EFFECTIVE TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE IN TEACHING QUADRATIC FUNCTIONS IN MATHEMATICS

BY

CHARLES DUZEPHI SIBUYI

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Department of Science, Mathematics and Technology Education

Faculty of Education

University of Pretoria

PRETORIA

Supervisor: Prof: G.O.M Onwu

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APPROVAL

This research work has been examined and is approved as meeting the required standards of scholarship for partial fulfilment of the requirements for the degree of Master of Education at the University of Pretoria.

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Supervisor **Date**

_____ _____

External Examiner **Date**

DEDICATION

I dedicate this dissertation to my father, Sibuyi Mabihana Willias, who passed on the 7th of February 2009 to be with his Lord and Saviour Jesus Christ.

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ABSTRACT

This study investigated the pedagogical content knowledge supposedly held by two FET mathematics teachers from Mpumalanga Province as they taught quadratic functions in grade 11 classes. The criterion for selecting the two teachers was that they had consistently produced good results (overall pass rate of 80% or more) in the grade 12 mathematics examinations of the National Senior Certificate for the past three years or more and thus, they were classed as effective. The two teachers prepared and taught lessons on quadratic functions in grade 11 whilst they were being observed. The study focused on teacher knowledge base as exemplified in the teachers' pedagogical content knowledge (PCK). Three elements of PCK were investigated; namely; (i) knowledge of the subject matter; (ii) knowledge of teaching strategies and (iii) knowledge of learners' conceptions. Qualitative research approach using the case study research method was used to collect qualitative data on the pedagogical content knowledge of the two teachers through lesson observations, lesson plan analysis and interviews. Analysis of the results suggests that the two teachers have adequate subject matter knowledge but have limited knowledge on the aspects of teaching strategies and knowledge of learners' pre-conceptions and misconceptions on the topics of quadratic functions that they taught. The study recommends that teachers be exposed to workshops that deal specifically with the various topic specific teaching strategies and knowledge of learners' pre-conception and misconceptions on the topic of quadratic functions.

Keywords: *Pedagogical content knowledge, Knowledge of teaching strategies, Knowledge of learners' conceptions, knowledge of the subject matter, quadratic functions*

LIST OF TERMS

Effective teachers: In this study, effective teachers refer to those teachers who have consistently produced good results, an average pass rate of 80% or more in the National Senior Certificate grade 12 mathematics examinations for the past three years or more as reflected in the district's grade 12 performance statistics.

Pedagogical Content Knowledge (PCK): In this study pedagogical content knowledge mean an amalgam of (i) specific content knowledge on quadratic functions, (ii) knowledge of teaching strategies and application and (iii) prior knowledge of learners' conceptions that allow a teacher to transform specific content knowledge in a more conceptually accessible version for the learners.

Subject matter knowledge: In this study subject matter knowledge (as displayed by the teacher) mean the correct application of mathematical concepts, facts and procedures, the reasons underlying mathematical procedures and the relationship between mathematical concepts during classroom teaching of quadratic functions.

Knowledge of learners' conceptions: Knowledge of learners' conceptions in the study is defined as the teachers' awareness of learners' prior knowledge which may include pre-conceptions, misconceptions, learning difficulties and correct conceptions they may have, which can be used by the teacher on the students behalf during classroom teaching and lesson planning for effective teaching.

Knowledge of Pedagogy: In the study, knowledge of pedagogy refers to knowledge of planning and organization of a mathematics lesson and teaching strategies for effective teaching of the particular topic under investigation.

Knowledge of curriculum: In the study; knowledge of curriculum refers to knowledge about learning goals for different grade levels for use to organise lesson planning and classroom teaching.

Procedural knowledge: Procedural knowledge is regarded as knowledge of mathematical rules, algorithms and procedures that a teacher uses to assist learners to learn how to solve mathematical problems quickly and efficiently because it is to some extent automated through drill work and practice.

Conceptual Knowledge: In this study, conceptual knowledge refers to knowledge, as displayed by the teacher, of the core concepts and principles and their interrelations in the mathematics domain.

Misconceptions: In this study misconceptions refer to pieces of wrong knowledge that may arise as result of learners' prior experience and learning both inside and outside of the classroom and effective mathematics teachers should have knowledge to diagnose and eliminate such wrong knowledge.

Teaching Strategies: In this study, teaching strategies refers to methods used by teachers to create learning environments and to specify the nature of the activities in which the teacher and learners will be involved during the lesson to ensure that the sequence or delivery of the lesson helps learners to understand the topic taught.

CHAPTER 1
INTRODUCTION AND BACKGROUND

1.1 INTRODUCTION

Skills in mathematical reasoning are becoming even more important than ever before in the workplace and everyday living, driven in part by emerging technologies and job demands. To assure that learners in schools gain the indispensable mathematical reasoning required in life, the new curriculum in South Africa, the National Curriculum Statement (NCS); calls for educators to assure that their learners participate during mathematics lessons and express their mathematical ideas. According to Brodie (2007:p.3), "getting learners to talk is seen as important because it (i) shows that learners are attending to the lesson; (ii) allows learners to express and clarify their own ideas; (iii) enables learners to share ideas with each other; and (iv) provides teachers with information about what learners know and do not know and how learners are thinking and trying to make sense of ideas. Teachers are encouraged to make their mathematics lessons more learner-centred by encouraging learners to contribute to the lesson."

To achieve this kind of approach to teaching, schools need quality teachers who have the appropriate knowledge about the art of teaching. Without doubt teachers are one of the most powerful influences on students' engagement with mathematics (Attard, 2011). Such teachers, according to Tanner (2003), "create experiences that help students make sense of the knowledge and skills being studied". According to Turnuklu and Yesildere (2007:p.1), "although a number of factors may influence the effective teaching of a particular subject, teachers play an important role in that success". Good teachers, Attard (2011) claims, can achieve high and consistent levels of engagement and effective learning. Contrary to common belief in society that a teacher who knows a particular subject very well is best suited to teach such a subject, research has shown that this belief is not necessarily true (Shulman 1986, 1987; Hill, Rowan and Ball 2005; Etkina, 2010). Various researchers such as Shulman (1986, 1987), Hill, Rowan and Ball (2005) and in particular, Etkina (2010:p.1) emphasise that "teachers of a specific subject should possess special understandings and abilities that integrate their knowledge of the content of the subject that they are teaching as well as having knowledge of the learners who are learning the content". Knowledge of the learners includes; amongst other things; having knowledge of what pre-conceptions,

misconceptions and difficulties that the learners might have about a topic to be taught. As educators know, teaching is a complicated practice that requires an interweaving of many aspects of specialised knowledge (Mishra and Koehler, 2006). Such specialised knowledge includes knowledge of pedagogy; knowledge of the subject matter and knowledge of the learners as explained above.

1.2 BACKGROUND OF THE STUDY

In South Africa, the general performance of learners in mathematics and Science in the National Senior Certificate examinations was recorded as being poor for the period between 2008 and 2011 (Report on National Senior Certificate Examination, 2011). In terms of this report, the percentage of learners in the whole country who managed to obtain a mark above the 40% pass level in mathematics in the 2011 Grade 12 final examination is shown in the table below.

YEAR	**2008**	**2009**	**2010**	**2011**
Total registered learners in Mathematics	300 008	290 407	263 034	224 635
% of learners who obtained passes above 40% pass level	29.9	29.4	30.9	30.1

The information in the table shows a decline in the number of learners taking mathematics at school and also their low levels of performance. This trend of poor performance in mathematics by South African learners can be traced back to earlier pre-democracy years of South Africa (CDE, 2004). In a research report released by the Centre for Development and Enterprise (CDE, 2004), educational planners and those involved in education in the country are concerned about the poor grade 12 mathematics results and the quality of education in mathematics and science offered in schools generally. Aside from the poor quality of education, there is a growing concern in the country about the dwindling numbers of learners leaving school with sufficiently good grades to enter mathematics and science-based courses at tertiary institutions. In this same report (CDE, 2004), statistical data reveal that between the period 1991 to 2003, enrolment in mathematics Higher Grade, a subject essential for entry into many tertiary education science courses, plummeted from 53 631 to 39 159. In Mpumalanga province for example, of the 359 schools that offered mathematics at higher

Grade level in 2003, only 19 schools managed to produce one or more candidates who obtained symbols in the 80% pass region. The CDE report (2004) goes on to conclude in rather ominous terms that failure to improve mathematics and science education is probably the most significant obstacle to African advancement in South Africa because this reality undermines the country's ambition for expanded economic growth, black empowerment and community development. Based on this information, it would be of interest to know what research would say about the possible causes of poor performance in mathematics by South African learners.

A more recent report by CDE (2011) that investigated the quantity and quality of South African school teachers indicates that South Africa is at or near the bottom of other developing countries when ranked in terms of student performances in mathematics and Science. In the same report, it is revealed that many of the existing teachers of mathematics and Science are not teaching the subjects well and are also poorly managed. This report points to poor teaching by teachers at schools as the main cause of poor performance in Mathematics and Science.

Other researchers in South Africa (Howie, 2003; De Clercq, 2008; CDE, 2004 and 2011) also identified various in-schools and out-of-school factors that impact on learner performance in mathematics. De Clercq (2008), in her study of teacher quality, appraisal and development, asserts that factors contributing to poor learner performance in mathematics in developing countries (such as South Africa), include teacher quality, the socio-economic background of learners and their communities, the context of schooling, poor school leadership and poor or under-resourced school facilities. The CDE (2004 and 2011) research reports also point to teacher quality, classroom environment and language of instruction as factors accounting for the poor performances of learners in Mathematics and Science.

The language of learning and teaching may also contribute to South African learners' poor performance in mathematics (Howie 2003). Based on data gathered from TIMSS (Trends in mathematics and Science Studies, 1999), a study was done Howie (2003) to assess whether language and other background factors affected secondary school pupils' (Grades 7, 8 and 12) performance in mathematics in South Africa. It was found that learners, whose proficiency in English (as a medium of learning and teaching) was good, performed significantly better in mathematics than learners who had a poor proficiency in English. In

most rural and township schools of South Africa, English is used as a language of learning and teaching although it is a second language for most learners (CDE, 2004).

The history of the South African education system may also be assumed to have had an impact on the current situation of poor performance in Mathematics at schools. Adler and Reed (2002), state that segregation, fragmentation, authoritarian and bureaucratic control of the curriculum, institutions and governance, inefficiency and inequality have characterised South African education for a long time. Each of these are said to have had a considerable effect on the present performance levels in Mathematics and Science, particularly, in the rural and township schools where the culture of teaching and learning is said to have virtually collapsed. Furthermore, Bush (2003) posits that years of struggle against apartheid inevitably affected schools, particularly those in the townships. Teachers formed teacher unions that played a key role in the political struggle and because educators were frequently absent from school to engage in protest activities, the culture of learning and teaching was not sustained.

Educator factors that have consistently been linked to poor performance in Mathematics and Science include teachers' knowledge of Mathematics and the skill of performing the teaching task (Ingvarson, Beavis, Bishop, Peck and Elsworth, 2004; Baumert, Kunter, Blum, Brunner, Voss, Jordan, Kusman, Kraus, Neubrand and Tsai, 2010). Hill, Rowan and Ball (2005) in their study that explored whether and how teachers' mathematical knowledge for teaching contributes to gains in learners' Mathematics achievement, found that teachers' mathematical knowledge was significantly related to learners' achievement. The knowledge about teaching and learning that teachers bring to the classroom has an impact on whether learners will access the topics that teachers teach.

Still on teacher factors, Baumert et al (2010) posit that the pool of alternative mathematical representations and explanations given by teachers to learners in the classroom are largely dependent on the breadth and depth of the teachers conceptual understanding of the subject, and that insufficient understanding of the mathematical content, limits the teachers' capacity to explain and represent that content to learners in a sense-making way. This is a deficit that cannot be offset by pedagogical skills alone. Anecdotal evidence suggests that efforts of teachers with limited conceptual understanding of the mathematics topics that they teach fall short of providing students with powerful mathematical experiences.

Although the general performance of most learners in the National Senior Certificate mathematics examination in most South African schools is poor, as already alluded to in the previous paragraphs, there are, however, some schools that are consistently producing good National Senior Certificate results (Grade 12) and high quality work (80% and above pass level) in Mathematics (CDE, 2004; NSTF, 2007). Mathematics teachers in such schools may be presumed to be doing something differently from what other teachers in less effective schools are doing. Effective teachers in such good performing schools were the focus of this study.

Influenced by the fact that research has pointed to pedagogical content knowledge of a teacher as having an influence on how teachers make a lesson topic accessible to their learners (Shulman, 1986), and the fact that research also points to poor teacher knowledge as being a contributory factor to poor performance (CDE, 2011), the interest of this study concerned investigating the pedagogical content knowledge held by two teachers who where classed as effective, as they taught topics of quadratic functions in Grade 11.

The concept of quadratic functions was chosen for this study for several reasons. First, it serves as an entry point to the study of polynomial functions in mathematics. Second, according to Zaslavsky (1997), the functions concept is the foundation for all mathematics fields. The third reason for selecting this concept is that it has many uses in career-related professions such as business, engineering and science where the concept is used for modelling ideal situations. In business it may be used to help in forecasting profit and loss. The U-shape of a parabola is incorporated in science in the construction of structures such as the parabolic reflectors of satellite dishes and car head lamps. Good insight into quadratic functions will enable learners to deal with different types of functions such as trigonometric functions, linear functions, exponential functions and logarithmic functions, leading to an understanding of real-life uses of this concept. Moreover, several mathematics teachers in the circuit where the study was conducted complain that learners are not performing well in this topic during examinations. On the basis of this explanation, the concept of the quadratic function was chosen for the study.

1.3 THE PURPOSE OF THE STUDY

The purpose of the study was to investigate the pedagogical content knowledge held by two teachers who were classed as effective since their learners have consistently achieved good

passes (average pass rate of 80% and above) in Grade 12 mathematics National Curriculum Statement examination in mathematics for the past three or more years.

Specifically, the study investigated what pedagogical content knowledge (PCK) these two teachers considered to be effective have with regard to the teaching of quadratic functions. The study also sought to determine how the teachers had acquired the PCK that they were using.

1.4 PROBLEM STATEMENT

The problem of this study was to determine what teacher knowledge base two effective mathematics teachers have and display in the context of teaching the topic on quadratic functions in Grade 11 Mathematics classrooms and how they acquired it (PCK).

The following research questions are derived from the problem statement.

1.5 RESEARCH QUESTIONS

- What pedagogical content knowledge do the two teachers display in teaching quadratic functions in Grade 11?

- How did the teachers develop the pedagogical content knowledge that they use in teaching quadratic functions?

1.6 SIGNIFICANCE OF THE STUDY

The study was undertaken because the issue of effective teaching of mathematics is of vital importance to the provision of quality education. It is believed that the results of the study will have beneficial application as it seeks to reveal the PCK held by the two participating teachers with regard to the teaching of quadratic functions. Such knowledge and skills could be used to develop in-service and pre-service teacher education programmes aimed at improving the quality of teaching of mathematics specifically, the teaching of quadratic functions. The teacher educational programmes developed would be of practical value, at the same time making a significant contribution to general educational theory.

Pedagogical content knowledge (PCK) remains a vague form of knowledge which cannot be easily isolated and studied separately from other teacher knowledge bases because it is

individualistic, is developed by the teacher and is unique to that teacher. However, it (PCK) provides researchers with a starting point for collecting and analysing data regarding teacher knowledge. It was felt that such knowledge would be useful to practising mathematics teachers and could be used to lay a pathway to finding improved methods for teaching topics in mathematics which learners may find difficult to understand. The scientific contribution of the study is that it would contribute to closing the knowledge gap if any, between teacher practices and their knowledge base which has an impact on effective teaching and the achievement of learners with regard to the topic of quadratic functions.

1.7 OVERVIEW OF CHAPTERS

Chapter one provides an introduction to the study, the background, significance and purpose of the study, the problem statement, the research questions and an overview of the content of the chapters in the study.

The second chapter reviews and discusses some aspect of the literature needed to justify the study. The findings and the research methodologies employed by other researchers in similar studies were examined. Special attention was given to findings as well as methods used in other studies that investigated the pedagogical content knowledge of teachers where the three elements, knowledge of subject matter, knowledge of teaching strategies and knowledge of learners' conceptions were used as a framework that was adopted for this research. The chapter concludes by discussing the conceptual framework designed for this study.

The third chapter outlines the study's methodology. It also describes the population and the procedures for sampling the two teachers for the case studies. The development of the research instruments are described as well as their validation. The data analysis is also explained. Ethical issues that were considered for this study are dealt with as well.

The fourth chapter of the study presents the findings. Each case study is narrated in terms of the data obtained from the observations, document analysis and interview schedules.

The fifth and last chapter discusses the findings as reflected in the data obtained from the two case studies using the framework developed in Chapter Two. Similarities and differences of the case studies are highlighted by focusing on the three themes identified for the study: knowledge of the subject matter, knowledge of teaching strategies and knowledge of

learners' conceptions. As a conclusion to the study; recommendations for appropriate teacher development and further research on the issue are offered.

1.8 CHAPTER CONCLUSION

This chapter discussed the background of the study which was mainly about the poor performance in Mathematics by most Grade 12 learners in the South African education system. Various possible factors causing poor performance were tabled. The purpose, research questions, significance of the study which emphasised the need to investigate the pedagogical content knowledge (PCK) held by the two case teachers were also discussed. The chapter ends with an overview of the chapters contained in this report.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, the concept of Pedagogical content knowledge (PCK) as applied in the teaching of mathematics is discussed. Each of the three elements of Pedagogical content knowledge that had been used as framework for this study are explained. The elements are; (i) knowledge of subject matter; (ii) knowledge of teaching strategies and (iii) knowledge of learners' conceptions are also explained. The conceptual framework for the study is also presented in this chapter.

2.2 What is Pedagogical Content Knowledge?

In a study of the knowledge bases that teachers must possess to teach effectively, Shulman (1987) identified pedagogical content knowledge (PCK) as one of the most important knowledge bases that teachers should possess in order to teach effectively. He maintained that having knowledge of the subject matter is not enough to teach it. Teachers need to possess pedagogical content knowledge as well. This knowledge base, PCK, must; according to Shulman (1987:p.8) include "knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of educational ends, purposes and values and their philosophical and historical bases". This has led researchers to now consider PCK as important as the subject matter knowledge. According to Shulman (1987), PCK depends on a teacher's subject matter knowledge, knowledge of pedagogy and on how the teacher transforms this knowledge into various forms that enable students in different learning environments to understand the subject matter. He acknowledges that pedagogical content knowledge is difficult to isolate and to measure.

Kwong, Joseph, Eric, Khoh, Gek and Eng (2007:p.28), indicate that "Shulman (1986; 1987) has suggested that pedagogical content knowledge (PCK) forms a unique and distinct knowledge domain of teacher cognition. PCK emphasises the manner in which teachers relate their subject matter knowledge (what they know about what they teach) to their pedagogical knowledge (what they know about teaching, how their learners' learn and the learners' conceptions) and how subject matter knowledge is part of the process of pedagogical

reasoning". Shulman (1986: p.9) defined PCK as "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others… . It also includes an understanding of what makes the learning of specific concepts easy or difficult, the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning environment". Thus, pedagogical content knowledge (PCK), according to Kwong et.al (2007:p.28) is viewed as that "distinctive knowledge domain of teaching that differentiates the expert teacher in a subject area from the subject expert". Furthermore, as Kwong et al (2007:p.28) asserts; "while general pedagogical knowledge can be generically applied to all teaching subjects, much of PCK is specific to individual topics in subjects". Darling-Hammond, 2000 indicate that "an emerging consensus is that teachers' knowledge of discipline-specific pedagogy is critical to being able to present topics within a range of subjects in a manner that learners will comprehend" while Kagan, 1992 and Reynolds, 1992 asserts that "studies have shown that novice teachers often struggle to present concepts in a manner understandable to their students because they have little or no PCK at their disposal". From the above paragraph, it can be seen that pedagogical content knowledge is an important knowledge base for teachers to have in order to teach mathematics topics effectively.

Mishra and Koehler (2006:p.1027) see pedagogical content knowledge as "that knowledge base which is concerned with the representation and formulation of concepts, pedagogical techniques, and knowledge of what makes concepts difficult or easy to learn, and knowledge of learners' prior knowledge" This is the same view that Shulman, 1986 and De Jong, 1999 hold regarding pedagogical content knowledge. Mishra and Koehler (2006:p.1027), further cite Rohaan, Taconis and Jochems, 2009 who see pedagogical content knowledge as that knowledge which "involves knowledge of teaching strategies that incorporate appropriate conceptual representations in order to address learner difficulties and misconceptions and to foster meaningful understanding". Carpenter, Fennema, Peterson and Carey (1988:p.386), see pedagogical content knowledge as the knowledge, held by a teacher, which "includes knowledge of the conceptual and procedural knowledge that students bring to the learning of a topic, the misconceptions about the topic that they may have developed, and the stages of understanding that they are likely to pass through in moving from a state of having little understanding of the topic to mastery of it. It also includes knowledge of techniques for assessing students' understanding and diagnosing their misconceptions, knowledge of

instructional strategies that can be used to enable students to connect what they are learning to the knowledge they already possess, and knowledge of instructional strategies to eliminate the misconceptions they may have developed"

From the above paragraphs, it can be seen that teachers need to have pedagogical content knowledge in order to teach their subjects effectively. It is for this reason that the study had interest in investigating the pedagogical content knowledge supposedly held by the two participating teachers.

2.2.1 Components of pedagogical content knowledge

The components of pedagogical content knowledge, according to Shulman (1986), comprise first, knowledge of the specific subject matter; second, knowledge of instructional strategies; third, knowledge of learners' conceptions; and fourth, an understanding of what makes the learning of a specific topic difficult or easy for learners. Shulman's (1986) fifth category of teachers' knowledge bases, curriculum knowledge, involves awareness of how topics are arranged both within a school year and over a given longer period and ways of using curriculum resources, such as textbooks, to organise a programme of study for students.

Pedagogical content knowledge, which is at the centre of this study, is an amalgam of a teacher's knowledge bases that Hagevik et al (2010) and Yusof and Zakaria (2010) say includes:

- knowledge of context, curriculum and assessment
- knowledge of student learning
- knowledge of instructional strategies and representations of Mathematics
- Knowledge of student understanding about concepts in Mathematics.

De Miranda (2008:p.17) described pedagogical content knowledge as "the knowledge of three knowledge bases coming together to inform teacher practice: namely, subject matter content knowledge, pedagogical knowledge and knowledge of context. Subject matter content knowledge is described as knowledge that is unique to mathematics teachers and separates, for example, an engineering and technology teacher from an engineer". Since different versions of the definition of PCK exist, for the purpose of this study, Shulman's

(1986) description of pedagogical content knowledge will be adopted as its theoretical framework in which he sees it as an amalgam of a teacher's knowledge base that includes:

1) Knowledge of the representation of subject matter for teaching

2) Knowledge of relevant instructional strategies

3) Knowledge of learners' conceptions (preconceptions and misconceptions).

This study focused on these three elements mentioned in the previous paragraph and were consciously integrated when observing how the two effective teachers displayed them when teaching quadratic functions in their respective Grade 11 mathematics classes. The choice of the three elements of PCK was influenced by the fact that they form the core of what Shulman (1986) indicated as teachers' PCK that would enable them (the teachers) to transform the subject matter in such a way that their learners would be readily able to access the content. First, the teacher needs to have a good grasp of the subject matter before being able to transform it. Second, the teacher needs a teaching strategy to use to make the subject accessible to the learners. Third, the teacher needs to have an idea of possible learners' conceptions that the learners may have about the topic in order to prepare explanations that will help to eliminate or reinforce the conceptions as is necessary. Furthermore, studying the PCK elements that a teacher holds cannot be done in isolation of each of its other elements since PCK is an amalgam within a teacher's total knowledge base that is uniquely constructed by the individual teacher. It is the intersection of knowledge of pedagogy, knowledge of learners' conceptions and subject matter knowledge (Mishra and Koehler, 2006), and is thus individualistic.

These three elements of PCK in this study were also used by Winsor (2003) in a study involving an investigation of pre-service mathematics teachers' knowledge of functions where he regarded them (the three elements of PCK) analogously as the legs of "a three-legged stool. The seat of the stool represents the PCK and each one of the legs represents subject matter knowledge, knowledge of learners and knowledge of instructional strategies. It is reasoned that the seat needs equal support from each leg while the legs need help from the seat to stand firmly". They are interdependent in relation to each other, and can thus not be studied in isolation although each has its own characteristics and function.

2.2.2 Pedagogical content knowledge in the teaching of mathematics

In a study conducted by Yusof and Zakaria (2010) that explored and described the level of pedagogic content knowledge of three teachers, focusing on the topic of functions at secondary level mathematics, it was found that the teachers who participated in that study lacked conceptual knowledge of topics on functions. Hence their lessons were inaccurate and lacked substantial clarity for the learners. In turn this inhibited understanding. As was the case with this study, these two scholars focused on how the lessons' contents were conveyed to learners. The PCK elements that were observed in that study involved the use of analogy, representation using symbols, examples, explanation or demonstration that were suitable for providing conceptual and procedural explanation or description, as well as noting the ways in which the teachers stimulated the teaching process. Specifically, the particular study sought to explore and describe the elements of teacher's PCK and also to determine the teacher's level of PCK on the topic of functions. A qualitative research approach based on a case study research design was used. The case study method used a combination of interviews, classroom observation and document analysis to collect data on the three teachers' PCK. In order to describe the teachers' PCK, the following guidelines, were used:

According to Yusof and Zakaria (2010:p.34), for "Level 1 PCK; at this level, the teacher explains the wrong concept or a concept which is not quite clear" to the learners. The teacher asks low level questions, the lesson is teacher-centred, and the teacher is unable to detect learners' difficulties with the topic. The teacher does not relate his lesson presentation to the learners' existing knowledge. For Level 2 PCK; at this level, the teacher explains the correct mathematical concepts but gives the same type of examples to back his explanations, no variety of examples are given. The teaching is seen to be teacher-centred. The teacher has an awareness of learners' difficulties but does not probe further through asking questions that allow learners to speak out their ideas about the topic. Level 3 PCK: The teacher's explanations of concepts are more accurate and clear along with the incorporation of suitable examples. Learners' participation is seen to be positive through the provision of relevant activities which provide both conceptual and procedural understanding of mathematics principles that are being studied"

In the current study the researcher observed the type of PCK that the two participating effective teachers used as they presented their lessons. The framework for observing the

teachers' PCK was based on the PCK elements that were given in earlier paragraphs but no classification of the PCK levels was done like what Yusof and Zakaria (2010) did.

The theme of Chick, Pham and Baker's study (2006) involved teachers' pedagogical content knowledge as they taught the subtraction algorithm. What became clear was that the teachers had a good lesson presentation but lacked knowledge of how to identify and correct students' misconceptions. A qualitative research approach using the case study research design was also used in that study. Data was collected via questionnaires, lesson observations and interviews and the following framework, containing these three attributes, was used to evaluate teachers' PCK. First, if the teacher had knowledge of the subject matter, this was evident when the teacher exhibited deep and thorough conceptual understanding of identified aspects of the subtraction algorithm; identified critical mathematical components within the concept of the subtraction algorithm that are fundamental for understanding and applying the concept; and displayed skills for solving a problem. Second, if the teacher had knowledge of instructional strategies and application, it would be evident if the teacher used appropriate activities during the instruction phase; used real life examples; applied different instructional strategies in the presentation if need be; and also used different representations in the instruction. Third, if the teacher had knowledge about learners' conceptions, evidence would be obvious if the teacher showed interest in the learners' prior knowledge; dealt with the learners' difficulties during the lesson; took care of possible learners' misconceptions about the topic during the lesson; and also had instruments to measure the level of learners' learning of the topic. Although this researcher has a different topic, namely quadratic equations, the same three components as those used by Chick et.al (2006) in their framework were adopted for this study.

Bukova-Güzel (2010) investigated pre-service mathematics teachers' pedagogical content knowledge by using solid objects and found that the participating teachers did not pay attention to possible student misconceptions. In the said study, data was collected through semi-structured interviews, analysis of lesson plans prepared by the students and video recordings of instructional applications. The framework for the analysis of PCK used in that study (Table 2.1) uses knowledge of teaching strategies, knowledge of learners and the curriculum. Two of these components, knowledge of teaching strategies and knowledge of learners were incorporated in this study.

Knowledge of teaching strategies and multiple representations	Knowledge of learner	Knowledge of curriculum
• Using appropriate activities in Instruction • Using real life examples and analogies in instruction • Utilising different instructional strategies in presentations • Making use of different representations in instruction (graphics, tables, formulas, etc.)	• Having knowledge of students' prior knowledge • Using real life examples and analogies in instruction • Having knowledge of the difficulties students will face during learning • Having knowledge of possible student misconceptions • Having knowledge of student differences	• Being aware of the elements of the mathematics curriculum (conception, purposes, etc.) • Being aware of the varieties of instructional tools in the mathematics curriculum and how to use them • Being aware of the instruments to measure student learning and how to use them

Table: 2.1: Bukova-Güzel's framework for pedagogical content knowledge Source: Bukova-Güzel (2010)

The remaining sections of this chapter will discuss each of the three identified elements of PCK and the conceptual framework used in this study. It will draw attention to how other researchers, in their respective studies, observed these elements as contributing to a teacher's PCK.

2.2.2.1 Knowledge of the Subject Matter and mathematics Teaching

One of the three elements of pedagogical content knowledge identified for this study is subject matter knowledge. According to Turnuklu and Yesildere (2007), knowledge of mathematics and knowledge of mathematical representations are related to content knowledge. The teaching process of mathematics topics starts from the teacher's understanding of what must be taught and how it must to be taught to the learners. Such a teaching process proceeds through a series of activities in which learners are given a series of instructions and an opportunity to learn, although ultimately the learning itself remains the learner's responsibility (Shulman, 1987). If the teaching action has been effective, it should end up with the learner having newly acquired comprehension (Shulman, 1987).

The study by Turnuklu and Yesildere (2007) that investigated pre-service primary teachers' competency of pedagogical content knowledge in mathematics, found that, to teach mathematics effectively, teachers ought to have a deep understanding of the mathematical knowledge of the topics that they teach. Their findings indicated that there is a link between knowledge of mathematics topics by the teacher and effective teaching of mathematics. They argue that, if a teacher has good conceptual understanding of mathematics topics, the influence on the quality of their instruction and the instructions used and provided would be positive. Mishra and Koehler (2006) agree with this conclusion as they contend that teachers who have a good understanding of the subject matter find different ways to represent it and make it accessible to learners.

It was clear to Turnuklu and Yesildere (2007) that the pre-service primary teachers' mathematical knowledge on the topics of fractions, decimal fractions and integers was mediocre, thus they could not assist their learners with the misconceptions that their learners displayed. These specific findings point to the fact that, for teachers to be able to present their mathematics topics in a way that will be understood by their learners, and be able to identify their learners' problems, they must have good subject matter knowledge specific to the topics. It is for this reason that this particular study has included subject matter knowledge as one of the components to be addressed.

Subject matter knowledge by teachers of any subject is important in teaching as evidenced by findings of Mishra and Koehler (2006) in a study focusing on developing a framework for investigating teacher knowledge. They posit that subject matter knowledge is knowledge about the actual subject matter that is to be taught and learned. Teachers need to know the subject matter very well. The mathematics content to be covered in high school mathematics is different from the mathematics in graduate computer science hence the purpose it serves has to be considered too. Mishra and Koehler (2006:p.1026) posit that "teachers must know and understand the mathematics that they teach, including knowledge of central facts, concepts, theories and procedures within a given topic; knowledge of explanatory frameworks that organise and connect ideas; and knowledge of the rules of evidence and proof". Though Turnuklu and Yesildere (2007) worked according to a framework that could be used to assess teachers' abilities to incorporate technological devices in teaching, they also emphasised the need for teachers to have a good understanding of the topics that they intend to teach, in order for them to select an appropriate technological device that could be used in

the teaching. The inclusion of subject matter knowledge as part of the framework used for this study is thus further justified by these findings.

A study conducted by Ball (1990) investigating the mathematical understanding that teachers-in-training bring to education, revealed that prospective teachers of mathematics in both secondary and elementary schools have limited understanding of the mathematics when teaching a lesson. The teachers' knowledge of the subject matter appeared to be rule-bound and thin. In the study, interviews and questionnaires were used to collect data from 252 prospective teachers. The interviews and questionnaires were designed to explore the participants' knowledge of mathematics and the teaching of it. In this study, interviews were also used to collect data about how the teachers intended to handle their lessons.

Ryan and McCrae (2005:p.641) see subject matter knowledge as more than just the knowledge of facts or concepts. To them, subject matter knowledge "requires knowledge of both the substantive structure (facts and their organising principles) and the syntactic structure (legitimacy principles for the rules) of a subject domain". They further indicate that teachers need to have a "good understanding of both the conceptual knowledge and the procedural knowledge of mathematics to be able to provide learners with clear explanations". Their study was particularly interested in the conceptual and procedural knowledge that the participants exhibited as they taught their learners. It is critical that there is a clear understanding of what the two types of knowledge mean in mathematics.

To bring clarity about what conceptual and procedural knowledge is in mathematics, reference is made to Schneider and Stern (2008:p.2) who "see conceptual knowledge as the knowledge of the core concepts and principles and their interrelations in the mathematics domain. It is knowledge that is rich in relationships. On the other hand, procedural knowledge in mathematics allows learners to solve problems quickly and efficiently because it is to some extent automated through drill work and practice. Procedural knowledge can thus be viewed as rules and procedures for solving mathematics problems". These two points are accommodated in this research initiative that also investigated the two participating teachers' display of both conceptual knowledge and procedural knowledge when they were presenting lessons on quadratic functions.

Zerpa, Kajander and Van Barneveld (2009:p.59) stress that "teachers need to have deep conceptual understanding of the mathematics that they are teaching their learners and must be

able to illustrate why mathematical algorithms work, and how these algorithms may be used to solve problems in real-life situations". Cockburn (2008) posits that having a sound understanding of mathematics is a crucial component of an effective teacher's repertoire. A good conceptual understanding of the topics will enable teachers to diagnose learners' misconceptions and misunderstandings easily (Kilić, 2011).

The relevance of teachers' domain-specific knowledge for effectiveness in teaching has also been emphasised repeatedly by Ball, Lubienski and Mewborn (2001). Following up on the work of Shulman (1986) and Kraus, Neubrand, Blum and Baumert (2005), three sub-dimensions of pedagogic content knowledge that are specifically important to mathematics teachers and that help to make the subject matter accessible to the learners, can be identified. These are: (i) tasks given to learners, (ii) using learners existing conceptions and prior knowledge and (iii) giving appropriate instructional support and guidance in the form of explanations, analogies, illustrations and examples that will enable learners to master the content.

Of prime importance during lesson observations in this study was how teachers displayed their subject matter knowledge of topics on quadratic functions at the same time ensuring that the learners were able to internalise the content during the lesson. The teachers' subject matter knowledge was assessed by checking: (i) the accuracy of mathematical facts; (ii) flexibility of presenting explanations displayed by the teachers; (iii) sequential presentation of facts;(iv) and hierarchical presentation of facts and also the (v) flow of ideas of presenting the topics by the teacher during the presentation of the lesson. Pedagogical content knowledge, like all other forms of knowledge is useful only when it is applied and inferred, which is why this researcher wanted to observe its application and not measure it as was done in the study by Hill et al (2004) where they developed a measure for a teacher's mathematics knowledge for teaching mathematics.

2.2.2.2 Knowledge of Instructional Strategies and Mathematics Teaching

According to Brodie (2007), the new curriculum that has been recently introduced into South African schools calls for learners to participate in mathematics lessons and to express their mathematical ideas. Teachers are encouraged to make their lessons more learner-centred by encouraging learners to contribute to the lesson. The choice of the instructional strategy to be used by the teacher is very important. Different lessons require different teaching methods.

According to Shulman (1987), the correct choice of such an instructional strategy does not depend on the teachers' knowledge of the subject matter only but also on the teacher's knowledge of the learners' level of understanding. Since this research investigated the use of instructional strategies during lessons, it is important to know what "good" teaching strategies the teacher used in mathematics teaching.

Lim (2007), in his study of the characteristics of mathematics teaching in Shanghai, noted that the success of a teacher in teaching a specific mathematics topic depends on the depth and breadth of the individual teacher's pedagogical content knowledge because, prior to the commencement of a lesson, a mathematics teacher needs to (i) plan the lesson; (ii) choose a teaching strategy; and (iii) select content that will suit the learners' level of understanding. These three activities are all assumed to be elements of PCK. Teachers with a sound knowledge of the elements of PCK, always select teaching strategies that are appropriate for the level of development of their mathematics learners. Cockburn (2008) asserts that, although content knowledge is central to an educator's effectiveness in teaching mathematics, the method of teaching plays an equally important role if any learning is to take place. In the case of this study the teaching strategies that the effective teachers used when they taught quadratic functions was also investigated.

Tanner (2003) posits that good instructional strategies should: (i) actively engage the learners; (ii) assist them in using their prior knowledge and skills to solve problems in mathematics; (iii) motivate the learners to participate during the lesson; and also;(iv) create an appropriate learning environment. According to Ingvarson, Beavis, Bishop, Peck and Elsworth (2004) excellent teachers of mathematics are aware of a wide range of effective teaching strategies and techniques for teaching and learning mathematics that promote the learners' enjoyment of the subject. Furthermore, such teachers usually choose teaching strategies that tend to create the best learning experience for every learner. The PCK of teachers according to De Miranda (2008:p.17) "involves knowing how to take advantage of different teaching approaches that make a learning experience most appropriate for the learners. This includes being flexible and adjusting instruction that takes into account various learning styles, abilities and interests. Knowing how to best teach a concept so that the learners will receive the best learning experience speaks to the essence of PCK. The different teaching approaches employed will vary from teacher to teacher and in differing contexts, but invariably will revolve around similar principles for each approach".

Westwood (2004:p.79) asserts that "studies have indicated that although expert teachers differ in their actual style of teaching and management, they all use instructional strategies that (i) maximize students' time and engagement in learning tasks; and (ii) encourage students' active participation during lessons. In addition, (iii) they ensure that students understand the work they are required to do; and, (iv) they set tasks and activities at the right level to ensure high rates of success. Expert teachers also (v) create a positive and supportive classroom environment; (vi) they are good managers of behaviour; and (vii) are skilled in motivating learners to learn". This study too, investigated how the teachers used their teaching strategies to benefit the learners. The teaching strategy that the participating teachers used during lesson presentation was investigated by checking the method used such as telling method, group work and self discovery teaching method.

It is important to know what is meant by an effective instructional strategy. An effective instructional strategy is one that triggers active learning by the learner (Eysink, de Jong, Berthold, Kolloffel, Opfermann and Wouters, 2009). Active learning, according to Eysink, et al. 2009), encompasses processes such as interpreting, exemplifying, classifying and organising the content by the learner.

Baumert et al (2009), in their study involving teachers' mathematical knowledge, cognitive activation in the classroom and student progress', mention three components of instructional strategies that are crucial for initiating and sustaining insightful learning processes in mathematics lessons. These three components are:

- Cognitively challenging and well-structured learning opportunities
- Learning support through monitoring of the learning process and individual feedback and adaptive instruction
- Efficient classroom and time management.

From the discussion so far, what is being implied is that teachers need to select teaching strategies that encourage discussion and justification of ideas in the content of the topic so as to demonstrate mathematical understanding (Eysink, et. al 2009). Furthermore, teachers need to support their learners through guided practice until they are independent, and confront misconceptions that learners may have about a given topic in mathematics (Tanner, 2003; Lim, 2007). According to Westhood (2004), the choice of a teaching strategy must also

encourage a disciplined learning environment that allows learners to listen to other learners' inputs and encourages the sharing of mathematical ideas as they unfold from the lesson.

The researcher in this investigation observed the kind of instructional strategies that the two effective teachers used in their mathematics lessons on quadratic functions. The intention was to see whether the strategies suited the topics that were being taught during that specific lesson, and whether the chosen teaching strategy was able to challenge the learners cognitively; the examples used as well as providing the best learning opportunities for the learners.

2.2.2.3 Knowledge of Learners' Conceptions, Preconceptions and Misconceptions in Mathematics Teaching.

According to Fennema and Franke (1992:148), "knowledge of learners is generally defined as knowing about the characteristics (conceptions, pre-conceptions, misconceptions and learning difficulties) of a certain group of learners, establishing a classroom environment and planning instruction accordingly to meet the needs of these learners". Smith, DiSessa and Roschelle (1993) think that learners do not come to class as blank slates. They come to the classroom with certain preconceptions about topics in mathematics. As they learn mathematics, the sense they make of what they are presented with can differ from what their teachers expect, and teachers may also not be aware of the total experience that these learners bring along to the class. From a constructivist view of learning, all learning involves the interpretation of phenomena, situations and events including classroom instruction, through the perspective of the learners' existing knowledge.

A study by Kiliç (2011) that investigated pre-service secondary teachers' knowledge of their students revealed that having strong subject matter knowledge is essential to becoming a good teacher, but it is not sufficient for effective teaching. The findings of the study revealed that teachers should, in addition, "know how to teach a particular mathematical concept to particular learners, how to represent a particular mathematical idea, how to respond to learners' questions, and what curriculum materials and tasks to use to engage students in a new topic" Kiliç (2011: p.18) . Kiliç (2011:p.18) cite An, Kulm and Wu (2004) who assert that since "PCK is perceived as knowledge of how to teach particular subject matter, knowledge of subject matter and knowledge of pedagogy is not enough to achieve an effective teaching practice without knowing the learners". Kiliç (2011) used classroom

observations, structured interviews, questionnaires and journals as data collection instruments. The results in the study by Kiliç (2011) showed that pre-service teachers have insufficient knowledge of learners' conceptions and "when the pre-service teachers were given examples of learners' errors and asked how to address them, the pre-service teachers tended to repeat how to carry out the procedures or explained how to apply a rule or mathematical fact to solve the problem" instead of explaining the correct concepts that would help eliminate the learners' errors. In this current study the two participating teachers' knowledge of learners' conceptions, knowledge of the subject matter and knowledge of teaching strategies were investigated using interviews, observations and lesson plan analysis as data collection instruments. Kiliç (2011) used the same data collection instruments.

As Tanner, Gene, Caro and Amy (2003) report about their study entitled 'Instructional strategies: how teachers teach matters', learners were seen to have varying knowledge and interest levels about the mathematics topic that their teachers brought to the classroom. Their prior knowledge (preconceptions) about the topic can interfere with their new learning experience. This could result in a concept being incorrectly understood by the learners. In this research study, teachers' lesson plans were analysed to check which concepts were to be taught, what prior learning was required of the learners and what possible misconceptions if any the teacher anticipated the learners might have about the topic being taught.

When teachers choose teaching strategies to help learners understand mathematics topics, it is important to consider what prior conceptions about the topic their learners in the class have acquired. As teachers transform the content to a more accessible form for the learners, documented misconceptions about the topic or content must be considered (Tanner, Gene, Caro and Amy, 2003). Knowledge of common mathematical errors and misconceptions of learners can provide teachers with insight into the learners' thinking as well as offering ideas that would serve as a focus for teaching and learning (Ryan and McCrae, 2009). Teachers need to do baseline assessment to find out what learners know about the topic. If teachers are familiar with what learners know, they can help build bridges between the known and the unknown, and this will help teachers to clarify misconceptions in order to assist learners in comprehending new information (Ryan and McCrae, 2009).

In order to investigate the misconceptions learners could possibly have about the topics that they were previously taught, it is important to know what misconceptions are. Misconceptions are pieces of wrong knowledge that may arise from learners' prior

experience and learning, both inside and outside of the classroom (Smith et al. 1993). Teachers who have poor subject matter knowledge of mathematics may also contribute to the development of such misconceptions in learners since misconceptions are faulty extensions of productive prior learning (Smith et al. 1993). Reasoning based on misconceptions leads to consistently wrong problem solutions (Kőrner, 2005). Such misconceptions are usually formed when the new lesson is not compatible with the learner's existing conceptual prior knowledge about the topic. Furthermore, because of their strength and flawed content, misconceptions interfere with learning new concepts.

Ingvarson et al. (2004) indicate that excellent teachers of mathematics have rich knowledge of how students learn mathematics. They have an understanding of current theories relevant to the learning of mathematics. Such educators have knowledge of the mathematical development of students including learning sequences, appropriate representations, models and language. Effective teachers are aware of the common misconceptions that learners may have regarding a particular topic and they structure their lessons in such a way so as to confront the misconception that the learners may have. Such teachers encourage discussion and justification of the ideas that learners may bring during a mathematics lesson (Ingvarson et. al, 2004).

According to Smith et al. (1993), for classroom instruction to be successful in eliminating misconceptions, teachers must present the correct concepts in clear opposition to the students' faulty conceptions. The chosen instruction (if the teacher has knowledge of learners' misconception) should "include demonstrations and activities that produce counter-evidence and plausible conceptual alternatives to target misconceptions. The confrontation of ideas through discussions in the classroom is then internalised by students as a psychological process of competition that finally results in the replacement of the misconception".

As already indicated, the key elements of Shulman's (1986) conception of PCK are knowledge of representations of subject matter on the one hand and the understanding of specific learning difficulties and student conceptions on the other. Obviously, these elements are intertwined and should be used in a flexible manner: It is assumed that the more representations teachers have at their disposal and the better they recognise learning difficulties, the more effectively they can deploy their PCK (Shulman 1987).

Teachers need to have good knowledge of possible difficulties that learners might experience when a particular mathematics topic is presented. This will enable the teacher to prepare possible explanations and examples that will enable the learners to access the content of the topic that is being taught. Knowledge of such learning difficulties will also allow the teacher to prepare possible prior content knowledge necessary for a particular topic that learners could easily link to the new knowledge.

Learning difficulties may arise due to the language of instruction in mathematics. Teachers need to choose a teaching strategy that allows them to use a language of teaching that the learners are familiar with. Howie (2003), Kanyongo et al (2007), Maree and Erasmus (2007) conducted studies on factors that affect the performance of learners in Mathematics in sub-Saharan Africa. Each of these studies revealed, among other factors, that learners whose proficiency in the language of teaching and learning was good performed better than learners whose proficiency in the language of teaching and learning was poor. Teachers who teach in the second language of the learners must have that awareness. Learners with a language problem may have difficulty in understanding written mathematical problems, communicating mathematical ideas orally and reading text.

2.2.3 Conceptual Framework for the Study

The review of relevant literature has shown that to teach effectively, teachers need to have both content knowledge and the pedagogy of the topics that they teach together with knowledge of the learners' conceptions (Deubel, 2009). The three elements of PCK, namely, knowledge of the subject matter, knowledge of teaching strategies and knowledge of learners' conceptions in this research synchronise with the views and constructs of PCK used by several researchers in this domain, like Shulman (1986), Hailm and Meerah (2002), De Jong (2003), Ball, Lubienski and Mewborn (2005); De Jong et al., (2005) and Kraus, Neubrand, Blum and Baumert (2005). Thus these three elements formed the core of the conceptual framework that was used in the development of data collection instruments as well as analysis of data for this study.

The proposed study intends to find answers to the following research questions:

1. What pedagogical content knowledge do effective teachers display with regard to teaching quadratic functions in Grade 11?

2. How did the teachers develop the PCK that they use when teaching quadratic functions?

The conceptual framework developed for this study was based on the three elements of pedagogical content knowledge that have been discussed in conjunction with the literature reviewed in this chapter. It is also presented in tabular format (Table 2.2) and indicates all the expected sub-items within each component. The PCK elements are listed as a heading for each column with the items that were observed for that particular component of PCK being listed in each column. Such an approach was used by other researchers such as Bukova-Guzel (2010) and Yusof and Zakaria (2010).

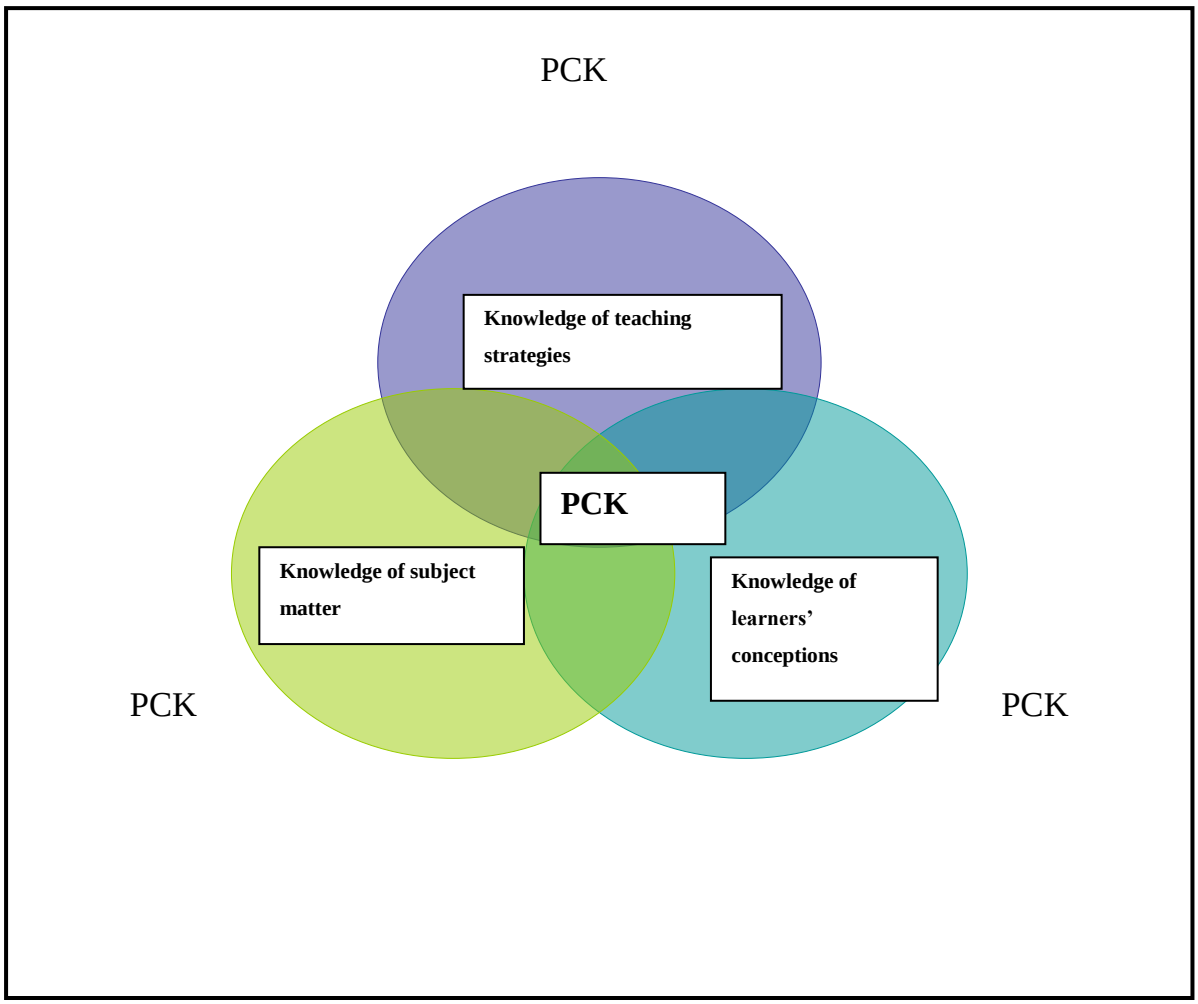


Figure 2.1: Schematic diagram showing the interdependence of the three PCK elements. PCK is at the centre of the intersection of the three elements. Each element can stand independently but as an amalgam they provide stability to the PCK construct.

a. Knowledge of subject matter	b. Knowledge of Teaching strategies	c. Knowledge of learners' conceptions
• Exhibits deep and thorough conceptual and procedural understanding of identified aspects of quadratic functions.(guided by checking (i) correctness of mathematical facts (ii) flexibility of explanations (iii) sequential representation of facts (iv) hierarchical presentation (v) easy flow of ideas • Identifies critical mathematical components within the concept of quadratic functions that are fundamental for understanding and applying the concept. • Displays skills for solving problems in the area of quadratic functions	• Using appropriate activities in Instruction. • Using real life examples and analogies in instruction • Utilises different instructional strategies in presentations	• Addresses learners' misconceptions. • Displays expectations of possible difficulties that learners may face during learning and address such. • Discusses learners' ways of thinking about a concept. • Being aware of the instruments to measure student learning and how to use them.

Table 2.2: A framework of PCK Elements used in the study Source: Bukova -Güzel (2010)

The PCK elements supposedly held by the two effective teachers in the study; were investigated by checking the sub-items indicated in each column of Table 2.2 as the teachers presented their lessons on quadratic functions to grade 11 learners.

2.3 CHAPTER CONCLUSION

In this chapter, the notion of pedagogical content knowledge was discussed with reference to the work of other scholars. The three elements selected for this study, namely, knowledge of the subject matter, knowledge of teaching strategies and knowledge of learners' conceptions, were also presented. The implication of each element for the teaching of mathematics was also considered. The chapter ends with a presentation of the conceptual framework that was used for both the collection and analysis of the data.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

In this chapter the research procedure that was used to investigate the research questions is described with reference to the research design, method and sampling procedures for the identification of the participating teachers, the research instruments and their validation, the data gathering process and the administration of the main study.

3.2 RESEARCH DESIGN AND METHOD

A qualitative research approach which focused on case study method was used in this investigation. In their respective studies, this method was also applied by Yusof and Zakaria (2010), Chick, Pham and Baker (2005) as well as Bukova-Guzel (2010) whose focus too was on teachers' pedagogical content knowledge (PCK). In all these studies qualitative research was fitting as the researchers endeavoured to describe an event in the social world from the standpoint of the individuals who were part of the ongoing event (Sinkovic et al., 2008). So, in this study, the research interest was to study the PCK held by the two participating teachers as they taught topics on quadratic functions in grade 11 and qualitative research approach was seen as being consistent with the intentions of the research questions.

The qualitative research approach in education, according to Mason (2006), enables the researcher, after data analysis, to understand and explore the richness, depth, context and complexity within which teachers in the research site operate. In this study the units of analysis were the two teachers who were considered as being effective in the teaching of mathematics based on their good performance (pass rate of 80% and above) in the Grade 12 external National Senior Certificate mathematics examination. The researcher collected data regarding the teachers' pedagogical content knowledge through direct observation, document analysis and interviews. The case study method allowed the researcher to gain insight into the type of pedagogical content knowledge the teachers were using as they taught. The advantage of the case study method is that it allows the researcher to focus and gain insight on a specific phenomenon but the disadvantage of this method is that findings cannot be generalized. Lessons were observed using an observation schedule. Verbal responses to the interviews

with participants were transcribed in written form so as to serve as a backup as well as for easy reference to what had transpired during each session devoted to data collection.

3.3 RESEARCH SITE AND POPULATION

The research site was located in a certain school circuit of the Mpumalanga Department of Education in South Africa. The population for this study comprised all mathematics teachers who came from schools that had obtained an average of 80% overall pass rate and an average of 80% in the National Senior Certificate Grade 12 mathematics examinations for the past three (or more) consecutive years.

As a point of entry to identify the two teachers who could be classified as "effective", statistical records of performance of schools from the Province's district education office in which this circuit fell, were used to compile a record of performance of schools in mathematics. The Chief Education Specialist (CES) for further Education and Training (FET) band in the district was consulted to supply this information. He supplied the researcher with schools that were seen to be closer to the set criterion of 80% pass rate. Finding schools with a pass rate of 80% was difficult as most schools that could be regarded as good performing schools around the district had an average performance of 70% pass rate in mathematics and the researcher had to settle for schools in this category. Of the 135 secondary schools in the district; only eight schools had such an average pass rate level in mathematics during the past three to four years. The eight schools formed the pool of schools from which the sample of teachers could be drawn.

In the pool of eight schools; there were fourteen teachers who taught mathematics in grades 10 to 12 and they formed the population from which the sample could be drawn.

3.4 SAMPLING PROCEDURE AND SAMPLE

From the eight identified schools which satisfied the performance criterion of 70% pass rate or more for the past three to four years, two strata of schools were formed, one comprising of schools from rural settings and the other stratum of schools from an urban setting. A second criterion was that teachers from these eight schools had to be teaching Grade 12 classes when such success was achieved. Two schools; with 5 teachers; were from a rural setting while the other six schools; with 9 teachers; were from an urban setting satisfied the two set criteria. A third criterion was that the two participating teachers should be from schools in the same

circuit. One teacher was then purposively identified from each stratum and letters inviting them to participate in the proposed study were sent to them. Two male teachers were found from the same circuit, one from a rural school and the other one from an urban school. This approach of selecting two teachers to do in-depth studies was employed by other researchers such as Chick and Harris (2007), Randall (2008) and Li and Yu (2009) when they conducted in-depth studies to explain teachers' pedagogical content knowledge in their own studies.

The profiles of the two participating effective teachers are given in tabular form (Table 3.1) to enable easy comparison of the similarities and differences between the two teachers.

	ITEM	**TEACHER A**	**TEACHER B**
1.	Educational qualifications	B.Sc (Mathematics and Statistics); ACE (Advanced Certificate in Education (Mathematics)	Diploma in Education; ACE Certificate in Education (Mathematics)
2.	Current school location	Township setting	Deep rural
3.	Gender	Male	Male
4.	Age	34	38
5.	Experience(at the time of the research)	18	12
6.	Grades teaching since appointed.	Grade 10 to 12	Grade 10 to 12
7.	Current studies (if any)	None	B.Ed (Educational Management)

Table 3.1: Profile of the Participating Teachers

Thus, the participants are qualified mathematics teachers and are presumed to have sufficient general subject matter knowledge to enable them to develop the specific content knowledge for teaching quadratic functions in school mathematics as can be seen from their qualifications. The general believe in society is that teachers who hold such qualifications, with the relevant years of experience, should be able to develop adequate content knowledge and PCK for teaching school mathematics.

3.5 DATA COLLECTION INSTRUMENTS

Data was collected via observation of the teachers' lesson presentation, lesson plan analysis and one-on-one interviews with each mathematics teacher to find answers to the research questions (see Table 3.2 to 3.5 for format and scoring of each instrument). Chick and Harries (2007) used observation schedule when they studied the pedagogical content knowledge of teachers by observing the examples that teachers gave to learners on the topic of ratios. Observations were also used by Kilic (2006) when he studied the components of pedagogical content knowledge that pre-service teachers gain through the method course offered at a certain university.

The format and how each instrument was used during data collection are now presented.

3.5.1 Classroom Observation Protocol

PCK ELEMENT TO BE OBSERVED	EVIDENT WHEN THE TEACHER….	OBSERVED PRACTICE DISPLAYED
a. Knowledge of the subject matter	1. Exhibits deep and thorough conceptual understanding of identified aspects of functions. 2. Identifies critical mathematical components within the concept of functions that are fundamental for understanding and applying that concept. 3. Displays skills for solving problems in the area of functions.	
b. Knowledge of Teaching strategies	1. Uses appropriate activities in Instruction 2. Uses real-life examples and analogies in instruction 3. Utilises different instructional strategies in presentations.	
c. Knowledge of learners' conceptions	1. Addresses learners' misconceptions 2. Displays expectations of possible difficulties learners may face during learning and address such. 3. Discusses learners' ways of thinking about a concept. 4. Shows an awareness of the instruments to measure student learning and how to use them	

Table 3.2: Observation protocol used in the study

Source: Adapted from Chick, Baker, Pham (2006)

To answer the first research question which sought to find out what pedagogical content knowledge (PCK) effective mathematics teachers display in the teaching of quadratic functions, a lesson observation protocol based on Yusof and Zakaria (2010) model was used to collect data on the two teachers' presentation of lessons. Yusof and Zakaria (2010) as well as Bukova-Guzel (2010) used the observation method to assess the teachers' PCK on mathematics. The following elements were assessed during lesson observations (see table 3.2):

- Knowledge of the content of the topic in which the teacher is engaged, that is, conceptual knowledge and procedural knowledge where the following were used to assess this element: accuracy of mathematical facts presented; flexibility of presentation; sequential representation of facts; flow of ideas and hierarchical presentation of facts.

- Knowledge of teaching strategies that enabled the teacher to present the lesson in a way that was comprehensible to the learners for which the following guidelines were used: organisation of the lesson; choice of examples; representations, use of chalkboard and appropriate teaching strategies.

- Knowledge of learners' conceptions (misconceptions and pre-conceptions) about the topic under discussion where the following were used to check the teacher's knowledge of this element: assessing learners' understanding; identifying errors learners made; addressing learners' difficulties, and determining sources of such difficulties; identification of misconceptions and elimination of them by probing questions; and using appropriate tasks.

The observation protocol schedule assessed how teachers presented their lessons in order to assist learners in comprehending the topic; how the teachers assessed their learners' after the lessons; the teaching strategies that they employed; and how the teachers dealt with misconceptions and learner difficulty during lesson presentation. Table 3.2 depicts the classroom observation protocol that was used during lesson observations.

3.5.2 Teachers Pre-lesson interview Questions

The teachers were interviewed before each lesson to find out more about their PCK. The purpose of these interviews was to find out how the teacher had organised the lessons, the

teacher's knowledge of key concepts to be taught, teaching strategies to be used, how the teacher planned to assist learners with difficulties, assessment tasks, any expectations of learners' misconceptions that the teacher might have had and also to triangulate lesson observation data. The questions that were used during the interviews are presented in Table 3.3 below.

ELEMENTS OF PCK FOR THIS STUDY	**QUESTION RELATED TO PCK ELEMENT**	**RESPONSES**
a. Knowledge of the subject matter	1. What are the key concepts in the lesson that you are about to teach? 2. Draw a concept map illustrating the sequence you will follow to teach these key concepts. 3. Does the lesson involve any procedural knowledge that the learners must know? If so, what does the procedure involve?	
b. Knowledge of teaching strategies and application	1. Which teaching strategy will you employ to ensure successful delivery of this lesson? 2. Why did you choose such a strategy? 3. In your selection of examples to be used in this lesson, have you selected real-life examples?	
c. Knowledge of learners' conceptions	1. What is the goal/aim of your lesson? 2. Which learners' prior knowledge is regarded as important before the above key concepts can be successfully taught to learners? 3. What possible learner misconceptions do you anticipate regarding this lesson? 4. How will you assist learners who experience difficulties with this lesson? 5. Have you prepared an assessment instrument to evaluate whether the goal of the lesson have been achieved?	

Table 3.3 Interview Questions: Source: Adapted from Chick, Baker, Pham (2006) (Probing teachers' pedagogical content knowledge: Lessons from the case of the subtraction algorithm)

3.5.3 Document Analysis: The lesson plan analysis

In addition to interviews and lesson observation, the teachers' lesson plans were analysed using a structured format (see Table 3.4). This was intended to check for other PCK attributes that may not have been observed during lesson presentation and also not mentioned during interviews. Such attributes include key concepts, teaching strategies and dealing with misconceptions.

ELEMENT OF PCK	CHECKED IN THE PREPARATION	OBSERVATIONS
Knowledge of subject matter	1. Are key concepts to be taught during the lesson indicated in the preparation? 2. Does the preparation indicate possible mathematics procedures to be taught to the learners? 3. Does the lesson preparation reflect accurate concepts associated with the topic quadratic functions?	
Knowledge of teaching strategies and application	1. Is the teaching strategy to be used stated in the preparation? 2. Are alternative teaching strategies to be used during the lesson reflected in the preparation? 3. Are examples to be used during the lesson indicated in the lesson preparation?	
Knowledge of learners' conceptions	1. Does the preparation reflect possible misconceptions that will be addressed during the lesson? 2. Does the preparation reflect the required learners' prior knowledge before the start of the new topic? 3. Are possible learners' difficulties reflected in the preparation? 4. Is an assessment instrument indicated in the preparation? 5. Is the goal of the lesson clearly stated in the preparation?	

Table 3.4: Lesson plan analysis: Guiding questions: **Source**: Adapted from Chick, Baker, Pham (2006): Probing teachers' pedagogical content knowledge: Lessons from the case of the subtraction algorithm).

3.5.4 Questionnaire on the Teacher's Mathematics Teaching Knowledge Development

This questionnaire sought to find out how the two mathematics teachers acquired the mathematics teaching knowledge that they have which helped them to teach effectively; by producing good results. The questions that were used are tabulated below (Table 3.5).

Item	Question to be asked
1	Did you receive any special training as a mathematics teacher after your initial teacher training?
2	(a) Do you attend workshops that focus on teacher development? (b) What have you gained from attending such workshops?
3	(a) Have you ever observed your colleagues when they were teaching a mathematics lesson? (b) What did you learn from such an observation?
4	What are your qualifications as a mathematics teacher?
5	For how long have you been a mathematics teacher?
6	How often do you review the lessons that you have taught?

Table 3.5 Pedagogical content knowledge development questionnaire: Source: Shulman (1987)

3.6 VALIDATION OF INSTRUMENTS

To ensure that the three data collection instruments (namely; observation schedule, interview questions and lesson plan analysis) have content and face validity, three experts in the mathematics department of a certain University were requested to scrutinise the instruments in order to establish the validity of both the content and the format of each research instrument. The experts worked independently of each other to scrutinize the instruments. Furthermore, a pilot study was done specifically to test the observation protocol and the consistency of the researcher's observations in one school that was amongst the eight that were regarded as having met the criteria for selecting an effective teacher. It was found that the researcher could use the instruments with consistency. The instruments were then modified and fine-tuned in consultation with the three experts. Language related errors were noted on the instruments and changes were effected to improve their usefulness.

3.7 PREPARING FOR THE MAIN STUDY

Preparation for the research process involved each of the following steps:

- Obtaining permission from the Mpumalanga Provincial Department of Education
- Obtaining permission from the principals of selected schools
- Validation of research instruments
- Obtaining consent letters from the participating teachers
- Obtaining consent letters from parents whose children would be taught by the participants. Parents were assured that if they so wish, they could withdraw their children from participating and that the lessons missed by their children would be repeated by the teacher. Such an arrangement was also agreed upon with the two participating teachers.
- Agreeing how data collection through observation, interviews and document analysis, all of which focused on each of the three elements of pedagogical content knowledge would be done with the two participating teachers and their school managers.

The identity of the two participating teachers was protected and codes were allocated to them. The code name of one of the teachers was Teacher A, and the other teacher's code name was Teacher B. In the research instruments, all data collected from the participating teachers is referred to according to their code names, Teacher A or Teacher B. This is in line with the principle that one of the cornerstones of research ethics is that respondents should be offered the opportunity to have their identity hidden in a research report (Oliver, 2003).

3.8 ADMINISTRATION OF THE MAIN STUDY

Before the data gathering process started, the researcher met with each of the participating teachers to discuss the whole research process and to clarify any issue that they might have been concerned about regarding the research process. The two teachers were anxious about the fact that the June examinations were about to be written, which meant that the research process could be interrupted for some days. They suggested that observations of the lessons

should take place on days when the teachers and learners were free from writing examinations. The researcher agreed to the teachers' input. The teachers also negotiated that the lessons to be observed and key discussions on lesson plan analysis and pre-lesson interviews should be based on lessons and topics that they (teachers) had already prepared according to their interpretation of the syllabus. Furthermore, the participating teachers agreed to repeat any lesson that may have been missed by any learner whose parent may have not concerted the learner to participate in the study. Lesson observations were done during school hours and during the mathematics period of the grade 11 class. Learners were given letters of consent to be given to their parents to sign if they consented. Fortunately, all parents of both set of schools signed the consent forms.

Data was collected over a period of two weeks from each of the two teachers due to the interruptions of the half-yearly examinations. Five lessons from each teacher were observed. The lessons were audio-taped; consent was obtained from the two teachers to do so. Pre-lesson interviews for each teacher were always done the day before the actual lesson presentation. Each recorded interview was transcribed as written notes. Each teacher taught only one grade 11 Mathematics class.

The teachers' demographic profiles regarding how they may have developed their mathematics teaching knowledge were collected through interviews on the day the last lesson was observed. Once data was collected; it was ready for analysis.

3.9 DATA ANALYSIS

Data collected was analysed using the conceptual framework according to the following categories:

- Knowledge of the subject matter (checking for the teacher's conceptual understanding of the topic; display of skills in problem solving (procedural knowledge) (Refer to Appendix A, B and C)

- Knowledge of teaching strategies (checking for use of appropriate activities; use of real-life examples; and use of different teaching strategies) (Appendix A, B and C)

- Knowledge of the learners' conceptions (checking for the teacher's ability to address the learners' misconceptions; expectation of possible learners' difficulties; discussion

of learners' ways of thinking; awareness of the instruments to measure student learning) (Appendix A, B and C)

- Teachers' PCK development (finding out about the qualifications of the teacher; teaching experience; workshop attendance; and peer observation during lessons)

The above stated categories and themes were investigated through the observation of lessons, interviews with the teacher and lesson plan analysis.

The next section will discuss the trustworthiness of the study.

3.10 TRUSTWORTHINESS OF THE STUDY

According to Sinkovic et al (2008), trustworthiness in a qualitative study aims to support the argument that the study's findings are worthy of receiving attention. In order to establish trustworthiness, credibility, dependability, transferability and confirmability need to be established.

Credibility focuses on establishing a match between the constructed realities of the participants and those represented by the researcher (Lincoln and Guba, 1989). To ensure credibility in this study, the lessons that were observed were video-taped and this ensured that the researcher could re-visit the lessons with ease to ensure that the reality that the researcher had recorded was not a fabrication. The researcher ensured that there was accurate reflection on the observations by cross-checking with the participants regarding what had been experienced during the lesson. Field notes reflected what transpired during the lesson. Peer debriefing was used to ensure that the items in the observation checklist did indeed relate to aspects of pedagogical content knowledge.

Dependability deals with the consistency of research results obtained over time. Dependability, according to Sinkovic et.al (2008) can be established by using different methods of data collection and different times of collecting the data on the same research problem. In this study, dependability was established by having prolonged and concentrated engagement with the participants about the study, two to three weeks in this case. In addition, pre-lesson interviews and also lesson plan analysis were used as evidence when collecting data about the teachers' PCK.

According to Rodwell and Byers (1997), confirmability can be established if the results can be linked to the data itself. It speaks to data management and the analysis of the data itself. In this study, confirmability was established by keeping the collected data that was used for interpretation safely, so that any interested researcher could access the data for inspection. In addition, an audit trail was done by independent critical readers whom the researcher had asked to evaluate the methods used for the gathering of the data.

Transferability refers to the applicability of the findings to another setting (Lincoln and Guba, 1985). As this was a qualitative study and no substantive generalisations could be made, the researcher gave thick description with enough detail of the findings so that readers could decide on their own whether the results of the study would be transferable to their own research contexts or not.

3.11 ELIMINATION OF BIAS

Bias occurs when interfering factors distort the truth or accuracy of the information. Bias can be easily eliminated if the sources of such bias are known. Sources of interfering factors could include the use of leading questions, incorrect recording of respondents' answers and situational factors such as discomfort or anxiety among participants.

All these contributing factors were carefully avoided starting from the construction of the questionnaires and preparation of all the other research instruments. The interview questionnaire did not contain leading questions and all the additional observations noted from the participants were verified with the participants before being finally recorded for further analysis. The respondents' identity was not be revealed, which further assisted in eliminating bias.

3.12 ETHICAL CONSIDERATIONS

Sometimes there is a sense of insecurity among human science practitioners that their approach is not as objective as that of the general scientist who deals with measurable and quantifiable phenomena. Bochner (2002) suggests that the human sciences are a little untidy and show signs of inferiority, stating that 'Traditionally we have worried much more about how we are judged as "scientists" by other scientists than about whether our work is useful, insightful, or meaningful – and to whom" (2002:259). Even other scientists who work with quantitative information needs to bear the human aspect of their research in mind thus ethics

would play a pivotal part in these studies too. Wherever people are involved in studies the ethical aspect raises its head. And, as we live in a constitutional democracy with a Bill of Rights, as formalised in 1996, this is an important consideration.

According to Schurink, Schurink and Poggenpoel (1998), important ethical considerations include:

- Voluntary participation on the part of those requested to be part of the data gathering process. Participants will be also informed that they can voluntarily leave the project whenever they choose to do so, and this without penalty.

- The participants will need to give their informed consent – this will mean that they will be informed of what the research entails and of how they can participate. Their superiors in the school hierarchy would also be included. This will include parents of learners whom the participants teach. Lessons missed by learners who could be withdrawn from the project would be repeated by the teacher. In this study all parents of learners in schools that were selected consented that their children take part in the research project.

- Confidentiality and anonymity should be assured in the contract drawn up between the researcher and the participants. For this study, the two participants were allocated code names, Teacher A and B respectively.

- Feedback regarding the results and findings of the research would need to be contractually arranged and the agreement be effected over time as the project progressed. For this project, as soon as the results have been certified as valid by the ethics committee of the University, the two participating teachers will be informed of the outcomes of the study.

- The competency of researcher should be assured, as well as the scientific soundness of project. For this researcher to have successfully defended the proposal, it reflects adequate competency levels to be able to manage this research project.

Ethical considerations of this nature reflect that even in the teaching of Mathematics the teachers participating in this Mpumalanga study have to be regarded with respect and dignity. Consent was sought from all participants including the learners' parents so they were aware

that such a study project was underway. All participants were free to choose to participate or not. Code names were used to protect their identities. Interview transcripts and all observational notes were kept in a locked, safe place to ensure that no one other than the researcher could access the information.

3.13 CHAPTER CONCLUSION

In this chapter the research methodology was detailed with regard to the research procedure, the research site, the population size, the sample, the data collection instruments, the validation of the research instruments and the research and data gathering processes. Data analysis was dealt with and the trustworthiness of the study was established. Attention was drawn to the elimination of bias and ethical considerations.

CHAPTER 4

DATA ANALYSIS AND RESULTS OF THE CASE STUDIES

4.1 INTRODUCTION

This chapter reports the findings on the pedagogical content knowledge of each of the two case teachers who are herein referred to as Teacher A and Teacher B. For each case study, findings on classroom observations (by first describing the lesson observations), pre-lesson interviews, and lesson plans analysis will be presented. The presentation will include an analysis of how their presumed pedagogical content knowledge (PCK) was used in the teaching of quadratic functions.

4.2. The Teachers' Pedagogical Content Knowledge

This section presents the findings regarding the participating teachers' PCK with reference to the three elements identified for this study: namely, knowledge of the subject matter, knowledge of teaching strategies and application and knowledge of learners' conceptions. Classroom lesson observations, pre-lesson interviews and lesson plan analysis were used to collect data about the participants' PCK on the topic of quadratic functions. A description of the observations made during each teacher's lessons presentation will be given and a link to one of the elements of PCK will be done if such a link did exist. Furthermore, a summary of the findings on pre-lesson interviews and lesson plan analysis on the three elements of PCK will also be given. The next paragraph will describe the lesson observation of Teacher A.

4.2.1 Description of Classroom Observations for Teacher A

The purpose of lesson observation was to examine the interaction patterns at work in the classroom for each of the teachers, namely how they used their content knowledge in teaching a particular topic on quadratic functions. The instructional skills and strategies used by the teachers, the ways in which they tried to identify learners' preconceptions and learning difficulties, and what they did to try to rectify these misconceptions, if any, were also examined. The topic in which most lessons were observed were on quadratic functions (how to draw the graph, how to sketch the graphs, determining the equation of a parabola given three points).

LESSON OBSERVATION: TEACHER A

DESCRIPTION OF LESSON OBSERVATION (TEACHER A)	CATEGORISATION/THEMES
Condition of the classroom There were 35 male learners and 20 female learners. Each learner had a desk, a textbook and a chair though there were many learners which limited movement between the rows of tables. Teacher A had a full view of all the learners. A chalkboard, a teacher's table and a duster were available in the classroom. Teacher A had his chalks and textbook.. Wall charts of Mathematics topic were hung on the walls	Despite the large class size, the classroom presented a highly conducive learning environment.
LESSON OBSERVATION: Grade 11 **Topic: How to draw the graph of a Quadratic function**	**CATEGORISATION/THEMES**
Line 1: Teacher A, standing in front of the class, introduced the topic of the day "*Today's lesson will be about drawing the graph of a parabola*";. He wrote the variations of a quadratic function on the chalk board, numbered as shown here (i) $y= ax^2$, where, $a\neq0$, $b=0$ and $c=0$ (ii) $y = ax^2 + bx$, $a\neq0$, $b\neq0$, $c=0$ and (iii) $y = ax^2 + bx + c$ and said; *"These are the three variations of a quadratic equation. Note that "a" the coefficient of x^2 can never be equal to zero. The simplest form of the equation is $y = x^2$".* He reminds learners of the graph of a straight line y =mx + b which they had learnt in grade 10 and that it is different from the quadratic function graph as he writes the equation of a straight line on the chalkboard. *"In grade 10 you learnt how to draw the graph of a straight line y= mx + b, and you needed to choose just three x-values to substitute in the equation to get coordinates to draw the graph. For a quadratic function, you need more x- values as you will see when I demonstrate later"* He ends his introduction with a question; *"Do you understand?"*	Teacher A used *content knowledge* to present the three variations of the equation using the lecture method. He did a review about the approach of how to draw the graph of a straight line and emphasises that it is different from the approach of drawing that of a quadratic function. His question of asking learners whether they understand his explanation does not allow learners to express their views on the given explanation.
Line 2: All the learners in the class responded to Teacher A's question asked in line 1 above " Yes"	Teacher A's *closed question* elicits chorus answer from learners s without an opportunity to explain their thinking.
Line 3: Teacher A then wrote the equation $y = x^2- 4$ on the chalk board followed by; "*Now I will show you how to draw the graphs of such functions. You must pay attention as I demonstrate how it is done. We will start by the graph of $y= x^2 -4$ as an example"*. He then asked *"In the given equation, what is the coefficient of x^2"*	Teacher A used a *topic specific example* to demonstrate how to draw the graph of a parabola using a table of values. He engages learners by asking them questions.

Line 4: A learner responds; "*Two, Sir"*. Teacher A does not accept the answer given and points at another learner to give the correct answer. He said; "*No, not correct! Any one else to help us?"*	Teacher A correctly rejects the learner's answer but does not probe further to allow the learner to explain why he/she thinks the answer is "2".
Line 5: Another learner responds to the question; " The coefficient of x^2 is one"	Learners are actively involved in the lesson as they participate in the development of the lesson.
Line 6: Teacher A accepts the solution given by the learner; *" Yes, the coefficient of x^2 in the equation $y = x^2 – 4$ is one"*	Teacher A accepts the learner's answer but does not probe the learner to find out how he got the correct answer. Teacher A uses his *content knowledge* here.
Line 7: Teacher A explains further whilst he writes on the chalkboard that if the coefficient of x^2 was zero the equation $y = x^2 - 4$ would become $y = -4$ which is a constant function and no longer a quadratic function. That is the reason $a \neq 0$ in the equation $y= ax^2 + bx + c$. *"If the coefficient of x^2 was equal to zero, the equation $y = x^2 - 4$ would become $y= 4$ in this case which is a constant function or just a linear function as you can see that the equation $y = ax^2 + bx + c$ becomes $y = bx + c$. Do you understand?"* He wrote down all these equations on the chalkboard for the learners to see.	Teacher A uses *content knowledge* to provide an explanation of why the coefficient of x^2 is never equal to zero and he further gives a conceptual reason why "a" must never be equal to zero in a quadratic function. If "a" is equal to zero, the quadratic function will become linear.
Line 8: The entire class responds with a "Yes"	Once again the class responds as a whole-chorus answer- without the opportunity of individuals expressing their comprehension of the coefficient of x^2 in a quadratic function. The teacher uses *inefficient questioning technique* to probe learners. The question usually require "yes" or "no" type of response (*pedagogic knowledge*)
Line 9: Teacher A then proceeds with the lesson:; *"Now, to draw the graph of $y = x^2 – 4$ we choose, say, 9 values of x; which will be substituted in the given equation to get the corresponding y- values and form coordinates written in the form (x;y)"* The following x-values were chosen; -4;-3;-2;-1;0;1;2;3;4 and were to be substituted in the equation $y = x^2 - 4$ to get y-values. Teacher A drew a table as shown and explains "*For each x-value, we are going to calculate the corresponding y- value so that we have a set of coordinates which we are going to plot on the graph paper that I will issue to you. Remember that coordinates are a set of ordered number pairs of x and y*	Teacher A demonstrates *knowledge of content* on the topic of quadratic functions. He predominantly uses procedural knowledge to provide explanations on how to draw the graph of a parabola.

<table>
<tr><td>written in the form (x;y)”

<table><tr><td>X</td><td>-4</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Y</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>(x;y)</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table></td><td></td></tr>
<tr><td>Line 10: A boy learner asked Teacher A why he chose 9 x-values; “Sir, how many x-values must one choose?”</td><td>The learners are participating in Teacher A’s lesson as they ask questions about the number of x-values that they can choose in order to calculate the y values.</td></tr>
<tr><td>Line 11: Teacher A responds to the learner’s question “You can choose as many x- values as you like, the minimum being seven but in your choice, you must include zero and there must be equal number of positive and negative x-values as you can also see from my choice. I chose 9 values just to help you see how it is done”. He re-wrote the set of numbers that he had chosen earlier and underlined each negative value and its corresponding positive value for the learner to see what he meant by equal number of positive and negative numbers. He further explained that for a quadratic function, they need to choose more values because three x-values would not be sufficient to reveal the curvilinear nature of a quadratic function. “You need more x-values so that you have enough coordinates to reveal the curvilinear nature of a quadratic function.”</td><td>Teacher A gives learners a procedure of how to choose independent values of x in order to calculate the corresponding y-values from a given quadratic function. The teacher’s content knowledge and knowledge of procedures assisted him in providing such an explanation. Teacher A demonstrates both procedural and conceptual knowledge approach for the question asked by the learner.</td></tr>
<tr><td>Line 12: Learner accepts the explanation without any further questioning which can be assumed that he was satisfied with the teacher’s explanation given.</td><td>Teacher A gave an explanation that seemed to have satisfied the learner but he does not probe the learner with a follow-up question to see if he (the learner) has indeed understood the number of x-values to choose and how they should be arranged.</td></tr>
<tr><td>Line 13: Teacher A proceeds to show the learners how to substitute the chosen x-values in the equation $y = x^2 – 4$ to get the corresponding y- value which he called the independent values of y. He starts by substituting x= -4 and say; “Now that we have the chosen x-values, we need to substitute these values in to the equation of the function to get the corresponding y-values and be able to form coordinate points that will be plotted on the graph. So, for x = -4, we get y= $(-4)^2$ -4 = 16 -4 = 12 and the coordinates are (-4; 12)” The calculated value and the corresponding coordinates were written in the table drawn in</td><td>Teacher A accurately demonstrates his procedural knowledge for calculating the corresponding y-values in order to plot the points. He uses an algorithm to determine the corresponding y values for plotting.</td></tr>
</table>

line 9.	
Line 14: A girl learner questions the calculated value of y as 12 and thinks the value should be -12 and say; "No sir, y must be equal to -12"	The learners are actively involved in the lesson and one of the learners exposes her *preconception or misconception* (line 14) regarding multiplication of integers and exponents.
Line 15: Teacher A responds to the learner's rejection of the calculated y-value by asking the learner a question; "*Why do you think so*?" as he moved closer to the learner's desk.	Teacher A asks for an explanation from the learner in order to get more insight into the learner's thinking; which is a good diagnostic instructional strategy (*pedagogic knowledge*)
Line 16: The leaner explains her thinking that displays that she is not able to distinguish or differentiate between -4^2 and -4x2. She has difficulty perhaps in manipulating exponential notations or does not have the knowledge that a negative number times a negative number is positive and that 'squared' means multiply a number by itself: " *Because -4 squared is -8 and -8-4 = -12"* as she said	The explanation demanded by the teacher of the learner's thinking and effective *pedagogical knowledge* of the teacher helps to expose the learner's misunderstanding and difficulty with exponential notations. At the same time it could be a language difficulty were "squared" means multiply by 2 to her.
Line 17: Teacher A responds by correctly explaining that -4^2 = -4x-4 =16 *"No, $(-4)^2$ is like -4x-4 = 16. In general $a^2 = a$ x a. Do you understand?"* The learner appeared not convinced but accepted the explanation as Teacher proceeded to substitute the next chosen value of x being -3 and said; *"For x =-3, we have y = $(-3)^2$ -4 = (-3x-3) – 4 = 9-4 = 5. Can you see that?"* [Whilst looking at the girl learner who had shown a misconception about squared numbers]	Teacher A correctly explains to the learner the correct concept. His *knowledge of the subject* matter assisted in giving the correct explanation and also how to deal with *learners' misconceptions* but to eliminate the learner's doubts, Teacher A should have given the learner a similar problem for her to solve using the newly acquired knowledge.
Line 18: Teacher A then allowed learners to do the rest of the calculations of the corresponding y-values from the remaining x-values on their own whilst offering support to the learners who were not confident in doing the calculations. He had instructed them to raise their hands in case of difficulties. He said; "*Now that you have an idea of how to calculate the coordinates, I want you to continue to find the corresponding y-values using the given x-values on own. If you experience problems, raise your hand and I will come and assist you."* Learners were randomly pointed at to give the corresponding y-value from a given x-value and ultimately a table as shown on the next page was formed.	Teacher displayed good *content knowledge* on the topic and concepts associated with quadratic functions. He also has shown good *knowledge of pedagogy* by the way in which he supported learners who experienced difficulties with certain concepts such as calculations involving exponential notations (line 16) and choosing x-values (line 10) during the lesson. Teacher A was observed assisting them as he moved around (line 14).

X	-4	-3	-2	-1	0	1	2	3	4
Y	12	5	0	-3	-4	-3	0	5	12
(x;y)	(-4;12)	(-3;5)	(2;0)	(-1;-3)	(0;-4)	(1;-3)	(2;0)	(3;5)	(4;12)

Line 19: Teacher A goes on to explain how to choose a suitable scale to draw the graph once coordinate points have been calculated. "*Now that we have all the required coordinate points, we can now plot the points on the graph papers which I am now handing out to you. To plot the points you need an X-Y plane which is drawn according to scale"* He explained to learners how to choose a suitable scale. "*Use your ruler to draw the X-Y plane with zero at the point of intersection. Consider a certain number of squares on your graph to represent I unit of length in each of the axis, for example 4 small squares to represent 1 unit. Calibrate each axis to accommodate the values associated with that axis. For example, your x-axis should accommodate values from -4 up to 4 units whilst your y-axis should accommodate values from -4 units up to 12 units. Do you understand?"*	Teacher A displays knowledge (*content knowledge*) of how to choose the appropriate scale (*content-specific procedural knowledge*) to draw the graphs but displays poor questioning techniques (*pedagogic knowledge)* to probe learners with regard to assessing whether his learners have understood his explanation of choosing a suitable scale.
Line 20: Class responds to the question in line 19 with a chorus; "Yes'	Teacher A did not ask any individually directed questions to ensure that overall the learners have a common or shared understanding of the concepts being taught. A *probing question* directed to a specific learner would have been appropriate.
Line 21: A girl Learner says; "Sir, I find it difficult to choose a suitable scale" On hearing this comment, Teacher A moved closer to the learner's desk.	Learners are actively engaged. A learner expresses difficulty in choosing a suitable scale for drawing the graph (*learner difficulty* in quadratic functions involving graphs).
Line 22: Teacher A explains to the learner how to choose a scale using the squares of the given graph paper. He said; "*To decide on a scale, choose a number of small squares to represent one unit. As an example, choose 5 small squares of your graph paper to represent one unit, then ten small squares will represent 2 units and so on"* He concludes with a non-probing type of question; "*Do you understand*?"	Teacher A is responding to learner's difficulty on selecting a suitable scale. He repeats the procedure (*content specific procedural knowledge*) on how to choose a scale while demonstrating on the learner's graph paper that he had earlier issued to them.

Line 23: Learner responds to Teacher A's question and says; "yes"	The teacher once again did not follow up with an example-a demonstration by the learner to see if the learner was able to choose a suitable scale for graphing. There is a need for follow-up questions in order assure learner comprehension.
Line 24: Teacher A in concluding the lesson outlines a procedure that learners should follow when they have to draw the graph of a quadratic function " *Take note that the following steps must be followed in the order shown on the chalkboard to draw the graph*"; he said as he numbered the steps from 1 to 5 (see appendix k). He gave learners a home work and said; "*For your home work, you will draw the graphs of the following functions: (i)* $y = x^2-9$ *(ii)* $y = x^2 -4x +4$*"*	Teacher A uses more of his *procedural knowledge* and gives a home work to conclude his post-activity lesson on how to draw the graph of quadratic functions. He uses more of a *procedural knowledge* approach which is amenable to the nature of the concept or topic taught. He also displays sound *conceptual knowledge* in explaining some aspects of the topic

Table 4.1 Description of Classroom observation for Teacher A

Before summarising Teacher A's observed PCK, an excerpt of the interviews held with Teacher A before the lesson will be presented. This will then be used to triangulate the PCK elements of Teacher A observed during lesson observations.

4.2.2 **Pre-Lesson Interviews.**

Pre-lesson interviews were done to collect data about the participating teachers' PCK in teaching quadratic functions in grade 11. The interview questions and responses (Table 4.3) were used to collect data on each of the three elements of pedagogical content knowledge of the two teachers. This was for the purposes of triangulation with the lesson observations described earlier in the preceding section 4.2.1**.**

Pre-lesson Interviews with Teacher A

Question Posed	**Response of Teacher A**
Line 1: Researcher: What are the key concepts in the lesson that you are about to teach?	*The key concepts in this lesson are the x-axis, y-axis dependent values, coordinates, dependent and*

	independent values
Line 2: Researcher: Draw a concept map illustrating the sequence you will follow to teach these concepts.	*Well, I will just give you how the lesson will flow from one aspect to the other: formula of equation → x-values → substitution to get y → choose scale → plot the points on Cartesian plane*
Line 3: Researcher: Does the lesson involve any procedural knowledge?	*I want to show the learners a procedure that they would use to get coordinate points to be able to draw graphs of a parabola*
Line 4: Researcher: Which teaching strategy will be employed to ensure successful delivery of the lesson?	*The lecture method is appropriate for this lesson because other methods such as group work would need that I move around the learners' desks and that is not possible given the size of the class"*
Line 5: Researcher: Why do you choose such a teaching strategy?	*The lecture method helps me to save time and it is appropriate to be used given the large size of the class. There is no room for movement and rearrangement of the sitting plan for learners to allow for group work would waste valuable teaching time.*
Line 6: Researcher: In your selection of examples for illustration of the topic or concept, have you selected real life examples?	*No, there are no real life examples but I have selected a question from the regional grade 11 final examination paper 1 to be used as an example.*
Line 7: Researcher: What is the goal/aim of your lesson?	*The goal of this lesson is to draw (according to scale) the graph of a given parabola using a table of values.*
Line 8: Researcher: Which learners' prior knowledge do you regard as important before the above topic can be successfully taught to learners?	*To draw the graph of a parabola, you must be able to substitute chosen values of x in to the equation of the function and be able to get coordinates. You also need to be able to choose a suitable scale to label your axis*
Line 9: Researcher: What possible learners' misconceptions do you anticipate regarding this particular lesson?	*I have no idea about the possible misconceptions that the learners might have regarding this lesson, but should such a situation arise during lesson presentation, I will deal with it in the classroom. I mean whatever misunderstanding the learners might bring to my attention during the lesson, I will assist the learners.*

Line 10: Researcher: How would you assist learners who experience difficulties with regard to this particular lesson (on any topic about quadratic function)?	*I will give individual attention to the learners who are experiencing difficulties with the lessons or I may as well refer such learners to the learners who have shown good understanding of the topics or I will repeat the lesson if the situation warranted that. To repeat the lesson would depend on the number of learners who need help.*
Line 11: Researcher: Have you prepared an assessment instrument to evaluate whether the goal of the lesson was achieved?	*Yes, I always have a class work or home work to gauge the level of learning that may have taken place.*
Line 12: Researcher: Thank you for your time, we will meet during lesson presentation.	*I thank you.*

Table 4.2: Pre-Lesson Interview questions and Responses of Teacher A

4.2.3 Summary of the PCK of Teacher A based on Pre-Lesson Interviews and Lesson Observations

A summary of Teacher A's PCK will now be presented based on the lesson observation description and the conducted interviews. Each of the three elements of Teacher A's PCK elements will be presented.

- **Knowledge of The subject matter of some aspects of quadratic functions**

Teacher A demonstrated that he has the required content knowledge in the teaching of quadratic functions topics during the actual lesson presentation as he was able to present accurate mathematical facts on the topics on quadratic functions (ref. section 4.2.1). This display of content knowledge is in line with what transpired during interviews when he was able to outline the concepts he was about to teach on how to draw the graph of a parabola, (ref. section 4.2.2, line 1). He said; "*The key concepts in this lesson are the x-axis, y-axis dependent values, coordinates, dependent and independent values"* These key concepts were explained during lesson presentation.

During interviews, he outlined a sequence on how he will teach the concepts of how to draw the graph of a parabola using a table of values. The concept map that he drew can be

presumed to have played a pivotal role in assisting him to present sequential explanations on the topic that he taught which helped learners to understand the concepts in the topics (ref. section 4.2.2, line 2). In some instances, he emphasized on procedural knowledge approach to his lesson on how to draw the graph of a parabola using a table of values where rules and algorithms were emphasized; he said during interviews; "*I want to show the learners a procedure that they would use to get coordinate points to be able to draw graphs of a parabola"* (ref. section 4.2.1, line 24 and 4.2.2, line 3). In the lesson about how to draw the graph of a parabola, he displayed skilful execution of the procedures of calculating the y-coordinate with no errors. He further skilfully explained to learners how to choose an appropriate scale to be able to draw the graph using the calculated coordinate points where both his procedural knowledge and conceptual knowledge were displayed (ref. section 4.2.1 line 11). The concept of drawing the graph of a parabola demands that the learner should understand the curvilinear nature of the parabola; otherwise, learners are inclined to join adjacent points of a parabola in a straight line which distorts the nature of a parabola.

Teacher A's demographic profile (Table 3.1) further suggests that he should have adequate knowledge about this topic since he holds a B.Sc degree in Mathematics and statistics and an Advanced certificate in education in mathematics education. Furthermore, he has 18 years of experience as a teacher which should have helped him develop knowledge of the subject matter and specifically, the topics on quadratic functions.

- **Knowledge of Learners' Misconceptions and Conceptions**

Whilst his content knowledge appeared to be adequate, his knowledge of dealing with learners' conceptions and misconceptions appear to be limited. He seldom asked probing questions to his learners during lessons on how to draw the graph of a parabola which could have helped him to gain some insight into the learners' misconceptions (ref. section 4.2.1, line 16). Though he used oral probing questions during the lesson on the topic of how to draw the graph of a parabola, most of the questions that he asked were of low order level, requiring a "Yes" or "No" response which did not assist the teacher in eliciting learner thinking, learning difficulty and misconceptions if any, and in enhancing teacher-learner discussion (ref. section 4.2.1, lines 8 and 20). It was only in one instance where Teacher A asked a learner to explain why that learner thought a particular answer to a given question was not the correct one (ref. see section 4.2.1, line 15).During this interaction with the learner, it was

discovered that the learner had misconceptions and difficulties regarding exponential notation, multiplication of negative numbers and language difficulty.

Teacher A did not appear to plan his lesson based on some anticipated learners' preconceptions about the topic that he was about to teach on quadratic functions. He acknowledged during interviews that he did not know the possible preconceptions or misconceptions that his learners might have as he taught them how to draw the graph of a parabola (ref. section 4.2.2, line 9). He said; "*I have no idea about the possible misconceptions that the learners might have regarding this lesson, but should such a situation arise during lesson presentation, I will deal with it in the classroom. I mean whatever misunderstanding the learners might bring to my attention during the lesson, I will assist the learners"* Misconceptions and learner difficulties were discovered when Teacher A asked probing questions and allowed learners to do exercises on their own or learners exposing their own misconceptions or difficulties themselves during the lesson activities (ref. section 4.2.1, line 16).Based on the observations just given, Teacher A's knowledge of learners' difficulties and misconceptions can be presumed to be limited on the topic of quadratic functions. But he adequately addressed any difficulties his learners experienced.

- **Knowledge of Teaching Strategies**

On the use of teaching strategies, he preferred using the telling method to deliver his lessons whilst occasionally using oral probing questions to determine learners' knowledge. Asked during the interviews about which teaching method he would use, he said; "*The lecture method is appropriate for this lesson because other methods such as group work would need that I move around the learners' desks and that is not possible given the size of the class"* He combined the lecture method with discussion method where learners were guided on how to draw the graph of a parabola.

He would explain a concept and then demonstrate how to use a specific procedure associated with solving problems in quadratic functions (ref. see section 4.2.1, line 9). Learners were actively involved in his lessons; some asked questions which the teacher tried to address. His apparent deficiency was primarily in lack of follow up examples to ensure understanding on the part of learners. Others were poor questioning technique such as asking questions not directed to a specific learner, which resulted in chorus answers from the entire class He however used some effective teaching strategies such as actively involving learners as he

delivered the lesson, handling of learners' questions and effective use of the chalkboard in drawing and explaining graphs. Quadratic functions demand that individual learners be able to carry out certain operations and skills of graph construction and interpretation on their own and teachers need to know that individual learners are able to do so, through proper inclusive assessment procedures and questioning techniques. Furthermore, home work was seen by the teacher as an extended opportunity for the learners to learn the concepts that he taught them (ref. see section 4.2.1, line 24). Learners were given an opportunity to work on their own which helped the teacher to diagnose learners' misconceptions and difficulties and also enhanced their learning opportunities on the topics. Overall the teacher's knowledge of teaching strategies was regarded as inadequate on the topic of quadratic functions.

4.2.4 Description of Classroom Observations for Teacher B

DESCRIPTION OF LESSON OBSERVATION TEACHER B (LESSON 1)	**CATEGORISATION/THEMES**
Condition of the classroom There were 12 female learners and 23 male learners The classroom represented a well organised learning environment with double desks neatly arranged in four columns and six rows as viewed from the position where the teacher was standing. Though some window panes were missing in some of the windows, the classroom was fairly warm. A chalkboard and a teacher's table with a box of white chalks and a duster were available. There was sufficient room for the teacher to move around.	The classroom displayed a fairly conducive learning environment though poorly resourced.
LESSON OBSERVATION: GRADE 11 **Topic: How to draw the graph of a quadratic function**	**CATEGORISATION/THEMES**
Line 1: **Teacher B:** Introduced the topic for the lesson about quadratic functions; "*Last year, in grade 10 you drew graphs of* $y = ax^2$*, which represent the simplest form of a quadratic function and I want us to quickly review certain facts about this type of graph*". He proceeds and asks learners; "*Can someone tell me the shape of the graph of* $y = ax^2$?" as Teacher B points to one of the learners for an answer.	Introduces the topic by referring learners back to the prerequisite knowledge taught the previous year. *Pedagogical Knowledge* of the teacher is helping to set the scene for the new lesson as the teacher checks what do the learners know and have that will help facilitate access to the new knowledge.

Line 2: The learner responds correctly to the question; " It is cup shaped or bell shaped"	The teacher engages learners through recall type questions and they appear to have the required pre-requisite prior knowledge.
Line 3: Teacher B accepts the learner's answer to the question and proceeds to ask another question *"Yes the shape is not like that of a straight line graph. Now, under which conditions is a graph of* $y = ax^2$ *cup shaped and under which conditions is it bell shaped?"* and points at another learner to give the answer.	Teacher B continues to use recall type questions to assess learners' pre-requisite knowledge and conceptions.(*Pedagogy*)
Line 4: A Learner responds to the question correctly by explaining the conditions under which graphs are cup shaped or bell shaped. The learner said; *"It will be cup shaped when it has a minimum turning point and bell shaped when it has a maximum turning point"*	The learners demonstrate that they still have knowledge of facts about the graph of a simple quadratic function from grade 10. This diagnostic questioning might assist the teacher in knowing that the learners are aware that a quadratic function is not a straight line graph at least.*(Pedagogy*)
Line 5: Teacher B appreciates the learner's response to the question and continues to clarify the conditions for a minimum turning point or a maximum turning point of the parabola using the cup and bell shape concept. He said; "*Well done! When the coefficient of* x^2 *is positive the graph has a minimum or is cup shaped and when the coefficient of* x^2 *is negative, the graph has a maximum and is bell shaped"* He then proceeded with the lesson of the day and said; "*To draw the graph of a parabola you need to choose 7 or 9 integer x-values which must include zero"*	Teacher B acknowledges positively the learner's response and proceeds to provide *conceptual knowledge* on the topic under discussion-. He has *knowledge of the subject matter* and has used the "cap" and "bell" concept to illustrate minimum and maximum turning points. These are real life examples (ref. line 4).
Line 6: Learner asks a question; "What do you mean by integer values?"	Learners are attentive to what teacher B is saying during the lesson as they ask relevant questions.
Line 7: " *They are whole numbers";* Teacher B explains to the learner what he meant by integer values and gave an example of such set of chosen x-values which are symmetrical about zero; namely; -4;-3;-2;-1;0;1;2;3;4. He concludes his explanation by asking the learner to choose his own set of x-values; *"Can you choose your own x-values"*	Teacher B displays *knowledge of the subject matter* and checks for understanding of the explanation from the learner through asking a question (*Pedagogy*).
Line 8: The learner respond to teacher B's question and say: "Yes, -6;-5;-4;-3;-2;-1;0;1;2;3;4;5;6;"	The learner understands the teacher's explanations and can apply his newly gained knowledge about how to choose x-values.

<table>
<tr>
<td>Line 9: Teacher B acknowledges the learner's choice of x-values and says; "That is good." He then chooses y = x^2-9 as the function to be used to illustrate how to use a table of values to draw the graph of parabola. He draws a table as shown below so that the corresponding values of y can be calculated.
<table>
<tr><td>X</td><td>-4</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Y</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>(x;y)</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
</td>
<td>Teacher B displays knowledge of content and pedagogy by the way he explains and handles learner question and answer and how he chooses the appropriate example to illustrate the topic of the day (Line 5, 6 and 8).</td>
</tr>
<tr>
<td>Line 10: "For x = -4; y = (-4)^2 -9 = 7"; he writes the sum on the chalkboard and then tells the learners to also do their own calculations using the approach that he had just demonstrated with x= -4. After a while, all the x-values were substituted and the corresponding y-values were obtained. The table was then completed as shown:
<table>
<tr><td>X</td><td>-4</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Y</td><td>7</td><td>0</td><td>-5</td><td>-8</td><td>-9</td><td>-8</td><td>-5</td><td>0</td><td>7</td></tr>
<tr><td>(x;y)</td><td>(-4;7)</td><td>(-3; 0)</td><td>(2;-5)</td><td>(-1;-8)</td><td>(0;-9)</td><td>(1;-8)</td><td>(2-5)</td><td>(3; 0)</td><td>(4;7)</td></tr>
</table>
</td>
<td>He involves learners in the calculation of the corresponding values of y using the equation of the function (pedagogy). He applies his procedural knowledge frequently punctuated conceptual knowledge (ref. Line 5) to provide explanations to the learners. Learners were seen to be enthusiastic in doing the calculations on their own. It can be presumed to mean that they made meaning of what they were doing.</td>
</tr>
<tr>
<td>Line 11: Teacher B explains to the learners how to choose a suitable scale to draw the parabola on a graph paper: "Now that we have the coordinates, we need to choose a suitable scale to use on your graph paper. Choose 1unit: 10 mm. If 1 unit: 10 mm, how many millimetres will represent 2 units; 3 units etc?" He further says to the learners; "I hope you still remember what coordinates are?</td>
<td>Teacher B explains to learners how to choose a scale which is an important part for the learners to be able to draw the graph. He uses procedural knowledge approach to deliver his explanation. The teacher has appropriate subject matter knowledge and is able to involve his learners (Pedagogy) as he explains how to choose a scale.</td>
</tr>
<tr>
<td>Line 12: A learner responds to Teacher B's question regarding coordinates and says; " A set of ordered number pairs x and y"</td>
<td>The learners have the appropriate pre-requisite knowledge.</td>
</tr>
<tr>
<td>Line 12: Before Teacher B responds to the answer given on coordinates, another learner responds to the question on the scale; "2 units will be 20 mm and 3 units will be 30 mm"</td>
<td>The learners are actively involved in the lesson. Teacher B asks questions that engage learners in expressing their ideas about the topic under discussion (Pedagogy).</td>
</tr>
</table>

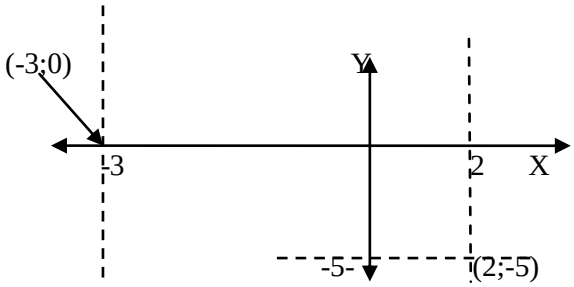
Line 13: Teacher B appreciates the learner's response to the question and says; "*Excellent*". He further gives learners information about how to use their chosen scale; "*You must calibrate both the X and Y axis with the same scale. In your graph paper, one small square is approximately 2 mm so 5 small squares will be equivalent to 10 mm"* Learners were then told to draw their graphs on the graph paper while Teacher B was moving around their desks to offer assistance.	Teacher B encourages learners by rewarding them with appropriate words such as "excellent" (*pedagogy*). He proceeds to give a conceptual meaning of how to choose a scale using the graph papers and he further allows the learners to do the graphing themselves as he moves around to offer support. A good display of the *knowledge of pedagogy.*
Line 14: A learner informs the teacher of the difficulty she is experiencing in plotting the graph; "Sir; I cannot plot the point (-3; 0). Show me how it is done"	The learners are not inhibited to expose their *difficulties* to teacher B. They are encouraged by the positive way in which he handles their questions. Good display of *teaching strategy*.
Line 15: Teacher B provides a conceptual way of identifying the coordinate point on the graph. "*I expected this type of difficulty from some of you. Most learners are unable to plot the correct coordinate points because of failure to read the x and y values correctly. X-values are read along the vertical lines whilst y-values are read along the horizontal lines. As an example, the (-3; 0) is at x= -3 on the x-axis since y=0 along the x-axis. The point (2; 5) is at the intersection of the point x=2 and y =5 see the sketch on the chalkboard.'* He explained while drawing the sketch below on the chalk board and also drawing horizontal and vertical dotted lines to illustrate the exact position of the coordinates. 	Teacher B uses *conceptual knowledge* approach to deal with *learners' topic-specific difficulty*. The teacher anticipated such difficulty in plotting correct coordinate points. He uses the chalk board to illustrate where to put coordinate points on the X-Y plane. He displays sound *mathematics knowledge* on drawing the graph of a parabola and provides accurate and logical explanations using both *conceptual and procedural knowledge*.
Line 16: Learner accepts explanation; *" I understand, I will plot the other points"*	The teacher gave a satisfactory explanation and a solution to the *learner's difficulty*.
Line 17: Teacher B indicates to his learners that he will give them assistance as he moves around their tables and concludes this lesson with a home work. He said; "*I will be moving around your desks to assist those who need assistance. For your home work you will draw the graphs of the following: (i) $y = x^2 - 4$ (ii) $y = x^2 - 2x$".* The lesson of the day was concluded.	Teacher B gave his learners home work; which serves to offer the learners an extended opportunity to learn how to draw the graph of a parabola using a table of values. A good display of *pedagogy*.

Table 4.3 Description of Classroom observations for Teacher B

4.2.5 Pre-lesson interviews with Teacher B

Pre-lesson interview excerpt of Teacher B will be indicated before a summary of his PCK as observed in the lesson observations (section 4.2.1) is discussed as was done with Teacher A.

Question Posed	**Response of Teacher A**
Line 1: Researcher: What are the key concepts in the lesson that you are about to teach?	*The key concepts in this lesson are the x-axis, y-axis, coordinates, and scale.*
Line 2: Researcher: Draw a concept map illustrating the sequence you will follow to teach these concepts.	*Well, my lesson is planned as shown* *Choose values of X → substitute in equation → get Y → form coordinates points ▶ choose scale coordinate points on graph paper*
Line 3: Researcher: Does the lesson involve any procedural knowledge?	*In a way, the lesson involves both conceptual knowledge and procedural knowledge. Knowing why certain things are done the way they are done and what must be done first before the next step.*
Line 4: Researcher: Which teaching strategy will be employed to ensure successful delivery of the lesson?	*I will first explain to the learners how to draw this graph then arrange the learners in groups of say five to six learners and then demonstrate how to draw this graph should be drawn using a table of values.*
Line 5: Researcher: Why do you choose such a teaching strategy?	*Demonstration and lecture method will allow me to give the learners a guided practice whilst group work will enable them to learn from each other.*
Line 6: Researcher: In your selection of examples for illustration of the topic or concept, have you selected real life examples?	*Well, in a way there is real life examples if you consider that the shapes of the quadratic functions can be described in terms of either cup shaped or bell shaped.*
Line 7: Researcher: What is the goal/aim of your lesson?	*The goal of this lesson is to draw (according to scale) the graph of a given parabola using a table of values.*
Line 8: Researcher: Which learners' prior knowledge do you regard as important before the above topic can be successfully taught to learners?	*Knowledge of the shape of the simplest quadratic function $y = ax^2$. This knowledge allows learners to form a good link between the expanded forms of the*

	quadratic function $y = ax^2 + bx + c$ and $y = ax^2$
Line 10: Researcher: What possible learners' misconceptions do you anticipate regarding this particular lesson?	*Generally, most learners find it difficult to choose an appropriate scale for a graph and secondly, they find it difficult to plot points correctly as they have a problem in reading the values of the system of axis in the X-Y plane.*
Line 11: Researcher: How would you assist learners who experience difficulties with regard to this particular lesson (on any topic about quadratic function)?	*I usually give guided practice first for all learners and having arranged them in groups helps me to move around each group and offer explanations to struggling groups. If all fails, I usually repeat the lesson during extra lesson times.*
Line 13: Researcher: Have you prepared an assessment instrument to evaluate whether the goal of the lesson was achieved?	*Yes, I always have a class work or home work to assess the level of learning that took place.*
Line 14: Researcher: Thank you for your time, we will meet during lesson presentation.	*You are welcomed.*

Table 4.4: Pre-Lesson Interviews and Responses of Teacher B

4.2.6 Summary of the PCK of Teacher B Based on Lesson Observations and Pre-lesson Interviews

A summary of Teacher B's PCK based on the description of the observed lessons and the pre-lesson interview is presented as follows.

- **Knowledge of The Subject Matter**

Teacher B displayed adequate knowledge of the subject matter as he presented lessons on quadratic functions in grade 11. Key concepts to be taught on the lesson about how to draw the graph of a parabola were clearly articulated during pre-lesson interviews as he provided them correctly without even referring to the text book. He said "*The key concepts in this lesson are the x-axis, y-axis, coordinates, coordinates and scale"* He combined procedural knowledge and conceptual knowledge in the presentation of his lessons to his learners (ref. section 4.2.4, lines 5, 10 and 13). Teacher B displayed comprehensive knowledge, both conceptual and procedural as well as flexibility in aspects of quadratic functions that he

taught. Conceptual knowledge approach was seen to be predominant in his presentation as he emphasised and insisted on learners' comprehension and understanding of the concepts rather than mere routine application of rules and algorithms of the topics taught. He avoided that his learners memorized procedures without any understanding. The explanations that he provided during the lessons were seen to help his learners to access the topics that he taught under quadratic functions. They were able to do exercises associated with the topic correctly (ref. section 4.2.4, line 15). He showed flexibility in his explanations since he was able to provide two different explanations on the same concept so as to make the topic under discussion accessible to his learners (ref. section 4.2.5, line 11 and 13).

Furthermore, Teacher B has the required professional qualifications to teach quadratic functions in grade 11 (Table 3.1) and has twelve years of experience as a mathematics teacher.

- **Knowledge of Teaching Strategies**

Teacher B demonstrated that he has insufficient knowledge of teaching strategies to teach quadratic functions in grade 11. Though he engaged learners with questions that assisted learners to express their mathematical thinking and knowledge (ref. see section 4.2.4, lines 4 and 8); it has to be mentioned that some of his questions were of low order level because they were mainly recall type of questions (section 4.2.4, line 2); but these could be justified in terms of accessing base knowledge before introducing new topic. Furthermore, as he asked questions, he pointed at learners who raised their hands and mostly left out those seemed to know the correct answers only and did not engage those who did not. It can only be presumed that he focused on getting the correct answers from the learners who appeared to know the answers. It would be good to get answers from any of the learners so as to pick up misconceptions and conceptions that learners might have regarding the topic.

He mainly used the lecture method where he was observed as the main imparter of information to learners to present his lessons. As he used this method, he would explain concepts to the learners in a way that assisted the learners to access the topic that he was presenting on quadratic functions. Group work was sometimes used by Teacher B in the delivery of his lessons to assist learners who were seen to be having difficulties with the topics. He also evaluated the learners' prior knowledge on the topic through oral questioning before presenting the new lesson (ref. section 4.2.4 line 1 and 3). He occasionally asked

learners questions that required learners to express their mathematical thinking or knowledge on the topic. His encouraging words such as "*well done"* or *"excellent"* encouraged learners to be involved as they were able to reveal their own misunderstanding without being coerced or inhibited (ref. section 4.2.4, line 14). Teacher B ensured that all the learners felt confident and safe enough to pose questions or answer them rightly or wrongly.

He used the chalkboard to the advantage of all learners as they were able to have a clear view of what was written on the chalkboard. The examples he used were relevant to the topics on quadratic functions. His main source of information was the school mathematics textbook used by his learners as well. It encouraged learners to also use the book to their advantage in knowing the page numbers where additional practice questions on each section on quadratic functions can be referred if need be.

- **Knowledge of Learners' Misconceptions and Conceptions**

While his knowledge of the subject matter appeared to be adequate, his knowledge of learners' conceptions with regard to the topic seemed to be inadequate. There were few instances where he came to class with an anticipation of the type of difficulties that his learners might experience about a topic (ref. section 4.2.4, line 15). During the pre-lesson interviews on the lesson about how to draw the graph of a parabola; he clearly articulated possible learners' difficulties and said; "*Generally, most learners find it difficult to choose an appropriate scale for a graph and secondly, they find it difficult to plot points correctly as they have a problem in reading the values of the system of axis in the X-Y plane"* There were few instances of course; where Teacher B asked probing or follow-up questions during the lesson, where he also discovered learners' difficulties and addressed them confidently and adequately(ref. section 4.2.4, line 5). These were rare.

4.2.7 **Lesson Plan Analysis of the participating Teachers**

This section focuses on the results of the analyses of the lesson plans prepared by the teachers in teaching quadratic functions. They were analysed to get more information about the teachers' PCK in terms of knowledge of the subject matter; knowledge of teaching strategies and knowledge of learners' misconceptions and conceptions. Data about these three elements were obtained from the lesson plan analysis using a set of guiding questions as shown in table 4.5 below. The two teachers used a common lesson plan template. Asked why they used a

common lesson plan template they said; *"We received these templates from the curriculum implementer of the district and all schools in the district are expected to use these templates for mathematics preparations."*

ELEMENT OF PCK	CHECKED IN THE PREPARATION	
a. Knowledge of subject matter	• Are key concepts to be taught during the lesson indicated in the preparations? • Does the preparation indicate possible procedures to be taught to the learners? • Does the lesson preparation reflect accurate concepts and procedures associated with the topic on quadratic functions?	
b. Knowledge of teaching strategies	• Is the teaching strategy to be used stated in the preparations? • Are alternative teaching strategies to be used during the lesson reflected in the preparations? • Are examples to be used during the lesson indicated in the lesson preparation?	
c. Knowledge of learners' conceptions	• Does the preparation reflect possible misconceptions that will be addressed during the lesson? • Does the preparation reflect the required learners' prior knowledge before the start of the new topic? • Are possible learners' difficulties reflected in the preparations? • Is an assessment instrument indicated in the preparations? • Is the goal of the lesson clearly stated in the preparations?	

Table 4.5: Guiding for Lesson Plan Analysis

An analysis of each teacher's lesson plan was done and findings for both teachers on each of the three elements of pedagogical content knowledge presented.

4.2.7.1 Lesson Plan Analysis of Teacher A

KNOWLEDGE OF THE SUBJECT MATTER

Guiding question	Observation made	Categorization/Theme
a) Are key concepts on quadratic functions to be taught indicated in the lesson plan?	Some of the key concepts on how to draw the graph of a quadratic function were indicated in the	Teacher's preparation reflects knowledge of the subject matter including concepts for the topic

	lesson plan	
b) Does the preparation indicate possible mathematical procedures for the topic to be taught?	The preparation indicates that the teacher will help learners to calculate the y-values and complete a given table of values.	Procedures to be taught were indicated but not all of them
c) Does the lesson preparation reflect accurate concepts associated with the topic on quadratic functions?	The lesson plan reflects accurate concepts associated with quadratic functions and especially on how to draw the graph of a parabola.	Teacher A has good knowledge of the subject matter

KNOWLEDGE OF TEACHING STRATEGIES

Guiding Question	**Observation Made**	**Categorization/Theme**
a) Is the teaching strategy to be used stated in the lesson plan?	The lesson plan reflects the teacher's activities as well as the learners' activities. The teacher will first do an exposition of how a graph of a quadratic function is drawn. Learners' will calculate the y-values from the chosen x-values	Teacher A used the lecture and demonstration methods to teach the rules.
b) Are alternative teaching strategies to be used during the lesson reflected in the preparation?	The lesson plan does not have an alternative teaching strategy indicated	Teacher A prefers to use one teaching method in a lesson. Does not demonstrate sufficient in flexibility in his teaching approach

Guiding Question	**Observation Made**	**Categorization/Theme**
c) Are examples to be used during the lesson indicated in the lesson preparation?	Examples to be used during the lesson were indicated in the lesson plan. For this lesson, $y= x^2-4$ was used as the main example. No everyday life examples	Teacher A prepares his examples in advance before the lesson. Does not use real-life examples to make concepts meaningful to learners.

KNOWLEDGE OF LEARNERS' CONCEPTIONS AND MISCONCEPTIONS

Guiding Question	**Observations Made**	**Categorization/Theme**

a) Does the preparation reflect possible misconceptions that will be addressed during the lesson?	The preparation does not reflect possible learners' misconceptions that will be addressed during the lesson.	Teacher A does not show that he is aware of possible misconceptions to be addressed.
b) Does the preparation reflect the required learners' prior knowledge required before the new topic?	Yes, the required learners' prior knowledge was reflected in the lesson plan for the topics to be taught on quadratic functions.	Teacher A knows the required prior knowledge of learners before teaching a new topic on quadratic functions.
c) Are possible learners' difficulties reflected in the preparation?	No, possible learners' difficulties were not indicated in the lesson plan.	Teacher A has no knowledge about the possible difficulties in the topic on quadratic functions
d) Is an assessment instrument indicated in the preparations?	Yes, a home work was given as an instrument to assess learning. Expand $y = x^2 - 9$ was given as a home work.	Teacher A knows that learning must be assessed after each lesson and uses past examination questions to assess his learners.
e) Is the goal of the lesson clearly stated in the preparation?	The lesson plan did not reflect the goal of the lesson.	The goal or objective of a lesson must always be stated for the sake of the learners and teachers as well, but this was not the case.

4.2.7.2 Lesson Plan Analysis of Teacher B

KNOWLEDGE OF THE SUBJECT MATTER

Guiding question	Observation made	Categorization/Theme
a) Are key concepts on quadratic functions to be taught indicated in the lesson plan?	Key concepts on how to draw the graph of a quadratic function were indicated in the lesson plan	Teacher's preparation reflects knowledge of the subject matter including concepts for the topic
b) Does the preparation indicate possible mathematical procedures for the topic to be taught?	The preparation mentions that the teacher will help learners to calculate the y-values, plot the points and also completion of a given table of values.	Procedures to be taught indicated in the plan.

c) Does the lesson preparation reflect accurate concepts associated with the topic on quadratic functions?	The lesson plan reflects accurate concepts associated with quadratic functions and especially how to draw the graph of a parabola.	Teacher B has good knowledge of the subject matter.

KNOWLEDGE OF TEACHING STRATEGIES

Guiding Question	Observation Made	Categorization/Theme
a) Is the teaching strategy to be used stated in the lesson plan?	The lesson plan reflects the teacher's activities as well as the learners' activities. The teacher will first do an exposition of how a graph of a quadratic function is drawn. Learners will be grouped and then calculate the y-values whilst the teacher will offer assistance	Teacher B used the lecture method of teaching. It may be due to the fact that he wanted to use that method for the observed lessons.
b) Are alternative teaching strategies to be used during the lesson reflected in the preparation?	Group work is mentioned as an alternative teaching strategy. For what? Group work in itself is meaningless without a particular purpose	Teacher B indicated only one teaching method in a lesson though he used group work as well. This might suggest that he only wanted to use the lecture method only in that lesson.

Guiding Question	Observation Made	Categorization/Theme
c) Are examples to be used during the lesson indicated in the lesson preparation?	Examples to be used during the lesson were not indicated in the lesson plan but there was an example. For this lesson, $y= x^2-9$ was used as the main example.	Teacher B did not reflect the examples to be used on the lesson plan and no real life example was used to enable concept understanding.

KNOWLEDGE OF LEARNERS' CONCEPTIONS AND MISCONCEPTIONS

Guiding Question	Observations Made	Categorization/Theme
a) Does the preparation reflect possible misconceptions that will be addressed during the lesson?	The preparation does not reflect possible learners' misconceptions that will be addressed during the lesson.	Teacher B may not have had misconceptions to be addressed or is not used to the practice of including knowledge of learner preconceptions in the lesson plan.

b) Does the preparation reflect the required learners' prior knowledge required before the new topic?	Yes, the required learners' prior knowledge was reflected on the lesson plan. Learners should be able to the graph of $y = ax^2$ as taught in grade 10 for what topic?	Teacher B knows the required prior knowledge of learners before this lesson on how to draw the graph of a parabola.
c) Are possible learners' difficulties reflected in the preparation?	No, possible difficulties were not written on the lesson plan.	Teacher B may have ignored indicating possible difficulties in the topic. However during lesson observations; he anticipated and identified possible learners' difficulties.
d) Is an assessment instrument indicated in the preparations?	Yes, a home work was given as an instrument to assess learning on how to draw a graph of a parabola using a table of values. The following equations were given: $y = x^2 - 4$ and $y = x^2 - 2x$	Teacher B knows that learning must be assessed after each lesson and uses the appropriate instrument. Past examination questions were used as home to assist learners to learn how to draw the graph of a parabola.
e) Is the goal of the lesson clearly stated in the preparation?	The outcome was indicated that the learners must be able to investigate, analyse, describe and represent the function.	The goal of the lesson must be given for each lesson to assist the teacher and the learners to remain focused.

Based on the presented lesson plan analysis, a joint summary of the PCK of the two participating teachers; Teacher A and Teacher B is given focusing on each of the three elements of pedagogical content knowledge as displayed in the lesson plans.

- **Knowledge of The subject Matter**

On analysing the two participating teachers' lesson plans (section 4.2.7.1 and 4.2.7.2) it was found that both teachers' lesson plans contained accurate information about the concepts on the topics on quadratic functions that they were about to teach which may imply that they have knowledge about the content of the topics they were about to teach. Both Teacher A and Teacher B did not indicate possible rules and algorithms which could be used to solve problems associated with the topics that they intended to teach they mentioned that there were procedures to be taught to learners during the pre-lesson interviews (see section 4.2.5). Lesson plans contained accurate information about the topics on quadratic functions.

The two teachers' lesson plans displayed that they both have adequate content knowledge on the topic of quadratic functions.

- **Knowledge of Teaching Strategies**

The analysis of the lesson plans of Teacher A and Teacher B revealed that they both have inadequate knowledge of teaching strategies to teach quadratic functions (section 4.2.1 and 4.2.4). They both relied heavily on the lecture method of teaching which puts them as the main imparters of the information about the topics on quadratic functions to their learners. The lecture method appeared to be adequate for presenting the topic on how to draw the graph of a parabola; given that it is fairly new to most learners. Teacher B occasionally varied the lecture method with group work as indicated in his lesson plan which was also observed during lesson presentation. How the teacher would approach the lesson activities was indicated on the lesson plan where the teacher's activities as well as the learners' activities were indicated. Specifically, Teacher A indicated in his lesson plans that he would first explain how a graph of a parabola is drawn and then guide learners to calculate y-values from the chosen x-values. Teacher B's teaching strategy was to first revise work that was done previously, explain and demonstrate how to use a procedure to calculate for the y- values and then organise learners in to groups and then monitor and assist learners in the various groups. The two teachers' knowledge of teaching strategies of teaching quadratic functions was seen as inadequate since they both relied on one teaching method, namely; the lecture method, to teach.

- **Knowledge of Learners' Misconceptions and Conceptions**

From the lesson plan analysis, it was noted that Teacher A did not reflect possible learners' misconceptions on the topic on how to draw the graph of a parabola using a table of values but he did indicate possible difficulties that his learners would experience. The difficulties that he anticipated (as per lesson plan) his learners would experience were about choosing the appropriate scale (appendix o). This somewhat contradicted what he said during lesson interviews when he said; "*I have no idea about the possible misconceptions that the learners might have regarding this lesson, but should such a situation arise during lesson presentation, I will deal with it in the classroom. I mean whatever misunderstanding the learners might bring to my attention during the lesson, I will assist the learners"* There was neither indication of possible learners' difficulties nor misconceptions on Teacher B's lesson

plans, but during interviews as well as during lesson observations, learners' difficulties were anticipated, identified and addressed (section 4.2.4, line 15 and section 4.2.6). It can be presumed that the inconsistency of indication of learners' misconceptions and learners' difficulties during interviews, lesson presentation and lesson plans, is as a result of the teachers not used to being asked to provide such information in their lesson plans and secondly not perhaps having adequate and systematic knowledge about learners' preconceptions regarding the topic on quadratic functions.

4.2.8 Development of the Pedagogical Content Knowledge of the Teachers

The study further wanted to find answers to the question of how the teachers may have developed the PCK that they were using in teaching quadratic functions. Interviews were held with each teacher as depicted in the tables shown below (Table 4.6 and 4.7) which display both the question posed and the responses to each question.

4.2.8.1 Interviews on Development of the Pedagogical Content Knowledge of Teacher A

QUESTION POSED	RESPONSE BY PARTICIPANT
Did you receive any special training as a mathematics teacher after your initial teacher training? [Previous question continued]	*Yes, after my initial teacher training (a B.Sc degree, majoring in Mathematics and Statistics) in secondary teaching), I registered and passed an advanced certificate (ACE) in Mathematics Education with the University of South Africa. This has helped me to master the content of most of the topics such as functions and others that I am supposed to teach my grade 10 to 12 learners.*
Do you attend workshops that focus on teacher development?	*Yes, in our district, curriculum implementers, who are specialists in mathematics teaching and employed at the district level, always arrange content and method workshops in mathematics that focus on how to teach certain topics such as quadratic functions and also about the content of some topics.*
Have you ever observed any of your colleagues when they are teaching a mathematics lesson?	*Yes I do. In most cases, during the IQMS (Integrated Quality Management System) development circles, we are bound to observe our peers to help in their assessment of the teaching standards that the department has set for educators to display as they teach. Doing such observations has helped to discover some mistakes that my colleagues do when teaching. These include teaching without a clearly stated goal, checking learners' prior knowledge and not giving feedback after an*

	exercise.
What have you learnt from such observation?	*I have learnt certain techniques of how to develop a lesson and how to conclude a lesson. I have also learnt how to deal with classroom control especially when there are learners who behave in an unacceptable way during lessons.*
What are your qualifications as a mathematics teacher?	*I have already told you that I hold a B.Sc degree specialising in Mathematics and Statistics and I also have an Advanced Certificate in Education specialising in the teaching of mathematics.*
For how long have you been a mathematics teacher?	*I have been teaching Mathematics for the past 18 years and mostly in grade 11 and 12 though I can also teach grade 10 classes.*
How often do you review the lessons that you have taught?	*I always make a review of the lessons that I have taught so as to check how well or badly I have presented the lesson. This helps me to plan better for the next lesson and identify areas where I need to improve my presentation.*
Thank you for your time!	*You are welcome!*

Table 4.6: How Teacher A may have developed the PCK

From the responses as noted in table 4.5 above including table 3.1 in chapter 3, Teacher A holds a B.Sc degree, majoring in Mathematics and Statistics and also holds an Advanced Certificate in Education (ACE) specialising in the teaching of Mathematics. Such qualifications may have helped shape Teacher A's content knowledge of quadratic functions. He has 18 years of experience as a mathematics teacher. Teacher A has continued to study for an advanced certificate in Education (ACE) with a higher institution of learning in South Africa. Such training may have helped shape the teacher's content knowledge of mathematics during the initial training as well as during further studies.

He has developed a practice of reviewing his lessons when after each lesson in order to check how well or badly he has presented the lesson and this helps him to plan better for the next lesson. The teacher attends workshops which focus on how certain mathematics topics including how the topic on quadratic functions should be taught to learners. He sometimes

observes his colleagues as they present lessons to learners and has learnt techniques on how to develop and conclude lessons and manage classrooms. In summary, initial teacher training, further studies, attending workshops, review of his own lessons and peer observations have helped shape Teacher A's pedagogical content knowledge. Although he has eighteen years of teaching mathematics experience he does not seem to have adequate learner knowledge on conceptions or potential learning difficulties on the topics of quadratic function that he taught grade 11 class (section 4.2.1). (Knowledge of these help the teacher to prepare suitable explanations that will assist learners in the topics on quadratic functions-discussion.)

4.2.8.2 Interviews on Development of the Pedagogical Content Knowledge of Teacher B

QUESTION POSED (TEACHER B)	RESPONSE TO THE QUESTION
Did you receive any special training as a mathematics teacher after your initial teacher training?	*Yes, I managed to obtain Mathematics I and II at the University of South Africa and also registered for an Advanced Certificate in Mathematics with the University of Limpopo*
Do you attend workshops that focused on teacher development?	*Yes, there are workshops that are arranged by the region that are compulsory to be attended by all mathematics teachers*
What have you gained from attending such workshops?	*One workshop that I attended focussed on content matter such as functions, data handling, transformation of functions and probability*
Have you ever observed your colleagues when they were teaching a mathematics lesson?	*Yes, with other colleagues in the Mathematics Department, we have agreed to observe each other and then do constructive criticism on how the colleague may have presented the lesson.*
What have you learnt from such observations?	*I have learnt different teaching strategies that can be incorporated in one lesson.*
What are your qualifications as a mathematics teacher?	*I hold a National Professional Diploma in Education, obtained at the University of North West. I also have an advanced certificate in Education, specialising in Mathematics Education. Currently, I have enrolled for a B.Ed in Mathematics Education with the University of North West*
For how long have you been a mathematics teacher?	*I have twelve (12) years teaching experience and as a mathematics teacher.*

How often do you review the lessons that you have taught?	*I try to reflect on almost all the lessons that I teach. Basically I check whether the approach I have used in the previous lesson was the appropriate one and also whether I cannot improve on the type of explanations that I have given to the learners.*

Table 4.7: How Teacher B may have developed the PCK.

From the above interview responses of Teacher B it can be noted that he received training as a mathematics teacher during his initial teacher training and has also furthered his studies by obtaining Mathematics I and II from a higher learning institution in South Africa. He also attends in-service workshops. Most of the workshops that he has attended focused on improving teacher pedagogic content knowledge in school mathematics regarding the new areas or topics of functions and data handling. It is hardly surprising that Teacher B was able to demonstrate adequate subject matter knowledge in teaching some aspects of quadratic functions.

Furthermore, Teacher B does observe some of his colleagues as they teach. From such observations it can be presumed that he learnt different teaching strategies that could be employed when he teaches his learners as he claimed. His knowledge of teaching strategies as observed when he presented lessons serves to confirm this claim. Teacher B holds a diploma in professional education, specialising in the teaching of mathematics. He has 12 years experience as a teacher of mathematics at grades 10 to 12 and is currently studying for a B.Ed in mathematics education. The teacher claims to always reflect on his previous lessons so as to see how best to improve that lesson in future. This claim concurs with Shulman (1987) where sources of PCK include teachers' reflections on lessons taught.

4.9 CHAPTER CONCLUSION

In this chapter the results of the two case studies were presented. Findings on the two teachers' knowledge of the subject matter; knowledge of teaching strategies and knowledge of learners' conceptions on the topic of quadratic functions were presented. The findings revealed that both teachers have adequate knowledge of the subject matter on the topic of quadratic functions but insufficient knowledge of teaching strategies and knowledge about learners' misconceptions and learners' difficulties on the topic of quadratic functions.

The two participating teachers have adequate professional qualifications to enable them to teach quadratic functions effectively in grade 11. The teachers attend workshops which help them to develop knowledge about the content on quadratic functions and teaching strategies. The two teachers also observe their peers and do lesson reviews to help improve on their teaching approaches.

CHAPTER 5

DISCUSSION OF RESULTS

5.1 INTRODUCTION

This chapter aims to comment on the findings from the two case studies in relation to the main research questions. Similarities and differences displayed by the two participating teachers with regard to their teaching approaches and practices are highlighted in accordance with the framework that guided the study.

Two main themes that were presumed to have an impact on effective teaching of quadratic functions were identified. The first theme, pedagogical content knowledge, with three sub-themes, endeavours to answer the first research question; "*what pedagogical content knowledge effective teachers display with regard to teaching quadratic functions in grade 11 classes?"* This theme focused on the teaching and learning process in the classrooms and how the participating teachers displayed knowledge of each of the three elements of pedagogical content knowledge as they taught lessons on quadratic functions in their grade 11 classes.

The second theme sought to find answers to the second research question; *"how did the teacher develop the pedagogical content knowledge that they use?"* The theme focused mainly on how the participating teachers could have developed the knowledge on the use of each of the three elements of pedagogical content knowledge, namely, knowledge of the subject matter, knowledge of teaching strategies and knowledge of learners' conceptions and misconceptions on the topic of quadratic functions.

5.2 DISCUSSION OF THEMES

5.2.1 Pedagogical Content Knowledge of the Participating teachers

This main theme has been divided into three sub-themes; namely; knowledge of the subject matter, knowledge of teaching strategies and knowledge of learners' conceptions and misconceptions. The discussion of findings on this theme, pedagogical content knowledge, is based on how the teachers displayed the three elements of PCK during lesson observations, lesson plan analysis and one-on-one interviews with the participating teachers.

Knowledge of the Subject Matter of Quadratic Functions

This sub-theme was aimed at finding out about the teachers' knowledge on concepts of quadratic functions, "facts and procedures, the reasons underlying procedures and the relationships between concepts" in quadratic functions as well as how the teacher deployed his knowledge during lesson presentation.

Based on the results from the three research instruments as triangulated, both teachers have adequate knowledge of quadratic functions topics. When the two teachers taught their respective grade 11 learners about how to draw the graph of a parabola using a table of values, concepts such as dependent y- values, independent x-values and scale were accurately explained by both teachers (ref. see section 4.2.1 and 4.2.4). Teacher A emphasised on the use of procedural knowledge to calculate the y-values from a given set of x-values in order to plot the graph. On the other hand, Teacher B used both procedural knowledge and conceptual knowledge (to a limited extent) to explain the concepts on how to draw the graph of a parabola (ref. see section 4.2.4, line 15). The use of procedural knowledge by Teacher A and the use of a combination of both procedural and conceptual knowledge by Teacher B were seen to be effective by the researcher because learners were able to successfully do problems associated with the concepts taught on their own.

While the use of procedural knowledge only by Teacher A was seen as effective (though insight of concepts by learners may not be guaranteed) in solving problems in quadratic functions, Star (2002) contends that the learning of procedures must be connected with conceptual knowledge in order to foster the development of understanding of the concepts, which is in line with what Teacher B was seen doing. Bosse' and Bahr (2008) indicate that if teachers of mathematics apply the alliance of factual knowledge, procedural proficiency and conceptual understanding, it provides a powerful way of learning quadratic functions by learners. Furthermore; learners who are only taught procedures without understanding the concepts, are often not sure when or how to use what they know and such learning is fragile and ineffective.

The two teachers' adequate knowledge of the subject matter on quadratic functions may be linked to their academic and professional qualifications including experience. Teacher A holds a degree in mathematics and has done an advanced professional course (ACE) in the teaching of mathematics. He has eighteen years of experience as a mathematics teacher in

grades 10-12. Similarly, Teacher B holds a diploma in mathematics teaching and has done an advanced professional course (ACE) in the teaching of mathematics. He has 12 years experience in the teaching of mathematics in grades 10 to 12.

In summary, both teachers have adequate knowledge of quadratic functions topics. Teacher A emphasises procedural knowledge to teach the topics whilst Teacher B incorporates both procedural and conceptual knowledge (to a lesser extent though) in the teaching of quadratic functions to bring effectiveness. The experience they have as teachers of grades 10-12, professional and academic qualifications is presumed to have contributed to the development of their subject matter knowledge on quadratic functions. The next paragraph will focus on knowledge of teaching strategies of quadratic functions by the two teachers.

Knowledge of Teaching Strategies of Quadratic Functions

This particular element of pedagogical content knowledge focused on investigating the two teachers' knowledge of the following: (i)how to plan and teach lessons using a variety of teaching strategies, (ii) how they engaged learners through questions and assessment tasks,(iii) how they used the chalkboard during lesson presentation on topics of quadratic functions.

Teacher A usually started his lessons by first reviewing the concepts which were dealt with during the previous lessons by posing questions to the learners (ref. see section 4.2.1). The questions were however mostly recall questions which required "yes" or "no" type of response (ref. see section 4.2.1). It can be assumed that the aim of this (questioning) activity was merely to check if the learners recall the concepts since there was no probing of the learners.

Similarly, Teacher B reviewed lessons taught during the previous day and also did corrections of exercises given the previous day. During such reviews and corrections, Teacher B also engaged his learners through oral probing (ref. see section 4.2.4). The difference with Teacher A regarding oral probing is that Teacher B sometimes asked his learners questions that allowed them to express their mathematical thinking on the topic of quadratic functions (ref. see section 4.2.4). Unfortunately, during such oral probing in the lessons observed, he only pointed at learners who had raised their hands presumably because he thought they knew the correct answers. This researcher thinks it would have been very

good had the teacher also tried to involve all the learners including those who did not raise their hands just to get their thinking about the topic on quadratic functions under discussion (own anecdote).

Both teachers used the telling or lecture method to present their lessons on quadratic functions. According to Anthony and Walshaw (2009), when a teacher uses the telling method, he or she is the main imparter of information while learners are passively listening. The same authors (Anthony and Walshaw) indicate that effective teachers encourage classroom exchanges in the form of carefully planned questions that encourage learners to speak out their mathematical ideas about the concepts on quadratic functions. In such an environment the teacher will be seen as guiding the learners on a topic about quadratic functions whilst the learners themselves are the main contributors in the lesson. The two teachers seldom used teaching methods that encouraged the learners to be the main speakers (ref. see section 4.2.1 and 4.2.4); instead the teachers were the main imparters of information. The over use of the telling method of teaching by the two teachers made their lessons on quadratic functions to be teacher-centred. On rare occasions (especially in the lessons observed), Teacher B sometimes allowed a learner to present a solution of a given exercise on the chalkboard whilst the other learners were allowed to ask questions to the presenter.

The two teachers used the chalkboard very well during each lesson on quadratic functions. Notes on the lesson were neatly developed on the chalkboard as the lesson progressed (ref. see section 4.2.1 and 4.2.4). Examples that were used during the lessons; in particular on the lesson about how to draw the graph of a parabola were written on the chalkboard for learners to see how to draw the graph of a parabola. According to Anthony and Walshaw (2009), effective teachers use tools and representation to bring about effectiveness in their teaching. Class work and home work assignments were given at the end of each lesson by each of the two participating teachers to assess the learners (ref. see 4.2.1 and 4.2.4).

In summary, the two teachers; Teacher A and Teacher B, used the telling method to present most of their lessons on quadratic functions. Teacher A, usually asked recall type of questions during lesson presentation whereas Teacher B sometimes posed questions that required his learners to speak out their mathematical thinking regarding topics on quadratic functions. The two teachers assessed their learners at the end of each lesson and gave them an additional opportunity to learn the concepts through home work. The next paragraph will focus on knowledge of learners'

Knowledge of Learners' conceptions (misconceptions and pre-conceptions)

This element of pedagogical content knowledge was intended to find out the teachers' knowledge of possible difficulties, errors, misconceptions and preconceptions that learners might have on the topics of quadratic functions.

Based on the five observed lessons for each teacher, both teachers, Teacher A and Teacher B assisted learners who had been identified as experiencing difficulties (through their failure of correct application of concepts) regarding certain concepts such as exponential notation and multiplication of integers and plotting of points on the graph respectively (ref. see 4.2.1 and 4.2.4). Errors, including learners' difficulties; were usually discovered when the teachers were doing reviews of lessons done the previous day. These errors and misconceptions were also discovered by the two teachers when doing corrections of exercises given and also when learners asked questions themselves.

The questioning technique of the two teachers was however not effective enough to reveal the learners' misconceptions (ref. Section 4.2.1 and 4.2.4). Teacher A did not ask probing questions that would help expose the learners' misconceptions about certain aspects of quadratic functions during most of his lesson presentation (ref. see section 4.2.1). Most of the questions that he asked during the lessons were mostly recall type of questions that required a 'yes" or "no" response as already alluded to in earlier paragraphs. It was only in one instance during a lesson about how to draw the graph of a parabola where Teacher A asked a learner a probing question which resulted in revealing that a the learner had difficulties in dealing with exponential notation and multiplication of integers (ref. see 4.2.1, line 15). This was the only instance where the teacher asked a probing question which led to the discovery of the learner's difficulties through questioning. According to Kilić (2011), "teachers need to identify learners' misconceptions and difficulties correctly and eliminate such misconceptions and difficulties by asking probing questions or using appropriate tasks. Moreover, teachers need to be able to determine the source of students' difficulties and errors in order to correct them effectively".

On the other hand, Teacher B sometimes asked his learners probing questions that allowed them to express their mathematical thinking. This observation was made when the teacher taught learners how to draw the graph of a parabola (ref. see 4.2.4, line 7). In that lesson he also came to class with an idea of possible learners' difficulties that his learners might

experience about how to draw the graph of a parabola (ref. see section 4.2.4, line 15). In this case he had some prior knowledge of students' potential learning difficulty and used this knowledge on the learners' behalf. This was one of the rare occasions where he came to class with knowledge of possible learners' difficulties on the topic of quadratic functions.

The two teachers' knowledge of learners' difficulties and misconceptions on the topic of quadratic functions appears to be limited, though Teacher B is better than teacher A on this aspect. They both neither engaged learners with probing questions with any measure of consistency ,nor came to their respective classes with a clear set of learners' difficulties and misconceptions about the topic on quadratic functions. The two teachers' failure to use probing questions in order to diagnose misconceptions and their habitual failure to come to class with a clear set of possible learners' difficulties mean that they do not have sufficient knowledge about this component of pedagogical content knowledge.

According to (An, Kulm, & Wu, 2004), "teachers who posses a strong knowledge base in this domain know which mathematical concepts are difficult to grasp, which concepts learners typically have misconceptions about and also know possible sources of their learners' errors". Moreover they are aware of how to eliminate those difficulties, errors and misconceptions.

In summary, the two teachers do oral probing to try to discover misconceptions that learners may have regarding the topic on quadratic functions but their questioning technique is not effective. The teachers mostly use recall type of questions though Teacher B, occasionally asked his learners thought provoking questions. The two teachers were able to deal with learners' misconceptions and learners' difficulties on the topics of quadratic functions once discovered. They used the learners' responses to identify the type of misconceptions that their learners might have had. It has been noted that the two teachers' knowledge of learners' misconceptions and difficulties is however very limited; though comparatively, Teacher B was better than Teacher A on this knowledge domain.

The next section will focus on how the teacher may have developed his pedagogical content knowledge.

5.2.2 Development of the Pedagogical Content Knowledge of the Participating Teachers

This theme, which helps to answer the second research question, was intended to reveal how the two teachers may have acquired the type of pedagogical content knowledge that they were using by analysing the information gleaned from the interviews conducted (ref. see sections 4.2.2 and 4.2.3).

Teacher A received further training on the teaching of mathematics after his initial teacher training programme. His initial teacher training is a BSc degree, majoring in Mathematics and Statistics and he also has an advanced certificate in Education, specialising in the teaching of mathematics. Teacher B holds a National Professional Diploma in Education obtained from a certain University. He obtained mathematics I and II at a University after his initial teacher training programme. He further obtained an advanced certificate in mathematics teaching. In terms of the Employment of Educators Act 1998 (South Africa), both educators are relevantly qualified to teach mathematics at grades 10-12 level.

Their qualifications may be the reason why their content knowledge of the subject matter on quadratic functions can be considered to be adequate. They both have a good grasp of the various topics of quadratic functions.

The two teachers attend workshops arranged by their district. Most of the workshops that Teacher A attended dealt with aspects of how to teach certain topics such as sequences and analytical geometry. For Teacher B, most of the workshops that he attended dealt with content matter such as functions, data handling and transformation of functions. It appears as if the workshops that they have attended did not discuss learners' conceptions and learners' difficulties on the topic of quadratic functions. The two teachers appear to have limited knowledge about knowledge of learners' conceptions and misconceptions.

Regarding their teaching experience which is also regarded as a source of pedagogical content knowledge (Shulman, 1987), Teacher A has 18 years experience as a mathematics teacher whilst Teacher B has 12 years. Their experience in teaching the subject may have contributed to their effectiveness in terms of getting good results in the grade 12 final mathematics examination. Surprisingly, despite their experience in teaching the subject, their knowledge of learners' conceptions and misconceptions is low. Both teachers observe other

mathematics teachers teaching quadratic functions and other topics in the grade 11 and 12 Mathematics syllabi.

In summary, the two teachers are relevantly qualified to teach mathematics in grades 10-12 and have the required content knowledge to teach quadratic functions. They also have appropriate experience in terms of long service but it is surprising that their knowledge of learners' misconceptions is limited though teacher B is better compared to Teacher A.

5.3 CONCLUSION

The study effectively investigated the pedagogical content knowledge held by two successful teachers whose selection to participate was based on their learners' performance in the Grade 12 Mathematics examination the past three years. Their average performance was in the region of a 70% pass rate over the past three years. A qualitative research approach using the case study was used. A framework to guide the study and assist in data collection was developed. The framework that was developed focused on three elements of pedagogical content knowledge as having influence on effective teaching of quadratic functions. Observation protocols, interviews and lesson plan analysis were used to gather data about the teachers' approaches to teaching quadratic functions as a way of collecting the teachers' pedagogical content knowledge. The data collected was triangulated via one-on-one interviews with the teachers before each of five lesson presentations, during lesson observation and through lesson plan analysis. In addition, the study attempted to find out how the teachers had developed the pedagogical content knowledge that they used in the teaching of quadratic functions.

The findings of the study

The findings of the study in relation to the research questions stated below are presented:

> *1. What pedagogical content knowledge do effective teachers display with regard to teaching quadratic functions in grade 11?*
>
> *2. How did the teachers develop the PCK that the y use when teaching quadratic functions?*

For the first research question, the two effective teachers were found to have adequate knowledge of the subject content knowledge on quadratic functions. They presented logical

and accurate lessons to their learners which made the topics on quadratic functions accessible to the learners. Exercises given to learners were usually taken from past examination question papers by both teachers. Teacher A used procedural knowledge to present his lessons on quadratic functions. Teacher B sometimes incorporated procedural knowledge and conceptual knowledge (to a limited extent) to teach. Drill work on past examination question papers was emphasised by both teachers. These practices may be presumed to have assisted the two teachers to produce good results in grade 12 mathematics National Senior Certificate examinations.

However, the study found that the teachers' knowledge of instructional strategies was limited though effective. Both teachers relied heavily on the lecture method. They used the lecture method in all the lessons presented, though Teacher B would occasionally employ the use of group work where learners were grouped in fours or fives to work on a solution of a given exercise and the teacher would then choose one learner from a volunteering group to present the solutions. No other teaching method was used by the teachers. Particularly noteworthy was the absence of probing questions that would help determine learners' preconceptions about the topic on quadratic functions or to make a note of them. From the literature review (ref. section 2.2.2.2), it was found that effective teaching strategies are able to maximize learners' time and engagement in learning tasks and encourage learners' active participation during lessons. The attributes mentioned above can only be realized if the teacher uses instructional strategies that can initiate and sustain insightful learning processes in the mathematics classroom lesson.

Findings on the teachers' knowledge of learners' conceptions and misconceptions on the topics of quadratic functions revealed that, the two teachers have limited knowledge on this knowledge domain. They were sometimes unable to detect the misconceptions which the learners had by analysing their (learners) responses to questions. These findings are consistent with the findings of Prediger (2010), who in a study which focused on diagnostic competencies of pre-service teachers on learners' misconceptions and difficulties; found that the teachers have difficulties in properly analysing their learners' responses to diagnose misconceptions. To resolve the problem of learner difficulties, such as the common issue of poor arithmetic skills, asking probing questions could well reveal the actual source of the cause of the learners' misunderstanding, rather than the teachers merely reiterating the procedures that they had taught. Most of the lessons were teacher-centred with questions

posed to the learners being recall type, closed questions requiring one word answers only. The type of questions posed mostly did not allow learners to explain their thinking so that any misconception could be noted and corrected. Learners' solutions were mostly used as a resource to deal with misconceptions. Both teachers selected exercises from past examination question papers for their learners and the learners were drilled on these. This may suggest that their success in producing good grade 12 results in the National Senior Certificate mathematics examinations is based on the drill and practice strategy solely for the purpose of passing the examination.

The teachers' effectiveness as defined in this study could be linked to their experience as mathematics teachers, their knowledge of the subject matter and their use of examination-related questions that allowed the learners to become fully acquainted with the form of questions that would be found in the final examination. So, knowledge of the subject matter, the use of procedural knowledge and drill work makes some teachers effective in terms of producing grade 12 results. The findings of Star (2005) in his study that investigated the role of procedural knowledge in mathematics learning partly support the results emerging from this current study. He (Star) found that procedural knowledge applied with insight of the concepts is a useful tool in mathematics learning by learners.

5.4 Limitation of the study

The limitations of this study need to be taken into account when considering the findings as the researcher was aware of them from the outset and care was taken to accommodate them in the best possible way.

- The study dealt with only two cases; hence generalisation would need to be handled with discretion.
- Investigating teachers' pedagogical content knowledge is challenging as it reveals itself in many places and ways, such as, in teachers' planning, classroom interactions, explanations, mathematical competency, and so on and a study of only one environment could lead to a limited perspective emerging.
- The study was restricted to one circuit only due to funding and time constraints.

5.5 Recommendations

The study has revealed that though the teachers are effective in terms of producing good mathematics results in Grade 12, their knowledge of teaching strategies and that of knowing their learners' conceptions and misconceptions on the topic of quadratic functions; is inadequate. The teachers' use of procedural knowledge and drill work contributes to their learners achieving good results in mathematics. To improve the two teachers' knowledge of learners misconceptions and conceptions on the topics of quadratic functions, they should use the learners' responses as a resource of revealing the learners' thoughts about a specific topic on quadratic functions. They need to ask questions that require learners to explain their ideas about a topic on quadratic functions. In addition, the way the teachers prepare their lessons needs to reflect possible learners' difficulties that would need to be addressed during the lesson. Possible learners' difficulties can be obtained from the learners' responses during oral probing and also from the learners' solutions of exercises given. Furthermore, the teachers' lesson plans should also indicate all key concepts, prior knowledge of learners, the teaching strategy to be used as well as possible misconceptions that the teacher thinks the learners might experience. The teachers need to be exposed to workshops that not only deal with the content knowledge of quadratic functions only but also focuses on knowledge domains of learners' misconceptions and knowledge of teaching strategies that could be used to effectively teach quadratic functions.

Recommendation for further studies

Based on the findings of this study, it has been noted that some successful teachers do not necessarily know or practise all the identified elements of pedagogical content knowledge. A good knowledge of the subject matter and use of drill work can bring success in teaching based on the teaching approach that teachers use. Teachers teach mathematics based on their understanding and belief of what they think mathematics is and how it should be taught. So, a study to investigate whether there is a relationship between the pedagogical content knowledge held by a teacher on a specific topic in mathematics and the teacher's mathematical beliefs is necessary.

REFERENCES

Addler, J., Pournara, C., Taylor, D., Thorne, B. & Moletsane, G. (2009) Mathematics and science teacher education in South Africa: a review of research, policy and practice in times of Change. *African Journal of Research in Mathematics, Science and Technology Education.* Special issue, 2009, pp. 28- 46.

Addler, J. (2002).Lessons from and in curriculum reform across contexts. *The Mathematics Educator*. Volume 12(2), pp. 2-5.

Addler, J. & Pillay, V. (2007). An investigation into Mathematics for teaching: insights from a case. *African Journal of Research in Science, Mathematics and Technology Education.* Volume 11(2), pp. 87-102.

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school mathematics teachers in China. *U.S. Journal of Mathematics Teacher Education*, 7, pp. 145–172.

Anthony, G. & Walshaw, M. (2009). Characteristics of Effective Teaching of Mathematics: A View from the West. *Journal of Mathematics Education.* Volume 2(2), pp.147-164.

Attard, C. (2011). "If you like the teacher, you'll get maths more: student talk about good mathematics teachers. *Curriculum Leadership: An Electronic Journal for Leaders in Education*. Volume 9 (6), pp. 5-23.

Ball, D. L., Lubienski, S. T. & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), Handbook of research on teaching (4th ed). New York: Mac Millan.

Ball, D.L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics.* Volume 8 (1), pp. 40-48.

Ball, D.L. (1991). Research on teaching mathematics: making subject matter knowledge part of the equation . In J. Brophy (Ed), *Advances in research on teaching:* Volume 2. *Teachers' subject matter knowledge and classroom instruction:* Greenwich, ET: JAI press.

Bochner, A.P. (2002). Criteria against ourselves. In N.K Denzin and Y.S. Lincoln (Eds)., *The Qualitative Inquiry Reader*. Thousand Oaks: Sage Publications.

Bosse', M.J. & Bahr, D.L. (2008). The State of Balance Between Procedural Knowledge and Conceptual Understanding in Mathematics Teacher Education. *International Journal of Mathematics Teaching and Learning*. Volume 25 (11), pp. 1-28.

Brink, H. (2006). Fundamentals of Research Methodology. (2nd Edition). Cape Town: Juta.

Brodie, K. (2007) Dialogue in mathematics classrooms: beyond question and answer method. *Pythagoras*. Volume (66) pp. 3-13.

Brown, G. & Atkins, M. (1988). Effective teaching in higher educatio*n*. London: Routledge.

Bukova-Güzel, E. (2010). An investigation of pre-service mathematics teachers' pedagogical content knowledge, using solid objects. *Scientific Research Essays.* Volume 5 (14), pp.1872-1880.

Carpenter, T.P., Fennema, E., Peterson, P.L. & Carey, D.A (1988).Teachers' Pedagogical Content Knowledge of Student's Problem Solving in Elementary Arithmetic . *Journal for Research in Mathematics Education*, Volume 19 (5), pp 385- 401.

Carroll, J. (2007). Teaching mathematics in primary schools: different types of teachers. Victoria University: Victoria, Wellington.

Centre for Development and Enterprise (CDE). (2004). From laggards to world class: reforming maths and science education in South Africa's schools. *Abridged.* Report on Research. Johannesburg: Centre for Development and Enterprise.

Centre for Development and Enterprise (CDE). (2011). Value in the Classroom: The Quantity and Quality of South Africa's Teachers. Abridged Report on Research. Johannesburg: Centre for Development and Enterprise.

Chick, H.L., Pham, T. & Baker, M.K. (2006). Probing teachers' pedagogical content knowledge: lessons from the case of subtraction algorithm . In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds). *Identities, Cultures and learning spaces* (Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australia), pp .139-146. Sydney : Merga

Cockburn, A.D. (2008). How can research be used to inform and improve mathematics teaching practice? *Journal of Mathematics Teacher Education.* Volume 11, pp. 343-347.

Connors, B. (1991). Teacher development and the Teacher. In P. Hughes (Ed.) *Teachers' professional development*. pp. 53-81. Camberwell, Victoria: Australian Council for Educational Research.

Darling-Hammond, L. (2006). Powerful Teacher Education. San Francisco: Wiley.

De Jong, O., Ahtee, M., Goodwin, A., Hatzinikita, V. & Koulaidis, V. (1999). An international study of prospective teachers' initial teaching conceptions and concerns: the case of teaching combustion. *European Journal of Teacher Education.* Volume 22(1), pp. 45-59.

De Miranda, M.A.(2008). Pedagogical Content Knowledge and Engineering and Technology Teacher Education: Issues for Thought. *Journal of the Japanese Society of Technology Education,* Volume 50 (1), pp. 17-26

De Vos, A.S. (Editor) (1998). Research at grass roots. A Primer for the caring professions. First Edition. Pretoria: Van Schaik.

Dekkers, P. & Mnisi, E. (2003). The nature of science - do teachers have the understandings they have to teach? *African Journal of Research in SMT Education,* Volume 7, pp. 21-34.

Denzin, N.K. & Lincoln, Y.S. (Editors.) (2002). *The Qualitative Inquiry Reader*. Thousand Oaks: Sage Publications.

Deubel P. (2009). Professional development. Accessed online at http//www.ct4me.net/professional-development.htm on 30-03-2009.

Doyle, W. (1984). Effective classroom practices for secondary schools. University of Texas, Austin.

Eilam, B. & Poyas, Y. (2009). Learning to teach: enhancing pre-service teachers' awareness of the complexity of teaching-learning processes. *Teachers and Teaching,* Volume 15(1), pp. 87-107.

Fennema, E & Franke, M.L.(1992). Teachers' Knowledge and its Impact. In Douglas A. Grouws (Ed), *Handbook of Research on Mathematics and Teaching*, New York, MacMillan, pp. 147-164

Greenwald, R., Hedges, L.V. & Laine, R. D. (1996). The effect of the school resources on student achievement. *Review of Educational Research*. Volume 66 (3), 361-396.

Grossman, P.L. (1990). The making of a teacher: teacher knowledge and teacher education. New York: Teachers College Press.

Handbook of research on teaching 4th Edition. pp. 433-456. New York: Macmillan.

Helsinki Declaration: Recommendation for conduct of clinical research. 1964. http://ohsr.od.nih.gov/helsinki.php3 . Accessed 20 July 2008.

Hiebert, J. (2007). Effective mathematics teaching: what does the research say? University of Delaware, Delaware.

Hill, H.C., Rowan, B & Ball, D.L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, Volume 42 (2), pp. 371-406.

Howie, S.J. (2003). Language and other background factors affecting secondary pupils' performance in mathematics in South Africa. University of Pretoria, Centre for Evaluation and Assessment.

Ibeawuchi, E.O. (2010). The role of pedagogical content knowledge in the teaching of Quadratic Functions. MEd dissertation: University of South Africa.

Ignacio, N.G., Nieto, L.J. & Barona E.G. (2006). The effective domain in mathematics learning. *International Journal of Mathematics*, Volume 1, pp. 16-32.

Ingvarson L., Beavis A., Bishop A., Peck R. & Elsworth G. (2004). Investigation of effective mathematics teaching and learning in Australian secondary schools. Commonwealth: Australia.

Ishmail, M.I. (2004). Impact of curriculum transformation on classroom practice in Northern Cape schools. University of Free State: Bloemfontein.

James A., Naidoo, J. & Benson H. (2008). Casme's approach to the sustainability of science education in South Africa. University of Kwa-Zulu-Natal: Durban.

Kanyongo, G.Y., Scheiber, J.B. & Brown, L.I. (2007). Factors affecting mathematics achievement among 6th graders in three sub-Saharan countries: the use of hierarchical linear models (HLM). *African Journal of Research in Science, Mathematics and Technology Education,* Volume 11, pp. 37-46.

Kilić, H. (2011). Pre-service secondary Mathematics Teachers' knowledge of students. *Turkish Online Journal of Qualitative Inquiry,* Volume 2(2), pp. 17-35.

Krauss, S., Neubrand, M., Blum, W. & Baumert, J (2005). The professional knowledge of German secondary mathematics teachers: investigations in the context of COACTIV Project: *Journal of Educational Psychology*. Volume 100(3), pp. 716-725

Kwong, C.W., Joseph, Y.K.K., Eric, C.C.M & Khoh, L.S. (2007). Development of Mathematics Pedagogical Content Knowledge in Student Teachers.*The Mathematics Educator,* Volume 10 (2), pp 27-54

Lim, C.S. (2007). Characteristics of mathematics teaching in Shanghai, China: through the lens of a Malaysian. *Mathematics Education Research Journal*, Volume 19 (1), pp. 77-89.

Lincoln, Y. & Guba, E. (1985). Naturalistic inquiry. Beverly Hills, CA: Sage.

Maharaj, A. (2007). Using a task analysis approach within a guided problem-solving model to design mathematical learning activities. *Pythagoras,* Volume 66 (4), pp. 34-42.

Maoto, S. & Wallace, J. (2006).What does it mean to teach mathematics for understanding? When to tell and when to listen. *African Journal of Research in Science, Mathematics and Technology Education,* Volume 10(1), pp. 59-70.

Maree J.G. & Erasmus C.P. (2007). Mathematics skills of Tswana-speaking learners in the North West province of South Africa. Unpublished MEd dissertation. University of Pretoria.

Maree, K. , Aldous. C., Hattingh, A., Swanepoel, A. & Van der Linde, M. (2006). Predictors of learner performance in mathematics and science according to a large-scale study in Mpumalanga. *South African Journal of Education*, Volume 26 (2), pp. 229-352.

Mason, J. (2006). Mixing methods in a qualitatively driven way. *Qualitative Research*, Volume 6 (1), pp. 9-25.

McKenzie, B. & Turbill, J. (1999). Professional Development, classroom practice and student outcomes: exploring the connections in early literacy development. University of Wollongong.

Mji, A. & Makgato, M. (2006). Factors associated with high school learners' poor performance: a spotlight on mathematics and physical science. *South African Journal of Education,* Volume 26 (2), pp. 253-266.

Muir, T. (2008). Principle of practice and teacher actions: influences on effective teaching of numeracy. *Mathematics Education Research Journal*, Volume 20 (3), pp. 78-101.

Mwakapenda, W. (2001).Using everyday experiences in teaching mathematics: a case study of change in Malawian classrooms. *Journal of Saarmste*, Volume 5(6), pp. 53-64.

Nation, J.R. (1997). Research Methods. Upper Saddle Valley, New Jersey: Prentice-Hall.

Nuremberg Code. US Government Printing Office. (1949). Trials of war criminals before the Nuremberg military tribunals under control. *Council Law* No. 10, Volume 12, pp. 181-182. Washington DC. Found on the Web site http://onlineethics.org/reseth/nuremberg.html Accessed 20 July 2008.

Oliver, P. (2003). The Student's Guide to Research Ethics. Library of Congress Cataloguing: In Publication Data: Great Britain.

Opolot-Okurut, C. (2005). Student attitudes towards mathematics in Ugandan secondary schools. *African Journal of Research in Science, Mathematics and Technology,* Volume 9 (2), pp. 167-174.

Polaki, M. V. & Nenty, H.J. (2001). Gender differences in mathematics performance attribution among first year students at National University of Lesotho:

implications for access to and performance in mathematics. *African Journal of Research in Science, Mathematics and Technology*, Volume 5(5), pp. 41-52.

Policy Handbook for Educators. (2003). Employment of Educators Act 76, 1998: Education Labour Relations Act.

Prediger, S. (2010). How to develop Mathematics for teaching and understanding: The case of Meaning of the Equal sign. *Journal of mathematics Teacher Education*, Volume 13(1), pp: 73-93

Ramatlapana, K. A. (2009). Provision of in-service training of mathematics and science teachers in Botswana: Teacher perspectives. *Journal of Mathematics Teacher Education.* Volume 12, pp. 153-159.

Randall, E. S. (2008). Teacher development and change in the context of teaching large and under-resourced science classes. MEd dissertation: University of Pretoria.

Republic of South Africa (Constitution) (1996). Constitution. Act No. 108 of 1996. Cape Town: Constitutional Assembly.

Rodwell, M. K & Byers, K.V. (1997). Auditing constructivist inquiry: perspectives of two stakeholders. *Qualitative Inquiry*. Volume 3 (1), pp. 116-135.

Rosenshine, B. V. (1986). Synthesis of research on explicit teaching. *Educational Leadership*, Volume 43(7), pp. 60- 69.

Ryan, J & McCrae, B. (2005) Subject matter knowledge: Mathematical errors and misconceptions of beginning Pre-service teachers. In: *Building connections: Theory, Research and Practices: Proceedings of the 28th Annual conference of the Mathematics Education Research Group of Australia.*

Saunders, M., Lewis, P. & Thornhill, A. (2003). Research methods for business students. (3rd Edition). England: Pearson Education Ltd.

Schneider, M. & Stern, E. (2008). Conceptual and procedural knowledge of mathematics problem: their measurement and their causal interrelations. Max Planck Institute for Human Development: Germany schools. London: Routledge Falmer.

Schurink, W.J., Schurink, E.M. & Poggenpoel, M. (1998). Focus group interviewing and audio-visual methodology in qualitative research. In: De Vos, A.S. (Ed.) 1998, *Research at grass roots. A Primer for the caring professions.* Pretoria: Van Schaik.

Setati, M. (2002). Researching mathematics education and language in multilingual South Africa. *The Mathematics Educator*. Volume 12(2), pp. 6-20.

Shulman, L. (1987). Knowledge and teaching: foundations of the new reform. *Harvard Educational Review*. Volume 57(1), pp. 1-22.

Shulman, S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher*. Volume 15(20), pp. 4-14.

Silver, E. A; Mesa, V.M ., Morris, K.A.; Star, J.R. & Benken, B.M. (2009). Teaching mathematics for understanding: an analysis of lessons submitted by teachers seeking NBPTS Certification. *American Educational Research Journal.* Volume 46(2), pp. 501-531.

Sinkovic, R. R., Penz, E. & Ghauri, P.N. (2008). Enhancing the Trustworthiness of a Qualitative Research in International Business. *Management International Review*. Volume 48 (6), pp. 689-714.

Sorensen, R. (2003). Effective Teaching in High School Mathematics. *Teachers and Teaching*. Volume 12(5), pp. 312-335.

Star, J.R (2005). Reconceptualising Conceptual Knowledge. *Journal for Research in Mathematics Education.* Volume 36 (5), pp. 404-411

Stewart, J. (2009). Investigating Relationship Between Mathematics Teachers' Content Knowledge, Their Pedagogical Knowledge and Their Learners' Achievement in Terms of Functions and Graphs. MEd dissertation: Nelson Mandela Metropolitan University.

Tardif, M., Gauthier, C., Gerin-Lajoie. D., Lenoir. Y., Lessard, C., Martin. D., Mujawamariya D. & Mukamurera, J. (1998). Professional Knowledge and Education Training. Laval University, Quebec.

Taylor, N. (2009). The state of South African schools Part 1: time and the regulation of consciousness. *Journal of Education*, Issue No 46, pp. 9 - 32

Terre Blanche, M. & Durrheim, K. (1999). *Research practice: applied methods for the Social Sciences*. Cape Town: University of Cape Town Press.

Tsui, A.B.M.(2009). Distinctive qualities of expert teachers. *Teachers and Teaching.* Volume 15 (4), pp. 421- 439.

Turnuklu, E.B. & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: pre-service primary teachers' perspectives. *IUMPST: The Journal*. Volume 1, pp. 1 - 13.

Van Driel, J.H., Verloop, N. & De Vos, W. (1998). Developing science teachers' pedagogical content knowledge. *Journal of Research in Science Teaching.* Volume 35(6), pp. 673-695.

Westhood, P. (2004). Effective teaching to reduce educational failure. University of Hong-Kong.

Wilson, P.S., Cooney, T.J. & Stinson, D.W. (2005). What constitutes good mathematics teaching and how it develops: nine high school teachers' perspectives. *Journal of Mathematics Teacher Education*, 2005(8), pp. 83-111.

Winsor, M.S. (2003). Pre-service mathematics teachers' knowledge of functions and its effect on lesson planning at the secondary level. Unpublished PhD Thesis, University of Iowa.

Yusof, Y. M. & Zakaria, E. (2010). Investigating secondary mathematics teachers' pedagogical content knowledge: a case study. *Journal of Education and Sociology* , March, 2010, pp. 32 -39

Zakaria, E. (2010). Analysis of students' errors in learning of quadratic equations. *International Education Studies.* Volume 3(3), pp .105-110

Zakaria, E. & Zaini, E. (2009). Conceptual and procedural knowledge of rational numbers in trainee teachers. *European Journal of Social Sciences,* Volume 9(2), pp.202-217.

Zaslavsky, O. (1997). Conceptual obstacles in the learning of quadratic functions. *Focus on Learning Problems in Mathematics*, Volume 19 (1), pp. 20- 44.

Zerpa, C, Kajender, A. & Van Barneveld, C. (2009). Factors that impact pre-service teachers' growth in conceptual mathematical knowledge during a mathematics methods course. *International Electronic Journal of Mathematics Education*, Volume 4 (2), pp. 57-76

APPENDIX A : Pre-Lesson Interview Questions

ELEMENTS OF PCK FOR THIS STUDY	QUESTION RELATED TO PCK ELEMENT	RESPONSES
a. Knowledge of the subject matter .	1. What are the key concepts in the lesson that you are about to teach? 2. Draw a concept map illustrating the sequence you will follow to teach these key concepts. 3. Does the lesson involve any procedural knowledge that the learners must know? If so, what does the procedure involve?	…… …… …… …….. …… …… …… …… ……
b. Knowledge of teaching strategies	1. Which teaching strategy will you employ to ensure successful delivery of this lesson? 2. Why did you choose such a strategy? 3. In your selection of examples to be used in this lesson, have you selected real-life examples?	…… ……... ……
4. Knowledge of learners' conceptions	1. What is the goal/aim of your lesson? 2. Which learners' prior knowledge is regarded as important before the above key concepts can be successfully taught to learners? 3. What possible learner misconceptions do you anticipate regarding this lesson? 4. How will you assist learners who experience difficulties with this lesson? 5. Have you prepared an assessment instrument to evaluate whether the goal of the lesson have been achieved?	…… …… …… …… …… ……... …….. …….. …… ……

Adapted from Chick, Baker, Pham (2006): Probing teachers' pedagogical content knowledge: Lessons from the case of the subtraction algorithm)

APPENDIX B : Classroom Observation Protocol

Lesson number:-_____ Topic_____ Duration of period:_____

1. What pedagogical content knowledge successful teachers display with regard to the teaching of quadratic functions?

PCK ELEMENT TO BE OBSERVED	EVIDENT WHEN THE TEACHER….	OBSERVED PRACTICE DISPLAYED
b. Knowledge of the subject matter	1. Exhibits deep and thorough conceptual understanding of identified aspects of functions. 2. Identifies critical mathematical components within the concept of functions that are fundamental for understanding and applying that concept. 3. Displays skills for solving problems in the area of functions.	**1**…… …… …… **2**…… …… …… …… …… **3** …… ……
b. Knowledge of Teaching strategies	1. Uses appropriate activities in Instruction 2. Uses real life examples and analogies in instruction 3. Utilizes different instructional strategies in presentations.	**1**…… …… **2**…… …… **3.** …… ……
c. Knowledge of learners' conceptions	1. Addresses learners' misconceptions 2. Displays expectations of possible difficulties learners may face during learning and address such. 3. Discusses learners' ways of thinking about a concept. 4. Shows an awareness of the instruments to measure student learning and how to use them	**1**…… …… **2**…… …… **3**…… …… **4**…… ……

Framework for observing Pedagogical Content Knowledge of teachers (Based on Chick, Baker, Pham (2006): Probing teachers' pedagogical content knowledge: Lessons from the case of the subtraction algorithm)

APPENDIX C: Guiding Questions on Lesson Plan Analysis

ELEMENT OF PCK	CHECKED IN THE PREPARATION	OBSERVATIONS MADE
a. Knowledge of subject matter	1. Are key concepts to be taught during the lesson indicated in the preparations? 2. Does the preparation indicate possible procedures to be taught to the learners? 3. Does the lesson preparation reflect accurate concepts and procedures associated with the topic on quadratic functions?	
b. Knowledge of teaching strategies	1. Is the teaching strategy to be used stated in the preparations? 2. Are alternative teaching strategies to be used during the lesson reflected in the preparations? 3. Are examples to be used during the lesson indicated in the lesson preparation?	
c. Knowledge of learners' conceptions	1. Does the preparation reflect possible misconceptions that will be addressed during the lesson? 2. Does the preparation reflect the required learners' prior knowledge before the start of the new topic? 3. Are possible learners' difficulties reflected in the preparations? 4. Is an assessment instrument indicated in the preparations? 5. Is the goal of the lesson clearly stated in the preparations?	

APPENDIX D: Interview questions on Pedagogical Content Knowledge Development of the Participating Teachers

Item	Question to be asked
1	(a) Did you receive any special training as a mathematics teacher after your initial teacher training?
2	(a) Do you attend workshops that focus on teacher development? (b) What have you gained from attending such workshops?
3	(a) Have you ever observed your colleague when he/she is teaching a mathematics lesson? (b) What did you learn from such an observation?
4	What are your qualifications as a mathematics teacher?
5	For how long have you been a mathematics teacher?
6	How often do you review the lessons that you have taught?

Adapted from Shulman (1987): Knowledge and Teaching: Foundations of the new reform.

APPENDIX E: Letter to request for Permission to conduct a Research

Enq: Sibuyi C.D
Cell: 082 499 8277
E-mail: cdsibuyi@vodamail.co.za

P.O Box 336
THULAMAHASHE
1365

18th February 2011

The Head of Department
Department of Education
Private Bag X11341
NELSPRUIT
1200

Sir/Madam

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN SOME OF YOUR SCHOOLS: MYSELF

The above matter bears reference;

1. I, Charles Duzephi Sibuyi, hereby request to conduct a research in some of your schools that will meet the requirements of the sampling technique that will be used in the study.
2. I am currently registered with the University of Pretoria, as an M.Ed (Assessment and Quality Assurance) student.
3. The title of my research study is: **Investigating Successful Teachers' Pedagogical Content Knowledge in teaching Quadratic Functions in School Mathematics** and I have successfully defended it.
4. The study will use a qualitative research design using a case study method where data will be collected from selected teachers.
5. Hoping for a favourable response to my request.

Yours Faithfully

_____

Sibuyi Charles Duzephi

APPENDIX F: Permission from The Provincial Department of Education

education
DEPARTMENT: EDUCATION
MPUMALANGA PROVINCE

Private Bag X 11341
Nelspruit 1200
Government Boulevard
Riverside Park
Building 5
Mpumalanga Province
Republic of South Africa

Litiko leTemfundvo Umnyango weFundo Departement van Onderwys Umnyango weze Mfundo

Enquiries: A.H Baloyi (013) 766 5476

Mr. C.D. SIBUYI
P.O. BOX 336
THULAMAHASHE
1365

RE: APPLICATION TO CONDUCT EDUCATIONAL RESEARCH IN SOME OF THE SCHOOLS IN THE PROVINCE.

Your application to conduct educational research on the topic: "Investigating Successful Teachers' Pedagogical Content Knowledge in teaching Quadratic Functions in School Mathematics" was received on the 21 February 2011.

Your detailed research proposal (questions, purpose and the background) gives an impression that your study will benefit the entire department especially the schools which are beneficiaries of the research study. Given the motivation and the anticipated report of the study, I approve your application to conduct your research in the selected schools of the department.

You are further requested to read and observe the guidelines as spelt out in the attached research manual. The importance of this study cannot be overemphasized; therefore you are expected to share your findings with the

department. It will be appreciated if you can present your findings in electronic form and make formal presentation to the Strategic Planning's' research unit.

For more information kindly liaise with the department's research unit @ 013 766 5476 or a.baloyi@education.mpu.gov.za.

The department wishes you well in this important study and pledge to give you the necessary support you may need.

RECOMMENDED/NOT RECOMMENDED.

28/02/2014

MS G. MASHITENG **DATE**
SENIOR MANAGER: STRATEGIC PLANNING,
RESEARCH & PROJECT COORDINATION.

APPROVED/NOTAPPROVED:

28/02/2014

MRS MOC MHLABANE **DATE**
HEAD OF DEPARTMENT

APPENDIX G: Letter to the Principals

Enq: Sibuyi C.D
Cell: 082 499 8277

P.O Box 336
THULAMAHASHE
1365
……..

The Principal

……..
……..
……..

Sir/Madam

REQUEST FOR PERMISSION TO CONDUCT A RESEARCH IN YOUR SCHOOL: MYSELF

The above matter refers;

1. I, Charles Duzephi Sibuyi, hereby request to conduct a research in your school.
2. I am currently registered with the University of Pretoria as an M.Ed student.
3. The title of my study is: **Investigating Successful teachers' pedagogical content knowledge when teaching quadratic functions in school mathematics.**
4. The research questions are :
    1. What pedagogical content knowledge do successful teachers display when they teach quadratic functions in grade 11?
    2. How did the teachers develop the pedagogical content knowledge that they use?
5. Your grade 11 mathematics teacher has been identified as being one of the successful teachers in the district based on his/her previous grade 12 mathematics results in the National Senior Certificate for the past three years.
6. Your teacher will be expected to prepare and teach five lessons based on quadratic functions. Each lesson will be observed and video taped. Two interviews, one before the start of lesson observations and the other one after all the lessons have been observed, will be conducted with your teacher. The teacher's lesson plans will also be analysed.
7. There will be no financial incentives for participating in the research but findings will be made known to your teacher. The teacher may withdraw at anytime that he/she feels like and the data collected before withdrawal will not be used any further.
8. The data collection instruments as well as consent forms to participate are herein attached for your attention.

Thanking you in advance.

Yours Faithfully

_____

Sibuyi Charles Duzephi

APPENDIX H: Letter to Invite Participants

Enq: Sibuyi C.D
Cell: 082 499 8277

P.O Box 336
THULAMAHASHE
1365

……

Mr/Ms/Dr/Hon……

……

Sir/ Madam

INVITATION TO PARTICIPATE IN A RESEARCH PROJECT: YOURSELF

The above matter refers;

1. I, Charles Duzephi Sibuyi, hereby invite you to be a participant in a research to be conducted in your school.
2. I am currently enrolled for an M.Ed degree with the University of Pretoria and this research project is a pre-requisite for me to be able to fulfil the requirements of the mentioned degree.
3. The title for my research is: **Investigating Successful teachers' pedagogical content knowledge when teaching quadratic functions in school mathematics.**
4. The research questions are :
    1. What pedagogical content knowledge do successful teachers display when they teach quadratic functions in grade 11?
    2. How did the teachers develop the pedagogical content knowledge that they use?
5. As a grade 11 mathematics teacher, you have been selected to participate in this research based on your previous good performance in grade 12 mathematics National Senior Certificate examination results for the past three years.
6. You will be expected to prepare and teach five lessons based on quadratic functions. All your lessons will be observed and will also be video taped. Two Interviews will be conducted with you, one before the start of lesson observations and the other, after the last observed lesson. Furthermore, all your lesson plans for the five lessons will be analysed.
7. **Kindly note that learners who may withdraw from the project, will have to be taught the lesson that they have missed due to their withdrawal.**
8. Neither risks nor health hazards are anticipated during your participation in this project.
9. There will be no financial benefit for participating but the findings of the study will be made known to you. More over, you are free to withdraw from participation at any time you feel like and the data collected from you before your withdrawal will not be used any further.
10. Data collection instruments and a consent form are herein attached for your attention.

Yours Faithfully

_____

Sibuyi Charles Duzephi

APPENDIX I: Letter to request for permission of Learner Participation from Parents

Enq: Sibuyi C.D
Cell: 082 499 8277

P.O Box 336
THULAMAHASHE
1365

To :

……..

……..

Sir/Madam

REQUEST FOR PERMISSION TO HAVE YOUR CHILD AS PART OF THE GROUP OF LEARNERS WHO WILL BE TAUGHT DURING OBSERVATIONS OF THEIR MATHEMATICS TEACHER

The above matter refers;

1. I, Charles Duzephi Sibuyi, hereby request for your permission to include your child…….(full names) to be part of the group that will be taught by their mathematics teacher when he/she is observed by me (a student researcher) as he teaches lessons on quadratic functions at school.
2. I am currently registered with the University of Pretoria as an M.Ed student.

3. The title of my study is: **Investigating Successful teachers' pedagogical content knowledge when teaching quadratic functions in school mathematics.**
4. The research questions are :
    1. What pedagogical content knowledge do successful teachers display when they teach quadratic functions in grade 11?
    2. How did the teachers develop the pedagogical content knowledge that they use?
5. The grade 11 mathematics teacher in the school where your child is attending has been identified as one of the successful teachers in the district based on his/her previous grade 12 mathematics results in the National Senior Certificate for the past three years and this research is based on such teachers.
6. The teacher will be expected to prepare and teach five lessons based on quadratic functions. Each lesson will be observed and video taped. Two interviews, one before the start of lesson observations and the other one after all the lessons have been observed, will be conducted with your teacher. The teacher's lesson plans will also be analysed.
7. There will be no financial incentives for participating in the research but findings will be made known to your child's teacher. You may withdraw your child from participating at anytime that you feel like and any data collected, which is directly linked to your child before the withdrawal of your child will not be used any further. In case your child is not part of the project, lessons missed by your child will be re-done by the teacher.
8. Consent forms to allow for voluntary participation of your child are herein attached for your attention.

Thanking you in advance.

Yours Faithfully

_____

Sibuyi Charles Duzephi

APPENDIX J: Informed consent form

(Form for parents to allow their children to participate in the research project)

(Must be signed by the parent of each learner research participant who accepts to participate, and be returned with the letter of acceptance of the invitation to participate in the research)

1 Title of research project: **Investigating Successful Teachers' pedagogical Content Knowledge when teaching Quadratic Functions.**

2 I,……, (full names) hereby voluntarily grant my permission for my child ……..(full names) who is in grade 11 at…….(full names of school) to participate as a learner when the mathematics teacher will be observed as part of data collection in the project as explained to me in the letter of invitation to participate in a research project by……..……..

3 The purpose, research procedures, objectives, possible safety and health implications have been explained to me and I understand them.

4 I understand my right to choose whether my child can participate in the project or not and that the information obtained will be handled confidentially.

5 I am aware that the results of the investigation may be used for the purposes of publication.

6 I am aware that it is within my rights to withdraw my child from participation in the project at any time I may feel like.

7 I am aware that in case I withdraw my child from the project, the subject teacher will teach my child the lesson that he/she may have missed.

(Upon signature of this form, the parent will be provided with a copy).

Signed: _____ Date: _____

Witness: _____ Date: _____

Researcher: _____ Date: _____

APPENDIX K: TEACHER A LESSON OBSERVATION DESCRIPTION

Excerpt 1: How to draw the graph of a Quadratic Function (Parabola)

[In each of the excerpts, names of learners that appear are not their real names. Pseudonyms have been used to protect their identities]

Line 1: Teacher A: *"Today's lesson will be about drawing the graph of a quadratic equation function. Such a function is of the form* $y = a x^2 + b x + c$ or $y = a x^2 + b x$ or $y = x^2$. *These are the three forms of the equation. Note that "a" the coefficient of* x^2*; can never be equal to zero. The simplest form of the equation is* $y = x^2$. *Do you understand?* **[The three forms of the equation were written on the chalkboard]**

Line 6: Learners: "Yes" […In a Chorus]

Line 7: Teacher A: *"Now I will show you how to draw the graphs of such functions. You must pay attention as I present how it is done. We will start by the graph of* $y = x^2 - 4$ *as an example. In the given equation, what is the coefficient of* x^2*? Raise your hand if you know the answer. A..h..h!!*[Humming and looking around the class]*Yes Lufuno".*

Line 11: Lufuno: "Two Sir".

Line 12: Teacher A: "*No! Any one else to help, Zenani, give us the solution."*

Line 13: Zenani: "The coefficient of x^2 is one".

Line 14: Teacher A: '*Yes, the coefficient of* x^2 *in the equation* $y = x^2 - 4$ *is one. Now, to draw the graph of the given quadratic function, we need to choose, say, nine values of x and substitute each one of them in the equation* $y = x^2 - 4$ *to get the corresponding value of y and then write the values of x and y in coordinate form; that is* (x; y).*We are going to draw a table of values and the coordinate as shown on the chalkboard. Lets choose the following x-values: -4;-3;-2;-2;-1;0;1;2;3;4 and we call the x-axis the independent variable. The y-axis will form the dependent variable since y-values depend on the x-values."*

Line 21: Mpendulo [A learner in the class**]**: "Sir, but why did you choose these x-values? Can I choose other x- values besides the ones that you have chosen?"

Line 23: Teacher A: '*You have asked me two questions neh? For your first question, I have decided to choose these values and for your second question; YES you can choose other*

values but there must be equal number of negative numbers and positive numbers including zero. Do you understand?"

Line 27: Mpendulo: "Yes"

Line 28: Teacher A: "*Now that we have the values of x, we need to substitute these values in to the equation* $y = x^2 - 4$ *to get the corresponding values of y and be able to form coordinates that will be plotted on the graph. So, for* $x=-4$; $y = (-4)^2 - 4 = 16 - 4 = 12$"

Line 31: Zenani [A learner in the class, interjects]: "No Sir, y must be -12"

Line 32: Teacher A: '*Why do you think so*?"

Line 33: Zenani: "Because -4 squared is -8 and -8-4 = -12"

Line 35: Teacher A: *"No, -4 squared is 16 and 16 – 4 = 12"*

Line 36: Zenani: "I don't understand this".

Line 37: Teacher A: "*You see Zenani; -4 squared is like -4x-4 = 16. Do you understand?"*

Line 38: Zenani:" Ok". [Showing doubts]

Line 39: Teacher A: *"Let's proceed. For* $x = -3$; $y = (-3)^2 - 4 = 5$. *So the coordinates are (-3; 5)"*

[Teacher A calculated the corresponding y-values using the chosen x-values and the table below was developed]

x-values	*-4*	*-3*	*-2*	*-1*	*0*	*1*	*2*	*3*	*4*
y-values	*12*	*5*	*0*	*-3*	*-4*	*-3*	*0*	*5*	*12*
(x;y)	*(4;12)*	*(-3;5)*	*(-2; 0)*	*(-1;-3)*	*(0;-4)*	*(1;-3)*	*(2; 0)*	*(3;5)*	*(4;12)*

Line 46: Teacher A: "*Now that we have all the coordinate points for our chosen x-values, we can now plot the points on the graph paper that I have given you. To plot the points, you need to first choose a scale that will be used to represent the x and y values. The graph of the*

function must be legible enough but not too big for your chosen scale. To decide on the scale, choose a number of small squares to represent one unit. You can choose, say, 5 small squares to represent one unit. So, five squares represent 1 unit then ten small squares, represent 2 units etc. Do you understand?" **[The graph shown on section 4.2.21 was drawn]**

Line 53: Class: "Yes!" [In a chorus]

Line 54: Learner: "Sir, I find it difficult to choose a scale"

Line 55: Teacher A: "*Ok, as I said before, decide on the number of small squares in your graph paper which will make one unit. As an example, 5 small squares may form 1 unit, so 10 small squares will be 2 units and so forth. Do you see that?"*

Line 58: Learner: "Yes"

Line 59: Teacher A: "*For your home work, you are to draw graphs of the following functions using a table of values*: (i) $y = x^2 - 9$ (ii) $-x^2 + 2x - 3$ (iii) $-x^2 + 2x + 8$. *Take note that the following steps must be followed in the order shown to be able to draw the graph of a quadratic function:*

1. *Know the equation of the function*
2. *Choose seven or nine values of x, with equal number of negative values and positives and must include a zero. [They are called the independent values. So the x-axis is the independent axis]*
3. *Substitute each value in to the equation of the function and get the corresponding value of y and write in coordinate form (x;y).*
4. *Decide on a suitable scale to draw the x and y values of your Cartesian plane.*
5. *Plot the points and then join them. Your graph is then drawn. Do not forget to label the axis as well as your graph by writing the equation of the function."*

Line 72: Teacher A*: "Make sure that you do your home work to gain confidence in drawing these graphs".* [The lesson was concluded by giving learners an exercise as a home work as the bell rung indicating the beginning of the next period]

APPENDIX K (Continued)

TOPIC: How to draw Sketch graphs of a Parabola

Line 1: Teacher A: **"***We have seen how we can draw a graph of a parabola using a table of values. All of you will have noted that it involves a lot of calculation for the corresponding values of y after having chosen the seven or nine values of x. Do you still remember how it is done?"*

Line 5: Learners: "Yes!!!" [In a Chorus]

Line 6: Teacher A: *"Today, I want to show you a much shorter method of drawing the graph but this time it is a sketch graph which is drawn not according to a scale. You do not need a scale to draw the sketch graph of a parabola. You only approximate the positions of the points".*

Line 10: Njabulo: [A learner in the class]: "How are we going to do that?"

Line 11: Teacher A: *"Be patient please. To draw a rough sketch of a parabola you need the following points :(i) the roots of the equation or the x-intercepts which we obtain when y=0; that is; when the function value is zero. You also need the (ii) y-intercept; which is obtained when x=0. The third point that is important is (iii) the axis of symmetry or the line of symmetry which we obtain by using the formula x=-b/2a. Lastly you need the (iv) the vertex or the turning point which is obtained when x=-b/2a. The value of –b/2a is substituted in to the main equation to get the y-value. Sometimes you can use the formula* $y = -(b^2 - 4\,a\,c)/4a$ *to get the y-value. In other words,* $(-b/2a; -[b^2 - 4\,a\,c]/4a)$ *are the coordinates of the turning point".*[Teacher A wrote the four points on the chalkboard as he spoke]

Line 20: Learner: "Sir, can you show an example of how to do this? It seems there are so many formulae to be used in order to draw the sketch graph and it looks difficult".

Line 22: Teacher A: "*Ok, I was just about to illustrate that with an example. Please pay attention! Given* $y = x^2 - 2x - 8$*, draw a rough sketch of this function. You do not have to fear; it is easy to do. Just watch. Four important steps must be followed in order to draw a sketch graph of a parabola. You do not have to do the steps in the same sequence as I have numbered them."*

(i) *"We proceed to get the x-intercepts which are obtained when y=0; that is; x^2- 2x -8=0 and we factorise the trinomial which becomes (x-4)(x-2)=0. This imply that x=4 or x=-2. Written in coordinate points; the x-intercepts for this graph are (4;0) and (-2;0). Do you understand?"*

(ii) *"The y-intercept is obtained when x=0; so for $y=x^2$ -2x -8; when x=0 we have $y=0^2$- 2(0) -8 = -8. The coordinates of the y-intercepts are (0;-8). Do you see that class?"*

(iii) *"For the axis of symmetry or line of symmetry x=-b/2a. By the way, who can tell me what the coefficients of x and x^2 are in the given equation respectively?"*

Line 36: Learner: "The coefficient of x is 2 and the coefficient of x^2 is 1".

Line 37: Another Learner: "No, the coefficient of x is -2 and that of x^2 is 1"

Line 38: Teacher A: "*Yes the coefficient of x is equal to the value of "b" which is -2 and the coefficient of x^2 is equal to "a" which is 1".*

(iv) *"For x= -b/2a, imply that x=-(-2)/2(1) =1. So $y = (1)^2$ -2(1) -8 = -9 hence the vertex of the parabola will be (1;-9). Any question?"*

Line 41: Class: "No question".

Line 42: Teacher A: "*I will now draw the rough sketch, using the coordinate points obtained from the four important steps that I have shown you."*

[The teacher drew the rough sketch as shown in section 4.2.1.1(c), figure 4.2 and concluded his lesson by giving learners an exercise and homework]

Appendix L: TEACHER A PRE-LESSON INTERVIEWS

TOPIC: HOW TO DRAW THE GRAPH OF A PARABOLA

1. KNOWLEDGE OF THE SUBJECT MATTER

Researcher: What are the key concepts in the lesson that you are about to teach?

Teacher A: *The key concepts in this lesson are the x-axis, y-axis, dependent values, and independent values*

Researcher: Draw a concept map illustrating the sequence you will follow to teach these concepts.

Teacher A: *Well, I will just give you how the lesson will flow from one aspect to the other. In this case it will follow the sequence as drawn:*

Formula for equation → choose values of X → substitute in equation → get Y → choose scale → plot → points on graph paper

Researcher: Does the lesson involve any procedural knowledge?

Teacher A: *I want to show the learners a procedure that they would use to get coordinate points to be able to draw graphs of a parabola*

2. KNOWLEDGE OF TEACHING STRATEGIES

Researcher: Which teaching strategy will be employed to ensure successful delivery of the lesson?

Teacher A: *The lecture method is appropriate for this lesson because other methods such as group work would need that I move around the learners' desks and that is not possible given the size of the class"*

Researcher: Why do you choose such a teaching strategy?

Teacher A: *The lecture method helps me to save time and it is appropriate to be used given the large size of the class. There is no room for movement and rearrangement of the sitting plan for learners to allow for group work would waste valuable teaching time.*

Researcher: In your selection of examples for illustration of the topic or concept, have you selected real life examples?

Teacher A: *No, I do not have real life examples but I have taken two questions from the Regional grade 11 final examination papers 1 to be used as examples.*

3. KNOWLEDGE OF LEARNERS' MISCONCEPTIONS

Researcher: What is the goal/aim of your lesson?

Teacher A: *The goal of this lesson is to draw (according to scale) the graph of a given parabola using a table of values.*

Researcher: Which learners' prior knowledge do you regard as important before the above topic can be successfully taught to learners?

Teacher A: *To draw the graph of a parabola, you must be able to substitute chosen values of x in to the equation of the function and be able to get coordinates. You also need to be able to choose a suitable scale to label your axis.*

Researcher: What possible learners' misconceptions do you anticipate regarding this particular lesson?

Teacher A: *I have no idea about the possible misconceptions that the learners might have regarding this lesson, but should such a situation arise during lesson presentation, I will deal with it in the classroom. I mean whatever misunderstanding the learners might bring to my attention during the lesson, I will assist the learners.*

Researcher: How would you assist learners who experience difficulties with regard to this particular lesson (on any topic about quadratic function)?

Teacher A: *I will give individual attention to the learners who are experiencing difficulties with the lessons or I may as well refer such learners to the learners who have shown good understanding of the topics or I will repeat the lesson if the situation warranted that. To repeat the lesson would depend on the number of learners who need help.*

Researcher: Have you prepared an assessment instrument to evaluate whether the goal of the lesson was achieved?

Teacher A: *Yes, I always have a class work or home work to gauge the level of learning that may have taken place.*

Researcher: Thank you for your time, we will meet during lesson presentation.

APPENDIX M: TEACHER A'S LESSON PLAN ANALYSIS

TOPIC: HOW TO DRAW THE GRAPH OF A PARABOLA

KNOWLEDGE OF THE SUBJECT MATTER

Guiding question	Observation made	Categorization/Theme
a) Are key concepts on quadratic functions to be taught indicated in the lesson plan?	Some of the key concepts on how to draw the graph of a quadratic function were indicated in the lesson plan	Teacher's preparation reflects knowledge of the subject matter including concepts for the topic
b) Does the preparation indicate possible mathematical procedures for the topic to be taught?	The preparation only mentions that the teacher will help learners to calculate the y-values and complete a given table of values.	Procedures to be taught not planned in advance
c) Does the lesson preparation reflect accurate concepts associated with the topic on quadratic functions?	The lesson plan reflects accurate concepts associated with quadratic functions and especially how to draw the graph of a parabola.	Teacher A has good knowledge of the subject matter

KNOWLEDGE OF TEACHING STRATEGIES

Guiding Question	Observation Made	Categorization/Theme
a) Is the teaching strategy to be used stated in the lesson plan?	The lesson plan reflects the teacher's activities as well as the learners' activities. The teacher will first do an exposition of how a graph of a quadratic function is drawn. Learners' will calculate the y-values from the chosen x-values	Teacher A used the lecture and demonstrate method of teaching
b) Are alternative teaching strategies to be used during the lesson reflected in the preparation?	The lesson plan does not have an alternative teaching strategy indicated	Teacher A prefers to use one teaching method in a lesson

APPENDIX N (continued)

Guiding Question	Observation Made	Categorization/Theme
c) Are examples to be used during the lesson indicated in the lesson preparation?	Example to be used during the lesson was indicated in the lesson plan. For this lesson, y= x^2-4 was used as the main example.	Teacher A prepares his examples in advance before the lesson.

KNOWLEDGE OF LEARNERS' CONCEPTIONS AND MISCONCEPTIONS

Guiding Question	Observations Made	Categorization/Theme
a) Does the preparation reflect possible misconceptions that will be addressed during the lesson?	The preparation does not reflect possible learners' misconceptions that will be addressed during the lesson.	Teacher A may not be aware of possible misconceptions to be addressed.
b) Does the preparation reflect the required learners' prior knowledge required before the new topic?	Yes, the required learners' prior knowledge was reflected on the lesson plan. Learners should be able to calculate y-values given the x-values	Teacher A knows the required prior knowledge of learners before this lesson.
c) Are possible learners' difficulties reflected in the preparation?	Yes, possible learners' difficulties were indicated in the lesson plan. Possible difficulties included learners having difficulty with choosing an appropriate scale.	Teacher A has some knowledge about the possible difficulties in the topic.
d) Is an assessment instrument indicated in the preparations?	Yes, a home work was given as an instrument to assess learning. y = x^2 -9 was given as a home work.	Teacher A knows that learning must be assessed after each lesson and uses the appropriate instrument.
e) Is the goal of the lesson clearly stated in the preparation?	The lesson plan did not reflect the goal of the lesson.	The teacher may not be attaching any value in writing the goal of the lesson in his lesson plans.

APPENDIX M (Continued)

LEARNING AREA:	**GRADE:** 11
DATE/WEEK: 2011-05-23	**DURATION:** 35 minutes
TOPIC *(For this lesson)*	QUADRATIC FUNCTIONS GRAPHS OF QUADRATIC FUNCTIONS
LEARNING OUTCOME(S): *(State in words)*	
ASSESSMENT STANDARD(S) **(State in words)**	
PREREQUISITE KNOWLEDGE: *(What knowledge do learners need to have before this lesson)*	Calculate y values given, x-values Knowledge of the x-y plane
RESOURCES:	Graph paper Classroom mathematics
LESSON PROCEDURE: **1. Introduction:** *(State how you will introduce the lesson)*	Teachers asks a learner to give the definition or general term of a quadratic function.
2. Main Activity: **(a) Teacher:** *(State in short what you will do as the teacher)*	Exposition on how to draw a table of values ie how to calculate y given x for $y = x^2 - 4$ for learners then draw $x = -4, -3, -2, -1, 0, 1, 2, 3, 4$ the x-y plane
(b) Learners' Activities : *(State the activities which learners will be engaged in to enhance their understanding of the topic during your presentation)*	Learners to calculate y values given x ie complete the table Educator to help learners to draw the x-y plane and indicate appropriate scale.
3. Conclusion: *(State how you will conclude the lesson)*	Give learners the homework for $y = x^2 - 9$
ASSESSMENT AND EVALUATION:	Learners had problems in drawing and using appropriate scales for the x-y plane.
Skills	(Please tick what is applicable in the lesson and give a brief explanation below)
Knowledge	
Values	
Attitudes	
REFLECTIONS:	
EXPANDED OPPORTUNITIES:	

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5							

x	-4	-3	-2	-1	0	1	2	3	4
y									

APPENDIX N: Excerpts of Teacher B's Lesson Observation Description

Topic: How to draw graphs of the form $y = ax^2$

Line 1: Teacher B: Last year, in grade 10, you drew graphs of $y= ax^2$, which represents the simplest form of a quadratic function. This year in grade 11, you are expected to draw the graphs of $y = ax^2 + bx$ and $y = ax^2 + bx + c$. Can someone tell me the shape of the graph of $y = ax^2$?

Line 4: Learner: It is cup shaped or bell shaped

Line 5: Teacher B: Correct. All graphs of quadratic functions are either cup shaped or bell shaped. When doe s the graph has a cup shape and when does it have a bell shape?

Line 7: Learner: When the coefficient of x^2 is positive it is cup shaped and when the coefficient of x^2 is negative it is bell shaped.

Line 9: Teacher B: Well done! When the coefficient of x^2 is positive we say the graph has a minimum or cup shaped whilst when the coefficient of x^2 is negative the graph has a maximum or bell shaped. Now, to draw the graph of a parabola, you need to choose x-values which are symmetrical about zero.

Line 13: Learner: What do you mean being symmetrical about zero?

Line 14: Teacher B: It is when you have equal number of negative values and positive values with zero at the centre of the two set of numbers, for example -4;-3;-2;-1;0;1;2;3;4. Can you choose your own numbers using the example given?

Line 17: Learner: Yes, -6;-5;-4;-3;-2;-1;0;1;2;3;4;5;6

Line 18: Teacher B: That is good. You do not have to choose such a lot of x-values. To illustrate the use of a table of values; suppose we have to draw the graph of $y = x^2 – 9$ and we choose our x-values to be -4;-3;-2;-1;0;1;2;3;4. We need to substitute our x-values in to the equation $y = x^2 -9$ in order to get the y-values and form coordinate points. Let's draw a table as shown on the chalkboard, and then calculate the corresponding value of y. For x= -4; $y = (-4)^2 -9= 16 -9=7$ [**Teacher B calculated the corresponding values of y together with the learners**]

X	-4	-3	-2	-1	0	1	2	3	4
Y	7	0	-5	-8	-9	-8	-5	0	7
(x;y)	(-4;7)	(-3;0)	(-2;-5)	(-1;-8)	(0;-9)	(1;-8)	(2;-5)	(3;0)	(4;7)

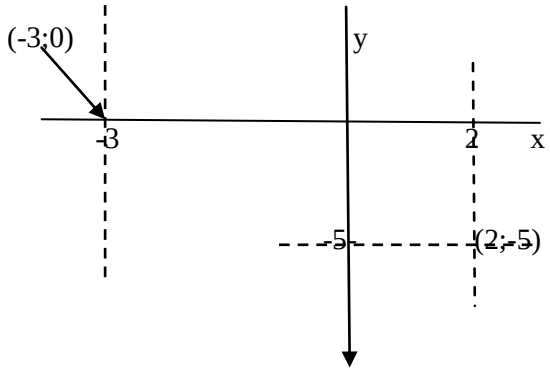
Line 27: Teacher B: Now that we have the values of x and y, we are now in a position to draw the graph of the parabola. We need to choose a suitable scale to use on our graph papers. As an example choose 1 unit: 10mm and then calibrate your x and y axis. If 1 unit: 10mm, how many millimetres will represent 2 units; 3 units etc?

Line 31: Learner: 2 units will be 20 mm and 3 units will be 30 mm

Line 32: Teacher B: Excellent! More over, you must calibrate both the x and y axis with the same scale. In your graph paper, one small square is approximately 2 mm, so 5 small square units will be equivalent to 10 mm.

Line 35: Learner: Sir, I cannot plot the point (-3;0). Show me how I can do it.

Line 36: Teacher B: Common problems experienced by learners in drawing graphs is their failure to read the x and y values. Take note that the x- values are read in a vertical way whilst the y-values are read along the horizontal direction. For (2;-5), the coordinate is the point of intersection of the x and y values. For the coordinate point (-3;0), the point is right at x= -3 since y=0 along the x-axis and x=0 along the y-axis. See diagrams on the chalkboard.



Do you see how you should plot the points?

Line 42: Learners: Yes!

Line 43: Teacher B: I will be moving around your desks to assist those who need assistance. For your home work you will do the following: (i) $y = x^2 - 4$ (ii) $y = x^2 - 2x$

[Lesson was concluded in this way]

Appendix O: TEACHER B'S PRE-LESSON INTERVIEWS

TOPIC: HOW TO DRAW THE GRAPH OF A PARABOLA

1. KNOWLEDGE OF THE SUBJECT MATTER

Researcher: What are the key concepts in the lesson that you are about to teach?

Teacher B: *The key concepts in this lesson are the x-axis, y-axis, coordinates, and scale.*

Researcher: Draw a concept map illustrating the sequence you will follow to teach these concepts.

Teacher B: *Well, my lesson is planned as shown*

Choose values of X → substitute in equation → ~~get~~ Y → ~~form~~ coordinates points → ~~choose~~ scale → coordinate points on graph paper

Researcher: Does the lesson involve any procedural knowledge?

Teacher B: *In a way, the lesson involves both conceptual knowledge and procedure knowledge. Knowing why certain things are done the way they are done and what must be done first before the next step.*

2. KNOWLEDGE OF TEACHING STRATEGIES

Researcher: Which teaching strategy will be employed to ensure successful delivery of the lesson?

Teacher B:*I will first explain to the learners how to draw this graph then arrange the learners in groups of say five to six learners and then demonstrate how to draw this graph should be drawn using a table of values.*

Researcher: Why do you choose such a teaching strategy?

Teacher B: *Demonstration and lecture method will allow me to give the learners a guided practice whilst group work will enable them to learn from each other.*

Researcher: In your selection of examples for illustration of the topic or concept, have you selected real life examples?

Teacher B: *Well, in a way there is real life examples if you consider that the shapes of the quadratic functions can be described in terms of either cup shaped or bell shaped.*

3. KNOWLEDGE OF LEARNERS' MISCONCEPTIONS

Researcher: What is the goal/aim of your lesson?

Teacher B: *The goal of this lesson is to draw (according to scale) the graph of a given parabola using a table of values.*

APPENDIX O (Continued)

Researcher: Which learners' prior knowledge do you regard as important before the above topic can be successfully taught to learners?

Teacher B: *Knowledge of the shape of the simplest quadratic function* $y = ax^2$. *This knowledge allows learners to form a good link between the expanded forms of the quadratic function* $y = ax^2 + bx + c$ *and* $y = ax^2$

Researcher: What possible learners' misconceptions do you anticipate regarding this particular lesson?

Teacher B: *Generally, most learners find it difficult to choose an appropriate scale for a graph and secondly, they find it difficult to plot points correctly as they have a problem in reading the values of the system of axis in the X-Y plane.*

Researcher: How would you assist learners who experience difficulties with regard to this particular lesson (on any topic about quadratic function)?

Teacher B:*I usually give guided practice first for all learners and having arranged them in groups helps me to move around each group and offer explanations to struggling groups. If all fails, I usually repeat the lesson during extra lesson times.*

Researcher: Have you prepared an assessment instrument to evaluate whether the goal of the lesson was achieved?

Teacher B: *Yes, I always have a class work or home work to assess the level of learning that took place.*
Researcher: Thank you for your time, we will meet during lesson presentation.

APPENDIX P: TEACHER B'S LESSON PLAN ANALYSIS

TOPIC: HOW TO DRAW THE GRAPH OF A PARABOLA

KNOWLEDGE OF THE SUBJECT MATTER

Guiding question	Observation made	Categorization/Theme
a) Are key concepts on quadratic functions to be taught indicated in the lesson plan?	Key concepts on how to draw the graph of a quadratic function were indicated in the lesson plan	Teacher's preparation reflects knowledge of the subject matter including concepts for the topic
b) Does the preparation indicate possible mathematical procedures for the topic to be taught?	The preparation mentions that the teacher will help learners to calculate the y-values, plot the points and also completion of a given table of values.	Procedures to be taught not planned in advance
c) Does the lesson preparation reflect accurate concepts associated with the topic on quadratic functions?	The lesson plan reflects accurate concepts associated with quadratic functions and especially how to draw the graph of a parabola.	Teacher B has good knowledge of the subject matter

KNOWLEDGE OF TEACHING STRATEGIES

Guiding Question	Observation Made	Categorization/Theme
a) Is the teaching strategy to be used stated in the lesson plan?	The lesson plan reflects the teacher's activities as well as the learners' activities. The teacher will first do an exposition of how a graph of a quadratic function is drawn. Learners will be grouped and then calculate the y-values whilst the teacher will offer assistance	Teacher B used the lecture and demonstrate method of teaching and also grouped his learners
b) Are alternative teaching strategies to be used during the lesson reflected in the preparation?	Group work is mentioned as an alternative teaching strategy.	Teacher B indicated only one teaching method in a lesson though he used group work as well.

APPENDIX Q(Continued)

Guiding Question	Observation Made	Categorization/Theme
c) Are examples to be used during the lesson indicated in the lesson preparation?	Examples to be used during the lesson were not indicated in the lesson plan but there was an example. For this lesson, $y= x^2-9$ was used as the main example.	Teacher B did not reflect the examples to be used on the lesson plan.

KNOWLEDGE OF LEARNERS' CONCEPTIONS AND MISCONCEPTIONS

Guiding Question	Observations Made	Categorization/Theme
a) Does the preparation reflect possible misconceptions that will be addressed during the lesson?	The preparation does not reflect possible learners' misconceptions that will be addressed during the lesson.	Teacher B may not be aware of possible misconceptions to be addressed.
b) Does the preparation reflect the required learners' prior knowledge required before the new topic?	Yes, the required learners' prior knowledge was reflected on the lesson plan. Learners should be able to the graph of $y = ax^2$ as taught in grade 10	Teacher B knows the required prior knowledge of learners before this lesson.
c) Are possible learners' difficulties reflected in the preparation?	No, possible difficulties were not written on the lesson plan.	Teacher B may have ignored indicating possible difficulties in the topic because during observations; he identified possible learners' difficulties.
d) Is an assessment instrument indicated in the preparations?	Yes, a home work was given as an instrument to assess learning. $y = x^2 -4$ and $y = x^2 – 2x$ was given as a home work.	Teacher B knows that learning must be assessed after each lesson and uses the appropriate instrument.
e) Is the goal of the lesson clearly stated in the preparation?	The outcome was indicated that the learners must be able to investigate, analyse, describe and represent the function.	The teacher understands that setting the goal of the lesson; helps him to remain focused on the goal as he teaches.

APPENDIX P (Continued)

BOMBANI HIGH SCHOOL LESSON PLAN TEMPLATE

LEARNING AREA: MATHEMATICS	**GRADE:** 11
DATE/WEEK:	**DURATION:** 35 minutes
TOPIC *(For this lesson)*	Sketching parabolas of the form $y = ax^2$
LEARNING OUTCOME(S): *(State in words)*	The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.
ASSESSMENT STANDARD(S) **(State in words)**	Learners should be able to draw parabolas of the form $y = ax^2$.
PREREQUISITE KNOWLEDGE: *(What knowledge do learners need to have before this lesson)*	Learners have drawn graphs of the form $y = ax^2$ in Grade 10.
RESOURCES:	Grid paper
LESSON PROCEDURE: **1. Introduction:** *(State how you will introduce the lesson)*	- Start by revising work done previously. - Write the function on the board and explain the parameters.
2. Main Activity: **(a) Teacher:** *(State in short what you will do as the teacher)*	- Explain and demonstrate how to calculate table of values. - Show learners how to plot and join points - Organise learners into small groups of four - Monitor and assist learners in groups.
(b) Learners' Activities : *(State the activities which learners will be engaged in to enhance their understanding of the topic during your presentation)*	- Drawing and calculating tables of values. - Plotting the points and drawing the graphs - Analysing their graphs and comparing with others.
3. Conclusion: *(State how you will conclude the lesson)*	- Summarise by highlighting the main concepts of the lesson.
ASSESSMENT AND EVALUATION:	Rubric to assess the graph Classwork
Skills ✓ **Knowledge** ✓ **Values** ✓ **Attitudes**	(Please tick what is applicable in the lesson and give a brief explanation below) Writing tables of values Plotting points & Drawing graphs.
REFLECTIONS:	
EXPANDED OPPORTUNITIES:	