Theory and application of quasi-elastic equations in terrain-following coordinates based on the full pressure field

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Summary

The thesis reports on the development of a new quasi-elastic nonhydrostatic model, cast in a terrain-following coordinate based on the full pressure field. The equations used are the σ coordinate analogue of the nonhydrostatic pressure coordinate equations formulated by White (1989). The equations are filtered of vertically propagating acoustic waves. However, since Lamb waves are present, the equations may be termed quasi-elastic. In contrast to similar quasi-elastic pressure-based models, the equations and the numerical solution procedure presented here are formulated independent of the use of a reference state thermodynamic profile. Thus, it is possible that the equations may be used to simulate atmospheric motion at spatial scales larger than the meso-scale.

A novel split semi-Lagrangian procedure is formulated to solve the quasi-elastic equations on a grid that is nonstaggered in both the horizontal and vertical. A nonstaggered grid is appealing to use in semi-Lagrangian discretizations of the atmospheric equations, since only one set of trajectories needs to be calculated during each advection time step. However, it is well known that the nonstaggered grid has poor gravity wave dispersion properties. In this study, this problem is alleviated by using high-order centered spatial differencing, and by applying a spatial filter to remove two-grid-interval waves from the grid. It is shown that large time steps (large Courant numbers) are allowed during the semi-Lagrangian advection step. This makes the method computationally attractive compared to explicit or split-explicit procedures that use an Eulerian approach to treat the advection terms. For situations where the fast moving gravity waves carry a non-negligible amount of the energy, the split semi-Lagrangian approach may even be computationally more efficient than the widely used semi-implicit semi-Lagrangian solution procedures. The thesis reports on a large set of bubble convection tests performed with the new kernel. It is concluded that the new model is worth developing further.

Teorie en toepassing van kwasi-elastiese vergelykings in terrein-volgende koordinate gebasseer op die volle drukveld

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Samevatting

Die proefskrif handel oor die ontwikkelling van 'n nuwe kwasi-elastiese nie-hidrostatiese model, in 'n terrein-volgende koordinaat gebasseer op die volle drukveld. Die vergelykings wat gebruik word is die σ-koordinaat analoog van die nie-hidrostatiese drukkoordinaat vergelykings geformuleer deur White (1989). Die vergelykings is gefilter van vertikaal voortplantende klankgolwe. Lamb-golwe is egter teenwoordig en daarom kan die vergelykings kwasi-elasties genoem word. In kontras met soortgelyke kwasi-elastiese drukgebasseerde modelle, is die vergelykings wat hier gebruik word onafhanklik van die gebruik van 'n termodinamiese verwysingsprofiel. Dit is dus moontlik dat die vergelykings gebruik kan word om atmosferiese sirkulasie op ruimtelike skale groter as die meso-skaal te simuleer.

'n Oorspronklike split semi-Lagrange prosedure is geformuleer om die kwasi-elastiese vergelykings op te los op 'n rooster wat in die horisontaal en vertikaal nie-verspringend is. So 'n rooster is aanloklik om te gebruik in die semi-Lagrangian diskretisering van die atmosferiese vergelykings, aangesien dit nodig is om net 'n enkele stel trajekte te bereken gedurende elke adveksie tydstap. Dit is egter welbekend dat die nie-verspringende rooster swak gravitasiegolf dispersie eienskappe het. In die studie word hierdie probleem hanteer deur hoe orde differensiasie te gebruik en deur 'n ruimtelike filter toe te pas wat twee-rooster-interval golwe van die rooster verwyder. Dit word aangetoon dat groot tydstape (groot Courant getalle) toegelaat word gedurende die semi-Lagrange adveksie stap. Dit maak die metode berekeningsgewys aantreklik in vergelyking met eksplisiete en split-eksplisiete prosedures wat 'n Euler benadering gebruik vir die adveksie terme. Vir situasies waar die vinnigbewegende gravitasie golwe 'n nie-weglaatbare hoeveelheid van die energie dra, kan die split semi-Lagrange benadering selfs meer berekeningseffektief wees as die gewilde semi-implisiete semi-Lagrange prosedures. 'n Groot reeks borrel konveksie eksperimente is uitgevoer met die nuwe kern en dit blyk die moeite werd te wees om die nuwe model verder te ontwikkel.

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Contents

List of Symbols

$\hat{a}_{\hat{u}}$	x component of the estimated acceleration
$\hat{a}_{\hat{v}}$	y component of the estimated acceleration
$\hat{a}_{\hat{\dot{\sigma}}}$	σ component of the estimated acceleration
c	phase speed
c_s	speed of sound
c_p	specific heat at constant air pressure
c_v	specific heat at constant air volume
f	Coriolis parameter
g	acceleration of gravity
h	surface elevation
	depth of fluid in the shallow-water equations (section 4.3.3)
H	mean depth of fluid in the shallow-water equations
$\tilde{H}$	vertical length scale
$H^{'}$	scale of the height perturbation
H_0	RT_0/g
k	wave number in x direction
K_s	horizontal diffusion coefficient applied to wind field
K_{Ts}	horizontal diffusion coefficient applied to temperature field
K_σ	vertical diffusion coefficient applied to wind field
$K_{T\sigma}$	vertical diffusion coefficient applied to temperature field
l	wave number in y direction
L	horizontal length scale
L_x	wave length in x direction
L_y	wave length in y direction
L_z	wave length in z direction
m	wave number in z direction
N	$\equiv \sqrt{g\kappa/H_0}$

p	pressure
p_{surf}	surface presure
p_{surf_ref}	reference surface pressure
p_{surf_ave}	mean sea-level pressure
p_0	$\equiv p_{surf\_ref} - p_T$
p_T	constant pressure at top of model
p_s	$\equiv p_{surf} - p_T$
$\hat{p}_s$	amplitude of wave-like solution for p_s
$p_0(\mathbf{x})$	pressure at lower boundary that is independent of time but may depend on horizontal position
p_{sr}	$\equiv p_0(\mathbf{x}) - p_T$
p_{STAN}	standard pressure level
P	pressure scale
r	$\equiv gp/RT$
r_{ref}	$\equiv gp/RT_{ref}$
r_l	relaxation coefficient
$\mathbf{r}$	vector moving with fluid
R	gas constant
R_A	relative phase speed
s	$\equiv (p/p_s)(g/RT)$
S_{ref}	$\equiv -dT_{ref}/dp + \kappa T_{ref}/p$, reference state static stability function
t	time
τ	time
T	temperature
T'	temperature perturbation
$\hat{T}'$	amplitude of wave-like solution for the temperature perturbation
T_0	temperature of isothermal atmosphere
T_{ref}	reference state temperature

T_{ref_ave}	mean reference state temperature
u	wind speed in x direction
$\hat{u}$	amplitude of wave-like solution for u (Chapter 3);
	estimated wind speed in x direction at time-level $\tau + \Delta t/2$ (Chapter 4)
$\tilde{u}$	horizontal velocity scale
U	constant wind speed in x direction
$\mathbf{u}$	three-dimensional velocity vector
$\hat{\mathbf{u}}$	estimated velocity at time-level $\tau + \Delta t/2$
v	wind speed in y direction
$\hat{v}$	estimated wind speed in y direction at time-level $\tau + \Delta t/2$
V	constant wind speed in y direction
$\mathbf{v}$	$\equiv (u, v)$, horizontal velocity.
w	wind speed in z direction
$\tilde{w}_{ref}$	$\equiv -R\omega T_{ref}/gp$, approximated vertical velocity
$\hat{w}$	$\equiv -R\omega T/gp$, approximated vertical velocity
x^*	x coordinate of departure point
y^*	y coordinate of departure point
z	geometric height
$z^{'}$	geometric height perturbation
z_{ref}	reference state geometric height
Δx	constant grid increment along x axis
Δy	constant grid increment along y axis
Δt	constant time-step
Δt_s	advection time-step
Δt_a	adjustment time-step
$\Delta\sigma$	constant grid increment along σ axis
α	specific volume (Chapter 1);

	$\equiv u\Delta t/\Delta x$ (Chapter 4 and 5)
α^*	$\equiv -\left(p_0/\theta_{ref}\right) d\theta_{ref}/dp$
β	$\equiv v\Delta t/\Delta y$
γ	c_p/c_v
ϵ	$\equiv (1/g)\, DW/Dt$, measure of the vertical acceleration
ζ	vertical component of the vorticity
θ	potential temperature
θ'	potential temperature perturbation
θ_0	homogeneous reference state potential temperature
θ_{ref}	reference state potential temperature
ϑ	numerical value of the local frequency
ϑ_T	analytic local frequency
κ	$\equiv R/c_p$, ratio of gas constant to specific heat at constant pressure
λ	latitude (Chapter 1);
	amplification factor (Chapter 5)
μ	wave number in z direction
ν	frequency of oscillation
π	hydrostatic pressure (Chapter 2);
	$\equiv \arccos(-1)$ (Chapter 3 to 5)
π_{surf}	hydrostatic pressure at the surface
π_T	hydrostatic pressure at model top
$\hat{\pi}$	$\equiv \hat{p}_s/p_0$
ρ	density
ρ_0	constant basic state density
ρ_{ref}	reference state density
σ	pressure-scaled vertical coordinate
σ^*	σ coordinate of departure point
$\dot{\sigma}$	$\equiv D\sigma/Dt$, vertical velocity in σ coordinates

$\hat{\dot{\sigma}}$	amplitude of wave-like solution for $\dot{\sigma}$ (Chapter 3);
	estimated vertical velocity in σ coordinates at time-level $\tau + \Delta t/2$ (Chapter 4)
ϕ	geopotential
$\phi^{'}$	geopotential perturbation
$\hat{\phi}'$	amplitude of wave-like solution for geopotential perturbation
ϕ_{ref}	reference state geopotential
χ	$\equiv \left[(p/RT)\,(\partial\phi/\partial p)\right]^{-1}$
ω	$\equiv Dp/Dt$, vertical velocity in isobaric coordinates
$\tilde{\omega}$	ω scale
ω_T	gravity wave frequency
Ω	ω/p
Ω_T	true frequency
Ω_N	frequency of waves in the numerical solution

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List of Abbreviations

AGCM	Atmospheric general circulation model
ALADIN	Aire Limitee Adaptation Dynamique Dyveloppement International
CAT	Clear air turbulence
C-CAM	Conformal-cubic atmospheric model
CFL	Courant-Friedrichs-Lewy
CSIR	Center for Scientific and Industrial Research
CSIRO	Commonwealth Scientific and Industrial Research Organisation
DARLAM	Division of Atmospheric Research Limited Area Model
HC	Horizontal advection Courant number
VC	Verical advection Courant number
MM5	Fith-generation Pennsylvania State University-National Center for Atmospheric Research Meso-scale Model
MP	Miller-Pearce
NCAR	National Center for Atmospheric Research
NCEP	National Centers for Environmental Prediction
NWP	Numerical weather prediction
PC	Personal Computer
RAMS	Regional Atmospheric Modelling System
SAWB	South African Weather Bureau
SAWS	South African Weather Service
SAST	South African Standard Time
SOR	Successive Over-Relaxation
UKMO	United Kingdom Meteorological Office
UCT	University of Cape Town
UP	University of Pretoria
WITS	University of the Witwatersrand
WRC	Water Research Commission