CHAPTER 4

APPLIED GENERAL EQUILIBRIUM (AGE) MODELING: A DESCRIPTION OF THE GTAP MODEL

4.1 Introduction

Let us go back to Chapter 2 and consider the first attempts to model trade. The famous Ricardian model immediately comes to mind. This model was very simple and consisted of two products and two regions. A more complicated analysis was just not possible and many important pieces of trade theory and economic theory were still ages away from being put together. With the advent of incredible new computing power however, modeling possibilities became endless and at last the comprehensive trade theory could really be tested for realistic situations.

This Chapter introduces a specific trade model, namely the GTAP frameworkfor modeling trade. This is used in this study to analyze the effects of the Free Trade Agreement between South Africa and the EU. As indicated later, the GTAP model is based on sound theoretical principles, and is basically a general equilibrium model, with Walras' Law and Armington Functions as its cornerstones. On the other hand, what makes this model so special is the computer package built around the theoretical framework. It is highly user-friendly and freely available, and makes the replication and extension of original research possible within a timeframe of minutes. This explains the popularity of the model and its widespread use. Before the actual model isdissected, a brief discussion of the development of general equilibrium theory and its functionality is presented.

4.1 Background to AGE Modeling

General equilibrium (GE) modeling has a long and distinguished ancestry (Kehoe et al,1991). Numerical applications of general equilibrium narrowly defined, began with the work of

Harberger (1962) and Johansen (1960). Harberger used a model with two production sectors, one corporate and the other non-corporate, calibrated to US data from the 1950s, to calculate the impact of US corporate income tax. Johansen used a model with 19 productionsectors, calibrated to Norwegian data from 1950, to identify the sources of economic growth in Norway over the period 1948–53.

Work on applied GE models received a crucial stimulus from the research of Scarf and Hansen (1973) on the computation of economic equilibria. Scarf developed an algorithm for calculating the equilibrium of a multi-sectoral GE model. Refinements of this algorithm are still used by some modelers. Probably the most significant consequences of Scarf's work, however, were to establish a close connection between applied GE research and the theoretical research of such economists as Arrow and Debreu (1954), and McKenzie (1981). Their work focused on the existence of equilibrium in very general models and inspired a generation of Yale graduate students to enter the applied GE field (Kehoe, 1996).

Two of Scarf's most prominent students were Shoven and Whalley (1972), who developed a calibrated, multi-sectoral general equilibrium framework to analyze the welfare impact of government tax policy. Early models in the Shoven–Whalley tradition were explicitly static, studying the determination of equilibrium in a single period. Later models studied the evolution of capital stocks over time in a framework where the people in the model either solve static problems (as in Johansen's model) or have myopic expectations (that is, they expect current relative prices to persist in the future).

Researchers working in the Shoven–Whalley tradition have stressed the importance of developing theoretical underpinnings for applied GE models and producing results that are meant to be compared with those of simpler theoretical frameworks (Kehoe, 1996). They havenot spent much time comparing their results with the outcomes of policy changes in the world. Whalley (1986), for example, contends that these models are not intended to forecast the values of economic variables, but rather to provide useful insights that may help policy makers to under-take more informed, and presumably more desirable, policy actions. This line of thoughthas led Whalley to suggest that the concept of positive economics should perhaps be altogether abandoned within applied GE modeling.

After Shoven and Whalley (1972), several other groups of researchers began using static applied GE models to do policy analysis. One such group centered around the World Bank and focused on developing countries; a survey of its work is presented by Dervis et al.(1982). Another group has come to prominence doing policy analysis in Australia (Dixon et al, 1992).

There is a large and expanding literature on multi-sectoral applied GE models, and each group of models tends to have a different focus, although employing the same basic principles. This thesis uses the GTAP framework of applied general equilibrium (AGE) modeling to analyze policy implications, specifically those concentrating on trade. GTAP stands for the Global Trade Analysis Project, which is admin-istered by the Center for Global Trade Analysis of Purdue University, USA. The next section will give a brief overview of GE analysis, drawing some comparisons with partial equilibrium analysis (PE).

4.3 An Overview of the GTAP GE Analysis

The GTAP framework uses an economy-wide simulation model which is constantly reviewed to encompass 50 commodities and more than 20 regions. The multi-region model captures the global economy and all its trade flows. It is furthermore typically based on neoclassical theories of firm and household behavior with a time frame long enough to achieve equilibrium in all markets. The framework is based on comparative static analysis, but dynamic versions are also available.

GE analysis has become popular for a number of reasons, in part because some of the limitations of PE can be avoided. PE analysis generally does not acknowledge finite resource endowments, whereas a subsidy in GE analysis pulls resources away from other sectors, making them scarce, and thereby increasing their price. PE analysis also does not indicate where subsidies come from, meaning the model does not punish those agents that have to pay for the subsidy. This could have profound welfare consequences where the subsidy may increase welfare of some at the expense of others. Income effects are not captured endogenously in PE and there is no link between factor income and expenditure. GE analysis also provides a consistency check through Walras' Law, which does not apply to PE analysis. If it is the aim of the researcher to study the impact of subsidies on food prices, and if the food sector is relatively small, then PE analysis is good enough. Why then is GE analysis ever necessary?

First, there is the issue of theoretical consistency. A well-constructed CGE model is not a black box. Walras's law provides a definitive computational check, which is used to great effect by most modelers. Second, the fundamental GE equations in the GTAP framework provide accounting consistency. There is thus no double counting. Key accounting identities in the model include:

- commodity and factor market clearing conditions;
- private and public household budget constraints; and
- balance of payments conditions.

Third there are the inter-industry effects. Sometimes it is valuable to have an exhaustive model to account for the internal effects in an economy. Especially in the agricultural and food sectors, inputs and outputs often become blurred. Finally, CGE results place an emphasis on the impact of a policy change on factors and households, and thus ultimately on people. Conventional supply-demand analysis on the other hand focuses on commodity prices, consumer and producer surplus. The GE analysis is thus more valuable in terms of welfare effects, which, especially with the GTAP framework, can be broken down into various effects for each commodity. To put this in perspective, GE analysis highlights the importance of relative levels of taxation and technological change in the various sector of the economy.

GE analysis should however be avoided when the study is only concerned with sectoral effects, and when there is a need to introduce a lot of complexity into the model. Data and time constraints can also prevent researchers from building GE models. They are however highly suitable when research questions cut across food and non-food sectors, such as trade liberalization, tax reform, and growth and technological change. GE analysis is also a must when inter-industry linkages are important.

The next section will focus on the GTAP framework, and leans heavily on the GTAP book,edited by Hertel (1997). Due to the richness of the theoretical basis on which the model is based, this study will not attempt to give a detailed account of the framework, but will rather give a general description of the model. More detail on relevant issues concerning the model will be presented in the last two sections of this Chapter.

4.4 The GTAP Model

4.4.1 Introduction

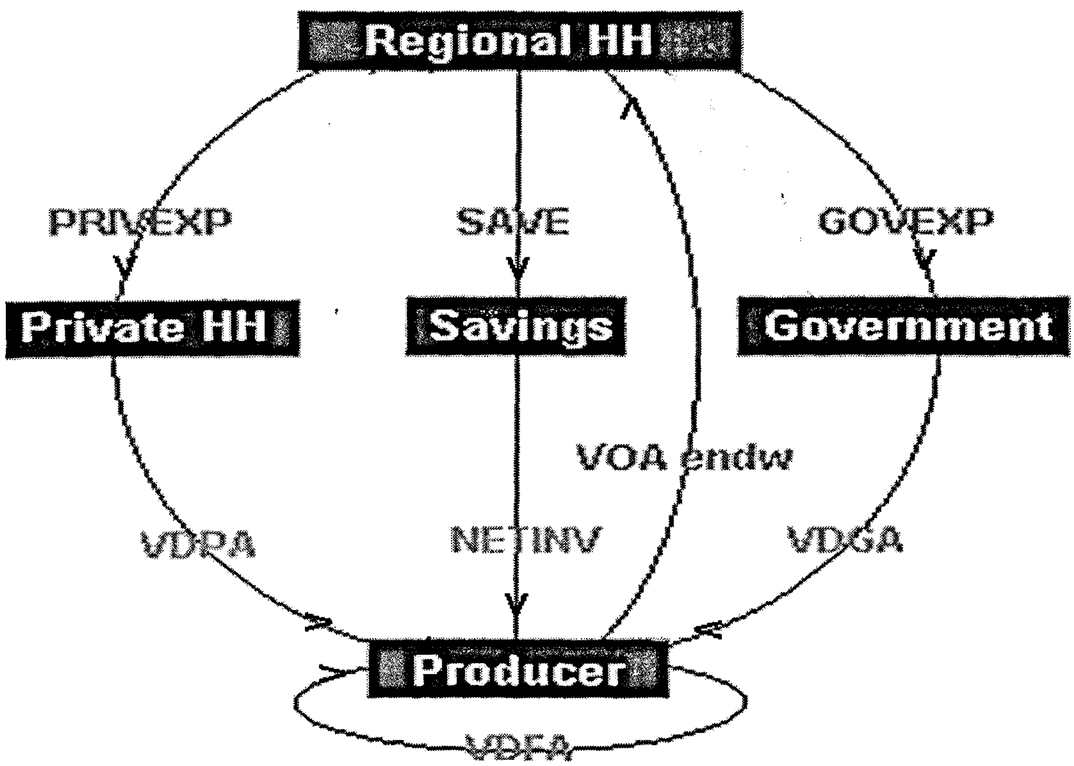
GTAP is a multi-regional applied general equilibrium (AGE) model, which captures world economic activity in 50 different industries in more than 20 countries. However, the theory behind the GTAP model is similar to that of other standard, multi-regional AGE models. As mentioned before, the underlying system of equations covers two different kinds of relationships. One part is concerned with accounting relationships, which ensure that receipts and expenditures of every agent in the economy are balanced. The other part of the equation system consists of behavioral equations based upon microeconomic theory. These equations specify the behavior of optimizing agents in the economy, such as demand functions.

The ever-increasing number of components necessary to build the GTAP framework, makes it very difficult to give an overview of the theory behind the model. The discussion will therefore initially consider a very simple closed economy structure , before moving on to a multi-region model of GTAP, where a trading sector will be introduced in the presence of taxes and subsidies.

4.4.2 One-Region Closed Economy

The following illustration will explain the basic concept of GTAP by focusing on the accounting relationships. The starting point in this exposition is a regional household associated with each country or composite region of GTAP (Figure 4.1). This regional household collects all income that is generated in the closed economy. According to a Cobb Douglas per capita utility function, regional income is exhaustively utilized over the three forms of final demand: private household expenditure (PRIVEXP), government expenditure (GOVEXP) and savings (SAVE). This approach represents the standard closure of GTAP in which at equilibrium each component of final demand gets a constant share of total regional income. Thus, an increase in regional income causes an equi-proportional change in private expenditure, government expenditure and savings.

Figure 4.1. One-Region Closed Economy without Government Intervention



Source: GTAP Lectures, 1999

Alternately, the level of government activities (GOVEXP), the level of savings (SAVE) or both components can be specified exogenously, so that private household income is calculated as a residual.

The focus now shifts onto the producers. The firms and the regional household, together with its three components of final demand, now build a closed economy. This makes it possible to take a closer look at the accounting identities specified in the GTAP model. Starting with the regional household, the top half of the figure shows that the available regional income consists of the value of output at agent's prices (VOA) paid by producers for the use of endowment commodities to the regional household. In order to give a clearer presentation, Figure 4.1 only displays the value flows in the economy. However, there are corresponding flows of ownership of assets, which pass through markets in the opposite direction. In the case described above, the value flow VOA has a corresponding flow of endowment commodities, going from the regional household back to the producers. This flow, like the other goods and service flows, is not included in the figures.

Figure 4.1 clearly indicates that available regional income is collected by the regional household and spent entirely on private household expenditure, government expenditure and savings. Modeling the components of final demand via this regional household has the advantage that no agent can spend more income than he/she receives. Besides, this concept of regional income is well suited to computing equivalent variation as a measure of regional welfare, which arises due to different policy scenarios (Hertel, 1997).

Having established the distribution of regional income, we are now in a position to consider the economic activities of other agents in the closed economy. First, Figure 4.1 shows the accounting relationship for the government, the private household and savings. According to this, the government spends its entire income on consumption goods, denoted as value of domestic government purchases, evaluated at agents' prices (VDGA). In order to model the behavior of the government, a Cobb Douglas sub-utility function is employed in GTAP. Thus, the assump-tion of constant budget shares, which we also find at the top-level nest of the utility tree, is applied once again. The second component of final demand is represented by the private household.

Corresponding to the accounting relationship, the private household uses all of its income to buy consumption commodities (value of domestic private household purchases, evaluated at agents' prices, VDPA). The constrained optimizing behavior of the private household is represented in GTAP by applying a CDE (constant difference of elasticity) function. This CDE function is less general than the fully flexible functional forms on the one hand, but more flexible than the commonly used CES functions on the other hand. It is easily calibrated using data on income and own price elasticities of demand (Hertel et al., 1991).

Considering the third component of final demand, the accounting relationship in Figure 4.1 shows that savings are completely used up in investment (NETINV). In GTAP the investment is savings-driven, according to the constant budget share of savings in the top-level nest of the utility tree. Given the static nature of the GTAP model, current investment is assumed not to be installed during the period under consideration, and therefore does not affect the productive capability of the industries in the model. However, investment represents a category of final demand and will affect the economic activity in the region through its effects on the demand structure. The mix of capital goods used for investment is treated in a manner analogous to the modeling of intermediate demand, which is discussed below.

Looking at the production side of the closed economy, Figure 4.1 also shows the accounting relationships of firms in GTAP. The producers receive payments for selling consumption goods to private households (VDPA) and the government (VDGA), intermediate inputs to other producers (value of domestic firm purchases, evaluated at agents' prices, VDFA) and investment goods to the savings sector (NETINV). Under the zero profit assumption employed in GTAP, these revenues must precisely match expenditures for intermediate inputs (VDFA) and primary factors of production (VOA).

The nested production technology in GTAP exhibits constant returns to scale and every sector produces a single output. Furthermore, it is assumed the technology is weakly separable, so that individual intermediate inputs and primary factors are used in fixed proportions to produce its output. Profit maximizing firms therefore choose their optimal mix of primary factors independently of the prices of intermediate inputs. Utilizing this type of separability also means that the elasticity of substitution between any individual primary factor and between any intermediate inputs is equal. Accordingly, the derived CES factor demand equations of domestically produced intermediate inputs and primary factors depend solely on the relative prices of intermediates and primary factors, respectively.

Among the primary factors, the GTAP model additionally distinguishes between endowment commodities which are perfectly mobile and those which are sluggish to adjust. In the former case, the factor earns the same market return regardless of where it is employed. In the case of sluggish endowment commodities, returns in equilibrium may differ across sectors.

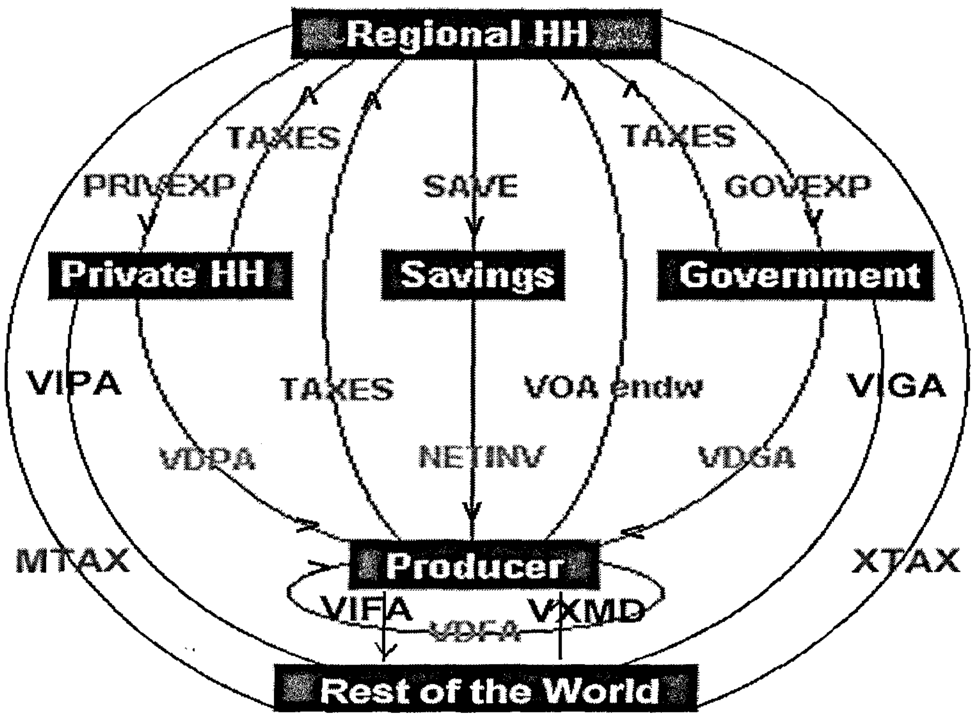
The complete accounting relationships in this one-region closed economy model form a simultaneous equation system in which one identity is redundant and can be dropped. In GTAP the savings–investment identity is not imposed. A separate computation of savings and investment therefore offers a consistency check on the accounting relationships and verifies that Walras' Law is satisfied. Since the model can only be solved for N– 1 prices, the price of savings is set exogenously, and all other prices are measured in proportion to this comparator.

4.4.3 Multi-Region Open Economy

Given knowledge of the theory behind the one-region version of the GTAP model, we are now in a position to integrate a trading sector into the model. All regions in the model except one are put together in a sector called "rest of the world". The single region is then used to show the changes in the model structure, which has to be done in order to model an open economy. Since these changes occur in every region of the multi-region model, a complete overview is given by this approach.

The rest of the world and the value flows initiated by this new agent are included in Figure 4.2. The Figure now represents a multi-region open economy in which the accounting relationships of all agents have changed. Considering the production side of the open economy, Figure 4.2 indicates that firms get additional revenues by selling commodities to the rest of the world. These exports are denoted by VXMD. On the other side, the producers now spend their revenues not only on primary factors and domestically produced intermediate inputs, but also on imported intermediate inputs, VIFA. Furthermore, the firms have to pay an additional consumption tax on imported inputs to the regional household. Since this tax expenditure is included in the TAXES flowing from the producer to regional household, the Figure does not show any change in this respect.

Figure 4.2. Multi-Region Open Economy



Source: GTAP Lectures, 1999

The GTAP model employs the Armington assumption in the trading sector, which provides the possibility of distinguishing imports by their origin and explains intra-industry trade of similar products. Thus, imported commodities are assumed to be separable from domestically produced goods and combined in an additional nest in the production tree. The elasticity of substitution in this input nest is equal across all uses. Under these circumstances, the firms first decide on the sourcing of their imports and, based on the resulting composite import price, they then determine the optimal mix of imported and domestic goods. In contrast to the closed economy, the multi-region model therefore includes separate conditional demand equations for domestic and imported intermediate inputs.

Figure 4.2 also shows the accounting relationships of the component of final demand in an open economy. Here, the government and private households not only spend their income on domestically produced commodities, but also on imported commodities. These are denoted as VIPA and VIGA, respectively. Furthermore, both agents have to pay additional commodity taxes on imports to the regional household, so that the accounting relationships of these two agents now include consumption taxes and expenditure for imported commodities. Analogous to the firms' behavior described above, the multi- region GTAP model includes conditional demand equations for imported commodities for the government and private households. Imported commodities and domestically produced commodities are also combined in a composite nest for the private household and the government household, respectively. The elasticity of substitution between imported and domestically produced goods in this composite nest of the utility tree is assumed to be equal across uses. Firms' and households' import demand equations therefore differ only in their import shares.

The accounting relationship of the third component of final demand, savings, has also changed. Since these variations cannot easily be represented in the Figure, the savings in figure 4.2 are denoted simply as global savings. In the multi-region version of the GTAP model, savings and investment are computed on a global basis, so that all savers in the model face a common price for this savings commodity. This means that if all other markets in the multi-regional model are in equilibrium, all firms earn zero profits, and all households are on their budget constraint, then global investment must equal global savings and Walras' Law will be satisfied.

Finally, we have to check the accounting relationships for the rest of the world. According to the Figure, the rest of the world gets payments for selling their goods to the private household, the government and the firms. These revenues will be spent on commodities exported from the single region to the rest of the world, denoted as VXMD, and on import taxes (MTAX) and export taxes (XTAX) paid to the regional household.

Trade generated tax revenues and subsidy expenditures are computed in a manner analogous to those raised by policy instruments used in the domestic market. The only difference is that now the tax or subsidy rates are defined as the ratio of market prices to world prices. If there is an import tax (subsidy), the market price is higher (lower) than the world price, so that the power of the *ad valorem* tax is greater (smaller) than unity. In the case of an export tax (subsidy), the market price lies below (above) the world price and the power of the *ad valorem* tax is smaller (greater) than unity.

The equations below briefly illustrate the discussion:

$$PM = PW \times T \tag{1}$$

$$\frac{\text{value of transaction at market price}}{\text{value of transaction at world price}} = T \tag{2}$$

where PM is the market price, PW the world price and T the tax rate.

GE analysis is always concerned about who pays the tax, and who receives the subsidy. This has profound consequences for the welfare analysis. Below is a brief summary of gainers and losers:

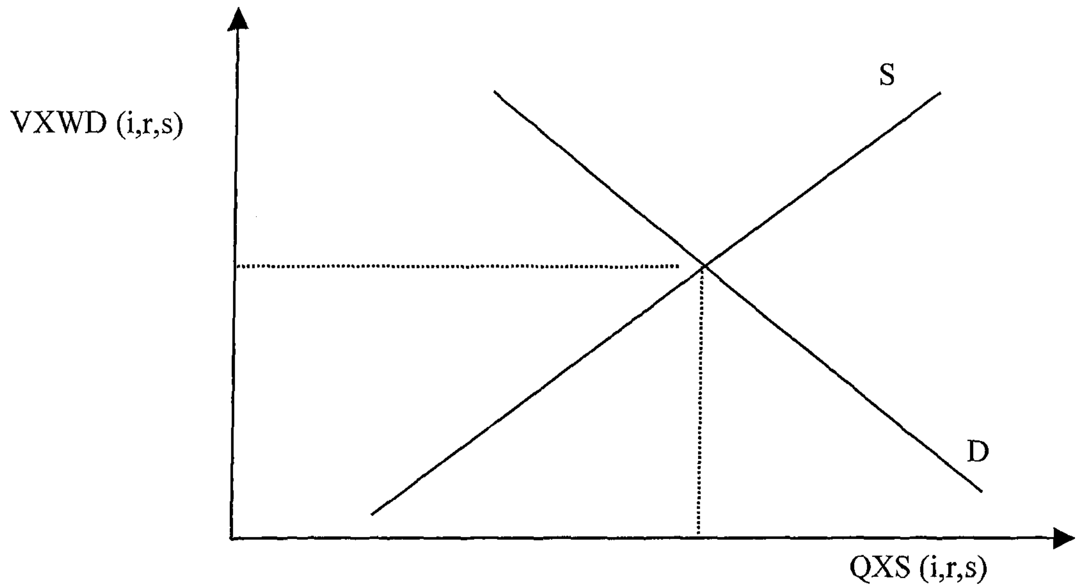
- export side $T < 1$ for a tax
  $T > 1$ for a subsidy
- import side $T > 1$ for a tax
  $T < 1$ for a subsidy

Once again these relations may be examined graphically. Figure 4.3 shows the market equilibrium without any interventions, where the value of exports supplied and demanded at world prices equals the value of exports in the local market at local market prices:

$$VXWD\ (i,r,s) = VXMD\ (i,r,s) \tag{3}$$

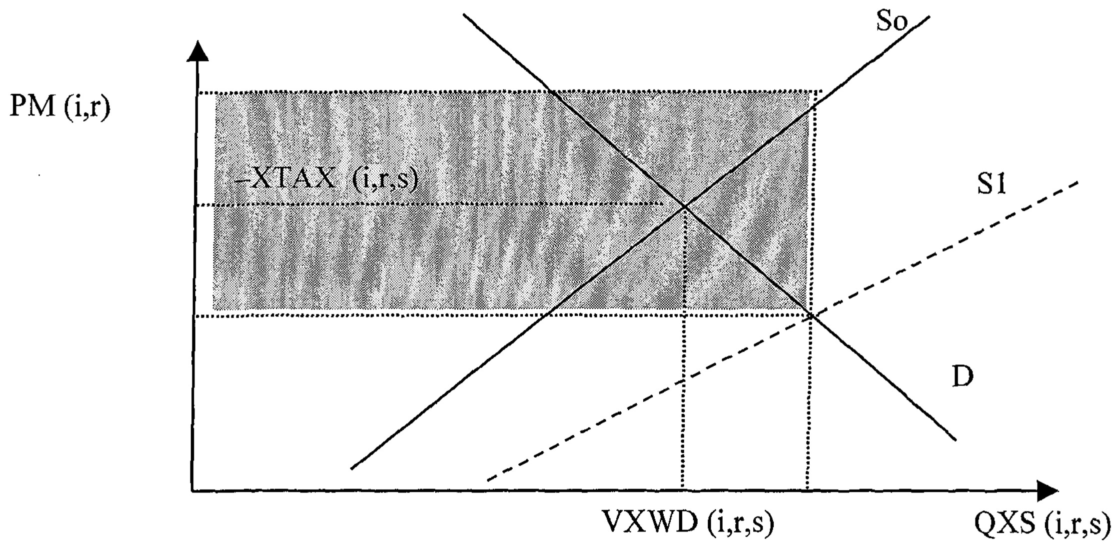
Figure 4.4 presents the situation where an export subsidy is imposed on the exports of commodity i from region r to region s. As a result of this intervention VXMD is now higher than the fob (free on board) price. In addition, the export subsidy calculated as VXWD – VXMD represents an expenditure paid by the regional household in region r to the producers of commodity i in region

Figure 4.3. Market Equilibrium



Source: GTAP Lectures, 1999

Figure 4.4. Export Subsidy



Source: GTAP Lectures, 1999

r. Hence, it is covered by the TAXES flowing from the regional household to the producers in Figure 4.2:

VXWD (i,r,s) = VXMD (i,r,s) + XTAX (i,r,s) (4)

The power of the export subsidy can be calculated as the ratio of the value of exports of commodity i from region r to region s, valued at the exporter's domestic market, by destination price (VXMD (i,r,s)) to the value of exports of commodity i from region r to region s, valued at the world prices, by destination (VSWD (i,r,s)):

TXS (i,r,s) = VXMD (i,r,s)/VXWD (i,r,s) (5)

TXS is bigger than one in the presence of an export subsidy. Thus, the domestic price of commodity i in region s is increased according to the price linkage relationship:

PM = PFOB / TXS (6)

The introduction of an import tax on the other hand drives a wedge between the world price and the local market price, whereby the domestic price is increased and the cif (cost, insurance and freight) price is decreased. Therefore, the power of the *ad valorem* import tax, TMS, calculated as the ratio of the value of imports of commodity i from region s to region r, valued at importers' domestic price (VIMS (i,s,r)) to the value of imports of commodity i from region s to region r, valued at cif price (VIWS (i,s,r)) is greater than one. Given the price linkage relationship

PMS = PCIF / TMS

the import tax revenues can be computed as follows:

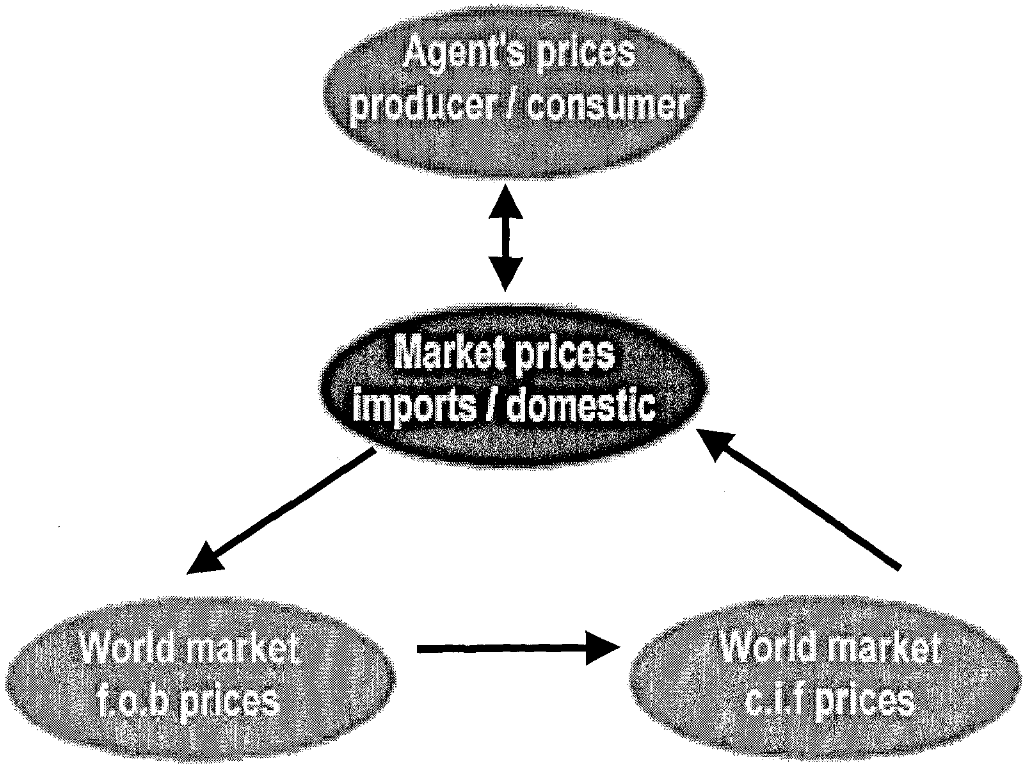
MTAX (i,s,r) = VIMS (i,s,r) - VIWS (i,s,r) (7)

These import taxes are paid by the purchaser of commodity i (private household, government and firms) in region r. Since tax revenues always accrue to the regional household these import taxes are included in the TAXES flowing from the private household, the government and the producers to the regional household in region r (Figure 4.2).

In the presence of an import subsidy the cif price of commodity i supplied from region s to region r exceeds the importer's domestic price. Accordingly, the power of the *ad valorem* import tax is less than one, and MTAX, calculated as the difference between VIMS and VIWS, is an expenditure that is withdrawn from the regional household.

Finally this Section is concluded by an overly simplified figure of the linkages between prices (Figure 4.5). The different taxes, which are imposed in various places by various agents, explain the differences in prices. These differences are governed in the GTAP framework by a complex set of equations, which are beyond the scope of this study. However, the next Section will explain in more detail the behavioral equations in the framework that explain some of the demand and supply responses of agents, concentrating on firm behavior. The last section of the Chapter will analyze the welfare system of equations constructed for the GTAP model.

Figure 4.5. Price Linkages



Source: GTAP Lectures, 1999

4.5 Behavioral Equations

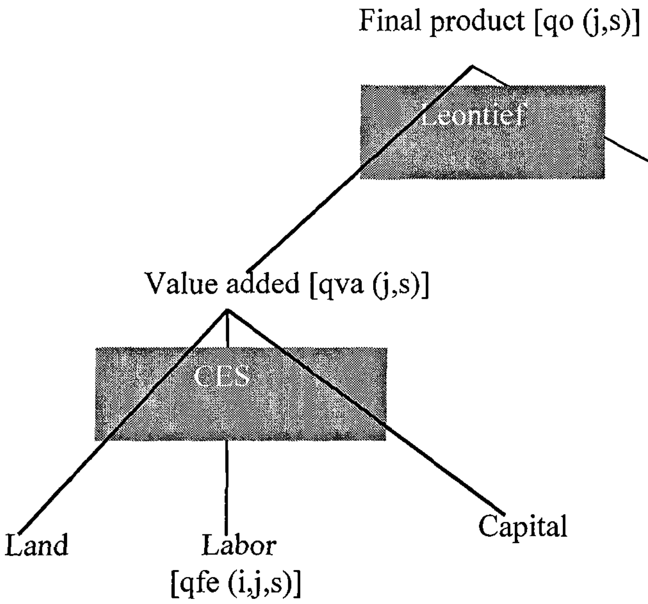
4.5.1 Firm Behavior

Figure 4.6 provides a visual display of the assumed technology for firms in each of the industries in the model. This kind of a production "tree" is a convenient way of representing separable, constant returns-to-scale technologies. At the bottom of the inverted tree are the individual inputs demanded by the firm. For example, the primary factors of production are land, laborand capital. Firms also purchase inter-mediate inputs. Some are produced domestically and some imported. With imports, the intermediate inputs must be from particular exporters. This sourcing occurs at the border, since information on the composition of imports by sector is unavailable. The model makes use of a constant elasticity of substitution (CES) nest between the firms' production tree bilateral imports.

The manner in which the firm combines individual inputs to produce its output depends largely on the assumptions that are made about separability in production. For example, it is assumedthat firms choose their optimal mix of primary factors independently of the prices of inter-mediate inputs. Since the level of output is also irrelevant, owing to the assumption of constant returns to scale, this leaves only the relative prices of land, labor and capital as arguments in the firms' conditional demand equations for components of value-added. By assuming this type of separability, the restriction that the elasticity of substitution between any individual primary factor and intermediate inputs be equal is imposed. This permits the model to draw the production tree, for it is this common elasticity of substitution that enters the fork in the inverted tree at which the intermediate and primary factors of production are joined. It also represents a significant reduction in the number of parameters that need to be provided in order to operationalize the model.

Within the primary factor branch of the production tree, substitution possibilities are also restricted to one parameter. This CES assumption is quite general in those sectors that employ only two inputs: capital and labor. However, in agriculture, where a third input, land, enters the production function, the model assumes that all elasticities of substitution are equal. This is certainly not true, but there is not enough information to calibrate a more general specification at this point (Hertel, 1997). In general, the behavioral parameters at each level in the production tree can be specified by the user of the model. However, the specific form of the equations used to represent firm behavior imposes the restriction of non-substitution between composite intermediates and primary factors.

Figure 4.6. The GTAP Production Tree



Source: GTAP Lectures, 1999

Turning to the intermediate input side of the production tree in Figure 4.6, it can be seen that the separability is symmetric, that is, the mix of intermediate inputs is also independent of the prices of primary factors. Furthermore, imported intermediates are assumed to be separable from domestically produced intermediate inputs. That is, firms first decide on the sourcing of their imports; then, based on the resulting composite import price, they determine the optimal mix of imported and domestic goods. This specification was first proposed by Armington (1969) and has since become known as the "Armington approach" to modeling import demand. However, it has been widely criticized in the literature. For example, Winters (1997) argue that the functional form is too restrictive. Although more flexible functional forms are preferable, this critique could apply just as well to every other behavioral relationship in the model. However at this stage a more flexible model would be nearly impossible to estimate in the context of a disaggregated global model.

The rest of this Section will be dedicated to explaining the equations that are incorporated into the production tree. There are different groups of equations involved, and each group of equations refers to one of the "nests" or branches in the technology tree discussed above. For each nest there are two types of equations. The first describes substitution among inputs within the nest. Its form follows directly from the CES form of the production function for that branch. The second type of equation is the composite price equation that determines the unit cost for the composite good produced by that branch. The composite price then enters the next higher nest in order to determine the demand for this composite.

The CES functional form was invented through an intuitive exposition that begins with the definition of the elasticity of substitution (Arrow et al. 1961). Consider the two input-case, where the elasticity of substitution is defined as the percentage change in the ratio of the two cost-minimizing input demands, given a 1 percent change in the inverse of their price ratio:

$$\sigma \equiv (Q_1/Q_2)/(P_2/P_1) \tag{8}$$

A familiar benchmark is the Cobb-Douglas case, whereby σ equals 1. In this case cost shares are invariant to price changes. For larger values of σ the rate of change in the quantity ratio exceeds the rate of change in the price ratio and the cost share of the input that becomes more expensive actually falls. Expressing equation (8) in percentage change form (lower case letters), we obtain:

$$(q_1 - q_2) = \sigma(p_2 - p_1). \tag{9}$$

In order to obtain the form of the regular demand equation, several substitutions are necessary. First, note that total differentiation of the production function, and use of the fact that firms' pay factors their marginal value product, gives the following relationship between inputs and output (i.e. the composite good):

$$q = \Theta_1 q_1 + (1 - \Theta_1) q_2 \tag{10}$$

where Θ_1 is the cost share of input 1 and $(1-\Theta_1)$ is the cost share of input 2. Solving for q_2 gives

$$q_2 = (q - \Theta_1 q_1)/(1-\Theta_1) \tag{11}$$

which may be substituted into (9) to yield

$$q_1 = \sigma(p_2 - p_1) + (q - \Theta_1 q_1)/(1-\Theta_1). \tag{12}$$

This simplifies to the following derived demand equation for the first input:

$$q_1 = (1-\Theta_1)\sigma(p_2 - p_1) + q \tag{13}$$

Note that this conditional demand equation is homogeneous of degree zero in prices, and the compensated cross-price elasticity of demand is equal to $(1-\Theta_1)*\sigma$. The final substitution required to obtain the CES demand equation introduces the percentage change in the composite price

$$p = \Theta_1 p_1 + (1-\Theta_1)p_2. \tag{14}$$

This is identical to the zero profit condition, except that both sides are divided by the value of output at agents' prices. Since all revenue is spent on costs, the resulting coefficients weighting input prices are the respective cost shares. From here, one can proceed in a manner analogous to that explored above, first solving for p_2 as a function of p_1 and p, and then substituting this into (13) to obtain

$$q_1 = (1-\Theta_1)\sigma\{(p - \Theta_1 p_1)/(1-\Theta_1) - p_1\} = q. \tag{15}$$

This simplifies to the following final form for the first input in this CES composite

$$q_1 = \sigma(p - p_1) + q. \tag{16}$$

The beauty of equation (16) is the intuition it offers, and the fact that its form is unchanged when the number of inputs increases beyond two (Hertel, 1997). This equation decomposes the change in a firm's derived demand into two parts. The first is the substitution effect. It is the product of the (constant) elasticity of substitution and the percentage change in the ratio of the composite price to the price of input 1. The second component is the expansion effect. Owing to constant returns to scale, this is simply an equi-proportionate relationship between output and input.

4.5.2 Tariff Reform

At this point it is useful to employ the linearized representation of producer behavior to think through the effects of a trade policy shock. Consider, for example, a reduction of the bilateral tariff on imports of i from r into s. This lowers $pms(i,r,s)$ via price linkage, as shown in equation (17):

$$pms(i,r,s) = tm(i,s) + tms(i,r,s) + pcif(i,r,s) \qquad (17)$$

Domestic users immediately substitute away from competing imports according to equation (18):

$$qxs(i,r,s) = qim(i,s) - \sigma_m(i) * [pms(i,r,s) - pim(i,s)] \qquad (18)$$

Also, the composite price of imports facing sector j falls via equations (19) and (20), thereby increasing the aggregate demand for imports through equation (21):

$$pim(i,s) = \sum_{REG} MSHRS(i,k,s) * pms(i,k,s) \qquad (19)$$

$$pfm(i,j,r) = tfm(i,j,r) + pim(i,r) \qquad (20)$$

$$qfm(i,j,s) = qf(i,j,s) - \sigma_D(i) * [pfm(i,j,s) - pf(i,j,s)] \qquad (21)$$

Cheaper imports serve to lower the composite price of intermediates through equation (22), which causes excess profits at current prices, via equation (23):

$$pf(i,j,r) = FMSHR(i,j,r) * pfm(i,j,r) + [1 - FMSHR(i,j,r)] * pfd(ijr) \qquad (22)$$

$$VOA(j,r) * ps(j,r) =$$

$$\sum_{ENDW} VFA(i,j,r) * pfe(i,j,r) + \sum_{TRAD} VFA(i,j,r) * pf(i,j,r) + VOA(j,r) * profitslack(j,r) \qquad (23)$$

This in turn induces output to expand, which then generates an expansion effect via equations (24) and (25):

$$qva(j,r) + ava(j,r) = qo(j,r) - ao(j,r) \qquad (24)$$

$$qf(i,j,r) + af(i,j,r) = qo(j,r) - ao(j,r). \qquad (25)$$

The expansion effect induces increased demands for primary factors of production via equation (26):

$$qfe(i,j,r) = afe(i,j,r) = qva(j,r) - \sigma_{VA}(j) * [pfe(i,j,r) - afe(i,j,r) - pva(j,r)] \qquad (26)$$

In a partial equilibrium closure, labor and capital might be assumed forthcoming in perfectly elastic supply from the sectors, so $pfe(i, j, r)$ is unchanged for $i =$ labor, capital. However, in the general equilibrium model, this expansion generates an excess demand via the mobile endowment market clearing condition equation (27), thereby bidding up the prices of these factors, and transmitting the shock to other sectors in the liberalizing region:

$$VOM(i,r) * qo(i,r) = \sum_{PROD} VFM(i,j,r) * qfe(i,j,r) + VOM(i,r) * endwslack(i,r) \qquad (27)$$

Now turn to region r, which produces the goods for which $tms(i,r,s)$ is reduced. Equation (18) may be used to determine the implications for total sales of i from r to s, given the responses of agents in region r to the tariff shock. Equation (28) dictates the subsequent implications for total output:

$$VOM(i,r) * qo(i,r) = VDM(i,r) * qds(i,r) + VST(i,r) * qst(i,r) + \sum_{REG} VXMD(i,r,s) * qxs(i,r,s) + VOM(i,r) * tradslack(i,r) \qquad (28)$$

At this point, the equations (24) and (25) again come into play, transmitting the expansion effect back to intermediate demands and to region r's factor markets.

4.6 The Decomposition of Welfare Changes in the GTAP Model

In this section an extension to the theoretical structure of the GTAP model of the world economy is discussed, which facilitates further analysis of welfare changes. This is accomplished by implementing a decomposition of the equivalent variation (EV) welfare measure currently employed in the model. The single-region version of the GTAP model (Hertel and Swaminathan, 1996) has proven to be very convenient for teaching and demonstration purposes, so the decomposition is first developed in this context and then extended to the multi-region case.

In a comparative static AGE model, with fixed endowments and technology, the only means of increasing welfare is by reducing the excess burden owing to existing distortions. Furthermore, any change in allocative efficiency may be directly related to taxes interacting with equilibrium quantity changes. Thus the following form for the single region decomposition of EV can be used:

EV_ALT = [.01/INCRATIO]*[sum(i,NSAV_COMM, PTAX(i) * qo(i))
+ sum(i,ENDW_COMM,sum(j,PROD_COMM, ETAX(i,j) * qfe(i,j)))
+ sum(j,PROD_COMM, sum(i,TRAD_COMM, DFTAX(i,j) * qf(i,j)))
+ sum(i,TRAD_COMM, DPTAX(i) * qp(i))
+ sum(i,TRAD_COMM, DGTAX(i) * qg(i))
+ sum(i,TRAD_COMM, VPA(i) - VPA(i)*INCPAR(i)) * up. (29)

The first term on the right hand side is a scale factor and will be discussed at greater length below. The following five terms in brackets (.) correspond to transaction tax instruments in the one-region model: PTAX represents a tax on output of good i; ETAX a tax on use of endowment i in industry j; DFTAX a tax on use of intermediate good i in industry; DPTAX a tax on private household consumption of good I; and DGTAX a tax on government consumption of good i. Each tax (subsidy) is paired with the relevant quantity change, which usefully defines the nature of the tax. For example, qfe (i,j) is the percentage change in derived demand by industry j for endowment commodity i. As mentioned above, ETAX(i,j) is the tax on endowment i usage in sector j. For those unfamiliar with GTAP notation, the other quantity changes are: qo(i)– change in supply of good I; qf(i,j) – change in derived demand for intermediate good i by sector j; qp(i) – change in consumer demand for good I; and qg(i) – change in government demand for good i.

The final term on the right hand side of equation (29) arises due to the non-homothetic nature of private household preferences in the GTAP model. VPA(i) is the value at agents' prices of private household purchases of good I; INCPAR(i) is an income expansion parameter from the CDE minimum expenditure function used to represent private household preferences in the model and is related to the income elasticity of demand for good I; and "up" represents the percentage change in private household utility. If good i is a superior (inferior) good such that INCPAR(i) is greater than (less than) one (which implies that VPA(i) – VPA(i) times INCPAR(i) is less than (greater than) one, and the change in household utility is positive) then a greater (smaller) proportion of household income is being spent on good i than before the model was shocked. This means that less (more) income is available to spend on all other goods and the overall effect on welfare is negative (positive).

The intuition behind much of the decomposition is quite straightforward. For example, it is welfare improving to increase the level of a relatively highly taxed activity, since this involves the re-allocation of a commodity or endowment from a low value use into a relatively high social marginal value usage. Conversely, if the simulation in question reduces the level of a subsidized activity, this will tend to benefit the economy in question, since it involves the re-allocation of resources away from a relatively low social marginal value product use. Furthermore, note that if there are no taxes in the initial equilibrium, and the nature of the shock is something other than a tax/subsidy intervention, then there will be no allocative efficiency effect from the simulation.

The decomposition offered in equation (29) is designed to provide as much detail on the sources of the welfare changes from policy experiments as possible. Not only can it show that a portion of the overall welfare change has resulted from decreased output (qo), but it also shows the components of the change in terms of output changes of specific commodities interacting with the output taxes or subsidies (PTAX) present in the model for each of the commodities in question. Likewise, if a model simulation resulted in an increase in the use of an intermediate input (qf) that is taxed (DFTAX), the decomposition clearly shows how this contributes positively to the overall welfare change. Finally, note that summation of all of the various terms in the decomposition equals the overall welfare change from the policy simulation under study.

4.7 Summary

It needs to stressed again at the end of this Chapter, that to model a field as complex as trade is a tremendous task, which needs a very complex model. However the model can never capture all the effects, and it will always just be an abstraction of the real thing. However, the GTAP model has been used to great advantage in many studies and is based on sound assumptions, which can be adjusted to suit the purpose of the individual modeler. For this study, the GTAP framework puts the researcher in a position to make more than an educated guess at the effects of the FTA between South Africa and the EU. The welfare analysis will clearly indicate the gains and losses experienced by the various regions, and thus ultimately show whether the agreement will be favorable to South Africa. The next two Chapters will explore this burning question.