours
I	last reporting year
L	lamp rated lifetime
N	lamp population size
Y	percentage of lamps left at rated lifetime
Z	Standard score of cumulative confidence
a	meter purchasing cost per unit
b	population difference equation vector
b	meter installation cost per unit
c	meter maintenance cost per unit per month
d	Consumer price index (inflation rate)
h	Step increment
n	number of observations
n'	number of effective observations
p	precision relative to the mean
r	Minimum Attractive Rate of Return,
	or investment interest rate
u	control input
x	population difference equation parameter vector
$\bar{x}$	sample mean
z	standard score of normal distribution
Γ	true cumulative standard deviation on energy use

Φ	surviving proportion of lamp population
$\bar{\chi}$	cumulative sample mean
α	population decay rated lifetime parameter
β	population decay slope parameter
γ	population decay initial value parameter
ϕ	PELP study data
ε	exponential decay factor
η	project inception cost
θ	true cumulative mean energy use
$\boldsymbol{\lambda}$	optimisation decision variable
μ	true mean energy use
	in a given year
ρ	lag 1 autocorrelation coefficient
σ	true standard deviation on energy use in a given year
ω	project operational cost

Subscripts

adjusted	finite population adjustment
B	baseline
J	Number of groups
j	group counter
k	year counter
t	time instant in years
unadjusted	not adjusted for finite populations
0	year 0

LIST OF ABBREVIATIONS

ASHRAE	American society for heating, refrigeration and air-conditioning engineers
AMS	Approved methodology for small scale
AR	Autoregressive
CDM	Clean development mechanism
CFL	Compact fluorescent lamp
CL	Confidence limit
CPI	Consumer price inflation
CPUC	California public utilities commission
CV	Coefficient of variance
CVRMSE	Coefficient of variance on the root mean square error
DSM	Demand side management
EE	Energy efficiency
EUL	Effective useful life
FEMP	Federal energy management program
i.i.d.	independently and identically distributed
IPMVP	International performance measurement and verification protocol
GA	Genetic algorithm
M&V	Measurement and verification
MCMC	Markov chain Monte-Carlo
MSE	Mean squared error
NaN	Not a number
NMBE	Normalised mean bias error
NPV	Nett present value
OLS	Ordinary least squares
PELP	Polish efficient lighting project
PT	Performance Tracking
R	South African rand
SA	Survival analysis
SANS	South African national standard
SI	Sensitivity index

TolCon	Tolerance on constraints
TolFun	Tolerance on function values
UNFCCC	United Nations framework convention for climate change
V	Volts
W	Watts

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

1.1 PROBLEM STATEMENT

1.1.1 Problem context

South Africa's national electricity utility, Eskom, oversees more than 700 Energy Efficiency (EE) and Demand Side Management (DSM) projects, and also supplies more than 95% of the electricity used in the country - which is over 45% of that used on the African continent [1]. In order to monitor project sustainability, the savings realised by these projects need to be measured and verified by an independent accredited Measurement and Verification (M&V) body for a number of years [2]. This is called performance tracking (PT).

International guidelines suggest that M&V costs should not exceed 10% of annual savings [3]. However, the time horizons on these projects span many years; for these projects to be eligible under the United Nations Framework Convention for Climate Change (UNFCCC) Clean Development Mechanism (CDM), the performance of lighting projects should be tracked for up to 10 years, whilst other projects may be tracked for up to 21 years [4]. Certain stringent statistical requirements on measured data also apply: for projects to be eligible for recognition under CDM guidelines, measured data should conform to a statistical confidence level of 90%, and a relative precision around the mean of 10%, known as the 90/10 criterion. Although other leading guidelines recommend an 80/20 level [5], [6], [7], the 90/10 criterion will be used for the purposes of this study. Due to the long (but finite) planning horizons, non-optimal PT may therefore affect total monetary savings or may report savings with inadequate confidence and precision. This could affect project viability or eligibility detrimentally. Savings may also be reported with confidence and precision values exceeding requirements, which increases monitoring costs unnecessarily.

M&V Engineers require two kinds of data to calculate energy usage in such projects: population survival data, and aggregated averaged daily energy use data [8]. Energy usage data are obtained from electricity meters installed in a statistically representative number of households, whilst population survival data are collected through surveys. For this study, population survival data are assumed to be known and thus no sampling design will be devised for this component of energy use, although a new population decay model will be proposed.

Both the International Performance Measurement and Verification Protocol and American Society for Heating, Refrigeration and Air Conditioning Engineers' (ASHRAE) Guideline make a useful distinction between the different kinds of uncertainty encountered during an M&V study. Measurement uncertainty occurs due to equipment inaccuracy: incorrect selection, calibration, installation or operation. Modelling uncertainty arises from inappropriate mathematical models being used: not considering all covariates, for example. Sampling uncertainty pertains to quantifiable uncertainties arising from not measuring the whole population. This study will focus on managing the latter kind of uncertainty in the context of M&V project cost.

1.1.2 Research opportunity

Research grounding the theory of M&V is underway [9], but there is a need to establish best practice by the application of statistics to the specific challenges in energy monitoring. Guidelines specify statistical sampling techniques, but do not consider the cost implications of sampling and how to minimise this.

Lighting projects were selected because they are both common - establishing a need for monitoring theory - and relatively simple to model through simple statistics and binomial working/failed states - providing an opportunity for a starting point from which more complex models may be devised. Although some work has been done on optimal monitoring, the current models are not practical yet, and present an opportunity for improvement.

1.2 RESEARCH OBJECTIVE AND QUESTIONS

This study focuses on monitoring plans for residential lighting retrofit projects where old incandescent lamps are replaced with energy saving Compact Fluorescent Lamps (CFLs), or where halogen downlighter lamps are replaced with Light Emitting Diode (LED) lamps.

The objective is to devise and implement improvements to current CFL retrofit monitoring models as a step towards a mature model accurate enough for cost-effective practical implementation.

Research questions may be formulated as follows:

- Could a more accurate and versatile population survival model for CFL populations be devised?
- How would using different population survival models affect the optimal metering plan?
- What is the nature of the solution space for such optimal metering plans?
- How would nett present value considerations affect optimal metering plans?
- How would exponential decay windowing functions affect metering?

1.3 OVERVIEW OF STUDY

The dissertation is organised in the following manner. Chapter 2 provides an overview of the literature relevant to this study. Chapter 3 describes longitudinal tracking models, and develops a mathematical model for population decay. Thereafter this model and other improvements are integrated into an existing sampling model. Chapter 4 applies this new model to an existing case study and presents results, which are discussed in Chapter 5. Chapter 6 concludes the dissertation, and an appendix provides the computer programs used for calculation.

CHAPTER 2

LITERATURE REVIEW

2.1 CHAPTER OVERVIEW

This chapter surveys performance tracking literature for approaches to energy monitoring, both in an academic context as well as for technical standards used in industry. Population decay models are also discussed, and solutions from other disciplines are considered. Finally, logistic population models are treated in more detail.

2.2 PERFORMANCE TRACKING IN LITERATURE

The theory of performance tracking is essentially the theory of reporting uncertainty, which is applied statistics within the constraints of the M&V context. To date, existing guidelines provide sound, but sub-optimal sampling frameworks for M&V studies. That is, no guideline specifies a cost-effective monitoring plan.

Very little literature pertaining to this specific problem exists. If M&V protocols do specify statistical bounds and discuss sampling, optimality is not mentioned. For example, the International Performance Measurement and Verification Protocol (IPMVP) [3] which is the internationally accepted guideline on best practice, does advise that the 90/10 criterion be used, but only advises that standard simple random and stratified sampling formulae be used. The Federal Energy Management Program (FEMP) guideline [10] uses the 80/20 and 90/10 levels in examples. It also advises that 10% more meters should be installed than is strictly necessary, in order to account for meter attrition. These formulae are applicable to single-sample projects, and do not consider repeated sampling. The South African National Standard (SANS) 50010: Measurement and verification of energy savings [11] does not place statistical requirements on M&V measurements, but only states that "a more rigorous study should not invalidate the result", implying that reported savings should be conservative. The Australasian guide follows the IPMVP and advises that measurement uncertainty should be balanced with cost, considering the point of diminishing returns as desirable [12]. It also follows the IPMVP's recommendation that M&V cost should not exceed 10% of project cost. Concerning uncertainty, it states that "quantified uncertainty should be expressed in a statistically meaningful way, by declaring both accuracy and confidence level".

The ASHRAE Guideline (14-2002) on Measurement of Energy and Demand Savings [6] is a comprehensive technical resource. Statistical bounds are project-dependent with confidence levels of 68%, 80%, and 90% considered standard, in conjunction with precision levels of 10%, 20%, or 50%. Most methods described in the guideline rely on covariates such as weather data affecting the thermal systems. Nevertheless, the fractional savings approach [13] advocated by ASHRAE represents an advanced and industry-specific method for evaluating PT uncertainty. Fractional savings are defined as the ratio of uncertainty on reported savings to reported savings. This approach has intuitive appeal but expresses the same quantity as the relative precision (10% in the 90/10 case). However, it does so by taking the number of monitoring points pre- and post-retrofit into account, as well as considering the ratio of energy savings to baseline energy use in the calculation. As such, the goodness of fit of an energy model may be judged relative to these criteria. The delineation and treatment of uncorrelated and autocorrelated data is also useful. It is shown that utility data representing monthly averages are not strongly correlated, whereas hourly or daily data may be so, especially with respect to temperature variation for thermal systems [14]. This is not as significant a consideration for lighting projects, but will become so when the methods of this study are applied to other EE and DSM projects, discussed in more detail below. The ASHRAE approach could not be followed for the problem at hand since this approach considers a single facility, and not the population of facilities as is the case for residential mass rollout, where a large number of lamps are retrofitted in multiple households over a large geographic area. Furthermore, the standard confidence/precision approach has been followed in order to conform to CDM requirements.

The State of California Public Utilities Commission (CPUC) M&V protocol [15] also follows a technical approach. PT projects are divided into three categories: Effective Useful Life (EUL), retention, and degradation studies, each with various levels of rigour. Degradation refers to the decrease in performance of a system due to technical or behavioural factors. For the purposes of this study it is assumed that degradation is negligible. Although CFL lumen output decreases over time, the electronic ballast ensures that the energy consumption (current drawn) does not increase [16]. EUL studies attempt to find the median life of an EE measure (CFL), and retention studies consider the percentage of measures retained in operable condition. Rigour levels include 90/30 and 90/10, but crucially also include statistical power considerations. This protocol also recognises the value of Survival Analysis (SA) for M&V studies, which will be discussed below. Since the protocol relies on survival analysis principles, it also requires the monitoring of a control group. This is a collection of similar residences or facilities where no intervention was implemented. The treatment group is compared to the control group, which is similar to doing the baseline measurement and post-implementation measurement simultaneously.

The UNFCCC AMS0046 Guideline [17] for monitoring EE lamp retrofit projects does provide a statistical sampling framework, but it has been observed that this framework is not practical, and adoption in industry has thus been poor [18]. Moreover, the framework aims at a sound sampling and recording protocol, rather than a treatment of the statistical computation methods. Other AMS guidelines also discuss sampling frameworks for M&V projects. AMS II.C (Demand-side energy efficiency activities for specific technologies) [19] discusses energy efficiency equipments such as lamps, ballasts, pumps, motors, and fans saving less than 60 GWh. It makes provision for constant-load equipment such as lamps, and recommends that a sample be taken conforming to the 90/10 criterion, at least for 90 days to establish a baseline. It also makes provision for energy use being monitored instead of operating hours for constant-load devices (par. 31). AMS II.J (Small Scale methodology: Demand-side activities for efficient lighting technologies) [8], specifically applicable to ballasted CFL's replacing incandescent lamps. The methodology specifies that two approaches may be used: the first is a "deemed-savings" approach where it is assumed that the lamps burn for 3.5 hours per day, and no measurements are done. For the second approach, monitoring is required: both for the operating hours and retention rates (lamps left after a certain period of time).

Recently, more focused studies of the problem have been conducted, specifically regarding the replacement of incandescent lamps with Compact Fluorescent Lamps (CFLs). The most notable pertains to two-sample meter cost minimisation models, where a CFL and a Light Emitting Diode (LED) group are combined in a stratified random sample weighted according to population sizes and solved using frequentist statistics [20]. This model was then applied to a case of a single population over multiple years, where population decay is also considered [21]. It was assumed that samples are independently and identically distributed (i.i.d.) because meters are placed in different households. However, this tacitly assumes that samples from different years, taken by the same meter in the same household, are independent. Such time-series data are usually autocorrelated and not independent unless it is a Gaussian ‘white noise’ process [22]. It has been shown that autocorrelation does exist in hourly and daily energy use data [14], but if observations have constant variance and a lag 1 autocorrelation of ρ is assumed (as has been in literature [13]), then the number of effective observations n' may be modified to [23]:

$$n' = n\frac{1-\rho}{\rho}. \tag{2.1}$$

However, given that the nature of the autocorrelation is unknown during the modelling phase, a simple model for i.i.d. measurements may be posited as a starting point, with a recommendation that future work investigate the possibility of autocorrelation and employ the concomitant statistical tools. Therefore the two aforementioned studies will be used as a basis for this contribution, and expanded to incorporate more advanced population decay, weighting, and economic considerations.

2.3 POPULATION MODELS IN LITERATURE

Various approaches to lamp survival modelling may be followed. First, notable methods that were not adopted will be reviewed briefly, after which the approach that has been decided upon will be discussed.

2.3.1 Miscellaneous population survival approaches

Time Series Forecasting is a set of mathematical techniques used to predict data such as future sales or stock prices given the past sales history or stock market fluctuations [24]. Markov chains, neural networks and general statistics also find applications within Time Series Forecasting, but will be discussed later. Time Series Forecasting relies on past data, and is especially useful for considering periodicity and seasonality. Although the method decided upon to predict lamp population decay could be viewed as a time series forecast, the techniques employed in the financial sector are not completely suitable to the problem at hand, as it occurs on the logistic scale (0;1).

The lifetime of CFL’s are thought to be strongly correlated to the number of ‘cold’ starts experienced, i.e. how many times they have been switched on and off [25]. However, results are not conclusive [26]. This finds an analogy in metal fatigue, where the fatigue life of a component can be predicted with accuracy, given a known fatigue curve, number of cycles and stress. Thorough treatment of the topic and methods used to predict fatigue do exist [27], but show that fatigue analysis is concerned with the relationship between the stress intensity and the number of cycles to failure. Because there is no parameter analogous to the stress intensity for CFL life, the methods of fatigue analysis are not relevant to M&V performance tracking.

Markov analysis, also called state-space analysis, is an elegant way to model how certain stochastic systems evolve over time. For instance, consider a workshop where a heterogeneous population of machines could be classified as new, standard, old or failed. When taking a Markov chain approach, each member of the population has a certain probability of transitioning to the next state given the current state. These transition probabilities are considered to be stationary as they are independent of previous states or events, and do not change with time. Eigenvalue techniques then provide equilibrium parameters. Although useful for repair-and-replace scenarios, it assumes a constant hazard rate. The hazard rate is also called the failure rate, but applies to non-repairable items and represents the number of failures expected in a specific time. Since ageing is an important factor for CFL's, the constant hazard assumption is invalid. Theoretically, the constant hazard assumption need not limit Survival Analysis using Markov chains [28], [29] as various non-parametric or semi-parametric likelihood approaches may be followed [28], with applications also discussed in literature [29]. A Bayesian approach to the problem may be taken by implementing a Markov Chain Monte Carlo (MCMC) algorithm, if data is available by which a posterior probability may be elicited [30]. Although these approaches warrant further investigation and may solve the problem at hand, they do so inelegantly because MCMC, for example, doesn't solve analytically but in a brute-force approximate way. Simpler methods may therefore be employed.

Clinical trial design and SA may also be used to model lamp population decay, as both human and lamp populations follow the same sigmoid decay curves. Like performance tracking for M&V projects, clinical trials track subjects over time, until they experience an 'event', whether it be cure, death, or some biological parameter reaching a predetermined 'endpoint'. These techniques have been implemented in many statistical packages [31]. Studies determining the smallest number of subjects necessary for a statistically significant result have been designed [32], and numerous resources are available for the engineer seeking to implement these principles ([33], [34], [35], [36], and [37]). An engineering experiment design approach has also been developed [38] and considers subject dropout (also called censoring). However, certain simplifying assumptions are made that are untenable for the current analysis. Both aforementioned texts emphasise the statistical power of the test ($1-\beta$, the complement of the Type II error, which is the probability of incorrectly failing to reject the null hypothesis); an important metric in such experiments which is not given much consideration in current

M&V literature. This is because power considerations are usually important when two-sample hypothesis testing is done. It should also be noted that as with the IPVMP approach, with the clinical trial approach the variance of the population still needs to be estimated beforehand in order to continue with calculations.

SA uses parametric and non-parametric statistics to describe the data and draw conclusions. However, Cox's seminal contribution [39] increased the scope of SA greatly by adding semi-parametric regression techniques by assuming proportional hazards, where the underlying shape of a survival function need not be known in order to analyse the data. Cox Regression has become synonymous with Survival Analysis, and is mentioned because for this study, it is not the SA technique of interest. The underlying shape of the distribution is of interest, and thus parametric techniques should be employed.

Since the current investigation focuses on spatial sampling (meters per sampling point) and not longitudinal sampling, SA is not immediately applicable to the question at hand. Also, many SA techniques can only be applied "ex-post", that is to say, after the study is completed. This is not compatible with M&V practice which requires reporting accuracy during the study, but such techniques may be incorporated in future, and these tools should feature more prominently in M&V literature although it falls outside the scope of the present study.

2.3.2 Basic population models

Let Φ_t express the proportion of functioning lamps surviving at time t, where N_0 denotes the initial population size and N_t the population size at time t:

$$\Phi_t = \frac{N_t}{N_0}. \tag{2.2}$$

Various models for population decay have been proposed [40]. By way of introduction, the simplest model is that of exponential decay:

$$\Phi_t = e^{-t} \tag{2.3}$$

Such a model is not realistic however, as it implies that the product has a constant hazard rate, which is an invalid assumption because of ageing, as discussed in Section 2.3.1.

The second population decay model considered is that in the UNFCCC CDM AMS-II.J Guideline, as implemented in current studies on which this study is based [21]. It is a straight line graph where

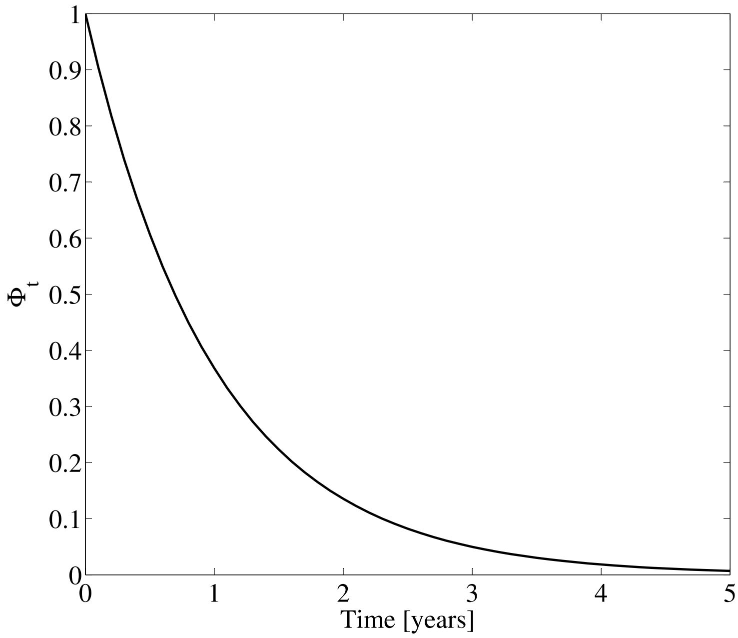

Figure 2.1: Graphic representation of equation (2.3)

H denotes annual operating hours and i denotes years. $Y = 50$ is recommended, where Y is the percentage of lamps left at the rated lifetime L :

$$N_t = \begin{cases} N_0 - i \times H \times \frac{100-Y}{100\times L} & \text{for } i \times H < L \\ 0 & \text{for } i \times H \geq L \end{cases} \tag{2.4}$$

Real populations of people, animals or engineering devices rarely follow straight-line curves, and equation (2.4) could at best be an approximation. Therefore a more sophisticated population model is necessary.

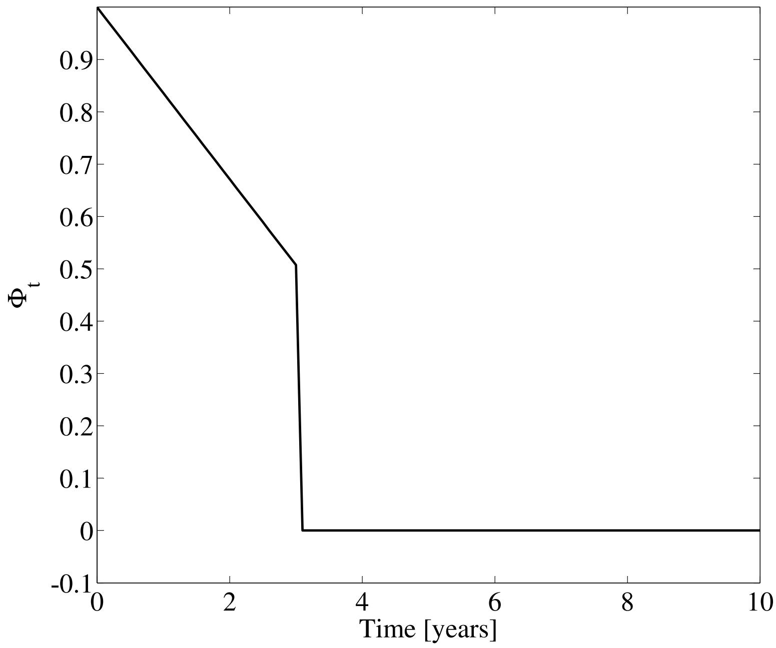

Figure 2.2: Graphic representation of the CDM population decay equation (2.4) for $Y = 50$, $i = 10$, $H = 1642.5$

2.3.3 Logistic population approach

Logistic population models are confined to the range [-1,1] and are therefore useful for modelling population proportions. These equations have been used to model the many different biological systems, as well as the human population proportions [41]. They have also been used to model adoption rates in energy systems [42].

Logistic models have sigmoid shapes - a feature expected by the CPUC SA approach, and also corroborated by empirical studies. For example, the Polish Efficient Lighting Project (PELP), conducted by the World Bank through the International Finance Corporation and UNFCCC, tracked 1.2 million lamps over a number of years in order to assess various aspects of such retrofit projects [43]. The main objective was to assess the immediate impact of the program due to CFL sales and distribution (CO_2 emissions reduction). The second objective was to assess the wider impact of the programme on the residential CFL market in Poland, specifically market penetration and sustainable market transformation due to non-subsidised CFL's being sold. Population survival was tracked as an intermediate variable, and is of interest to this study.

The results correlated with another study conducted by the contractor in the New York area [44], and it was found that the decay rate was approximately 6.2% p.a. The result is a logistic population decay curve shown in Figure 2.3. Some questions do still remain regarding their research methodology. However, since this is the best data available, and since the shape of the curve does agree with that described above, it is reasonable to use this data for the purpose of modelling.

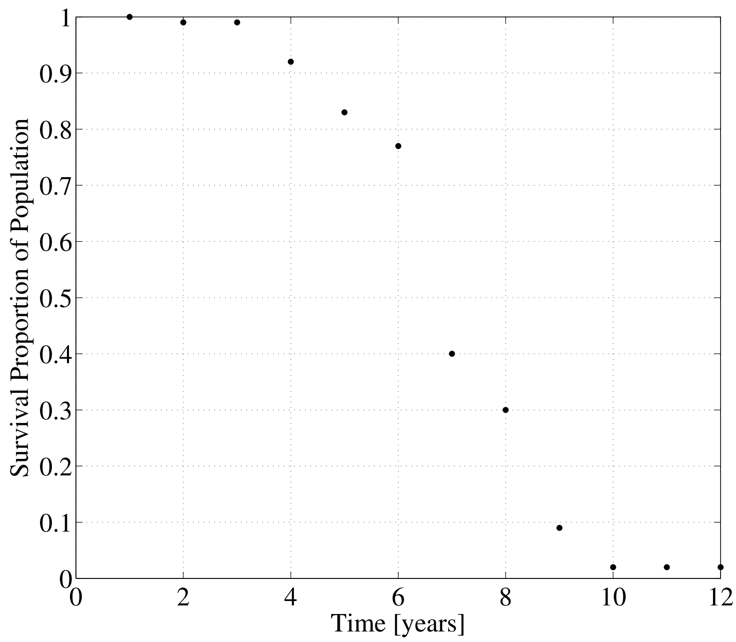

Figure 2.3: PELP Data [43].

CFL's currently installed by Eskom have median lifetimes of 6,000, 8,000, and 10,000 hours [45]. Because of these lifetimes, doing such a comprehensive study in South Africa is prohibitively expensive, and thus the PELP study is regarded as having the most reliable data for the South African context [45], given similar project characteristics. The following model has been proposed to fit these data:

$$\Phi_t = \frac{1}{1+e^{t-L}}. \tag{2.5}$$

This model is part of a family of logistic populations first proposed by Verhulst [46], and take the form

$$\Phi_t = \frac{1}{1+e^{-t}}. \tag{2.6}$$

These models were developed to describe biological population dynamics. The main contribution was to show the limiting effect that the carrying capacity of the land has on population growth, where

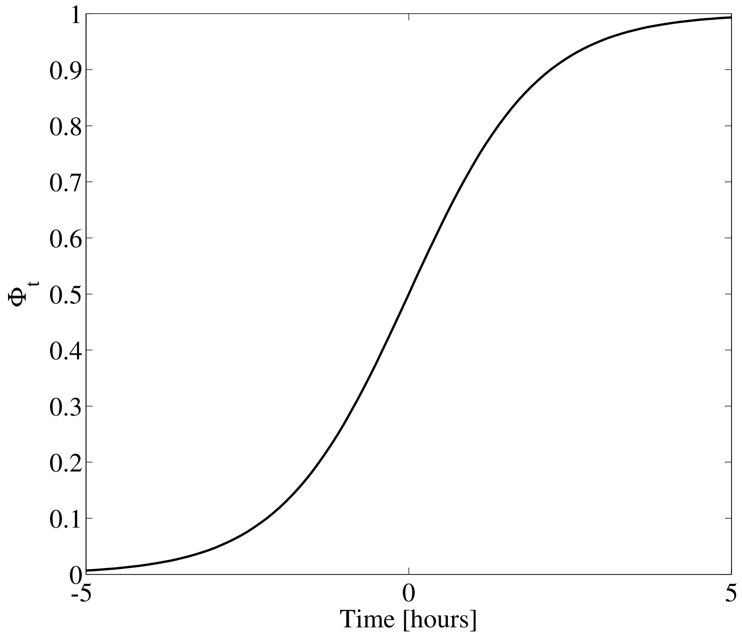

Figure 2.4: Graphic representation of the logistic function equation (2.6). Note that in this case population growth is indicated, although population decay may also be modelled.

population growth was seen as exponential and unconstrained previously. It may also be used as a population decay model by altering the sign of the exponent. Since the standard logistic equation is a symmetrically odd function centred around (0, 0.5), the $-L$ term was introduced in order to accommodate different lamp lives as is illustrated in Figure 2.5. As can be seen, the population decay rate is constant, and the parameter L only shifts the curve along the time axis. This means that although the curve would model populations with this decay rate with great accuracy (the PELP data happen to have such a decay rate), populations with different decay rates cannot be modelled adequately. It is for this reason that a more versatile population survival curve is needed.

2.4 CHAPTER SUMMARY

The two focus areas of this research are performance tracking and lamp population survival models. Although some academic literature on performance tracking exists, the application to optimal tracking in multi-year studies has not matured yet. Various solutions to lamp population decay has been considered, and the further development of a logistic function is deemed to be the most appropriate.

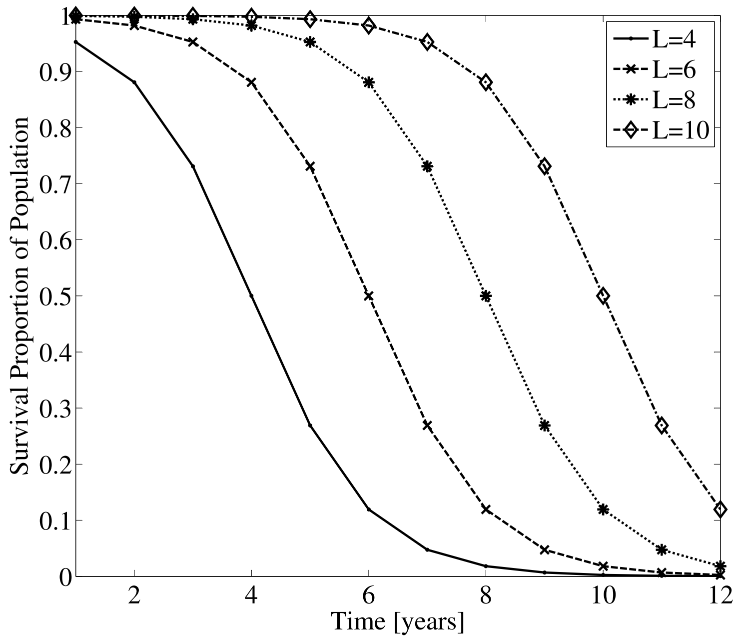

Figure 2.5: Decay curves from equation (2.5) for different lamp lives *L*.

Model	Advantages	Disadvantages
Time Series Forecasting	• Wealth of literature. • Can account for periodicity.	• Not easily adapted to logistic scale (0;1).
Fatigue Analysis	• Used for metal fatigue failure.	• Stress-magnitude parameter difficult to quantify
Markov Chain Analysis	• Simple matrix algebra approach. • Can be used with Monte-Carlo techniques.	• Constant hazard rate assumption invalid. • Non-constant hazard rate models too complicated.
Survival Analysis	• Large amount of literature exists. • A mature field. • Used for similar medical studies. • Used by the CPUC. • Accounts for censoring.	• Control group necessary. • Results only available ex-post.
Exponential Logistic Model	• Simple step-ahead forecasting. • Amenable to control techniques.	• Assumes a constant hazard rate.
UNFCCC CDM AMS-II.J	• Accepted by the UNFCCC. • Simple	• Assumes linear population decay. • Only considers population up to 50%
Logistic models	• Fits known data (PELP study) well. • Only one unknown parameter. • Within the range (-1;1). • Used to model population decay.	• Single-parameter models will not fit populations with different hazard rates well.

CHAPTER 3

LONGITUDINAL TRACKING MODEL DEVELOPMENT

3.1 CHAPTER OVERVIEW

A new decay model is formulated and written in different formats useful for engineering applications. The existent sampling model is discussed and assumptions identified. Improvements to the existing model, including the exponential windowing functions and time-value of money considerations, are discussed and implemented.

3.1.1 Population decay model formulation

As shown in Table **??** and Section 2.3.3, logistic models are widely used for population modelling due to their flexibility, simplicity and [-1;1] range. However, using a single-parameter logistic model logistic model prevents different population decay rates to be modelled.

The proposed improvement to equation (2.5) is similar to Lotka's reformulation of Verhulst's model as a dynamic equation with additional parameters [47]. As previous work suggests, the difference equation form of this decay model is especially applicable to the engineering context [48].

It is proposed that CFL decay be described by

$$\frac{d\Phi}{dt} = -\beta\Phi(1-\gamma\Phi). \tag{3.1}$$

which is also called the logistic growth equation [42]. In discrete (or difference) form it is written as

$$\Phi_{k+1} = \beta\gamma\Phi_k^2\Delta t - \beta\Phi_k\Delta t + \Phi_k, \tag{3.2}$$

and in general form it may be written as:

$$\Phi_t = \frac{1}{\gamma + \alpha e^{\beta t}}, \tag{3.3}$$

which could be written as:

$$\Phi_t = \frac{1}{\gamma + e^{\beta t - L}}, \tag{3.4}$$

where:

$$\alpha = e^{-L}. \tag{3.5}$$

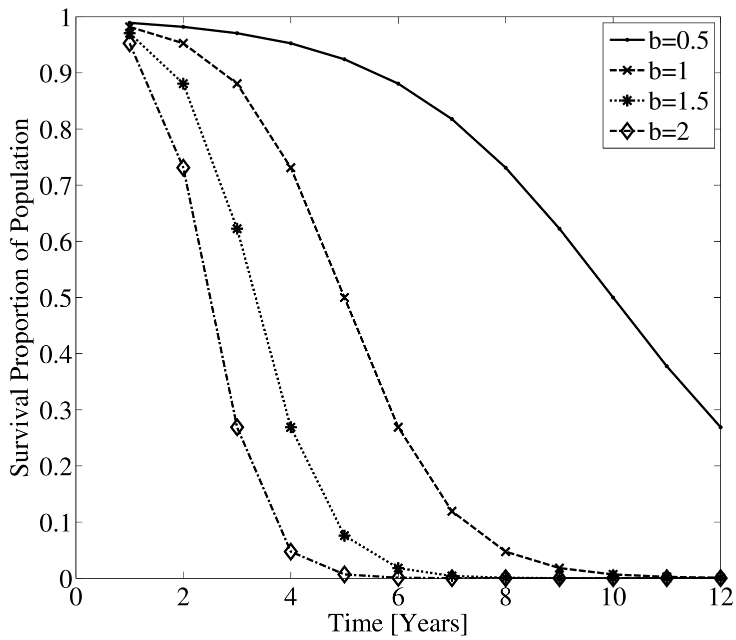

Figure 3.1: Decay curves for equation (3.4) using $L = 5$, $c = 1$, and varying b.

3.1.2 Parameters in the CFL population context

Not only does the logistic equation fit known data well, but the parameters have physical significance and correspond to factors affecting population decay.

The population decay parameter β indicates rate and is in the unit /time. The inflection point of the sigmoid curve is located at $\frac{\ln\alpha}{\beta}$. It is apparent that equation (3.1) is similar to equation (2.5) for $\beta = \gamma = 1$. However, the parameter β in equation (3.1) allows different population decay rates to be modelled as shown in Figure 3.1. This is important because it is not necessarily the case that the decay rate is always close to unity (even though this is shown to be the case for the PELP study [43]). With different operating conditions, different manufacturing quality control, or different technologies, it is expected that the assumption of $\beta = 1$ would not necessarily hold. A steeper slope would indicate a smaller manufacturing and operating variance, with more lamps failing closer to the mean lamp life. Thus the parameter β increases the flexibility of the model significantly, and it is possible that other technologies may also be modelled in this way. It is conceivable that a list of factors contributing to CFL population decay be drawn up, and β be predicted. However, much data will be needed in order to make ascertain the contribution of the covariates in such a way as to make meaningful predictions.

For population growth equations, γ represents the carrying capacity of the land. In this model, is approximately inversely proportional to the starting population because at $t = 0$,

$$\Phi_0 = \frac{1}{\gamma + e^{-L}}, \tag{3.6}$$

but for any realistic L,

$$\Phi_0 \approx \frac{1}{\gamma} = \Phi_0. \tag{3.7}$$

Although $\gamma = 1$ should always be the case (implying a starting population proportion of 1, i.e. all lamps working), the model fit has been found to vary in practice. These variations may be ascribed to project phenomena rather than true population behaviour. For example, Free Ridership (where subsidised CFLs would have been installed without the subsidy) may cause some units to be installed at a later date than project inception, since they are stored by home owners first whilst lamps purchased previously are used. This would alter the initial population in a way that can be accounted for by γ. However, γ cannot be used to account for a whole monitoring project starting after $t = 0$, e.g. setting $\gamma = 2$ if project monitoring starts where $N = 0.5$, as at time $t = 6.8$ in Figure 3.2. If this were the case, the decay rate $\frac{dN}{dt}$ would approach 0^- asymptotically where it should be maximum as at $t = 6.8$ (the point of inflection of the logistic decay curve). Also, altering the value of γ may not be used to compensate for non-homogeneous populations, for example where only half of the population is composed of CFLs. In such a case, stratified random sampling should be used, where each sub-group is homogeneous and considered separately.

3.1.3 Discrete formulation

The difference equation formulation means that fewer data points need to be used for the identification of system parameters, since the parameter L need not be determined. This also means that the lamp installation date is not relevant for determining the state of the system at $t = k+1$, as Φ_{k+1} is not a function of t, but only of Φ_k and parameters β and γ. Of course, the more information available the better, but strictly speaking it is not necessary.

The discrete form of equation (3.2) holds three advantages [48]: First, estimated savings at Φ_{k+1} informs the project manager of the feasibility of continuing the project. Second, since survival data are binomially distributed, and for binomial data the sample size $n = f(\Phi)$, this provides information about the estimated population size (and thus required sample size) at the next time step. Third, control techniques may be applied to the problem of lamp replacement by reformulating equation (3.2) as

$$\Phi_{k+1} = \beta\gamma\Phi_k^2 - \beta\Phi_k + \Phi_k + u_k \tag{3.8}$$

for $\Delta t = 1$, where u_k is a control input. This finds practical application in a scenario where a project developer is paid by Eskom based on the savings realised in a certain EE project. The developer would then want to optimise his control inputs (replacing lamps) over the duration of the project in order to ensure that he maximises profit. Since original publication [48], a preliminary version of this approach has been implemented [49].

3.1.4 System identification

During the system identification phase, the model formulated above is tested to determine how well it fits known data.

System parameters β and γ can be identified using a least-squares approach with decision variables β and γ. Let ϕ_t be the surviving proportion of lamps at time t in the PELP study. For year I as the last reporting year, the objective function is defined as

$$\min \frac{1}{n} \sum_{t=2}^{I} (\Phi_t - \phi_t)^2, \tag{3.9}$$

where Φ_t is defined by equation (3.2). The function considers data from $t = 2$ and onward since $t = 1$ is assumed to be known such that $\Phi_1 = \phi_1$. It is found that the PELP data can be characterised by $\beta = 0.921$ and $\gamma = 0.986$, with a Mean Squared Error (MSE) of 0.0015. The result is shown in

Figure 3.2. By way of comparison, the optimal least-squares fit of equation (2.5) yields $L = 6.866$, with an MSE of 0.0368.

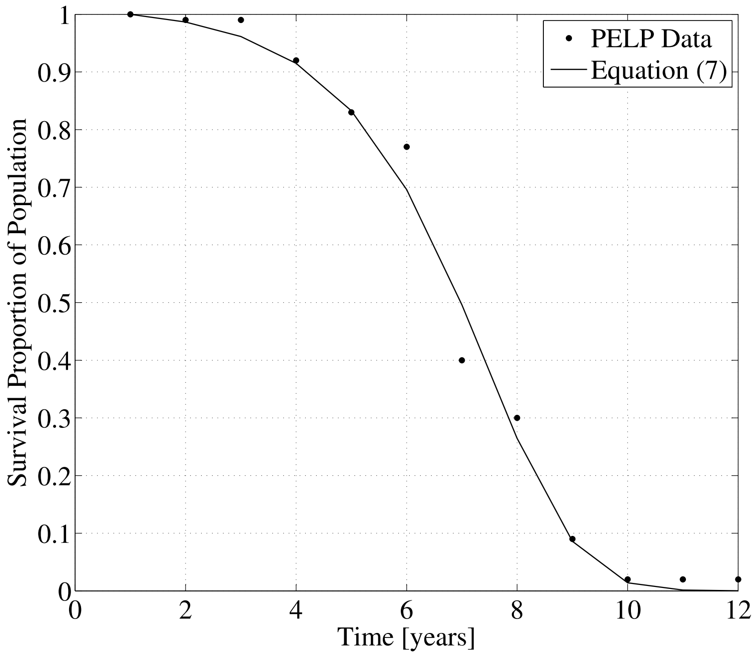

Figure 3.2: Least-squares fit to PELP data of equation (3.2)

3.1.4.1 Goodness of fit measures

At this point a short discussion of goodness-of-fit metrics is warranted, since much confusion exists regarding the appropriate metrics that should be used to describe model uncertainty for time-series data.

Most engineers use the coefficient of determination R^2 as the default goodness of fit measure. However, the R^2 value is dependent on the slope of the curve: R^2 changes with the gradient of the straight line fitted to the data [13], even if the CV of the data remains the same. In this case CV refers to the mean variation in the data not explained by the model, normalised with respect to the mean of the dependent variable. R^2 may therefore be used to compare how well different models fit to the same data set or how well a model fits a given data set (e.g. baseline energy use). However, if the slope of the energy use curve changes, then comparing R^2 values would be invalid. R^2 may therefore not be used to draw comparisons between models fitted to different data sets or as an absolute index of how well a given model fits a given set of data.

The Coefficient of Variation on the Root Mean Squared Error (CVRMSE) is prescribed by ASHRAE Guideline 14-2002 Section 5.2.11.3 (Modelling uncertainty) [6]. However, models with different 'dispersions' on the data may have the same CVRMSE values if their means also vary proportionally [13]. Therefore the CVRMSE value is also not an adequate statistic by itself, unless the only criterion for goodness of fit is the uncertainty in model prediction for the same model with different data sets [13]. Furthermore, the CVRMSE is the RMSE normalised with respect to the mean value. Therefore this metric may only be used where the mean is stationary. The mean of the population decay curve is not stationary, and it is conceivable that the CVRMSE would change from year to year as the mean changes, even if the goodness of fit of the model does not change.

Like the CVRMSE, the Normalized Mean Bias Error (NMBE) used by ASHRAE 14-2002 (equation 5.4) is normalised with respect to the arithmetic mean of a sample of observations, and therefore unsuitable for models with non-stationary means.

The considerations mentioned above change when autocorrelation is present. Autocorrelation occurs when the errors at different measuring points are not independent of each other, and residuals are not normally distributed - a key assumption for normal regression. This is most commonly caused by time-dependent changes in a building (central heating, for example) or by not considering key energy governing factors. (Dry bulb) temperature changes are easily measured and data is readily available, but humidity may also affect cooling load, and is usually not measured. When plotting a regression curve of a dependent variable against an independent variable in a scatterplot, the temporal dependence is not shown since data points measured consecutively are not plotted consecutively. However, autocorrelation is still present in the autocorrelation of the residuals and will affect model accuracy.

When using standard Ordinary Least Squares (OLS) estimators for autocorrelated data, the estimates of the regression parameters (in this case β and γ) may not be biased, but standard techniques may overestimate confidence on these parameters greatly. Using an autoregressive (AR) model (one where the state at the previous time is taken into account, as is done in this study) may then improve estimates on the error bounds [14].

Because of the considerations listed above, selecting a metric to determine goodness of fit for this case is not simple. It was decided to use the MSE. The reasons for doing so are that the MSE is simple and easily understood, that the AR model implemented should account for autoregression, and that the resultant fit is satisfactory. The caveats are that the MSE may be biased [14] and that it may overestimate the error bounds. However, since this model will not be used to estimate error bounds, this consideration is not important for the current study. Since it is a non-linear curve, the R^2 and CVRMSE figures listed above were not used, although they could just as easily have been implemented in the objective function instead of the MSE. The hybrid prediction method [14] was not used because it is not as widely recognised, although it does represent a viable alternative to the MSE approach.

3.1.4.2 Residuals

One method of evaluating the appropriateness of a model is to investigate the nature of the residuals. Ideally, the residuals should be normally distributed with no discernible pattern with respect to time.

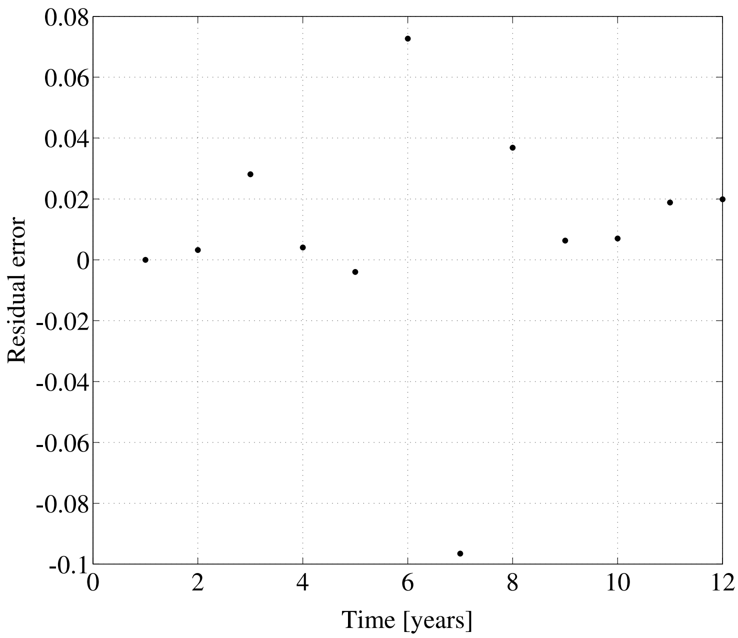

Figure 3.3: Plot of equation (3.2) with parameters as determined in Section 3.1.4

As can be seen in Figure 3.3 the residuals are not completely random or evenly spaced around 0, and exhibit a slight positive gradient, implying that there may be a missing linear term in equation (3.2). However, two outliers are notable in years 6 and 7, and do skew the residuals. From Figure 2.3 it can be deduced that these two do not follow the general trend of the curve, and thus outliers on the residual plot are to be expected. When plotting these residuals on a normal probability plot

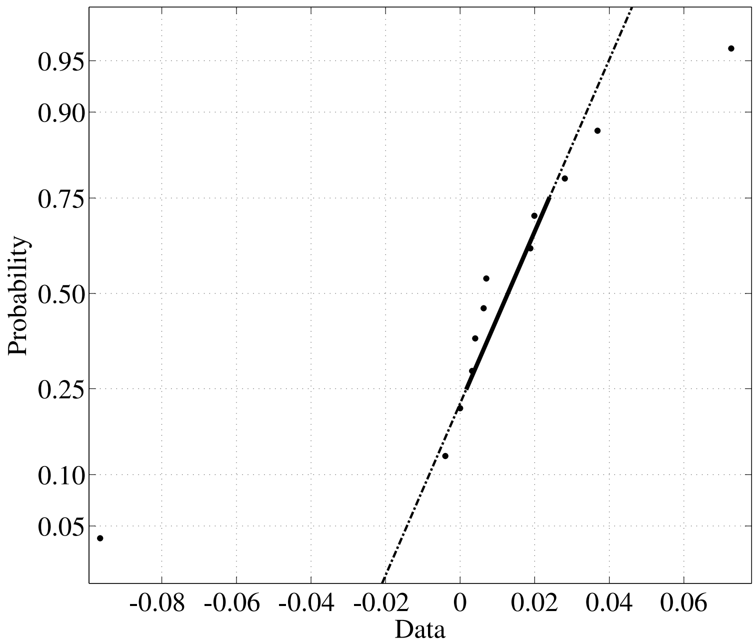

Figure 3.4: Normal probability plot of equation (3.2) with parameters as determined in Section 3.1.4

(Figure 3.4), however, one can see that the assumption of normality is not violated significantly, as all residuals fall fairly close to the straight line - especially the second and third quartiles, which are indicated by the thicker line. Since the residuals are not completely randomly distributed around the normal probability line, some autocorrelation may still be present. However, this component is relatively small, and therefore it is concluded that even though the model may be improved, the fit is satisfactory.

3.1.4.3 Incremental step-ahead forecasting

It is noted that in practice, parameters β and γ may not be known accurately beforehand. Let D_k be the data available at time k, in this case the sample population proportions. Then:

$$(\Phi_{k+1}|D_k) = f(\Phi_k, \beta_k, \gamma_k), \tag{3.10}$$

where β_k and γ_k are determined incrementally at each time $t = k$ so that

$$(\beta_k, \gamma_k) = (\beta, \gamma | D_k). \tag{3.11}$$

Since equation (3.2) has two parameters instead of one, more data is needed to define the equation (3.2) than would be the case for equation (2.5). However, it is argued that a two-parameter model such as equation (3.2) is not merely convenient, but necessary since equation (2.5) assumes $\beta = 1$, i.e. all populations decay at the same rate (the PELP population decay rate). As such, it appears accurate for predicting the decay of the data in question, but would not be able to predict other population survival curves satisfactorily.

The accuracy of equation (3.10) would be improved if β and γ are known or expected from previous studies. Since $\gamma \approx 1$ for all cases, it is recommended that this assumption be made in the early stages of project monitoring with unknown parameters. This reduces the number of unknown parameters to one, thereby reducing the amount of data needed to define the model accurately.

In future, population decay models may be improved by taking sample sizes into account. In the current context D_k only contains the proportion of lamps surviving at time k according to a sample taken. However, D_k can also include more information such as the sizes of samples corresponding to each population proportion, as well as censoring information. Since larger samples should carry more weight when calculating model parameters than smaller samples, more accurate parameter estimation can be achieved. Such considerations can be incorporated by the use of conditional probability methods, but is outside the scope of the current study.

3.2 SAMPLING MODEL FORMULATION

When calculating energy savings for a lighting retrofit project with a total number of groups J there are two main components that constitute energy use. The first is the average daily energy use per device E_j, and the second number of surviving devices N_j [50]. Taking a baseline energy use per device E_B, this may then be expressed as

$$E_B = \sum_{j=1}^{J} (N_j E_j). \tag{3.12}$$

N_j is affected by population decay as described in Section 2.3, and E_j is the energy use determined directly from energy meter measurements, and not from occupancy schedules or usage hour estimates and lighting system efficacy measurements. Meter measurements will be the focus of the rest of this study.

A statistical model for describing sequential metering samples has been formulated previously [21]. As the aforementioned model was used as the basis for the current research, the assumptions and key equations will be reproduced below.

The assumptions on which this model is based have been stated as [21]:

1. Metering data are independent and normally distributed.

2. The lamp population does not decay during the CDM-specified 90-day baseline period.

3. During the reporting period maintenance is performed on active meters only.

There are also additional latent assumptions in the aforementioned model. It is recommended that the sensitivity of the model to these assumptions be investigated and in a future study:

4. The mean energy use (the integral of the daily load profile) is stationary throughout the study.

5. Samples of the same population, taken in different years, can be treated as independent.

6. Statistical power (Type II-errors) is not considered in calculating sample size

7. The meter purchasing cost is constant in future value terms. That is, R4,032 would purchase a meter in any given year.

The assumption of normally distributed metering data in 1 is made implicitly in the IPMVP [3], ASHRAE 14-2002 [6], FEMP [10], and seminal literature on the subject [13]. It is the responsibility of the M&V practitioner to verify the validity of this assumption and there may be cases where normality does not hold. However, given the central limit theorem and the number of independent variables which influence energy usage, assuming a normal distribution is more valid than assuming any other distribution at project inception.

The assumption of a stationary mean is potentially significant. First it should be noted that since the same lamps are being measured repeatedly, continuity is expected. Seasonal effects should be visible in the month-to-month energy use, however, since annual energy use is considered for calculation, it may be neglected. At a finer surveillance resolution, a model correcting for seasonality and other periodic autocorrelative effects should be implemented. Assuming a stationary mean for modelling purposes simplifies calculation, and is the preferred choice in the absence of data to the contrary. It would also be possible for account for varying parameters during actual studies, using non-routine adjustments. This is because the sampling interval allows for recalculation to be done where equation (3.16) could be rewritten as a less elegant summation with different mean and standard deviations for different years. Likewise, θ_K and Γ_K below may be adjusted when this is deemed necessary.

3.2.1 Calculation of mean and variance

Because of the assumption of a stationary mean in Point 4 above, the cumulative sample mean $\bar{\chi}$ varies according to

$$\bar{\chi}_K \sim \text{Normal}[\theta_K, \Gamma_K^2], \tag{3.13}$$

where the true mean θ_K is defined as

$$\theta_K = \frac{\sum_{i=1}^{K} N_i \mu_i}{\sum_{i=1}^{K} N_i}, \tag{3.14}$$

and the cumulative sampling standard deviation is defined as

$$\Gamma_K = \sqrt{\sum_{i=1}^{K} \frac{\sigma_i^2}{n_i} \cdot \frac{N_i^2}{\left(\sum_{i=1}^{K} N_i\right)^2}}. \tag{3.15}$$

Because of the assumption of a stationary mean, samples taken at different times may be combined to give a cumulative sample size with which calculations may be done.

Assuming that a given sampling mean for year i follows the distribution $\bar{X}_i = \text{N}(\mu_i, \sigma_i^2/n_i)$, the cumulative sample mean in year K is defined as

$$\bar{\chi}_K = \frac{\sum_{i=1}^{K} N_i \bar{X}_i}{\sum_{i=1}^{K} N_i}. \tag{3.16}$$

By substituting the variables defined above into the standard score transformation

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \tag{3.17}$$

and rearranging, we find that

$$Z_K = \frac{\bar{\chi}_K - \theta_K}{\Gamma_K} \tag{3.18}$$

and from equation (3.19)

$$P_K = \frac{\bar{\chi}_K - \theta_K}{\bar{\chi}_K} \tag{3.19}$$

where P_K and Z_K denote the cumulative precision and standard scores up to the Kth crediting year.

Practically, however, the true mean μ and true standard deviation σ are not known; only the sampling mean $\bar{x}$ and sampling standard deviation s are known. In order to do simulations for meter placement planning, these values have to be assumed. By realising that if the coefficient of variation, CV, is defined as

$$\mathrm{CV} = \frac{\sigma}{\mu}, \tag{3.20}$$

then

$$s = \bar{x}CV. \tag{3.21}$$

It should be noted that the CV value is fundamentally different to the coefficient of variance on the root mean squared error CVRMSE, although in this case they may be the same. The coefficient of variation described above is a measure of the variance of a dataset relative to the mean of the data, in other words the ‘spread’. A variance of 4 is relatively big if the mean is 10, but is relatively small if the mean is 100. In the first case, $\mathrm{CV} = 0.2$, in the second, $\mathrm{CV} = 0.02$. CVRMSE is measured relative to the model predicted value, or the estimator. If it is an unbiased estimator, $\mathrm{MSE} = \sigma^2$, in other words, the variance is equal to the mean squared error. But this is not always the case, as there are many biased estimators used in statistics.

By substituting equation (3.21) into equation (3.17), the way in which confidence and precision have been formulated in previous studies [21] has been that for year i,

$$Z_i = \frac{\sum_{j=1}^{i} \frac{N_j z_j}{\sqrt{n_j}}}{\sqrt{\sum_{j=1}^{i} \frac{N_j^2}{n_j}}} \tag{3.22}$$

and

$$P_i = \frac{\sum_{j=1}^{i} \frac{CV_j N_j z_j}{\sqrt{n_j}}}{\sum_{j=1}^{i} N_j}. \tag{3.23}$$

It should be noted that this formulation does not allow for sample sizes of zero, i.e. for studies where there are years where no samples are taken. This presents a problem when optimising, as not considering sample sizes of zero constrains the problem unnecessarily. Thus, Z_i may be formulated as

$$Z_i = \begin{cases} \text{equation (3.22)} & \forall\, n_i > 0 \\ 0 & \forall\, n_i = 0 \end{cases} \tag{3.24}$$

There is no need to constrain P_i since it is a function of z_i for years subsequent to a zero-sample year, and it may therefore be left as undefined or 'Not a Number' (NaN) in Matlab:

$$P_i = \begin{cases} \text{equation (3.23)} & \forall\, n_i > 0 \\ \text{undefined} & \forall\, n_i = 0 \end{cases} \tag{3.25}$$

3.2.2 Sample size calculation

The well-known standard normal sampling formula shows that

$$n_0 = \frac{z^2 \mathrm{CV}^2}{p^2}, \tag{3.26}$$

where z is the standard score corresponding to a given confidence level P(z).

The relative precision p is defined as the maximum difference between the confidence limits (CL) and the mean, normalised with respect to the mean:

$$p = \frac{|\mathrm{CL} - \mu|}{\mu}. \tag{3.27}$$

Therefore, for the 90/10 criterion (90% confidence interval, 10% precision), and assuming a CV of 0.5 as is usually assumed in M&V for homogeneous samples [3], [10] the required sample size would be

$$n_{unadjusted} = \frac{1.645^2 \cdot 0.5^2}{0.1^2} \approx 68. \tag{3.28}$$

For small populations, it is necessary to include a finite population adjustment:

$$n = \frac{n_{unadjusted,i} N}{n_{unadjusted,i} + N}. \tag{3.29}$$

This adjustment affects the sample size for $n/N > 5\%$ [51]. By combining equation (3.26) and equation (3.29), the required sample size is found to be

$$n_i = \frac{z_i^2 CV_i^2 N_i}{z_i^2 CV_i^2 + N_i p_i^2}. \tag{3.30}$$

3.2.3 Project cost calculation

Project costs may now be calculated, and consist of two parts: the project inception cost, and operational costs.

Let a_i be the meter purchasing cost in year i, b_i be meter installation cost, and c_i be the monthly meter maintenance cost, all expressed in per unit Present Value. The project inception cost (which includes 3 months' maintenance in baseline period) would then be

$$\eta = (a_0 + b_0 + 3c_0)n_0, \tag{3.31}$$

and project operational costs would be

$$\omega = \begin{cases} \sum_{i=1}^{I}[12c_i n_i - B_i(a_i + b_i)] & \forall B < 0 \\ \sum_{i=1}^{I}(12c_i n_i) & \forall B > 0 \end{cases} \tag{3.32}$$

where B_i is the backup meters (meters previously installed, but no longer needed) available in year i, defined as

$$B_i = \max(B_{i-1}, 0) + n_{i-1} - n_i. \tag{3.33}$$

3.2.4 Optimisation formulation

Therefore one could formulate the mathematical optimisation program for a project up to year I as:

Decision variable:

$$\boldsymbol{\lambda} = (z_1, p_1, ...z_I, p_I) \tag{3.34}$$

Objective function:

$$\text{Min } \eta + \omega \tag{3.35}$$

Constraints:

$$Z_i \geq 1.645 \; \forall \; i \; \in \; \boldsymbol{\delta} \tag{3.36}$$

$$P_i \leq 10\% \; \forall \; i \; \in \; \boldsymbol{\delta}, \tag{3.37}$$

where $\boldsymbol{\delta}$ represents the set of reporting years. For example, if it is required that savings be reported in years 1, 5, and 10, $\boldsymbol{\delta} = (1, 5, 10)$.

3.2.5 Model improvement

The model thus far is standard theory, but may be improved upon by implementing the changes discussed below.

3.2.5.1 Non-linear decay model

The first improvement that has been implemented is a non-linear population decay model as described in Section 2.3. Previously, the CDM population decay model was used [21], as described in equation (2.4). The usefulness of such a model has been argued for in Section 3.1.2.

3.2.5.2 Exponential windowing function

Previously, all measurements in the time series were weighted equally as a legacy of single-time, multiple-sample models. However, in practice data obtained during earlier measurements are not accorded the same weight as data obtained more recently. For example, in year 10 of a study, a relatively large sample size in year 1 could outweigh more recent data. In such a case, an exponential windowing function as described in equation (3.38) would compensate for the age of the measure. Therefore an exponential decay window, akin to exponential smoothing functions used in time series analysis, has been introduced. This can be thought of as a moving weighted average.

Whereas the cumulative mean distribution was formulated as equation (3.14), for an exponential decay factor ε, it is now written as

$$\bar{\chi}_i = \frac{\bar{X}_K N_K}{N_K} + \frac{\sum_{i=1}^{K-1} \bar{X}_i N_i (1-\varepsilon)^{K-i}}{\sum_{i=1}^{K-1} N_i (1-\varepsilon)^{K-i}}, \tag{3.38}$$

thereby weighting measurements not only by sample size, but also by recency.

3.2.5.3 Time-value of money considerations

Because the kind of projects under investigation have long planning horizons, it is prudent to consider the time-value of money when calculating project cost. Two factors were taken into account: the depreciation of meter purchasing values, and the opportunity cost incurred from spending money early in the project, when that money could have been invested to generate interest.

Let $d = 6.44\%$ be the Consumer Price Inflation (CPI) [52], and $r = 10\%$ be the Minimum Attractive

Rate of Return, or investment interest rate. For year n, the true meter purchasing cost is calculated as

$$a_n = \frac{a_0}{(1+d)^n} + \frac{a_0(1+r)^{K-n}}{(1+d)^K}. \tag{3.39}$$

It is assumed that the meter purchasing cost stays constant in future value terms, i.e. Ra_0 would purchase a meter in any given year. Due to inflation, however, the meter purchasing cost declines in real terms according to the inflation rate. This is the first term in equation (3.39).

Furthermore, it is assumed that the money used to purchase a meter could be invested at a rate of return of $r = 10\%$ until the end of the study in year K. This is the opportunity cost incurred by purchasing the meter in year n. This is the second term in equation (3.39).

Since meter installation and maintenance costs are labour costs, it is assumed that they will increase with inflation, and thus stay constant in real value terms. However, opportunity costs are still taken into account so that

$$b_n = b_0 + \frac{b_0(1+r)^{K-n}}{(1+d)^K}, \tag{3.40}$$

and

$$c_n = c_0 + \frac{c_0(1+r)^{K-n}}{(1+d)^K}. \tag{3.41}$$

3.3 CHAPTER SUMMARY

The logistic population decay model represents an improvement on current lamp population decay models. It may be formulated as a difference equation, allowing incremental step-ahead prediction to be applied to the problem. System Identification performed on benchmark data shows that the model is satisfactory, although selecting a goodness of fit metric is challenging. The existing sampling formulation is found to have critical latent assumptions. Improvements to the current model, namely an exponential windowing function and considering the time value of money, are added to the model to increase practicality for implementation.

CHAPTER 4

CASE STUDY

4.1 CHAPTER OVERVIEW

In this chapter the theory developed thus far is applied to a real-world project. Firstly, the code was validated by investigating the nature of the solution space and testing the code against previously established solutions. Certain changes are made and improvements implemented, and new solutions are compared in terms of cost. A statistical sensitivity analysis is also conducted to determine the influence of various parameters on the the expected project cost.

4.2 BACKGROUND

A previous case study and model [21] is used as a benchmark to ensure fair comparison. In this case study, incandescent lamps were replaced with CFLs in a number of provinces in South Africa [53] by distributing them free of charge to households in the Western Cape, Northern Cape, Gauteng, Limpopo, Mpumalanga and the Free State. Installation was ensured by counting and crushing certificates, verifying the destruction of previously installed incandescent lamps. The relevant parameters are listed in Table 4.1. The scenario described above is deemed appropriate for the decay and sampling model proposed in this study. The crediting period is assumed to be 10 years with reporting every second year complying to the 90/10 criterion. The baseline measurement period is 3 months, and it is assumed that no population decay occurs during this time.

The smoothing parameter was set at $\varepsilon = 0.25$ (a time constant of 4). The 0.25 value was selected since values between 0.2 and 0.3 are recommended in industry as a starting value for time series analysis [54], [55]. Since this isn't a classic time series analysis, one should not expect the same values to hold.

However, since this seems like a reasonably realistic way to weight sampling results from previous years, it has been adhered to for the purposes of this study, although ε should be case-dependent and determined by the M&V engineer's judgement.

4.2.1 Monitoring plan

The UNFCCC CDM guideline AMS II.J (Demand side activities for efficient lighting technologies, small-scale methodology) [8] Par. 19 defines how monitoring should take place for the case study. Operating hours O_j (in the standard) may be determined by "the average measured value determined from measurements of a representative sample conducted once, prior to or concurrent with the first ex post monitoring survey." Therefore the load profile is measured instead of operating hours and power is used. This also simplifies the calculation, since uncertainty calculations are combined in one measurement rather than two. AMS II.C (Demand-side energy efficiency activities for specific technologies) [19] confirms this approach in par. 31 where it states that:

"If the project equipment installed has a constant current (ampere) characteristic, monitoring shall consist of monitoring either the 'power' and 'operating hours' or the 'energy use' of the equipment installed using an appropriate method. Appropriate methods include... Metering the 'energy use' of an appropriate sample of the project equipment installed."

For this investigation, it is assumed that an energy meter placed on the lighting circuit measures the energy used on a per-second basis, and averages over thirty seconds to save two energy use values per minute. In cases where meters with current transformers only are used, spot measurement of the utility supply voltage should be done.

4.2.2 Model validation

First, the nature of the search space was investigated. The previously reported case study was used with its reported optimal solution (labelled 'Solution 1'), a line section was drawn to another solution ('Solution 2'), and the resultant cost function plotted in Figure 4.1. This proved that the search space is both stepped and discontinuous, and explains the finding that solutions given by gradient methods such as the interior-point algorithm are sensitive to the initial solution $\boldsymbol{\lambda}_0$. The stepped nature of

Table 4.1: Case Study Parameter Values (before changes)

Parameter name	Value
Reporting years	$\boldsymbol{\delta}$ = (2,4,6,8,10)
Meter purchasing cost	a_0 = R 4,032
Meter installation cost	b = R420
Meter maintenance cost (per year)	c = R122
Coefficient of variation	CV = 0.5
CPI inflation rate	d = 6.44%
Investment interest rate	r = 10%
Exponential decay factor	ε = 0.25
Population size	N_0 = 607,559
Rated lamp life	L=20,000 hours
Daily usage	4.5 hours
Incandescent Lamp power rating	100W
CFL power rating	20W

the solution space can be attributed to the effects of rounding; decimal sample sizes may be found using equation (3.30), but these need to be rounded to integer values for the cost function. The discontinuous nature may be explained by the fact that confidence and precision pairs interact in a complex way within $\boldsymbol{\lambda}$ to produce the cumulative confidence and precision levels.

Thereafter, a cross section was drawn at a random dimension of the solution, in this case $\boldsymbol{\lambda}_7$, and this was made to vary with Δh. It can be seen in Figure 4.2 that for $\boldsymbol{\lambda}_7$ the algorithm did indeed converge on the optimal solution, and that it is constrained on one side. Although not shown here, this is also the case for the other dimensions of $\boldsymbol{\lambda}$.

The cost calculation was validated by reprogramming the cost equations into Microsoft Excel and comparing the costs of various sampling plans to the results given by the Matlab subroutine.

Given the findings discussed above, it was decided that a Genetic Algorithm (GA) is appropriate for solving the problem at hand. The optimisation parameters are shown in Table 4.2. In order to

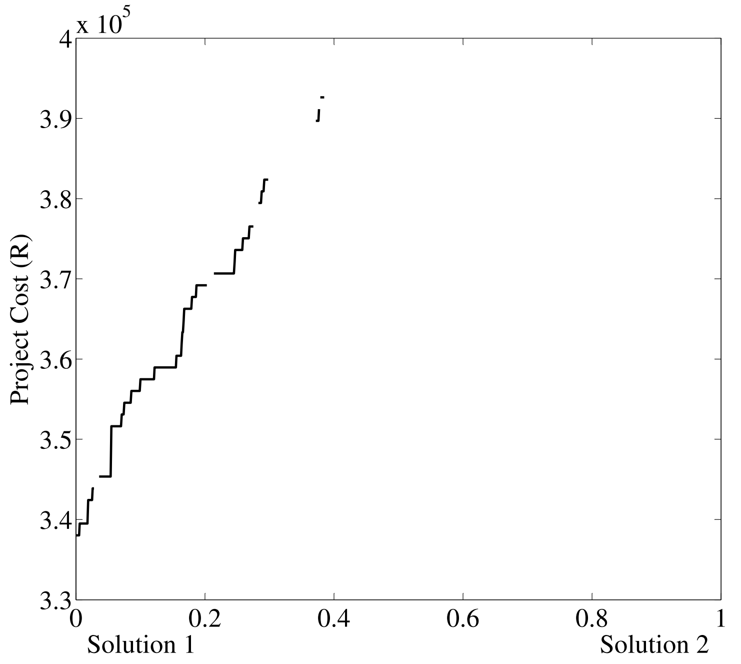

Figure 4.1: Line section through search space for Solution $1 + h \times (Solution2 - Solution1)$

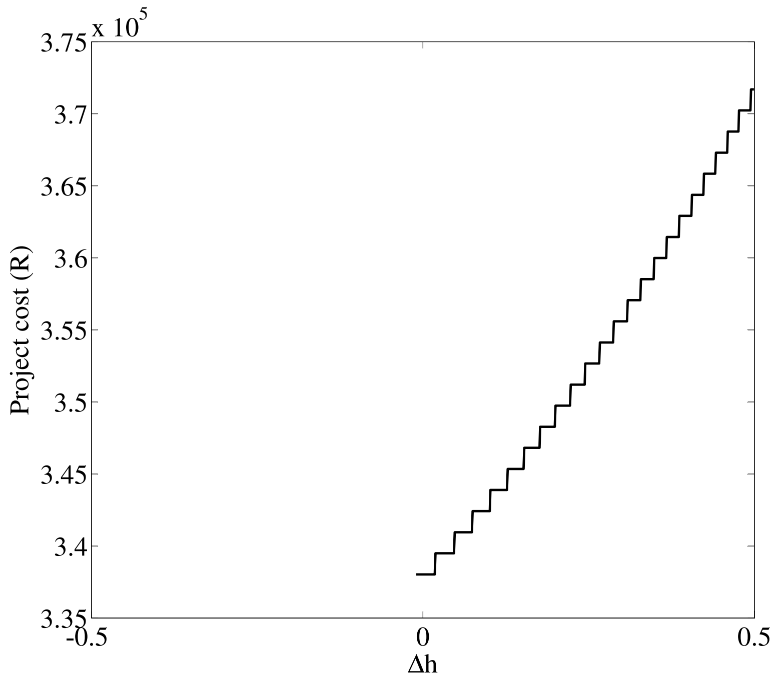

Figure 4.2: Cross section of solution at $\boldsymbol{\lambda}_7 + \Delta h$

validate the code, the model parameters were altered to $\boldsymbol{\delta} = (3, 8)$ and $\varepsilon = 0.99$, effectively constraining the model to two different sampling problems. As would be expected, the model converged

Table 4.2: Optimisation Parameter Values

Parameter name	Value
Heuristic	`ga.m` (Genetic Algorithm)
TolFun	10^{-12}
TolCon	10^{-12}
Population Size	500
$\boldsymbol{\lambda}_0$	(1.645, 0.1, 1.645, 0.1, ...)

on the solution of 67 meters for years 5 and 8, and 0 meters in other years (adjusted from 68 using equation (3.30)).

In order to establish a basis for comparison, the original problem was first solved using the GA. Since `ga` is a heuristic, it does not guarantee that a global optimum will be found. However, the optimisation parameters were set such that the heuristic converged reliably to high-quality solutions with the parameters listed in Table 4.2. No improved solutions to the ones originally published for this case study were found, because of the alteration made in the way the algorithm rounds decimal sample sizes. For a discussion of this, see Section 5.2. The closest the heuristic came to the original value of R338,028 was R339,942 - 0.59% higher than previous studies [21]. The sampling plan for this value is as follows:

$$\mathbf{n} = (34, 34, 34, 13, 9, 8, 8, 4, 3, 5, 2). \tag{4.1}$$

It is also proposed that the reporting years be changed from $\boldsymbol{\delta} = (2, 4, 6, 8, 10)$ to $\boldsymbol{\delta} = (1, 2, 4, 6, 8, 10)$. This is because one would expect the reported energy usage during the baseline phase (part of year 1) to adhere to the 90/10 criterion just as much as any other reporting year. In fact, if statistical power were a consideration, it would be more cost-effective to increase the baseline measurement accuracy, rather than to compensate on all subsequent accuracies to achieve the combined error margin. This may be a topic for future investigation. With this new constraint to year 1 as well as the baseline period (assumed to be taken together [21]), a solution is found to be:

$$\mathbf{n} = (68, 68, 28, 16, 8, 8, 6, 6, 4, 4, 2), \tag{4.2}$$

at a cost of R545,760. It is noted that adhering to the 90/10 criterion during the baseline phase adds significantly to project costs, because the cumulative effect of repeated sampling cannot be used to reduce individual sample sizes. Previously, the sample sizes in the first two years were allowed to be

34 and 34, increasing the cumulative confidence in precision in the first reporting year (year 2) to the 90/10 level. But because reporting in year 1 should actually adhere to the 90/10 criterion, the sample size in year 1 is forced to be 68, doubling the sampling cost for the first year. Since the same reporting accuracy is required in year 2, this sample size is kept for the second year also.

4.2.3 Model comparison

4.2.3.1 Non-linear PELP model

The first model improvement considered was that of the non-linear PELP population decay rate model proposed in equation (3.1). The PELP model predicts 8.3% more electricity savings than the CDM population decay rate model for the current case study parameters. The optimisation heuristic could not find a better solution to this problem than the one found for the CDM decay curve. The two sampling regimes could be different, though, with the CDM curve requiring smaller samples because of the finite population adjustment. More research is warranted in this area, but if the sampling plan proposed for a CDM model were used when in fact the PELP study is closer to the true decay shape, the 90/10 criterion would not be adhered to because the population difference would require a different finite population adjustment factor as incorporated into equation (3.30). The population would be undersampled, and savings underestimated.

The parameters of the PELP model were set to induce the same population at the end of the study than was present in the original study. As such, $\beta = 0.543, \gamma = 0.99$. The comparison is shown in Figure 4.3

The model converged to the following result:

$$\mathbf{n} = (68, 68, 28, 16, 8, 8, 6, 5, 4, 4, 2) \tag{4.3}$$

at a cost of R545,760. The confidence levels, precision levels, and yearly costs are plotted in Figure 4.4, Figure 4.5 and Figure 4.6 respectively.

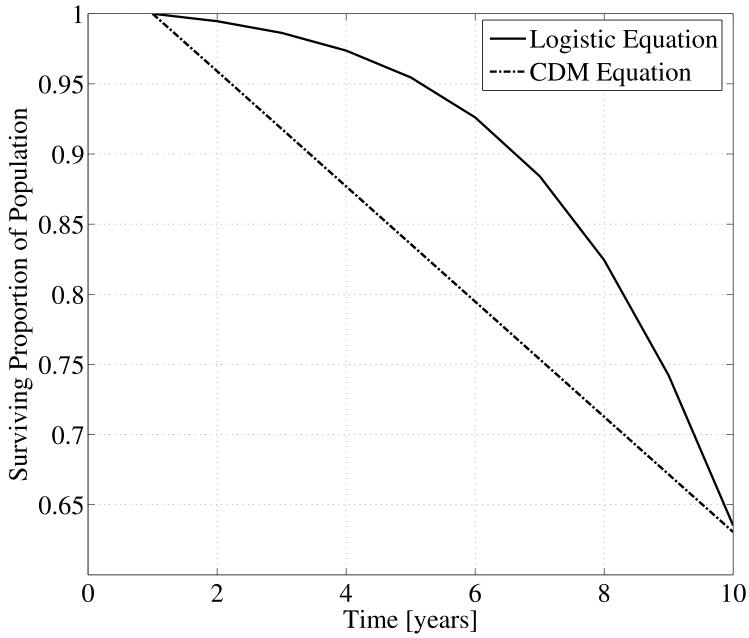

Figure 4.3: Comparison between CDM (equation (2.4) with $H = 1,642.5$; $Y = 50$; $L = 20,000$) and logistic decay (equation (3.2) with $\beta = 0.543$; $\gamma = 0.99$) curves

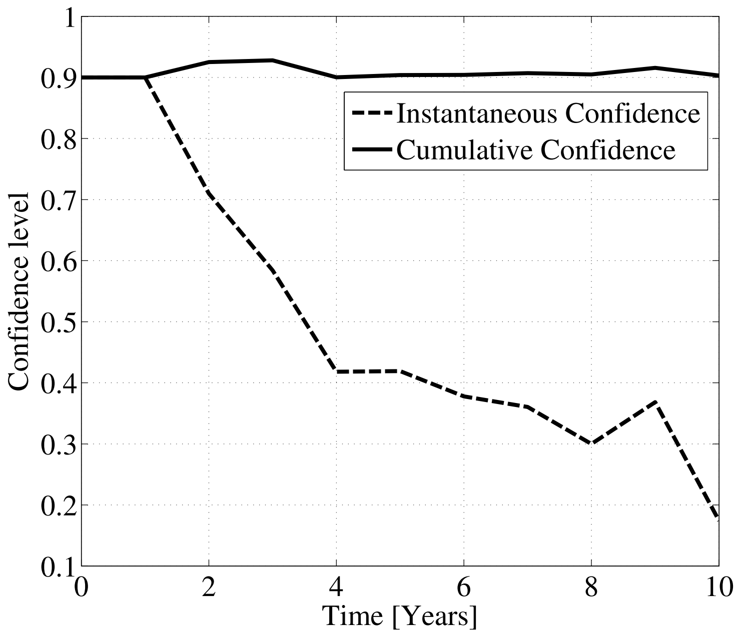

Figure 4.4: Confidence levels for the model with non-linear population decay taken into account as in equation (4.3).

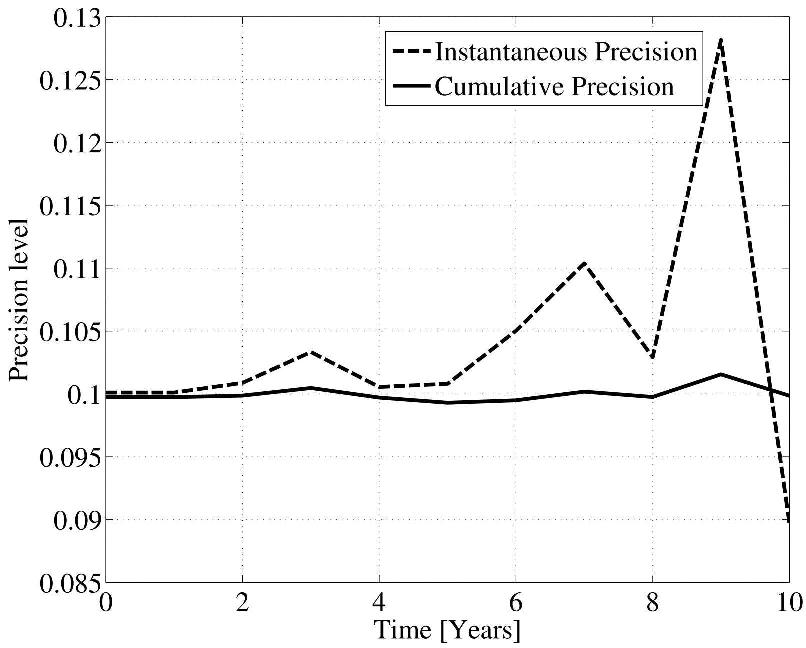

Figure 4.5: Precision levels for the model with non-linear population decay taken into account as in equation (4.3).

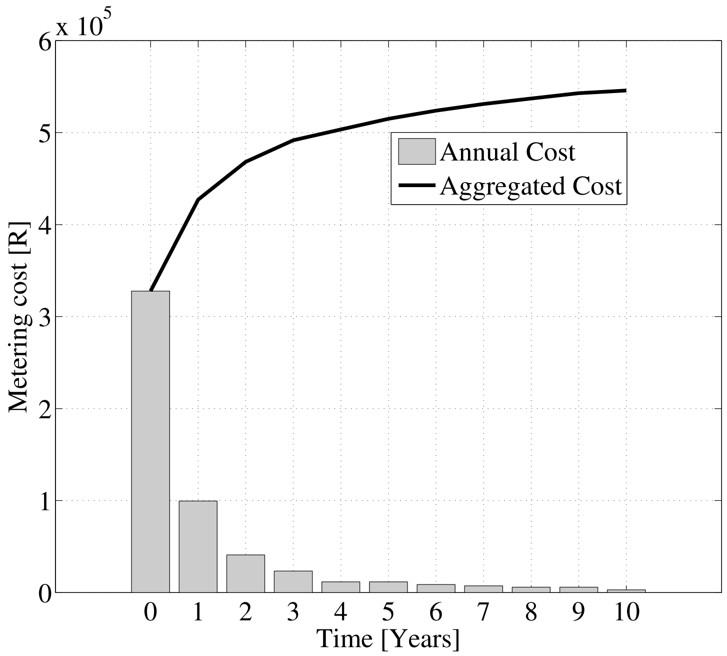

Figure 4.6: Costs for the model with non-linear population decay taken into account as in equation (4.3).

4.2.3.2 Exponential Windowing

Second, and additionally, the exponential windowing function of equation (3.38) was introduced. As would be expected, since older and more recent results are not weighted equally, more samples would be required to adhere to the 90/10 criterion. The result was

$$\mathbf{n} = (68, 68, 33, 16, 28, 21, 19, 16, 16, 15, 20) \tag{4.4}$$

at a cost of R696,552. The confidence levels, precision levels, and yearly costs are plotted in Figure 4.7, Figure 4.8 and Figure 4.9

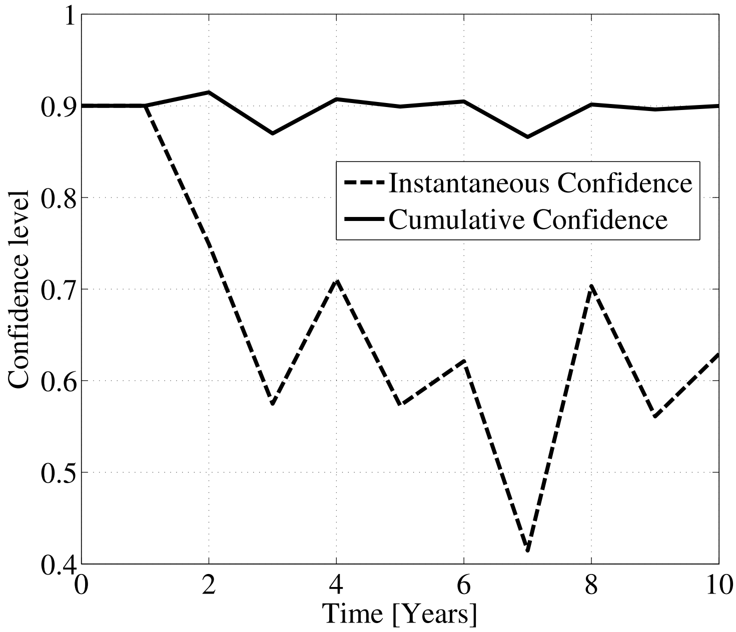

Figure 4.7: Confidence levels for the model with non-linear population decay and exponential windowing taken into account as in equation (4.4).

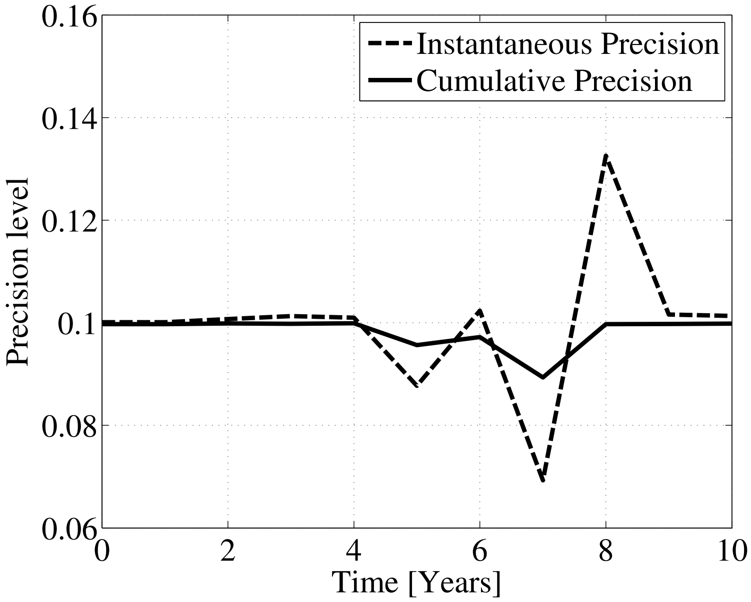

Figure 4.8: Precision levels for the model with non-linear population decay and exponential windowing taken into account as in equation (4.4).

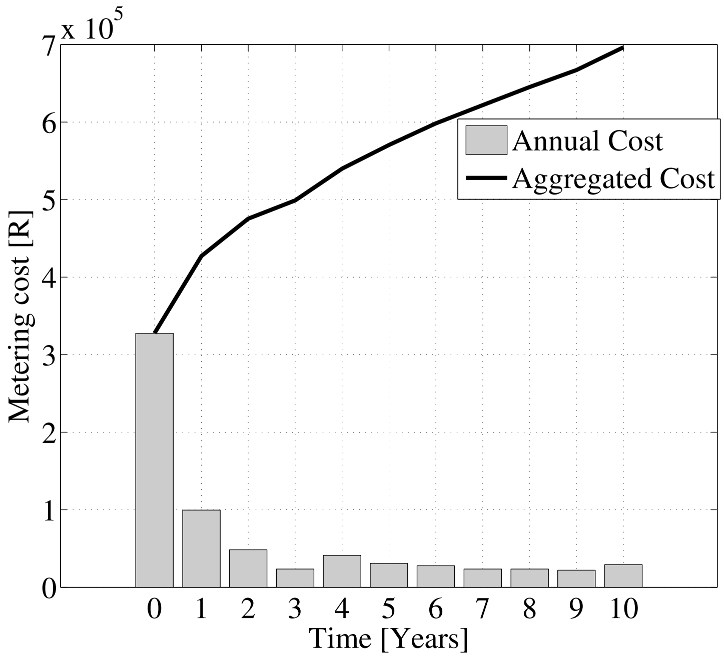

Figure 4.9: Costs for the model with non-linear population decay and exponential windowing taken into account as in equation (4.4).

4.2.3.3 Nett present value considerations

Last, the time-value of money was also taken into account. Since costs are now calculated in a different way with equation (3.39)-equation (3.41) these results cannot be compared to previous results. A sampling regime for this consideration is:

$$\mathbf{n} = (68, 68, 32, 19, 22, 14, 27, 15, 18, 17, 19) \tag{4.5}$$

at a cost of R1,459,121. The confidence levels, precision levels, and yearly costs are plotted in Figure 4.10, Figure 4.11 and Figure 4.12

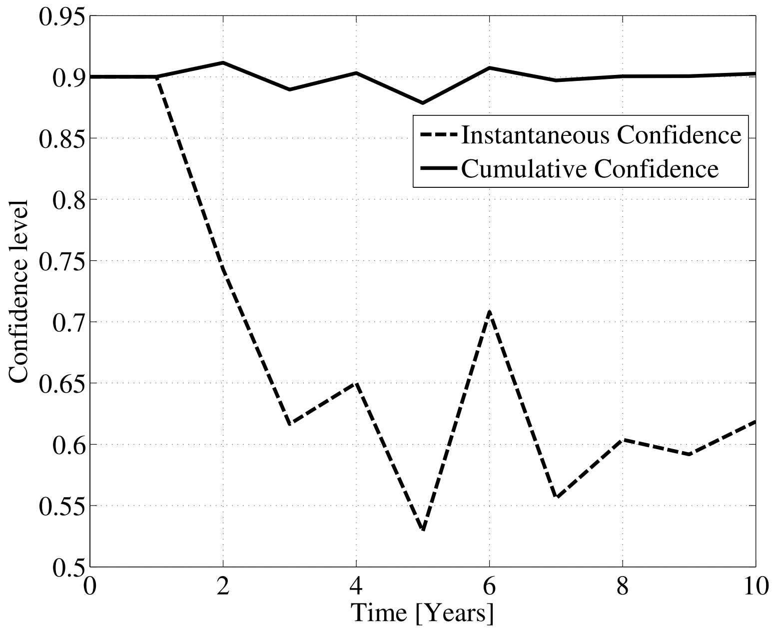

Figure 4.10: Confidence levels for the model with non-linear population decay, exponential windowing and the time value of money taken into account as in equation (4.5).

It is noted that if the rounding is set to `ceil` instead of `round`, a solution is found to be

$$\mathbf{n} = (68, 68, 30, 17, 24, 22, 18, 20, 14, 14, 23) \tag{4.6}$$

at a cost of R1,455,759. However, since rounding using the `ceil` function is not considered optimal, this solution is not considered the benchmark, but is included for academic purposes. This is elaborated upon in the next chapter.

The cost of equation (4.2) in terms of the economic model is R1,199,061 showing that the proposed

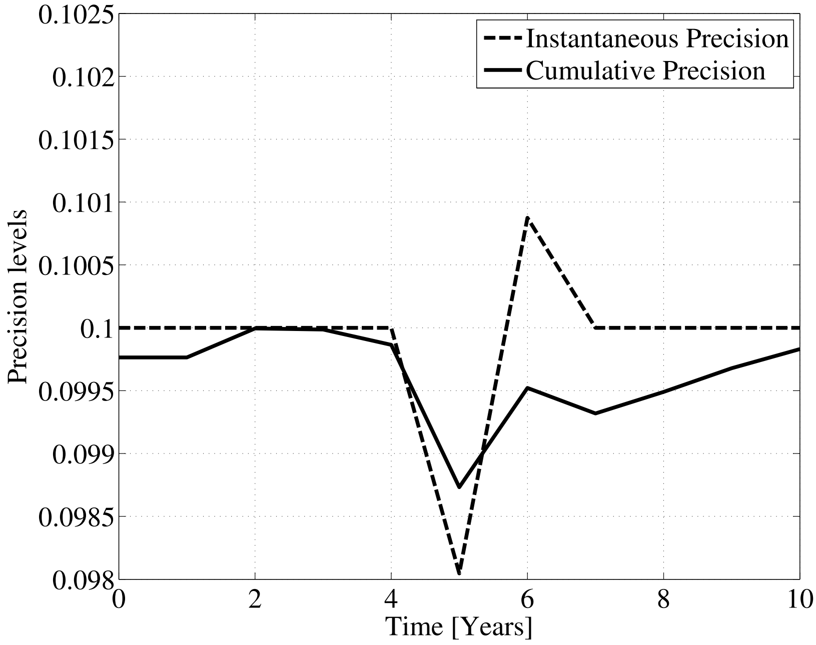

Figure 4.11: Precision levels for the model with non-linear population decay, exponential windowing and the time value of money taken into account as in equation (4.5).

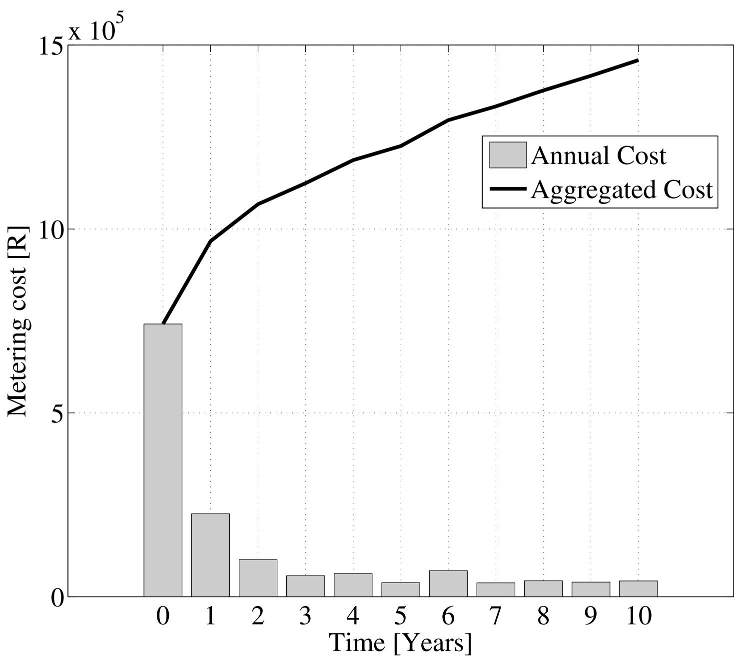

Figure 4.12: Costs for the model with non-linear population decay, exponential windowing and the time value of money taken into account as in equation (4.5).

changes increase monitoring costs by 21,4% in NPV terms. This may be taken to mean that earlier models underestimated true project costs by this amount.

4.2.3.4 Sample size comparison between different models

A comparison of the three proposed modifications (excluding the improvement on the previous optimal solution equation (4.1) or the correction of the baseline reporting requirement equation (4.2)) is plotted in Figure 4.13.

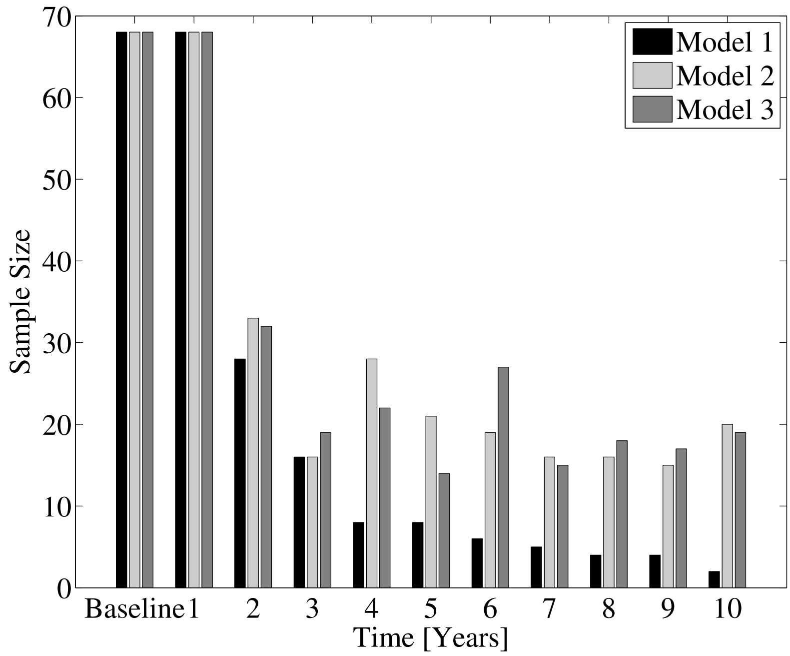

Figure 4.13: Comparison of three proposed model modifications. Model 1: equation (4.3), Model 2: equation (4.4), Model 3: equation (4.5)

4.3 SENSITIVITY ANALYSIS

A sensitivity analysis was performed to investigate which variables are most critical to the expected total cost reported by the model. Inflation rate, interest rate, meter purchasing cost, meter installation cost, meter maintenance cost, confidence level, precision level, and coefficient of variance were considered.

These factors may have complex interactions that are difficult to model analytically. A screening test was therefore conducted to identify the most important variables. For the purposes of the sensitivity analysis, the model output is assumed to be linearly dependent on the input factors. The base value corresponds to the value in Table 4.1 and the low and high values under consideration are listed in Table 4.4. They are changed one at a time whilst all other variables are kept at their base values.

The Sensitivity Index (SI) method was selected as it is one of the simplest, but also most reliable sensitivity analysis techniques [56]. It does not require a detailed knowledge of the parameter distributions or random sampling schemes like Monte Carlo techniques do. The direct sensitivity method using partial derivatives of the model differential equation is the most accurate sensitivity analysis technique but 'is typically much more demanding to implement than other sensitivity methods and yet provides only comparable results' [56]. Statistical factor techniques like the one described below are considerably simpler, but static, analyses.

The SI has been defined in literature [56] as follows. Let D be the total expected project cost in the context of sensitivity, according to an optimal metering scheme. Then:

$$SI = \frac{D_{max} - D_{min}}{D_{max}}. \tag{4.7}$$

However, it should be noticed that even this metric is not completely satisfactory, as the index awards a score between 0 and 1 based on the model output of the size of the 'high' case relative to the 'low' case. It is therefore suggested that the sensitivity index be normalised with respect to the base value and not the highest value, in the following manner:

$$SI = \frac{D_{max} - D_{min}}{D_{base}}. \tag{4.8}$$

The sensitivities of the parameters under consideration are reported in Table 4.3.

The variable ranges may be selected in three ways. Take inflation as an example. First, it may be varied according to historical inflation data for South Africa. This will provide specific insight for Energy Services Companies in South Africa, but may not be useful for other countries where inflation may be close to zero. Second, all variables considered may be changed by a given percentage, and the effect on the model analysed. However, not all variables are expected to vary according to the same percentage value. Therefore, the third option of altering each variable within reasonable bounds is adopted.

Table 4.3: Sensitivity Indices for parameters under consideration

Parameter name	Symbol	Sensitivity Index
Precision level	p	3.99
Coefficient of variance	CV	1.89
Confidence level	P(z)	1.07
Interest rate	r	0.63
Inflation rate	d	0.52
Meter maintenance cost	c	0.33
Meter purchasing cost	a	0.24
Exponential decay factor	ε	0.14
Meter installation cost	b	0.03

Inflation is varied between 0% and 10%, to make this study applicable to a wider range of countries. Since the base case has an inflation rate of 6.44%, the low value of the minimum acceptable rate of return (interest rate on investments) has been set to 6.44%, but varied to 20%. The exponential decay rate, as well as the three metering costs, were varied by 30%. The lowest confidence level specified by ASHRAE [6] is 68%, and the highest 95%. Similarly, precision is varied between 30% and 5%. The coefficient of variance is varied between 0.25 and 0.75 as these values are encountered in practice [57].

Table 4.4: Sensitivity Analysis parameter ranges

Parameter name	Symbol	Low Value	Base Value	High Value
Inflation	d	0%	6.44%	10%
Interest Rate	r	6.44%	10%	20%
Exponential decay factor	ε	0.175	0.25	0.325
Meter purchasing cost	a	R2,822.40	R4,032	R5,241.60
Meter installation cost	b	R294	R420	R546
Meter maintenance cost (per year)	c	R85	R122	R158.60
Confidence level	P(z)	68%	90%	95%
Precision level	p	30%	10 %	5%
Coefficient of variation	CV	0.25	0.5	0.75

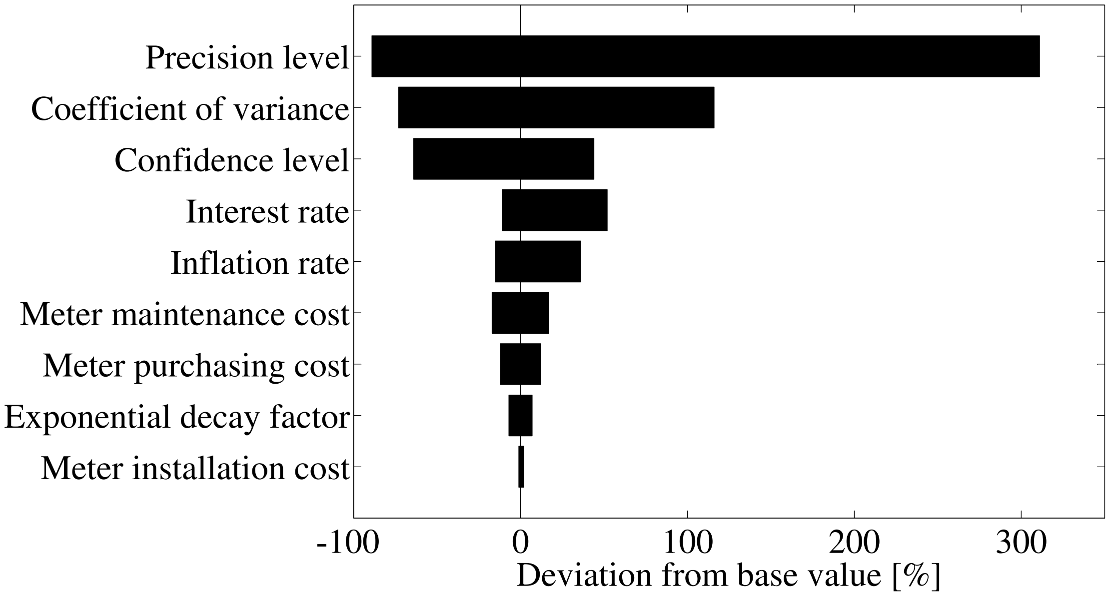

Figure 4.14: Tornado diagram showing sensitivity of expected project cost to various parameters. 'High', 'low', and 'base' values are given in Table 4.4.

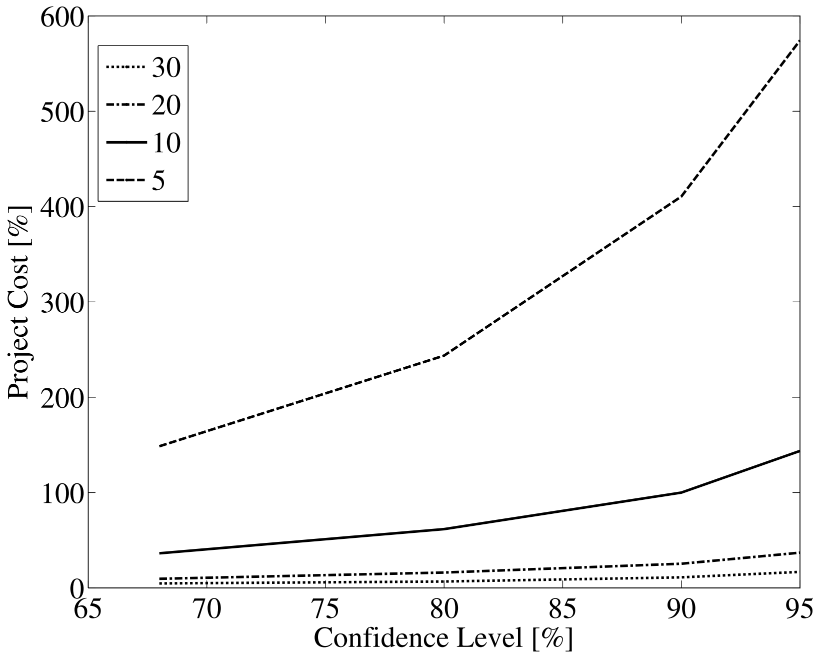

Figure 4.15: Sensitivity of project cost to various precision levels, relative to the 90/10 case.

It should be noted that the results from this analysis are obtained using a genetic algorithm. The GA is a heuristic, and does not guarantee that the results reported are globally optimal.

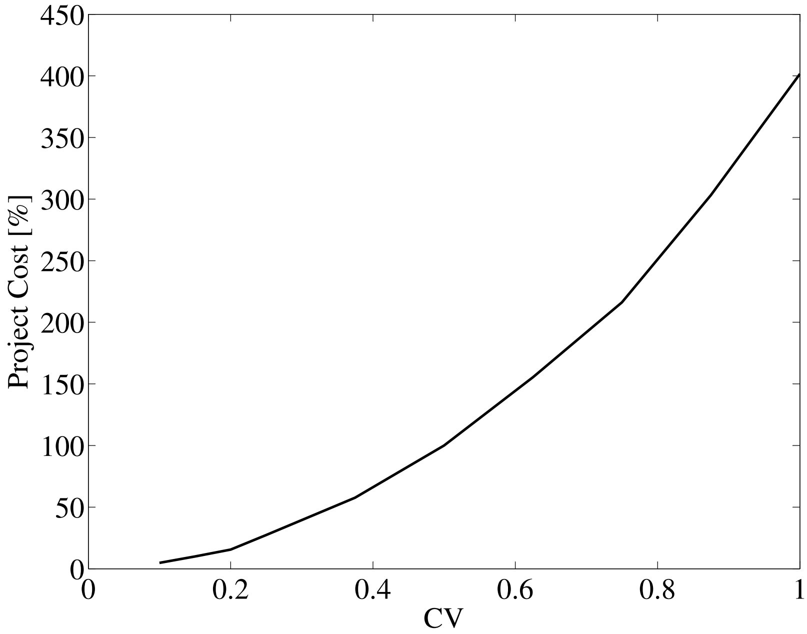

Figure 4.16: Sensitivity of project cost to various CV levels, relative to CV= 0.5 (at 100%).

4.3.1 Interaction Effects

Varying model parameters (factors) one at a time may provide some insight into the effect that a certain parameter has on the model, since one expects the model to be close to the baseline most of the time. However, relying only on such graphs is poor engineering practice since interactions between factors are frequently very significant [58]. Interaction occurs when the effect of one factor on the model differs at different levels of another factor. Other than taking the partial derivatives of the model differential equation, the correct approach when experimenting with multiple factors is to vary them simultaneously in a systematic way called factorial experimentation.

Reporting precision, CV, and reporting confidence were identified as the most important factors during the initial screening. Although it has been shown in Figure 4.15 and Figure 4.16 that the expected project cost varies non-linearly with these factors, a two-level experimental design was selected. Since such an analysis assumes linear relationships, the results of this design will not yield accurate model regression coefficients, but will indicate the relative magnitude of the main effects of the factors and interactions. The experiment design is thus three-factor and two-level, or a 2^3 factorial design. Since a computer model is used, single replication is considered for model analysis as there should not be variance around the mean of the optimal sampling plan solutions to each case. Other experimental design procedures such as randomisation, blocking, analysis of variance, or fractional designs will complicate the experimental design and analysis without adding value, and have not been used. Therefore the averaged main effects of reporting confidence, reporting precision, and CV have been considered orthogonally, along with their interactions. In order to make effects unitless and thus comparable, factors are coded as in Table 4.5. Traditionally, factors are named a, b, and c, and their effects A, B, and C [58]. The interaction between a and b would be denoted ab, for example. However, since these symbols denote other values in this study, the notation of CV, P(z), and p will be used for CV, confidence level and precision level respectively. The interaction between precision and confidence will therefore be denoted pP(z), for example.

The experiment was conducted using the standard order (Yate's order), with factors varied orthogonally as shown in Table 4.6. The results are shown in Figure 4.17. It should be noted that the calculation of the averaged effects in the last column is quite involved, but available in literature [58].

Table 4.5: Factorial design variable coding

Coded value	CV	Confidence	Precision
0	0.25	0.3	0.68
1	0.75	0.05	0.95

4.4 CHAPTER SUMMARY

A case study previously published in literature was considered as a benchmark. An investigation of the solution space showed stepped discontinuity, and thus a genetic algorithm was chosen rather than a gradient method. The various improvements proposed in this paper were implemented sequentially and was found to increase project cost markedly. The sensitivity analysis revealed that the parameters that have the biggest influence on expected project cost are reporting precision level, coefficient of variance, and reporting confidence level.

Table 4.6: Factorial experiment order and results

	Precision	**Confidence**	**CV**	**Project Cost**	**Effect**
(1)	0	0	0	27,490	
p	1	0	0	538,123	6,289,834
P(*z*)	0	1	0	69,625	3,925,635
*p*P(*z*)	1	1	0	2,097,272	3,708,442
CV	0	0	1	141,751	5,310,013
*p*CV	1	0	1	4,793,903	5,020,694
P(*z*)CV	0	1	1	534,001	3,124,992
*p*P(*z*)CV	1	1	1	18,502,908	2,949,935

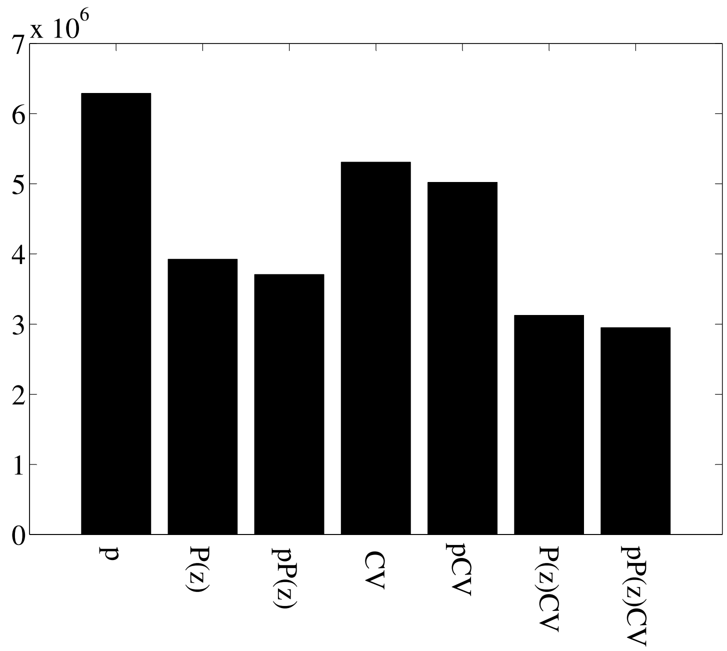

Figure 4.17: Averaged main effects and interactions of reporting precision level (p), reporting confidence level (P(z)), and CV.

CHAPTER 5

DISCUSSION

5.1 CHAPTER OVERVIEW

This chapter provides reviews outstanding questions regarding the implementation of the model to a real-world case study. The effect of the proposed improvements on the economic performance of the sampling project is also discussed, as well as the sensitivity of the model to certain factors. Lastly, the application to other technologies as well as the applicability of the model to real-world scenarios is assessed according to the validity and impact of the assumptions made during model development.

5.2 MODEL IMPLEMENTATION

Because of the way in which the model has been formulated with $\boldsymbol{\lambda}$ as the decision variable containing the parameters z and p for every year, sample size is not optimised directly. Although there are valid reasons for this (discussed below), it presents a problem in that z and p map many-to-one to an n_i-value through equation (3.30). In effect, the optimisation function then searches for a solution with high z_i and low p_i that still map to the lowest n_i possible, since cost is determined by n_i.

However, there are good reasons for fomulating the problem this way. First, it may then be compared with previous models. Second, the alternative is not obviously preferable. One may be able to re-arrange equation (3.30) to give confidence as a function of sample size and precision, or precision as a function of sample size and confidence (assuming a CV value in each case). But both confidence and precision are variable, because the one may be traded off against the other. Fixing confidence, for example, is not an option since a large sample (and confidence) in one year may be traded off against a smaller one in another year for economic reasons. Therefore optimising for precision and confidence simultaneously in the current formulation of the problem, although not efficient, is justifiable.

An observation concerning rounding is also necessary. Previously, rounding was done using the `ceil` function in Matlab to convert the decimal n value to an integer for cost and accuracy calculation. This function rounds up, so that 1.0001 will be rounded to 2. The rationale behind this approach is that since decimal samples are not realistic, the next whole number should always be selected for the sample size. This is normally the case when employing equations such as equation (3.30). Since the pre-rounded figures are used to calculate precision and confidence through equation (3.22) and equation (3.23), rounding interacts with the model in a complex way; it is not simply that always rounding up will be conservative, since increased confidence can lead to decreased precision. In the end, both the `round` and the `ceil` function supply similar results to the problem through optimisation, as the algorithm works within the context of the rounding selected. However, it is believed that `round` is the preferable alternative, since due to calculation and multiplication at the limit of working precision (as the optimisation algorithm does) may produce slight inaccuracies such as where 0 is expressed as 0.000001. It is reasonable in such a case to round down rather than to round up. Another way in which `ceil` can be seen to deliver sub-optimal results is when performing the model validation test of Section 4.2.2. If $\boldsymbol{\delta} = (5,8)$ with $\varepsilon = 0.99$ and `ceil` is used, the optimal result would be $\mathbf{n} = (1,1,1,67,1,1,1,1,67,1,1)$, which is obviously sub-optimal, as no samples should be taken in the years other than 3 and 8. Therefore solutions using `round` are considered in this study.

5.3 IMPACT OF IMPROVEMENTS ON MODEL PERFORMANCE

As expected, number of meters never exceed 68 in any given year, as indicated by equation (3.28). However, each subsequent model improvement considered does increase the cost of metering above the previous case.

In this case, the optimal solution for the linear and non-linear population decay curves were identical, although this is not necessarily always the case. Depending on the size of the population, the increased metering cost due to using the sigmoid decay curve rather than the CDM straight-line curve may be offset by the increase in true savings reported. If a larger lamp population were considered in the study, and the finite population adjustment factor thus made no difference, the sampling plans for both population decay models would be identical. In this case greater and more accurate savings are reported with no additional cost due to this modification.

The second improvement - the exponential windowing function - does not allow the model to rely on high initial sample sizes to support much lower sample sizes in later years. As such, sample sizes during the later years of the study are notably higher than before, although the algorithm stabilises on sample sizes of between 15 and 20 per year.

The last improvement, taking the time-value of money into account, also tends to shift metering towards the end of the study. This is because meter cost was modelled as depreciating, reinforced by opportunity costs which also penalise early expenditure on ‘expensive’ meters, when money could be invested and spent later on ‘less expensive’ meters. It is interesting to note that introducing this consideration does not increase sample sizes significantly, although it does skew their size distribution towards the later years of the study. The increased cost of this consideration in NPV terms is 0.5% when compared to the exponential windowing function modification. However, if ε were changed to a different value, this comparison may well show a much greater discrepancy.

5.4 MODEL SENSITIVITY

From the sensitivity analysis it is evident that the model is more sensitive to statistical parameters such as confidence, precision and assumed CV, than to economic parameters such as metering costs. Inflation rate and interest rate are still notable, since the project developer has little control over these figures. They must therefore be forecast accurately.

Since confidence and precision values are usually specified by legislation, M&V Engineers cannot select these values, although they are sometimes negotiated at project inception if CDM requirements do not need to be adhered to. Figure 4.15 shows that monitoring projects with lower precision values will almost always dominate projects with higher precision values in economic terms, regardless of the confidence level. Relative to the 90/10 case, reporting at a precision level of 5% would increase project costs fourfold, and halving the precision to 20% would decrease project costs to 25% of the original value.

It is also important to note the sensitivity of the model to the coefficient of variance, as this value is estimated by engineering judgement before project commencement. Assuming a CV of 0.25 when the CV is actually 0.5 will decrease the expected cost of the project to 27% of the true cost, but will detrimentally affect reporting accuracy. On the other hand, if a CV of 0.5 is assumed, but it is later found that the CV is actually closer to 0.75, monitoring costs need to double in order to adhere to the contractually agreed reporting accuracy levels. Therefore CV should be tested over a short period of time, and then updated, bearing in mind that seasonal effects may affect this value. In long-term projects the CV value should be evaluated on an annual basis.

The interaction effects revealed by the factorial experimental design shows that one-factor-at-a-time sensitivity approaches do not do justice to the complexities of these projects. Figure 4.17 indicates that precision and CV are still the most important factors, but that the interaction between these two factors is more significant than the main effect of the confidence level. This is not only because those two factors dominate expected project cost, but also because at different CV levels, the same precision level specification affects the model differently. Other interactive effects are also large relative to the main effects. One should bear in mind that this analysis technique assumes a linear relationship between expected project cost and the factors and that it is static. However, setting up a linear regression model using these factors will not work, as illustrated by Figure 4.15 and Figure 4.16. Nevertheless, it does show that if a non-linear regression model were set up, the interactive terms such as x_1x_2 or x_1x_3 should not be discarded. If it is possible to derive the model differential equation, this will shed more light on the interaction between these factors.

5.5 APPLICATION OF MODEL TO OTHER TECHNOLOGIES

Although this model may be applied to technologies other than lighting, caution should be exercised in such cases. This model is derived from first principles, but certain assumptions have been made that need to be adhered to ensure compliance to the 90/10 reporting criterion. First, lighting retrofit projects usually have large population sizes, making them suitable for statistical analysis. Second, normally distributed residuals are also assumed; this may not always the case for other technologies, and the engineer involved should consider this possibility. Third, lighting technologies are relatively insensitive to seasonality effects such as outside air temperature. For heating technologies where such covariates may be significant, it is recommended that an optimal sampling model be derived from the ASHRAE models [6] described in Section 2.2. Simple power meters may therefore not be suitable for such studies, and a combined uncertainty analysis is then warranted.

5.6 APPLICABILITY OF MODEL TO REAL-WORLD SCENARIOS

How applicable is this model to real-world sampling scenarios? The accuracy of the model depends on the validity of the assumptions made before implementation. Although the NPV-calculations are not complicated, they do add realism to the cost minimisation calculation. However, the applicability of the whole economic model depends on the pricing structure of the contractor supplying the meters. Often certain 'packaged' schemes are sold and thus price is not directly proportional to the number of meters installed, or the maintenance aspect of the project is calculated differently to the way this model calculates it. Nevertheless, a proportional model most accurately reflects true costs incurred, and is thus the preferable model within a research context.

Computer simulations of long-term projects are at best an approximation of a complex system. Installing the exact number of meters recommended by the algorithm would carry the risk of undersampling in the event of a meter failing, data being corrupted, or other unforeseen circumstances interfering with data integrity. A greater risk is that of underestimating the coefficient of variance - a common problem in M&V practice [57]. And yet building in a 'buffer' is precisely what the optimisation approach seeks to avoid. In this regard, using the sampling as a guide, there is no substitute for an experienced M&V practitioner's judgement on the expected variance within the population to be monitored as well as on data reliability.

More significant concerns pertain to the assumptions made at a fundamental level when this model was formulated originally, as alluded to in Section 3.2.

The first assumption of concern is that the mean daily energy use is stationary throughout the study. Since the current focus is on residential lighting projects, occupancy change should not affect energy consumption as much as it would hot water consumption, for example. One could also argue that baseline adjustments should be done in order to compensate for such (and other) changes over the monitoring period. However, meter readings are done prior to baseline adjustment, and thus means and standard deviations used to calculate sample size at a given confidence and precision level would still be affected. Assuming a stationary mean for the current study is reasonable. Seasonal effects should also be visible in the month-to-month energy use. However, since annual energy use is considered for calculation, seasonal effects may be neglected. At a finer surveillance resolution such a model should be implemented, also correcting for other periodic autocorrelative effects. It would also be possible to account for varying parameters during actual studies as the sampling interval allows for recalculation to be done where equation (3.16) could be rewritten as a less elegant summation with different mean and standard deviations. Likewise, θ_K and Γ_K may be adjusted when this is deemed necessary.

The second assumption is that samples taken by the same meter, in different years, are independent, as the assumption for normal sampling (used in the model) is that samples are i.i.d. If there is autocorrelation, then the number of effective samples reduce dramatically according to equation (2.1). This is applicable to hourly, daily or monthly energy use. However, all things being equal, annual energy use should not be affected by autocorrelative phenomena such as seasonality, and may be assumed to be i.i.d.

5.7 CHAPTER SUMMARY

Various challenges to model implementation have been overcome, specifically in relating confidence and precision to sample size, as well as the question of rounding. Improvements in the model accuracy resulted in a cost increase of 21.4% in NPV terms. Certain model assumptions need to be investigated further, but are considered valid and is expected to have a small impact on results. The model sensitivity analysis shows that assumptions of CV made during the project planning phase have a large cost implication, as does the selected confidence and precision reporting level.

CHAPTER 6

CONCLUSION

A number of improvements to current sampling design studies have been proposed and implemented, altering optimal sampling regime shapes and expected project costs significantly.

The improved CFL population survival model was found to fit known data very well, with a mean squared error (MSE) of 0.0015 compared to an MSE of 0.0368 for the previous model. The new model in discretised form also allows for optimal control theory to be applied to the problem, and reports greater savings than existing linear models.

By plotting line sections through the search space and investigating the gradient around known solutions, the nature of the search space was proven to be stepped and discontinuous, and the Genetic Algorithm was determined to be an appropriate heuristic for optimisation. The model was validated by applying it to certain test cases and checking performance against their known solutions. This approach allows for greater confidence in optimisation results.

A more accurate cumulative sampling function was devised and implemented. This function allows for the exponential decay of weights on past data, thereby increasing the relative contribution of more recent data during calculation. An exponential decay factor of 0.25 was selected for the current study, but it is recognised that this value will vary between projects.

An economic model incorporating the time-value of money was also implemented. This model not only accounts for inflation, but also takes investment opportunity costs and the difference between labour and capital costs into account. Using a minimum acceptable rate of return of 10% and an inflation rate of 6.44% based on Consumer Price Inflation, it was found that the project costs for optimal sampling plans are 21.4 % higher in Nett Present Value (NPV) terms than previously calculated, although this figure is dependent on project-specific assumptions.

Neither the improved population survival model nor the addition of NPV considerations altered the sampling plans in a notable way, contributing 0% and 0.5% respectively. However, not weighting all samples equally did have a significant effect, contributing the bulk of the increased project cost.

The expected project cost is sensitive to the reporting precision, coefficient of variance, and reporting confidence, with modified sensitivity indices of 3.99, 1.89 and 1.07 respectively. Large interaction effects between these factors exist.

6.1 RECOMMENDATIONS

Although numerous improvements to the aforementioned changes may be made, it is recommended that future work focus on the latent assumptions identified in existing literature on longitudinal CFL sampling design: that the mean energy usage is stationary throughout the study, that samples taken in different years are independent, the possibility of considering statistical power during sample size calculation, and the structure of meter pricing and contracting schemes. Future research may also incorporate Bayesian methods and binomial data techniques.

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APPENDIX A

MATLAB PROGRAMMING CODE

A.1 SAMPLEOPTIM.M

```
%   Author: Herman Carstens. Naming conventions based on code by
    Xianming
%   Ye in order to assure continuity.
%   Department of Electrical, Electronic and Computer Engineering
%   Date: 2013/10/07

clear all;
close all;
%% Variable declaration
% Initial values
a = 4032*ones(1,11);     %Meter purchasing Cost
b = 420*ones(1,11);      %Meter installation Cost
c = 122*ones(1,11);      %Meter maintenance Cost
Year=10;                 %Length of study
smoothingfactor = 0.25;  %Exponential smoothing coefficient
delta = [1 2 4 6 8 10];  %Reporting years

%Reference sequence (Should be called lambda)
x_ref=[1.645
    0.100105813
    1.148569001
```

```
    0.10073342
    0.797595167
    0.101293138
    1.059205978
    0.100988871
    0.794130446
    0.087695381
    0.88042801
    0.102345577
    0.545535986
    0.069281733
    1.043751244
    0.132554463
    0.773859636
    0.101610681
    0.894999911
    0.101335949];

%% Program controls
expoption = 1;  %1 for exponential windowing
survcurve = 1;  %1 for PELP, 0 for CDM
npv = 0;        %1 for time-value of money calc
figures = 1;    %1 to display figures
optimise = 0;   %1 to optimse, 0 to analyse existing x_ref

%% NPV calc
%Discount price of meter by CPI inflation: Assume meter price stays
%constant but in Present Value thus decreases by CPI.
%Also, factor in an opportunity cost

if npv
    inflation = 0.0644;
    interest = 0.1;
```

```
    metercost = a(1);
    installationcost = b(1);
    maintenancecost = c(1);
    opportunity = 0;
    for i=1:Year
        meter_cost_depreciated(i) =metercost/((1+inflation)^(i-1));
        opportunity_cost_purchasing(i) = (metercost*(1+interest)^(
           Year-i))/((1+inflation)^Year);
        opportunity_cost_installation(i) = (installationcost*(1+
           interest)^(Year-i))/((1+inflation)^Year);
        opportunity_cost_maintenance(i) = (maintenancecost*(1+
           interest)^(Year-i))/((1+inflation)^Year);
        a(i)=meter_cost_depreciated(i)+opportunity_cost_purchasing(
           i);
        b(i) = b(i)+opportunity_cost_installation(i);
        c(i)=c(i)+opportunity_cost_maintenance(i);
    end
end

N0=607559;  %Initial Population
cv= 0.5;    %Coefficient of variance

%% Survival Rate variable declaration

k=4.5;             %daily burning time (hours)
H=k*365;           %Annual burning time (hours)
L=20000;           %Rated life span (hours)
y=zeros(1,11);     %The population that has decayed

%% PELP Survival
if survcurve
    SurRate = zeros(1,11);
    SurRate(1:2) =1;
```

```
    beta=0.543;
    gamma=0.99;
    for t=3:1:11
        SurRate(t) = beta*gamma*SurRate(t-1)^2-beta*SurRate(t-1)+
           SurRate(t-1);
    end
end

%% CDM Surival
if ~survcurve
    for t=1:1:10
        if t*H<=L
            y(t+1)=(t).*H./(2*L);
        else
            y(t+1)=1;
        end
    end

    y(1)=0;
    y=[y(1) y(1:end-1)];
    SurRate=1-y;
end

%% Plot survival
%SurRate=[1 SurRate(1:end-1)];

N=zeros(1,Year+1);

for t=1:1:Year+1
    N(t)=N0*SurRate(t);
end

figure(1);
```

```
plot(N,'k','linewidth',3);
grid on;
hold on;
xlabel('Time: Year')
ylabel('Population Size')
legend('N(i)');
N=N(2:end);

%% Optimisation
if optimise

    %Initial value
    x0=[1.645
        0.1
        1.645
        0.1
        1.645
        0.1
        1.645
        0.1
        1.645
        0.1
        1.645
        0.1
        1.645
        0.1
        1.645
        0.1
        1.645
        0.1
        1.645
        0.1]';
    %Upper and lower bounds
```

```
    lb=[0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;]';
    ub=[3;1;3;1;3;1;3;1;3;1;3;1;3;1;3;1;3;1;3;1;];

    optionsga = gaoptimset('InitialPopulation',x0,'Tolcon',1e-12,'
        tolfun',1e-12,'PlotFcns',@gaplotbestf,'PopulationSize',50,'
        PopInitRange',[0; 2]);

    [x,fval,exitflag] = ga(@(x)MeteringCostFunction(x,cv,a,b,c,N)
        ,20,[],[],[],[],lb,ub,@(x)MeteringConstraintFunction(x, N,
        cv, smoothingfactor,expoption,a,b,c,delta,Year),optionsga)
    [TotalConf, confidence, TotalPre, precision, CumCost, Cost,
        Sample, Backup, BaseCost,B,n,Z] = cumcalc(x,expoption,cv,N,
        Year,a,b,c,smoothingfactor);

else
    if ~optimise
        x=x_ref;    %Perform subsequent calculations with given
            value
    end
end
%% Output Confidence and Precision
%This is the main function of the program
MeteringCostFunction(x,cv,a,b,c,N)
[TotalConf, confidence, TotalPre, precision, CumCost, Cost, Sample,
    Backup, BaseCost,B,n,Z] = cumcalc(x,expoption,cv,N,Year,a,b,c,
   smoothingfactor);

%% Figures
if figures
    t=0:1:Year;

    %Adding Baseline value
    confidence=[confidence(1) confidence];
```

```
precision=[precision(1) precision];
Sample=[Sample(1) Sample];
CumCost=[BaseCost, CumCost];
Cost=[BaseCost, Cost];

%Confidence
figure(2);
plot(t, confidence,'k--','linewidth',3);
set(gca,'FontSize',22,'Fontname','Times')
grid on;
hold on;
plot(t, TotalConf,'k','linewidth',3);
xlabel('Time [Years]','fontsize',22,'Fontname','Times')
ylabel('Confidence level','fontsize',22,'Fontname','Times')
h_legend=legend('Instantaneous Confidence','Cumulative
    Confidence');
set(h_legend,'FontSize',22,'Fontname','Times');

%Precision
figure(3);
plot(t, precision,'k--','linewidth',3);
set(gca,'FontSize',22,'Fontname','Times')
grid on;
hold on;
plot(t, TotalPre,'k','linewidth',3);
set(gca,'FontSize',22,'Fontname','Times')
xlabel('Time [Years]','fontsize',22,'Fontname','Times')
ylabel('Precision level','fontsize',22,'Fontname','Times')
h_legend=legend('Instantaneous Precision','Cumulative Precision
    ');
set(h_legend,'FontSize',22,'Fontname','Times');

%Sample size, backup meter population size
```

```
    figure(4);
    plot(t,Sample,'k--','linewidth',3);
    grid on;
    hold on;
    plot(t, Backup,'k','linewidth',3)
    set(gca,'FontSize',22,'Fontname','Times')
    xlabel('Time [Years]','fontsize',22,'Fontname','Times')
    ylabel('Sample size','fontsize',22,'Fontname','Times')
    h_legend= legend('Adopted meters','Backup meters');
    set(h_legend,'FontSize',22,'Fontname','Times');

    %Cost
    figure(5);
    bar(t, Cost);
    grid on;
    hold on;
    plot(t, CumCost,'k','linewidth',3);
    set(gca,'FontSize',22,'Fontname','Times')
    xlabel('Time [Years]','fontsize',22,'Fontname','Times')
    ylabel('Metering cost [R]','fontsize',22,'Fontname','Times')
    h_legend=legend('Annual Cost','Aggregated Cost');
    set(h_legend,'FontSize',22,'Fontname','Times');
end
```

A.2 CUMCALC.M

```
%   Author: Herman Carstens. Naming conventions based on code by Xianming
%   Ye in order to assure continuity.
%   Department of Electrical, Electronic and Computer Engineering
%   Date: 2013/10/07

function [TotalConf, confidence, TotalPre, precision, CumCost, Cost, Sample, Backup, BaseCost, B,n, Z] = cumcalc(x_passed,expoption,cv,N,Year,a,b,c,smoothingfactor)

%%Variable Declarations
n=zeros(1,Year);                %Decimal Sample size
zscore=zeros(1,Year);           %instantaneous standard score
precision=zeros(1,Year);        %instantaneous precision
confidence=zeros(1,Year);       %instantaneous confidence
Sample=zeros(1,Year);           %rounded sample size
Cost=zeros(1,Year);             %Annual Cost
CumCost=zeros(1,Year);          %Total Cost
B=zeros(1,Year);                %Backup meters
numz=zeros(10,1);               %z-calculation numerator
denz=zeros(10,1);               %z-calculation denominator
nump=zeros(10,1);               %precision numerator
denp=zeros(10,1);               %precision denominator

%% Instantaneous confidence and precision levels
for i=1:1:Year
    zscore(i)=x_passed(2*i-1);
    precision(i)=x_passed(2*i);
    if zscore(i)==0 && precision(i)==0 %This circumvents the problem of
         %dividing by zero for z=0.
```

```
        n(i)=0;
    else
        n(i)=((zscore(i)^2)*(cv^2)*N(i))/((zscore(i)^2)*(cv^2)+(precision(i)^2)*N(i)); %Finite population adjustment
    end
    confidence(i)=normcdf(zscore(i),0,1)-normcdf(-zscore(i),0,1);
    Sample(i)=round(n(i));
end
%Sample=[]; %Insert sample size here for a quick calculation. Note that
%confidence and precision will be inaccurate, as x_passed is needed as per
%above.

%% Cost calculation
BaseCost=(a(1)+b(1)+3*c(1)).*Sample(1); %Cost of baseline period
Cost(1)=12*c(1)*Sample(1);              %Cost of first year
CumCost(1)=BaseCost+Cost(1);            %Total cost of first year
B(1)=0;
for i=2:1:Year
    B(i)=max(B(i-1),0)+Sample(i-1)-Sample(i);    %Backup meter calculation
    if B(i)<0                                     %This is a rare condition
        Cost(i) = 12*c(i)*Sample(i)-B(i)*(a(i)+b(i));
    else
        Cost(i)=12*c(i)*Sample(i);
    end
    CumCost(i)=CumCost(i-1)+Cost(i);
end
Backup=[0 B];
%% Cumulative confidence and precision calculation
```

```
if expoption   %For the case where exponential windowing is
    considered
    for i=1:1:Year
        for j = 1:i
            if Sample(j)  ~= 0
                numz(i)=numz(i)*(1-smoothingfactor)+N(j).*zscore(j)
                    ./sqrt(Sample(j));
                nump(i)=nump(i)*(1-smoothingfactor)+cv.*N(j).*
                    zscore(j)./sqrt(Sample(j));
                denz(i)=denz(i)*(1-smoothingfactor)+N(j)^2/Sample(j
                    );
            else
                nump(i)=1;
                numz(i)=0;
            end
            denp(i)=denp(i)*(1-smoothingfactor)+N(j);
        end
        z(i)  = numz(i)/sqrt(denz(i));
        Conf(i)=normcdf(z(i),0,1)-normcdf(-z(i),0,1);
        if Sample(i)~=0
            p(i)  = nump(i)/denp(i);
        else
            p(i)  = NaN;
            Conf(i)=NaN;
        end
    end
end

if ~expoption   %No exponential windowing isn't considered
    for i=1:1:Year
        for j = 1:i
            numz(i)=numz(i)+N(j).*zscore(j)./sqrt(Sample(j));
            nump(i)=nump(i)+cv.*N(j).*zscore(j)./sqrt(Sample(j));
```

```
            denz(i)=denz(i)+N(j)^2/Sample(j);
            denp(i)=denp(i)+N(j);
        end
        z(i) = numz(i)/sqrt(denz(i));
        Conf(i)=normcdf(z(i),0,1)-normcdf(-z(i),0,1);
        p(i) = nump(i)/denp(i);
    end
end

%% Cumulative vectors
Z=[z(1) z(2) z(3) z(4) z(5) z(6) z(7) z(8) z(9) z(10)];
TotalConf=[Conf(1) Conf(1) Conf(2) Conf(3) Conf(4) Conf(5) Conf(6)
   Conf(7) Conf(8) Conf(9) Conf(10)];
TotalPre=[p(1) p(1) p(2) p(3) p(4) p(5) p(6) p(7) p(8) p(9) p(10)];
```

A.3 METERINGCOSTFUNCTION.M

```
%   Author: Herman Carstens. Naming conventions based on code by Xianming
%   Ye in order to assure continuity.
%   Department of Electrical, Electronic and Computer Engineering
%   Date: 2013/10/07
%%
function totalcost= MeteringCostFunction(x,cv,a,b,c,N)
Year=10;
n=zeros(1,Year);
B=zeros(1,Year);

%Calculate n. This needs to be done since this function is called fromt the
%optimisation function.
for i=1:1:Year
    n(i)=x(2*i-1).^2*cv.^2*N(i)./(x(2*i-1).^2*cv.^2+x(2*i).^2*N(i));
end

Sample=round(Sample);

%% Cost Calculation
%(Same calculation as in cumcalc)

BaseCost=(a(1)+b(1)+3*c(1)).*Sample(1); %Cost of baseline period
Cost(1)=12*c(1)*Sample(1);              %Cost of first year
CumCost(1)=BaseCost+Cost(1);        %Total cost of first year
B(1)=0;

for i=2:1:Year
```

```
    B(i)=max(B(i-1),0)+Sample(i-1)-Sample(i);   %Backup meter calculation
    if B(i)<0                                  %This is a rare condition
        Cost(i) = 12*c(i)*Sample(i)-B(i)*(a(i)+b(i));
    else
        Cost(i)=12*c(i)*Sample(i);
    end
    CumCost(i)=CumCost(i-1)+Cost(i);
end
totalcost = CumCost(end);
end
```

A.4 METERINGCONSTRAINTFUNCTION.M

```
%   Author: Herman Carstens. Naming conventions based on code by Xianming
%   Ye in order to assure continuity.
%   Department of Electrical, Electronic and Computer Engineering
%   Date: 2013/10/07
function [c,ceq] = MeteringConstraintFunction(x, N, cv, smoothingfactor,expoption,a,b,c,delta,Year)
Year=10;
n=zeros(1,Year);
[TotalConf, confidence, TotalPre, precision, CumCost, Cost, Sample, Backup, BaseCost,B,n,Z] = cumcalc2(x,expoption,cv,N,Year,a,b,c,smoothingfactor);
c=[TotalPre(delta+1)'-0.1
    0.9-TotalConf(delta+1)'];
ceq = [];
% No nonlinear equality constraints:
```