Chapter 1

Introduction

1.1 Background to the research

1.1.1 Hydrostatic and nonhydrostatic atmospheric models

1.1.1.1 Unapproximated and fully-elastic equations

It is believed that the equations of momentum, mass and energy conservation for a perfect gas (e.g. Kasahara, 1974; Haltiner and Williams, 1980; Burger and Riphagen, 1990) govern atmospheric motion over a wide range of spatial scales: from small turbulent eddies of a few millimeters in diameter, to synoptic and planetary waves stretching over thousands of kilometers (e.g. Haltiner and Williams, 1980; Holton, 1992). In fact, this set of nonlinear partial differential equations, applied in unapproximated form over the full depth of the atmosphere, is thought to offer the only possibility for the development of a universal atmospheric model (Laprise 1992, Caya and Laprise, 1999; Davies et al. 2003, White et al., 2005). That is, these equations may be used to describe atmospheric motion over the full range of spatial scales it occurs, over any portion of the atmosphere. Based on the fundamental laws of physics, they remain equally applicable in an unknown stellar atmosphere. The unapproximated atmospheric equations are too complicated to be solved analytically. However, these equations may be simplified until qualitative interpretation of physical and dynamical processes is feasible, or until analytic solutions are obtainable. This has become an important study field, called "Dynamic Meteorology". The approximations introduced are normally only valid for atmospheric circulations that occur at a specific spatial scale. For example, the quasi-geostrophic equation set (e.g. Hoskins and Pearce, 1983; Holton, 1992) is widely applied to qualitatively describe atmospheric circulation over the mid-latitudes of Earth. Numerical solutions of the unapproximated equations or an approximated set may also be obtained. This has become another important study field in Meteorology, called "Numerical Atmospheric Modelling".

The universal applicability of the unapproximated equations apparently renders them ideal for the numerical simulation of atmospheric circulation. However, there are currently only a few models that use these equations. An important reason for this, is that acoustic waves form part of the solution set of the unapproximated equations (e.g. Haltiner and Williams, 1980; Room, 1998). Consider, for example, the case of motion that occurs in the $x-z$ plane only. If uniformity is assumed in the lateral direction (y), and if the Coriolis, friction and diabatic terms are neglected, the unapproximated equations reduce to (Miller and White, 1984):

$$\frac{Du}{Dt}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0 \tag{1.1}$$

$$\frac{Dw}{Dt}+\frac{1}{\rho}\frac{\partial p}{\partial z}+g=0 \tag{1.2}$$

$$\frac{D\ln\rho}{Dt}+\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{1.3}$$

$$\frac{D\ln T}{Dt}-\kappa\frac{D\ln p}{Dt}=0. \tag{1.4}$$

Equations (1.1) and (1.2) are the horizontal and vertical components of the momentum equation, and (1.3) and (1.4) are the mass continuity and thermodynamic equations, respectively; u and w are the wind components in the x and z directions, respectively. The density ρ, pressure p and temperature T are related by the perfect gas law $p=\rho RT$; R is the gas constant, and $\kappa=R/c_p$ with c_p the specific heat at constant pressure. The total derivative is $D/Dt \equiv \partial/\partial t+u\partial/\partial x+\partial+w\partial/\partial z$; all partial derivatives with respect to x and t are carried out at constant z. If (1.1)-(1.4) are linearized around an isothermal basic state of no motion in hydrostatic balance , it may be shown that waves travelling at speed

$$c_s=\sqrt{\frac{c_p}{c_v}RT_0} \tag{1.5}$$

are part of the solution set of the linearized equations (e.g. Haltiner and Williams, 1980). Here c_v is the specific heat at constant volume, T_0 is the temperature of the isothermal reference state and (1.5) is the famous formula for the speed of sound.

An equation set that contains sound waves as part of its solution set is termed "fully-elastic" or "fully-compressible". Acoustic waves, however, are believed to have no significant influence on the atmospheric processes that determine weather and climate (e.g. Haltiner and Williams, 1980). The presence of these fast travelling waves implies computational penalties during the numerical solution of the fully-elastic equations (e.g. Room, 1998; Davies et al., 2003). Until fairly recently, these computational constraints have rendered fully-elastic equation sets impractical for general use. However, with the advent of ever faster computers, in combination with the development of economical semi-implicit time integration schemes (e.g. Robert, 1969; Tanguay et al., 1990), the unapproximated equations have become practical and increasingly popular over the last few years. This popularity is linked to a general trend to develop universal models that may be applied at all spatial and time scales (e.g. Laprise, 1992; Wood and Staniforth, 2003; Davies et al., 2005; White et al., 2005). It might well be that the unapproximated equations are the only option for the eventual development of a truly universal model (Davies et al., 2003). The use of various approximated equation sets, however, remains relevant for theoretical and modelling studies, as well as for operational use. The main classes of approximated equations are discussed in the following sections.

1.1.1.2 The hydrostatic approximation and hydrostatic models

With the advent of computers in the late 1940s, the operational use of atmospheric equations for the purpose of numerical weather prediction (NWP) became computationally feasible. When applied to obtain regional or global predictions, these models were restricted by computational constraints to horizontal spatial resolutions that varied from a few hundred kilometers in the early 1950s, to a few tens of kilometers at present. At these resolutions, many atmospheric circulation patterns can not be fully resolved. For example, individual thunderstorms occur at spatial scales ranging from a few hundred meters to a few kilometers (the micro and small scales). Gravity waves and sea-breezes, which occur at the meso (a few tens of kilometers) or smaller scales, are other important circulation systems that can not be properly resolved when the model resolution is in the order of a few tens of kilometers or coarser. Untill the early 1990's, computational constraints did not allow for the operational use of the fully-elastic equations in NWP and climate simulation models. Approximated equation sets were derived from the fully-elastic unapproximated equations in order to specifically describe the scales of motion that could be resolved at the model resolutions used operationally. Although the resulting equation sets do not possess the property of universal applicability, they are computationally much less expensive to solve numerically.

Probably the single most fundamental simplification introduced to the fully-elastic equations, has been the "hydrostatic approximation". A basic scale analysis of the unapproximated equations (e.g. Holton, 1992) indicates that the vertical acceleration term in the vertical momentum equation is of fundamental importance to describe small and micro-scale circulations, such as the updrafts and downdrafts in individual thunderstorms. However, for systems occurring at scales of a few tens of kilometers (meso-scale) to the synoptic-scale and larger, the vertical acceleration term is of negligible importance. At these scales, there exists an almost perfect balance between the downward gravity force and upward pressure gradient force in the atmosphere. This balance is referred to as "hydrostatic balance". Neglecting the vertical acceleration term from the vertical momentum equation is called the "hydrostatic approximation". An equation set that employs the hydrostatic approximation, in combination with associated simplifications introduced to the remaining equations, is called a "hydrostatic" equation set. For example, on making the hydrostatic approximation, the vertical momentum equation in the two-dimensional fully elastic equations (1.1)-(1.4) for an inviscid, nonrotating adiabatic atmosphere reduces to:

$$\frac{\partial p}{\partial z} = -\rho g. \tag{1.6}$$

Equation (1.6) is the well-known "hydrostatic equation". The majority of atmospheric models applied operationally since the 1950s until the early 1990s solved hydrostatic equation sets, and have evolved into highly sophisticated "hydrostatic primitive equation models". These models describe both nearly-geostrophic and gravity wave motion, and apart from the hydrostatic approximation, also include the spherical geopotential and shallow atmosphere approximations (see the recent discussion by White et al., 2005). Acoustic waves may usually be filtered from a given hydrostatic equation set, by means of applying appropriate lower and upper boundary conditions (e.g. Miller and White, 1984). This implies a significant computational advantage over the fully-elastic equations during the time-integration of a hydrostatic model.

At the typical horizontal resolutions employed by hydrostatic models, circulation with nonhydrostatic features (primarily convection and gravity waves) can not be resolved explicitly. The hydrostatic approximation is therefore valid, but the contribution of nonhydrostatic processes to the momentum, mass and energy budgets of the atmosphere needs to be incorporated into the models statistically. Examples of these statistical representations are cumulus or convective and gravity-wave-drag parameterization schemes. Intensive development and use of hydrostatic models at meteorological centers world wide for more than five decades have caused these models to become highly evolved and sophisticated.

1.1.1.3 The anelastic approximation and nonhydrostatic models

"Nonhydrostatic" equation sets span the fully-elastic equations and all approximated sets in which the vertical acceleration term is retained. Nonhydrostatic models have been used for research purposes since the 1960s (e.g. Ogura and Charney, 1962; Dutton and Fichtl, 1969). The majority of these models made use of the anelastic approximation, which filters sound waves from the resulting equation set, whilst retaining the vertical acceleration term. In its most extreme form (the so called Boussinesq approximation), the anelastic assumption consists of the incompressibility assumption and the assumption that the basic state density field is independent of height (e.g. Mahrt, 1986; Tritton, 1988). The two-dimensional Boussinesq equations for an inviscid nonrotational and adiabatic atmosphere are:

$$\frac{Du}{Dt} + \frac{1}{\rho_0}\frac{\partial p}{\partial x} = 0 \tag{1.7}$$

$$\frac{Dw}{Dt} + \frac{1}{\rho_0}\frac{\partial p}{\partial z} - \frac{\theta^{'}}{\theta_0}g = 0 \tag{1.8}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{1.9}$$

$$\frac{D\theta^{'}}{Dt} + w\frac{d\theta_0}{dz} = 0. \tag{1.10}$$

Here $\theta^{'}$ is the deviation of the potential temperature from a reference state $\theta_0(z)$. The constant basic state density ρ_0 has replaced ρ in the horizontal momentum equation. The vertical momentum equation (1.8) is nonhydrostatic in nature and (1.10) is a form of (1.4) in terms of the potential temperature deviation. Note that the incompressibility assumption results in the simplified continuity equation (1.9), compared to the continuity equation (1.3) of the fully-elastic equations. A linear analysis indicates that the Boussinesq equations are filtered of sound waves. Equation sets with this property are called "anelastic".

Various anelastic equation sets have been derived in which the Boussinesq approximation is relaxed to some extent (Ogura and Phillips, 1962; Wilhelmson and Ogura, 1972; Clark and Peltier, 1977). The absence of sound waves in the solution set of the anelastic equations implies a significant computational advantage over the fully-elastic equations during numerical integration (e.g. Davies et al., 2003). The anelastic approximation, however, at least when applied in height-based coordinates, is not deemed appropriate to simulate flow at large spatial scales. A primary reason for this, is that anelastic equation sets in height-based coordinates significantly misrepresent the internal Rossby modes, at wavelengths typically encountered in NWP and climate simulation models (Davies et al., 2003). Studies with anelastic nonhydrosatic models were therefore mostly concerned with the circulation characteristics of micro to meso-scale circulation systems such as thunderstorms and mountain waves. A closely related group of equations that are filtered of vertically propagating acoustic modes, but retain the Lamb waves, have also been developed (Miller, 1974; Miler and White, 1984; White, 1989; Durran, 1989). Some of these quasi-elastic equation sets seem to be more widely aplicable than the anelastic equations (White, 1989; Davies et al., 2003, also see Chapters 2 and 3).

The approximations introduced to obtain anelastic or quasi-elastic equation sets require that a diagnostic elliptic equation (often in the pressure or geopotential perturbation) needs to be solved at each time step during numerical solution (see Chapter 3). Solving a three-dimensional elliptic equation is computationally very intense (see Chapter 4). It may be noted that the computational advantage gained by the bigger time steps allowed during numerical solution of anelastic and quasi-elastic equation sets, is partially cancelled by solving the computationally expensive elliptic equation. It may even be argued that an efficient time-splitting scheme for the fully-elastic equations, that avoids solving an elliptic equation, may be computationally just as efficient as using a filtered equation set. However, for the case where the vertical resolution is higher than the horizontal resolution, vertically propagating sound waves are very restrictive on the time-step used in explicit or split-explicit solutions of the fully-elastic equations. Here filtered equations sets offer a clear computational advantage (e.g. Davies et al., 2003; also see Chapter 5). Also, solving an elliptic equation may allow better coupling between the dynamics of a model and physical parameterizations, than would be possible in time-splitting procedure where the use of an elliptic equation is avoided. Thus, solving an elliptic equation may be advantageous during the numerical solution of the fully-elastic equations, which would again render the use of filtered equation sets advantageous from a computational perspective. In the case of global modelling at resolutions where the hydrostatic approximation is valid, the semi-implicit solution of the fully-elastic equations, which allow the use of large time steps and also require the solution of an elliptic equation, is at present regarded as the most efficient solution procedure (e.g. Tapp and White, 1976; Tanguay et al., 1990; Davies et al., 2005; also see Chapter 5).

With the advent of ever faster computers in the 1990s, it became possible to integrate numerical atmospheric models operationally at resolutions where the hydrostatic assumption is not valid (horizontal resolutions finer than 10 km, approximately). However, even present computational constraints limit operational simulations at such fine resolutions to be performed over only relatively small model domains. At most of the large meteorological centers of the world, intensive research was initiated to convert the highly sophisticated and evolved hydrostatic NWP models into nonhydrostatic models (Janjic et al., 2001). This is generally thought to be the best way to proceed in order to obtain a nonhydrostatic model suitable for operational NWP. The well developed framework (physical parameterization schemes, data assimilation procedures, etc.) of the hydrostatic model is retained, but the hydrostatic equation set is replaced with nonhydrostatic (usually the fully-elastic) equations (Gallus and Rancic, 1996; Janjic et al., 2001; Davies et al., 2005). The main alternatives are to develop an existing anelastic model until it is suitable for integration of a wider range of spatial scales and for relatively long integration periods (White, 1989), or to develop a completely new model based on the fully-elastic equations (Tanguay et al., 1990).

Against the background of the worldwide research on nonhydrostatic models, this thesis reports on the development of a dynamic kernel (an equation set and the numerical procedure to solve it) for a new nonhydrostatic atmospheric model. The equation set used has never been applied before in atmospheric modelling, although closely related equation sets have been used. The unique characteristics and advantages of the equation set are discussed. A numerical solution procedure was developed which allows for efficient numerical solution of the equation set. The suitability of the dynamic kernel to simulate nonhydrostatic circulation in the atmosphere is illustrated by means of a large set of numerical tests. In the following sections, the status quo of atmospheric modelling in South Africa is discussed, and it is motivated why the research effort to develop a new atmospheric model was made.

1.1.2 Numerical atmospheric modelling in South Africa

In the 1960s, a small group of scientists from the Center for Scientific and Industrial Research (CSIR) and the South African Weather Bureau (SAWB) became the first South Africans to develop a three-dimensional NWP model. This gridpoint model was applied over the southern hemisphere and formulated in terms of the quasi-geostrophic barotropic equations. When the model was first used operationally in the 1970s, it was integrated over only two or three levels in the vertical because of computational constraints (Riphagen, personal communication). The research group was headed by Burger of the CSIR. In later years, he and Riphagen of the CSIR (and later SAWB), were to write a number of telling papers on how to obtain energy-consistent approximations of the atmospheric equations (Burger, 1991; Burger and Riphagen, 1979, 1981, 1990, 1999). In a study of the equations of motion expressed in an arbitrary vertical coordinate sytem, Burger and Riphagen (1990) retained in the components of the momentum equation the Coriolis terms that vary with the cosine of the latitude (the $\cos\lambda$ terms) (see the recent discussion by White et al., 2005) and other terms not included in the hydrostatic primitive equations. The omission of the $\cos\lambda$ terms has been called the "traditional approximation" (Eckart, 1960). The paper by Burger and Riphagen (1990) preceded a recent international trend in global atmospheric modelling, in which the hydrostatic primitive equations (that contain the traditional approximation) are replaced by more complete quasi-hydrostatic (White et al., 1995) or fully elastic nonhydrostatic (Dudhia and Bresch, 2002; Davies et al. 2005; White et al., 2005) equation sets that contain the full Coriolis force.

In South Africa, the earlier theoretical investigations of Riphagen and Burger led to the the development of a new filtered quasi-geostrophic model based on energy conservation principles (Burger and Riphagen, 1978, 1979; Riphagen and Burger, 1978). The model ran operationally at the SAWB, as a backup model for a hydrostatic primitive equation model that was obtained from an international institution. In the early 1980s, a five-level, split-explicit, hydrostatic primitive equation model for hemispheric prediction was developed at the CSIR (Riphagen, 1984). The performance of the model was evaluated over an 18 month semi-operational run at the SAWB, and it was found to be comparable to that of the international model used at the time. A theoretical investigation of the computational stability of the model was carried out, which resulted in a longer time step being allowed (Riphagen and Burger, 1986).

The development of the hydrostatic primitive equation model by Riphagen (Riphagen, 1984; Riphagen and Burger, 1986) provided an excellent foundation for future model development in South Africa. Unfortunately, a change in philosophy at the CSIR in the mid-1980s caused the organisation to become more commercially driven, and research into NWP became a low priority. Riphagen joined the SAWB in 1986, however, here it was decided to obtain model codes from international institutions, rather than to maintain and improve a locally developed code. This decision was based on the believe that the NWP research group at SAWB, which was small in comparison to international modelling groups, would not be able to keep up with all the development in NWP taking place at the time. Today, with the vision of hindsight, this decision was probably not the correct one to make. Relatively small research groups that were active at the same time in countries such as Australia, have managed to develop atmospheric models that are today state-of the art in atmospheric modelling. The SAWB, however, has become dependent on Northern Hemisphere organisations for obtaining numerical models. To apply these model codes locally has proven to be a labour intensive task in itself. Expensive license fees also became a reality. To obtain the supercomputers that are required to run the computing-intensive model codes of Northern Hemisphere countries in South Africa, has placed an additional financial burden on the SAWB.

Since the majority of meteorologists in South Africa were employed by the SAWB (and today by the South African Weather Service (SAWS)), only a few further studies in the field of numerical meteorology took place in the country. These were mostly concerned with the numerical schemes and data assimilation systems applied in some of the international models that became operational at the SAWB (e.g. Riphagen, 1989, 1999; Tennant et al., 1997; Riphagen et al., 2002). Today South Africa lacks meteorologists skilled in the field of numerical atmospheric modelling.

Since the late 1980s, a number of international atmospheric models has been run in-house at the SAWB, to obtain numerical predictions on the short to seasonal time scales (e.g. Riphagen, 1993, 2005). Probably the most important of these models is the ETA-model from the National Centers for Environmental Prediction (NCEP), which was first used in 1992. The ETA model became the base model for the daily weather forecasts issued by the SAWB and the later SAWS. The fifth generation Pennsylvania State University-National Center for Atmospheric Research (NCAR) Meso-scale Model (MM5) also runs operationally at the SAWS at present, where it is is mainly used for the generation of aviation forecasting products. In 2004 changes at NCEP persuaded the SAWS to obtain a new model for operational runs, this time from the United Kingdom Meteorological Office (UKMO). The UKMO model (Davies et al., 2005) still needs to be operationally implemented at the SAWS.

Some universities in South Africa have also managed to obtain and implement model codes from international institutions. At the University of the Witwatersrand (WITS) sensitivity experiments with the Regional Atmospheric Modelling System (RAMS) were performed in the 1980s and 1990s (Crimp et al., 1998). The regional climate model DARLAM (Division of Atmospheric Research Limited Area Model) was also applied to perform climate simulations over southern Africa (Joubert et al., 1999). Since then the climatology group at WITS has found different focus areas. The University of Cape Town (UCT) has become more active in the field, and has been performing climate simulations with MM5 since the 1990s. Recently, five-year MM5 simulations of present and future climate over Southern Africa was performed and described by the UCT group (Hewitson et al., 2005). Computational constraints associated with the expensive-to-integrate MM5 have so far prevented the UCT group from performing longer simulations. Regarding research into numerical atmospheric modelling, the meteorology group at the University of Pretoria (UP) is probably the most active South African university.

1.1.3 Atmospheric modelling activities at the University of Pretoria

Atmospheric modelling research at UP was initiated in the early 1990s when the CSIRO4 atmospheric general circulation model (AGCM) was obtained through a license agreement between UP and the Commonwealth Scientific and Industrial Research Organisation (CSIRO) in Australia. The model was installed on a local super computer (CONVEX C-120). A number of model simulations were performed, including a 20-year control run, as well as selected experiments describing the interaction between the ocean surface and atmosphere over the Indian and Atlantic Oceans (Van Heerden et al., 1995).

In 1995, with the assistance of CSIRO researchers, the CSIRO Mark 2 AGCM with an R21 spectral resolution was installed on a CRAY-EL94 super computer located at the SAWB by Rautenbach. A number of experiments were performed in order to investigate ocean-atmosphere interaction (Jury et al., 1996; Jury et al., 2000). The model's ability to simulate present day climate over southern Africa was also investigated. The AGCM became a useful tool to produce experimental seasonal forecasts of rainfall over the southern African region (Rautenbach, 2003). In his PhD-thesis Rautenbach, with the assistance of Gordan from the CSIRO, introduced a hybrid (sigma/pressure) vertical co-ordinate to the dynamic formulation of the CSIRO9 Mark II AGCM (Rautenbach, 1999). In this study it was indicated that the hybrid vertical co-ordinate system contributes to improved climate simulation in the upper levels of the atmosphere.

Regional climate modelling has numerous unexplored applications over the southern African region (Engelbrecht and Rautenbach, 2000). Seen against this background an agreement was reached in 1999 between UP and the CSIRO to install the regional model DARLAM on a suitable computer at UP. With this prospect in mind the author (Engelbrecht) visited the CSIRO Atmospheric Research during January 2000. With assistance from CSIRO scientists McGregor and Katzfey, DARLAM was installed on a computer at UP. The first climate simulations performed with DARLAM at UP are described by Engelbrecht (2000) and Engelbrecht et al. (2002).

In 2002, intensive research on atmospheric model development commenced at the University of Pretoria, as part of a project sponsored by the South African Water Research Commission (WRC). During the course of the project, the hydrostatic Conformal-Cubic Atmospheric Model (C-CAM) of CSIRO was implemented on computers at UP. Various sensitivity studies were performed in order to improve the model's ability to simulate rainfall over southern Africa (Rautenbach et al., 2005). With the assistance of CSIRO scientists McGregor and Thatcher, the research group at UP managed to run C-CAM operationally for routine NWP over southern Africa (Rautenbach et al., 2005). UP is the first institution in South Africa, other than the SAWS, to accomplish this. An important aspect of the WRC project, was the development of a nonhydrostatic kernel for a new atmospheric model, which contributed to the development of an a nonhydrostatic kernel for C-CAM. The present thesis contributed largely to this research aspect of the WRC project (Rautenbach et al., 2005). In a subsequent WRC project concerned with the hydrological impact of global warming on South Africa, C-CAM was also used to perform two 30-year simulations of climate and climate change over Southern Africa (Engelbrecht, 2005).

1.2 Motivation for the research

1.2.1 Recent developments in nonhydrostatic atmospheric modelling

The development of nonhydrostatic models has been ongoing for about four decades. It started with different mesoscale investigations (e.g. Ogura and Charney, 1962; Dutton and Fichtl, 1969; Miller and Pearce, 1974; Tapp and White, 1976; Klemp and Wilhelmson, 1978; Pielke, 1984). The resolution of operational NWP models, however, has until fairly recently been limited by computing power and time constraints of the operational environment to resolutions where the hydrostatic approximation is almost perfectly valid. Operational forecasts therefore mostly relied on hydrostatic models until recently. With computer systems becoming faster and more affordable over the past decade, a corresponding increase in the resolution of both NWP and climate simulation models occurred. This, as well as the growing requirements of model precision, has brought the transition of the highly developed existing hydrostatic models to nonhydrostatic models into the limelight. Over the past decade many atmospheric research institutions have started to replace operational hydrostatic models with nonhydrostatic versions (Gallus and Rancic, 1996; Davies et al., 2005). Model upgrading is much easier if connections with the existing hydrostatic model is preserved during the development of the nonhydrostatic model (Janjic et al., 2001). In this regard, preserving the vertical coordinate of the hydrostatic model (usually some type of pressure coordinate) is essential.

With operational model resolutions now beyond the hydrostatic limit (less than about 10 km in the horizontal) over relatively small domains, convection is at least partially resolved (and partially explicitly simulated) by the nonhydrostatic models. Convection can probably only be fully resolved at model resolution of about two orders of magnitude higher - that is, about 100 m in the horizontal. It is therefore likely that for many years to come, operational nonhydrostatic models will function at resolutions where convection can only be partially resolved. The use of convection parameterization schemes at these resolutions is a relatively unexplored study field in atmospheric modelling. In fact, the parameterization schemes applied in hydrostatic models have generally been designed to function at resolutions where convection cannot be resolved at all. These parameterization schemes need to be reviewed and probably modified in order to function well beyond the hydrostatic limit. Theoretical studies of the explicit simulation of moist convection is another growing study field and nonhydrostatic models are a primary tool for these investigations. Explicit simulations of moist convection will benefit the design of convection parameterization schemes for application at model resolutions where convection can not be fully resolved.

Meteorological research in South Africa would greatly benefit by having easy access to a nonhydrostatic model. Setting up high-resolution simulations requires thorough knowledge of the nonhydrostatic model's code. Since the codes of international models are extensive and highly evolved, using such models correctly requires thorough research in itself. These models are usually formulated in terms of certain map projections of Earth, which complicates the design of numerical experiments by users not fully familiar with the model equations and the projection used. As an alternative to studying the code from an international model, a new nonhydrostatic model may be developed from scratch. Such a research effort might simultaneously contribute to the development of an international model, for example, from hydrostatic to nonhydrostatic dynamics.

To summarize, the development of a new nonhydrostatic model, and/or thorough knowledge of an existing nonhydrostatic model code, will make it possible for meteorologists in South Africa to contribute to four important, interlinked and growing fields in numerical atmospheric modelling:

- The development of nonhydrostatic models in general (equation sets, vertical coordinates, numerical solution procedures, etc.).
- Conversion of existing hydrostatic (mostly pressure-based) hydrostatic models into nonhydrostatic models.
- The study of moist convection by means of explicit numerical simulations with a nonhydrostatic model.
- The development of convection parameterization schemes for nonhydrostatic models that are applied at resolutions where convection is only partially resolved.

1.2.2 Nonhydrostatic circulation systems over South Africa

Atmospheric circulation systems exhibiting nonhydrostatic features frequently occur over southern Africa. In particular, severe thunderstorms often occur over the Highveld and eastern escarpment of South Africa (Garstang et al., 1987), whilst mountain waves are regularly observed over the Lesotho Drakensberg (De Villiers, 1998) and the mountains of the southwestern Cape (see Fig. 1.2). The characteristics of these systems have never been studied by means of high-resolution, nonhydostatic model simulations. The reasons for this deficiency in meteorological studies over South Africa are clear: the non-existence of a locally developed nonhydrostatic model, and a shortage of local meteorologists skilled in the use of an international nonhydrostatic model. In this subsection of the thesis, the most important nonhydrostatic circulation systems occuring over South Africa are reviewed, and it is illustrated that there is a need for nonhydrostatic model simulations to aid the study of the characteristics of these systems.

1.2.2.1 Convective rainfall over South Africa

The Summer rainfall region of South Africa receives most of its precipitation in the form of convective rainfall. Quite often the convection occurs east of upper air troughs that pass over the country from west to east. In fact, 80% of the rainfall over the summer rainfall region occurs when tropical-temperate troughs (a westerly trough in combination with an easterly wave) moves over southern Africa (Harrison, 1984, 1986). During these events tropical air enters the country, mostly from Botswana or Namibia. The thunderstorms that occur under these circumstances are organised in a north-west to south-east alignment, extending from Namibia to the east coast of South Africa. However, convective thunderstorms also occur frequently over the Highveld and eastern escarpment regions purely in response to surface heating and low-level convergence. Thus, convection over South Africa may occur under a wide range of circumstances. However, surprisingly little research has been performed up to date on the climatological properties of South African thunderstorms. In particular, the dynamical characteristics of thunderstorms occurring in South Africa are not well understood.

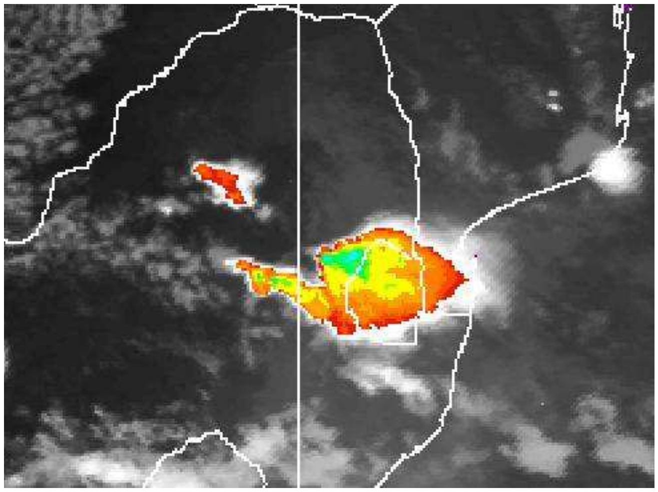

Figure 1.1: A Meteosat 7 colour enhanced infrared satellite image showing a severe thunderstorm over Swaziland and the Lowveld of South Africa. Storm splitting may have occured, since two overshooting tops are indicated by the light blue regions (coldest cloud top temperatures).

1.2.2.2 Thunderstorms in South Africa

There have been few investigations that attempted to quantify the typical frequencies and intensity of thunderstorms over South Africa, and no proper thunderstorm climatology is available. Most studies have been concerned with the frequency of occurrence of hail-producing thunderstorms. These studies largely relied on the use of radar data (Carte and Held, 1987; Held, 1978; Held, 1982). The radar-based and earlier studies (Schulze, 1972), indicate that there are typically more than 80 rain days over the high plateau of KwaZulu-Natal, about 70 over Gauteng, and between 20 and 30 over the KwaZulu-Natal coastal areas. Hail-producing thunderstorms annually destroy about 1% of South Africa's crops (Visser, 2000) and a single hail event may cause damage of millions of rands to buildings and cars (Visser, 2000). There are no formal studies on the dynamics of severe thunderstorms over South Africa available in the literature. The little that is known about the dynamics of these storms, was derived from general thunderstorm circulation theory (e.g. Klemp, 1987; Doswell, 1991; Holton, 1992) applied to the southern hemisphere (Doswell, 1991; De Coning et al., 2000) and from radar-based case studies of individual storms (Visser, 2000; De Coning et al., 2000). Figure 1.1 shows an outbreak of wedged-shaped (possibly severe) thunderstorms over the Drakensberg mountains in South Africa.

A wind that turns counter clockwise with height (backs) is typical of the development of rotating thunderstorms in the southern hemisphere (e.g. Doswell, 1991; Holton, 1992). This type of wind profile commonly occurs over eastern

South Africa in the austral summer. Ridging high-pressure systems would typically induce a north-easterly low-level flow over eastern South Africa, with an upper air trough simultaneously causing north-westerly flow at 700 and 500 hPa. Often a jet stream, or at least very strong westerly winds, are to be found at the same time at about 200 hPa. Thus, the development of rotation in thunderstorms occurring over eastern South Africa, and the associated storm splitting (e.g. Holton, 1992) should be fairly common. Unfortunately, the absence of Doppler radar systems in the country hampers the identification and study of these events. However, with high-resolution Meteosat Second Generation (MSG) (Meteosat 8) satellite images recently becoming available in South Africa, the identification of the occurrence of at least storm-splitting is possible. The long record of ordinary radar data that is available for the Highveld and eastern Free State is also a valuable source of information to gain more insight into the dynamics of South African thunderstorms. However, the development of rotating storms is less easily identified using ordinary radar data, and the ideal solution for the study and real-time forecasting of rotating thunderstorms would be use of Doppler radar in the country (De Coning et al., 2000). A high-resolution nonhydrosatic numerical atmospheric model may be used to aid the study of the dynamics of rotating and severe thunderstorms occurring over South Africa, in combination with the radar and satellite data.

Supercell thunderstorms in South Africa

Supercell thunderstorms (rotating storms associated with heavy rainfall, severe hail and sometimes tornadoes) rarely occur in South Africa (De Coning et al., 2000; Admirat et al., 1985). This statement is supported by studies concerned with the nature of hail producing thunderstorms over South Africa (Held, 1978; Held, 1982; Carte and Held, 1978). However, the typical wind profile associated with the development of the severe cyclonic (clockwise) rotating supercell thunderstorms in the southern hemisphere (Doswell, 1991; De Coning et al., 2000), occurs fairly often over eastern South Africa (see the previous paragraph). Other environmental conditions apparently prevent rotating thunderstorms from developing into severe supercell storms on a regular basis. However, Visser (2000) argues that the frequency of occurrence of supercell thunderstorms over South Africa may be underestimated due to the poor observational network in the country. These aspects require further study, and environmental conditions favourable for supercell formation over South Africa need to be quantified. A high-resolution nonhydrostatic model may be of great help in such investigations.

Tornadoes

A tornado is defined as a violently rotating column of air with small diameter extending from a thunderstorm to the ground (Goliger et al., 1997). Tornadoes are often accompanied with damaging winds, heavy rainfall and hail. The CSIR data base of tornadic activity (Goliger et al., 1997) contains 200 reports of tornadic events over South Africa, dating from 1905 to 1996. Most tornadoes have been observed over the eastern escarpment areas and over Gauteng (probably as a result of the high population density in the latter area). Most of these tornadoes occurred between November and March, typically between 16:00 and 19:00 South African Standard Time (SAST) (Goliger et al., 1997). They varied in intensity from F0 to F3 (see Fujita, 1973a,b for a discussion of the Fujita-Pearson scale to evaluate tornado intensity). The classification of tornadoes in the above data base was mostly based on assesessment of the damage caused. Many of the severe storms that occur in the country are never surveyed for damage or evidence of tornadoes. In fact, the mere identification of tornadic events occuring in South Africa is challenging (De Coning and Adam, 2000) and their frequency of occurrence is most probably underestimated.

During the 1998/99 summer season, several severe thunderstorms occurred over the eastern escarpment. At least three of these severe storm events were accompanied by tornadoes (De Coning and Adam, 2000). The circulation properties of two of the tornadic events, occurring at Harismith and Mount Aliff, were studied in some detail by De Coning and Adam (2000) and Visser (2000). The study of the Harismith tornado utilized the hydrostatic ETA model at 48 km horizontal resolution over South Africa. The model was only found to be helpful in identifying the area where thunderstorms occurred on the specific day. The radar data were more illuminating, and showed evidence that storm splitting (a feature associated with rotation in severe thunderstorms, Klemp, 1987; Holton, 1992) occurred. The surface observation network, in combination with model data, showed evidence that the wind was backing with height (which is inducive to the formation of clockwise rotation in thunderstorms in the southern hemisphere). For the Mount Aliff tornado, De Coning and Adam (2000) found that typical stability and wind shear indices, as simulated by the ETA model, correctly identified the area where the tornado occurred as one favourable for severe thunderstorms and/or tornadoes. The severe thunderstorm that spawned the tornado was too far away from the nearest radar station for thorough analysis by means of radar data.

The occurrence of an intense tornadic event may cause wide-spread damage and loss of life. The Harismith tornado, for example, was estimated to have caused damage of between between 3 and 4 million rands; 21 people died and 350 were injured by the Mount Aliff tornado (De Coning and Adam, 2000). More research is needed in order to identify the atmospheric characteristics that are inducive of tornadic events over South Africa. A nonhydrostatic model may be invaluable in such investigations.

1.2.2.3 Mountain waves

When the prevailing wind over the Lesotho Drakensberg mountains is northwesterly, mountain wave clouds are often observed in the lee of the mountains.

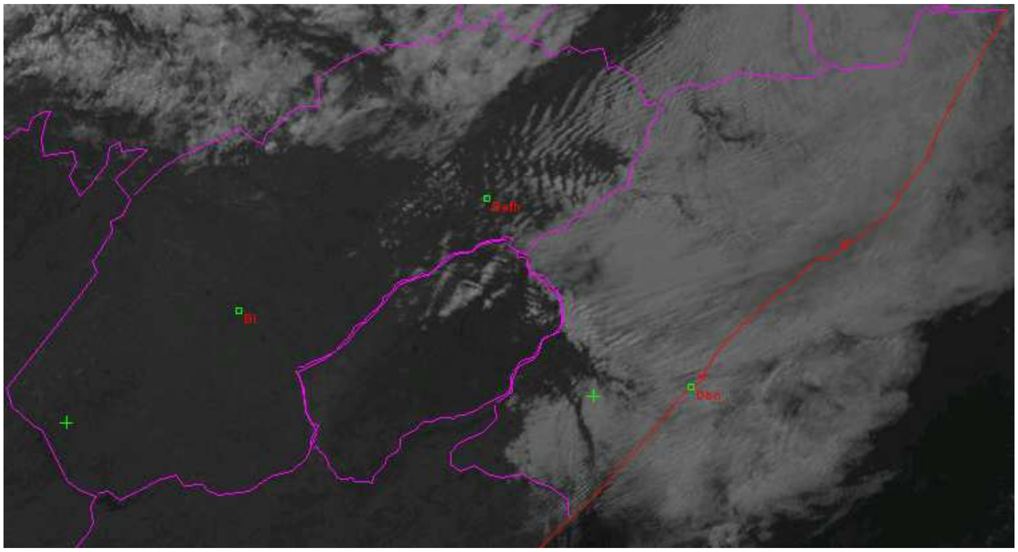

Figure 1.2: A Meteosat 8 visible satellite image showing the formation of mountain waves downstream of the Drakensberg region of South Africa and Lesotho.

These clouds form when the air mass present over the mountain is stably stratified. As air is forced to rise over the mountain barrier, a series of oscillations is induced as the air decelerates and accelerates in the stable environment. If the air is sufficiently moist, condensation takes place where the air is rising, in this way forming the mountain wave clouds. If the air is not sufficiently moist (as is often the case with north-westerly flow over the Lesotho Drakensberg) mountain waves may still develop, without the mountain wave clouds being present. Aircraft flying over the area under such conditions may experience clear air turbulence (CAT) in the rising and descending portions of the mountain waves. The occurrence of CAT over the Lesotho Drakensberg has caused injuries to passengers in aircrafts flying over the area on a number of occasions (De Villiers, 1998, 2001). The synoptic-scale conditions favourable for the occurrence of mountain waves over the Lesotho Drakensberg have been studied by De Villiers (1998; 2001). Except for these studies, no attempt has been made to quantify the stability and wind shear conditions conducive to the occurrence of mountain waves over the Lesotho Drakensberg. A nonhydrostatic model is essential to the study the dynamics of these waves in some detail.

It may also be noted that mountain waves occur frequently over the mountains of the southwestern Cape, in the north-westerly flow ahead of an approaching cold front. Such an event is depicted in Fig. 2, an infrared MSG satellite image which shows the occurrence of mountain waves over a region of more than 100 km^2 over the south western Cape. Although weather forecasters in the country are well aware of the potential aviation hazards implied by events like this (De

Villiers, 1998, 2001), the dynamics of the south-western Cape mountain waves, as for the Lesotho mountain waves, remain to be the studied.

1.2.2.4 Modelling nonhydrostatic circulation systems occurring over South Africa

Running a NWP model operationally at resolutions beyond the hydrostatic limit over a typical nested model domain including South Africa, is not feasible within the bounds of present computing resources available in the country. However, for research purposes, studies at these resolution are possible, at least over limited areas of the country (the Highveld, for example). It should be possible to apply some of the nonhydrostatic atmospheric models used in South Africa for studies of this kind, in particular, the MM5 model may be used. To set up a nonhydrostatic model at high resolution over a specific area, however, is fairly challenging. Multiple nesting is required, and to initialize the atmospheric state for a specific experiment, careful consideration is needed to prevent the generation of spurious gravity and sound waves (Mendez-Nunez and Carroll, 1994). There is currently a lack of expertise in the country to design experiments of this kind. Still, there is a clear need for research on the application of an existing or a newly developed nonhydrostatic model in order to study the characteristics of thunderstorms and mountain waves that occur over South Africa. This study is an attempt to develop modelling capacity for future investigations into the characteristics of nonhydrostatic circulation systems occurring over South Africa.

It may finally be noted that studies into air quality standards over South Africa may benefit from the application of a high-resolution nonhydrostatic model. At present the input of air pollution dispersion models in South Africa consists largely of surface observations, since upper air balloon soundings and vertical wind profile observations by sodar are only available at a small number of locations in the country. The input from the ETA model, which runs operationally at the SAWS at a horizontal resolution of 32 km over the country, is used as aditional input. However, high-resolution (horizontal resolutions of about 1 km or finer over the Higveld, for example) simulations of the three-dimensional wind field in the boundary layer may be used as input for a Lagrangian dispersion model, in this way improving simulations of pollution advection and dispersion. This provides further motivation for a research effort aimed at developing nonhydrostatic modelling capacity in the country.

1.3 Objectives of the research

Taking into account the status of numerical atmospheric modelling in South Africa, the research links between UP and CSIRO Atmospheric Research, the current worldwide research into the development of nonhydrostatic models, and the potential applications of nonhydrostatic models over southern Africa, the research outlined in this thesis has two main objectives:

To identify a nonhydrostatic equation set in terrain-following pressure-based (σ) coordinates suitable for implementation in an existing hydrostatic σ coordinate model, and to study the characteristics of the equation set.

This was attained through a literature study of the large variety of nonhydrostatic equation sets developed over the last four decades. Three groups of nonhydrostatic equation sets were considered: fully-elastic, quasi-elastic and anelastic. Most existing hydrostatic NWP and climate simulation models employ pressure-based vertical coordinates. The most convenient way of obtaining nonhydrostatic versions of these models is by the use of a formulation of nonhydrostatic equations cast in pressure-based coordinates. In particular, the development of a nonhydrostatic equation set in pressure-based terrain-following coordinates, may facilitate the conversion of the hydrostatic CSIRO model C-CAM to nonhydrostatic dynamics. Therefore the literature studied focussed on the different nonhydrostatic equation sets formulated in pressure-based coordinates. The research led to the formulation of a quasi-elastic equation set in σ-coordinates that has not been used before. The unique characteristics of this equation set were studied, largely by the use of linear perturbation analysis and numerical experiments.

To develop a dynamic kernel for a new nonhydrostatic atmospheric model based on the quasi-elastic equation set.

A time-split semi-Lagrangian method was developed to solve the quasi-elastic equation set, in two or three spatial dimensions. The method is shown to be more efficient than the explicit methods used previously to solve similar nonhydrostatic equation sets. The three-dimensional model code that was developed opens the door for further model development and capacity building in the field of numerical atmospheric modelling, at UP and in South Africa.

1.4 Organisation of the report

The present Chapter sets the scene for the thesis and motivates why the research effort to develop a nonhydrostatic kernel for a new atmospheric model was made. In Chapter 2 the different types of vertical coordinates that may be used to formulate nonhydrostatic equation sets are reviewed. The use of pressure-based coordinates is discussed in particular, since a pressure-based nonhydrostatic model is formulated and applied in the thesis. The fully-elastic (unapproximated) equations in pressure and σ coordinates are stated, and it is motivated why these equation sets are not regarded suitable for numerical atmospheric modelling. The fully-elastic equations based on the hydrostatic pressure field are then stated, and it is explained why this equation set has become a popular choice for the development of universal models. Different approximated non-hydrostatic pressure-based equation sets, which may be termed anelastic and quasi-elastic, are reviewed. These equation sets employ vertical coordinates that are based on the full pressure field. They offer computational advantages over the fully-elastic equations, but seemingly at the cost of a loss in universal applicability. The scales of motion that the different approximated sets apply to are stated, and the computational advantages offered by each set are mentioned.

In Chapter 3 nonhydrostatic models employing vertical coordinates based on the full (nonhydrostatic) pressure field as vertical coordinate are studied. The first model of this kind, formulated in p coordinates, was developed by Miller (1974) and Miller and Pearce (1974). The characteristics of the Miller-Pearce (MP) model, which is limited for application at the meso-scale, are reviewed. The MP model in p coordinates was extended by White (1989) for application at larger-scales. This equation set of White (1989) is transformed to σ coordinates in Chapter 3. An elliptic equation that is used to diagnose the geopotential distribution is also derived. The characteristics of the σ coordinate equation set are studied, largely by means of linear stability analysis. It is shown that the equation set implies an energy equation similar to that of the hydrostatic equations, with the additional representation of vertical kinetic energy. The equations are shown to be filtered of vertically propagating acoustic waves and it is illustrated that the Lamb waves in the model are significantly retarded. This implies that the derived equation set offers computational advantages over other nonhydrostatic equation sets based on the fully-elastic equations.

A time-split, semi-Lagrangian procedure to solve the quasi-elastic equations in two or three spatial dimensions on a nonstaggered grid is developed in Chapter 4. The semi-Lagrangian method used to calculate the departure points of air parcels is discussed, as well as the associated interpolation schemes applied in two or three spatial dimensions. A forward-backward scheme that handles the fast travelling waves during the adjustment step is described. It is shown that it is essential to apply a spatial filter at the smallest resolvable scales, in order to avoid problems with stationary two-grid-interval waves. An iterative procedure is formulated which efficiently solves the elliptic equation in the geopotential, in two or three spatial dimensions. In Chapter 5 the accuracy and stability properties of the split semi-Lagrangian solution procedure applied to the quasi-elastic equations in σ coordinates is demonstrated with a series of convective bubble experiments in two and three spatial dimensions. The experiments also function to illustrate that the dynamic core functions well at resolutions far beyond the hydrostatic limit. A new numerical test was designed, in which the development of rotation in an environments with wind shear is simulated. The results obtained for this test comform well with the linear theory of storm splitting. Conclusions on the research are drawn in Chapter 6.

1.5 New aspects of the research

The most important novel aspects of the research may be summarized as follows:

- The formulation of a nonhydrostatic equation set in terrain-following coordinates based on the full (nonhydrostatic) pressure field - an equation set which has not been used before in atmospheric modelling.
- A theoretical analysis of the derived equation set, showing its unique characteristics and advantages. The analysis indicates the equation set may be termed quasi-elastic.
- The development of a novel time-split semi-Lagrangian solution procedure which efficiently solves the quasi-elastic equations on a nonstaggered grid.
- The first numerical simulations of bubble convection in three spatial dimensions. A new numerical test was designed, in which nonhydrostatic splitting flow is simulated in an environment with vertical wind shear.

Thus, a novel dynamic kernel has been developed, providing the framework for a new locally-developed meso-scale atmospheric circulation model.

Chapter 2

Nonhydrostatic models in pressure-based coordinates

2.1 Introduction

Many different types of vertical coordinate systems have been applied successfully in atmospheric modelling (see, for example, Haltiner and Williams, 1980). Pressure-based vertical coordinates have been the most popular choice, for operational weather prediction and climate simulation, over the last four decades. By pressure-based coordinates are meant "pure" pressure coordinates (Eliassen, 1949), and terrain-following pressure-scaled (σ) coordinates (Phillips, 1957). Pressure-based vertical coordinates offer some unique advantages over other vertical coordinate systems. Firstly, atmospheric measurements are taken at constant pressure levels during balloon soundings. Interpolation of the observed variables at constant pressure levels to model levels in the vertical is most accurately and conveniently achieved if the model is using pressure-based coordinates. More importantly though, is that the atmospheric equations in pressure coordinates assume a simpler form than in most other coordinate systems (e.g. Miller and White, 1984; Room et al., 2001). This is true in particular for hydrostatic primitive equation models cast in pressure coordinates. In this framework, the atmosphere is non-divergent and the continuity equation reduces to a diagnostic equation (see, for example, Haltiner and Williams, 1980; Holton, 1992). In pressure-based coordinates, density is also eliminated from the prognostic equations, which is advantageous since density is not readily measured in the atmosphere. With the elimination of density, the momentum equations and thermodanamic energy equation simplify in addition to the simplifications to the continuity equation. Finally, many conceptual models used in weather forecasting have been developed in terms of the distribution of certain variables at constant pressure levels.

The popularity of the hydrostatic primitive equation models for operational weather prediction and climate simulation during the last four decades has been strongly linked to computational constraints of the operational environment (see the discussion in Chapter 1). These constraints implied that models could only be integrated operationally at resolutions where the hydrostatic approximation is valid (resolutions coarser than approximately 10 km, e.g. Daley, 1988). Therefore, until recently, all operational models were hydrostatic models. As discussed in the previous paragraph, pressure-based vertical coordinates are the most popular choice for application in operational hydrostatic primitive equation models. Thus, the constraints of the operational environment, and the advantages offered by pressure-based coordinates to hydrostatic modelling, have made hydrostatic primitive equation models in pressure-based coordinates the natural choice for operational atmospheric modelling. Only in the 1990s did faster computers start to allow simulations beyond the hydrostatic limit, although only over limited areas of Earth. Even at the large meteorological centers of the world, global (nonhydrostatic and hydrostatic) weather forecasting models are at present still integrated at resolutions where the hydrostatic assumption is valid.

Despite the computational constraints on obtaining operational simulations beyond the hydrostatic limit, the development of nonhydrostatic models has been ongoing for more than four decades. Most of these models were developed purely for research purposes, to study the properties of meso-scale circulation systems exibiting nonhydrostatic circulation features (e.g. Ogura and Charney, 1962; Dutton and Fichtl, 1969; Miller and Pearce, 1974; Tapp and White, 1976; Klemp and Wilhelmson, 1978; Pielke, 1984). The majority of nonhydrostatic models developed employed vertical coordinate systems based on the geometric height (z-based coordinate systems) (e.g. Ogura and Charney, 1962; Ogura and Phillips, 1962; Lipps and Hemler, 1982; Shutts and Gray, 1994; Smolarkiewicz et al., 2001). A recent review of the various types of z-based coordinate models, from the perspective of a normal mode analysis, is given by Davies et al. (2003). An important conclusion from this study is that only the fully-elastic z-based equations are suitable for modelling at spatial scales ranging from the microscale to the large-scale. All approximate sets, including anelastic, quasi-elastic and hydrostatic z-based equations, appear to have limitations to the spatial scales for which they may be applied (Davies et al., 2003).

In 1974 Miller developed the first nonhydrostatic model employing a pressure-based vertical coordinate. The model was used to study cumulonimbus convection by Miller and Pearce (1974) and became known as the Miller-Pearce (MP) Model. An important property of the MP model is its anelastic nature in pressure coordinates. The filtering of sound waves is achieved by introducing approximations to the fully-elastic pressure coordinate equations from the basis of a scale analysis (Miller, 1974; also see section 2.3.1). Miller and White (1984) derived the MP model from the basis of a formal power series expansion, and also stated the quasi-elastic σ coordinate equivalent of the model. The MP model in presure coordinates and also in σ coordinates was used in numerous studies of the numerical simulation of convection and airflow over mountains (e.g. Miller and Pearce, 1974; Xue and Thorpe, 1991; Miranda and James, 1992).

In the early 1990s it was realized that the operational use of models beyond the hydrostatic limit would soon become computationally feasible (e.g. Tanguay et al., 1990). It was envisaged that the new generation of models would be able to simulate atmospheric proceses ranging from the micro- to the large-scale. By this time the hydrostatic primitive equation models, all cast in pressure-based vertical coordinates, were highly evolved and sophisticated (Janjic et al., 2001). It was realized that the most convenient way of obtaining nonhydrostatic models suitable for operational use, would be to convert existing hydrostatic pressure-based models to nonhydrostatic models (Laprise, 1992; Bubnova et al., 1995). The only pressure-based nonhydrostatic model developed at the time was the MP model. However, originally designed to obtain simulations of nonhydrostatic micro- and meso-scale processes (Miller, 1974), the MP model is not deemed for appropriate for simulations at larger scales (White, 1989; see also section 2.3.1 and Chapter 3). In fact, it is believed that for a model to be able to simulate atmospheric motion at spatial scales ranging from the micro- to the large-scale, it needs to be be fully-elastic (Laprise, 1998; Davies et al., 2003; Davies et al., 2005). However, the suspicion that the full pressure field may be singular in the case of severe vertical acceleration (Laprise, 1992, 1998), has prevented the development of a fully-elastic model based on the full pressure field as vertical coordinate. Instead, Laprise (1992) suggested the use of hydrostatic pressure as vertical coordinate for the fully-elastic equations. This novel idea soon became popular, and a number of fully-elastic pressure-based vertical coordinate models were developed (e.g. Bubnova et al., 1995; Gallus and Rancic 1996; Janjic et al., 2001). It was even suggested that the fully-elastic equations based on the hydrostatic pressure is the only manageable option for operational nonhydrostatic NWP (Bubnova et al., 1995). However, a fully-elastic global model empoying a z-based vertical coordinate, suitable for both operatonal NWP and climate simulation, has recently been developed at the UKMO (Davies et al., 2005).

The purpose of the present chapter is to introduce the different nonhydrostatic equation sets cast in pressure-based vertical coordinates. These include the fully-elastic equations based on the hydrostatic pressure field, and the MP model that is based on the full pressure field. Other variants discussed include the equations used in the MM5 model (Dudhia, 1993; Dudhia and Bresch, 2002), and an anelastic σ coordinate equation set that was developed from the MP model (Room et al., 2001). Most importantly, however, is an extension of the MP model in pressure coordinates by White (1989), that is presented in section 2.3.3. This equation set is discussed in much more detail in Chapter 3, and its

σ coordinate equivalent forms the basis of the new numerical model developed in Chapter 4. Against the background of the various types of pressure-based vertical coordinates that have been developed, the particular choice of vertical coordinate and associated equation set applied in the present study is motivated in section 2.4.

2.2 Fully elastic equations in pressure-based vertical coordinates

In this and the following sections, the various nonhydrostatic equation sets employing pressure-based coordinates are stated. The emphasis is on showing how the MP model and its extension by White (1989) are related to other nonhydrostatic pressure-based equation sets. For the sake of simplicity, the Coriolis, friction and diabatic terms are neglegted, and the motion is taken to be independent of the transverse coordinate y. In practical modelling applications, the Coriolis, friction and diabatic terms must be included and the motion generally also depends on y. Extension to these more general cases is straightforward and will not be described here. However, with meso-scale applications in mind, a more general three-dimensional quasi-elastic equation set including Coriolis terms is formulated in Chapter 3, for the specific choice of vertical coordinate used in the thesis.

2.2.1 The fully-elastic equations with the full pressure field as vertical coordinate

The fully-elastic equations (1.1) to (1.4) with geometric height as vertical coordinate, for a nonrotating, frictionless and adiabatic atmosphere, have been stated in chapter 1. These equations may be transformed to full pressure coordinates to obtain (Miller and White, 1984):

$$\frac{Du}{Dt} + (1+\epsilon)\left(\frac{\partial \phi}{\partial x}\right) = 0, \tag{2.1}$$

$$\frac{RT}{p} + (1+\epsilon)\left(\frac{\partial \phi}{\partial p}\right) = 0, \tag{2.2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial \omega}{\partial p} - \frac{D\ln(1+\epsilon)}{Dt} = 0, \tag{2.3}$$

$$\frac{DT}{Dt} - \kappa\frac{T}{p} = 0. \tag{2.4}$$

Here p denotes the full (true) pressure field, which may be nonydrostatic. $\omega = Dp/Dt$; $\phi = gz$ (the geopotential); $D/Dt = \partial/\partial t + u\partial/\partial x + \omega\partial/\partial p$; all partial differentiations with respect to x and t are carried out at constant p, whilst ϵ is a measure of the vertical acceleration defined as

$$\epsilon \equiv \frac{1}{g}\frac{Dw}{Dt} = \frac{1}{g^2}\frac{D\phi}{Dt}. \tag{2.5}$$

2.2.2 The fully-elastic equations in σ coordinates based on the full pressure field

The σ coordinate based on the full pressure field may be defined as

$$\sigma = \frac{p}{p_{surf}}, \tag{2.6}$$

whith p the full (possibly nonhydrostatic) pressure and p_{surf} the corresponding surface pressure. Following Miller and White (1984), (2.1) to (2.4), or alternatively (1.1) to (1.4), may be transformed to σ coordinates defined by (2.6) to obtain:

$$\frac{Du}{Dt} + (1+\epsilon)\left(\frac{\partial \phi}{\partial x}\right) + RT\left(\frac{\partial \ln p_{surf}}{\partial x}\right) = 0, \tag{2.7}$$

$$\frac{RT}{\sigma} + (1+\epsilon)\left(\frac{\partial \phi}{\partial \sigma}\right) = 0, \tag{2.8}$$

$$\frac{\partial u}{\partial x} + \frac{\partial \dot{\sigma}}{\partial \sigma} + \frac{D \ln p_{surf}}{Dt} - \frac{D \ln(1+\epsilon)}{Dt} = 0, \tag{2.9}$$

$$\frac{DT}{Dt} - \kappa T\left(\frac{\dot{\sigma}}{\sigma} + \frac{D \ln p_{surf}}{DT}\right) = 0. \tag{2.10}$$

Here

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + \dot{\sigma}\frac{\partial}{\partial \sigma}; \tag{2.11}$$

all partial derivatives with respect to t and x are carried out at constant σ and $\dot{\sigma} = D\sigma/Dt$, whilst $\phi = gz$ is the geopotential; ϵ is defined by (2.5).

It would be possible to develop a fully-elastic nonhydrostatic model that uses the full pressure field, or a terrain following coordinate based on the full pressure field, as vertical coordinate. However, such a coordinate runs the risk of becoming singular in cases of severe vertical accelerations (Laprise, 1992; Bubnova et al., 1995; Laprise, 1998). Therefore, a fully-elastic numerical model using a vertical coordinate based on the full pressure field has never been realized.

2.2.3 The fully-elastic equations in σ coordinates based on the hydrostatic pressure field

The use of the hydrostatic pressure as vertical coordinate for fully elastic models was first suggested by Laprise (1992). Using the hydrostatic pressure as vertical coordinate avoids the potential singularity of coordinates based on the full pressure field. This coordinate system has the additional advantage that it leads to a formulation of the fully elastic equations that closely parallels that of the hydrostatic primitive equation models using pressure based coordinates (Laprise, 1992). Thus, by the choice of hydrostatic pressure as vertical coordinate, the conversion of a hydrostatic model formulated in terms of the pressure field to a nonhydrostatic model can be conveniently achieved (Laprise, 1992; Bubnova et al., 1995; Gallus and Rancic, 1996).

Laprise (1992) and Bubnova et al. (1995) stated a form of the fully elastic equations in a generalized vertical coordinate that is monotonically linked to the pressure that would have occurred, should the atmosphere have been in hydrostatic equilibrium. An alternative formulation, directly based on the hydrostatic pressure as vertical coordinate, is given by Janjic et al. (2001). In the latter formulation, the σ coordinate is defined by

$$\sigma = \frac{\pi - \pi_T}{\pi_{surf} - \pi_T}. \tag{2.12}$$

Here π represents the "actual" hydrostatic pressure that would have occurred, should the atmosphere have been in hydrostatic equilibrium. Thus, π is a function of space and time, whilst π_{surf} and π_T are the hydrostatic pressures at the surface and at the top of the model atmosphere. In terms of the vertical coordinate defined by (2.12) the formulation of the fully-elastic, two-dimensional equations for an adiabatic, frictionless, nonrotational atmosphere is (see Janjic et al., 2001):

$$\frac{Du}{Dt} = -\left(1+\epsilon\right)\frac{\partial \phi}{\partial x} - \alpha\frac{\partial p}{\partial x}, \tag{2.13}$$

$$\frac{Dw}{Dt} = g\left(\frac{\partial p}{\partial \pi} - 1\right), \tag{2.14}$$

$$\frac{D\mu}{Dt} + \mu\left(\frac{\partial u}{\partial x} + \frac{\partial \dot{\sigma}}{\partial \sigma}\right) = 0, \tag{2.15}$$

$$c_p\frac{DT}{Dt} = \alpha\frac{Dp}{Dt}. \tag{2.16}$$

Here (2.13) is the horizontal momentum equation, (2.14) the vertical momentum equation, (2.15) is the continuity equation and (2.16) the thermodynamic energy equation. As before, ϵ is defined by (2.5). The symbols for the full (nonhydrostatic) and hydrostatic pressures are p and π, respectively. The total derivative is defined by (2.11) , but with σ defined by (2.12). The vertical velocity may be written as $w \equiv (1/g)\, D\phi/Dt$. All other variables have their usual meaning.

The fully elastic equations based on the hydrostatic pressure field have been successfully implemented in the limited-area models Aire Limitee Adaptation Dynamique Dyveloppement International (ALADIN) in France (Bubnova et al., 1995) and ETA in the USA (Gallus and Rancic, 1996). A new model based on these equations was developed by Janjic et al. (2001). The even more recently developed Weather Research and Forecasting model (WRF) also employs the fully elastic equations cast in the σcoordinate based on the hydrostatic pressure (Skamarock et al., 2005).

The form of the vertical coordinate used in the fully-elastic MM5 model may finally be noted. Here a σ coordinate based on a constant in time, hydrostatic mean background pressure field is used (Dudhia, 1993). The regional MM5 model has primarily been used for short-range NWP and meso-scale studies (Dudhia, 1993). Bubnova et al. (1995) have stated that the choice of vertical coordinate used in the MM5 model may limit the application of the model to relatively short time scales over relatively small domains. This limitation is due to the constant in time σ coordinate, that depends on a mean temperature profile through the link between the mean reference hydrostatic surface presure and the terrain height (Bubnova et al., 1995). Long integration periods over large areas may involve a great change of air masses, for which a coordinate system depending on a mean temperature profile is not suitable (Bubnova et al., 1995). However, it should be noted that a global version of the MM5 model has been developed (Dudhia and Bresch, 2002). Employing the same vertical coordinate based on a hydrostatic mean background pressure as the regional version of the model, the global model has been used for real-time medium range forecasting (Dudhia and Bresch, 2002). In these experiments the model used a horizontal resolution of about 128 km, which is well within the limits of validity of the hydrostatic approximation. It remains to be clarified whether using the σ coordinate based on the hydrostatic mean background pressure limits the spatial and time scales for which the model may be applied.

2.3 Approximated nonhydrostatic equation sets based on the full pressure field

The suspicion that the relationship between the full pressure field and height may not always be monotone, has prevented the development of a numerical model that employes the fully-elastic equations based on the full pressure field. However, a group of approximated nonhydrostatic equation sets based on the full pressure field have been under development since the 1970s, and have been applied successfully in meso-scale numerical models. For these approximated equation sets, the potential singularity of a vertical coordinate based on the full pressure field is apparently not reached. In this section, these equation sets are briefly reviewed in stated. For the purpose of introducing the various approximated equation sets, it is sufficient to work in two spatial dimensions as before. The new (three-dimensional) numerical model that is developed in Chapters 3 to 5, is based on an equation set that is closely related to the equation sets introduced in this section.

2.3.1 The Miller-Pearce model

Miller (1974) developed the first nonhydrostatic atmospheric model that employed a pressure-based vertical coordinate. This model uses the full pressure field as vertical coordinate, and was initially derived from a scale analysis of the fully-elastic equations based on the full pressure field.

Following the analysis of Miller (1974), a reference state which is a function of pressure only and is in hydrostatic balance may be defined:

$$\frac{d\phi_{ref}}{dp} = -\frac{RT_{ref}}{p}. \tag{2.17}$$

The thermodynamic variables may be expressed as deviations from a reference state as follows

$$\phi = \phi_{ref} + \phi^{'}(x, p, t), \tag{2.18}$$

$$T = T_{ref} + T^{'}(x, p, t). \tag{2.19}$$

Writing

$$\phi = gz \tag{2.20}$$

as before, it follows that

$$z = z_{ref} + z^{'}. \tag{2.21}$$

As before, z represents the geometric height above sea level of a certain pressure level.

Expanding the fully-elastic equations (2.1) to (2.4) without approximation around the reference state gives (Miller and White, 1984):

$$\frac{Du}{Dt} = -(1+\epsilon)\frac{\partial\phi^{'}}{\partial x}, \tag{2.22}$$

$$-\frac{RT^{'}}{p} = \frac{\partial\phi^{'}}{\partial p} + \epsilon\left(\frac{-RT_{ref}}{p} + \frac{\partial\phi^{'}}{\partial p}\right), \tag{2.23}$$

Table 2.1: Scale analysis of the horizontal momentum equation

Du/Dt	$=$	$-\partial\phi^{'}/\partial x$	$-\epsilon\partial\phi^{'}/\partial x$
$\tilde{u}^2/L$		$gH^{'}/L$	$\left(gH^{'}/L\right)\left(\tilde{H}\tilde{u}^2/L^2 g\right)$

$$\frac{\partial u}{\partial x}+\frac{\partial \omega}{\partial p}=D\left[\ln\left(1+\epsilon\right)\right]/Dt, \tag{2.24}$$

$$\frac{DT^{'}}{Dt}=\omega S_{ref}+\kappa\omega T^{'}/p. \tag{2.25}$$

Note that u, w and ω keep their original definitions since the reference state is defined as a state of no motion. $S_{ref}=S_{ref}\left(p\right)$ is the reference state static stability function and is defined as

$$S_{ref}=-\frac{dT_{ref}}{dp}+\kappa\frac{T_{ref}}{p}. \tag{2.26}$$

In his scale analysis, Miller (1974) considered the case where the vertical length scale $\tilde{H}$ and horizontal length scale L in the dynamical system are similar. That is,

$$\tilde{H}\sim L. \tag{2.27}$$

Let $\tilde{u}$, $\tilde{\omega}$ and $H^{'}$ be the characteristic values of u, ω and $z^{'}$. The pressure scale is denoted by P. It follows that

$$\frac{D}{Dt}\sim\frac{\tilde{u}}{L} \tag{2.28}$$

and

$$\epsilon\sim\frac{\tilde{H}}{g}\left(\frac{\tilde{u}}{L}\right)^2. \tag{2.29}$$

A scale analysis of the horizontal momentum equation (Table 2.1) and continuity equation (Table 2.2) gives

$$\tilde{u}^2\sim gH^{'} \tag{2.30}$$

and

$$\frac{\tilde{u}}{L}\sim\frac{\tilde{\omega}}{P}, \tag{2.31}$$

provided that the length scale of the system is much less in magnitude than the horizontal velocity scale.

Table 2.2: Scale analysis of the continuity equation

$\partial u/\partial x$	$+\epsilon\partial u/\partial x$	$+\partial\omega/\partial p$	$+\epsilon\partial\omega/\partial p$	$=$	$D\epsilon/Dt$
$\tilde{u}/L$	$(\tilde{u}/L)\left(\tilde{H}\tilde{u}^2/L^2g\right)$	$\tilde{\omega}/P$	$\left(\tilde{H}\tilde{u}^2/L^2g\right)(\tilde{\omega}/P)$		$(\tilde{u}/L)\left(\tilde{H}\tilde{u}^2/L^2g\right)$

Table 2.3: Scale analysis of the vertical momentum equation

$(1/g^2)\frac{D}{Dt}\left(D\phi^{'}/DT\right)$	$(1/g^2)\,\omega^2\partial^2\phi_{ref}/\partial p^2$	$+\left(\partial\phi^{'}_{ref}/\partial p\right)D\omega/Dt$	$=$	$\chi-1$
$\left(H^{'}/L\right)^2$	$\left(H^{'}/L\right)\left(\tilde{H}/L\right)$	$\left(H^{'}/L\right)\left(\tilde{H}/L\right)$		$H^{'}/\tilde{H}$

In order to perform a scale analysis of the vertical momentum equation, it is convenient to rewrite (2.23) in the form

$$\frac{1}{g^2}\frac{D}{Dt}\left(\frac{D\phi^{'}}{Dt}\right)+\frac{1}{g^2}\omega^2\frac{\partial^2\phi_{ref}}{\partial p^2}+\frac{1}{g^2}\frac{\partial\phi_{ref}}{\partial p}\frac{D\omega}{Dt}=\chi-1, \qquad (2.32)$$

where

$$\chi=-\left(\frac{p}{RT}\frac{\partial\phi}{\partial p}\right)^{-1}. \qquad (2.33)$$

The scale analysis of the vertical momentum equation presented in Table 2.3 makes use of (2.30) and (2.31), with the additional assumption that

$$H^{'}\ll\tilde{H}. \qquad (2.34)$$

It follows that the first term in (2.32) may be neglected in comparison with the remaining terms.

From the scale analysis in (2.1) to (2.3) a consistent approximated equation set based on the full pressure field is is obtained:

$$\frac{Du}{Dt}=-\frac{\partial\phi^{'}}{\partial x}, \qquad (2.35)$$

$$\frac{R}{g}\frac{D}{Dt}\left(\omega T_{ref}/p\right)=-\left(gp/RT_{ref}\right)\frac{\partial\phi^{'}}{\partial p}-g\frac{T^{'}}{T_{ref}}, \qquad (2.36)$$

$$\frac{\partial u}{\partial x}+\frac{\partial\omega}{\partial p}=0, \qquad (2.37)$$

$$\frac{DT^{'}}{Dt} = \omega S_{ref} + \kappa\omega T^{'}/p. \tag{2.38}$$

Note that the unapproximated form of the thermodynamic energy equation is used, but that approximations have been introduced to the continuity and momentum equations. Note that (2.35) to (2.38) are formulated in terms of a reference temperature profile. An important property of (2.35) to (2.38) is that the equations are filtered of vertically propagating acoustic waves, whilst Lamb waves may be filtered by an appropriate choice of lower and upper boundary conditions (Miller, 1974). The equations may therefore be termed anelastic. During the explicit numerical solution of these equations, the complete absence of sound waves greatly alleviates the the restriction on the time step as imposed by the Courant-Friedrichs-Lewy (CFL) condition. The equation set is the p coordinate counterpart of the anelastic height-based equations of Ogura and Phillips (1962), although there is no precise mathematical equivalence (Miller and White, 1984). Miller and Pearce (1974) used (2.35) to (2.38) to study cumulonimbus convection, and the equations became known as the Miller-Pearce (MP) model.

Miller and White (1984) extended the work of Miller (1974) by deriving (2.35) to (2.38) from a formal power series expansion. They also derived the corresponding σ coordinate equations:

$$\frac{Du}{DT} + \frac{\partial\phi^{'}}{\partial x} - \sigma\left(\frac{\partial\phi^{'}}{\partial\sigma}\right)\frac{\partial\ln p_{surf}}{\partial x} = 0 \tag{2.39}$$

$$\frac{R}{g}\frac{D}{Dt}\left[T_{ref}\left(\frac{D\ln p_{surf}}{Dt} + \frac{\dot{\sigma}}{\sigma}\right)\right] + g\frac{T^{'}}{T_{ref}} + \frac{\sigma g}{RT_{ref}}\frac{\partial\phi^{'}}{\partial\sigma} = 0 \tag{2.40}$$

$$\frac{\partial u}{\partial x} + \frac{\partial\dot{\sigma}}{\partial\sigma} + \frac{D\ln p_{surf}}{Dt} = 0 \tag{2.41}$$

$$\frac{DT^{'}}{Dt} - \left(\frac{D\ln p_{surf}}{Dt} + \frac{\dot{\sigma}}{\sigma}\right)\left(p_{surf}\sigma S_{ref} + \kappa T^{'}\right) = 0. \tag{2.42}$$

Here σ is defined by (2.6) in terms of the full pressure field; the material derivative is as stated for the fully-elastic σ coordinate equations based on the full pressure field. Lamb waves are present in the solution set of (2.39) to (2.42) but vertically propagating sound waves are absent (Miller and White, 1984), rendering the equations quasi-elastic. The absence of vertically propagating sound waves alleviates the CFL stability restriction on time steps, especially when high vertical resolution is used (Xue and Thorpe, 1991). Equation set (2.39) to (2.42) is similar to the formulation of the anelastic equations in normalized height coordinates (Gal-Chen and Somerville, 1975), although no precise mathematical equivalence exists beteen the two sets (Miller and White, 1984). The MP model in p and σ coordinates has been used in numerous studies of the numerical simulation of convection and airflow over mountains (e.g. Miller and Pearce, 1974; Xue and Thorpe, 1991; Miranda and James, 1992). The formulation of the MP model in terms of a reference temperature profile may limit its application to the meso-scale, and more specifically to regions where the horizontal gradient on constant pressure levels is rather weak (White, 1989). More details on the applications, characteristics and limits of applicability of the MP model can be found in Chapter 3.

2.3.2 Anelastic terrain-following equations

The presence of Lamb waves in the σ coordinate form of the MP model implies computational penalties compared to the corresponding anelastic p coordinate equations (Miller and White, 1984). Room et al. (2001) have modified the MP model in σ coordinates in order to filter the Lamb waves in addition to the filtered vertically propagating acoustic waves. The main feature of the anelastic model formulated by Room et al. (2001) is that the lower boundary is given by a fixed pressure distribution $p_0(\mathbf{x})$ that is independent of time. This implies that the domain of the model is fixed in pressure coordinates:

$$p_T \leq p \leq p_0(\mathbf{x}). \tag{2.43}$$

Here p_T is the model top, which may be chosen as $p_T = 0$. The pressure p_0 is the mean background pressure at the surface, which may be calculated from the barometric formula (Room et al., 2001):

$$p_0(\mathbf{x}) = p_{surf\_ave} \exp^{(-g/R)\int_0^{h(\mathbf{x})}\left(1/T_{ref\_ave}(z)\right)dz}. \tag{2.44}$$

Here p_{surf_ave} is the mean sea-level pressure, R is the gas constant for dry air, g is the gravitational acceleration, T_{ref_ave} is the mean background temperature and $h(\mathbf{x})$ is the surface elevation.

The σ coordinate used is defined as

$$\sigma = \frac{p - p_T}{p_{sr}}, \tag{2.45}$$

where $p_{sr}(\mathbf{x}) = p_0(\mathbf{x}) - p_T$.

When the MP model in pressure coordinates (2.35) to (2.38) is tranformed to σ coordinates defined by (2.45), the equations of motion obtained are, with the exception of the continuity equation, the same as the MP model in σ coordinates. (Note, however, that the σ coordinate in the MP model is based on the full surface pressure field, which is a prognostic variable in the MP model). The total derivative of the surface pressure in σ coordinates defined by (2.45) reduces to

$$\frac{D \ln p_{sr}}{Dt} = u \frac{\partial \ln p_{sr}}{\partial x}, \tag{2.46}$$

and the continuity equation transforms to

$$\frac{\partial u}{\partial x} + \frac{\partial \dot{\sigma}}{\partial \sigma} + u \frac{\partial \ln p_{sr}}{\partial x} = 0. \tag{2.47}$$

Writing (2.47) in flux form and integrating from the model top to the model surface gives

$$\nabla \cdot \left(p_{sr} \int_0^1 u d\sigma \right) = 0. \tag{2.48}$$

Equation (2.48) replaces the surface pressure tendency equation that arises in the MP model in σ coordinates (see Chapter 3). Room et al. (2001) have shown that the Lamb waves are filtered by the application of (2.48) in combination with fixing the lower boundary p_0. The complete removal of acoustic modes implies a significant computational advantage over the σ coordinate MP model, especially when an explicit procedure is used to solve the equations (Room et al., 2001). The set of anelastic σ coordinate equations has been termed the nonhydrostatic adjusted dynamics (NHAD) model (Room et al., 2001).

A potential disadvantage of the NHAD model may result from its main feature, the fixed surface pressure profile. Some meso-scale and synoptic scale weather systems may cause large variations in surface pressure from the mean barometric state (2.44). For example, in cases such as these, the pressure $p_0(\mathbf{x})$ may be greater than the actual surface pressure. This implies that calculations at the lowest model level will be carried out below the actual terrain height, which is an unphysical situation. In order to perform calculations at the model level(s) below the actual terrain height, assumptions on certain variables need to be introduced. For example, a temperature and geopotential profile "below ground level" will need to be introduced. In the case where $p_0(x)$ is less than the actual surface pressure, the atmospheric layer below $p_0(x)$ will not be represented in the model. Thus, the use of the $p_0(x)$ profile may limit the application of the NHAD model to meso-scale circulation systems where variations in the surface pressure from the mean barometric state are small.

In the hydrostatic σ coordinate models and nonhydrostatic models based on the full pressure field (such as the MP model) the σ coordinate used is based on the actual surface pressure simulated by the model. The surface $\sigma = 1$ therefore always follow model terrain height precisely. It is interesting to note that this is not the case for the fully-elastic models based on the actual hydrostatic pressure field. Here nonhydrostatic effects may cause the actual surface pressure simulated to be less or greater than the simulated hydrostatic surface pressure. However, these variations of the actual surface pressure from the hydrostatic surface pressure are likely to be less than variations from the average barometric pressure. Thus, a problem similar to that described above for the NHAD model, although probably of much smaller significance, may occur in the fully-elastic models based on the actual hydrostatic presure. However, for fully-elastic models such as the MM5, where the vertical coordinate is based on the mean background hydrostatic pressure, problems at the lower boundary similar to that described for the NHAD model may be expected (see also Bubnova et al., 1995 and section 2.2.3).

2.3.3 White's extension of the MP Model

Both the MP model and Room's anelastic σ coordinate equations are formulated in terms of a reference temperature and geopotential profile. This may limit the application of these models to situations where the temperature profile is rather homogeneous (White, 1989). An extended p coordinate equation set based on the MP model, but formulated independently from a reference profile, was developed by White (1989):

$$\frac{Du}{Dt} + \frac{\partial \phi}{\partial x} = 0, \tag{2.49}$$

$$\frac{R}{g}\frac{D}{Dt}(\frac{\omega T}{p}) + g + \frac{gp}{RT}\frac{\partial \phi}{\partial p} = 0, \tag{2.50}$$

$$\frac{\partial u}{\partial x} + \frac{\partial \omega}{\partial p} = 0, \tag{2.51}$$

$$\frac{DT}{Dt} - \kappa\frac{\omega T}{p} = 0. \tag{2.52}$$

Here p represents the full pressure field as in the MP model, and all the other symbols have their usual meaning. White's equation set is discussed in more detail in Chapter 3. The equation set appears to be more suitable than the MP and NHAD models for application in regions such as frontal zones, where the horizontal temperature gradient is steep. It may possibly also be applied in large-scale modelling (White, 1989) (as opposed to meso-scale modelling). White's equations appear not to have been used before in a numerical atmospheric model. In Chapter 3 the p coordinate equations of White are transformed to σ coordinates. It is shown that the resulting equation set is closely linked to the MP model in σ coordinates, in the sense that it is also quasi-elastic. In Chapter 4, a new meso-scale numerical model based on the σ coordinate equations of White is formulated.

2.4 Discussion

In this chapter the various pressure-based nonhydrostatic equation sets developed have been discussed. The numerical models based on these equation sets may be divided into the following four groups (Room et al., 2001):

1. The MP model that employs the full (nonhydrostatic) pressure as the vertical coordinate (Miller, 1974; Miller and Pearce, 1974; Miller and White, 1984). The MP model contains approximations that make it anelastic in pressure coordinates and quasi-elastic in σ coordinates. There have been numerous applications of the MP model to the numerical simulation of convection and airflow over mountains (e.g. Miller and Pearce, 1974; Xue and Thorpe, 1991; Miranda and James, 1992). The formulation of the MP model in terms of a reference thermodynamic profile, that depends on pressure alone, may limit its application to cases where the horizontal temperature gradient on pressure surfaces is weak (White, 1989). The properties of the MP model are discussed in more detail in Chapter 3.

2. Fully-elastic models that employ the hydrostatic component of the pressure field as the vertical coordinate (Laprise 1992). These equations and coordinate system have been implemented in the Meteo-France operational limited-area model, ALADIN (Bubnova et al., 1995), the nonhydrostatic version of the ETA model (Gallus and Rancic, 1996), the model developed by Janjic and Gerrity (2001) and the WRF model (Skamarock et al., 2005).

3. Models which use the hydrostatic mean background field as the vertical coordinate. This coordinate frame is employed in the nonhydrostatic extension of the Penn-State National Center for Atmospheric Research model (Dudhia 1993, Dudhia and Bresch, 2002).

4. Models that may be regarded as hybrids of the three groups described. Most noteworthy is the model of Room et al. (2001), which makes use of the actual pressure field and a mean background field of surface pressure to define a terrain-following vertical coordinate. This model is unique in the sense that it is the only anelastic σ coordinate model that has been developed so far. However, its formulation in terms of a constant-in-time surface pressure distribution and a reference thermodynamic profile is likely to limit its application to the meso-scale.

There has been no realization of a numerical model using the fully-elastic equations based on the full pressure field. This is due to the suspicion that such a coordinate might become singular in the presence of severe vertical advection (Laprise, 1992, 1998; Bubnova et al., 1995). It appears also as if White's (1989) extension of the MP model in pressure coordinates has not been realized as a numerical model. This equation set is likely to be more generally applicable than the MP model (see section 2.3.3 and Chapter 3). In Chapter 3, this equation set is transformed to σ coordinates, and a new numerical model based on the σ coordinate equations is formulated in Chapter 4.