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Fourier transform based investment styles on the Johannesburg Stock Exchange

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ABSTRACT

Share price periodicity and calendar effects have been well documented for stock exchanges. If these market anomalies are persistent and of sufficiently high amplitudes, the use of frequency analysis will allow investors to earn abnormal returns.

This research study examined the use of the discrete Fourier transform combined with prior exponential growth and momentum periodicity as an investment style. A graphical time series approach was used to evaluate performance of the examined styles. The time series consisted of the JSE top 160 shares from December 1985 to October 2013.

A momentum-Fourier transform investment style is identified that outperforms most if not all documented univariate ranked investment styles on the JSE for the analysed timeframe. Returns of 27.6% per annum are achieved. It is found that both examined momentum styles are enhanced by using the Fourier transform as a noise filter. Combining prior exponential growth rate and the Fourier transform failed to produce favourable results.

Keywords: discrete Fourier transform, momentum periodicity, time series

DECLARATION

I declare that this research project is my own work. It is submitted in partial fulfilment of the requirements for the degree of Master of Business Administration at the Gordon Institute of Business Science, University of Pretoria. It has not been submitted before for any degree or examination in any other University. I further declare that I have obtained the necessary authorisation and consent to carry out this research.



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Date

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ABBREVIATIONS

CADGR Compounded Average Daily Growth Rate

CAGR..... Compounded Annual Growth Rate

CR..... Cumulative Returns

DFT Discrete Fourier Transform

EMH Efficient Market Hypothesis

FFT Fast Fourier Transform

GDP Gross Domestic Product

IDFT Inverse Discrete Fourier Transform

JSE Johannesburg Stock Exchange

RMS..... Root Mean Square

ROC Rate of Change

STFT Short Time Fourier Transform

1 INTRODUCTION

1.1 Background

Can history be used to predict the future? If it is possible, humankind is yet to find the golden formula that can be used to simply and persistently beat the stock market. Due to the allure of the possible existence of such a formula there has always been a demand for economic and financial time series research in search of factors that can be used to predict share prices. Share price forecasting is an actively studied and hotly debated field with fundamental analysts in one field and technical analysts in the other.

Stock market and share price periodicity is one of the fields being researched that tries to explain the cross sectional spread of returns in stock markets. Share price periodicity refers to the rise and fall of share prices at regular intervals. These intervals or periods can be days, weeks, months or years or fractions thereof. Recent journal articles have argued that contrary to the Efficient Market Hypothesis (EMH), there is a significant degree of periodicity in stock exchanges (Auret & Cline, 2011; Figelman, 2007; Moller & Zilca, 2008). This frequency information is not necessarily reflected in share prices, leading to possible mispricing and time arbitrage.

Public perception would indeed have us believe that there is a considerable amount of periodicity in stock markets and in business cycles. Some of these cycles include the approximately 10-year property cycle in South Africa, a 21 to 22 month peak gold price cycle (Cafariello, 2013), interest rate cycles and Kondratieff cycles (Bodie, Kane & Marcus, 2011). Business and recession cycles have even been compared to sunspot cycles (Kaminska, 2009).

1.2 Research problem

If share prices periodicity exists, is persistent enough and of sufficiently large amplitude, it should be possible to use these cycles to beat the market. It is however possible that each stock is subject to many independent frequencies resulting in interactions that will make them hard to identify by casual observation. If these price patterns are however as repeatable and regular as is believed, it should be possible to identify and predict them using spectral analysis tools such as Fourier transforms. Fourier transform analysis can be used to transform a time series dataset into a set of

frequencies with corresponding phases and amplitudes. The transformed data can be used to identify major frequencies and filter out higher frequency noise.

Many researchers and practitioners have tried to apply frequency and spectral analysis to identify and predict share price movements without success. Little empirical evidence could however be found in academic literature regarding the use of Fourier transform analysis for share price forecasting. Furthermore, there is conflicting research evidence around the effectiveness and accuracy of stock market predictions using any form of analysis and the use or existence of periodic stock market cycles.

The efficient market hypothesis (EMH), introduced by Fama (1970), states that prices in stock markets always "fully reflect" available information. According to the EMH it is impossible for investors to consistently beat the market by using the available information at the time the investment is made. Another theory that hypothesises that it should be impossible for investors to beat the market is the random walk theory. According to this theory "the future path of the price level of a security is no more predictable than the path of a series of cumulated random numbers" (Fama, 1965).

There is however a large body of research conflicting with the EMH and the random walk theory. A number of studies have documented distortions in share returns that seem to follow a seasonal pattern. These periodic market distortions or anomalies include amongst others, the January effect (Thaler, 1987), the September effect (Hulbert, 2013), end of tax-year effects (Pekkala, Polk & Ribeiro, 2007), a weekend effect (Keim & Stambaugh 1984), a Halloween indicator (Bouman & Jacobsen, 2002) and a seasonal affective disorder (SAD) effect (Kamstra, Kramer & Levi, 2003). Fama and French (1988) found that for long holding returns, 25% to 40% of the variation in returns could be predicted by past returns. Malkiel (2003) on the other hand, dismissed many of the market anomalies as being too small to profit from.

Many researchers and practitioners have devoted a substantial amount of attention to invent market-timing strategies in an attempt to beat the market (Arshanapalli, Switzer & Panju, 2007). Trying to time the market illustrates the belief by some researchers and investors that there is seasonality and a degree of periodicity and predictability in share prices.

There are essentially three major stock analysis disciplines used by trading professionals and fund managers. These disciplines include technical analysis, behavioural finance analysis and fundamental analysis.

Technical analysts are advocates of the idea that periodic cycles, amongst other things, together with trading volumes and moving averages can be used to predict future share prices (Bodie *et al.*, 2013). Technical analysts or chartists use methods such as Dow theory or trend analysis and various mathematical methods and series such as Fibonacci numbers. The majority of academics do not consider technical analysis to be based on science and robust research. Lo, Mamaysky and Wang (2000) did however identify some technical indicators that might have practical value.

Behavioural finance analysts believe that investors are not always rational, that they over react to new information and that psychological biases cause prices to deviate from their fair levels. A well-documented behaviour is the disposition effect, which refers to the tendency of investors to hold on to losing investments. This effect can lead to share price momentum regardless of fundamental values following a random walk (Bodie *et al.*, 2011). Many researchers, including Muller and Ward (2012), have found that momentum has historically been a successful quantitative method to beat the market.

It remains however difficult to quantify, measure and explain investor behaviour. In the early 1700s, one of the brightest minds in history lost a fortune investing in South Sea Bubble shares. Ex-Post Sir Isaac Newton made the confession: "I can calculate the motions of heavenly bodies, but not the madness of people."

The forecasting methods used by successful fund managers would therefore be helpful in establishing if periodic functions are of any use in this context. Fund managers need to forecast share returns as a profession and one would expect them do so using some form of analysis. The fact that they are a highly educated group of people also provides some credibility to the forecasting methods used by them. In the United States, 25% of graduating seniors at Harvard University, 24% at Yale and 46% at Princeton were starting their careers in financial services in 2006 (Shiller, 2013).

These professionals, who take account of the advice of thousands of market analysts, should be best equipped to achieve returns in excess of average market returns on a

risk-adjusted basis and thereby beating the market or achieving positive alpha on portfolios.

Fund managers however fail to beat the market consistently. Fama and French (2010) conducted a study of U.S. equity mutual fund managers from 1984 to 2006. It was found that actively managed U.S. equity mutual funds achieved returns close to the market before deduction of costs. After deduction of costs however, the funds performed worse than the market on average. Du Preez (2011) found the same for South African fund managers where more than 50% could not beat their respective benchmarks in recent times.

In the study by Fama and French (2010) there were a small number of funds whose managers beat the market after adjusting for costs and risks. There is however no way of telling if their results are the result of skill or luck. Also, the knowledge that a manager had skill *ex post* is of no use. It has to date been impossible to determine which managers will possess skill in the future (Ferri, 2012).

The failure by fund managers to beat the market implies that no simple known method exists to forecast share prices, assuming that all methods, including fundamental analysis, technical analysis, behavioural finance and the use of spectral analysis have been experimented with by them. The argument against forecasting share returns using historical data is as follows - if there were known methods of predicting share prices the knowledge of such predictive theory would quickly be arbitrated away. On the other hand there appears to be no superior method of managing funds and forecasting returns. Share trading practitioners using technical analysis, behavioural finance analysis methods and fundamental analysis perform equally bad at beating the market.

1.3 Research objectives

The objective of this research will be to establish the effectiveness of Fourier transforms to identify and predict share price cycles and to form market-beating portfolios in combination with momentum periodicity. The ability of Fourier transform analysis to identify and forecast market anomalies will be tested at a share price level. There is limited documented academic research investigating the use of Fourier transforms for this purpose as could best be established.

The methods will be tested using time series data from the Johannesburg Stock Exchange (JSE). The results will be compared with previous time series research by Muller and Ward (2012) who examined a range of investment styles on the JSE.

2 LITERATURE REVIEW

2.1 Business cycles

In macroeconomic terms, business cycles are defined as the fluctuation of the gross domestic product (GDP) growth rate above or below the long-term average. It is a recurring pattern and has always been part of economic activity to varying degrees (Bodie *et al.*, 2011). If there is a degree of regularity or periodicity in business cycles it implies that leading indicators, such as the stock market price index, must also have a degree of regularity.

Economists use a set of leading indicators to forecast what is likely to happen to the economy in 12 to 15 months. As mentioned above, one of the leading indicators is the stock market price index. Share prices are forward looking, pricing equity with future returns in mind. The stock price index is therefore synchronised with other leading indicators making other leading indicators less useful for forecasting share returns. When leading indicators predict an upturn in economic activity the stock market would have reacted already (Bodie *et al.*, 2011).

Auerbach (1982) found that in the United States, the index comprising of the 12 leading indicators is unstable over time. He also found that only about half of the leading indicators are significant in predicting cyclical variability of business activity. The theory supporting the use of the series of leading indicators were also found to be lacking. The S&P stock price index was however found to be a significant indicator.

A great deal of research has been done to establish the beginning and end dates of expansions and contractions (Colander, 2010). The research by Hamilton (1989) in this field has been cited in over 5700 academic articles and yet, the academic economics profession failed to predict the 2008 global financial crises. Fukuda (2009) examined the performance of US and Japanese professional forecasters at predicting growth cycle turning points. With all the available research, Japanese forecasters did no better than a simple time series model at forecasting turning points. US forecasters however outperformed the time series model. Colander *et al.* (2009) finds that this is due to a misallocation of research efforts in economics. In particular, the persistence of formulating models that neglect the key elements driving outcomes in the real world

markets such as investment in financial products rather than physical investments and the increased connectivity of the global financial system.

The length of these expansions and contractions can however be irregular with irregular timing and amplitudes that varies as suggested by Keynes who stated that recessions in employment would be more violent but last for a shorter time period than expansions (Bodie *et al.*, 2011). Westlund and Öhlén (1991) examined the symmetry of business cycles. They found that there is no significant empirical evidence suggesting asymmetry in the business cycles of Sweden, Canada, France, Italy, the United Kingdom and West Germany. This degree of symmetry might therefore also be present in leading indicators like the stock market price index. This would allow the use of spectral analysis and trigonometric functions, which are symmetrical, for share price movement modelling and forecasting purposes.

2.2 Market anomalies

In the seminal work by Fama (1965), the random walk theory hypothesised that share prices move randomly around its intrinsic value and recurrence of historical patterns has no significance. In contrast with Fama's random walk model is a series of cyclical and periodical market anomalies that suggests that there are definite and regular patterns in share prices. If the amplitude, persistence and periodicity of these anomalies are strong enough, they should be identifiable with spectral analysis tools such as Fourier transformations.

The EMH (Fama, 1970) states that prices in stock markets always "fully reflect" available information. "Available information" can refer to three different categories that result in three versions of the EMH. The three versions are the weak, semi strong and strong forms of the hypothesis. The weak form of the hypothesis upholds that all possible trading data including price history, trading volume and returns are already reflected in share prices. This implies that no form of price and chart analysis will be of any benefit in share price forecasting. The semi strong form asserts that on top of all trading data, all publically available information regarding a firm's financial statements, quality of management, patents, prospects and earnings forecasts is already reflected in its share price. The strong form of the hypothesis states that share prices reflect all weak form and semi strong information as well as information only available to company insiders (Bodie *et al.*, 2011).

Fama and French (1970) reviewed weak form testing of the EMH and tested if speculators could find trends or patterns in past share prices that would allow them to beat the market. The study concluded that any anomalies are in line with the “fair game” model and prices simply reflect expected returns. On the other hand, Conrad and Kaul (1988) and Lo and MacKinlay (1988) established that weekly returns of the NYSE are positively serially correlated over short horizons. The correlation coefficients for short-term returns were however small and did not clearly indicate any market beating opportunities.

Fama and French (1988) tested the New York Stock Exchange (NYSE) over the 1926 to 1985 period for long-term correlation. Large negative autocorrelations for return horizons beyond a year was found which indicated the effect of mean reversion. Mean reversion is a theory that suggests that prices and returns eventually move back towards the historical mean or average (Mean Reversion, n.d.). They estimated that the predictable variation is about 40% of three to five year return variances for portfolios of small firms and about 25% for portfolios of large firms.

2.2.1 Periodical market anomalies

If share price variations are not periodic, spectral analysis and forecasting of share prices using periodic functions is unlikely to yield any useful results. A number of academic studies related to market anomalies have however shown some periodicity in share prices. This strengthens the case for the use of frequency analysis and forecasting methods.

In seminal research by Rozeff and Kinney (1976), seasonal patterns were found in the NYSE using an equally weighted index from 1904 to 1974. The returns for January were higher than for any other month. This is referred to as the January effect - a general increase in share prices during the month of January. Thaler (1987) looked at the results of various research papers that examined this effect and found that the effect is more pronounced for small firms. Moller and Zilca (2008) recently tested returns on the NYSE, NASDAQ and AMEX and found that the effect is still present. Although the magnitude of the effect is unchanged from initial research by others, the effect is shorter in duration and only present in the first part of January. Auret and Cline (2011) could however not find any evidence of the January effect on the JSE.

Another seasonal pattern that has been observed is the tendency of share returns to underperform in summer compared to winter months. This was tested and confirmed by Bouman and Jacobson (2002) for 19 major stock exchanges, including the JSE, over the period 1970 - 1998. One explanation of this anomaly is that investors tend to sell their securities towards holidays or vacations for liquidity reasons.

Kamstra *et al.* (2003) found a relationship between length of day and share returns that is further proof of stock market periodic behaviour. It is however directly opposed to the research of Bouman and Jacobson (2002) who examined the role of seasonal affective disorder (SAD) in the seasonal variation of share returns. SAD refers to a medical condition where shorter days may lead to depression. Depression, in turn, has been shown to lead to heightened risk aversion that can lead to a selloff of shares and lower returns. SAD effects are more pronounced the further North the market is. Southern Hemisphere markets are six months out of phase with Northern hemisphere markets and are likely to be less affected by this effect being further away from the South Pole and with comparatively longer winter days.

The holiday effect is the tendency for stock markets to rise ahead of long weekends. It is one of the oldest and most consistent of all seasonal market anomalies. According to Brockman and Michayluk (1998) and Lakonishok and Smidt (1988) it is associated with approximately 30 to 50% of the total return on the market.

The turn of the month effect refers to the tendency of share prices to rise at the turn of a month and fall in the middle of the month. This effect is also present at the turn of the week and is referred to as the Monday effect or weekend effect where the average return from Monday close to the previous Friday close is negative (Keim & Stambaugh, 1984; Lakonishok & Smidt, 1988).

Baker and Wurgler (2007) found a degree of periodicity in investor sentiment and its effect on the NYSE using various investor sentiment indexes. They showed that it is possible to measure investor sentiment and that sentiment cycles have distinctive and regular effects on individual firms and on the stock market as a whole.

Seasonal, monthly, weekly and daily market anomalies are also known as calendar effects. As could best be established, little research exists for calendar effects of African stock exchanges. In one of the few studies in this field for Africa, Alagidede

(2008) found high and significant returns in days preceding a public holiday for South Africa as well as higher returns in February than any other month. As far as a day of the week effect is concerned, Mbululu and Chipeta (2012) found no day of the week effect for the JSE although this was done at index level and not at individual share price level. They did however find a Monday effect for the basic materials sector. Plimsoll, Saban, Spheris and Rajaratnam (2013) examined calendar effects for the JSE at share price level for stocks included in the top-40 index. They find significant day of the week effects on at least one day of the week for ten of the top-40 firms but fail to identify any effects at index level. Coutts and Sheikh (2002) examined the existence of the weekend, January and pre-holiday effects in the All Gold Index on the Johannesburg Stock Exchange over an 11-year period and failed to identify these anomalies.

It appears that at least for the JSE, calendar effects are more pronounced at firm level than at index level. This implies that there will be some value in analysing price frequencies at firm level and using these in forecasting.

2.2.2 Momentum periodicity

Testing of the weak form of the EMH in the intermediate-term revealed evidence of a momentum effect where recent performance of particular stocks and the market aggregate continues over time (Jegadeesh & Titman, 1993). Momentum is simply the difference between the current closing price of a share and the closing price N periods ago. Momentum is usually normalised using the rate of change (ROC) formula to calculate the relative strength of the momentum effect (Price Rate of Change – ROC, n.d.) as shown in Equation 1. Muller and Ward (2012) showed that momentum with a 12-month formation period and three month holding period is a pervasive phenomenon and a successful investment style on the JSE over the period 1986 to 2011. They also found evidence that suggested that momentum investment strategies might work better for holding periods of shorter than three months. Formation periods of between one and 18 months were scrutinised and a period of 12 months were found to be the optimum. This suggests the existence of a persistent 12-month period in share prices.

Equation 1: Price rate of change

$$ROC = \frac{CP - CPn}{CPn}$$

Where

CP = Closing price

CPn = Closing price n periods ago

The periodicity of momentum was documented by Figelman (2007). Persistent long-term yearly periodicity and intermediate-term quarterly periodicity for momentum effects were identified. These effects were found to be even stronger than the three known momentum effects, which are intermediate-term effects and short and medium term reversal effects.

The reversal effect refers to stock losers rebounding and winners fading back (Bodie *et al.*, 2011). Figelman (2007) found that share returns tended to exhibit a pattern every 12 months. In the intermediate term, share returns tended to exhibit a pattern every three months. It was found that there is a strong reversal effect in January months, even for the intermediate term, which exhibits momentum in most other months. All momentum effects, except for quarterly periodicity, were found to be greater in months of earnings announcements. Bildik and Gülay (2007) found the same momentum-January effect for the Istanbul Stock Exchange. This is further proof that price frequency analysis might be a fruitful approach to beat the market.

The highest returns identified by Figelman (2007) in the univariate analysis for stock in the S&P 500 were achieved by ranking portfolios by the return of that individual month's return, 12 months before ranking. Returns of just over 9% were achieved by rebalancing monthly. The holding period was also examined and it was found that a holding period of three months with rankings based on the cumulative return of the past 12 months yielded the highest returns of around 8%.

2.3 Spectral analysis

Spectral analysis or frequency domain analysis in statistics and signal processing is the calculation of the strength of the frequency components that are included in a time-domain signal (Hayes, 1996). Share price data forms such a time domain signal when recorded over time as a time series. The output of the various spectral analysis tools is used to identify any periodicities in the analysed data. When data is transformed from the time domain to the frequency domain, any periodicity will be indicated as peaks at the corresponding frequencies. The magnitude of the peaks indicates the amplitudes of the identified frequencies.

The tools used to convert data from the time domain to the frequency domain are called transforms. To reverse or inverse transformations and transform data from the frequency domain to the time domain, frequency, amplitude and phase shift of each frequency is required (Hayes, 1996).

In the study of time-periodic signals, engineers find it informative to examine the various spectra or frequencies of a waveform. If the input data is periodic with fundamental periods, the plot of the fundamental frequencies is called the frequency spectrum. The bandwidth refers to the number of fundamental frequencies that are present (Zill & Cullen, 2000). For a time series of daily closing share prices, the frequency can be no higher than 250 times per year for a year with 250 trading days.

2.3.1 Fourier transforms

The Fourier series, as described by Equation 2, is a trigonometric polynomial. One of the transformation tools used to move data from the time domain to the frequency domain is the Fourier transform which decomposes a function into a Fourier series, in other words the sum of a number of sine or cosine wave frequency components (Zill & Cullen, 2000).

Equation 2: Fourier series

The Fourier series of a function $f(x)$ defined on the interval $(-p, p)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{p} x + b_n \sin \frac{n\pi}{p} x$$

Where

$$a_0 = \frac{1}{p} \int_{-p}^p f(x) dx$$

$$a_n = \frac{1}{p} \int_{-p}^p f(x) \cos \frac{n\pi}{p} x dx$$

$$b_n = \frac{1}{p} \int_{-p}^p f(x) \sin \frac{n\pi}{p} x dx$$

There are generally three Fourier transform pairs. These are the Fourier transform, the Fourier sine transform and the Fourier cosine transform and their three inverse transforms. The Fourier cosine transform is shown in Equation 3 and the inverse Fourier cosine transform in Equation 4 (Zill & Cullen, 2000).

Equation 3: Fourier cosine transform

$$\mathcal{F}_c\{f(x)\} = \int_0^{\infty} f(x) \cos ax dx = F(\alpha)$$

Equation 4: Inverse Fourier cosine transform

$$\mathcal{F}_c^{-1}\{F(\alpha)\} = \frac{2}{\pi} \int_0^{\infty} F(\alpha) \cos ax d\alpha = f(x)$$

The Fourier transform will transform a function with infinitely wide bandwidth to an infinite number of sine or cosine wave frequency components (Mathews & Fink, 1999). As discussed above, a time series consisting of daily closing prices of a stock will not have bandwidth that is infinitely wide. The Discrete Fourier transform (DFT) as shown in Equation 5 is used to transform a finite list of equally spaced samples of a function to the frequency domain. The DFT input and output sequences are both finite and is therefore more appropriate as a share price analysis tool. For an input sample of n observations, the DFT can only accurately calculate the amplitudes of frequencies

smaller than $n/2$ over the sample period. Its highest possible frequency resolution is therefore half the number of input samples. Analysing a daily closing price time series sample with the DFT will result in the identification of frequencies of no higher than $n/2$ per sample period. This is equivalent to a period of no shorter than two days. The analysed frequencies are integers smaller than $n/2$ but larger than zero (Zill & Cullen, 2000). Frequencies with periods longer than the sample period will therefore not be identified with the DFT. For a given stock's daily closing price time series sample of 250 trading days the highest frequency for which the DFT will accurately calculate an amplitude is 125 times over the sample period. The DFT calculates amplitudes for positive integer frequencies. There will therefore be no frequency in the output that contains a fraction of a complete frequency. An example of a frequency that the DFT will not calculate an amplitude for is 21.5 times per sample period. It is however possible to identify periods that are not integers. A frequency of 20 times per 250 day period has a corresponding period of 12.5 days. The DFT is therefore an appropriate tool for time series spectral analysis of daily closing share prices where it is not intended to trade more frequently than once every two days.

Equation 5: Discrete Fourier transform

$$F(\alpha) = \sum_{n=-\infty}^{\infty} f(nT)e^{i\alpha nT}$$

Where

T = sampling rate or length of the sampling interval

The DFT will provide detailed amplitude and phase data for the frequencies in a time series sample. This data can be used to replicate the time series using a series of cosines as discussed above. Extrapolating these cosines past the end of the time series will however result in the time series frequency pattern being replicated continuously over the extrapolated period (Hayes, 1996). Where a time series that includes continuous growth over the sample period is concerned, the DFT is not appropriate on its own as a tool to extrapolate expected behaviour past the end of the sample period. This problem is discussed in section 4.5.2 and illustrated in Figure 2.

The fast Fourier transform (FFT) is an algorithm that is used to compute the DFT and its inverse that is significantly less computationally intensive than using the DFT

directly. The computation of a DFT with n samples requires n^2 computations. The fast Fourier transform employs matrix factorisation to reduce the number of computations to $(n)\ln(n)$. This method can however only be used if the number of samples that are being transformed are equal to a positive integer power of 2; that is 2, 4, 16, 32 and so forth (Zill & Cullen, 2000). This restriction makes the FFT less useful for share price periodicity analysis where the sample sizes or number of observations in a time series can be any integer number.

2.3.2 Spectral analysis of share prices

Benhmad (2013) found that spectral analysis techniques are crippled by the underlying assumption of stationarity. Stationarity refers to the joint probability distribution of a data series that does not change over time. Since share prices exhibit growth over time the probability distribution cannot be assumed to be stationary over time. Most economic and financial time series are non-stationary due to various factors such as a change in interest rates, inflationary effects and business cycles. According to Benhmad (2013), Fourier analysis is therefore not useful for analysing periodic and stationary signals whose moments change over time.

Benhmad (2013) used wavelet transforms, another spectral analysis technique, to test cross correlation and co-movement between various international stock exchanges. The method proved to be beneficial for determining successful portfolio diversification on an international level. The method was however only tested for stock exchange indices and it is unsure if this method will provide successful forecasting results for an individual share.

Wavelets were also used by Aguiar-Conraria and Soares (2011) for time series analysis and to test for cross correlation between business cycles in the Eurozone. Soares and Conraria (2011) found that although wavelet analysis is frequently used in signal and image processing, quantum mechanics, geophysics, medicine and biology it is still emerging in financial and economic studies.

Gabor (1946) introduced the initial concept of Short Time Fourier Transform (STFT) to overcome the limitations of the standard Fourier transform. The STFT uses an arbitrary but fixed-length window for analysis, over which the actual non-stationary signal is assumed to be approximately stationary (Benhmad, 2013). A Fourier transform is

performed for each window and the magnitudes and phases for each point in time and frequency is added to a matrix. The windows are usually selected to overlap to reduce discontinuities and spikes at the boundaries of windows. For share price forecasting the length of the window will however be critical. A short window can enhance time information at the expense of frequency information as opposed to a long window that will enhance the frequency resolution at the loss of non-stationary time information (Benhmad, 2013).

Wilcox and Gebbie (2008) conducted spectral analysis of the JSE from 1993 to 2002 by investigating the serial correlation, periodic, aperiodic and scaling behaviour of eigenmodes of correlation matrices. Although their sample suffers from survivorship bias they did identify strong periodic components including a weekly, fortnightly and a quarterly effect. This suggested long term memory effects on the JSE that might be exploited using the DFT as an analysis and forecasting tool.

Hong and Zhang (2012) developed an adaptive Fourier decomposition algorithm that was shown to predict bull and bear markets for the Shanghai composite index. The algorithm is however based on Dow theory that is not supported by underlying scientific theory or peer reviewed academic research. The algorithm uses the Dow primary trend as basis for its prediction and superimposes the cycles identified using Fourier transforms on this trend line. The Dow theory primary trend is simply a straight line between the troughs of a share price time series. Hong and Zhang (2012) did not compare their results with those obtained by other spectral analysis or momentum derived methods to prove that it is in fact an improvement.

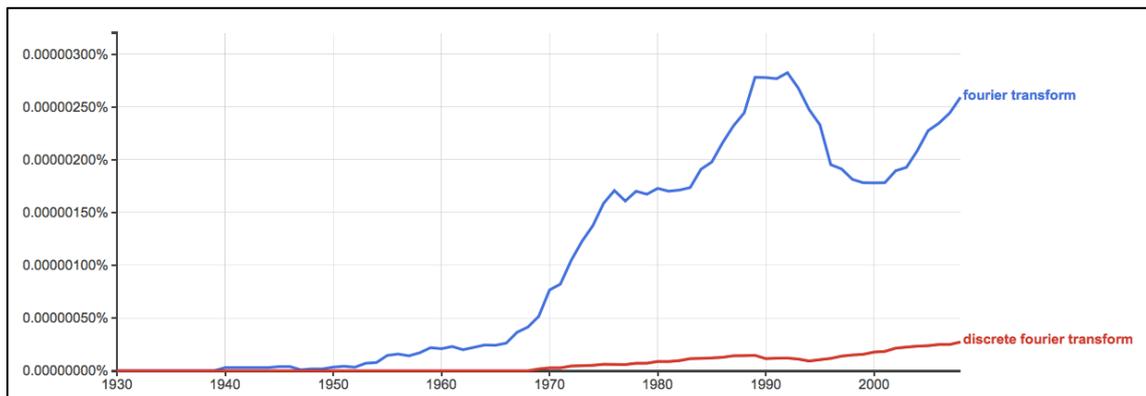
Chen, Chen, Fan and Chen (2012) combined a fuzzy time-series forecasting model with the FFT by using the Cooley–Tukey FFT algorithm. Cooley-Tukey is a divide and conquer algorithm that allows the FFT to be used with data samples that are not equal to 2^n with n any integer between 1 and infinity. The method improved the root mean square error between forecasted and realised values for the Taiwan stock exchange index from 1997 to 2003. This is yet another example of a spectral analysis forecasting technique that proved fruitful at index level but was not tested at individual share price level

The Fourier transform has been used to study share price volatility (Barucci, Magno & Mancino, 2012; Malliavin & Mancino, 2002). Barucci *et al.* found that the Fourier

estimator generally exhibits better performance compared with other realized volatility measures and performed well at filtering noise.

Use of the DFT on its own as a share price history analysis and forecasting tool appears to have received limited attention in academic research articles. Whether this is because failed results are not being published or an actual lack of research is unknown. Interest in both Fourier transforms and the DFT does however appear to be increasing. Figure 1 shows the occurrence of the terms “Fourier transform” and “discrete Fourier transform” in over 5.2 million books digitized by Google. The books include both academic and non-academic literature. There has been a sharp increase in the mentioning of “Fourier transform” since 2000 and mentioning of “discrete fourier transform” has been increasing steadily since 1995. Although interesting, this does not necessarily imply increased use of Fourier transforms in financial research.

Figure 1: Occurrence of the terms "fourier transform" and "discrete fourier transform" in Google books



(Google Ngram, 2013).

2.4 Investment styles

Investors typically group capital assets into funds with the aim of reducing volatility of the fund. When an investor groups equity together according to certain factors or characteristics he is said to make use of a certain investment style. The aim of following such a style is to exploit the cross sectional spread of returns in a bourse (Christopherson and Williams, 1997). If a spectral analysis method can be used to successfully group equity into winning and losing portfolios, it can be used as an input factor to an investment style.

Investment styles are traditionally based on different equity styles. According to Christopherson and Williams (1997) an equity style is an investment belief held by a group of managers who believe that following it will add value. For the style to be valid, it should result in the clustering of factor tilts or portfolio characteristics among those portfolios that share the style. Muller and Ward (2012) demonstrated that the styles they examined for the JSE successfully ranked shares into portfolios with realised returns corresponding with the rank of the portfolio. In most instances the returns for the top portfolio would be higher than the second, the second higher than the third and so forth. There are instances however where the shares ranked into the middle portfolios perform better than the top and bottom portfolios or where the bottom portfolio performs the best and *vice versa*. This was demonstrated by Muller and Ward (2012) for the Return on Equity investment style for instance. The shares ranked into the middle portfolio outperformed the other portfolios by a significant margin.

Equity styles can be grouped into various categories. The Frank Russell Company (in Christopherson & Williams, 1997) defines four major categories including value, growth, market orientated style and firm size. Morningstar Inc., an investment research firm in North America, Europe, Australia, and Asia does not consider market orientation to be a style. Their equity style box orders equity only according to firm size and value versus growth (Morningstar Style Box, n.d.). For the JSE, Muller and Ward (2012) categorises styles into financial ratio based styles, market based styles and behavioural finance based styles but exclude firm size as a style. They found no evidence of a size-based effect on the Johannesburg Stock Exchange.

Mutooni and Muller (2007) found that value stocks outperformed growth stocks across the size spectrum during the 20-year period, 1986 to 2006. They also found that the timing of using different styles was a potentially more profitable strategy than buying and holding a simple fixed style strategy. Fourier analysis methods might therefore be utilised to successfully time the inclusion and exclusion of equity of different styles in portfolios.

2.5 Literature review conclusion

It is apparent from the literature review that there exists a significant degree of periodicity in the stock market. The market anomalies presented in section 2.2 are all periodic and can mostly be attributed to behavioural finance effects. Successfully exploiting these anomalies using spectral analysis will therefore result in the identification of a new behavioural finance based investment style utilising quantitative analysis.

The literature is however conflicting as to whether or not this periodicity is persistent enough to have been used for time arbitrage. It is also unclear what the amplitude variation of these frequencies are and how they interact and interfere with each other. As could best be established, seasonal periodicity has been investigated more than non-seasonal periodicity. Non-seasonal periodicity refers to periods with lengths other than a week, month or year. These periods will for example not start at the same time of year every year.

Limited empirical evidence could be found demonstrating the efficacy of using the DFT for the forecasting of returns of individual shares and using the results to construct a market beating portfolio. Many studies that investigated the use of Fourier transforms for forecasting purposes focussed on the movement and periodicities of indices and industries as a whole rather than looking at individual shares.

One can reasonably expect share prices to grow at an exponential rate in the long run. Any forecasting method should therefore take this behaviour into account. Literature shows that using the DFT on its own to analyse share price frequencies and extrapolating the identified cycles past the end of the sample period will result in the replication of the share price pattern without growth. Forecasting actual share prices using the DFT will therefore require superimposing the extrapolated cycles on some form of expected growth.

3 RESEARCH QUESTIONS AND HYPOTHESES

The three research questions explore three methods of using the DFT as a tool to construct share portfolios. These questions are specific to the JSE and refer to daily closing prices of shares. It will investigate share price periodicity at a stock level and not at industry or index level.

Successfully ranking shares into a market beating portfolio will mean that the method constitute an investment style. A good investment style will result in a large cross section of returns between the best and worst portfolio with the best portfolio achieving returns above the stock market average and the worst portfolio achieving returns below market average.

3.1 Question one

The method examined in question one combines the DFT with different measures of prior exponential growth to forecast actual share prices. The forecast is calculated by superimposing the extrapolated prior growth on the extrapolated IDFT of the sample data.

Has there been sufficient periodicity in daily share prices to use share price movement frequencies, combined with prior exponential growth, as a market beating investment style?

3.2 Question two

Question two investigates if the DFT can be used to calculate seasonal price momentum periodicity measured over a 12 to nine month prior period to construct a market beating portfolio.

Can Fourier transform frequency analysis of share price movement in the 12 to nine month period prior to portfolio construction be used as a market beating price momentum investment style?

3.2.1 Hypothesis one

Question two lends itself to statistical comparison between the examined method and a method using prior momentum without frequency analysis. This will investigate if the frequency analysis added any additional returns to the prior momentum ranking method.

Hypothesis: The null hypothesis states that the difference between the monthly returns of a portfolio ranked according to 12 to nine month prior momentum and a portfolio ranked according to the DFT to calculate ROC in the same period is equal to zero.

$H_{1,0}$: $\mu_{d,1} = 0$ The population median of the difference d is zero
 $H_{1,A}$: $\mu_{d,1} \neq 0$ The population median of the difference is not zero

3.3 Question three

Question three investigates if the DFT can be used to calculate seasonal price momentum periodicity measured over a 12 month prior period to construct a market beating portfolio.

Can Fourier transform frequency analysis of share price movement in the 12 month period prior to portfolio construction be used as a market beating price momentum investment style?

3.3.1 Hypothesis two

The results obtained in investigating question three lend itself to statistical comparison with the results obtained in investigating question two.

Hypothesis: The null hypothesis states that the difference between the monthly returns of a portfolio ranked according to 12 month prior momentum and a portfolio ranked according to the DFT to calculate ROC in the same period is equal to zero.

$H_{2,0}$: $\mu_{d,2} = 0$ The population median of the difference d is zero
 $H_{2,A}$: $\mu_{d,2} \neq 0$ The population median of the difference is not zero

4 RESEARCH METHODOLOGY

The methodology followed in this research was based on that of Muller and Ward (2012) who examined style based effects on the JSE over the period 1986 to 2011. The same database and time series timeframe used in their study was used for this study to ensure that results could be directly compared with previously studied investment styles for the JSE.

4.1 Research design

Quantitative deductive research was conducted using a quasi-experimental time series design (Saunders & Lewis 2012). Twenty-eight years of share price data was analysed by ranking shares into portfolios using Fourier transform analysis based forecasting and portfolio ranking. The data was presented and analysed using a graphical time series approach to reveal any differences in portfolio returns as was done by Muller and Ward (2012). Hypothesis testing was done quantitatively to determine statistically significant differences between quarterly portfolio returns of portfolios constructed using the methods described in section 4.5.3 and 4.5.4.

The use of the quasi-experimental time series research design is widespread in studies comparing the performance of investment portfolios (Schoeman, 2012). The traditional approach to test the returns of portfolios in this type of study has been to report average monthly or quarterly portfolio returns and to use t-tests to test for significant differences in the results (Muller & Ward, 2012). This method however assumes a unimodal normal distribution that cannot necessarily be assumed. A Wilcoxon signed-rank test was therefore used for testing of the stated hypothesis. This method can produce dependable results regardless of the underlying distribution.

The use of average monthly or quarterly returns for statistical analysis is however viewed as methodologically weak compared to cumulative returns presented and compared graphically. This is comparable to average abnormal returns revealing relatively little compared to cumulative abnormal returns in event studies (Muller & Ward, 2012).

The average monthly or quarterly returns approach can be compared in some way to the real world example of style drift. One style might outperform another in the short

term but switching styles might not necessarily be cost effective and profitable. Where performance-chasing fund managers changes portfolio styles based on short-term average results, the results are often counterproductive (Brown, Harlow & Zhang, 2009; Cumming, Fleming & Schwienbacher, 2009).

4.2 Unit of analysis

For the graphical time series, the unit of analysis was cumulative portfolio returns. The relative size of the differences between portfolios was compared as well as the persistence of the effects. For the hypothesis tests the unit of analysis was monthly returns for each portfolio calculated as ROC as shown in Equation 1. Using a normalised unit for the hypothesis test ensures comparability.

4.3 Population and sampling

Every stock exchange is unique. Dutt and Humphery-Jenner (2013) quotes a large number of research papers documenting the differences in securities laws and regulations between stock exchanges and how these differences influence the way in which traders behave. This difference in behaviour influences the efficiency and liquidity of financial markets. Research conducted for one stock exchange will therefore not necessarily be applicable to other stock exchanges. From the literature review it is apparent that stock market periodicities can be attributed to investor behaviour. The literature review also showed that not all periodic effects are present in all markets. Therefore, if periodicities are in fact behavioural effects, they are likely to differ between stock exchanges. The population was therefore chosen to include only the shares of companies listed on the JSE mainboard. This prevented the dilution of effects that might be present within this population with the effects present in other stock exchanges.

The sample was the share prices of the 160 largest companies listed on the JSE mainboard from January 1986 to September 2013. The timeframe of the time series was chosen to coincide with the timeframe used by Muller and Ward (2012) to enable the comparison of results.

The top 160 shares represent 99% of the market capitalisation of the JSE. Shares falling outside this sample are considered too small and too illiquid to be of interest to institutional investors (Muller & Ward, 2012). The selected sample ensured that the research remained consistent with other rigorous research conducted for the JSE in this field (Muller & Ward, 2012 and Ward & Muller, 2012). Figelman (2007) used the same approach in similar time series research that used only the S&P 500. This sample represented between 55% and 75% of the NYSE. Shares of larger companies are more liquid with quoted prices closer to their executable prices. This is especially important for portfolios of any significant size. It is therefore important to use large-cap shares in studies like these since the simulated investment strategies involve portfolios of equally weighted shares. The results would otherwise be dominated with illiquid small-cap and micro-cap stocks (Figelman, 2007). Furthermore, Mutooni and Muller (2007) found that limited research has been done on non-main-board securities and that they may possess a large amount of non-systematic risk.

4.4 Dataset preparation

Access to an exclusive JSE price database was provided by Mr C. Muller (personal communication, May 24, 2013). The database is not publically available. This is the same database used by Muller and Ward (2012) and allowed for direct and accurate comparison of the investment styles examined by them with the styles examined in this report.

Data in the database was obtained from Sharenet and dividend pay-out data from INET using the dividend pay-out historical time series. The dataset was checked for data errors and where daily price movements exceed 40% the price movement was treated as zero. The database have been rigorously tested by Muller and Ward (2012) and has been proven to be sound and without discrepancies.

All shares listed on the JSE mainboard from January 1986 to September 2013 were included in the dataset used for this research. These included the shares of delisted and newly listed companies to control for survivorship bias. Portfolios constructed exclusively of companies that survive till the end of the time series will have distorted returns (Deaves, 2004). If a time series suffers from survivorship bias, it would be wrong to say that returns are only attributable to the tested variables. Survivorship is an additional variable in such instances.

Share splits or consolidations that caused changes in share prices were backwards adjusted in the dataset. Where a company in the sample unbundled a subsidiary, the returns of the newly listed company are included with those of the original holding company until the end of the holding period. The holding company and unbundled company were treated as separate companies for review and rebalancing at the end of the holding period. Delisting and listing of companies were treated in the same manner with newly listed shares included and delisted shares dropped at the start of the next quarter. The price used for calculating the returns of a delisted share is taken as the price on its last day of trading. Name changes were tracked and followed through the sample.

Dividends and scrip dividends were included in share returns. Share buy-backs are however not included as they only affect those shareholders leaving the company. Shares given to managers as compensation were ignored.

4.5 Data processing

A proprietary “style engine”, coded in VBA within Microsoft Excel was used to process and manipulate the dataset, which resides in a Microsoft Access database. This is the same “style engine” that was used in the studies by Muller and Ward (2012). The system enabled simple selection of start date, end date, number of portfolios required, the review period and number of months of history to be processed for each forecast.

Transaction costs related to rebalancing of portfolios were ignored on the grounds that it would have been immaterial and approximately the same between portfolios. Mutooni and Muller (2007) however acknowledged that neglecting transaction cost is a limitation. Mackintosh (2011) also points this out for the research conducted by Baker, Bradley & Wurgler (2011), stating that monthly transaction cost would be too high to repeat their return results. The exclusion of transaction cost is however necessary to compare the results of this study with the research done by Muller and Ward (2012).

4.5.1 Portfolio construction procedure

In brief, the procedure involved constructing equally weighted equity portfolios at a specific date in history. This date was the start of each holding period. The fixed

holding period was chosen as three months for this study to remain comparable with the research of Muller and Ward (2012). The use of a three-month holding period was also motivated by the research of Figelman (2007) who found the highest returns are earned when holding a portfolio for three months. The returns demonstrated by Figelman (2007) were however achieved using 12 months prior returns to rank portfolios. The methods used for this research project do not use 12 months prior price data in every instance.

The portfolios were constructed based on a ranking calculated using a price data sample over a fixed length historical window period. This sample period would be the period leading up to the start of each holding period. After holding the portfolios for three months, the returns would be calculated and the shares within portfolios would be adjusted according to a new ranking based on a revised historical data sample. The revised historical data sample would comprise of the fixed length historical period leading up to start of the new holding period. This process was repeated for data from 1 January 1986, to the most current date at the time of the study. The first date in the chosen dataset had to be the start of the first holding period minus the fixed historical sample period length used for the ranking of shares and construction of the first set of portfolios. The first portfolio was therefore constructed at the end of December 1986 and the dataset started on 1 January 1986.

The top 160 JSE main board companies were ranked according to three separate Fourier ranking methods, corresponding with the three research questions of section 3. Shares with the highest rank would be predicted to have the highest returns at the end of the holding period. Five equal weighted virtual portfolios were constructed according to their calculated rank. Each portfolio contained 32 shares

The return for each of the 32 shares in the respective portfolios was calculated on a daily basis. Dividends were included in the return calculation. The value of each portfolio was then calculated from a base of 1.0. On the last day of each holding period the value of each portfolio was retained. As mentioned above, the ranking and portfolio construction process was then repeated for each holding period in the database using a revised sample of top 160 companies every time.

As mentioned above, the rank for each share was calculated using three different methods of applying Fourier analysis. The first ranking method used growth correction and Fourier transforms for each of the 160 shares. The second and third ranking

methods combined yearly momentum periodicity (Figelman, 2007) with Fourier transforms for each of the 160 shares.

4.5.2 Ranking method one

Portfolio ranking method one examined the question posed in research question one. This method used an index calculated as the predicted share price at the end of the holding period divided by the actual price at the beginning of the holding period to rank shares into portfolios.

At the start of every holding period, the price data for each of the 160 shares in the history sample was flattened by dividing by an exponential growth factor and then transformed to the frequency domain using a DFT. This improved on the method used by Hong and Zhang (2012) that assumed linear instead of exponential growth. The transformed data consisted of a set of frequencies with corresponding phases and amplitudes. The transformed data was then transformed back to the time domain, using a limited set of frequencies, extrapolated to the end of the holding period and multiplying each price in the time series by the relevant exponential growth factor. Transforming the frequency domain data back to the time domain is also known as an inverse-DFT or IDFT.

By not flattening the data, or removing growth, the extrapolation of the IDFT would result in direct replication of historical data at incorrect price levels that excluded growth. This resulted in a step change in price on the first day of extrapolation and would have resulted in an unusable price prediction as shown with the red line in Figure 2.

To flatten the data perfectly for each share and extrapolate a forecast without abrupt price changes, the price on the first and last day of the analysed historical sample period had to be equal. To achieve this, the price for each day in the history period was divided by an exponential growth factor based on the compounded average daily growth rate (CADGR) as shown in Equation 6. This method flattened the price data to a level in line with the starting price of the history sample as shown in green in Figure 2.

Equation 6: Data flattening factor

$$\text{Flattening factor } n = \text{CADGR}^n$$

Where

$$n = \text{position of price in sample from 0 to } N$$

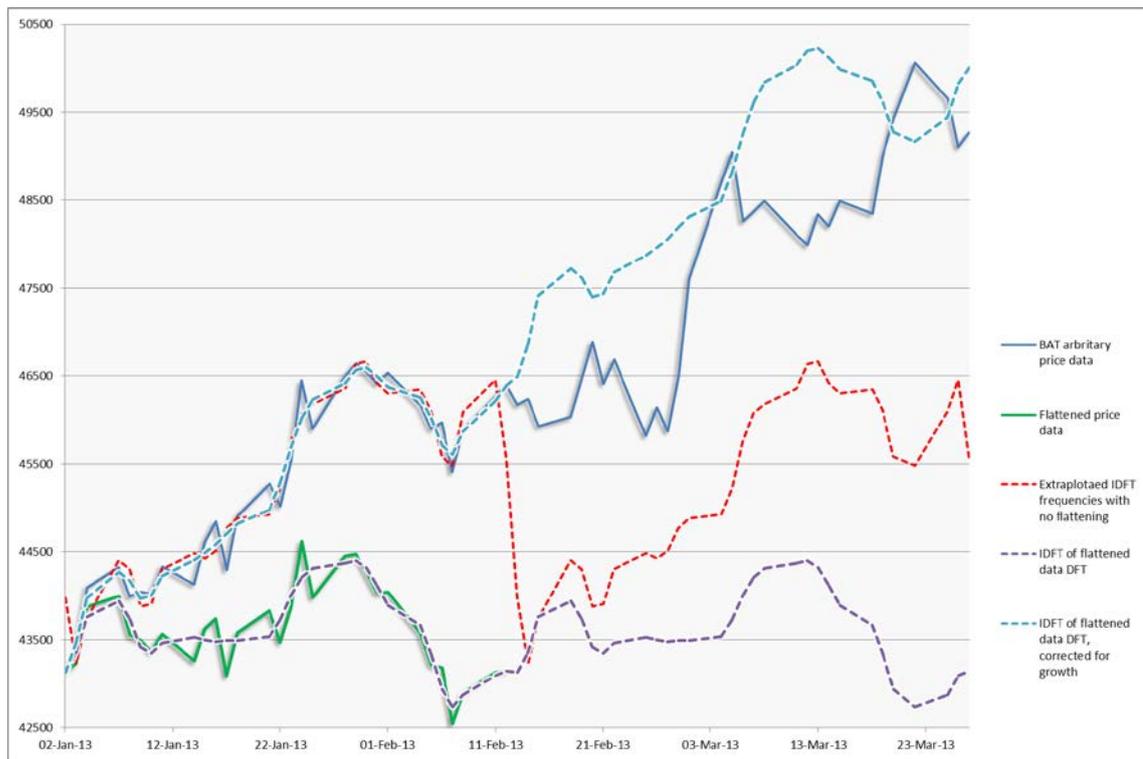
$$\text{CADGR} = \sqrt[N]{(\text{Growth over sample period} + 1)}$$

$$\text{Growth over sample period} = \frac{\text{Last price of history period}}{\text{First price of history period}} - 1$$

$$N = \text{number of daily prices in history period}$$

The IDFT of the flattened data was then extrapolated to the end of the holding period as shown in Figure 2. The extrapolated IDFT was corrected for growth as described above by multiplying each extrapolated price with the flattening factor of Equation 6 but with n starting at $N+1$ and ending the number of trading days in the holding period. The extrapolated IDFT corrected for growth is shown in Figure 2 in light blue. This method assumes that prior exponential growth will continue in future. It is therefore based on price momentum.

Figure 2: IDFT forecast using flattened and un-flattened price data



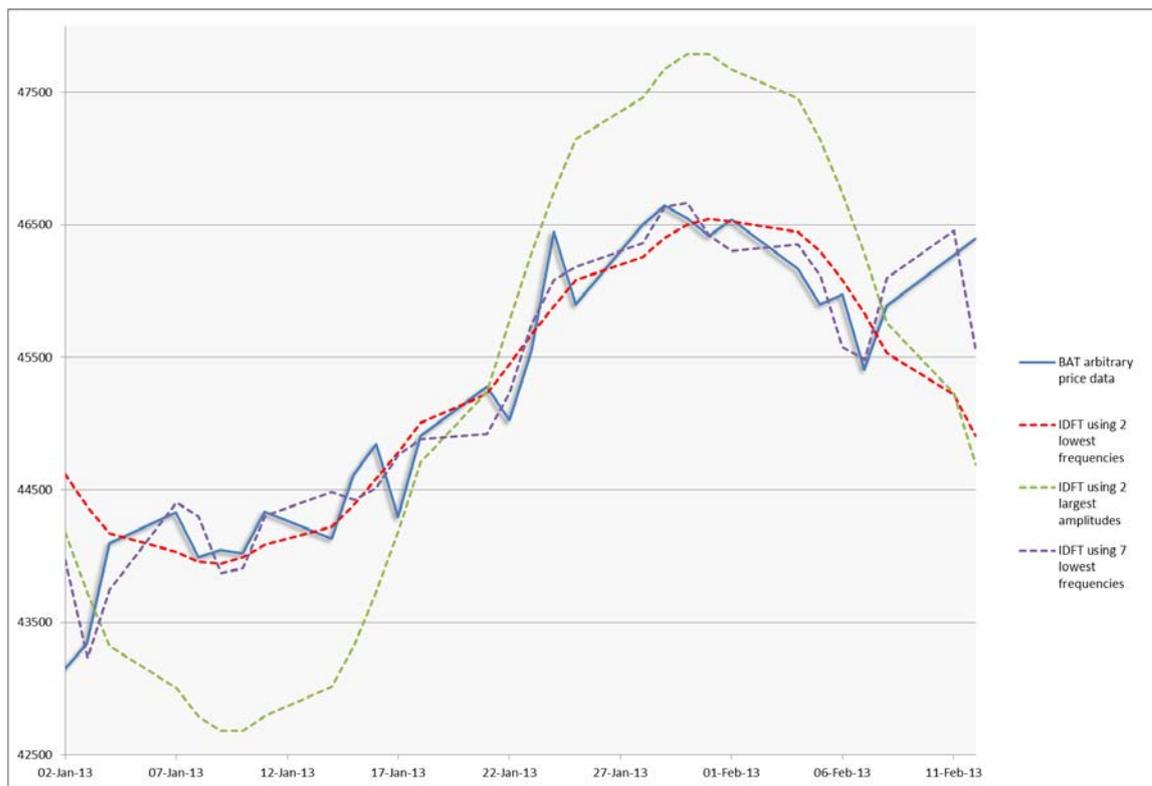
Since the flattening factor was used to add growth to the extrapolated prices, its magnitude was critical for prediction accuracies. In an attempt to improve the prediction accuracy, growth over the sample period was calculated using two methods. The first growth correction method used only the first and last day of the analysed historical period to calculate growth as mentioned above.

The second growth correction or flattening method used the average share price at the beginning and end of the analysed historical period. It is exactly the same as the first method in all other respects. This method does result in a small step change in price on the first day of extrapolated prices. This step is however immaterial since only the extrapolated price on the last day of the holding period is of interest. The number of prices used to calculate the average was kept to a fixed percentage of 2% of the length of the sample period. This method filtered out price spikes or outliers that happened to be at the start and end of the sample period. These outliers could otherwise result in unrealistic growth rates.

4.5.2.1 Ranking Method One parameter optimisation

The combination of frequencies used to extrapolate price data was seen as critical to get accurate share price predictions. By using only lower frequencies, high frequency noise could be filtered out. By using only the frequencies with large amplitudes, major periodical trends could be identified. By using an increasing number of frequencies the IDFT would get closer to replicating the historical behaviour exactly. This is shown in Figure 3 for an arbitrary 30-day price sample period of British American Tobacco. In this instance, using the two lowest frequencies did not yield a good estimation of the actual price. It is clear that including seven frequencies instead of two results in a better estimation of the actual price. A perfect estimate of the price would however include all the noise present during the sample period and extrapolation of noise was considered to be a poor strategy for price prediction.

Figure 3: IDFT for British American Tobacco for arbitrary 30 day sample period



The optimal combination of price flattening rate and set of frequencies used for the extrapolation of the IDFT could not be tested with the “style engine” due to computational intensiveness and time available for this research. The optimisation of these parameters was therefore examined by testing its accuracy of predicting share prices.

A DFT for the share price data was calculated using a history sample of 12, 18, 24, 30, 36, 42, 48, 60, 72, 84, 96, 108 and 120 months after flattening the sample using the two methods discussed above. The IDFT was then calculated using either the amplitudes of all the relevant frequencies, only the lowest 30 frequencies, only the lowest three and only the three frequencies with the highest amplitudes. A summary of the parameter combinations that were tested is shown in Table 1. Parameter combination number six excluded any IDFT extrapolated data and tested the prediction accuracy of the exponential growth rate over the preceding period only.

Both growth rate calculation methods were tested using all given DFT frequencies as the first two tests. Subsequent to these tests only the method with the highest share price prediction accuracy, using average beginning and ending prices, were used in further parameter combinations

This pre-test trial comprised of over 4100 tests per analysed share price. Due to computational intensity and the available timeframe for this research the share prices of only two companies were analysed. These were Redefine Properties and Standard Bank Group. These two companies were selected based on the fact that they are in different industries and should be subject to different market cycles.

Table 1: Parameter combinations tested for ranking method one prior to portfolio ranking testing in the “style engine”

Parameter Combination #	Number of frequencies used for IDFT extrapolation	Growth rate calculation method
1	All	Using spot price at period beginning and end
2	All	Using average price at period beginning and end
3	30 lowest	Using average price at period beginning and end
4	3 frequencies with largest amplitudes	Using average price at period beginning and end
5	3 lowest frequencies	Using average price at period beginning and end
6	None	Using average price at period beginning and end

A normalised prediction error was calculated as the difference between the predicted and actual price, divided by the actual price. The error was expressed as a percentage. This process was repeated every three months from Aug 2000 to May 2013. This constituted 53 tests for every combination of parameters. The average absolute size of the prediction errors was calculated for each combination of parameters by calculating the root mean square (RMS) of the errors as shown in Equation 7. The same approach

was used in study by Chen *et al.* (2012) to evaluate forecasting errors. By using the RMS equation the size of the deviation was centred around zero rather than around the mean, as would be the case if the standard deviation was used

Equation 7: Root mean square of errors

$$\sqrt{\frac{1}{n}(error_1^2 + error_2^2 + \dots + error_n^2)}$$

Where

n = number of errors

The combination of parameters yielding the lowest average absolute error was used in the “style engine” to calculate the expected price of each share at the end of each holding period. As mentioned, the Fourier prediction index was calculated using the share price that would be forecast at the end of the holding period divided by the price at the start of the holding period.

4.5.3 Ranking method two

Portfolio ranking method two examined the question posed in research question two. This method was based on the research by Figelman (2007), who found that returns for a period were similar to the returns for that same specific period 12 months before.

The highest returns that Figelman (2007) demonstrated to be possible were achieved with a holding period of one month and ranking based on the ROC of a share in the same month, 12 months prior to ranking. As mentioned, the holding period chosen for this research was three months to allow direct comparison with the research results of Muller and Ward (2012). The ranking method used by Figelman (2007) was therefore adjusted to use three months of data between 12 months and 9 months prior to the holding period. An example would be the returns and price periodicity for Sasol Ltd for the period January to March 2012 being used to forecast the returns of Sasol shares for the period January to March 2013.

Instead of calculating the growth as ROC in the normal manner like Figelman (2007) did, it was calculated using an IDFT and only the amplitudes of the lower frequencies

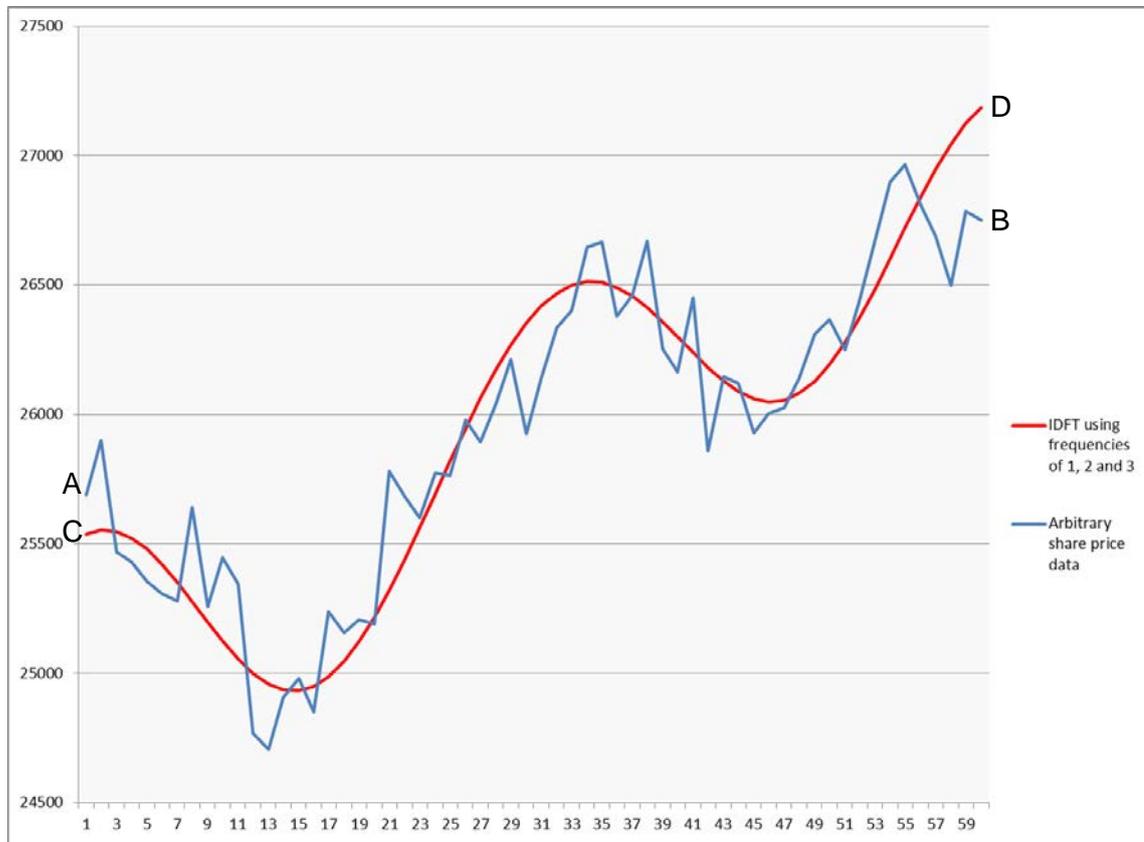
that the DFT identified for the sample period 12 months prior the prediction period. This method filtered out higher frequency noise. In contrast to method one it does not base rankings on predicted share prices but rather on prior returns performance and periodicity.

The sample period was transformed to the frequency domain using the DFT without removing growth first. Since the frequency domain data was not used to extrapolate time domain data for the holding period the subtraction of growth was not necessary. The IDFT for this sample was calculated next using only the three lowest frequencies of the DFT. The three lowest frequencies are once, twice and three times per sample period. For a three-month sample period these frequencies are equal to periods of one month, 1.5 months and three months per sample period. An example is shown in Figure 4 for an arbitrary three-month share price sample period. Choosing only the three lowest frequencies resulted in filtering of a sufficient amount of higher frequency noise whilst still matching the major periodicity of the period.

Figelman's (2007) method for a three month holding period and three month sample period would use the share prices at points A and B in Figure 4 to calculate ROC as the ranking method. It is possible that the prices on these days could have been outliers that spiked either upward or downwards. The price spikes could have resulted in an over optimistic or over pessimistic ranking when the ROC was calculated. Using a DFT-IDFT pair to estimate the low frequency periodicity of this period eliminates price spikes in the ROC calculation for ranking.

The prices used for ranking shares were taken as the first and last prices estimated with the IDFT. An example of these prices is shown in Figure 4 as points C and D.

Figure 4: IDFT fitted on arbitrary share price data using frequencies of 1, 2 and 3 / sample period



Returns for a comparative set of portfolios was constructed using the modified version of Figelman’s (2007) ranking method described above but without DFT noise filtering. This ranking method used the share closing price 12 months and 9 months prior to the start of the holding period. An example of the prices used by this momentum periodicity ranking method is shown in Figure 4 as points A and B. The returns achieved by this set of portfolios were compared to the portfolios ranked using ranking method two.

4.5.4 Ranking method three

It was not originally intended to test a third ranking method due to the computational intensiveness of testing Fourier derived ranking methods with the “style engine”. A third ranking method was however tested but only for the top portfolio was simulated. The third ranking method examined the question posed in research question three.

Ranking method three was essentially the same as ranking method two with only the sample period being different. The sample period was chosen to be the 12 months prior to the beginning of the holding period. This was based on the research by Figelman (2007) and Muller and Ward (2012). Both studies found that ranking shares according to 12 month prior returns and a holding period of three months yielded the highest returns of momentum investment styles.

The portfolio ranked using ranking method three was compared to the top portfolio ranked using ranking method two.

4.6 Data analysis

Data was analysed using a graphical time series approach to reveal any differences in portfolio returns. The cumulative index of cumulative returns of each portfolio was plotted over the timeframe of the research and the results were compared visually. Cumulative returns (CR) were calculated using Equation 8.

Equation 8: Cumulative returns for portfolios

$$CR_{T_2} = \prod_{i=T_1}^{T_2} (1 + R_{T_1}) - 1$$

Where

CR_{T₂} is the cumulative returns at time T₂

R_{T₁} is the actual portfolio returns at time T₁

Further to the cumulative returns of each portfolio that was plotted a price-relative between the highest and lowest ranked portfolios was plotted. The price relative was calculated as the value of the highest ranked portfolio divided by the value of the lowest ranked portfolio on each day. This compares the difference between the highest and lowest ranked portfolios and represents the spread of returns that were possible using this ranking methodology. An upward slope of the price relative represented the periods where the highest ranked portfolio outperformed the lowest ranked portfolio and vice-versa. Where the slope of the price relative was zero the performance of the highest and lowest ranked portfolios were equal.

The Compounded Annual Growth Rate (CAGR) was calculated for each portfolio at the end of the research timeframe using Equation 9. This value was shown on the time series graphs as a convenient way of comparing the returns of the portfolios over the full timeframe using a single value.

Equation 9: Compounded annual growth rate

$$CAGR = \left(\frac{P_{iT_f}}{P_{iT_0}} \right)^{\left(\frac{1}{\text{number of years}} \right)} - 1$$

Where

P_{iT_f} is the value of portfolio i at the end of the timeframe T_f

P_{iT₀} is the value of portfolio i at the beginning of the timeframe T₀

The market capitalisation weighted ALSI total return index (J203T) was also plotted as a benchmark. A price-relative between the highest ranked portfolio and the J203T was calculated and plotted in the same manner as the highest-lowest portfolio price-relative.

4.7 Hypothesis testing

Financial time series data suffers from kurtosis, clustering of volatilities and non-normal distributions which requires the use of nonparametric test statistics (Berry, Gallinger & Henderson, 1990; Ledoit & Wolf, 2008). For this reason the Wilcoxon signed rank test was used for hypothesis testing. This test assumes no specific population shape except approximate symmetry and it assumes that data is continuous (Weiers, 2011).

The Wilcoxon signed rank test was used to compare paired samples of monthly portfolio returns of the two sets of portfolios ranked according to the methods described in section 4.5.3. The Wilcoxon signed rank test for paired samples can be used for dependent samples. The hypothesis test did not imply that one method is superior to the other so a two-tail test was performed (Weiers, 2011).

Hypothesis testing was performed at the 5% significance level. This corresponds to similar tests in literature. The null hypothesis was rejected when the p-value was less than 5%.

4.8 Research limitations

The growth rate at which data is flattened in the first ranking method has a significant influence on forecast accuracy. The investigation conducted to establish the optimal combination of parameters to use is also not necessarily statistically significant.

The results and findings are limited to the top 160 shares by market capitalisation on the JSE for the studied period. The results will not necessarily hold true for shares traded before or after the studied period or for shares of market capitalisation lower than the top 160. Using the results as a forecasting strategy for future performance will ignore future external effects and behavioural changes of investors. External factors can include a number of items such as changes to trading regulations.

The omission of transaction costs and taxes means that the results would not be a true reflection of the returns that would have been earned in practice. The rebalancing of portfolios only every quarter would however result in lower costs than would be required for monthly rebalancing as examined in other time series studies such as the method proposed by Figelman (2007). All 32 shares in a portfolio would also not necessarily be traded every quarter.

The research did not take account of the gap between bid and ask prices of shares. By only using the top 160 shares of the JSE it is assumed that shares are more liquid with quoted prices closer to their executable prices.

The ranking methodology might have worked better in some industries than others. The research methodology did however not allow the investigation of the degree of industry specific seasonality or periodicity. Such an investigation was considered an unnecessary complication to an otherwise simple investment style.

5 RESULTS

The results are presented in the order of the research questions and hypotheses. This is also the sequence in which research was conducted. The ranking of shares according to ranking method one required optimisation of the method's parameters as discussed in section 4.5.2.1 and are presented first. This is followed by the results obtained by using portfolio ranking method one. The results obtained by using ranking method two is presented third and is compared with the results obtained by using the Figelman (2007) derived method discussed in section 4.5.3. The results are compared visually and statistically. Lastly the results obtained by using ranking method three are presented. It is compared with the top performing results of method two both visually and statistically.

The graphs of the results are shown alongside the ALSI total return index (J203T). The returns of the top portfolio relative to the lowest ranked portfolio and relative to the J203T are also shown. The CAGR for each series is shown on the right hand side of each graph. Each portfolio starts at the beginning of the examined period with a value of one.

The non-parametric Wilcoxon signed-rank test was used to perform two tailed statistical testing. Testing was performed on portfolio pairs as per the stated hypotheses. The tested portfolio pairs were the highest ranked portfolios ranked according to the methods being compared as stated in the hypotheses. The z-statistic for each pair was calculated and statistical significance at the 5% level was evaluated using the associated p-values.

5.1 Ranking parameter optimisation results

The parameters presented in section 4.5.2.1 were tested for their effect on share price prediction accuracy for the share prices of Redefine Properties and Standard Bank Group. The RMS of the prediction errors over the period Aug 2000 to May 2013 were calculated for every test and is presented in Table 2 and Table 3. Each RMS value in the table was calculated using 53 forecasting errors over the timeframe. The parameter combination number in Table 2 and Table 3 refers to the parameter combinations presented in Table 1.

Table 2: RMS of prediction error percentages for Redefine Properties Limited

		Number of months history used in sample period												
		12	18	24	30	36	42	48	60	72	84	96	108	120
Parameter combination #	1	15%	16%	18%	17%	18%	17%	18%	18%	16%	15%	15%	15%	16%
	2	15%	17%	18%	15%	17%	15%	16%	16%	14%	14%	13%	11%	14%
	3	15%	17%	18%	15%	17%	16%	16%	16%	15%	14%	13%	12%	14%
	4	15%	16%	18%	15%	17%	16%	15%	14%	15%	14%	13%	13%	13%
	5	16%	17%	18%	15%	17%	15%	15%	13%	13%	13%	12%	13%	14%
	6	12%	12%	13%	12%	13%	12%	12%	12%	12%	13%	11%	12%	13%

Table 3: RMS of prediction error percentages for Standard Bank Group Limited

		Number of months history used in sample period												
		12	18	24	30	36	42	48	60	72	84	96	108	120
Parameter combination #	1	15%	16%	18%	17%	18%	17%	18%	18%	16%	15%	15%	15%	16%
	2	15%	17%	18%	15%	17%	15%	16%	16%	14%	14%	13%	11%	14%
	3	15%	17%	18%	15%	17%	16%	16%	16%	15%	14%	13%	12%	14%
	4	15%	16%	18%	15%	17%	16%	15%	14%	15%	14%	13%	13%	13%
	5	16%	17%	18%	15%	17%	15%	15%	13%	13%	13%	12%	13%	14%
	6	12%	13%	13%	12%	13%	12%	12%	12%	12%	13%	11%	12%	14%

Parameter combination six consistently gives the lowest prediction error RMS value for both tested stocks. This is true for using all the tested number of months of history and parameter combinations except when using 108 months (nine years) history and parameter combination two. The exact difference between parameter combination two and six at 108 months is however less than 0.6%. A sample period of 108 months also does not necessarily yield the lowest prediction errors for the other parameter combinations. It was therefore assumed that the lower error achieved at 108 months and parameter combination two, relative to parameter combination six, was not material.

Parameter combination six uses no frequency information to predict share prices. It is purely based on the exponential growth rate of the prior period. It appears that adding frequency information to the expected growth rate introduces randomness that reduces the prediction accuracy.

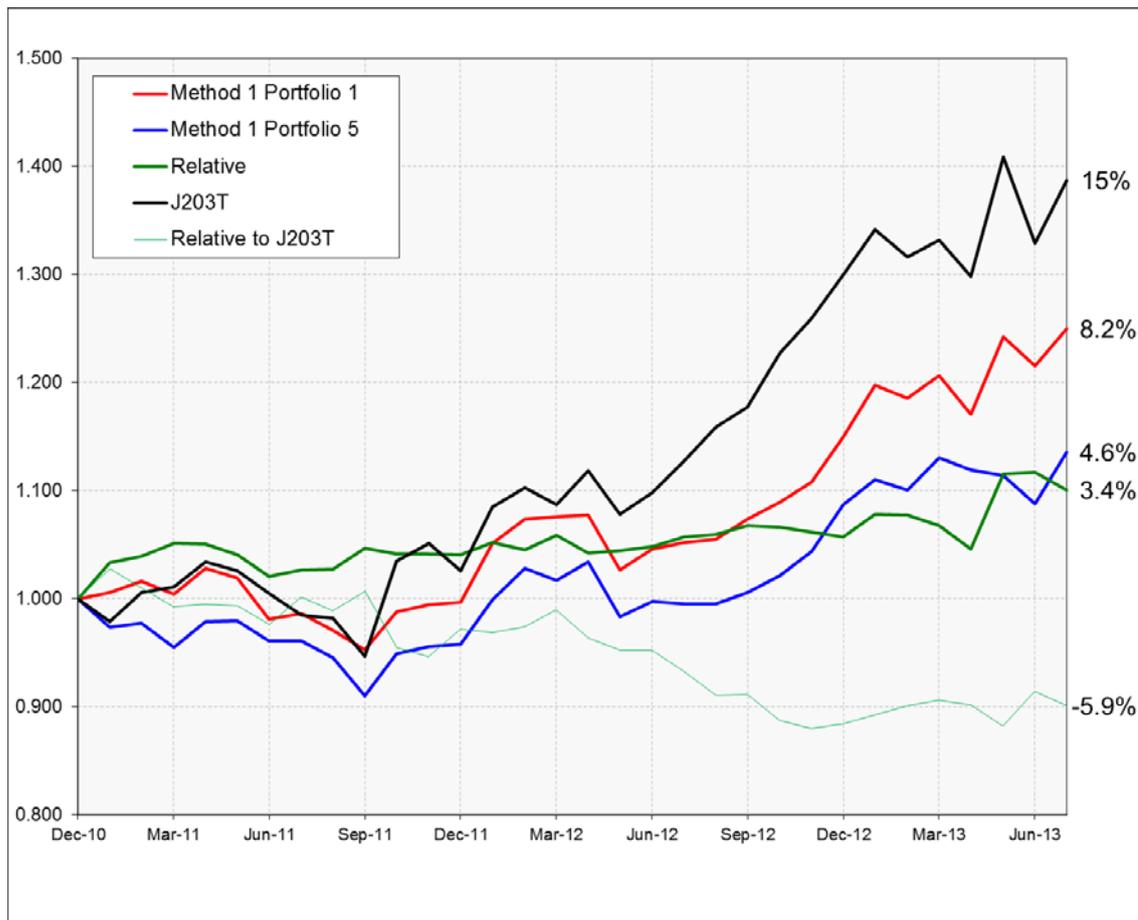
Since the aim of research question one was to investigate the use of share price movement frequencies, parameter combination three was selected to be used with ranking method one. Parameter combination three uses the 30 lowest frequencies over the sample period and exponential growth calculated by using the average price at the sample period's beginning and end. This parameter combination gives the lowest prediction error RMS value at a sample period of 12 months.

5.2 Ranking method one results

Figure 5 shows the cumulative returns of the highest and lowest ranked portfolios ranked according to method one. Results for the timeframe December 2010 to July 2013 shows that the method consistently fails to beat the market with the top ranked portfolio earning returns of 5.9% less per annum than the ALSI total return index. The cumulative returns for this period were 0.25 for portfolio 1, 0.14 for portfolio five and 0.39 for the J203T. The method was therefore not tested further back than December 2010. Portfolios two, three and four were also not examined based on the poor performance of portfolio one.

Method one does result in the returns of the highest ranked portfolio being slightly higher than that of the lowest ranked portfolio. The difference in CAGR between the highest and lowest ranked portfolios is however only 3.4% which is poor compared to the various other styles examined by Muller and Ward (2012).

Figure 5: Ranking method one - portfolio cumulative returns



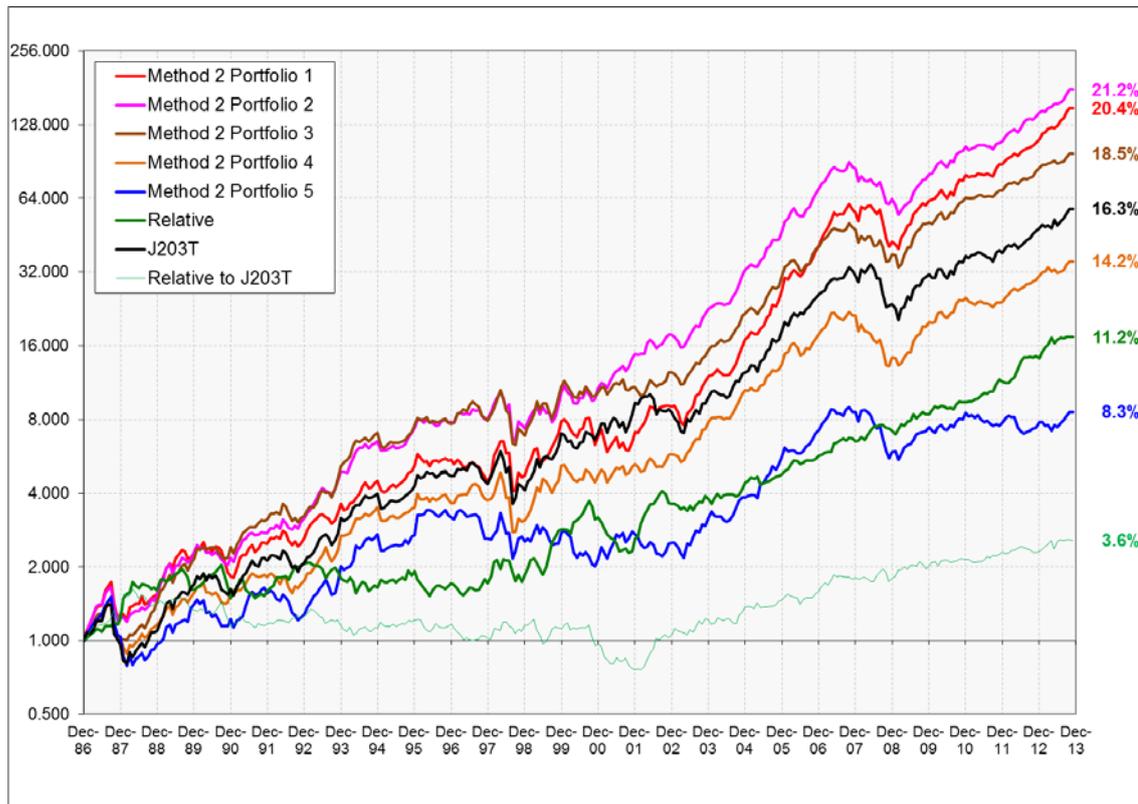
5.3 Ranking method two results

Figure 6 shows the cumulative returns of portfolios ranked according to method two. During the period December 1986 to December 2001 the method fails to rank shares in the top 3 quintiles according to their expected returns. The top three portfolios do however outperform the J203T during this period but the difference between returns of the highest and lowest ranked portfolios is negligible. Portfolio 3 outperformed the market by a significant margin until mid-1997 but tracks the market thereafter.

The best performing portfolio, portfolio two, achieved a CAGR of 21.2%. This is lower than the best performing investment style, momentum, examined by Muller and Ward (2012) that achieved 26.1%. The timeframe used by Muller and Ward (2012) was December 1986 to December 2011. The CAGR for portfolio two over this same timeframe is 20.6%. It therefore does not beat the momentum style examined by Muller and Ward (2012).

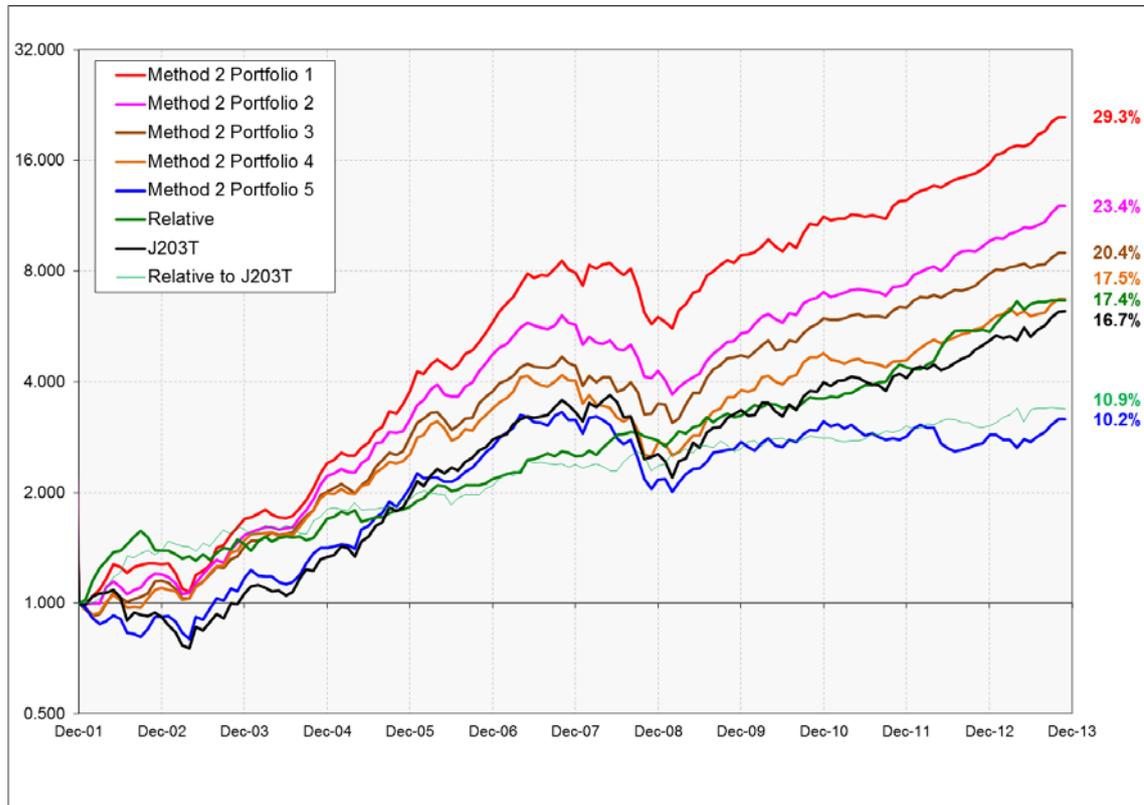
Since December 2001 however, the method has performed remarkably well. The highest ranked portfolio has consistently outperformed the lowest ranked portfolio as well as the J203T. This is evident from the slope of both relative lines which rises steadily since December 2001. It is interesting to note that the 2007-2008 global financial crisis had little to no effect on the relative outperformance of portfolio one. This is evident from the lack of a trough or dip in the relative line during this period.

Figure 6: Ranking method two - portfolio cumulative returns – Dec 1986 to Oct 2013



To examine the performance of ranking method two for the period December 2001 to October 2013, the portfolio return results were plotted again starting in December 2001 with a value of one as shown in Figure 7. For this period the method yielded a high CAGR of 29.3%. The method also managed to rank shares according to their actual returns performance. This is evident from the fact that the highest ranked portfolio performed the best, the lowest ranked portfolio the worst and the other portfolios performed in accordance with their rank. For this period the highest ranked portfolio has consistently outperformed the J203T. The relative outperformance flattened off however in the last three to four years. Cumulative returns for this period for portfolios one to five were 20.01, 11.01, 7.98, 5.72 and 2.16 respectively.

Figure 7: Ranking method two - portfolio cumulative returns – Dec 2001 to Oct 2013



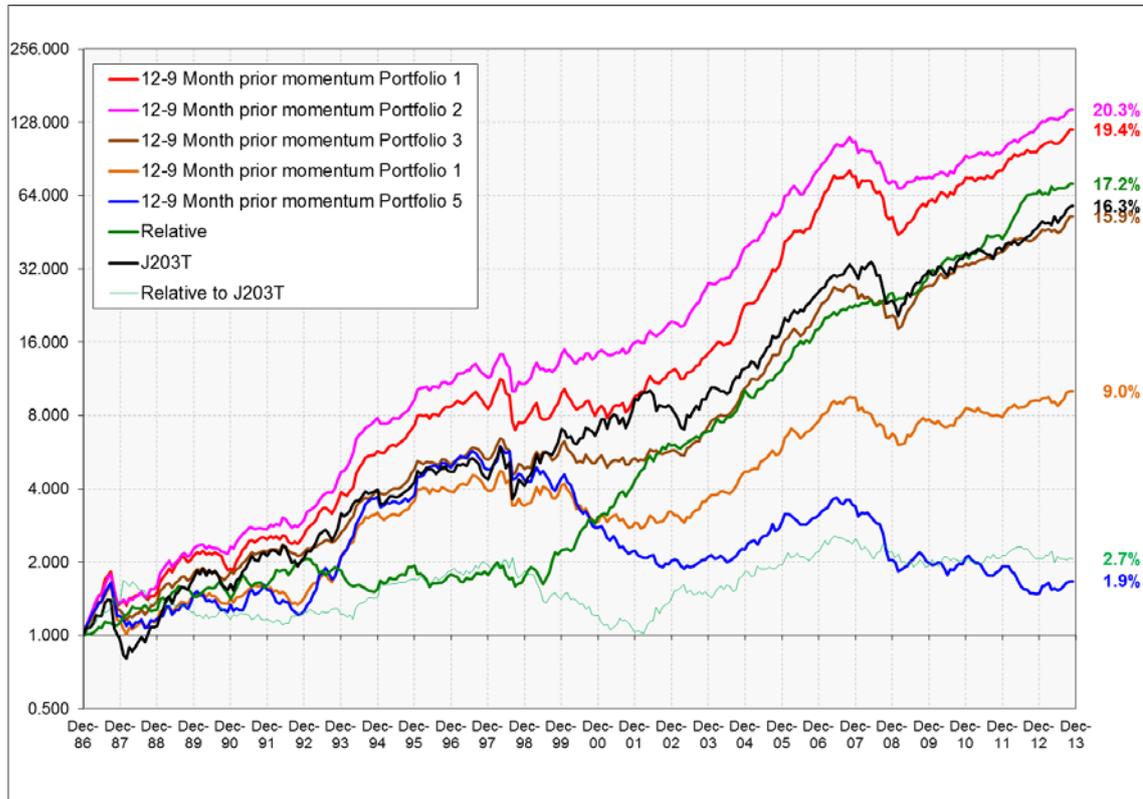
Portfolios ranked according to 12 to nine months prior momentum without DFT noise filtering were used to compare the results achieved with method two with. Figure 8 shows the cumulative returns of portfolios ranked according to 12 to nine months prior momentum. This method performs only marginally worse in terms of CAGR of the highest three ranked portfolios compared to method 2 for the timeframe December 1986 to October 2013.

During the period December 1986 to December 1999 the method fails to rank shares in the top 3 quintiles according to their expected returns as was the case for method two. Like with method two, this method fails to establish a difference of returns between the highest and lowest ranked portfolios over the period December 1986 to December 1999. This is evident from the horizontal relative line.

Similar to method two this method starts to perform well relative to the J203T from December 2001. The relative outperformance flattens off from December 2006 effectively turning the highest ranked portfolio into an index tracking portfolio. This indicates that the DFT used in method two adds some value in the last six to seven

years since the highest ranked portfolio of method two continues to outperform the J203T. Since mid-2006 the method performs well at grouping shares that underperform into the lowest ranked portfolio.

Figure 8: Ranking method using 12-9 month's prior momentum- portfolio cumulative returns – Dec 1986 to Oct 2013

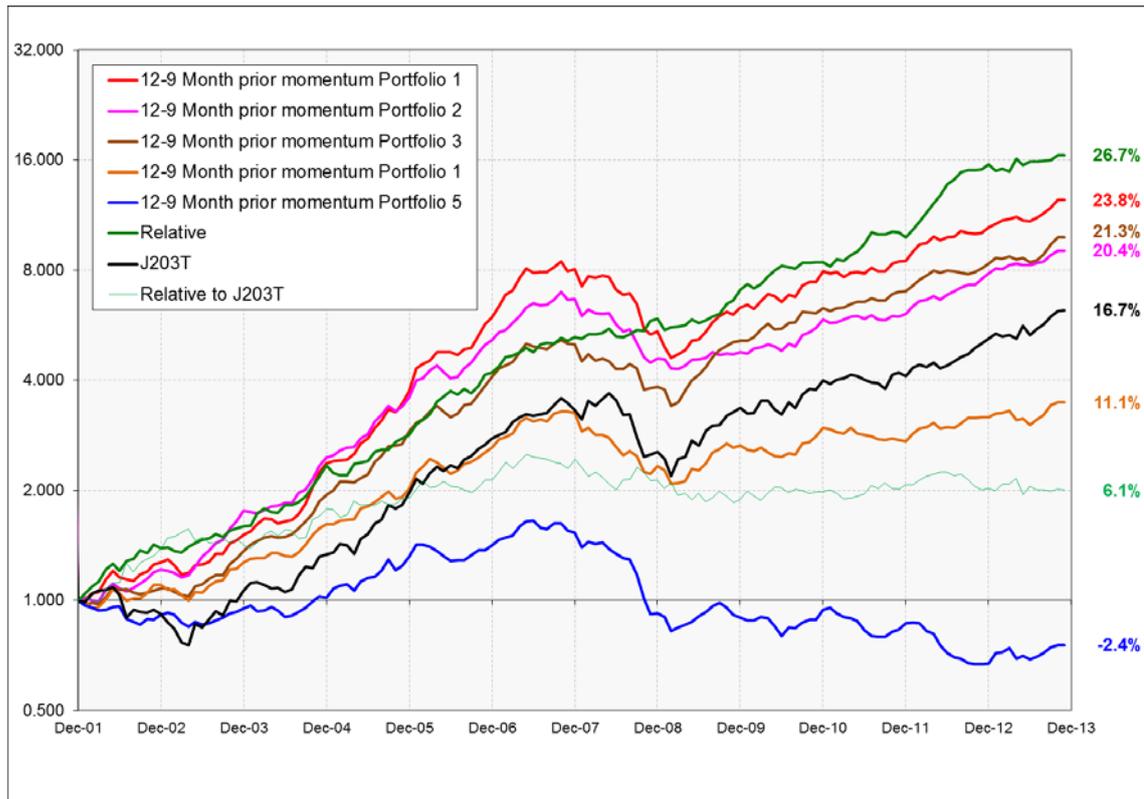


The 12 to nine months prior momentum method was also examined for the December 2001 to October 2013 timeframe as a comparison to method two for the same timeframe. Figure 9 shows the cumulative returns of portfolios ranked according to the 12 to nine months prior momentum method from December 2001 to October 2013 starting in December 2001 with a value of 1. For this period the returns of the 12 to nine months prior momentum ranked portfolios were significantly lower than that of the portfolios ranked according to method two.

The cumulative returns of the highest ranked portfolio using the prior momentum method was 11.45 compared to the highest ranked portfolio ranked using method two that achieved cumulative returns of 20.01. This is almost double the returns over the examined timeframe.

The 12 to nine months prior momentum ranking method performs worse than method two at ranking portfolios over this timeframe. The returns for portfolio three is higher than the returns of portfolio two. This is unlike method two that achieved returns for portfolios relative to their quintile.

Figure 9: Ranking method using 12-9 month's prior momentum- portfolio cumulative returns – Dec 2001 to Oct 2013



5.3.1 Hypothesis one test results

Hypothesis one was examined for two separate timeframes - firstly for the period December 1986 to October 2013 and secondly for the period December 2001 to October 2013.

Hypothesis two compared the returns of the top portfolios ranked according to method two and the 12 to nine months prior momentum method using a Wilcoxon signed-rank test. Table 4 indicates the statistical significance of the examined z-statistics. The z-statistic was found to be statistically significant at the 5% level for the second timeframe that was examined.

Table 4: Hypothesis one test results for examined timeframes

Difference between monthly returns of two tested methods over the period	z-statistic	Wilcoxon Signed-Rank p-value	Sample size
December 1986 - October 2013	0.841	0.200	322
December 2001 - October 2013	1.650	0.049 [*]	143

Note: ^{*} indicates statistical significance at the 5% level

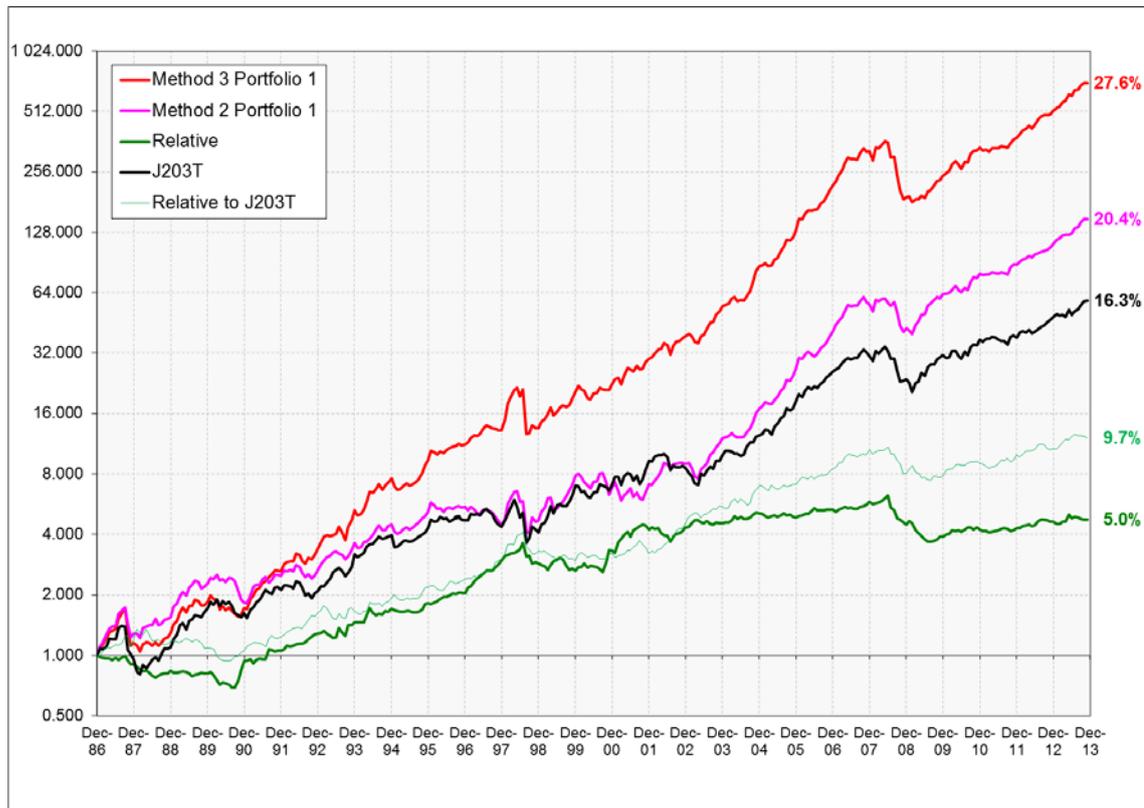
5.4 Ranking method three results

Figure 10 shows the cumulative returns of highest ranked portfolios ranked according to method two and three for the period December 1986 to October 2013. The portfolio ranked according to method three achieved a CAGR of 27.6% over this timeframe and outperformed the top portfolio ranked using method two.

The highest yielding univariate ranked investment style examined by Muller and Ward (2012) over the timeframe December 1986 to December 2011 was momentum. The momentum style achieved a CAGR of 26.1% and cumulative returns of 329.5. For the same timeframe, method three achieved a CAGR of 26.8% and the cumulative returns were equal to 378.5. This is marginally better than the momentum style.

The line indicating performance relative to the J203T indicates that the top portfolio ranked according to method three consistently outperformed the market. The slope of this line shows persistent performance but also indicates that method three fails to outperform the market after market crashes in 1997 and 2007 to 2008.

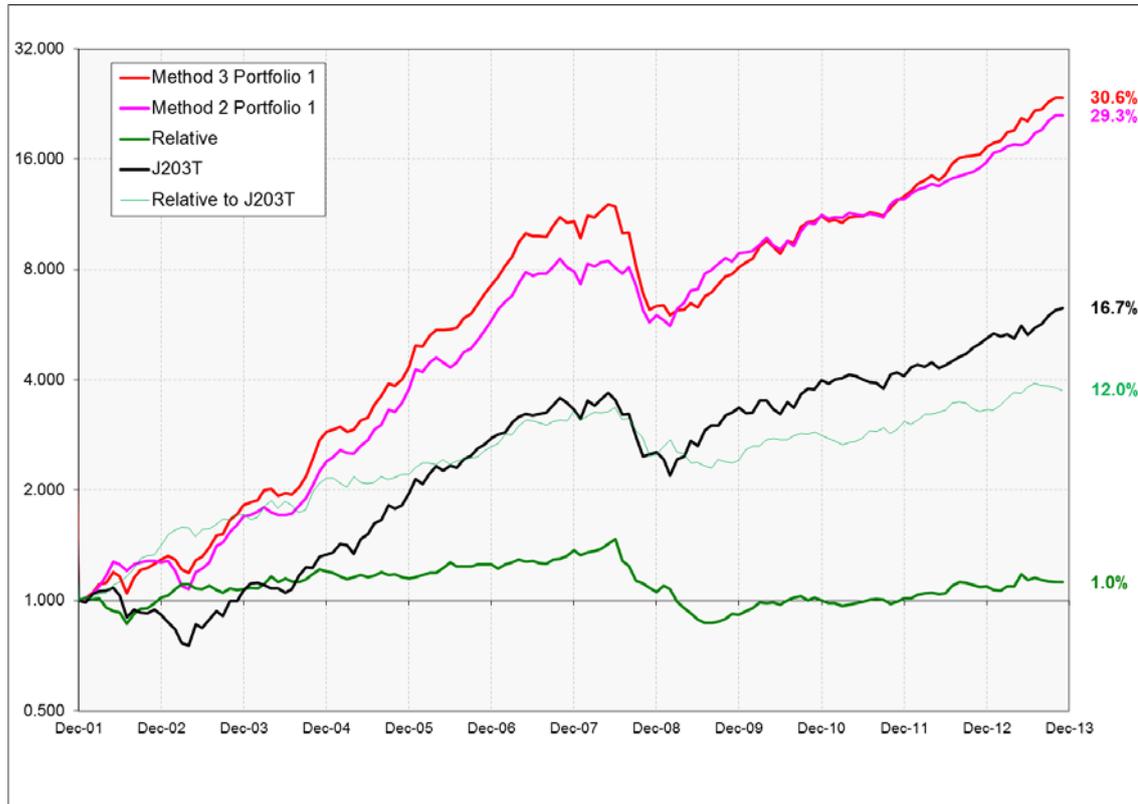
Figure 10: Ranking method two and three – top portfolio cumulative returns – Dec 1986 to Oct 2013



Method three outperforms method two during the first part of the examined timeframe but from December 2001 the difference in performance is relatively flat as is evident from the horizontal slope of the relative line. These methods are therefore also compared for the timeframe December 2001 to October 2013 as shown in Figure 11.

Figure 11 shows that the difference between the returns of the top quintiles ranked according to method two and three is small for the timeframe December 2001 to October 2013. Method three outperforms method two slightly until the market crash of 2007 to 2008. From mid-2008 method three starts to outperform method two again slightly. The cumulative returns for the top portfolios ranked according to method two and three for this timeframe is 20.01 and 22.56 respectively.

Figure 11: Ranking method two and three – top portfolio cumulative returns – Dec 2001 to Oct 2013



5.4.1 Hypothesis two test results

Hypothesis two was examined for two separate timeframes - firstly for the period December 1986 to October 2013 and secondly for the period December 2001 to October 2013. Hypothesis two compared the returns of the top portfolios of method two and three using a paired Wilcoxon signed-rank test. Table 5 indicates the statistical significance of the examined z-statistics. The z-statistic was found to be statistically significant at the 5% level for the first timeframe that was examined and not for the second timeframe.

Table 5: Hypothesis one test results for examined timeframes

Difference between monthly returns of two tested methods over the period	z-statistic	Wilcoxon Signed-Rank p-value	Sample size
December 1986 - October 2013	2.698	0.003 [*]	322
December 2001 - October 2013	1.108	0.134	142

Note: ^{*} indicates statistical significance at the 5% level

6 DISCUSSION

The results are discussed in the order of the research questions. The results show that there is considerable value in combining momentum investment strategies with spectral analysis methods on the JSE. Two of the three ranking methodologies that were examined produced favourable results and outperformed the market by a considerable margin.

6.1 Using price frequency and prior exponential growth as an investment style

The results presented in section 5.2 shows that ranking method one performs poorly as an investment style. For the tested timeframe the answer to research question one is therefore no. The top ranked portfolio tracked, but did not beat the market for a short period and then failed to beat it thereafter.

The optimisation of parameters for method one showed that prior exponential growth of a share price performs better at predicting share prices than combining share price periodicity with prior exponential growth. This was true for all of the tested sample periods. It was found that the prediction accuracy is somewhat influenced by the history timeframe used for spectral analysis and prior growth calculation. The results are however not necessarily statistically significant and more rigorous testing using more stocks will be required to rule out the probability that the RMS of the prediction errors for the two companies were similar because of chance.

It was known before conducting this test that extrapolating cosines identified with the DFT would result in a direct replication of the analysed historical prices as was shown in Figure 2. The frequencies predicted for the holding period would therefore be exactly the same as the first part of the analysed historical timeframe. Increasing the timeframe of the analysed history therefore added little value other than being able to filter out increasingly lower frequencies with an increase in analysed timeframe. The more frequencies that were filtered out the closer the extrapolated cosines would be to the predicted exponential growth rate.

By increasing the analysed historical timeframe the start date of the analysed period had to be moved. By moving the start date, the set of frequencies replicated for the holding period were therefore also moved.

The fact that all the historical timeframes that were examined were multiples of six months, inadvertently resulted in only investigating seasonal periodicity. Although Figelman (2007) found that seasonal periodicity works well as an investment strategy, it performed poor in this instance. The seasonal price momentum effects identified by Figelman (2007) are supported by the periodical market anomalies that have been extensively discussed in literature. All the periodical market anomalies discussed in section 2.2.1 are seasonal. Any seasonal periodicity momentum based investment style using periods of 12 months should therefore produce favourable results.

In understanding why method one failed, the differences between Figelman's (2007) method and method one needs to be considered. The major difference between the methods is the fact that Figelman (2007) used only the ROC of the analysed historical timeframe to rank shares. If the growth correction step were to be removed from method one, and only use the DFT with all given frequencies, method one and Figelman's (2007) method would produce the same results. This indicates that the assumption of prior exponential growth continuing in future introduced a degree of prediction volatility that reduced its accuracy.

A better approach to search for periodicity would have been to keep the holding period fixed and to examine the length of price history that would give the most accurate prediction results. Investigating the historical timeframe length at increments of one day would search for period lengths of any number of days.

6.2 Twelve to nine month prior price frequency as an investment style

For the tested timeframe the answer to research question two is yes. The results presented in section 5.3 showed that it is possible to beat the market by a significant margin by only using publically available trading data. This is in contrast with the weak form of the EMH (Fama, 1970). The seasonal price momentum periodicity of the JSE exploited in this investment style supports the literature presented in section 2.2.1. The literature reviewed in section 2.2.1 identified a range of seasonal market anomalies or

calendar effects. Based on the analysis of the graphical time series in section 5.3, ranking method two can be classified as a successful investment style. The results are also aligned with the findings of Wilcox and Gebbie (2008) who identified quarterly periodicity effects on the JSE.

A specific periodicity was examined with method two. This was again seasonal periodicity by looking back 12 to nine months. The spectral analysis tool, the DFT, helped to filter out spikes in daily prices but was not used to identify periodicity beyond the assumed periodicity. The ability of the DFT to filter noise is in line with the findings of Barucci *et al.* (2012) and Malliavin and Mancino (2002) where it was used to filter out noise in volatility data.

Method two performs well as a predictor of poor returns performance since December 2009. The 12 to nine months prior momentum style however performs even better at predicting poor performance also from December 2009. This might therefore be a useful style to use for short selling shares.

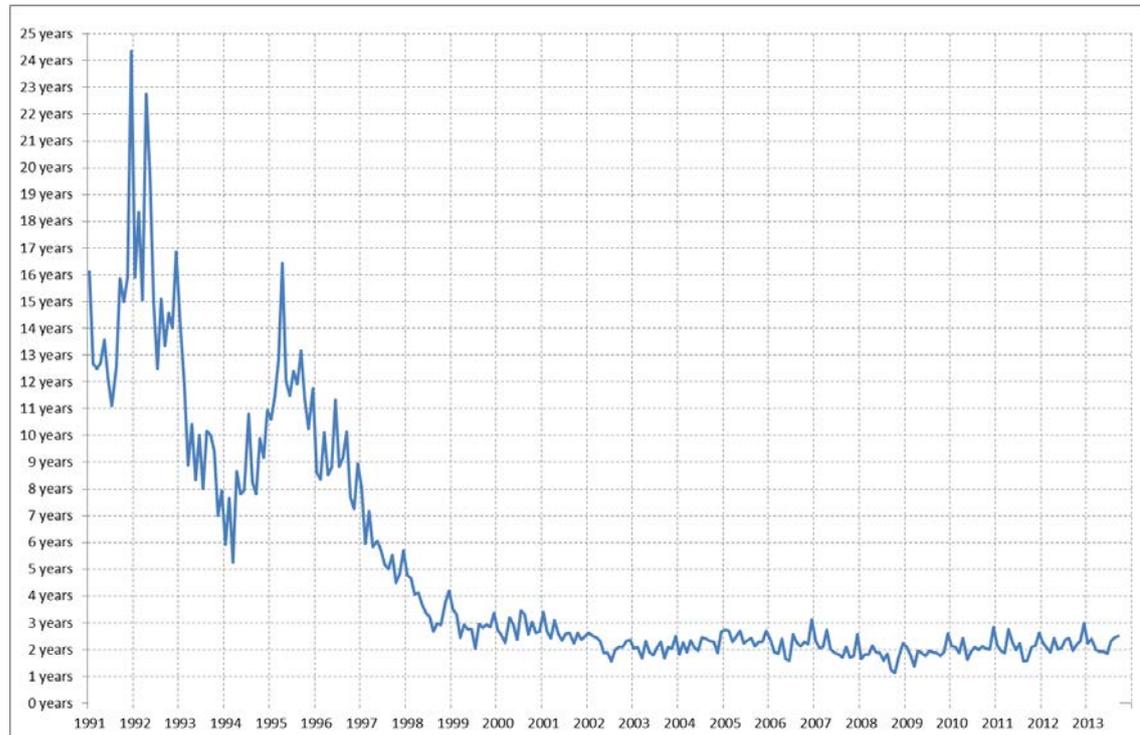
It is clear from Figure 6 that the performance of method two can be divided into two distinct periods for the analysed timeframe. Before December 2001 the style behaved differently than after this date. It would not have been possible to identify this distinct change in style behaviour without the use of a graphical time series. The test statistic for hypothesis one shows that for the timeframe from December 1986 to October 2013 there is no significant difference between method two and the momentum periodicity style it is based on.

An important observation is the fact that the style lost virtually no value relative to the market during the market crash of 2007-2008. Most other styles examined by Muller and Ward (2012) for this period exhibits some loss of value during this period.

In an attempt to understand the reason why the behaviour of method two changed in December 2001, the average holding period for shares traded on the JSE were examined. Figure 12 shows the average holding period for a stock on the JSE over the period 1991 to 2013. It is clear that the average holding period dropped sharply from 16 years at the beginning of 1995 to roughly three years in mid-1999. At the time that method two starts to behave differently as an investment style, the holding period drops from roughly two and a half years to two years. This implies that for method two to work, shares need to be held for a maximum of two years. This supports the findings of

Bildik and Gülay (2007) who found that self-funding strategies increases the degree by which momentum strategies beat the market. The graph does however not explain why the top portfolio of method two started to track the market more closely over the last three to four years of the timeframe.

Figure 12: Average holding period (in Years) for a stock on the JSE



6.3 Validity of hypothesis one

Hypothesis one examined if the frequency analysis used in method two added any additional returns to the momentum periodicity style it is based on. As mentioned above, analysis of the graphical time series suggested that frequency analysis only started to add value to the underlying style during December 2001.

For the timeframe December 1986 to October 2013 the null hypothesis was accepted at the 5% significance level. This outcome implies that any outperformance of method two as an investment style over the 12 to nine months prior momentum style was purely a product of chance. Since this is a statistical test there is however a chance that this conclusion is wrong.

For the timeframe December 2001 to October 2013 the null hypothesis was rejected at the 5% significance level. This outcome implies that any outperformance of method two as an investment style over the 12 to nine months prior momentum style cannot be attributed to chance. The outperformance of method two over the underlying momentum method is therefore statistically significant for the timeframe from December 2001 to October 2013.

6.4 Twelve month prior price frequency as an investment style

Research question three examined the use of the DFT to calculate seasonal price momentum over a 12 month period to construct a market beating portfolio. The results presented in section 5.4 shows that the answer to question three is yes. Method three also outperformed method two as a ranking method for the period December 1986 to October 2013 but there was little difference between the performance of method two and three as investment styles for the period December 2001 to October 2013.

Method three appears to be less sensitive to average holding periods of shares than method two. It is likely that this is due to the period used for the calculation of price momentum and that shorter periods used for the calculation of ROC is more sensitive to share holding period lengths.

The method performed marginally better than any other univariate ranked investment style examined by Muller and Ward (2012). It persistently outperformed the market and can be classified as a market beating investment style.

6.5 Validity of hypothesis two

Hypothesis two examined if there is a difference in performance between the top portfolios ranked according to method two and method three. Section 5.3 showed that method two started to perform better from December 2001. The hypothesis was therefore checked again for the two timeframes used for testing of hypothesis one.

For the timeframe December 1986 to October 2013 the null hypothesis was rejected at the 5% significance level. This outcome implies that any difference in investment style

performance between methods two and three is statistically significant and cannot be attributed to chance

For the timeframe December 2001 to October 2013 the null hypothesis was accepted at the 5% significance level. This outcome implies that any difference in investment style performance between methods two and three was purely a product of chance. The graphical time series shown in Figure 11 indicates however that method three performs better than method two except when a market crash occurs. Therefore, if the time between market crashes is long, method three will outperform method two and *vice versa*.

7 CONCLUSION

7.1 Findings

Using the DFT combined with prior exponential growth fails to predict share prices accurately. The results obtained in this research indicate that using prior exponential growth as an indicator for future exponential growth might be a more accurate method of forecasting future share prices when used on its own. The fact that the DFT merely replicates the sample period when it is extrapolated means that unless a sample period beginning date is chosen that correlates with the holding period there is no value in using the DFT as a predictor of future returns. The DFT cannot identify periods that are longer in duration than the sample period. Since lower share price frequencies also tended to have large amplitudes, the inability of the DFT of finding frequencies lower than a given sample period, might be a major disadvantage. Failing to identify the largest frequencies will adversely affect prediction accuracy.

The methodology used in this research could only establish if share price periodicity exists at an examined period length. In other words, it was only tested if share prices are correlated with historical prices going back six months, 12 months, 18 months and so forth. The methods used by Wilcox and Gebbie (2008) might prove more useful to find price periodicity. It is also possible that, other than seasonal or calendar effects, there is not enough share price periodicity on the JSE to be used for time arbitrage.

The results obtained in this research provide empirical evidence of momentum periodicity on the JSE for the period December 1986 to October 2013. It also provides evidence that the JSE is not an efficient market and that it is subject to behavioural finance effects such as price momentum.

The most successful investment style in terms of cumulative returns that was examined used 12 month prior price momentum, calculated after the data was filtered for noise, to rank shares and construct a winning portfolio for a three month holding period. The data was filtered for noise by transforming it to the frequency domain with the DFT and then transforming it back to the time domain with the IDFT using only frequencies of one, two and three. From December 2001 to October 2013 there was however no statistically significant difference between the returns of portfolios constructed using this method and a method using the same noise filtering and price momentum for the

period 12 to nine months prior to portfolio construction. The method using 12 to nine months prior price momentum was less volatile and lost less value during the 2007-2008 market crash.

It was found that the DFT-IDFT noise filtering method improved the returns of all investigated price periodicity based investment styles. Returns may be improved further by using a more concentrated top portfolio with fewer shares.

7.2 Implications

This research contributes to literature by proving that noise filtering by means of frequency analysis can improve the returns achieved with price momentum investment styles. With further refinement these returns may be improved further.

This research proves that the 12 to nine months prior price momentum investment style combined with DFT noise filtering is one of the most successful univariate ranked investment styles on the JSE for the period December 2001 to December 2013. The style performs well even during the market crash of 2007-2008. This investment style has not been documented before for the JSE in academic literature as could best be established.

This research also proves that 12 to nine months prior price momentum style could have been successfully used in shorting strategies by using the bottom ranked portfolio of this style. Over the period December 2001 to October 2013 it achieved a CAGR of minus 2.4%.

7.3 Recommendations

Some industries or some individual shares in different industries might have a higher degree of periodicity than others. This might be due various external factors. If this is true, periodicity based portfolio ranking methods might work better by constructing portfolios consisting only of these high-periodicity shares. If shares in these industries happen to yield high returns, following such a strategy will be a successful investment style. Examining the periodicity of each stock and industry will however complicate the proposed method as an investment style

The DFT method essentially acted as a noise filter before calculating ROC and ranking shares in a momentum based investment style. Another method for filtering noise is the least squares fitted exponential line. It will be worthwhile to examine the use of this method as an investment style and comparing it with method three of this research.

It was interesting to note that the RMS of the prediction error for method one examined in this research report was lower relative to other periods when using 108 months history. This may imply that a nine year cycle exist for the shares of the two analysed companies. More rigorous testing is however required to establish if periodicity other than seasonal periodicity or calendar effects are present on the JSE.

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