

Nanofluid viscosity: A simple model selection algorithm and parametric Evaluation

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ABSTRACT

In this paper, a study is undertaken to investigate the degree of variability between empirical nanofluid viscosity data and the viscosity model predictions. It is seen that there is a high degree of variability in the compared data, which suggests that a wide range of constitutive factors need to be incorporated into the models in order to adequately account for the rheological behavior of nanofluids. A selection algorithm is also proposed as a simple but effective tool for the model selection process in different nanofluidic conditions/compositions; consequently the odds of redundancy are substantially reduced.

Keywords: nanofluid, viscosity, model, algorithm

INTRODUCTION

Descriptively, nanofluids are colloidal dispersions containing solid particles measuring between 1 and 100nm. The idea behind the production of nanofluids entails developing a superior heat transfer fluid which can overcome the shortcomings in the heat transfer and frictional pressure drop performance of traditional fluids. Nanofluids can potentially improve the design and efficiency of thermal systems; however their performance limitations have so far raised a number of concerns due to the small size of nanoparticles which often clog flow channels. Anomalous behavior in the viscosity of nanofluids has also been reported [1-5].

Importantly, there are growing areas of potential application of nanofluids which can be majorly categorized into [6]: heat transfer applications; automotive applications; electronic applications; and biomedical applications. The first three categories can be considered as heat transfer engineering applications since they are pertinent to increased energy efficiency in engineering systems. There is also a wide interest in the use of nanofluids as sensors and optical fillers [7]. These vast areas of potential application make the current thrust of research into the viscous behaviour worthwhile, importantly emphasising the need to determine those properties which can be predicted and controlled for their efficient performance.

Relatively, empirical investigations into the viscosity of nanofluids have shown that existing theories, models and correlations are inadequate. The parameters which have so far been applied in modeling nanofluid viscosity include[2-5]: volume fraction/concentration, temperature, packing fraction, thickness of the nanolayer, particle shape/aspect ratio, aggregate radius, interparticle spacing and the capping layer. Although, there are empirical data on the effects of electromagnetic, electroviscous, and dispersion energy phenomena as well as the influence of density and polarity of base fluids on viscosity, existing models do not take into account the above effects. Notably, many of the existing formulas are

based on Einstein pioneering work [2, 8], since they relate viscosity to the volume fraction alone, and tackle partially the influence of base fluid and nanoparticle properties, as well as the mechanism of solid-liquid interaction.

Also, a number of model selection algorithms have been developed in order to enhance the selection and evaluation of models in wide-ranging applications. Pond et al [9] employed an evolutionary model selection technique with a genetic algorithm to justify sequence evolution; and described a likelihood-based approach for an evolutionary model selection process. The procedure employed a genetic algorithm (GA) to quickly explore a combinatorially large set of possible time-reversible Markov models with a fixed number of substitution rates. However, the method is complex due to its structural components as to be employed in nanofluid viscosity models selection. Castle et al [10] surveyed a number of common Model Selection Algorithms (MSAs), and demonstrated how they related to each other, as well as identified factors that clarified their performances attributes. Mukherjeet et al [11] advanced a cosmological model selection algorithm for a wide array of models. A new evidence algorithm known as *nested sampling*, which combined accuracy, generality of application and computational feasibility, was applied to some cosmological data sets and models, it was found that a five-parameter model with a Harrison-Zel'dovich initial spectrum was preferable. The composite architecture of this model does not however make it suitable for use in nanofluids viscosity model selection.

Despite an increase in theoretical investigations into the viscosity of nanofluids, there is still little understanding of factors that lead to anomalous enhancement in the viscosity: this ostensibly leads to differing and divergent approaches in the model formulation. Consequently, this article investigates the influence of variable parameters of the viscosity models and compares the predictions with empirical viscosity data, as well as proposes the use of a selection algorithm for the models on account of widely varying nanofluidic compositions/conditions and formulations.

EXPERIMENTAL AND THEORETICAL MODELS

A number of theoretical investigations have been conducted into the suspension rheology of colloidal dispersions. The fundamental work by Einstein [12] into the analysis of infinite dilute suspensions of hard spheres based on the rotational motion of the spherical shear particle, led to the following relation

$$\eta = \eta_o (1 + [\eta]\phi) \tag{1}$$

where $[\eta]$ is the intrinsic viscosity of the suspension. This model is specific to a case of $\phi < \sim 0.01$. For $\phi > \sim 0.01$,

Batchelor [13] obtained the following model for the relative viscosity

$$\eta_r = \eta/\eta_o = 1 + [\eta]\phi + k_H ([\eta]\phi)^2 \tag{2}$$

where k_H is the Huggin's coefficient. This coefficient also known as the interaction parameter, accounts for interparticle interaction, as opposed to pure hydrodynamic effects [13]. The semi-empirical relationship proposed by Krieger and Dougherty [14] for shear viscosity covering the full range of particle volume concentrations is given by

$$\eta_r = \eta/\eta_o = (1 - \phi/\phi_m)^{-[\eta]\phi_m} \quad (3)$$

The modified Krieger-Dougherty equation for the relative viscosity η_r is expressed as [14]:

$$\eta_r = (1 - \phi_a/\phi_m)^{-[\eta]\phi_m} \quad (4)$$

where $\phi_a = \phi/\phi_{ma}$, ϕ_{ma} is the packing fraction of the aggregates. The viscosity is assumed to follow a power law with a constant index, D . Consequently, ϕ_a is expressed as $\phi_a = \phi(a_a/a)^{3-D}$, where a_a/a is the ratio of effective radii of aggregates and primary nanoparticles. If nanoparticles are assumed to form aggregates and $\phi_m \rightarrow \infty$, considering the theory of variable packing fraction within the aggregate structure, the following expression for the relative viscosity of nanofluids was obtained [16]

$$\eta_r = \left(1 - \frac{\phi}{0.605} \left(\frac{a_a}{a} \right)^{1.2} \right)^{-1.5125} \quad (5)$$

The equation above is based on the theory that attributes the increase in viscosity to the aggregation state of nanoparticles. According to this theory, the aggregation phenomenon is more relevant for small particle dispersion. It was however observed that aggregation state alone could not describe the viscous behaviour of nanofluids. The particle size distribution (PSD) was seen to play a role in modelling the viscosity. If the PSD is discrete, the relative viscosity of an assemblment of non-interacting monomodal suspensions can be calculated as the product of each independent viscosity [17, 18]

$$\eta_r = \prod_{i=1}^m \eta_r(\phi_i) \quad (6)$$

where m denotes the number of classes of different average particle sizes which are considered in a nanofluid system, and ϕ_i is the corresponding particle fraction. The viscosity of each monomodal suspension can be related to ϕ_m (i.e, the maximum particle fraction), using the following expression [19]

$$\eta_r(\phi_i) = \left[1 - \frac{R\phi_i}{1 - \phi_i/\phi_m} \right]^{3.3\phi_m} \quad (7)$$

in which R is an adjustable parameter. For a polydisperse mixture of spherical particles, the procedure by Servais et al [18] and Muralidharan and Runkana [20] can be applied to determine ϕ_{\max}

$$\phi_m = \min(P_i) \quad (8)$$

where P_i stands for the packing fraction of each class size i , calculated as

$$P_i = \sum_{j=1}^n \phi_{ij} \nu_j \quad (9)$$

ϕ_{ij} is the volume fraction and ν_j is the binary packing coefficient. The procedures for calculating these two quantities are detailed in [17, 19]. Also, Brinkman [21] obtained the following one-parameter model for predicting the relative viscosity of nanofluids:

$$\eta_r = \frac{\eta_{nf}}{\eta_{bf}} = \frac{1}{(1 - \phi)^{2.5}} \quad (10)$$

where η_{nf} is the dynamic viscosity of the suspension and η_{nf} is the viscosity of the base fluid. Frankel and Acrivos [22] obtained the following model for the relative viscosity

$$\eta_r = \frac{9}{8} \left[\frac{(\phi/\phi_m)^{\frac{1}{3}}}{1 - (\phi/\phi_m)^{\frac{1}{3}}} \right] \quad (11)$$

in which the parameter ϕ_m was empirically determined. Lundgren [22] obtained the following model based on Taylor series, for calculating the relative viscosity

$$\mu_r = \left[1 + 2.5\phi + \frac{25}{4} \cdot \phi^2 \right] \quad (12)$$

If the second (or higher) order(s) of ϕ is neglected in the foregoing model, it simplifies to the Einstein's equation [12]

$$\eta_r = 1 + 2.5\phi \quad (13)$$

The generalized form of the Lundgren model [23] was proposed by Graham [24], this agrees well with the Einstein model [12] for low values of ϕ

$$\eta_r = 1 + 2.5\phi + 4.5 \left[\frac{1}{(h_i/d_p) \cdot (2 + h_i/d_p) \cdot (1 + h_i/d_p)^2} \right] \quad (14)$$

where d_p and h_i respectively are the particle diameter and interparticle spacing. The basic premise underlying the above formulas is the *linear fluid* assumption.

Nguyen et al [2] proposed the following empirical models for Al₂O₃/water nanofluid of particles sizes, 47 and 36nm, respectively

$$\eta_r = 0.904e^{0.1483\phi} \quad (15)$$

and

$$\eta_r = 1 + 0.025\phi + 0.015\phi^2 \quad (16)$$

The following empirical models based on temperature were proposed by Nguyen et al [2] for the Al₂O₃ /water nanofluid of (47 and 36nm) nanoparticle sizes

$$\eta_r = 1.125 - 0.007T \quad (17)$$

$$\eta_r = 2.1275 - 0.0215T + 0.0002T^2 \quad (18)$$

For higher volume fractions however, they could not provide a correlation which could simultaneously take into account varying temperatures, particle concentrations, and particle size effects on the relative viscosity of different nanofluid systems.

Avsec and Oblak [24] employed the concept of statistical nanomechanics in modeling the relative viscosity. In the work, they noted that the following models due to Cheng and Law [25] and Ward [26], gave very good agreement for two-phase flow with particles larger than 100nm

$$\begin{aligned} \eta_r = 1 + (2.5\phi) + \left(\frac{35}{8} + \frac{5}{4}\beta\right)\phi^2 + \left(\frac{105}{16} + \frac{35}{8} + \frac{5}{12}\beta^2\right)\phi^3 \\ + \left(\frac{1155}{128} + \frac{935}{96}\beta + \frac{235}{96}\beta^2 + \frac{5}{48}\beta^3\right)\phi^4 \dots \end{aligned} \quad (19)$$

and

$$\eta_r = 1 + (2.5\phi) + (2.5\phi)^2 + (2.5\phi)^3 + (2.5\phi)^4 \dots \quad (20)$$

where β is a diffusion exponent. The above equations were reported to be of little importance in nanoscale viscosity calculations. As a result this led to the following improved model known as the Renewed Ward (RW) model cited in [25]

$$\eta_r = 1 + (2.5\phi_e) + (2.5\phi_e)^2 + (2.5\phi_e)^3 + (2.5\phi_e)^4 \dots \quad (21a)$$

and

$$\phi_e = \phi \left(1 + \frac{h}{r} \right)^3 \quad (21b)$$

where ϕ_e is the effective volume fraction, h is the thickness of the nanolayer and r is the particle radius.

Hosseini and Ghader [8] proposed the following relations based on Eyring's viscosity [29] and the non-random-two-liquid (NRTL)[30] models for the prediction of the effective viscosity for a (two-fluid component) nanofluid:

$$\xi = \ln(\eta V) = \phi_1 \phi_2 \left(\frac{A_{21} G_{21}}{\phi_1 + \phi_2 G_{21}} + \frac{A_{12} G_{12}}{\phi_2 + \phi_1 G_{12}} \right) + \phi_1 \xi_1 + \phi_2 \xi_2 \quad (22a)$$

$$G_{ij} = \exp(-\alpha A_{ij} / RT) \quad (22b)$$

where $A_{ij} = a_{ij} + b_{ij} T$

Also, for a three-component nanofluid, the following model was proposed [8]

$$\begin{aligned} \xi = \ln(\eta V) = & \phi_1 \left(\frac{\phi_2 G_{21}}{\phi_1 + \phi_2 G_{21} + \phi_3 G_{31}} A_{21} + \frac{\phi_3 G_{31}}{\phi_1 + \phi_2 G_{21} + \phi_3 G_{31}} A_{31} \right) \\ & + \phi_2 \left(\frac{\phi_1 G_{12}}{\phi_2 + \phi_1 G_{12} + \phi_3 G_{32}} A_{12} + \frac{\phi_3 G_{32}}{\phi_2 + \phi_1 G_{12} + \phi_3 G_{32}} A_{32} \right) \\ & + \phi_3 \left(\frac{\phi_1 G_{13}}{\phi_2 + \phi_1 G_{13} + \phi_2 G_{23}} A_{13} + \frac{\phi_2 G_{23}}{\phi_3 + \phi_1 G_{13} + \phi_2 G_{23}} A_{23} \right) \\ & + \phi_1 \xi_1 + \phi_2 \xi_2 + \phi_3 \xi_3 \end{aligned} \quad (23)$$

the above models apparently require the knowledge of the effective viscosities of the constituent fluids and the relevant variables, as such may not be extended to multi-component nanofluids as they would require complex calculations.

Hosseini et al [28] employed the local composition theory to correlate the shear viscosity of nanofluids. The following correlation was proposed for the relative viscosity [25]:

$$\ln\left(\frac{\eta_{nf}}{\eta_{bf}}\right) \approx a + b\left(\frac{T_o}{T}\right) + c\left(\frac{T_o}{T}\right)^2 \quad (24)$$

where η_{bf} and T_o represent the viscosity of the base fluid and the reference temperature of the nanofluid. T is the temperature of the nanofluid while a , b and c are dimensionless curve-fitting constants.

Hosseini et al [30] obtained a new dimensionless model for predicting the relative viscosity of nanofluids. It was considered as a function of various dimensionless groups which contain the following parameters: (i) viscosity of the base fluid (ii) the hydrodynamic volume fraction of nanoparticles (iii) diameter of nanoparticle (iv) thickness of the capping layer, and (v) temperature. Using an analytical technique, the following model was obtained [30]:

$$\frac{\eta_{nf}}{\eta_{bf}} = \exp\left[m + \alpha\left(\frac{T}{T_o}\right) + \beta(\phi_h) + \gamma\left(\frac{d}{1+r}\right)\right] \quad (25)$$

where ϕ_h is the hydrodynamic volume fraction of nanoparticles, d is the nanoparticle diameter, r is the thickness of the capping layer, T_o is a reference temperature, and T is the measured temperature. α , m and β are empirical constants.

Masoumi et al [32] developed a new analytical model for calculating the viscosity of nanofluids. The critical assumption was that the relative velocity between the base fluid and nanoparticles was of significance underscoring the Brownian motion concept. A correction factor was incorporated into the model so as to account for simplifications made to the applied boundary conditions

$$\eta_{eff} = \eta_{bf} + \frac{\rho_p V_B d_p^2}{72C\delta} \quad (26)$$

with the constraints

$$\begin{aligned}
 \phi &< b/a, \text{ and} \\
 b &= c_1 d_p + c_2 \\
 a &= c_3 d_p + c_4
 \end{aligned}
 \tag{27}$$

where ρ_p is the particle density, δ is the distance between the centres of particles, V_B is the Brownian velocity, C is a correction factor, and d_p is the particle diameter. The constant C is defined by

$$C = \eta_{bf}^{-1} \left[(c_1 d_p + c_2) \phi + (c_3 d_p + c_4) \right]
 \tag{28}$$

The Brownian velocity V_B is given by

$$V_B = \frac{1}{d_p} \sqrt{\frac{18K_b T}{\pi \rho_p d_p}}
 \tag{29}$$

K_b is the Boltzmann constant and T denotes temperature. However the steps required to obtain N may limit its applicability in various nanofluid systems.

In order to model the influence of shear rate (i.e. shear velocity) and temperature on the apparent viscosity for empirical results of CuO/water and CuO/ethylene glycol nanofluid systems with 1.5 and 2.5% volume fractions of CuO, the following equation was proposed [32]

$$\begin{aligned}
 \log(\eta) &= A_0 + A_1 \log(\gamma) + A_2 \log^2(\gamma) \\
 &+ A_3 T \log(\gamma) + A_4 T + A_5 T^2
 \end{aligned}
 \tag{30}$$

where η is the apparent viscosity, and $A_0 \dots A_5$ are coefficients which were determined using the least square method.

Noni et al [34] proposed the following model for Al₂O₃/Water nanofluid

$$\eta_r = 1 + b \left(\frac{\phi}{1-\phi} \right)^n \quad (31)$$

where b is an empirical parameter, which is influenced by two factors; the first is related to electromagnetic effects, this could vary depending on the interaction between various phases, type of additives used as well as the composition and properties of the nanofluid. The second is related to mechanical-geometrical aspects of the particle, i.e., the specific surface area, density and apparent sphericity.

The following model by Brenner and Cordiff [35] was applied to dilute suspensions containing relatively large rod-like nanoparticles of Ethylene glycol/titanate (EG-TNT) nanofluids

$$[\eta](\infty) = \frac{0.312r}{\ln 2r - 1.5} + 2 - \frac{0.5}{\ln 2r - 1.5} - \frac{1.872}{r} \quad (32)$$

where $[\eta]$ is the intrinsic viscosity and r is the aspect ratio. However, it was found out that the classical models under-predicted the measured data. Similarly, the following empirical model was proposed from results of plots of shear viscosity vs. temperature reciprocals for the EG/TNT nanofluid [35]:

$$\ln \eta = A + B \cdot \frac{1000}{T + C} \quad (33)$$

where η is the shear viscosity, T is the absolute temperature, and A , B and C are constants which were determined from the plots.

Namburu et al [35] obtained the following model for estimating the viscosity of nanofluids composed of copper oxide nanoparticles suspended in a mixture of ethylene glycol and water:

$$\log(\eta_{nf}) = A e^{-BT} \quad (34)$$

where A and B are empirical constants. The following correlation was proposed by Kulkarni et al [36], to predict the viscosity of copper oxide nanoparticles suspended in water in the temperature range of 5-50C

$$In\eta_{nf} = A\left(\frac{1}{T}\right) - B \quad (35)$$

However, the model is limited to a narrow range of temperature and parametric variables. The model given by Kulkarni et al [38] for predicting temperature- and volume fraction-dependent viscosity, given by

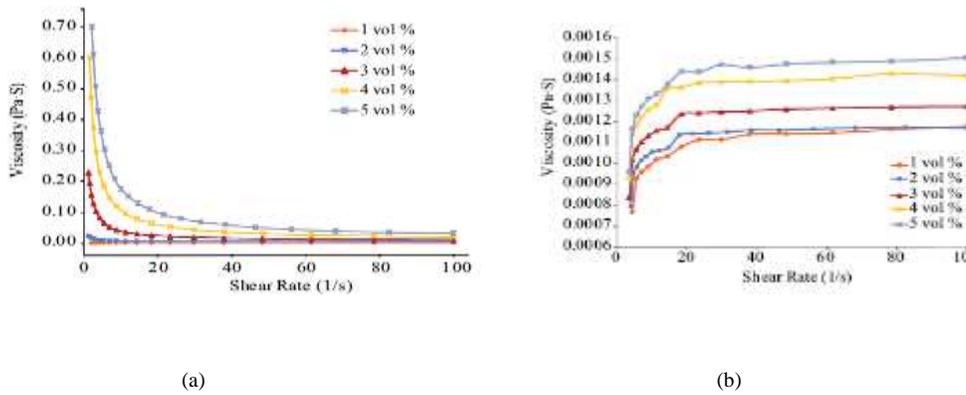


Figure 1 Plots of Al₂O₃/water nanofluids viscosities (a) before re-ultrasonication, and (b) after reultrasonication. Duan et al [37]

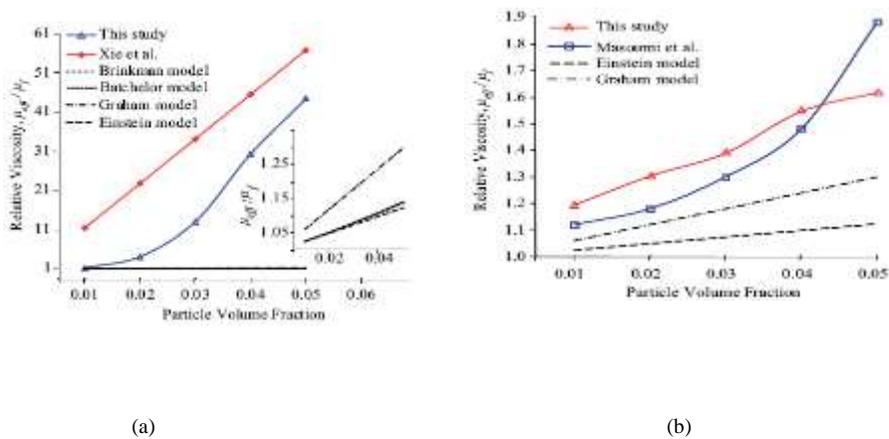
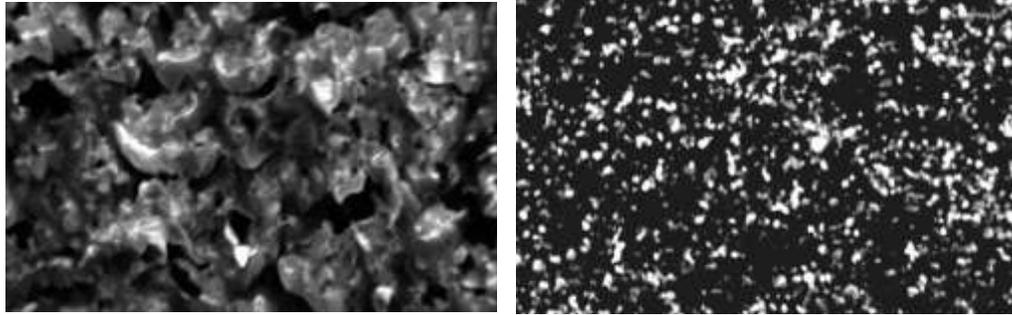


Figure 2 Plots of Al₂O₃/water nanofluids relative viscosities (a) before re-ultrasonication, (b) after reultrasonication. Duan et al [39]



(a)

(b)

Figure 3 SEM microstructure of the dried Al₂O₃/water nanofluids (a) before reultrasonication, and (b) after reultrasonication. Duan et al [37]

$$\begin{aligned} \ln \eta_{eff} = & \exp(-(2.8751 + 53.54\phi - 107.12\phi^2) \\ & + (1078.3 + 15857\phi + 20587\phi^2)(1/T)) \end{aligned}$$

(36)

Effects of Ultrasonication

To show that the viscosity of nanofluids can be affected by nanoparticle aggregation, Duan et al [39] conducted an empirical study on the effects of ultrasonication on the viscosity of Al₂O₃/water nanofluids. The experimental investigation employed the process of ultrasonication to disperse the nanoparticle aggregates of the 2-week old nanofluid while the control was not treated with ultrasonication. The results indicated that the treated fluid behaviour changed from non-Newtonian to Newtonian after the dispersion process (Fig. 1a and 1b). Initially, the viscosity of the nanofluid increased almost linearly before it reached a constant value, and afterward assumed the Newtonian behaviour with increasing shear rate values. This development supports the hypothesis surmising that increase in the viscosity of nanofluids could be attributed to a high level of nanoparticle aggregation. Comparing the magnitude of the plots, The viscosity values of the aggregated nanoparticles had an order of magnitude of 10⁻¹, while that of the dispersed nanofluid is 10⁻³. This result supports the supposition of theoretical standpoints which are based on the aggregation effects. The plots of the relative viscosity of the nanofluids are given in Figures 2a and 2b. From the obtained spectral-electron microscopy (SEM) images of the nanoparticles, before and after ultrasonication (Fig. 3a and 3b), a significant level of agglomeration is observed after the 2-week interval, and dispersion of the nanoparticles is also noticeable in the aftermath

of the ultrasonication. Despite this outcome, it is important to conduct more detailed studies on nanoparticle agglomeration, as well as its effects on the viscous behaviour of nanofluids

GENERIC ALGORITHM FOR SELECTION OF CONFORMING MODELS

Given the diversity of models which are used in predicting the viscosity of nanofluids, there is a necessity to computerize the model selection process due to the fact that a wide range of parameters are hardly accounted for in most of the models thus limiting their applicability in wide ranging conditions. An algorithmic approach is thus proposed, in an attempt to simplify the selection and categorization of the models on the basis of user-defined criteria. To define the selection criteria for the viscosity models, the following sequential steps are essential:

Step 1: define a list of the relevant models

Step 2: define arrays and sub-arrays for the models

Step 3: define criteria for selection

Step 4: implement search and sort operation

Step 5: select appropriate model (s)

Importantly, the algorithm for the model selection involves creating arrays and sub-arrays for each model which are fundamental to storing the model attributes. From the figure, typical models are designated as R, S, etc, each with attributes expressed as R1, R2...R6 and S1,S2...S6, respectively. A sub-array would hold additional information concerning each of the array objects, these can be denoted as r1 and r2. The system of arrays and sub-arrays defined for each model can store the following: the empirical or theoretical nature of the model, its error margin; mechanism of formulation, applicability to a specific or wide range of nanofluids, the range of parametric variables, the presence of dispersion energy coefficients, etc.

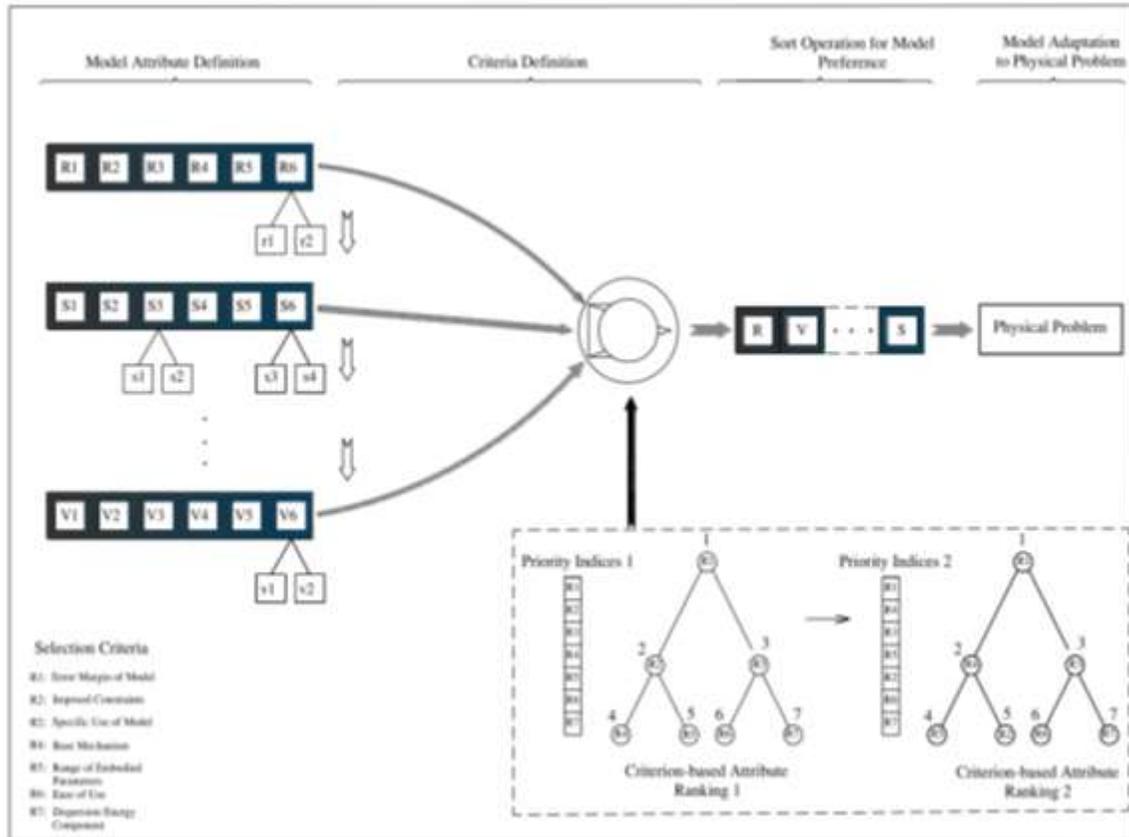


Figure 4: Generic algorithm for selection of appropriate nanofluid viscosity models

The algorithm also has a provision to scan in sequence those attributes for each model. The next implementation step is the definition of the selection criteria: this involves a logical description of priority indices for the attributes- inset of Figure 4. The parameters are allocated index numbers ranging from 0 to 7, for instance, to show priority - where 0 precedes 1 in terms of priority. The two ranking flow diagrams shown in the figure 3 give different priority patterns and index allocation. Supposing the models were stored in a database, it would be logical step to search and sort each models in terms of the user-defined priority indices. The actual adaptation of the model to a problem is the last step in the sequence. The particular need for this algorithm arises in solving real world problem where for instance, a case could arise in which volume fraction ϕ and temperature, T are requisite variables for application, it is thus plausible to select those models defined on the basis of ϕ and T , which are also specific to the nanofluid, or a range of considered nanofluids. The algorithm can contain an infinite range of parameters in its search routine, and can be recursively implemented using off-the-shelf programming packages. This ultimately can be used to build up of a database of conforming models - i.e, models with attributes which are in accordance with pre-defined search criteria - and filter-off

redundant ones, on the basis of the defined criteria. The algorithm can be useful in cases where a range of nanofluids are subject to varying operating conditions, in that it has the advantage of (i) selecting conforming models using its search and sort mechanism,(ii) adaptability to situations involving a variety of nanofluids and diverse models, and (iii) it is flexible. Importantly, if the last criterion - that based on the dispersion energy coefficient – were to be implemented, none of the reported models would pass for selection, thus presenting a challenge for future research.

DATA

Fig. 5 shows a typical performance plot of the algorithm, which gives a sort operation of thereported models on the basis of defined criteria. In the figure, EI,E2...EN refers to the equation1, equation 2 ... equation N, respectively, of the foregoing section. The cogency of the algorithm is tested in the sort application for the

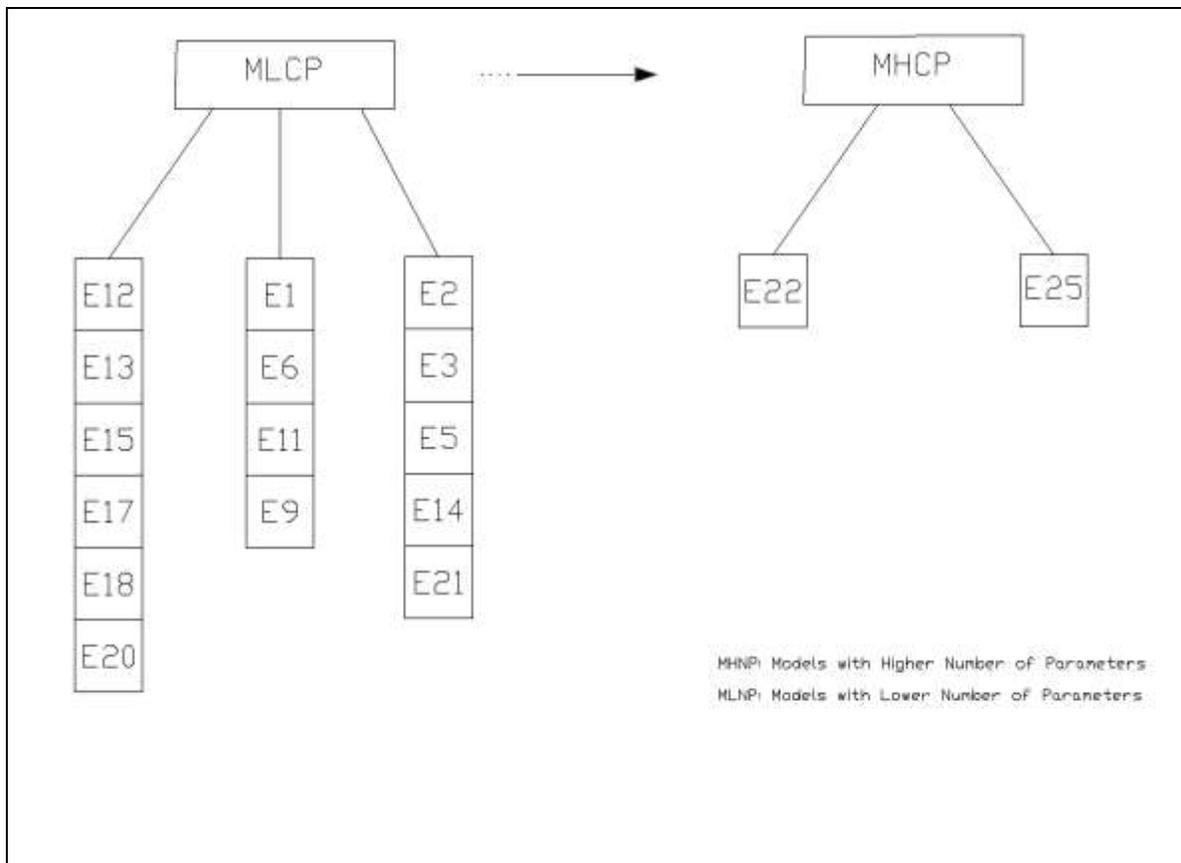


Figure 5: Nanofluid Model Sort Application

nanofluid models, and the sort process terminates after a finite number of steps owing to the finite dimensions of the problem; a decision can then be made for a particular model selection for a given application.

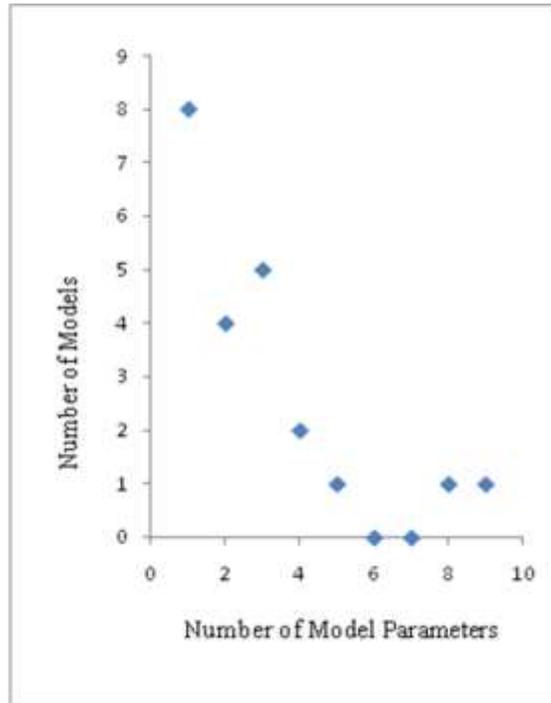


Figure 6: Scatter of Model Detection Performance vs. Number of Constituent Parameters

Fig 6, obtained from a frequency distribution of the model parameters, shows the model detection performance against the number of constituent parameters so as to justify the adaptation of a criteria-based selection process. It indicates also that few of the models have relatively a high number of parameters, with a considerable number of the models having the number of constituent parameters greater than three (3). This shows a significant variability in the reported models, hence the necessity for elimination of redundant models for a narrow spectrum of criteria-based outcomes.

RESULTS

The extent and degree of variability between the model predictions and empirical results are discussed. In Fig. 7, the prediction of the model proposed by Kulkarni et al [36] is compared with the empirical data of Nguyen et al [2] for CuO nanofluids. From the results, the trends of the model predictions are consistent with the trends of the empirical data however there is a wide margin in the viscosity values for decreasing temperatures. Notably, the model proposed by Kulkarni et al [36] model incorporates ϕ and T as independent variables, which most of the other models do not incorporate. If a

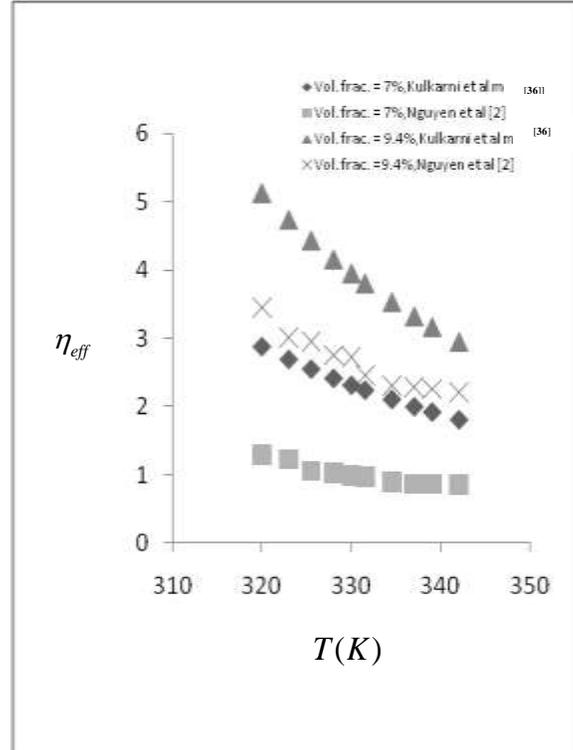


Figure 7: Comparison of empirical and model results for η_{eff} vs. T

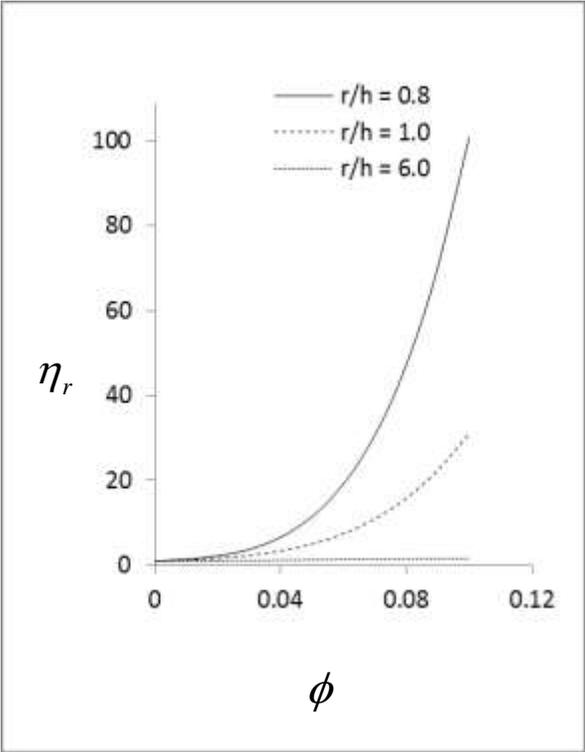


Figure 8: Effects of increasing ϕ and r/h values on η_r for the renewed Ward model [25]

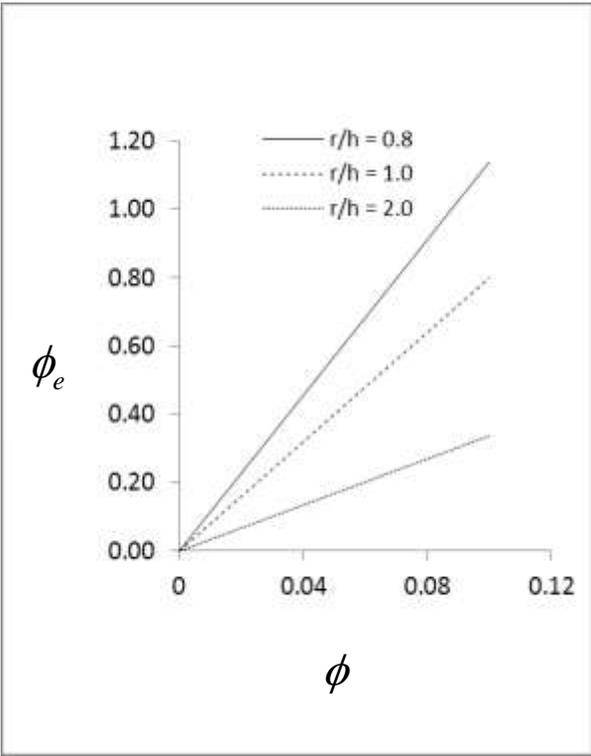


Figure 9: Effects of increasing ϕ on ϕ_e for the renewed Ward model [24]

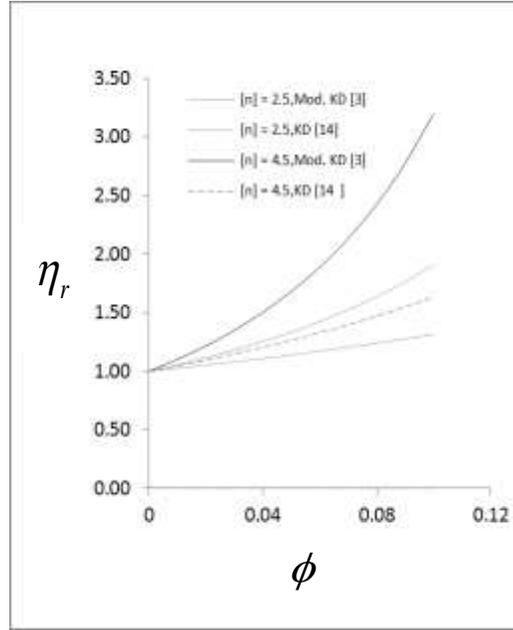


Figure 10: Effects of variation ϕ on η_r for the modified K-D [3] and K-D models [14]

correction factor were applied to the Kulkarni et al [36] it could predict the empirical data of the studied nanofluid.

In Fig. 8, the effects of increasing values of η_r on r/h are investigated. The ratio r/h compares the change in the nanoparticle radius to the thickness of the nanolayer; hence it gives an indication of the relative change in the value of the nanoparticle dimensions with respect to the nanolayer. In the figure, it is seen that increasing volume fraction significantly enhances η_r when r/h is minimum. At $r/h = 0.8$, η_r increases with ϕ , exponentially. When $r/h \gg 1$, there is no discernible change in η_r values. This could be as a result of decreased formation of the nanolayer. From the figure, it could also be deduced that nanolayer formation to some extent influences the viscosity. Little is however known of the interactions between nanolayers and the thermo-physical properties of solid/liquid nano-suspensions [40]. Choi et al [40] suggested that nanolayers act as a thermal bridge between a solid nanoparticle and the liquid medium in a case of thermal conductivity enhancement. Additional investigation is however required to understand the effects of nanolayer

formation on viscosity enhancement in nanofluids. Also, the dependence of ϕ_e on ϕ is shown in Fig. 9, for the renewed Ward model [25] where it is seen that there exists a linear relationship between the two variables ϕ_e .

In Fig 10, the effects of the intrinsic viscosity $[\eta]$ and volume fraction ϕ on the relative viscosity η_r are plotted for the Krieger and Dougherty [15] and the modified Krieger and Dougherty [14] models. Apparently, η_r increases with $[\eta]$ for a given ϕ value. The intrinsic viscosity $[\eta]$ gives an indication of the solute (nanoparticle) contribution to the base fluid viscosity. Most likely, the entanglement of nanoparticles owing to increased volume fraction will result to increasing intrinsic viscosity values, which in turn could lead to an improvement in the effective viscosity. Studies on the effects of size, size distribution and shape of nanoparticle on the intrinsic viscosity in relation to the base fluid density, polarity and temperature are however limited. Einstein [12] proposed an intrinsic viscosity value of 2.5 for spherical nanoparticles. However, this is not adequate for other shapes. More empirical investigation is thus needed to understand the contribution.

CONCLUSION

This paper examined major advancement in modelling the viscosity of nanofluids and suggested the use of an algorithm-based approach in selecting the divergent models. There were some observed inconsistencies in the model formulations and predicted data, consequently the following salient points are underscored:

- the relative viscosity is independent of temperature for very low volume fractions, but decreases with increasing temperature in high volume fractions
- the relative viscosity increases with increasing volume fractions
- the model studies show large variations in the predicted results and there is an apparent lack of standardization
- the effects of ultrasound application on size and distribution of aggregates is rarely examined.

Notably, the effects of electrostatic charges on viscosity, otherwise known as electroviscous effects, have been investigated [39, 40]. The empirical observations show that in micro-channels, nanoparticles can pick up

individual charges and consequently this affects the viscosity of nanofluid. However, these effects are not incorporated into any of the reported models: it is important therefore to conduct broad empirical investigation and theoretical formulation into these observed phenomena, this ostensibly would lead to more approximate models.

NOMENCLATURE

η – Shear viscosity (Centipoise)
 T – Temperature (K)
 ϕ – Volume fraction
 P – Particle concentration
 c – Number of rods per unit volume ($1/m^3$)
 b – Width of rod (m)
 L – Length of particle (m)
 a – Effective radius of aggregates (m)
 p – Packing fraction
 h – Thickness of nanolayer (m)
 r – Radius of nanoparticle (m)
 d – Diameter of nanoparticle (m)
 ρ – Density (kg/m^3)
 δ – Distance between centers of nanoparticles (m)
 V – Velocity (m/s)
 K_b – Boltzmann constant (kgm^2/Ks)
 γ – Shear rate ($1/s$)
 \tilde{T} – Non dimensional temperature
 Ra_f – Raleigh number
 Nu – Nusselt number
 D – Diameter (m)

Subscripts

o – basefluid
 cr – critical
 r – relative
 m – maximum particle volume fraction
 ma – packing fraction of aggregates
 i – class size
 nf – nanofluid
 bf – basefluid
 e – effective volume fraction
 h – hydrodynamic
 p – particle
 B – brownian
 eff – effective

ACKNOWLEDGEMENT

This author acknowledges the assistance of University of Pretoria, while carrying out part of this work in the University under a fellowship grant.

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