

Chapter 4: Theoretical Framework

Many studies have linked retirement funds to economic activity through its impact on saving, the labour market and the performance of the economy. Kotlikoff (1979, 1987) distinguishes the effects of saving for retirement programs in Keynesian and Barro models and the saving effects in the life cycle model. He argues that in the Keynesian model where individuals consume the same fraction (constant marginal propensity to consume) of disposable income, retirement programs have no saving effects in the case where transfers from young to old are being made, since it does not change aggregate consumption.

In Barro's (1974) model of intergenerational altruism¹⁹, retirement funds are also predicted not to affect saving. In this model the altruistic motive passes through generations, with individuals alive at present acting as if they will live infinitely, connected by a chain of operative intergenerational transfers. By acting as if they will live indefinitely, these agents maximise their utility functions, taking into account the utility of future generations (or of an entire dynasty in the Becker and Barro, 1988 terminology). In other words they internalise the utility functions of all their descendants. The solution to this optimisation problem holds that the consumption of a member of generation i depends on bequests received from the older generation, the labour income it receives and the rate of return of assets. Consumption is not affected by the introduction of social security and saving is thus also not affected. Hence, in the Barro (1974) model, changes in social security of generation i are just offset by increased bequests²⁰.

The effects of social security on saving indicate that changes in economic activity can only be caused by changes in productivity. In particular, the Barro model assumes the golden rule of the Samuelson (1958) and Diamond (1965) models, where only factor rewards and population changes can alter the level of equilibrium output. Under all these conditions,

¹⁹ See also the model of a dynastic family Becker and Barro (1986 and 1988) or of a safe family insurance setup model of Ehrlich and Lui (1998).

²⁰ See also Chapter 2, section 2.2.3

the level of saving is equal to that required to maintain the level of capital-labour ratio compatible to the general equilibrium or steady state of the economy.

4.1 Financing and Managing Retirement Programs

Since retirement contributions accumulated²¹ are not sufficient to finance the benefits promised to retirees, social security programs are mostly financed through taxes levied on the income of the current working population. Employers also contribute a significant share depending on the country's legislation²². The traditional PAYG system is financed in this way. In a model of balanced growth or golden rule, this represents an efficient method of intergenerational transfers of wealth. The present value of the individual's contributions (taxes), equals the net present value of benefits received. However, in practice the present value of an individual's contributions is less than the present value of an individual's benefits, implying that the economy is not growing at its steady state growth rate. Omitting other factors such as race, sex, number of children, marital status, etc. as in Bennett (1979), the relationship between taxes paid and benefits received can be written as:

$$\text{Present value of benefits} = \sum_{t=1}^n B(1+r)^{-t} > \text{present value of taxes} = \sum_{t=1}^n T(1+r)^{-t}$$

Where: B = benefits; T = taxes paid; r = interest rate and t is the number of years the individual contributes to social security after entering the labour market.

Given a production function with constant returns to scale, where inputs comprise labour and capital, if $r \phi n + F_l(K, L)$ the employee will receive lower returns than when the contributions were invested in alternative assets. In this case, n is the growth rate of the population and $F_l(\cdot)$ is the labour productivity growth.

²¹ In practice in a pure pay-as-you-go system there is no real accumulation of contributions since contributions from the young generation (currently working individuals) are used to finance the benefits of the old generation (the currently retired individuals).

²² See also Annexure 2 of Chapter 3 for legislation on the contributions to retirement funds.

Furthermore, the population grows at different rates during different stages of development. This causes the funds required to pay for benefits to be far less than the amount that is actually contributed, thus putting strain on the soundness of the PAYG system of financing retirement programs. When these programs are financed by an unfunded PAYG system, the trust fund can only pay a few months of benefits. In order to compensate for the balance required, current taxes are used to finance current benefits, which in the case of demographic changes, exerts pressure on government to search for alternative methods of financing the deficit in retirement expenditure.

By contrast, in a funded retirement system, each individual contributes an amount to a trust fund, which is “just sufficient to meet all its future insurance obligations arising from the past contributions of all currently living workers and retirees” (Kotlikoff, 1987). This method of financing retirement programs by providing workers with the monetary value of annuities and insurance benefits, equal to their contributions, is considered fairly based.

It was mentioned that the rate of return (r) on retirement funds’ investment in a steady state economy should equal the sum of the population growth rate (n) plus the productivity growth rate ($F_l(K, L) = g$):

$$r = n + g$$

Considering that individuals live in two periods, the first period being the working period (with younger people contributing to retirement funds as in Diamond, 1965) and the second when individuals retire (for simplicity of exposition the possibility that an individual can work part of his/her second period is ruled out)²³. In period 1 a young person contributes a fraction τ of his/her earnings to social security and the benefits he/she receive as social security when retired at time $t + 1$ are given as:

²³ For a model with individuals working part of his/her second period see, Hu (1979), vol. 69(3), where the in the second period an individual works a fraction $(1 - \alpha_{t+1})$ of the time and then retire. His/her net earnings in this period are: $(1 - \alpha_{t+1})(W_{t+1} - T_{t+1})$ with a pension of $\alpha_{t+1}P$.

$$B = \tau W_t(1+n)(1+g)$$

Where: $(1+n)$ is the increase in number of workers for each beneficiary in period 2 and $(1+g)$ is the increase in the earnings of each worker in period 2 relative to period 1.

Now consider that the amount channelled to social security (τW_t) is channelled to alternative savings, earning the steady state interest rate (r), then in period $t+1$ the individual could have received $\tau W_t(1+r)$. By comparing the two rates of return one could state that if $r < n + F_l(K, L)$ the worker is worse off under social security, that is the substitution effects or opportunity cost between the two is high. In this analysis the factor rewards are ignored, which may alter the final results of the impact of social security programs. If $r > n + F_l(K, L)$, the introduction of unfunded social security can be a Pareto improvement. Thus, in this chain, all future generations benefit since there is an infinite number of participants, namely all future generations, as in Barro (1974) and Becker and Barro (1986 and 1988). However, if $r = n + F_l(K, L)$, Pareto improvement no longer exists.

The way in which retirement programs are financed, raises the problem of how they should be managed and who should provide such retirement schemes. According to Hagemeyer (2000) the providers of retirement funds can be public or private institutions. These institutions can be classified as social security funds, central, state or local governments, autonomous and self-administered pension funds, insurance companies, mutual benefit societies, public and private employers and private welfare and assistance institutions. In this study the focus is on retirement funds, managed by public and private institutions. Since panel data econometric techniques are being used and different countries have different administrators of their retirement systems, a distinction is only made between public and private administered funds.

Based on available literature, the way in which retirement systems are being managed have implications for the performance of the economy and more fundamentally, on the behaviour of workers' consumption-saving decisions. In many countries, pension system

administrators are compelled by law to invest a certain percentage of retirement contributions in government bonds, in other words, a captive market based on the assumption that government acts as assessor. While government assets are less risky, it follows that other capital market assets are more risky²⁴. However, there is a trade-off between investing in secure assets (such as government bonds, treasury bills and other lower risk assets) and more risky assets (stock market, firm's shares, etc.) since the latter allows individuals to enjoy higher returns on their contributions to retirement funds than the former. The decision by government to compel institutions managing social security funds to invest a significant portion of their members' contributions into government bonds and other assets with a lower market risk, could contribute to the prevention of moral hazard, which poses a threat to the stability of retirement schemes.

Thus, through regulation, government has the responsibility to ensure that workers' contributions are being managed in a fairly and safely. Along with a regulatory function, many governments also provide services, such as a means test, food stamps, health care, etc. to the elderly and disabled people.

4.2 The Effects of Retirement Funds in the Life Cycle Model

Since the overlapping generations model of Samuelson (1958) and the emergence of models of consumption in the life cycle of an individual by Modigliani and Brumberg (1954) and Ando and Modigliani (1963), many studies have been conducted to analyse the effects of these models on the real economy. The Ando and Modigliani (1963) model has served as a framework for many researchers in this field. These life cycle models predict that the introduction of retirement schemes reduces saving, since it reduces disposable income, given the fact that individuals tend to maintain their consumption patterns over their entire life span. Given the life cycle hypothesis predictions, it is possible to foresee that social security will have implications on the performance of the economy, if it is assumed to operate under neoclassical fundamentals.

²⁴ Chapter 3, section 2.5 discusses the different saving alternatives for old age.

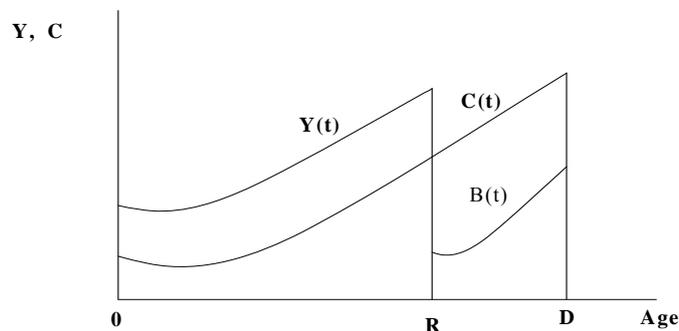
The common framework used in life cycle models is a simple world where individuals live in two periods. In the first period they work, pay taxes (assumed to be the contributions to social security only) and consume the net income of these taxes. In the second period, the individuals, when old, receive transfers (equivalent to taxes paid when young, plus returns) from the now young generation. The income received is consumed and for the sake of simplicity no bequests are considered.

It is assumed that social security returns are equal to the rate of interest if invested in alternative assets. This assumption requires that:

$$\int_0^R W_t \tau e^{-rt} dt = \int_R^D V_t e^{-rt} dt \quad (1)$$

Where: W is the income the individual receives during his/her working age; τ is the tax rate for funding retirement; V is the benefit received when the individual is retired; r is the interest rate, t represents the time dimension, $[0, R]$ is the duration of the working period interval and $[R, D]$ the interval during which the individual receives benefits. The Figure 6 illustrates this model.

Figure 6: Life-cycle of an individual contributing to an old age security



Where: C is consumption

Y is income

y(t) is the income of an individual during lifetime earnings, which is equivalent to W in the equation

c(t) is the entire lifetime consumption of an individual

B(t) is the benefits received during the retirement of an individual, which is equivalent to V in the equation

The utility and production functions in the neoclassical model are as follows:

$$\text{Utility function} \rightarrow U = C_1^\alpha C_2^\beta; \beta = 1 - \alpha \quad (2)$$

$$\text{Production function} \rightarrow Y = K_t^\rho L_t^{1-\rho} \quad (3)$$

Where: K is the capital stock of the economy, L is the labour force, C_1 and C_2 are consumption in periods 1 and 2, respectively, α , β , and ρ are the consumption and production elasticities, respectively.

The lifetime utility function (2) of the individual living in time t is a function of consumption in period 1 (when young) and period 2 (when old). Optimising the utility function, subject to a budget constraint:

$$C_1 + \frac{C_2}{1+r_2} = W_1(1-\tau_1) + \frac{V_2}{1+r_2} \quad (4)$$

yields the following consumption demand functions for the young individual:

$$C_1 = \beta \left[W_1(1-\tau_1) + \frac{V_2}{1+r_2} \right] \quad (5)$$

and the saving (supply of capital) of an individual of the young generation is:

$$S_2 = W_1(1 - \tau_1) - C_1 \quad (6)$$

alternatively this relationship can be expressed as:

$$S_2 = W_1(1 - \tau_1) - \beta \left[W_1(1 - \tau_1) + \frac{V_2}{1 + r_2} \right] \quad (6')$$

if assumed that social security is financed by a PAYG system, the revenue per young individual is represented by:

$$V_1 = \tau_1 W_1(1 + n) \quad (7)$$

Where: n is the population growth rate.

Given the revenue per young individual we can re-write (6') as:

$$S_2 = W_1(1 - \tau_1) - \beta \left[W_1(1 - \tau_1) + \frac{\tau_1 W_1(1 + n)}{1 + r_2} \right] \quad (8)$$

The optimisation problem (3) of a representative firm in the economy yields the usual demand for factors of the type:

$$W_1 = (1 - \gamma) k_1^\gamma \quad (9)$$

$$r_1 = \gamma k_1^{\gamma-1} \quad (10)$$

The equilibrium condition in the capital market is given by:

$$K = \frac{S}{1 + n} \quad (11)$$

Combining the expressions above yields the general equilibrium changes in factor rewards, and thus the steady state of capital can be written as:

$$(1+n)k^* = w^*(1-\tau^*) - V\left[w^* - \frac{(r^*-n)\tau^*w^*}{1+r^*}\right] \quad (12)$$

This equation is comparable to the Samuelson (1975) equation:

$$(f(k) - rk) - C_1 = k(1+g) \quad (13)$$

Which corresponds to the level of saving of the young with a pool of savings from which, interest and principal are consumed by retired individuals. Substituting (9) into (12) and differentiating with respect to τ yields:

$$\frac{(1+n)\partial k^{1-\alpha}}{\partial \tau} = -(1-\alpha)V\left(1 + \frac{n - \alpha k^{\alpha-1}}{1 + \alpha k^{\alpha-1}}\right) \quad (14)$$

This implies that the introduction of unfunded retirement schemes crowds out the steady state capital stock. Intuitively it implies that the benefits paid to start up retirees (first receivers) raises consumption by the same amount as the benefits paid. Assuming no distortions in the economy, the loss in the present value of benefits of the initial young generation as a result of the implementation of retirement schemes is $\frac{V(r-n)}{(1+r)}$ which is equivalent to a fraction of the reduction in consumption of the young generation. If assumed that the benefits paid ($V(1+n)$) are greater than the fractional reduction in the consumption of the young, total consumption in the initial period increases and saving is crowded out, leading to a new steady state with lower levels of saving.

In a general equilibrium situation, low levels of capital lead to a drop in wages, since the capital-labour ratio declines with increases in the interest rate. The latter benefits the older generation, while the present younger generation is worse off due to lower wages and investment is further crowded out due to the higher interest rate.

In this model social security affects the performance of the economy by constraining capital formation and interest rate increases, lowering the wages of the young generation

and thus reducing welfare. If generations are linked through a chain of intergenerational transfers, future generations will also be worse off, possibly resulting in a poverty trap.

4.3 Deficit Finance and Retirement Funds

The way the government deficit is financed is important to assess its effects on the capital market. If government has to borrow heavily it may crowd out investment, since such borrowing could result in increased interest rates. In models assuming that the provision of social security should only be funded by compulsory contributions, a lack of sufficient contributions can be explicitly considered as government debt policy (Kotlikoff, 1987). Thus, government borrows from the young to pay benefits to retired individuals, with the benefits exactly reflecting the principal (taxes paid when young) plus interest.

With government borrowing, the representative young generation can be portrayed as:

$$C_2 = S_1(1 + r_1) - T_1 \quad (15)$$

$$C_1 + \frac{C_2}{1 + r_2} = W_1 - \frac{T_2}{1 + r_2} \quad (16)$$

$$K_2 = \frac{(W_1 - C_1) - D_1}{1 + n} \quad (17)$$

$$T_1 = \tau_1 W_1(1 + r_2) - \tau_2 W_2(1 + n) \quad (18)$$

$$D_1 = \tau_1 W_1 \quad (19)$$

Where: D is the stock of official government debt to social security; T is the special tax levied on retirees and K_2 the funding per retiree.

The results show that financing retirement benefits through taxes or debt does not affect the economy differently; it reduces the supply of capital as can be seen from equation (17).

4.4 The Effects of Social Security on Fertility

The effects of social security on fertility and through this channel on saving and economic growth have been researched extensively theoretically and empirically. Unfortunately the results of the research are inconclusive because they are conflicting. This section explores the theoretical framework of the effects of social security on fertility in SSA countries based on the model from Boldrin, De Nardi and Jones (2005). This model initially assumes a PAYG system as is the case in most of the SSA countries, with agent i born in period $t-1$ maximising the following utility function:

$$U_{t-1} = u(c_t^m) + \zeta u(c_t^o) + \beta u(C_{t+1}^o) \quad (20)$$

Subject to the following constraints:

$$d_t^i + s_t + c_t^m + a_t n_t \leq (1 - \tau_t) \omega_t (1 - b_t n_t) \quad (21)$$

$$c_t^o \leq d_t^i + \sum_{\substack{j=1 \\ j \neq i}}^{j=n_{t-1}} d_t^j + (1 - \xi) R_t x_t + T_t^o \quad (22)$$

$$c_{t+1}^o \leq \sum_{j=1}^{j=n_t} d_{t+1}^j + (1 - \xi) R_{t+1} x_{t+1} + T_{t+1}^o \quad (23)$$

$$x_{t+1} \leq \xi R_t x_t / n_{t-1} + s_t \quad (24)$$

where: T_t^o is the benefit payment to retirees; τ_t is the social security tax rate; c_t^m is the consumption of the labour force (middle-aged person in the author's terminology) in period t ; c_t^o is the consumption of a retiree; s_t are the savings; n_t is the number of children; d_t^i is the level of support the agent i gives to his/her parents; x_t is the amount of capital stock each retiree controls in period t ; w_t is the wage rate; R_t is the gross return on capital in period t . If it is assumed that the retirement benefit T_t^o received when retired,

equals the funding thereof, and if taxes levied are: $(1 - \tau_t)w_t(1 - b_t n_t)$; then $T_t^o = n_{t-1} \tau_t w_t (1 - b_t n_t)$ and rewriting the budget constraint (21) we obtain:

$$d_t^i + s_t + c_t^m + \theta_t(\tau)n_t \leq (1 - \tau_t)w_t \quad (25)$$

The argument behind equilibrium equations (20-24) is that a representative agent i chooses the level of donation d to his parents so as to maximise his utility function (20). In the safe family social insurance setup (with no default) as in Ehrlich and Lui (1998), the optimisation problem implies that the elderly receive optimal transfers for consumption from their offspring. In this environment the introduction of social security will affect the choice of the number of children (n) and saving (s). The need for support from children decreases with improved social security, thereby reducing the level of donations required. But this reduction will depend on the level of compensation parents expect to receive from their children.

If social security benefits grow faster than the level of donation (d) received from children, parents will tend to reduce the number of children and increase investment in them, since the level of compensation or donation expected from children to parents may be correlated to the level of human capital accumulated. This reflects the choice regarding the number of children and quality referred to in the Ehrlich and Lui (1998) model, which implies that parents become more concerned about the quality of their children's education. In this model the level of support (compensation or donation) is related to the level of investment in human capital. A higher social security tax rate will initially increase the rate of return on the number of children relative to investment in human capital, because the added tax burden increases the ratio of emotional benefits relative to material compensation or donation from the children.

In SSA countries where families are still strongly connected by dynastic family structures as in Becker and Barro (1988), also known as the safe family insurance setup with no default as in Ehrlich and Lui (1998), social security may also negatively affect fertility, inducing parents to pay more attention to the quality of life rather than the number of their children.

However, the quality of education, lack of job opportunities, poverty and many other exogenous factors may constrain the performance of social security programs in these countries. This means that fertility rates fall much slower than in other regions in the world as parents still rely heavily on children as an important source of provision during retirement. Furthermore, the weaknesses of and highly imperfect financial markets²⁵ also hinder access to financial services to many African families, most of them living in rural areas. Boldrin, De Nardi and Jones (2005) argue that the development of financial markets reduces the value of within-family support during old age and, therefore, causes a decrease in fertility rates. As seen in many empirical studies (see the discussion in section 2.2.3) a negative relationship is expected between social security and fertility. However, the relationship is not as strong as in some other regions in the world because the weak financial markets would induce parents to choose other forms of retirement.

In many sub-Saharan African countries, children form part of the production function of a dynastic family or safe family insurance structure. They assist in breeding animals (mainly looked after by young boys) and crop farming and many other home tasks (predominantly the young girls). Even if parents are concerned about the education of their children, the great distances between schools and homes is a real constraint to the quality of children's education.

4.5 The Labour Market and Retirement Funds

Besides reducing the formation of capital as a result of rising interest rates, retirement funds also affect labour markets through income and substitution effects. The income effect is due to the fact that individuals expect windfall benefits that motivate them to retire early (interpreted as if their budget constraints have moved to a higher level). Payroll taxes induce the substitution effect between leisure and consumption and in some countries, means tested payments also lead to increased enjoyment of leisure. In fact, retirement programs may, to some extent, reduce the supply of labour not only of the elderly, but also of younger people who may choose to retire early. Boskin and Hurd

²⁵ See also section 3.5.3 in Chapter 3.

(1984) found such evidence in the United State. The reduction of the supply of labour of experienced individuals leads to economic inefficiency, lower productivity and increased costs due to the recruitment of new labour and their training.

Kotlikoff (1987) stresses that the linkages between tax payments and benefits received, and the way in which workers perceive these linkages, plays an important role in assessing the impact of such an increase in effective labour taxes. He further argues that even if the perception was correct, there would still be a considerable labour supply distortion.

These results suggest that economies are potentially exposed to a degree of voluntary unemployment, since individuals may choose to retire early. If this is the case the capital-labour ratio will rise freeing up capital. Wages will also rise, thereby increasing the opportunity cost of leisure and thus, eventually influencing the individual to choose work rather than retirement, depending, of course, on the utility derived from enjoying leisure. At the same time the rise in the capital-labour ratio (freeing up of capital) reduces the interest rate, which implies increased levels of investment. The final equilibrium of the economy will depend on the adjustment process back to the steady state equilibrium of the capital-labour ratio.

4.6 Main Insights and Concluding Remarks

This chapter investigated the theoretical effects of social security on economic variables. It argues that the way in which retirement programs are being managed has implications for the performance of the economy and fundamentally on the behaviour of workers' consumption-saving decisions. In this regard, social security financed through debt or taxes in the life cycle model will crowd-out the supply of capital. The final point of equilibrium of the economy will thus depend on the response of economic fundamentals, for example, the extent to which future generations will be affected by success of saving of the current generation.

The chapter also investigated the extent to which the size of donation or compensation depends on the amount of investment parents are prepared to make to the quality life of

their children. The main conclusion is that the size of the donation correlates to the amount of investment parents are prepared to make on their children. However, the distances between school and homes may contribute to the poor quality education of children in many SSA countries; even when parents are concerned with the education of their children. In this process the final equilibrium of the economy will depend on the adjustment process back to the steady state equilibrium of the capital-labour ratio.

Chapter 5: Methodology

In the previous chapter, the theoretical linkages between retirement programs and the economy were outlined. In this chapter models are structured to test the impact of retirement funds on saving, economic growth and fertility in SSA countries.

5.1 Model Specifications

This section discusses the specifications of the models in a panel data form. Three models are specified, namely: (i) a savings model for the testing of the effects of retirement benefits on savings in SSA countries; (ii) a growth model for the testing of the effects of retirement benefits on growth in these economies and (iii) a fertility model to test for the effect of retirement benefits on fertility rates in SSA countries.

5.1.1 Model specification for saving

Recent studies like the Health and Retirement Study (HRS) (Gustman and Steinmeier, 1998) using survey data to study the effects of social security within a certain cohort group, used a variant of the Ando and Modigliani (1963) model. Other studies used the basic Ando and Modigliani model extended by Feldstein (1974) and some modified versions of the latter, for example the Barro (1978) specification. However, few studies have been conducted using a cross-country panel data analysis to test for the effects of social security on saving for developing countries (to the best of my knowledge, to date none have been done for sub-Saharan African countries).

Recognising the advantages of panel data analysis when comparing the effects of social security programs in the different sub-Saharan African countries with only limited data available, a bootstrapping regression fixed effects model will be applied to the Ehrlich and Kim (2005) model for saving:

$$Rinv_{it} = \beta_{1it} RBen_{it} + \beta_{2it} RGC_{it} + \beta_{3it} GPC_{it} + \beta_{4it} RDef_{it} + \beta_{5it} RNX_{it} + u_{it} \quad (1)$$

Where: $\rightarrow u_{it} = \mu_i + \varepsilon_{it}$ is the error term component, with vectors of country specific fixed effects (μ_i) error component and ε_{it} the usual white noise error. The subscript i indicates country and t indicates a time dimension.

RBen is the social security to GDP ratio

RGC is the government consumption to GDP ratio

GPC is the per capita economic growth rate

RDef is the government surplus/deficit to GDP ratio

RNX is the net exports to GDP ratio

Rinv is the investment to GDP ratio

β is the vector of the coefficients; $\beta_1 < 0$, $\beta_2 < 0$, $\beta_3 > 0$, $\beta_4 < 0$, and $\beta_5 > 0$

The investment to GDP ratio is used as a proxy for saving in country i . This specification is a version of the life cycle hypothesis and is based on national identity. This model will test the hypothesis that retirement funds reduce national saving in the sub-Saharan African countries.

5.1.2 Model specification for growth

It has been emphasised that retirement funds affect the growth of economies in a number of ways: firstly, by reducing the supply of capital, either through a reduction in private savings or through higher interest rates if financed by borrowing. Secondly, they stimulate the substitution effects between consumption and leisure, that is, they affect labour supply. Thirdly, they reduce fertility if parents are concerned with the education of their children (Zhang and Zhang, 2004 and Ehrlich and Kim, 2005). In the most recent growth models, human capital is seen as endogenous rather than exogenous as considered in neoclassical models, like the Ramsey (1928) and Solow (1956) optimal growth models. From the endogenous perspective, the assumption about investment in human capital is pro-growth.

For this reason, this section investigates to what extent retirement funds may influence the economic growth rates of some selected sub-Saharan African countries as stated in the following specification:

$$GPC_{it} = \beta_{1it}SEC_{it} + \beta_{2it}RGC_{it} + \beta_{3it}RBen_{it} + \beta_{4it}INFL_{it} + \beta_{5it}Open_{it} + u_{it} \quad (2)$$

Where: $\rightarrow u_{it} = \mu_i + \varepsilon_{it}$ is the error term component, with vectors of country specific fixed effects (μ_i) error component and ε_{it} is the usual white noise error. The subscripts i and t are as defined before.

GPC, RGC, RBen, and u_{it} have been defined previously

SEC is secondary education attainment

INFL is the inflation rate

Open is openness of country i , defined as exports plus imports divided by GDP

β is a vector of coefficients as explained earlier; $\beta_1 > 0$, $\beta_2 < 0$, and $\beta_3 > 0$

This model seeks to test the hypothesis that retirement funds are positively related to economic growth as in Zhang and Zhang (2004), Wigger (1999) and Ehrlich and Kim (2005). It is expected that the coefficient of the retirement benefit to GDP ratio would be positive as in the above studies.

5.1.3 Model specification for fertility

While retirement funds have shown adverse effects on the growth rates of economies, a number of studies seem to indicate consistent empirical results in relation to fertility (see Zhang and Zhang, 2004 and Ehrlich and Kim, 2005)²⁶. The results of studies on the effects of retirement programs on fertility are consistently that it reduces fertility per woman and, therefore, increases investment in human capital per child²⁷. Thus, two different model specifications from Zhang and Zhang (2004) and Ehrlich and Kim (2005) were combined to get the specification below. This specification allows the investigation of the effects of social security on fertility rates in sub-Saharan Africa, by applying a fixed effect one way error component model:

²⁶ See also Chapter 2, section 2.2.3

²⁷ In the case of many dynastic families in SSA countries the expected reduction of fertility may not increase investment in human capital (at least in the initial phase of growth of social security programs) because of low income per capita that tend to slow down the reduction of fertility rates.

$$F_{it} = \beta_{1it}RBen_{jt} + \beta_{2it}RGC_{it} + \beta_{3it}GPC_{it} + \beta_{4it}open_{it} + u_{it} \quad (3)$$

Where: $\rightarrow u_{it} = \mu_i + \varepsilon_{it}$ is the error term component, with vectors of country specific fixed effects (μ_i) error component and ε_{it} the usual white noise error.

F is the fertility rate and all other variables have been specified earlier.

β is the vector of the coefficients as explained earlier; $\beta_1 < 0$, $\beta_2 > 0$, $\beta_3 < 0$, and $\beta_4 > 0$

Other variables are specified as before

In this model the hypothesis to be tested is: paid retirement benefits, reduce fertility rates in SSA countries. The outcome of the model will be viewed against the background of the evaluation of the reforms in social security programs as proposed by the World Bank Report (1994).

5.2 Data

This study comprises fourteen sub-Saharan African countries for whom consistent data on retirement benefit payments for at least ten years, were available.

The World Bank development indicators provide a comprehensive data on countries' investment and government consumption, surplus/deficit and net exports as a share of GDP, growth rates, etc. However, given the fact that no significant number of observations on fertility rates in many (all) sub-Saharan African countries exists, population growth is used as a proxy of net fertility, calculated as the gross fertility rate minus the mortality rate. Other sources of data, such as country statistics of the IMF were used to complement the required statistical information.

The data obtained from the World Bank CD-R statistics are in US dollars, but the majority of the country specific data of the IMF are in the various countries' currencies. However, this does not constitute a problem since only ratios are used, thereby avoiding discrepancies resulting from using data in their levels. It should be stated though that

research is not aimed at filling data gaps, but rather that the effect of social security on the performance of the SSA economies is measured using existing data whatever its quality.

5.3 Econometric Technique

The technique used in this study is based on the application of panel data analysis techniques. The advantage of using panel data analysis is that it allows for the quantification of the dynamics of adjustment as found in Baltagi (2001):

- (i) Panel data gives the ability to control for heterogeneity between countries of different characteristics and history.
- (ii) It allows for less collinearity among the variables, more degrees of freedom and more efficiency than time-series.
- (iii) It enables one to identify and measure the effects that are not detectable in pure time-series or cross-section data.
- (iv) It allows for the constructing and testing of more complicated behavioural models than pure cross-section or pure time-series.

It, therefore, applies a one-way error component model where the vector of country specific fixed effects (μ_i) of the error is unobservable but estimable. This technique allows for the control of unobservable country specific factors that may affect the economies of these countries but which are not captured by the variables in the model, thus allowing to control for the heterogeneity of the countries included in the study.

By controlling for all these factors, it is possible to identify policy influences that affect countries in a similar way. A test to check for countries' poolability is performed to evaluate the possibility of generalisability of the parameters estimated in the behavioural equation. Country's poolability means that the set of factors affecting one country may be similar to those affecting other countries because they are assumed to be defined by the

same parameters. Therefore, within the use of Least Square Dummy Variable (LSDV) fixed effects seem to be appropriate for this study.

Moreover, this study also considers the observation made by Mooney and Duval (1993) that the traditional parametric inference based on coefficients in a regression model relies on distributional conditions and assumptions that may not hold true for a given set of data. This constraint may result in biased estimates used to make inferences about the true population coefficient. Taking this into consideration the bootstrapping technique based on the Monte Carlo²⁸ sampling has been widely used in recent empirical studies. In this way a probability density function (PDF) known as an empirical probability distribution (EPD), can be constructed, which describes the distribution of coefficient of the true population.

Thus, bootstrapping being a non-parametric technique offers an important advantage to traditional probability density functions. It constructs an accurate distribution density function based on the data on hand by evoking a sufficiently large number of replications (re-sampling) with replacement, which in turn allows it to approximate the distribution of estimators.

Although panel data analysis as described above has important advantages, it is not without limitations and the two most important ones for this study are the short time-series dimension and the distortion of the measurements, which may increase computational difficulties.

5.4 Bootstrapping Steps

Bootstrapping in this study is based on random components of the panel regression models specified in sections 5.1.1, 5.1.2 and 5.1.3. These random components are the unrestricted

²⁸ For Monte Carlo evidence on numerical performance of bootstrapping based tests see: Horowitz, J.L. (1995) and specifically on Wald test (used here) see: Gregory and veall (1985), Lafontaine and White (1986) and others for the discussion.

residuals from the error component $u_{it} = \mu_i + \varepsilon_{it}$. That part of the error component subject to resampling is white noise ε_{it} . The procedure is as follows²⁹:

(i) A panel regression: $Y_{it} = X_{it} \beta + u_{it}$ is estimated

(ii) Determine the random component of the error term (the unrestricted residuals):

$$\varepsilon_{it} = Y_{it} - (X_{it} \beta + \mu_i)$$

(iii) The vector of the responses to the resampling process is given as:

$$Y_{bit}^* = \hat{Y}_{it} + \hat{\varepsilon}_{bit}, \text{ where } \hat{Y}_{it} = X_{it} \hat{\beta} + \mu_i$$

(iv) The vector of bootstrap responses regressed for each cross-section and case-wise on exogenous variables is created to estimate a bootstrapped vector of

$$\text{coefficients } Y_{bit}^* = X_{it} \hat{\beta}_b^* + \mu_{bi} + \varepsilon_i$$

(v) The bootstrapped regression coefficients for each resample and each cross-section are $B \times k$ matrices, where B is the number of replications and k is the number of coefficients in each resample and cross-section used to estimate the distribution of $\hat{\beta}_k$.

After the bootstrapping procedure, confidence intervals are determined using the bootstrapped coefficients to make inference about the distribution of the true coefficient in the population under investigation. An α -level confidence interval, that is $[(1-\alpha) \times 100] \%$, is determined such that it possibly includes the true value of the parameter of the population investigated. The confidence interval is interpreted as the interval that carries a $[(1-\alpha) \times 100] \%$ certainty that it would include the true value of the distribution of the population.

²⁹ For more information on bootstrapping steps see also Money, C.Z. and Duval, R.D. 1993. *Bootstrapping: A nonparametric approach to statistical inference*. Sage Publications, International Educational and Professional Publisher. Newbury. London. New Delhi.

Thus, such confidence intervals contrast the traditional parametric confidence interval (it is here where the power of bootstrapping lies), which assumes that the distribution of the parameter of the population is known and that it has a normal or student t distribution; an assumption that may not always hold true.

Apart from defining confidence intervals, the determination of the magnitude of the bias of the estimated coefficients is important in validating the results of bootstrapping. The coefficient of regression is biased if the assumption of zero correlation ($H_o : E(\varepsilon_{it}/X_{it}) = 0$) between the error term, in this case ε_{it} , and the exogenous variable is violated. The magnitude of the bias of the estimated coefficient $(\hat{\beta})$ is determined as follows:

$$\text{Bias}(\hat{\beta}) = \beta - E(\hat{\beta})$$

Therefore, the distribution of bootstrapped sampling can be used directly to determine the magnitude of the bias of the estimated coefficient. Following Efron (1982), a good approximation of the bias of the estimated coefficient $(\hat{\beta})$ is simply the difference between the expected value of the bootstrapping sampling distribution and the estimated coefficient $(\hat{\beta})$:

$$\text{Bias}(\hat{\beta}) = \hat{\beta} - \frac{\sum \hat{\beta}_b}{B}$$

Where $\hat{\beta}_b$ is the bootstrapped coefficient. This is equivalent to using the percentage standardised bias computed as follows:

$$\text{Bias}(\hat{\beta}) = \left(\frac{\hat{\beta} - \hat{\beta}_b}{\sigma_{\hat{\beta}_b}} \right) \times 100 \quad \%$$

This is the expression used to determine the magnitude of the bias, where $\sigma_{\hat{\beta}_b}$ is the standard deviation of the bootstrapped coefficient $(\hat{\beta}_b)$. The coefficient is biased if the

percentage standardised bias is greater than 28 per cent (see Bun and Kiviet, 2001 for the discussion on the acceptable levels of bias).

Next, the empirical results will be discussed to analyse the impact of retirement programs on saving, per capita growth and fertility.