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DETECTION PROBLEMS OF VORTICAL STRUCTURES

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ABSTRACT

Main conceptual problems faced in detection of vortical structures are dealt with and discussed on the background of a brief review of existing vortex-identification schemes.

INTRODUCTION

A large number of existing vortex definitions, vortex-identification methods and vortex-core visualization techniques is proportional to the number of their inherent problems and shortcomings as well as apparent vortex misinterpretations found in their fluid mechanics applications. The paper presents an update on the main applicability requirements as validity of detection algorithms for compressible flows and/or variable-density flows, determination of the local vortex intensity, determination of the integral vortex strength, vortex-axis identification, specific vortex-axis requirements: existence and uniqueness for each connected vortex region, the subjective choice of threshold in the vortex-boundary identification vs. physically defined boundary, allowance for an arbitrary axial strain rate vs. orbital compactness, ability to provide the same results in different rotating frames, and inherent bias towards shearing motion.

The Appendix contains a brief survey of typical existing vortex-identification schemes in the form of two summarizing tables, containing both region-type methods [1-15], Table 1, and line-type schemes [16-22], Table 2, the latter being characterized by the search for vortex-core lines. A brief explanation of some well-established methods and the associated symbols is also included. It should be noted that the region-type and line-type methods may be effectively combined as shown, for example, in [23, 24].

VALIDITY FOR COMPRESSIBLE FLOWS

The widely used region-type vortex-identification scheme, employed in LES (large-eddy simulations) and compressible flows for a long time, is *Q*-criterion (e.g. [25-29]). However, as mentioned in [15], the *Q*-criterion suffers from ambiguity as it offers two ways of extension for compressible flows which have different physical meaning, the second invariant of the velocity gradient tensor $\nabla \mathbf{u}$, and the quantity $\left(\|\mathbf{\Omega}\|^2 - \|\mathbf{S}\|^2\right)/2$, assuming standard decomposition $\nabla \mathbf{u} = \mathbf{S} + \mathbf{\Omega}$. Both choices of extension cannot avoid dependence on a non-zero divergence. Apart from this ambiguity, both extensions of *Q*-criterion cannot distinguish between expansion and compression as shown in [30]. Consequently, the only correct way how to use the *Q*-criterion is to redefine this criterion *a priori* in terms of a deviatoric part of strain-rate tensor **S** to read (subscript D denotes deviatoric quantity)

$$Q_\mathrm{D} = \left(\|\mathbf{\Omega}\|^2 - \|\mathbf{S}_\mathrm{D}\|^2\right)/2 > 0 \, . \tag{1}$$

The (region-type) Δ-criterion and the associated λ_{ci}-criterion are directly extendable to compressible flows. Both criteria fulfil a specific condition which can be conveniently expressed for compressible flows in terms of the second and third invariants of the deviatoric part of $\nabla \mathbf{u}$ by the form [30]

$$\Delta = \left(\frac{Q_\mathrm{D}}{3}\right)^3 + \left(\frac{R_\mathrm{D}}{2}\right)^2 > 0 \, . \tag{2}$$

It is well-known that the λ_2-criterion (based on the search for a pressure minimum across the vortex) was originally tailored for incompressible flows. The use of the quantity $\mathbf{S}^2 + \mathbf{\Omega}^2$ as an approximation of the pressure Hessian for compressible fluids requires discarding other terms [8] besides the unsteady irrotational straining and viscous effects originally removed from the strain-rate transport equation valid for incompressible flows only. These additional terms are related to a non-zero divergence and non-zero density gradients.

The following conclusion can be drawn: From the most popular region-type vortex-identification schemes (Q, Δ, λ_2, and λ_{ci}) only the Δ-criterion and the closely associated λ_{ci}-criterion are directly extendable to compressible flows.

The analysis of [13] which provides a frame-independent (this aspect is treated in more detail in a later section) definition of a vortex, is limited to 3D incompressible flows only. To identify vortices in 3D compressible, variable-density flows governed by the baroclinic term (i.e. the normalized cross product of a density gradient and pressure gradient) in the vorticity equation, it is proposed in [14] a Galilean invariant scheme, eigen helicity density. The recent method of [15] aims at the extraction of shearing motion near a point through the decomposition $\nabla \mathbf{u} = \mathbf{S} + \mathbf{\Omega} = (\mathit{residual\ tensor}) + (\mathit{shear\ tensor})$ by maximizing the following shear-indicating scalar quantity: $\left|S_{12}\Omega_{12}\right| + \left|S_{23}\Omega_{23}\right| + \left|S_{31}\Omega_{31}\right|$. Only off-diagonal terms of **S** are employed and hence the scheme is not affected by a non-zero uniform dilatation in the case of compressible flows. A specific portion of vorticity labelled *residual* vorticity which is obtained in [15] after the extraction of shearing motion is proposed to represent a local intensity of the swirling motion of a vortex. This kinematic measure is free of compressibility and variable-density effects. There is a modification of this method which emphasizes the kinematic role of local corotation of line segments near a point, see [31]. Regarding the line-type vortex-identification schemes [16-22] (see Appendix, Table 2), they provide a vortex skeleton in terms of vortex-core lines and hence are usually free of direct compressibility effect.

LOCAL VORTEX INTENSITY NEAR A POINT AND THE BIAS TOWARDS SHEARING MOTION

The quantity representing a local vortex intensity near a point is needed to describe the inner structure of a vortex. It is desirable to determine the vortex boundary by the condition requiring zero or negligible intensity at the boundary, Figure 1.

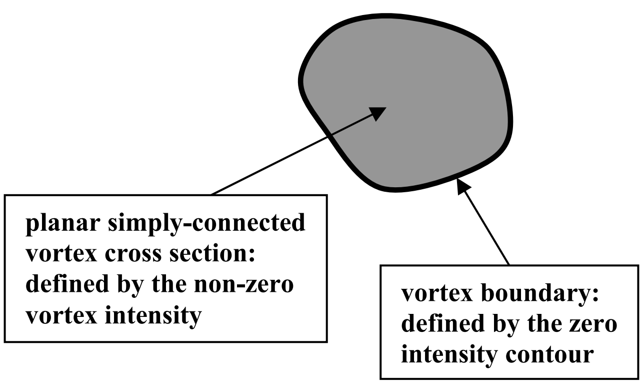

Figure 1 Generally defined vortex cross section

What is the local intensity of a vortex in 2D in terms of earlier published measures? The widely used $\nabla \mathbf{u}$-based region-type identification criteria (Q, Δ, and λ_2) degenerate in 2D incompressible flow to the same one [6]. This criterion reads in terms of magnitudes of planar vorticity ω and planar strain rate s as follows: vortex region is identified by $\omega^2 - s^2 > 0$ what implies the condition $|\omega| > |s|$. The quantity, $\omega^2 - s^2$, the second invariant of planar $\nabla \mathbf{u} \equiv \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix}$, is just the Okubo-Weiss parameter [32-34] used to distinguish elliptic and hyperbolic flow regions. The elliptic flow regions for $|\omega| > |s|$ coincide with the vortex regions determined by the degenerated planar versions of criteria Q, Δ, and λ_2.

The λ_{ci}-criterion provides a local vortex intensity as the angular frequency of revolutions of spiraling streamlines in a local reference frame moving with the examined point. In 2D, the local vortex intensity by λ_{ci}-criterion is $\sqrt{\omega^2 - s^2}$ under the condition $|\omega| > |s|$, thus for all the main region-type criteria (Q, Δ, λ_2, and λ_{ci}) it is sufficient to examine the quantity $\omega^2 - s^2$.

The bias of the quantity $\omega^2 - s^2$ has been already indicated in [15]. For the purpose of an updated analysis below, the bias towards shearing motion is conveniently expressed in terms of rotating line segments near a point as shown in Figure 2. Let us consider an arbitrary point inside the vortex region (Figure 1) by the condition $|\omega| > |s|$, then the local kinematics near a point can be for incompressible flow ($u_x + v_y = 0$) depicted by the upper part of Figure 2. The relevant quantities ω and s can be expressed in the form of angular velocities of line segments as

$$\omega = \left(v_x - u_y\right)/2 = \Omega_{\mathrm{AVERAGE}}, \tag{3}$$

$$|s| = \left(\sqrt{4u_x^2 + \left(u_y + v_x\right)^2}\right)/2 = \left(\left|\Omega_{\mathrm{HIGH}}\right| - \left|\Omega_{\mathrm{LOW}}\right|\right)/2. \tag{4}$$

It follows directly for $|\omega| > |s|$ valid inside a vortex

$$\begin{aligned} \omega^2 - s^2 &= \left[\Omega_{\mathrm{AVERAGE}} - \left(\left|\Omega_{\mathrm{HIGH}}\right| - \left|\Omega_{\mathrm{LOW}}\right|\right)/2\right] \\ &\quad \cdot \left[\Omega_{\mathrm{AVERAGE}} + \left(\left|\Omega_{\mathrm{HIGH}}\right| - \left|\Omega_{\mathrm{LOW}}\right|\right)/2\right] \\ &= \Omega_{\mathrm{LOW}} \cdot \Omega_{\mathrm{HIGH}} \end{aligned} \tag{5}$$

The quantity $\omega^2 - s^2$ apparently mixes in (5) the effect of a non-zero rigid-body rotation given by Ω_{LOW} with the effect of superimposed shearing motion through Ω_{HIGH} and fails as a candidate for the local vortex intensity due to this bias. Needless to say that vorticity itself, expressing an average angular velocity of fluid elements, cannot distinguish between shearing motions and the actual swirling motion of a vortex and misrepresents vortex geometry.

The residual vorticity (obtained after the extraction of shearing motion) introduced in [15] and taken as the vortex intensity can be interpreted through the least-absolute-value angular velocity of line segments, see Figure 2, so as to obtain

$$\omega_{\mathrm{RES}} = \omega - \omega_{\mathrm{SH}} = (\operatorname{sgn}\omega)\left[|\omega| - |s|\right] = \Omega_{\mathrm{LOW}} \quad \mathit{for}\ \ |\omega| \ge |s|, \tag{6}$$

$$\omega_{\mathrm{RES}} = 0 \quad \mathit{for}\ \ |\omega| \le |s|. \tag{7}$$

The non-zero residual vorticity is just the unbiased measure of the corotation of line segments as given by non-zero Ω_{LOW}.

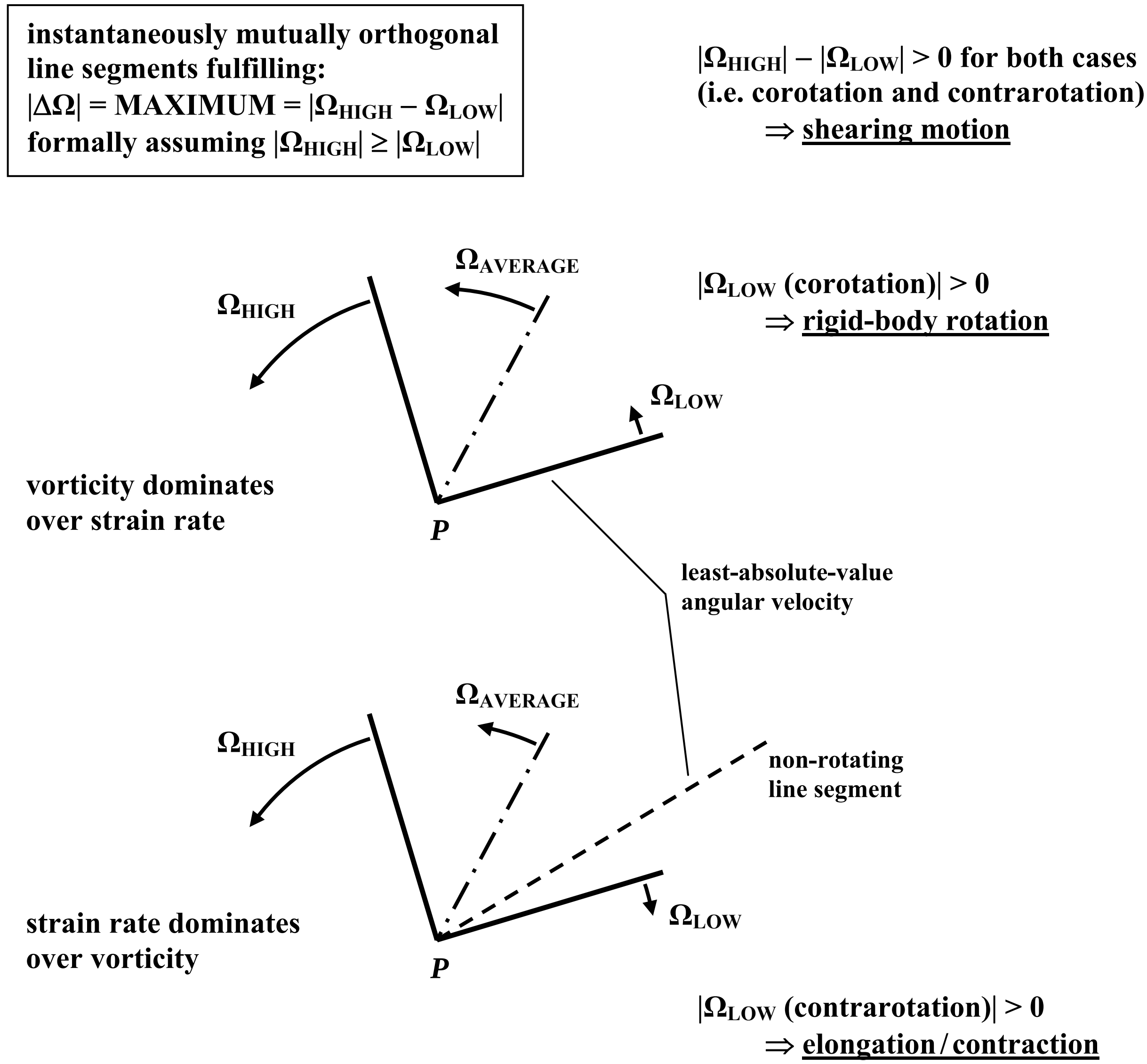

Figure 2 The local kinematics near a point in terms of angular velocities of line segments (in 2D)

The popular Okubo-Weiss parameter $\omega^2 - s^2$ absorbs — unlike the residual vorticity — shearing effects as it has been clearly expressed through the angular velocity of line segments by (5). The $\omega^2 - s^2$ is biased towards $\left[|\omega| + |s|\right]$ and fails to describe 2D vortex regions correctly inside a vortex.

The physical meaning of $\omega^2 - s^2$ for flow geometry and streamline patterns remains untouched including a useful relationship to flow dynamics (e.g. towards turbulence production in wakes [35], among others). However, this quantity is not suitable for vortex identification in 2D. A similar problem of the bias towards shearing motion is expected in 3D vortex identification where obviously both the vortex boundary and inner vortex structure are subjected to the detection tool (i.e. the concept of local vortex intensity) employed.

Unlike vorticity, the residual vorticity [15] aims to distinguish between shearing motions and the actual swirling motion of a vortex. Consequently, it correctly captures the vortical motion near a no-slip solid boundary, non-rotating relative to a reference frame, by diminishing to zero at the boundary. The 3D boundary conditions for the shear-vorticity vector $\boldsymbol{\omega}_{\mathrm{SH}}$ and the residual-vorticity vector $\boldsymbol{\omega}_{\mathrm{RES}}$, $\boldsymbol{\omega} = \boldsymbol{\omega}_{\mathrm{SH}} + \boldsymbol{\omega}_{\mathrm{RES}}$, are shown (for general 3D case) in Figure 3.

The vorticity decomposition $\boldsymbol{\omega} = \boldsymbol{\omega}_{\mathrm{SH}} + \boldsymbol{\omega}_{\mathrm{RES}}$ helps to qualitatively distinguish between vortex sheets and tubes in terms of different vorticity parts. A nice example is just a vortex ring impulsively generated from a tube opening (e.g. [36]), schematically depicted in Figure 4. Cylindrical "vortex" sheets (and any other "vortex" sheets) are characterized by high values of $\boldsymbol{\omega}_{\mathrm{SH}}$ and negligible values of $\boldsymbol{\omega}_{\mathrm{RES}}$ while vortex cores (vortex centers) are characterized vice versa, by relatively high values of $\boldsymbol{\omega}_{\mathrm{RES}}$ and negligible values of $\boldsymbol{\omega}_{\mathrm{SH}}$.

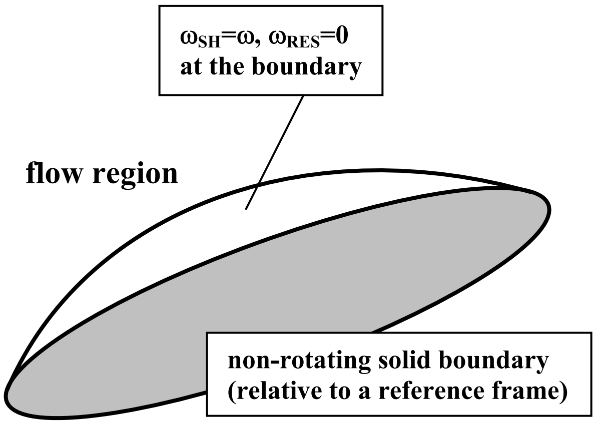

Figure 3 Boundary conditions for the shear vorticity and the residual vorticity

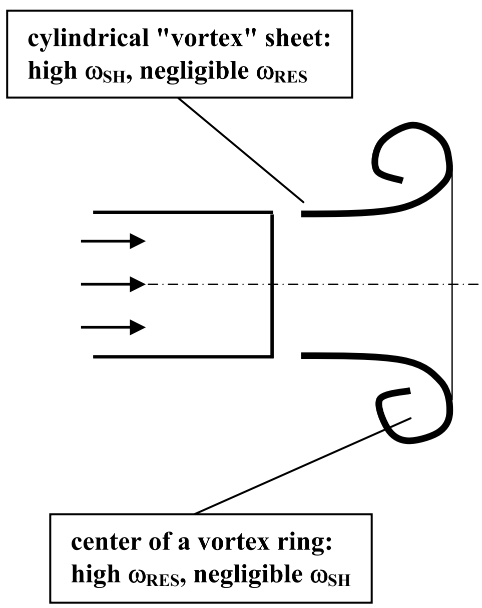

Figure 4 Vorticity characteristics for vortex sheets and tubes

THE INTEGRAL STRENGTH OF A VORTEX

The integral strength of a vortex is usually calculated as the circulation along the vortex boundary, or equivalently, due to Green's theorem, as the surface integral of vorticity over the vortex region. However, vorticity is misrepresenting the local vortex intensity as the vorticity is biased towards shearing motion. Consequently, one obtains a net circulation for the region of a simple shear due to a net vorticity. In spite of this fact, such vortex measures as the frequently employed initial circulation or downstream circulation still represent basic characteristics of vortical structures in free shear flows.

A residual circulation can be introduced as the (planar) surface integral in a similar manner as circulation Γ. It reads

$$\Gamma_{\mathrm{RES}} = \int_A \omega_{\mathrm{RES}}\, \mathrm{d}A \,. \tag{8}$$

For an arbitrary threshold value the region of the residual vorticity forms a subdomain of the vorticity region.

The concept of the residual circulation is particularly simple if applied to 2D or quasi-2D problems. Let us briefly recall a downstream behavior of jets in crossflow. The secondary-flow vortical structures form a counter-rotating vortex pair (CVP), even for twin jets in crossflow with a limited nozzle separation [37]. For three basic nozzle arrangements of twin jets in crossflow (tandem, side by side, and oblique at 45º) as well as for the single jet in crossflow, the residual circulation of the CVP (averaged for the asymmetric oblique case) indicates almost universal constant downstream behavior. This behavior strongly differs from that of the markedly decreasing conventional circulation. One can conclude that the turbulent vorticity transport across the CVP centerline and the corresponding circulation decay deal predominantly with the shear vorticity rather than the residual vorticity, at least within the measured downstream range. This might be a plausible explanation for the well-known fact that the counter-rotating vortex pair of a jet in crossflow persists far downstream. The application of the residual circulation is expected to reveal interesting flow features, as that already mentioned, in other flow problems.

Quite different vortex-strength models can be derived on the basis of different local vortex intensities as shown in [38] in 2D. The vortex region, vortex boundary and, consequently, the (integral) vortex strength are directly dependent on the choice of the concept of local vortex intensity. The vortex strength is usually calculated for the planar simply-connected vortex cross section defined by the non-zero vortex intensity and bounded by the zero (or negligible) intensity contour (Figure 1). In addition, interestingly enough, the integral approach may be considered not only in the cross-section sense but even in volumetric sense covering the whole 3D vortex region. Volumetric characteristics may offer a better insight into the evolutionary process of an individual vortical structure considered as a whole (and not, as usually, to be characterized in a few representative cross-sections only).

ARBITRARY AXIAL STRAIN RATE VS. ORBITAL COMPACTNESS

Let us recall the interesting but controversial idea of the vortex-identification requirement of allowance for an arbitrary axial strain rate. In [39] it is performed an analytical diagnosis of four local region-type vortex-identification criteria, demonstrated by the Burgers and Sullivan vortices, indicating that the Q-criterion [3] and λ_2-criterion [6] may cut a connected vortex into broken segments at locations with strong axial stretching. They emphasized the following vortex-identification requirements: a generally applicable vortex definition should be able to identify the vortex axis and allow for an arbitrary axial strain. The swirling-strength λ_{ci}-criterion of the study [9], based on the Δ-criterion [1, 2, 4], was further enhanced in [12]. According to [12], rapid radial spreading out (or, similarly, axial stretching out) of instantaneous streamlines may not appear to qualify the region as a vortex, as depicted in a simplified manner in Figure 5. In [12], a local approximation of the non-local property proposed in [8] is included, requiring that the swirling material points inside a vortex have bounded separation remaining small. They introduced an idea of orbital compactness of a vortex in terms of the so-called spiraling compactness of the motion projected onto the vortex plane given by the complex-conjugate eigenpair of $\nabla \mathbf{u}$.

The allowance for an arbitrary axial strain rate [39] became a subject of intensive debate [40], [41] as this requirement, basically, does not conform to the orbital compactness proposed in [12]. For an incompressible flow the axial strain rate is directly related to the spiraling compactness [12, 40]. According to [12, 40], the spiraling compactness requires for vortex-identification purpose an appropriate threshold dictated by the length and time scales of the given problem. Following [41], however, adding a threshold value to the local axial strain rate or to the orbital compactness is subjective and cannot be rationalized.

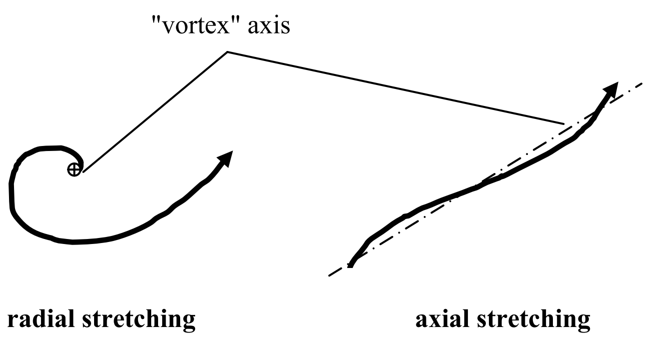

Figure 5 Vortex stretching

THRESHOLD VS. PHYSICALLY DEFINED BOUNDARY

The role of threshold, including that mentioned in the previous section, is not negligible and have to be taken into account. Regarding the vortex boundary, there are typical local (i.e. applied point by point) region-type discriminative criteria (as Q, Δ, λ_2, and λ_{ci}) which provide physically defined boundary. However, practical applications of the most popular criteria employ a nonzero threshold to remove noisy edges. The vortex surface with a positive threshold appears significantly smoother [9]. For example, in the eduction of longitudinal vortices in wall-turbulence [42] and in the study of a neutrally stratified planetary boundary-layer flow [43] it is employed a non-zero threshold for λ_2 contrary to the original λ_2-criterion. This practical aspect is emphasized in [19]. Moreover, the study of the relationship between local identification schemes [12] shows that all of the popular local criteria, given the proposed usage of threshold, result in a remarkable vortex similarity.

The choice of threshold may be important even for line-type methods. The search for vortex-core lines usually needs the vortex-intensity threshold (and/or relative-angle threshold or other criteria) which terminates the process of growing the skeleton. The typical line-type method is predictor-corrector scheme [17] with further modifications [23] and [24]. Starting from the seed point the skeleton is grown in both directions depending on the threshold adopted in the algorithm.

FRAME INDIFFERENCE

Though Galilean-invariant quantity, vorticity is not objective and provides different results in different rotating reference frames. The property of frame indifference (that is, both translational and rotational independence, [44]) is not fulfilled.

All the typical $\nabla \mathbf{u}$-based vortex-identification schemes (Q, Δ, and λ_2) are not objective. This fact has motivated a new vortex definition [13] which is objective relative to an arbitrarily rotating reference frame. This definition should help in situations where there is an unclear choice for a reference frame (e.g. vortical flows in rotating tanks)

Regarding vorticity decomposition, it is claimed in [45] that objective information is contained in the vorticity tensor. Unfortunately for vortex identification, it seems that the objective portion of vorticity is always strongly related to the objective strain-rate tensor and the deformational aspects of the flow. The residual vorticity of [15] is not objective and depends on the angular velocity of an observer's reference frame.

The following is just for illustrative purposes only. Let us consider a simple flow example with rotationally unclear choice of reference frame: the two-dimensional merging of two identical co-rotating vortices schematically shown in Figure 6. In this case, one should cautiously use the residual vorticity or any other $\nabla \mathbf{u}$-based identification scheme. The vortex-axis uniqueness for each connected vortex region may be broken. In this respect, the disappearance of individual single vortices separated by a vortex boundary is correctly viewed in the rotating reference frame stuck on the rotating center-to-center connecting line while the evolution of the resulting vortex is better viewed in the non-rotating reference frame.

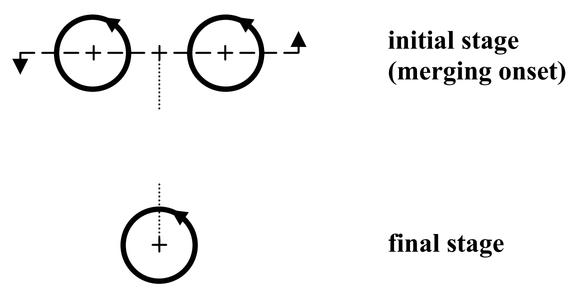

Figure 6 Corotating vortices: an unclear choice of reference frame

CONCLUSION

The longstanding conceptual problems faced in the detection of vortical structures have been discussed. Due to these problems, there is still no consensus on the generally acceptable and rigorous definition of the distinct flow phenomenon of a vortex. One should always be aware of the limitations of the scheme selected for data processing.

Following [39], it should be stated that owing to their universality, kinematic criteria are preferred if they work well. From the kinematic viewpoint, an easy-to-understand local vortex intensity has been recently introduced in terms of the local corotation of line segments in [31].

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APPENDIX

A brief survey of typical existing vortex-identification schemes is presented below, though this is not a complete list.

Table 1 Region-type vortex-identification methods

Author/s & Year **Basic characteristics**
Dallmann (1983) [1] Δ-criterion: complex eigenvalues of $\nabla\mathbf{u}$
Vollmers et al. (1983) [2] Δ-criterion: complex eigenvalues of $\nabla\mathbf{u}$
Hunt et al. (1988) [3] Q-criterion: second invariant of $\nabla\mathbf{u}$
Chong et al. (1990) [4] Δ-criterion: complex eigenvalues of $\nabla\mathbf{u}$
Berdahl & Thompson (1993) [5] swirl parameter, similar to λ_{ci}-criterion
Jeong & Hussain (1995) [6] λ_2-criterion: eigenvalues of $\mathbf{S}^2+\mathbf{\Omega}^2$
Portela (1997) [7] scheme based on set theory
Cucitore et al. (1999) [8] non-local (particle-trajectory) method
Zhou et al. (1999) [9] swirling-strength λ_{ci}-criterion: complex eigenvalues of $\nabla\mathbf{u}$
Sadarjoen & Post (2000) [10] advanced streamline method
Jiang et al. (2002) [11] scheme based on combinatorial topology
Chakraborty et al. (2005) [12] enhanced swirling-strength λ_{ci}-criterion
Haller (2005) [13] objective frame-independent vortex definition
Zhang & Choudhury (2006) [14] Galilean-invariant eigen helicity density
Kolář (2007) [15] triple decomposition of $\nabla\mathbf{u}$: residual vorticity

Q-criterion [3]: Vortices of an incompressible flow are identified as connected fluid regions with a positive second invariant of the velocity-gradient tensor $\nabla\mathbf{u}$, $\nabla\mathbf{u}=\mathbf{S}+\mathbf{\Omega}$, $\mathbf{S}$ is the strain-rate tensor, $\mathbf{\Omega}$ is the vorticity tensor (in tensor notation below the subscript comma denotes differentiation),

$$Q \equiv \frac{1}{2}\left(u_{i,i}^2 - u_{i,j}u_{j,i}\right) = -\frac{1}{2}u_{i,j}u_{j,i} = \frac{1}{2}\left(\|\mathbf{\Omega}\|^2 - \|\mathbf{S}\|^2\right) > 0 \tag{A.1}$$

that is, as the regions where the vorticity magnitude prevails over the strain-rate magnitude.

Δ-criterion [1, 2, 4]: Vortices are defined as the regions in which the eigenvalues of $\nabla\mathbf{u}$ are complex and the streamline pattern is spiraling or closed in a local reference frame moving with the point. To guarantee complex eigenvalues of $\nabla\mathbf{u}$ the discriminant Δ of the characteristic equation should be positive

$$\Delta = \left(\frac{Q}{3}\right)^3 + \left(\frac{R}{2}\right)^2 > 0\,. \tag{A.2}$$

where Q and R are the second and third invariants of $\nabla\mathbf{u}$, Q is given by (A.1), R is defined by $R \equiv \mathrm{Det}(u_{i,j})$. The Δ-criterion (A.2) is valid for incompressible flows only.

The Q-criterion is clearly more restrictive than Δ-criterion (cf. (A.1) and (A.2)).

λ_2-criterion [6]: This criterion is formulated on dynamic considerations, namely on the search for a pressure minimum across the vortex. The quantity $\mathbf{S}^2+\mathbf{\Omega}^2$ is employed as an approximation of the pressure Hessian after removing the unsteady irrotational straining and viscous effects from the strain-rate transport equation for incompressible fluids. Vortex region is defined as a connected fluid region with two negative eigenvalues of $\mathbf{S}^2+\mathbf{\Omega}^2$ (that is, if these eigenvalues are ordered, $\lambda_1 \ge \lambda_2 \ge \lambda_3$, by the condition $\lambda_2 < 0$).

λ_{ci}-criterion [9, 12]: The Δ-criterion has been further developed into the so-called swirling-strength criterion denoted as λ_{ci}-criterion. The time period for completing one revolution of the streamline is given by $2\pi/\lambda_{ci}$ [12]. The two criteria, Δ and λ_{ci}, are equivalent only for zero thresholds (Δ=0 and λ_{ci}=0).

Table 1 Line-type vortex-identification methods

Author/s & Year **Basic characteristics**
Levy et al. (1990) [16] extrema of normalized helicity density
Banks & Singer (1995) [17] vorticity-predictor and pressure-corrector scheme
Sujudi & Haimes (1995) [18] eigenvectors of $\nabla\mathbf{u}$, tetrahedral cells
Kida & Miura (1998) [19] sectional-swirl & pressure-minimum scheme
Roth & Peikert (1998) [20] parallel-vectors (higher-order) method
Strawn et al. (1999) [21] lines of maximum vorticity
Roth (2000) [22] generalization of earlier line-type methods

Probably the most detailed study of various line-type methods was conducted in [22]. However, there is a number of recent papers on this subject, for example [23, 24, 46].

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