Pseudo-Elliptic Function Bandstop filter with shunt Foster Sections

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ABSTRACT – *Filters with transmission zeros in the stopband have been shown to have superior rates of cutoff. Conventional Cauer filters give excellent performance but are relatively large and not always practically realizable. In this paper, the design of a pseudo-elliptic bandstop filter which employs only shunt Foster sections and unit elements is presented. The filter is of the same size as a Chebyshev filter of the same order, but has a performance comparable to a Cauer filter.*

Keywords – *Pseudo-elliptic function filter, shunt Foster sections*

1 INTRODUCTION

The Cauer filter, which employs transmission zeros at real frequencies in the stop band, ensures the designer of an equiripple passband return loss, as well as an equiripple insertion loss in the stop band [1]. While these structures are realizable for moderate to narrow bands, they are relatively large, with stubs in the shunt Foster sections of thousands of ohms. These sections often require short circuits, and while the length of a planar realization of a Cauer filter of order N would be $(N-1)\lambda_0/4$, the width would be of the order of $\lambda_0/4$.

Attempts have been made in the past to reduce the size; the most notable recently has been the introduction of a modified Chebyshev filter [2] where an equiripple stopband response was obtained by progressively adjusting the resonant lengths of a Chebyshev prototype until a satisfactory response is obtained. However, the upper passband return loss deteriorated rapidly. Because of the fact that the lines were not commensurate, the harmonic response beyond the upper passband is unpredictable.

In this paper, a new approach is described in which a structure consisting of shunt open circuit stubs spaced by unit elements is converted to a cascade of shunt Foster sections and unit elements. The resultant filters have equal ripples in both the stopband and the passband, and an overall performance comparable to that of a Cauer filter. Additionally, the structure is extremely compact.

2 SHUNT FOSTER SECTIONS

The Brune section produces transmission zeros at real frequencies and under certain conditions degenerates to the shunt Foster section. Consider a shunt open circuited stub as shown in Fig. 1(a). It resonates at a frequency f_0 determined by its length $l = \lambda_0/4$. The transmission line stubs shown in Fig. 1(b) are each of length $l = \lambda_0/8$, so the total length remains $\lambda_0/4$. The input impedance to the cascade of the two sections is given by

$$\begin{aligned} Z_{in} &= Z_1 \frac{(Z_2/S)+SZ_1}{Z_1+S(Z_2/S)} = \frac{1}{S}\frac{Z_1 Z_2}{Z_1+Z_2} + S\frac{Z_1^{\,2}}{Z_1+Z_2} \\ &= Z_C/S + SZ_L, \end{aligned}$$

where $S = j\tan\left(\frac{\pi}{4}\frac{f}{f_0}\right)$. (1)

The circuit will resonate at the frequency f_{00} where the input impedance to the cascade is zero:

$$0 = Z_C / S_{00} + S_{00} Z_L$$

$$\tan\left(\frac{\pi}{4}\frac{f_{00}}{f_0}\right) = \pm\sqrt{\frac{Z_C}{Z_L}} = \pm\sqrt{\frac{Z_2}{Z_1}}$$

$$f_{00} = f_0 \frac{4}{\pi}\tan^{-1} \pm\sqrt{\frac{Z_2}{Z_1}}. \quad (2)$$

If $Z_1 = Z_2$, then $f_{00} = f_0$. For $Z_2 < Z_1, f_{00} < f_0$. For $Z_2 > Z_1, f_{00} > f_0$. Note that the antiresonance of both circuits occurs at $2f_0$ as shown in Fig. 2. The responses repeat symmetrically about $2f_0$, i.e. if $f_{00} < f_0$, then $f_{00} > f_0$ above $2f_0$, and vice versa.

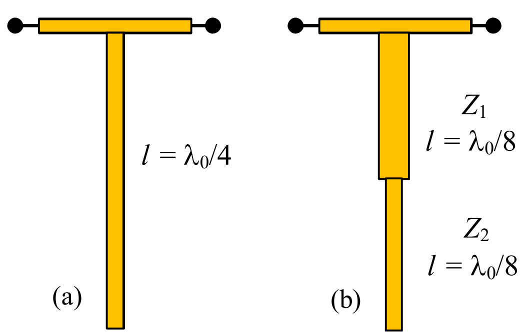


Figure 1 (a) Single stub, and (b) split stub forming a shunt Foster

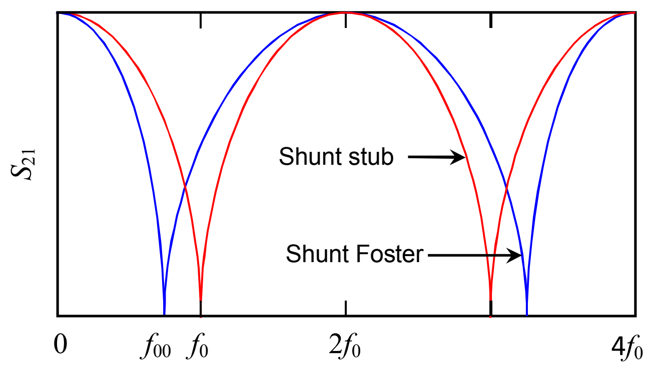


Figure 2 Transmission response of shunt stub and shunt Foster section.

3 PROTOTYPE

The filter design is based on obtaining a suitable bandstop structure consisting of a cascade of shunt open circuit stubs and unit elements, and then to modify the structure by replacing the shunt stubs by shunt Foster sections. In order to have a realistic starting point for impedance values for the stubs and unit elements, a Chebyshev prototype is developed. The shunt stubs of the Chebyshev filter are then replaced by shunt Foster sections that resonate at the transmission zero frequencies of a Cauer filter.

3.1 Chebyshev Prototype

The procedure for obtaining a realizable Chebyshev filter is well described [1]. A 5th order prototype with 20.3 dB return loss passband ripple was chosen, and scaled for a cutoff frequency of $0.85f_0$, i.e. a 30% bandwidth. After 50Ω-impedance scaling, and the application of two Kuroda transforms from each end, the structure shown in Fig. 3 is obtained; the element values are shown in Table 1.

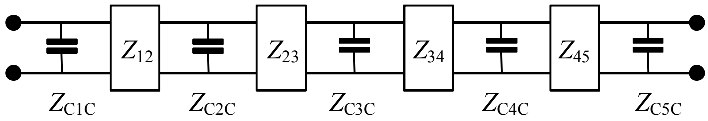


Figure 3 Realizable 5th order Chebyshev bandstop filter

TABLE 1
IMPEDANCE LEVELS FOR CHEBYSHEV PROTOTYPE

Element	Z_{C1C}	Z_{12}	Z_{C2C}	Z_{23}	Z_{C3C}	Z_{34}	Z_{C4C}	Z_{45}	Z_{C5C}
Impedance, Ω	317.3	59.4	141.1	57.1	116.5	57.1	141.1	59.4	317.3

3.2 Transmission zeros

The shunt open circuit stubs Z_{C1} to Z_{C5} in Fig. 3 are now converted to shunt Foster sections as shown in Fig. 4, with the values Z_{C1} to Z_{C5} and Z_{L1} to Z_{L5} to be determined.

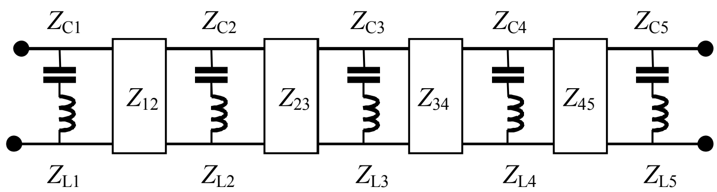


Figure 4 Resonated prototype with shunt stubs replaced by shunt Fosters.

In order to do so, positions for the transmission zeros must be determined. This is done by choosing a suitable Cauer filter as a prototype; in this case a choice was made for a filter designated C0505 with Θ = 43° [3]. It has a passband return loss ripple of 21.9 dB and stopband equiripple of -36.5 dB. The elements of the Foster sections in the Cauer filter are commensurate, i.e. they are all of length $l = \lambda_0/4$. Consequently, two sets of two transmission zeros, are formed at $\{f_{\infty 2-}, f_{\infty 2+}\}$, and $\{f_{\infty 4-}$ and $f_{\infty 4+}\}$, as shown in Fig. 5.

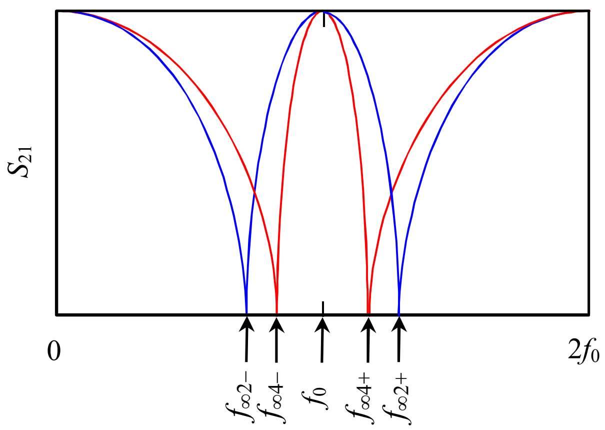


Figure 5 Positions of transmission zeros for Cauer filter.

The shunt stubs of the filter in Fig. 3 are now replaced by Foster sections that are calculated to resonate at the same frequencies as the Foster sections in the Cauer filter, except that all the elements in the latter sections are of length $l = \lambda_0/8$, and they are symmetrical about $2f_0$ rather than f_0, as shown in Fig. 6.

The centre frequency of the filter was chosen to be 2.0 GHz, and both the Chebyshev and Cauer filters were scaled for a band edge frequency of 1.70 GHz, or a relative bandwidth of 30%. The outermost sections of the Chebyshev filter have the highest impedance, and would thus give the sharpest zeros. Consequently, $f_{\infty 1}$ is placed at $f_{\infty 2-}$, $f_{\infty 4}$ at $f_{\infty 2+}$, etc., as shown in Table 2. The positions of the zeros for the pseudo-elliptic filter are determined by the ratios of Z_2/Z_1, calculated from the positions of the Cauer filter zeros.

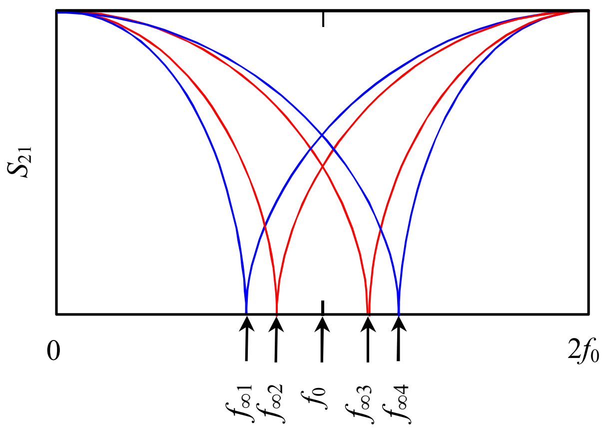


Figure 6 Positions of transmission zeros for pseudo-elliptic filter.

TABLE 2
POSITIONS OF TRANSMISSION ZEROS

Cauer Filter	$f_{\infty 2-}$	$f_{\infty 4-}$	f_0	$f_{\infty 4+}$	$f_{\infty 2+}$
Pseudo-Elliptic	$f_{\infty 1}$	$f_{\infty 2}$	f_0	$f_{\infty 3}$	$f_{\infty 4}$
Ratio Z_2/Z_1	1.4289	1.2691	1	0.7880	0.6999
Frequency (GHz)	1.774	1.848	2.000	2.152	2.226

3.3 Impedance levels.

It is clear from Figs 5 and 6 that there could be a reasonable approximation to the slope of the Foster sections of the Cauer Filter by the pseudo-elliptic approach, up to the first transmission zeros. However, in the stopband, the responses differ radically. It is to be expected that some form of compensation of the transmission zeros would be necessary in the stopband. Furthermore, the band edges would differ, and a choice exists as to where the overall response should have a predetermined value. This could be at either of the band edges, or at the centre frequency. In this case it was decided to match the Chebyshev response at the lower band edge.

The impedance values for Z_{L1} and Z_{L2} are taken to be the same as the Chebyshev values Z_{C1C}, Z_{C2C}, and Z_{C1}, Z_{C2} calculated from (2). In order to realize zeros above f_0, Z_{C4} and Z_{C5} are chosen equal to Z_{C2C} and Z_{C1C}, and Z_{L4} and Z_{L5} calculated from (1). The unit element values are left unchanged. Note that the centre stub, Z_{C3C} remains unchanged. However, in Table 3, it is shown as Z_{C3}, Z_{L3}. The reason for this will become clear later. The stub and unit element impedances are shown in Table 3.

The filter can now be realized as a cascade of compound Schiffman spurline sections as shown for one section in Fig. 7. Fig. 7 shows a calculation of return loss and insertion loss performance *vs* frequency for the element values in Table 3. It is clear that the frequency response is distorted, due to the choice of the match at the lower cutoff frequency. It is compared to the Cauer C0505 filter used for the positioning of the transmission zeros.

TABLE 3
ELEMENT VALUES FOR PSEUDO-ELLIPTIC FILTER

Element		$Z_{C1/L1}$	Z_{12}	$Z_{C2/L2}$	Z_{23}	$Z_{C3/L3}$	Z_{34}	$Z_{C4/L4}$	Z_{45}	Z_{C5L5}
Z_L Ω	Z_{UE}	317.3	59.4	141.1	57.1	116.5	57.1	179.2	59.4	453.4
Z_C Ω		453.4		179.2		116.5		141.1		317.3

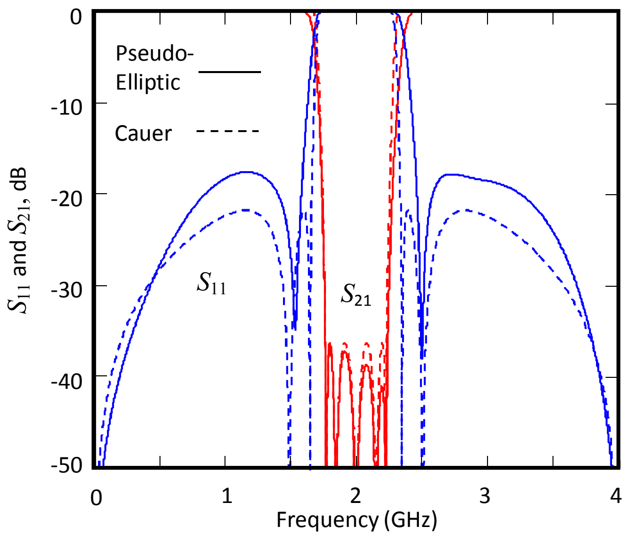
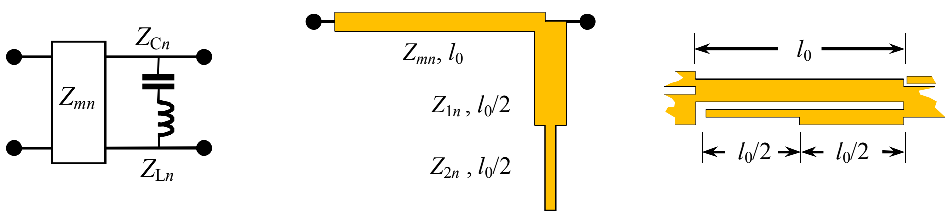


Figure 7 Return loss and insertion loss for pseudo-elliptic prototype.

6. CONSTRUCTION AND MEASUREMENT

The physical construction of the filter entailed realizing each of the pairs of Foster sections and unit elements as compound Schiffman spurlines [4]. Fig. 8(a) shows one such section, and its representation in terms of the parameters of section 2 in Fig. 8(b). The Schiffman spurline equivalent is shown in Fig. 8(c).



(a) (b) (c)

Figure 8 (a) Shunt Foster and unit element. (b) transmission line equivalent. (c) Coupled line realization.

The cascade of sections was modelled in Sonnet, and the positions of the zeros at the centre frequency and at $f_{\infty 3}$ shifted slightly lower in frequency. This entailed changing the centre stub to a Foster section as well, but the implication is trivial. Additionally, it was necessary to adjust the impedances of the unit elements in order to improve the overall impedance match, as well as the internal impedance transformation; values of Z_{12}, Z_{L2}, Z_{23}, $Z_{C3/L3}$, Z_{34}, and Z_{45} were modified and the new impedance levels are tabulated in Table 4. The calculated responses are shown in Fig. 9.

TABLE 4
MODIFIED ELEMENT VALUES FOR PSEUDO-ELLIPTIC FILTER

Element		$Z_{C1/L1}$	Z_{12}	$Z_{C2/L2}$	Z_{23}	$Z_{C3/L3}$	Z_{34}	$Z_{C4/L4}$	Z_{45}	Z_{C5L5}
Z_L Ω	Z_{UE}	317.3	54.7	144.5	56.0	118.8	56.0	179.2	54.7	453.4
Z_C Ω		453.4		179.2		116.3		141.1		317.3

Because there are five Foster sections, and only four unit elements, a 50Ω unit element is introduced at one end, in order to realize the end section. The final constructed filter is shown in Fig. 10. Note the symmetry of narrow and wide lines about the centre section. The filter performance was measured on a network analyzer, and the measured responses are shown in Fig. 11.

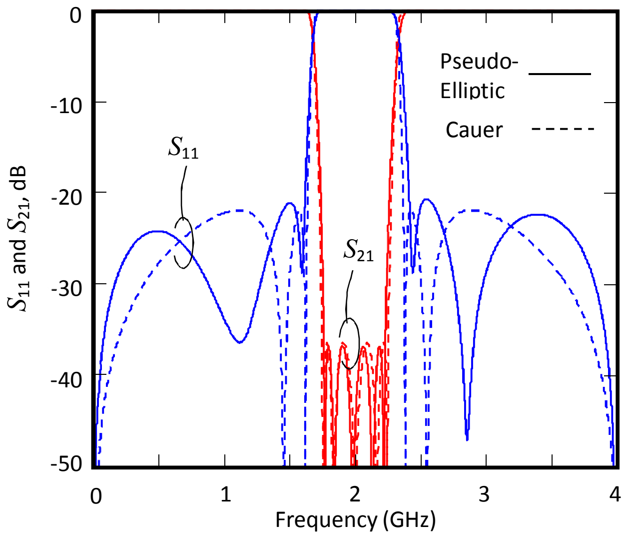


Figure 9 Calculated S_{11} and S_{21} for modified filter.

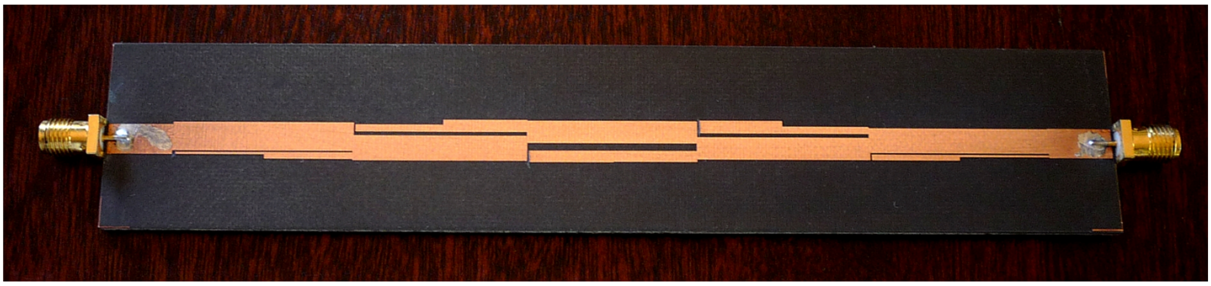


Figure 10 Constructed filter.

7. DISCUSSION

The constructed filter shows small but obvious differences from the theoretically calculated filter. In the main, the stopband is narrower at the minimum attenuation level of the ripples, whle the minimum attenuation level is lower. This is due to the transmission zeros not appearing at the exact design frequencies, because of end effect fringing between the spurline coupled stubs and the main line. This could readily be corrected by trimming the stub ends.

A certain amount of bandwidth expansion is noted; this is caused by shifting the transmission zeros of the Chebyshev filter from the centre frequency outwards to appriximate the equiripple response. The initial Chebyshev design was for a relative bandwidth of 30%, while the theoretically calculated bandwidth was 33.1% and the measured bandwidth was 35%. As the mathematical modelling of the structure is extremely simple, the expansion of bandwidth could very easily be compensated.

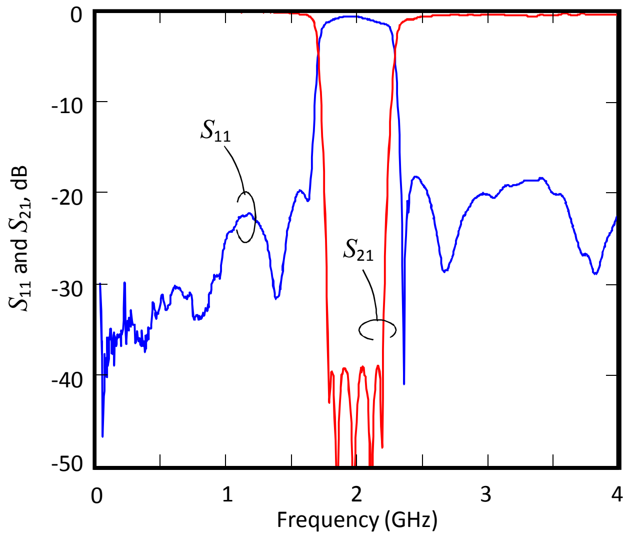


Figure 11 Measured responses of d S_{11} and S_{21} for constructed filter. Comparison with 5th order Cauer filter shown.

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