NUMERICAL ANALYSIS OF NATURAL CONVECTION HEAT TRANSFER FROM HORIZONTAL FIN ARRAY WITH RECTANGULAR NOTCH AT THE CENTRE.

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ABSTRACT

The problem of increasing the rate of heat transfer between structures and surrounding confronts the engineers in all the phases of technological development. Natural convection cooling with the help of finned surfaces often offers an economical, noiseless and trouble free solution. Natural convection heat transfers from horizontal rectangular fin arrays with and without rectangular notch at the center have been analyzed numerically by using finite difference technique. The results are obtained for notched array, having different percentage depth and are compared to without notch array over a range of fin spacing, fin heights and notch percentages. It is found that the heat transfer rate in notched fin array is better than without notched fin array. The fluid flow analysis shows that the air velocity increases in the upward direction with notched fin array arrangement. Computational data is obtained for the range of parameters - Pr = 0.7, Gr = 10^4 - 10^7, S/H = 0.1- 0.4, L/H= 2 and 4, Notch percentage – 20%, 50%, 60% , 80%.

NOMENCLATURE

L/H- Length to height ratio
S/H- Fin space to height ratio
INH- Notched height
IAFH= Apparent Fin Height  INH= Notch height
NDPER= Notch Depth %
Gr- Grashof number
Nua- Average Nusselt number over fin surface
Nub- Average Nusselt number over base surface
Pr- Prandtl number
T- Absolute Temperature
U, W - velocity in x and z direction
\(\beta\)- Volumetric expansion coefficient
\(\rho\)- Density of the fluid
\(\psi\)- Dimensionless stream function
\(\xi\)- Dimensionless vorticity

INTRODUCTION

Heat transfer from fin arrays is sum of total effect of heat transfer from fin base and fin flats. The fin flats are having comparatively larger surface area for heat transfer. However, the entire surface area is not that much effective but the portion of surface which is in contact with maximum temperature difference is responsible for larger amount of heat transfer. In this respect, the most practical flow pattern i.e. single flow chimney is considered (Fig.1). In single chimney flow pattern central portion of fin flat becomes ineffective due to the presence of heated air.

Fig.1-Single chimney flow pattern ( L/H<5 )
Thus rectangular fins (Fig.2) are modified by removing central fin portion by cutting rectangular notch and adding it at the array entrance on the two sides where it is more effective (Fig.3). In this case surface area of fin remains same, only geometry of fin changes, thus by keeping same material cost there is increase in heat dissipation.

Fig.2-Rectangular Fins without Notch  Fig.3-Rectangular Fins with Notch

Fig.4 shows 20% Notch depth configuration.

\[ NDPER = \frac{(IAFH - INH)}{IAFH} \times 100 \]
\[ NDPER = \frac{(10 - 8)}{10} \times 100 = 20\% \]

The analysis of this problem requires the simultaneous solution of 2 dimensional equations of continuity, momentum and energy.
Continuity Equation
\[ \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \] ---(1)

Momentum Equation
\[ \frac{u}{u} \frac{\partial u}{\partial x} + \frac{w}{w} \frac{\partial u}{\partial z} = -1 \frac{\partial P}{\partial x} + \rho \frac{\partial}{\partial z} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} + g \cdot \beta \cdot (T - T_\infty) \right) \] ---(2)
\[ \frac{\partial P}{\partial y} = 0 \] ---(3)

\[ \frac{u}{u} \frac{\partial w}{\partial x} + \frac{w}{w} \frac{\partial w}{\partial z} = -1 \frac{\partial P}{\partial z} + \gamma \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) + g \cdot \beta \cdot (T - T_\infty) \] ---(4)

Energy Equation
\[ \frac{u}{u} \frac{\partial T}{\partial x} + \frac{w}{w} \frac{\partial T}{\partial z} = K \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) \] ---(5)

The above equations are non-dimensionalised by introducing a non-dimensional stream function \( \Psi \) and vorticity \( \xi \) and with following substitution.

\[ U = \frac{\partial \Psi}{\partial z}, \quad W = -\frac{\partial \Psi}{\partial x} \] and
\[ \xi = \frac{\partial W}{\partial x} = -\frac{\partial U}{\partial z}, \quad \theta = (T - T_\infty)/(T_\infty - T_\infty) \]

Thus above equations further reduced in sets of three simultaneous second order partial differential equations.

Energy Equation:
\[ U \frac{\partial \theta}{\partial x} + W \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \] ---(6)

Vorticity transportation equation:
\[ U \frac{\partial \xi}{\partial x} + W \frac{\partial \xi}{\partial z} = \frac{1}{Pr} \left( \frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial z^2} \right) - Gr \frac{\partial \theta}{\partial z} \] ---(7)

Stream function:
\[ -\xi = \frac{\partial^2 \Psi}{\partial x^2} \] \[ -\frac{\partial^2 \Psi}{\partial z^2} \] ---(8)

Referring Figure5, boundary conditions in terms of new variable \( U, W, \xi, \psi \) and \( \theta \) are given in the following Table1.

<table>
<thead>
<tr>
<th>Surfaces</th>
<th>( \theta )</th>
<th>( \psi )</th>
<th>( \xi )</th>
<th>( U )</th>
<th>( W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABCD (Fin Base)</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
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<tr>
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</tr>
</tbody>
</table>

Table1 - Boundary conditions with Notched Fin

**METHOD OF SOLUTION**

Equations (6) to (8) are solved numerically using a finite difference technique. The central finite difference procedure is used for second order terms while the upwind difference scheme is used for non-linear terms. These differential equations are converted in FORTRAN code, which is found to be the most suitable computer language for solving much involved iterative numerical procedure.

Initially energy equation is solved to get the temperature distribution at each point in the domain. Using these temp. values, the vorticity distribution and then stream function values at solid boundary are calculated. Finally the
nusselt numbers are calculated using temperature distribution and convergence criterion is tested. After convergence the velocity components $U$ and $W$ are computed. For this a simple algorithm based on Gauss Siedel iterative method is used. Flowchart for computational procedure of horizontal fin array with and without notched fin arrays is shown in Fig. 6.

2) Variation of $Nub$ against $S/H$ with variable notch depth 20% and 50% as parameter is shown in Fig.8 and Fig.9 respectively. It is seen that with increase in notch depth, $Nub$ increases. By providing notch an increase about 10% to 30% has been obtained in the value of $Nub$. Also $Nub$ is maximum for higher Gr Number.

3) Fig.10a, 10b show the Isotherm profiles in the central plane of Fin domain for the case of $L/H = 2$, $S/H = 0.1$, $INH = 6$ and $Gr = 10E6$ with and without notch array. The values of temperatures are smaller in case of arrays with notch than the arrays without notched fin. This can be attributed due to increased fluid flow in this case making the fin domain cooler.

**RESULTS AND DISCUSSION**

**Heat Transfer Characteristics**

1) The variation of $Nua$ against $S/H$ with Gr number as parameter is shown in Fig 7. It is observed that $Nua$ decreases as $S/H$ increases. $Nua$ is maximum for higher Gr Number.

2) Velocity profiles of $U$ and $W$ Components

Fig.12 shows variation of $U$ component of velocity in X-Z plane and Fig.13 shows variation of $W$ component of velocity Z-X plane for the parameter $L/H = 2$, $S/H = 0.1$, $INH=3$ and $Gr = 10E6$. For this a simple algorithm based on Gauss Siedel iterative method is used. Flowchart for computational procedure of horizontal fin array with and without notched fin arrays is shown in Fig. 6.

**Fluid Flow Characteristics –**

1) Stream Function Plots

Fig.11a, 11b show stream function plots over entire domain of with and without notch configuration respectively for $Gr = 10E6$, $L/H = 2$, $S/H = 0.1$ and $INH= 6$. From above figures closed loop of stream function lines are observed. It indicates incoming cold air flow, its entry into heated array and formation of hot fluid plume in the central zone of fin array going in upward direction. Since the computations are carried out over a large distance the Natural Convection effects dies down and same air enters as fresh air forming a closed loop pattern.

It is also observed that for 20% and 50% notch depth configuration the stream function values are higher for arrays with Rectangular Notch at the center of fin array than without notched arrays because of augmentation of fluid flow. But above 50% notch depth (i.e. for 61% and 80% notch depth arrays) stream function values of without notched arrays are greater than notched arrays.
$10^7$ for arrays with and without notch at the center of fin arrays.

From U – velocity plots the formation of boundary layer is clearly observed as expected. U – Velocity components show a symmetric profile and an increase in magnitude in upward direction. Also it is seen that the values of U velocities are maximum for arrays with notch.

Fig.14 shows variation of U velocity for different notch height (INH). From graph it is clear that for 80% notch depth U velocity is higher than 20%, 50%, 61% notch depth.

CONCLUSIONS

The important findings and observations regarding heat transfer and fluid flow characteristics are as follows –

1. Base Nusselt number increases with decreasing ratio of S/H for a particular Gr and L/H ratio. This trend is observed for both the arrays with and without notched fin configuration.

2. There is an enhancement in the rate of heat transfer with notched fin arrays by about 10% over without notched fin arrays.

3. Stream function values are higher for notched array than without notch array for the same condition.

4. The increasing trend of velocity component in upward direction shows the increase in natural convection effect.

REFERENCES


Fig. 7- Variation of Nua against S/H for Gr number as parameter, L/H=2, Notch depth= 50%

Fig. 8- Variation of Nub against S/H for Notch depth as parameter, L/H=2, Gr=10^5

Fig. 9- Variation of Nua against S/H for Notch depth as parameter, L/H=2, Gr=10^7

Fig. 10a Isotherm plot for L/H=2, S/H=0.1, Gr=10^6- With Notch
Fig. 10b Isotherm plot for $L/H=2$, $S/H=0.1$, $Gr=10^6$. Without Notch

Fig. 11a Stream function plot over entire domain for $L/H=2$, $S/H=0.1$, $Gr=10^6$. With Notch

Fig. 14 U velocity (thousands) in Z direction for $L/H=2$, $S/H=0.1$, $J=1$, $I=9$, INH as parameter (INH8= 20% Notch Depth)

Fig. 11b Stream function plot over entire domain for $L/H=2$, $S/H=0.1$, $Gr=10^6$. Without Notch

Fig. 12 U velocity (thousands) in Z direction for $L/H=2$, $S/H=0.1$, $J=5$, INH=3, $Gr=10^6$. With (I) and without (WI) Notch

Fig. 13 W velocity (thousands) in X direction for $L/H=2$, $S/H=0.1$, $J=5$, INH=3, $Gr=10^6$. With (K) and without (WK) Notch