Appendix 1. Sobolev Spaces

The space $\mathcal{L}^2(\Omega)$

Consider an open subset $\Omega$ of $\mathbb{R}^n$. The space $\mathcal{L}^2(\Omega)$ consists of functions $f$ such that $f^2$ is Lebesgue integrable on $\Omega$. The first result is well known.

Theorem 1

The space $\mathcal{L}^2(\Omega)$ is a Hilbert space with inner product

$$(f, g) = \int_{\Omega} fg = \int_{\Omega} fg \, d\mu$$

where $\mu$ is the $n$-dimensional Lebesgue measure.

Theorem 2

The space $\mathcal{L}^2(\Omega)$ is separable (See [Ad, Th 2.15, p 28]).

Theorem 3

$C_c^\infty(\Omega)$ is dense in $\mathcal{L}^2(\Omega)$ (See [Ad, Th 2.13, p 28]).
The one-dimensional case

Suppose \( \Omega \) is a bounded open interval. The Sobolev spaces \( H^m(\Omega) \) are subspaces of functions in \( L^2(\Omega) \) with weak derivatives up to order \( m \) in \( L^2(\Omega) \).

Definition

For \( f \) and \( g \) in \( H^m(\Omega) \),

\[
[f, g]_m = (f^{(m)}, g^{(m)}) \quad \text{for} \quad m = 0, 1, \ldots
\]

For \( m \geq 1 \), the bilinear form \( [\cdot, \cdot]_m \) has all the properties of an inner product except that there exist functions \( f \neq 0 \) such that \( [f, f]_m = 0 \).

Definition

For \( f \) in \( H^m(\Omega) \),

\[
|f|_m = \sqrt{[f, f]_m} \quad \text{for} \quad m = 0, 1, \ldots
\]

The function \( |\cdot|_m \) is a semi-norm for \( m \geq 1 \).

The two-dimensional case

Suppose \( \Omega \) is a bounded open convex subset of \( \mathbb{R}^2 \). The Sobolev spaces \( H^m(\Omega) \) are subspaces of functions in \( L^2(\Omega) \) with weak partial derivatives up to order \( m \) in \( L^2(\Omega) \).

Remark

It is not necessary to require that \( \Omega \) be convex, but it is sufficient for our purpose. In the theory it is usually assumed that \( \Omega \) is star shaped or has the cone property.
Definition

For $f$ and $g$ in $H^m(\Omega)$,

$$[f, g]_m = \sum_{i+j=m} (\partial_i^1 \partial_j^2 f, \partial_i^1 \partial_j^2 g) \quad \text{for} \quad m = 0, 1, \ldots$$

For $m \geq 1$ the bilinear form $[\cdot, \cdot]_m$ has all the properties of an inner product except that there exist functions $f \neq 0$ such that $[f, f]_m = 0$.

Definition

For $f$ in $H^m(\Omega)$,

$$|f|_m = \sqrt{[f, f]_m} \quad \text{for} \quad m = 0, 1, \ldots$$

The function $| \cdot |_m$ is a semi-norm for $m \geq 1$.

The boundary

Recall that a curve is called smooth if its parametrization has a continuous derivative. The boundary of $\Omega$ is called piecewise smooth if it consists of a finite number of smooth curves.

For a vector valued function $r$ such that $r_i \in C^1[a, b] \quad \text{for} \quad i = 1, 2$, the range $C$ of $r$ defines a smooth curve in the plane.

Suppose that $C$ is a part of the boundary of $\Omega$. A function $f$ is Lebesgue integrable on $C$ if $f \circ r \sqrt{(r'_1)^2 + (r'_2)^2}$ is Lebesgue integrable on the interval $[a, b]$.

A function $f$ is in $L^2(C)$ if $f^2$ is Lebesgue integrable over $C$. The inner product for $L^2(C)$ is defined by

$$(f, g)_C = \int_C fg \, ds = \int_a^b (f \circ r) \, (g \circ r) \sqrt{(r'_1)^2 + (r'_2)^2} \, ds.$$

When necessary, we use the notation $(f, g)_\Omega$ and $(f, g)_\Gamma$ to avoid confusion.
Sobolev spaces of vector valued functions

Definition

\( u \in L^2(\Omega)^2 \) if \( u_i \in L^2(\Omega) \) for \( i = 1, 2 \).

\( u \in L^2(\Gamma)^2 \) if \( u_i \in L^2(\Gamma) \) for \( i = 1, 2 \).

\( u \in H^k(\Omega)^2 \) if \( u_i \in H^k(\Omega) \) for \( i = 1, 2 \).

\([u, v]_{m,2} = [u_1, v_1]_m + [u_2, v_2]_m \) for \( u \in L^2(\Omega)^2 \) and \( v \in L^2(\Omega)^2 \).

\(|u|_{m,2} = \sqrt{[u, u]_{m,2}} \) for \( u \in L^2(\Omega)^2 \).

The function \(| \cdot |_{m,2} \) is a semi-norm for \( m \geq 1 \).

When we need to distinguish between domains, we will use superscripts \( \Omega \) and \( \Gamma \) in the cases of a double subscript, e.g. \( \| \cdot \|_{m,2}^\Omega \) and \( \| \cdot \|_{m,2}^\Gamma \).

General definitions and results

Suppose \( \Omega \) is a bounded open interval or a bounded open convex subset of \( \mathbb{R}^2 \).

Notation

\( H^0(\Omega) = L^2(\Omega) \) and \( H^0(\Omega)^2 = L^2(\Omega)^2 \).

Definition

The inner product for \( H^m(\Omega) \) is defined by

\[(f, g)_m = \sum_{k=0}^{m} [f_k, g_k] \] for \( m = 0, 1, \ldots \)
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Definition

The norm for $H^m(\Omega)$ is defined by

$$\|f\|_m = \sqrt{(f, g)_m} \text{ for } m = 0, 1, \ldots$$

Definition

The inner product for $H^m(\Omega)^2$ is defined by

$$(f, g)_{m,2} = \sum_{k=0}^{m} [f, g]_{k,2} \text{ for } m = 0, 1, \ldots$$

Definition

The norm for $H^m(\Omega)^2$ is defined by

$$\|f\|_{m,2} = \sqrt{(f, g)_{m,2}} \text{ for } m = 0, 1, \ldots$$

Theorem 4

The space $H^m(\Omega)$ is complete (See [Ad, Th 3.2, p 45]).

Theorem 5

$C^m(\bar{\Omega})$ is dense in $H^m(\Omega)$ with respect to the norm of $H^m(\Omega)$.

(See [OR, Th 2.10, p 53].)

Theorem 6

The space $H^m(\Omega)$ is separable (See [Ad, Th 3.5, p 47]).
Theorem 7 (Rellich)

For any nonnegative integer, the embedding of $H^{m+1}(\Omega)$ into $H^m(\Omega)$ is compact (See [Ad, Th 6.2, p 144]).

Notation

$$\partial^\alpha = \partial_1^{\alpha_1} \partial_2^{\alpha_2} \ldots \partial_n^{\alpha_n}, \text{ where}$$

$$|\alpha| = \alpha_1 + \alpha_2 \ldots \alpha_n$$

and $|\alpha|$ denotes the order of the derivative.

Theorem 8 (Sobolev’s lemma)

Let $m$ be any nonnegative integer. If $u \in H^p(\Omega)$ where $p > n/2$, then $u \in C^m(\overline{\Omega})$ and

$$\|\partial^\alpha u\|_{\text{sup}} \leq \|u\|_p \quad \text{for} \quad |\alpha| \leq m.$$  

(See [OR, Th 3.10, p 80].)

Remarks

1. Theorems 4 to 8 are also true for vector valued functions. The proofs are all trivial.

2. When we need to distinguish between different domains, say $\Omega$ and $\Gamma$, they will appear as superscripts, for instance $\|\cdot\|_k^\Omega$ and $(f, g)_{m,2}^\Gamma$. 
Appendix 2. Inequalities

The one-dimensional case

Proposition 1

Consider any $u \in C^1[0,1]$. For any two points $x$ and $y$ in $[0,1]$,

$$|u(x)| \leq \|u'\| + |u(y)|.$$ 

Proof

Assuming that $x > y$ (without loss of generality), we have

$$u(x) = \int_y^x u' + u(y).$$

But $|\int_y^x f| \leq \|f\|$ for any $f \in L^2(0,1)$. This follows from the Cauchy-Schwartz inequality

$$\left(\int_y^x fg\right)^2 \leq \left(\int_y^x f^2\right)\left(\int_y^x g^2\right)$$

by choosing $g = 1$. The rest is obvious.

Theorem 1

For any $u \in C^1[0,1]$ with a zero in $[0,1]$ we have

$$\|u\| \leq \|u'\|.$$
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Proof

Suppose \( u(y) = 0 \), then \(|u(x)| \leq \|u'\|\) by Proposition 1.

Hence \( \|u\|_{sup} \leq \|u'\| \). The rest is obvious since \( \|u\| \leq \|u\|_{sup} \).

Proposition 2

For any \( u \in C^1[0, 1] \), \( |u(0)| \leq \sqrt{2} \|u\|_1 \).

Proof

Let \( g(x) = 1 - x \) and \( v = gu \) and consider the fact that
\[
u(0) = v(0) = -\int_0^1 v' + v(1) .
\]

Since \( v(1) = 0 \),
\[
|u(0)| = \left| \int_0^1 (u'g + ug') \right| \leq \|u'\|\|g\| + \|u\|\|g'\| \leq \|u'\| + \|u\|.
\]

Using the inequality \((a + b)^2 \leq 2a^2 + 2b^2\), it follows that
\[
|u(0)|^2 \leq 2|u'|^2 + 2|u|^2.
\]

The two-dimensional case

Suppose \( \Omega \) is a bounded open convex subset of \( \mathbb{R}^2 \) with a piecewise smooth boundary. The following result is referred to as the Poincare-Friedrichs inequality or Friedrichs’s inequality or Poincare’s inequality.

Theorem 2

Suppose \( \Sigma \) is a part of the boundary of \( \Omega \) with nonzero length. Denote the set
\[
\{ u \in C^1(\overline{\Omega}) \mid u = 0 \text{ on } \Sigma \}
\]
APPENDIX 2

by $F(\Omega)$. There exists a constant $c_F$ such that, for each $u \in F(\Omega)$,

$$\|u\| \leq c_F|u|_1.$$ 

Proof

See e.g. [Br, p 30].

Corollary

Suppose $\Sigma_1$ and $\Sigma_2$ are parts of the boundary of $\Omega$ with nonzero length. Denote the set

$$\{ u \in C^1(\bar{\Omega})^2 \mid u_1 = 0 \text{ on } \Sigma_1 \text{ and } u_2 = 0 \text{ on } \Sigma_2 \}$$

by $F(\Omega)^2$. There exists a constant $c_F$ such that for each $u \in F(\Omega)^2$,

$$\|u\|_{0,2} \leq c_F|u|_{1,2}.$$ 

Note that $\Sigma_1$ and $\Sigma_2$ may overlap and even be equal.

Theorem 3 (Korn’s inequality)

Suppose $b_B$ is the bilinear form for the Reissner-Mindlin plate. There exists a constant $c_\Omega$ such that

$$|u|^2_{1,2} \leq c_\Omega b_B(u, u)$$

for each $u \in V$.

Proof

See e.g. [Br, p 288-289].
Appendix 3. Trace

The one-dimensional case

Recall that for each $u \in H^1(0,1)$, there exists a sequence $\{u_n\} \subset C^1[0,1]$ such that $\|u_n - u\|_1 \to 0$ as $n \to \infty$.

**Theorem 1**

For each $u \in H^1(0,1)$, there exists a unique real number $\gamma_u$ with the following property: For each sequence $\{u_n\} \subset C^1[0,1]$ such that $\|u_n - u\|_1 \to 0$ as $n \to \infty$, we have

$$\lim_{n \to \infty} u_n(0) = \gamma_u.$$

**Proof**

Due to Proposition 2 in Appendix 2, $\lim_{n \to \infty} u_n(0)$ exists for each sequence $\{u_n\} \subset C^1[0,1]$ such that $\|u_n - u\|_1 \to 0$ as $n \to \infty$. Also due to this proposition, the limit is independent of the choice of the sequence $\{u_n\}$.

**Theorem 2**

The mapping $\gamma$ is linear and bounded. In fact,

$$|\gamma u| \leq \sqrt{2} \|u\|_1.$$
Proof

The linearity follows from the properties of limits and the estimate from Proposition 2 in Appendix 2 by considering the limits.

Remark

The mapping $\gamma$ is a bounded linear functional.

Theorem 3

For any $u \in H^1(0,1)$,

$$\|u\| \leq \|u'\| + |\gamma u|.$$  

Proof

Consider any $u \in C^1[0,1]$. Proposition 1 in Appendix 2 implies that

$$\|u\|_{\text{sup}} \leq \|u'\| + |u(0)|.$$  

Consequently,

$$\|u\| \leq \|u'\| + |\gamma u|.$$  

The same inequality holds for each $u \in H^1(0,1)$ since $C^1[0,1]$ is dense in $H^1(0,1)$.

The two-dimensional case

Definition (Trace operator $\gamma$)

For $u \in C(\bar{\Omega})$, the function $\gamma u$ is the restriction of the function $u$ to $\Gamma$.

Theorem 4

The trace operator $\gamma$ can be extended to a bounded linear operator mapping $H^1(\Omega)$ onto $L^2(\Gamma)$ and $\|\gamma u\|_{\Gamma} \leq K\|u\|_{1,\Omega}$. 

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Proof

This result is a special case of results in [OR, p 141-142].

Definition

For $u \in H^1(\Omega)$, we define $\gamma u$ by

$$\gamma u = \langle \gamma u_1, \gamma u_2 \rangle.$$ 

Theorem 5

Suppose $\Omega$ is the open rectangle $0 < x_1 < 1, 0 < x_2 < a$. $\Gamma$ is the side where $x_2 = 0$ and $\gamma_0 u = u(\cdot, 0)$. Then there exists a constant $K$ such that

$$\|u\|_{\Omega} \leq K|u|_1 + K\|\gamma_0 u\|_{\Gamma}$$

for all $u \in H^1(\Omega)$.

Proof

Proposition 1 Appendix 2 implies that for each $x_2 \in [0, a],$

$$|u(x)|^2 \leq 2a^2 \int_0^a \left[ \partial_2 u(x_1, \cdot) \right]^2 + 2[u(x_1, 0)]^2.$$

Therefore

$$\int_0^a [u(x_1, \cdot)]^2 \leq 2a^2 \int_0^a \left[ \partial_2 u(x_1, \cdot) \right]^2 + 2[u(x_1, 0)]^2.$$

Integration with respect to $x_1$ yields

$$\|u\|^2_{\Omega} \leq 2a^2 \|\partial_2 u\|^2_{\Omega} + 2\|\gamma_0 u\|^2_{\Gamma}.$$

The result follows.
Appendix 4. The spaces $C^k(J; Y)$

Consider $J = (a, b)$ or $J = [a, b)$. Let $Y$ be any Banach space and consider a function $u$ with values in $Y$. Let $t$ be any interior point of $J$.

**Definition (Derivative)**

Suppose there exists a $v \in Y$ such that

$$
\lim_{h \to 0} \left\| h^{-1}(u(t+h) - u(t)) - v \right\|_Y = 0,
$$

then $v$ is the derivative of $u$ at $t$. We write $u'(t)$ for the derivative.

It is obvious how to adapt the definition for the case $t = a$. The derivative (function) $u'$ and the second order derivative $u''$ are defined in the usual way.

**Notation**

$C^k([0, \infty); Y)$ and $C^k((0, \infty); Y)$
Appendix 5. Proofs

All the results and proofs are from [V4] and presented here for completeness.

Plate-beam system

Proposition 1

There exists a constant $K_T$ such that

$$
\|w\|^2 + \|\phi\|^2 \leq K_T \left( \|\phi'\|^2 + \|w' - \phi\|^2 \right)
$$

for each $(w, \phi) \in T(0, 1) \times C^1[0, 1]$

Proof

Suppose it is not true. Then there exists a sequence $\{(w_n, \phi_n)\}$ such that

$$
\|w_n\|^2 + \|\phi_n\|^2 = 1,
$$

while

$$
\|\phi'_n\|^2 + \|w'_n - \phi_n\|^2 \to 0 \quad \text{as} \quad n \to \infty.
$$

• We prove first that for $n$ sufficiently large, $\phi_n$ does not have a zero.

Suppose $\phi_n$ does have a zero. Then $\|\phi_n\| \leq \|\phi'_n\|$. We also have $\|w_n\| \leq \|w'_n\|$ since $w_n(0) = 0$. Consequently

$$
\|w_n\| \leq \|w'_n\| \leq \|w'_n - \phi_n\| + \|\phi_n\| \leq \|w'_n - \phi_n\| + \|\phi'_n\|.
$$

This implies that

$$
\|w_n\| + \|\phi_n\| \leq \|w'_n - \phi_n\| + \|\phi'_n\| \quad \text{for each} \quad n.
$$
This is a contradiction.

• We now show that $\|\phi_n\| > 1/2$ for $n$ sufficiently large.

If it is not true, then

$$\|w_n\| \leq \|w'_n\| \leq \|w'_n - \phi_n\| + \|\phi_n\| \leq 1/4 + 1/2 < 3/4$$

for $n$ sufficiently large. Consequently

$$\|w_n\|^2 + \|\phi_n\|^2 < 9/16 + 1/4 = 13/16 < 1.$$  

This is a contradiction.

• Next we show that $\int_0^1 \phi_n > 1/10$ for $n$ sufficiently large.

We may assume without loss of generality that $\phi_n > 0$. Writing $\phi$ for $\phi_n$, we have

$$\phi_{\text{max}} - \phi_{\text{min}} \leq \|\phi'\| \leq 1/20$$

and

$$\phi_{\text{max}} \leq 21/20$$

for $n$ sufficiently large. Consequently

$$\phi_{\text{min}}^2 = \int_0^1 \phi_{\text{min}}^2 = \int_0^1 (\phi_{\text{max}}^2 - \phi^2) + \int_0^1 \phi^2$$

$$= \int_0^1 \phi^2 - \int_0^1 (\phi^2 - \phi_{\text{min}}^2) \geq 1/4 - 1/10 > 1/10.$$  

Therefore

$$\int_0^1 \phi \geq \int_0^1 \phi_{\text{min}} > 1/10.$$  

• $\int_0^1 w'_n > 0$ for $n$ sufficiently large.

$$\left| \int_0^1 w'_n - \int_0^1 \phi_n \right| \leq \int_0^1 |w'_n - \phi_n| \leq \|w' - \phi\| \leq 1/20.$$  

• Finally we obtain a contradiction.

Since $\int_0^1 w'_n > 0$ and $w_n(0) = 0$, we have $w_n(1) > 0$ which is a contradiction.

$\Box$
Corollary

There exists a constant $K$ such that

$$\|w\|_1^2 + \|\phi\|_1^2 \leq K \left( \|\phi'\|_1^2 + \|w' - \phi\|_1^2 \right).$$

**Proposition 2**

$$\|w\|_1^2 + \left( \|\psi\|_{0,2}^\Omega \right)^2 \leq K \left( \|\nabla w + \psi\|_{0,2}^\Omega \right)^2 + K \left( \|\psi\|_{1,2}^\Omega \right)^2 + K \|\gamma_0 \psi_1\|_I^2$$

for each $(w, \psi) \in T_1(\Omega) \times T_2(\Omega)$.

**Proof**

Since $\psi_2$ is zero on a part of the boundary, we may use the Friedrichs inequality $\|\psi_2\|_\Omega \leq c_F \|\psi_2\|_1^\Omega$. We also use

$$\|\psi_1\|_\Omega \leq c_1 \|\psi_1\|_1^\Omega + c_1 \|\gamma \psi_1\|_I$$

(see Theorem 5 Appendix 3). Combining the two inequalities, we have

$$\left( \|\psi\|_{0,2}^\Omega \right)^2 \leq c_2 \left( \|\psi\|_{1,2}^\Omega \right)^2 + c_2 \|\gamma \psi_1\|_I^2.$$ 

Since $w$ is zero on a part of the boundary, $\|w\|_\Omega \leq c_F \|w\|^\Omega_1$, using the Friedrichs inequality. Therefore

$$\|w\|_1^2 + \left( \|\psi\|_{0,2}^\Omega \right)^2 \leq K \left( \|\nabla w + \psi\|_{0,2}^\Omega \right)^2 + K \left( \|\psi\|_{1,2}^\Omega \right)^2 + K \|\gamma_0 \psi_1\|_I^2,$$

Consequently

$$\|w\|_1^2 + \left( \|\psi\|_{0,2}^\Omega \right)^2 \leq K \left( \|\nabla w + \psi\|_{0,2}^\Omega \right)^2 + K \left( \|\psi\|_{0,2}^\Omega \right)^2 \leq K \left( \|\nabla w + \psi\|_{0,2}^\Omega \right)^2 + K \left( \|\psi\|_{1,2}^\Omega \right)^2 + K \|\gamma_0 \psi_1\|_I^2,$$

where $K$ is a generic constant depending on $c_1$ and $c_F$.

**Corollary**

$$\left( \|w\|_1^\Omega \right)^2 + \left( \|\psi\|_{1,2}^\Omega \right)^2 \leq K \left( \|\nabla w + \psi\|_{0,2}^\Omega \right)^2 + K \left( \|\psi\|_{1,2}^\Omega \right)^2 + K \|\gamma_0 \psi_1\|_I^2.$$
Theorem 1

The inertia space $X$ is a separable Hilbert space and $V$ is dense in $X$.

Proof

Since $C^\infty_0(\Omega)$ is dense in $L^2(\Omega)$, we have that $T_1(\Omega)$ is dense in $L^2(\Omega)$ and $T_2(\Omega)$ is dense in $L^2(\Omega)^2$. Also $T(I)$ is dense in $L^2(I)$. We conclude that the space $T$ is dense in the space $X$ equipped with the inner product

$$(u, v)_{L^2} = (u_1, v_1)_\Omega + (u_2, v_2)^\Omega_{0, 2} + \sum_{j=3}^6 (u_j, v_j)_I.$$

The norms $\|u\|_X$ and $\|u\|_{L^2}$, where

$$\|u\|^2_X = c(u, u) \quad \text{and} \quad \|u\|^2_{L^2} = (u, u)_{L^2},$$

are equivalent. Therefore $T$ is a dense subset of $X$ with respect to the inertia norm and $T \subset V \subset X$.

Theorem 2 (Korn’s inequality)

$$b_B(u_2, u_2) \geq K|u_2|_{1, 2}^2 \quad \text{for each } u \in T.$$

Theorem 3

There exist constants $c_1$ and $c_2$ such that

$$\|u\|_X \leq c_1\|u\|_{H^1} \leq c_2\|u\|_V$$

for each $u \in T$.

Proof

From the corollary to Proposition 1, we have

$$\|u_3\|^2_1 + \|u_4\|^2_1 + \|u_5\|^2_1 + \|u_6\|^2_1 \leq C_T b_T(u, u).$$
Rewrite the corollary to Proposition 2.

\[
(\|u_1\|_1^2 + \|u_2\|_2^2) \leq K (\|\nabla u_1 + u_2\|_{0,2}^2 + K (|u_2|_{1,2}^2) + K \|\gamma_0 u_{21}\|_I^2).
\]

Combining the results, we have

\[
\|u\|_{H^1}^2 \leq C (\|\nabla u_1 + u_2\|_{0,2}^2 + C (|u_2|_{1,2}^2) + C b_T(u, u),
\]

using Proposition 1 again. Now use Korn’s inequality.

**Nonmodal damping**

**Theorem 1**

For each \(y \in H\) there exists a unique \(x \in H\) such that

\[
x_2 = y_1
\]

\[
b(x_1, v) + a(x_2, v) = -c(y_2, v) \quad \text{for each } v \in V.
\]

**Proof**

Let

\[
g(v) = -a(y_1, v) - c(y_2, v) \quad \text{for each } v \in V,
\]

then \(g\) is clearly a linear functional on \(V\). Furthermore

\[
|g(v)| \leq K \|y_1\|_V \|v\|_V + c \|y_2\|_X \|v\|_X \quad \text{for each } v \in V,
\]

showing that \(g\) is bounded. The result follows from the well known theorem of Riesz.

**Theorem 2**

\(\Lambda\) is bounded.
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Proof

Consider any \( y \in H \) and suppose \( x = \Lambda y \).

\[
\begin{align*}
x_2 &= y_1 \\
b(x_1, v) + a(x_2, v) &= -c(y_2, v) \quad \text{for each } v \in V.
\end{align*}
\]

It follows that

\[
\|x_2\|_X \leq K \|x_2\|_V = \|y_1\|_V
\]

and

\[
\|x_1\|_V^2 = b(x_1, x_1) \\
\leq |a(x_2, x_1)| + |c(y_2, x_1)| \\
\leq K \|x_2\|_V \|x_1\|_V + K \|y_2\|_X \|x_1\|_V
\]

Consequently

\[
\|x_1\|_V \leq K \|y_1\|_V + K \|y_2\|_X \leq K \|y\|_H.
\]
Bibliography


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