

## Chapter 6

# Conclusions and recommendations

### 6.1 Recent developments in nonhydrostatic atmospheric modelling

Nonhydrostatic models have been developed and used for research purposes since the early days of numerical atmospheric modelling (e.g. Ogura and Charney, 1962; Ogura and Phillips, 1962; Dutton and Fichtl, 1969; Furukawa, 1973; Miller, 1974; Miller and Pearce, 1974). However, with the advent of ever faster computers, the era of operational nonhydrostatic atmospheric models has dawned. That is, NWP and climate simulation models may be applied at resolutions beyond the hydrostatic limit, where convection can at least be partially resolved. This has led to a renewed worldwide research effort into the development of nonhydrostatic atmospheric models (e.g. Laprise, 1992; Dudhia, 1993; Bubnova et al., 1995; Gallus and Rancic, 1996; Janjic et al., 2001; Smolarkiewicz et al., 2001; Dudhia and Bresch, 2002; Davies et al., 2005). A primary aspect of this international research effort, is the quest to find a “universal model” that may be applied at any spatial resolution, globally or over a limited area, and for time-scales ranging from the typical life-time of meso-scale systems to the extensive periods required for climate simulation. The need for a universal model stems partially from the labour involved in maintaining and developing different model codes suitable for application at different spatial scales. Many believe that the fully-elastic (unapproximated) equations provide the only option for the development of a universal model (Laprise, 1992; Caya and Laprise, 1999; Davies et al., 2003; White et al., 2005). However, these equations contain fast moving acoustic waves as part of their solution set, which may imply computational penalties when the equations are solved numerically (e.g. Room et al., 1998; Davies et al., 2003). Apart from the fully-elastic models, two other main classes of nonhydrostatic models have evolved, namely anelas-

tic (e.g. Ogura and Charney, 1962; Dutton and Fichtl, 1969; Furukawa, 1973; Miller, 1974; Mahrt, 1986; Tritton, 1988, White, 1989; Room et al., 2001, Smolarkiewicz et al., 2001) and quasi-elastic models (e.g. Ogura and Phillips, 1962; Wilhelmson and Ogura, 1972; Clark and Peltier, 1977; Miller and White, 1984; Durran, 1989). The anelastic equations are completely filtered of sound waves (whilst the quasi-elastic equations are partially filtered), and therefore offer computational advantages over the fully-elastic equations (e.g. Room et al., 1998). However, at least for the anelastic and quasi-elastic equations in height-based vertical coordinates, the property of universal applicability is lost (Davies et al., 2003). These equation sets result when approximations are introduced to the fully-elastic equations, with the aim of eliminating the terms responsible for the generation of sound waves. Unfortunately, the approximations may also effect the gravity and Rossby waves of the resulting equation sets, causing the properties of these waves to be different from those of the fully-elastic equations and the true atmosphere at the synoptic scale and larger (Davies et al., 2003). Indeed, traditionally the anelastic and quasi-elastic equation sets have been used for the study of meso-scale circulation systems for relatively short integration periods (e.g. Ogura and Charney, 1962; Miller and Pearce, 1974; Miranda and James, 1992; Dudhia, 1993; Gallus and Rancic, 1996). They remain important today to be used for this purpose (Davies et al., 2003), just as the hydrostatic equations remain important for the study of synoptic and large scale circulation. The different classes of nonhydrostatic models are reviewed in Chapter 1 of the thesis.

Another aspect that has received much attention in recent research effort into nonhydrostatic models, is the choice of vertical coordinate system to be used. Most of the early nonhydrostatic models developed for research purposes employed height-based coordinates (e.g. Ogura and Charney, 1962; Furukawa, 1973), whilst the highly evolved operational hydrostatic models developed over the last four decades are all cast in pressure-based vertical coordinates. The most convenient way to obtain a nonhydrostatic model suitable for operational use, is to convert an existing operational hydrostatic model to a nonhydrostatic model (Janjic et al., 2001). This can be most conveniently achieved if the pressure-based vertical coordinate system of the hydrostatic model is still used for the nonhydrostatic model. In fact, until fairly recently, it was believed that nonhydrostatic models formulated in terms of a vertical coordinate based on the hydrostatic pressure (Laprise, 1992; Bubnova et al., 1995) would offer the only practical option for the development of a fully-elastic nonhydrostatic model suitable for universal application (Caya and Laprise, 1999). However, research efforts at the UKMO over the last few years have led to the development of a fully-elastic model employing a height-based vertical coordinate system (Davies et al., 2005).

The first nonhydrostatic model employing a pressure-based vertical coordinate was developed by Miller (1974) and Miller and Pearce (1974). The MP model

differs from the recently developed fully-elastic models based on the hydrostatic pressure field, in the sense that it is anelastic and employs the full pressure field as vertical coordinate. A  $\sigma$  coordinate form of the MP model was developed by Miller and White (1984). The latter model is filtered of vertically propagating acoustic waves, but contains Lamb waves as part of its solution set. Its quasi-elastic nature still offers significant computational advantages over the fully-elastic equations (Miller and White, 1984; Gallus and Rancic, 1996; Room et al., 2001). A closely related anelastic  $\sigma$  coordinate model (the NHAD model) was developed by Room et al. (2001). Numerical models have been developed based on the MP equations, in both pressure and  $\sigma$  coordinates, and have been used for the study of convection (e.g. Miller and Pearce, 1974) and gravity waves (Miranda and James, 1994; Miranda and Valente, 1997). The MP model has never been applied at larger spatial scales, mainly because it is formulated in terms of a reference thermodynamic profile that limits its application to cases where the temperature distribution on pressure surfaces is fairly uniform (e.g. White, 1989; Caya and Laprise, 1999). For example, the MP equations are probably not applicable to frontal zones, or at large spatial scales where significant variations in temperature from the reference state are possible. The NHAD model is subject to the same limitation. The different types of pressure-based nonhydrostatic models are reviewed in Chapter 2 of the thesis.

With the limitations of the MP model in mind, White (1989) produced a closely related equation set that still employs the full pressure field as vertical coordinate, but is formulated independently of a thermodynamic reference profile. In Chapter 3 of this study, the  $\sigma$  coordinate equivalent of the equation set of White (1989) is derived, by means of a coordinate transformation of the pressure coordinate equations of White (1989) into a  $\sigma$  coordinate based on the full pressure field. The  $\sigma$  coordinate equation set is shown to be quasi-elastic, with the speed of the Lamb waves depending on the choice of model top  $p_T$ . The frequency equation for the gravity wave equations in the quasi-elastic equations is similar to the corresponding equation of the fully-elastic equations, whilst the energy equation of the fully-elastic equations is similar to that of the hydrostatic equations (see Chapter 3). These results, and the fact that the equations are formulated independent of a reference profile, suggests that the quasi-elastic  $\sigma$  coordinate equations based on the full pressure field may be used at spatial scales larger than the meso-scale.

## 6.2 A novel dynamic kernel based the split semi-Lagrangian formulation of the quasi-elastic equations

A novel dynamic kernel based on this set of quasi-elastic  $\sigma$  coordinate equations is formulated in Chapter 4 of the thesis. The new numerical model is the first realization of White's (1989) idea of a nonhydrostatic model based on

the full pressure field, that may be applied at spatial scales larger than the meso-scale. The numerical solution procedure applied to solve the quasi-elastic equations is closely linked to that used to solve the hydrostatic equations, in the sense that both the surface pressure tendency and vertical motion fields are calculated from the continuity equation (see Chapter 4). In most nonhydrostatic models, including the numerical realizations of the MP model (e.g. Xue, 1989; Xue and Thorpe 1991), the vertical motion field is calculated from the vertical momentum equation. Because the present approach is in harmony with the solution procedure typically used in hydrostatic models, it facilitates the convenient transformation of an existing hydrostatic model to a nonhydrostatic model based on the quasi-elastic equations. The numerical procedure used to solve the quasi-elastic equations involves the use of an elliptic equation in the geopotential (derived in Chapter 3), which is solved with SOR. Similar equations are solved in the MP and NHAD models (Xue, 1989; Xue and Thorpe, 1991; Room et al., 2001).

The most distinguishing feature of the numerical solution procedure developed, is that it uses a split semi-Lagrangian scheme to solve the quasi-elastic equations on a nonstaggered grid (see Chapter 4). The numerical realizations of the MP and NHAD models employ explicit solution procedures on staggered grids (e.g. Xue, 1989; Xue and Thorpe, 1991; Room et al., 2001). The use of the semi-Lagrangian technique to solve the advection process in the newly developed model offers a significant computational advantage over explicit procedures. The model remains stable at large Courant numbers, whereas the advection time-step in explicit solution procedures is always limited by a CFL-type stability condition. The fast propagating gravity and Lamb waves in the quasi-elastic equations can be treated accurately during an adjustment procedure, which may employ a smaller time-step than that used in the advection step.

It is advantageous from a computational perspective to use a nonstaggered grid when a semi-Lagrangian scheme is used for the advection process, since only one set of trajectories needs to be calculated. Semi-Lagrangian procedures applied on staggered grids require the calculation of more than one set of trajectories (for the staggered and nonstaggered positions), which implies that more than one set of spatial interpolations needs to be performed to evaluate the values of variables at the different sets of departure points. Unfortunately, the nonstaggered grid (or Arakawa A-grid) is known to have poor gravity wave dispersion properties (e.g. Winninghoff 1968; Mesinger and Arakawa 1976; Arakawa and Lamb, 1977; Schoenstadt, 1978), which renders its use during the adjustment step of the present procedure problematic. If second order differencing is used to calculate the spatial derivatives that occur in the adjustment step equations, the phase speed of the gravity waves in the quasi-elastic equations are retarded considerably near the shortest resolvable scales. In fact, for the horizontal spatial discretization the two-grid-interval waves are stationary. However, it is shown in the thesis that fourth order discretization may be used to significantly

improve the gravity wave dispersion properties of the quasi-elastic equations on the nonstaggered grid. Stationary two-grid-interval waves, and numerical noise that originate near the shortest resolvable scales, may be sufficiently controlled by the use of explicit diffusion and the highly scale-dependent Shapiro (1975) spatial filter. The successful and economical implementation of the split semi-Lagrangian procedure to solve the set of nonhydrostatic quasi-elastic equations on a nonstaggered grid, is one of the main aspects of the research that is novel. However, note that high order spatial differencing and filtering have been used in semi-implicit semi-Lagrangian discretizations of the hydrostatic equations (Purser and Leslie, 1988; Leslie and Purser, 1991). McGregor (2005) has recently developed an alternative approach for application in the hydrostatic variable resolution model C-CAM, where a semi-Lagrangian procedure is used on a nonstaggered grid. By transforming the wind components via a reversible interpolation scheme, the geostrophic adjustment process may then be carried out on a staggered grid.

The new split semi-Lagrangian kernel was tested by a series of numerical experiments described in Chapter 5. All the simulations in the thesis were performed on a single processor personal computer, so that high-resolution simulations where the flow is adequately resolved, were limited to two spatial dimensions by computational constraints. When working in two spatial dimensions, there is the additional advantage that the available computing power allowed grid-converged solutions to be obtained for diffusion-limited problems. This facilitated a comparison of the solutions obtained to the results obtained in similar studies performed with different equation sets and numerical solution procedures. Cold and warm bubble experiments were performed, in two and three spatial dimensions. These numerical experiments have shown that the split semi-Lagrangian formulation of the quasi-elastic equations is robust, and may be used to accurately describe highly nonlinear and nonhydrostatic flow. The solutions obtained correspond well to the two-dimensional results obtained using Eulerian formulations of the fully-elastic equations (Straka et al., 1993; Gallus and Rancic, 1996; Janjic and Gerrity, 2001). An interesting new three-dimensional numerical experiment was devised, where a warm bubble rises in an environment with vertical wind shear. The results obtained in this test correspond to the linear theory of storm splitting. A three dimensional experiment of air flow over a bell shaped mountain was performed, illustrating that the split semi-Lagrangian formulation performs well for the case of nonzero topography.

With regard to the optimum choice of numerical settings to be used in the split semi-Lagrangian scheme, the experiments indicated that there is little or no advantage in going beyond the  $D_2$  scheme of McGregor (1993) for the calculation of advection departure points. Fourth order spatial differencing of the adjustment step equations provides a significantly improved gravity wave phase speed representation compared to second order spatial differencing. For the

high-resolution experiments performed at small values of explicit diffusion, two-grid-interval noise develops unless the Shapiro spatial filter is applied. For these high-resolution tests the Shapiro filter effectively removes the two-grid interval waves from the nonstaggered grid, whilst having a negligible damping effect at longer wave lengths. The use of the filter prevents the development of nonlinear instability, allowing the use of significantly larger advection time-steps than is allowed in nonfiltered simulations. The combined use of explicit diffusion and the Shapiro filter on the nonstaggered grid generally allow solutions that are smooth and free of any signs of numerical noise. However, when relatively small adjustment time-steps are used, the filter should be applied only every few adjustment time-steps to prevent the solution from becoming excessively damped. Similarly, for the three-dimensional tests performed at relatively low spatial resolutions, where wavelengths near the shortest resolvable scales carry an important part of the energy of the motion, application of the filter may lead to excessively damped solutions. Fortunately, it was found that explicit diffusion sufficiently controls the two-grid-interval noise for these relatively low resolution simulations on the nonstaggered grid. Finally, the SOR procedure used to solve the elliptic equation provides reasonably efficient solutions for application of the model in both two and three spatial dimensions, generally requiring only a few iterations per adjustment time-step. However, potential computational advantages of a solution procedure based on the FFT technique (e.g. Xue, 1989) over the present SOR procedure is an aspect that requires further investigation.

The fully elastic equations are nowadays mostly solved with semi-implicit techniques, that slow down not only the sound waves, but also the gravity waves (e.g. Tanguay et al., 1990; Davies et al., 2003). With a split semi-Lagrangian approach, with explicit spatial filtering, selective damping of only the smallest resolvable waves is achieved. The remaining fast travelling waves are handled during the adjustment step that employs a (necessarily small) time step. However, the semi-Lagrangian procedure still allows a large time step to be used during the advection step. When the fast travelling gravity waves are important for their own sake, for example when they carry an important part of the energy of the dynamic system studied, the split semi-Lagrangian approach allows them to be handled accurately during the adjustment step. However, they may be artificially damped by a semi-implicit procedure employing a large time-step. In order to treat the gravity waves accurately, a small time step will have to be used in the semi-implicit solution procedure. In this way, the economical efficiency of the procedure is lost as it will now become equivalent in time step to a purely explicit solution procedure. However, in the split semi-Lagrangian approach, the advection time step may still be large, making the method more economical than a semi-implicit approach for cases where damping of the gravity wave phase speed is not desirable.

## 6.3 Ongoing research

### 6.3.1 Marginally resolved flow

The operational use of atmospheric models beyond the hydrostatic limit has opened up numerous unexplored areas of research. For example, the performance of models at spatial resolutions where convection is marginally resolved will be of practical importance for many years to come. Although it is the advent of faster computers that has made the operational use of atmospheric models beyond the hydrostatic limit possible, computational constraints are still limiting the resolution of operational regional and global nonhydrostatic models to a few kilometers (in the horizontal) at best. At these resolutions, convection can only be marginally resolved. Conversely, hydrostatic models are applied at resolutions where convection is not explicitly resolved. In these models, convection parameterization schemes are used to represent the effects of convection on the atmosphere. Thus, an important new focus area in numerical atmospheric modelling, is the development of numerical solution procedures (including parameterisation schemes) for nonhydrostatic models, that functions well at spatial resolutions where convection is marginally resolved.

The development of a truly “universal model” is still far from being a practical reality. It is unlikely that a numerical scheme will be found that performs best from an accuracy and computational perspective at all spatial resolutions. More likely is that certain schemes will offer computational advantages and produce more accurate results at specific spatial scales. There is also no guarantee that the convection schemes used in hydrostatic models will function well at resolutions where convection is marginally resolved. In fact, a necessary condition for development of a universal model, is the development of a parameterization scheme that may be applied at all spatial scales. The present research effort facilitates the participation of scientists in South Africa into the current international research effort into the development of nonhydrostatic models.

### 6.3.2 A quasi-elastic universal model?

It should also be asked if a set of anelastic or quasi-elastic equations exist that is suitable for universal application. The recent normal mode analysis performed for various approximated height-based equation sets has shown that none of these sets has the property of universal applicability (Davies et al., 2003). The question arises if a universally applicable filtered equation set based on the hydrostatic or full pressure field may be constructed. The formulation independent of a reference thermodynamic profile of the set of quasi-elastic equations derived in the thesis allow their application at scales larger than the meso-scale. It remains to be investigated how the approximations introduced to formulate these equations affect the representation of Rossby waves in the equation set. There also seems to have been no attempt to formulate filtered nonhydrostatic equations based on the hydrostatic pressure field. A normal mode analysis of

such filtered equations, and the equation sets based on the full pressure field described in the thesis, may provide more clarity on their potential application at large spatial scales. A Hamiltonian approach may provide a useful way of formulating such equation sets (e.g. Salmon and Smith, 1994). These research aspects fall beyond the scope of the present study, which is primarily about the formulation of a split semi-Lagrangian scheme to solve the quasi-elastic equations on a nonstaggered grid, and applications at the micro and meso-scales.

### 6.3.3 The study of nonhydrostatic circulation systems and continued model development

The dynamics of many interesting nonhydrostatic circulation systems occurring over South Africa has never been studied in detail. Examples include the frequent occurrence of mountain waves over Lesotho and the south-western Cape (De Villiers, 1998, 2001), and the development of severe thunderstorms over the Highveld and eastern escarpment areas (e.g. De Coning and Adam, 2000; De Coning et al., 2000). The newly developed model may be used to study the dynamics of these systems, possibly in combination with the high resolution MSG satellite pictures that recently became available in the country. More generally, the new model may be used to gain deeper insight into convective storm-splitting, interaction of updrafts with vertical shear of the horizontal wind, the development of mesocyclones in supercell storms, the merging of two cumulus cells, cold pools in two-dimensional squall lines, roll convection, airflow over mountains, etc.

Already, the new kernel has been tested for applications over regions of nonzero topography, by a series of experiments involving airflow over a three-dimensional bell-shaped mountain. The split semi-Lagrangian procedure performed well in these preliminary tests, giving typical results of three-dimensional splitting flow or breaking gravity waves, depending on the size of Froude number used. The simple lower boundary condition for the geopotential stated in Chapter 4 was applied successfully in these experiments. The SOR procedure required a similar number of iterations for convergence, per second of integration time, compared to the number of iterations required for convergence in the three-dimensional bubble convection tests (see Chapter 5). Thus, at least in these preliminary tests, nonzero topography didn't cause the SOR procedure to be computationally more expensive.

All the numerical experiments in the thesis have been performed on a single PC (see Chapter 5). Having a numerical code that runs on a PC is an advantage, in the sense that the researcher involved have total control over the numerical experiments to be performed. There is no dependency on external factors such as available computer time on a super computer, or on other researchers to assist with the design and set-up of numerical experiments (as may be the case at some of the large computing centers of the world). This provides the



freedom of improving the model code, by adding variables such as water vapor and cloud water, improving the lateral boundary conditions, adding a frictional boundary layer into the model, etc. Still, some recent advances in computational capacity at research institutions in South Africa may facilitate the potential real-atmosphere and three-dimensional applications of the new model. At UP, a PC cluster was obtained in 2005 for general use of scientists in the Faculties of Natural and Agricultural Sciences and Engineering. Unfortunately, the software needed for parallel computing possibilities and human capacity to maintain the system are still lacking. At a national level, the Center for High Performance Computing (CHPC) was established in 2005. The center strives to make super computing facilities (in the form of a powerful PC cluster) more accessible to South African researchers. The present model code will need to be modified for its potential future implementation on a PC cluster.

#### **6.3.4 Implementation of the quasi-elastic equations in C-CAM**

The nonhydrostatic equation set developed was introduced to C-CAM during a visit of the author to CSIRO Marine and Atmospheric Research in January and February 2004. In C-CAM the quasi-elastic equation set is solved with a modified version of the semi-implicit semi-Lagrangian method used previously in the model to solve the hydrostatic equations in  $\sigma$ - coordinates. The new nonhydrostatic dynamical kernel of C-CAM was used successfully to perform a cold bubble simulation in the conformal cubic grid (Rautenbach et al., 2005). This illustrated the suitability of the developed quasi-elastic equation set for use in nonhydrostatic modelling, and that the equations may be introduced to an existing hydrostatic model with relative ease.

### **6.4 The new Nonhydrostatic Sigma coordinate Model (NSM)**

In summary, a new nonhydrostatic model based on the quasi-elastic equations has been developed. The equation set cast in a terrain-following coordinate based on the full pressure field has not been applied before in atmospheric modelling. The equations are formulated independent of a reference profile, making them suitable for application at spatial scales larger than the meso-scale. However, in the present study the model is applied only to simulate highly nonhydrostatic and nonlinear flow at the micro - and meso-scales. The split semi-Lagrangian procedure provides accurate solutions for the case of adequately resolved flow and is stable at large Courant numbers. For the case of marginally resolved flow, the solutions obtained are damped, but smooth, stable and useable. The main advantage of the split semi-Lagrangian solution procedure may be at high spatial resolutions, where the accurate representation of gravity waves requires the use of small adjustment time steps. At these

resolutions, both explicit and semi-implicit procedures require the use of small time steps. Although requiring a similarly small adjustment time step, the split semi-Lagrangian procedure may still be applied at large advection time steps, in this way offering a computational advantage over other methods. The newly developed model has the potential of being used for the study of nonhydrostatic circulation systems, and for research into the improvement of numerical techniques and parameterization schemes used in nonhydrostatic models. The name “Nonhydrostatic Sigma coordinate Model (NSM)” is proposed for the model developed in the thesis. It is concluded that the new model is worth developing further.