# EXPLORING MATHEMATICAL LITERACY: THE RELATIONSHIP BETWEEN TEACHERS' KNOWLEDGE AND BELIEFS AND THEIR INSTRUCTIONAL PRACTICES


by

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# Summary

South Africa is the first country in the world to offer Mathematical Literacy as a school subject. This subject was introduced in 2006 as an alternative to Mathematics in the Further Education and Training band. The purpose of this subject is to provide learners with an awareness and understanding of the role that mathematics plays in the modern world, but also with opportunities to engage in real-life problems in different contexts. A problem is the beliefs some people in and outside the classroom have regarding this subject such as teachers believing ML is the dumping ground for mathematics underperformers (Mbekwa, 2007). Another problem is the belief of some principals that any non-mathematics teacher can teach ML. In practice there is Mathematics teachers who teach ML in the same way that they teach Mathematics; non-Mathematics teachers who in many cases lack the necessary mathematical content knowledge and skills to teach ML competently; and Mathematics teachers who adapted their practices to teach ML using different approaches than those required for teaching Mathematics. Limited in-depth research has been done on the ML teachers, what they believe and what knowledge is required to teach this subject effectively and proficiently.

The purpose of this study is to investigate the way in which ML is taught in a limited number of classrooms with the view to exploring the relationship between ML teachers' knowledge and beliefs and their instructional practices. According to Artzt, Armour-Thomas and Curcio (2008) the instructional practice of the teacher plays out in the classroom where teachers' goals, knowledge and beliefs serve as the driving force behind their instructional efforts to guide and mentor learners in their search for knowledge. To accomplish this aim, an in-depth case study was conducted to explore the nature of teachers' knowledge and beliefs about ML as manifested in their instructional practices. A qualitative research approach was used in which observations and interviews served as data collection techniques enabling me to interpret the reality as I became part of the lives of the teachers.

My study revealed that there is a dynamic but complex relationship between ML teachers' knowledge and beliefs and their instructional practices. The teachers' knowledge, but not their stated beliefs were reflected in their instructional practices. Conversely, in one case, the teacher's instructional practice also had a positive influence on her knowledge and beliefs. It was further revealed that mathematics teacher training and teaching experience played a significant role in the productivity of the teachers' practices. The findings suggest that although mathematical content knowledge is required to develop PCK, it is teaching experience that plays a crucial role in the development of teachers' PCK.

Although the study's results cannot be generalised due to the small sample, I believe that the findings concerning the value of teachers' knowledge and the contradictions between their stated beliefs and practices could possibly contribute to teacher training. Curriculum decision-makers should realise that the teaching of ML requires specially trained, competent, dedicated teachers who value the subject. This exploratory study concludes with recommendations for further research.

**Key words:** Mathematical literacy; Teachers; Learners; Curriculum; Instructional practice; Tasks; Discourse; Learning environment; Mathematical content knowledge; Pedagogical content knowledge; Beliefs.

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# List of abbreviations

| | |
|---|---|
| **ACE** | Advanced Certificate in Education |
| **BEd** | Baccalaureus Educationis |
| **BTech** | Baccalaureus Technologiae |
| **CAPS** | Curriculum and Assessment Policy Statement |
| **DoE** | Department of Education (South Africa) |
| **FET** | Further Education and Training |
| **GET** | General Education and Training |
| **HED** | Higher Education Diploma |
| **MCK** | Mathematical content knowledge |
| **ML** | Mathematical Literacy (the subject) |
| **NCS** | National curriculum statement |
| **OBE** | Outcomes-based education |
| **OECD** | Organisation for Economic Co-operation and Development |
| **PCK** | Pedagogical content knowledge |
| **PISA** | Programme for International Student Assessment |
| **QCDA** | Qualifications and Curriculum Development Agency |
| **QCE** | Queensland Certificate in Education |
| **QG** | Queensland Government |
| **QSA** | Queensland Studies Authority |
| **RME** | Realistic Mathematics Education |
| **TIMSS** | Trends in International Mathematics and Science Study |
| **UK** | United Kingdom |
| **UP** | University of Pretoria |
| **US** | United States |

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# Chapter 1

# Introduction and contextualisation

## 1.1 Introduction

When referring to mathematical literacy[1] a clear distinction is required between the international and national perspectives. Internationally mathematical literacy refers to *the competence of individuals* (Christiansen, 2006, p. 6), which ranges from a competence demonstrated in word problems to a critical or democratic competence and whose purpose may be *mathematics as a tool in gaining insights into oppression, inequalities, and exploitation; … to become aware of the effects of applying mathematical models in society … and a third component has to do with mathematics as a 'gate-keeper', i.e., access to further education* (p. 6). In South Africa, according to the Department of Education (DoE, 2005) mathematical literacy on national level refers to a fundamental subject where learners are provided with learning opportunities to consolidate and extend their basic mathematical skills.

South Africa is the first country in the world to have Mathematical Literacy (ML)[2] as a school subject (Christiansen, 2007). This subject was introduced in 2006 as an alternative to Mathematics in the Further Education and Training band (FET)[3]. The purpose of this subject is to provide learners with an *awareness and understanding of the role that mathematics plays in the modern world*, but also with *opportunities to engage in real-life problems in different contexts* (DoE, 2003a, p. 9). The DoE (2003a) defined ML as follows:

> *Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems* (p. 9).

In the implementation of this relatively new subject teachers are crucial as agents of change. Bearing this in mind, it is essential to plumb the depths of their knowledge and beliefs regarding the subject in

[1] The words mathematical literacy (no capital letters) refer to a competency in applying mathematical knowledge.
[2] Mathematical Literacy (ML) refers to the South African school subject.
[3] The FET band includes Grade 10 through to Grade 12.

order to understand their instructional practices. An overview of the international and national perspectives on mathematical literacy, the experiences of ML teachers and the existing paucity in the literature now follows.

### 1.1.1 International perspective on mathematical literacy

With the emphasis on globalisation and the information explosion in mind, mathematical literacy should imply the empowerment of learners to meet the demands of living in the 21st century (Gellert, Jablonka & Keitel, 2001; Queensland Government, 2007b; Skovsmose, 2007). Although different terminology such as 'numeracy' and 'quantitative literacy' is also used internationally, Jablonka (2003) prefers to use the term "mathematical literacy" to focus attention on *its connection to mathematics and to being literate*, in other words it refers to *a mathematically educated and well-informed individual* (p. 77). International comparative studies such as the Organisation for Economic Co-operation and Development's (OECD) programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS) have heightened international awareness of the value and significance of mathematical literacy. It is not just internationally, but also at national level that there is a growing concern about learners' mathematical literacy skills (DoE, 2003a). PISA's purpose is to measure *how well students can apply their knowledge and skills to problems within real-life contexts* while the purpose of TIMSS is to *measure the mathematics and science knowledge and skills broadly aligned with curricula of the participating countries* (National Centre for Education Statistics, 2008). At national level there is also a growing concern about learners' mathematical literacy skills (DoE, 2003a).

Jablonka (2003) investigated different international perspectives on mathematical literacy and found that the perspectives basically differ according to the stakeholders' underlying principles and values. On the one hand there are researchers who accentuate the formal application of mathematics by mathematicians to real-world contexts, demanding a high level of mathematical knowledge and the competence to use and apply that knowledge (Gellert et al., 2001; Hope, 2007; Jablonka, 2003; Skovsmose, 2007). Other researchers are persuaded that everyone needs some basic level of literacy to empower them to make well informed decisions in their daily lives, whether personally, to care for their families or to contribute meaningfully in their workplace or society (McCrone & Dossey, 2007; McCrone, Dossey, Turner & Lindquest, 2008; Powell & Anderson, 2007; Skovsmose, 2007). Internationally mathematical literacy in schools refers to a competency or skill to apply mathematical knowledge and is embedded in the subject Mathematics.

### 1.1.2 National perspective on mathematical literacy

The results of TIMSS 2003 show how poorly mathematics is understood and conceptualised by learners in South Africa (Bloch, 2009; De Meyer, Pauly & Van de Poele, 2005). In the past learners who could not perform well in Mathematics in the General Education and Training (GET)[4] band *usually stopped studying Mathematics, thus contributing to a perpetuation of high levels of innumeracy* (DoE, 2003a, p. 9). With that in mind ML for Grades 10-12 was introduced in 2003 and implemented for the first time in 2006.

The Constitution of the Republic of South Africa speaks of human rights, social justice and provides a basis for transformation and development in South Africa (DoE, 2003a). Mathematics as a discipline, with its inherent potential to develop critical thinking, is a significant role player in the realisation of the DoE's (2003a) vision to create internationally competitive and creative learners and thinkers. Guided by these statements the DoE's (2003a) purpose with ML was to introduce a subject that would bring mathematics to all people and to ensure that *citizens of the future are highly numerate consumers of mathematics* (p. 9). The emphasis is on the knowledge needed to be a self-managing person, a contributing worker and a participating citizen. It is clear from the definition of ML on p. 1 that the focus of ML is on the applicability of mathematics in everyday life situations.

Teachers play a valuable and important role in ensuring the success of a newly introduced subject such as ML which depends largely on the teachers' training, experience, knowledge and perceptions of the subject. In the DoE's (2009) **Report of the task team for the review of the implementation of the National Curriculum Statement** (NCS), it is emphasised that *teachers need absolute clarity on what they are required to teach* (p. 16). Teachers need to *regain confidence in their practice, and authority as subject specialists in the classroom* (p. 16). Teachers and schools reported that *newly qualified teachers have deficiencies in respect to their subject or learning area specialisations and it would appear that they often have not been adequately prepared in respect to appropriate methodologies* (p. 55).

### 1.1.3 The experiences of ML teachers

The intentions and purpose of the DoE (2003a) are admirable, but due to implementation problems, among other things, not all ML teachers share the DoE's sentiments. In Sidiropolous' (2008) study on the implementation of ML in South African schools, she found that the threat experienced by qualified mathematics teachers regarding their 'status identity' undermines the proper implementation of the

[4] The GET band includes Grade R through to Grade 9.

subject. She also found that teachers do not understand and value this new curriculum which involves understanding not only the concept of mathematical literacy but also the nature of mathematics*, its transformative purpose and possibilities* (p. 254). Teachers need to understand *the sudden shift from content, to context and content as a process* (p. 254).

ML requires a different teaching approach to that of Mathematics as the nature of ML is contextualised and de-compartmentalised (De Villiers, 2007; North, 2005; Venkat & Graven, 2007; Graven & Venkat, 2007). Researchers are concerned about teachers' knowledge and competency to use and apply an approach based on mathematical modelling (Brown & Schäfer, 2006; Glover & King, 2009; Vermeulen, 2007). The focus in research on the issue of teachers' knowledge and competency in ML should therefore be on teacher education. ML student teachers should experience practically what it means to develop an understanding of mathematics in context through, for example, an activity and investigation-based approach (Brown & Schäfer, 2006; Vithal & Bishop, 2006). Glover and King (2009) regard teacher professional development as an important component of curriculum reform and draw attention to issues that need to be addressed such as teachers' beliefs, self-efficacy and knowledge. They refer to subject knowledge, pedagogical content knowledge (PCK) and curriculum knowledge that need to be improved.

### 1.1.4 Silence in the literature addressed in this study

We have a unique situation in South Africa in that ML refers to a subject and not a skill or competency per se, as is the case internationally. Although the literature informs this study regarding international views, purposes and definitions of mathematical literacy as competency or skill, there is a gap in the literature regarding ML teachers' knowledge and beliefs. Existing literature on national level mainly focuses on the vision and purpose of the subject, the curriculum implementation process, curriculum issues such as the relevancy of the outcomes and assessment criteria, the content-context debate and which approaches to use. The silence in the literature this study attempts to address concerns ML teachers' knowledge and beliefs but also the relationship between their knowledge and beliefs and their instructional practices. It is this gap that this study aims to fill.

## 1.2 Rationale for the study

A rationale firstly addresses how the researcher developed an interest in the topic and secondly why the study is worth doing (Vithal & Jansen, 1997). As far as my personal interest is concerned, I have been involved in teacher training for the past twenty years, teaching Mathematics to undergraduate student teachers. I took a particular interest in the introduction of ML as subject because I value and appreciate the subject's vision, purpose and content. As I am interested in how people in and outside the school environment experience and view ML, I make a point of talking to people about the topic. Through my involvement in Teaching Practice which forms part of the undergraduate students' curricula at the University of Pretoria (UP) and being a member of the data collection team for the FET Implementation Project, I have been required to visit many local schools where I also have had the opportunity to talk to principals, teachers and learners. What I learnt and experienced is not at all what I had anticipated.

As far as the possible value of the study is concerned, I regard it as imperative to conduct a study which focuses on ML teachers' instructional practices, as the findings from the study will suggest ways to improve current teachers' practices and will also inform and enrich my own teacher training practice. Koellner et al. (2007) believe that to achieve the vision for school mathematics, no factor is more important than the teacher. A teacher requires a sound knowledge of subject content, principles and strategies, needs to believe in the potential of the subject and the learners, and should maintain a positive attitude. It is also the teacher who is to enthuse and motivate the learners about mathematics and the role it plays in their lives. Through the study I purpose to contribute to our understanding of how the subject is currently taught in South African schools and what the nature of the ML teachers' knowledge and beliefs is, as well as to help people gain insight into the value of ML. Awareness needs to be created of ML as an important subject and the necessity of having knowledgeable and positive teachers teaching this subject. As long as negative perceptions prevail regarding any subject, that subject will not be taken seriously and will not fulfil its purpose. Through the findings of the study I endeavour to show that ML is not a subject with a lower status, but is a different subject with a different emphasis and different requirements compared with Mathematics.

## 1.3 Statement of the problem

Many learners, teachers and parents have negative attitudes towards ML and regard it as an inferior subject. According to Mbekwa (2007) some teachers regard the subject as a dumping ground for mathematics underperformers. ML learners are ridiculed for having to take a subject that is considered a waste of time. ML is a relatively new subject in which an entirely different teaching approach is required: mathematical content should be taught in terms of real-life situations. For this to occur successfully, specific skills are required of the ML teacher. This study aims to address the problem concerning ML teachers' instructional practices. There are three groups of ML teachers: a) Mathematics teachers who teach ML in the same way that they teach Mathematics; b) non-Mathematics teachers who in many cases lack the necessary mathematical knowledge, skills and beliefs to teach ML competently; and c) Mathematics teachers who adapted their practices to teach ML using different approaches than those required for teaching Mathematics.

## 1.4 The purpose of the study

Curriculum developers and teachers are still in the process of addressing implementation problems and determining the required standard of the subject. Limited in-depth research has been done concerning the ML teachers, what they believe and what knowledge is required to teach this subject effectively and proficiently. The purpose of this study is to investigate the way in which ML is taught in a limited number of classrooms with the view to establishing the relationship between ML teachers' knowledge and beliefs and their instructional practices. To accomplish this aim, an in-depth study will be conducted to explore the nature of teachers' knowledge and beliefs of mathematics and ML as manifested in their instructional practices. These findings will then be used to investigate the possible implications thereof for teacher training and theory building.

## 1.5 Research questions

With the rationale, statement of the problem and purpose as background, the following research questions were formulated:

**Main question:**

What is the relationship between Mathematical Literacy teachers' knowledge and beliefs and their instructional practices?

**Subquestions:**

1. How can ML teachers' instructional practices be described?
2. What is the nature of ML teachers' knowledge and beliefs?
3. How do ML teachers' knowledge and beliefs relate to their instructional practices?
4. What are the possible implications of the findings from Questions 1, 2 and 3 for teacher training?
5. What is the value of the study's findings for theory building in teaching and learning ML?

## 1.6 Methodological considerations

This study was initiated by my interest in the relationship between teachers' knowledge and beliefs and their instructional practices. To answer the research questions a qualitative research approach will be used as it concerns *specific meanings, emotions and practices that emerge through the interactions and interdependencies between people* (Hogan, Dolan & Donnelly, 2009, p. 4). The research design is a case study as it observes *effects in real contexts, recognising that context is a powerful determinant of both cause and effect* (Cohen, Manion & Morrison, 2001, p. 181). My research paradigm is social constructivism[5] and is based on the epistemological assumptions that *social life is a distinctly human product* and that *human behaviour is affected by knowledge of the social world* (Nieuwenhuis, 2007, p. 59-60). This study is subjective in nature with the nominalist position as ontological assumption: reality is understood through words and is the product of individual consciousness (Cohen et al., 2001). Observations and interviews serve as data collection techniques to enable me to interpret the reality by becoming part of the lives of the teachers. The data are analysed according to the categories identified in the conceptual framework.

## 1.7 Definition of terms

The following are operational definitions of terms used in this study:

- Beliefs: This term refers to a viewpoint or a way of thinking, or even a preconceived idea a person holds. Beliefs can be interpreted as *mental constructs that represent the codification of people's experiences and understandings* (Schoenfeld, 1998, p. 19). Beliefs about teaching and learning can be located on a perspective continuum from traditional (instrumentalist view), to formalist (Platonist view), to a constructivist perspective (problem solving) (Dionne 1984; Ernest, 1988).
- Contextualised mathematics: This term is similar to realistic mathematics education (RME) where mathematics is seen as a human activity that is connected to reality and relevant to society. RME is founded upon the principles of using real-world contexts, bridging the gap between abstract and applied mathematics, allowing learners to develop their own problem-solving strategies, and making connections to other disciplines (Freudenthal as cited in Van den Heuvel-Panhuizen, 1998).
- Instructional practice: This term refers to the qualitative dimensions of teacher behaviour regarding their teaching. These dimensions involve *teachers' abilities to model cognitive strategies in meaningful and purposive activities, promote classroom dialogues, adjust instruction as required and establish classroom communities in which students collaboratively and cooperatively participate in enquiry-related activities* (Englert, Tarrant & Mariage, 1992, p. 62). A framework used to observe and describe teachers' instructional practices is built on three observable aspects of mathematical lessons namely tasks, discourse and the learning environment (Artzt, Armour-Thomas & Curcio, 2008).
- Learners: This term refers to school learners.

[5] Social constructivism is discussed under Section 3.2.1: Research paradigm.

- Pedagogical content knowledge: This term refers to the knowledge teachers need in the teaching profession that goes beyond having mathematical content knowledge (MCK) only. PCK includes knowledge of what learners do not understand, why they do not understand it and what can be done to rectify the situation. It is the knowledge needed to notice, predict and understand learners' misunderstandings and to assist and guide them to better understanding (Ball, Thames & Phelps, 2005; Shulman, 1986).
- Productive practice: A practice where the teacher listens to learners' mathematical thinking and aims to use it to encourage conversation that revolves around the mathematical ideas in the sequenced problems (Franke, Kazemi & Battey, 2007, p. 226).

## 1.8 Possible contribution of the study

Although workshops have been offered by the DoE and papers have been published and presented by national academics on various issues concerning the implementation of ML, there is no evidence of in-depth empirical research that has been conducted on ML teachers' knowledge and beliefs and the relationship between ML teachers' knowledge and beliefs and their instructional practices. This study will thus contribute to this new field and fill the gap in literature. The study further contributes to ML theory and practice as the findings will be implemented in undergraduate teacher training programmes at the University of Pretoria. Furthermore it is important to contribute to the vision and success of the DoE's (2003) endeavour to change the current situation where South Africa's adult population has a very low level of literacy and mathematical proficiency. To accomplish this, teachers as well as the community need to realise the place and value of ML in the school curriculum as well as the need for ML teachers to build their own 'status identity'. On a more personal level, this study will also contribute to the development and enrichment of my own practice of preparing ML student teachers. The personal experiences and findings will provide me with a broader and deeper knowledge and understanding of the subject ML, of ML teachers' knowledge and beliefs, and to what extent these knowledge and beliefs relate to their instructional practices.

## 1.9 Limitations of the study

The case study, like any other research method, has its weaknesses. In this study a limitation is that the primary data are gathered from a relatively small number of teachers who will be observed on three different occasions and will be interviewed three times. Cohen et al. (2001) mentioned that *results may not be generalisable except where other readers/researchers see their application* (p. 184). It will therefore not be possible to generalise the findings, but one may still acknowledge the value of the findings obtained from rich in-depth involvement in specific cases as *they provide insights into other, similar situations and cases* (p. 184).

During the data gathering process the Hawthorne effect will be taken into account as teachers naturally try to impress an observer in class or an interviewer during the time of observation and interviewing. To a certain extent they may even feel threatened by being observed during their lesson presentations, despite my reassurances. As they do have busy schedules, some may experience it as extra work and an intrusion into their privacy and they may not be as dedicated to the project as would be ideal. Despite the normal human preconceived ideas and perceptions, I will try to guard against being biased and selective and try to be objective during the data gathering and analysis stages in an effort to avoid the halo effect during the data analysis. Cohen et al. (2001) describe the halo effect as a cognitive bias in which the researcher's knowledge or perception of the person or situation exerts an influence on subsequent judgement. The fact that more than one data collection technique will be used minimises this problem. Another way these limitations are minimised is to properly prepare the teachers before the commencement of the data collection process and the establishment of a positive relationship with them, emphasising the value of honest and true data and their anonymity during the whole process.

## 1.10 Summary

This chapter provides an overview of the problem and rationale for the study, the research questions, methodological considerations and the possible contribution and limitations of the study. The different meanings attached to mathematical literacy both internationally and nationally have been discussed. Internationally mathematical literacy refers to *the competence of individuals* (Christiansen, 2006, p. 6) whereas nationally mathematical literacy mainly refers to a subject involving mathematical skills for solving contextual problems. The subject was introduced in 2006 and schools are experiencing a lack of qualified teachers. The purpose of this study is to investigate, by means of a case study, the way in which ML is taught with the view to determining the relationship between ML teachers' knowledge and beliefs and their instructional practices.

## 1.11 The structure of the thesis

The thesis consists of five chapters. Chapter 1 is summarised above. Chapter 2 provides an in-depth analysis of the findings in the relevant literature and also explains the conceptual framework on which the study is based. In Chapter 3 details regarding the methodology of the study are given. The data analysis strategies are discussed as well as the trustworthiness and ethical considerations of the study.

Chapter 4 includes the presentation of findings from the data obtained through class observations and interviews conducted with the ML teachers. The findings are also analysed and discussed according to the research questions based on the conceptual framework and the literature and trends are identified and explained. Chapter 5 contains the conclusion and implications and comprises a chapter summary, verification of the research questions, a reflection on the study, the conclusions, recommendations and limitations of the study.

# Chapter 2

# Literature review and conceptual framework

## 2.1 Introduction

This literature study is a critical and integrative synthesis of various researchers' findings, justifying this research endeavour. It is imperative to remember that South Africa is the only country offering ML as a compulsory alternative to Mathematics in Grades 10 to 12. As the study concerns the ML teachers and the relationship between their knowledge and beliefs and their instructional practices, the literature review begins with a comparison of the international and national perspectives of mathematical literacy. Comparisons are made between the different conceptions of mathematical literacy; the contexts in which mathematical literacy can be applied; international studies measuring learners' mathematical knowledge and literacy skills; meanings and definitions of mathematical literacy; and the role mathematical literacy plays in some school curricula. Following the review on mathematical literacy is a discussion of the meaning of teachers' instructional practices and the value of various approaches to teaching. Moving to the core of the problem, literature regarding teachers' knowledge and beliefs about the subject they teach are discussed. Attention is given to the different domains of teachers' knowledge, teachers' belief systems and the relationship between their knowledge and beliefs and their instructional practices. The literature review concludes with the conceptual framework which is based on concepts and theories from relevant work in the literature[6].

## 2.2 Mathematical literacy

Mathematical literacy is not a clearly defined term and internationally there exists a range of different conceptions of mathematical literacy that are discussed in this section. As mathematical literacy (ML) is a school subject in South Africa, it is important to understand the motivation and purpose of ML in the South African curriculum and to compare it with the role mathematical literacy plays internationally.

[6] Several direct quotations are used in the literature review to avoid nuance chances of meaning to the matter under discussion.

## 2.2.1 International perspectives on mathematical literacy

In this section I mention the different terminology being used for mathematical literacy, compare different conceptions of mathematical literacy, discuss different contexts in which mathematics could be applied and refer to some international comparative studies that measure learners' mathematical literacy skills in order to derive a general meaning or definition of mathematical literacy.

There is an expanding body of literature that uses the terms "mathematical literacy" and "numeracy" as synonyms (Jablonka, 2003). The National Council on Education and the Disciplines however uses the term "quantitative literacy" to stress *the importance of enquiring into the meaning of numeracy in a society that keeps increasing the use of numbers and quantitative information* (Jablonka, 2003, p. 77). Jablonka prefers to use the term "mathematical literacy" *to focus attention on its connection to mathematics and to being literate*, in other words *to a mathematically educated and well-informed individual* (p. 77).

In a comprehensive study by Jablonka (2003) in which different international perspectives on mathematical literacy were investigated, she found that the perspectives basically differ according to the stakeholders' underlying principles and values. In her opinion there is a direct connection between a conception of mathematical literacy and a particular social practice. She acknowledges the difficulty of pointing out the distinct meaning of mathematical literacy as it varies according to the *culture and context of the stakeholders who promote it* (p. 76). The different conceptions of mathematical literacy relate to a number of relationships and factors. One of the relationships is between *mathematics, the surrounding culture, and the curriculum* (p. 80) while another is between *school mathematics and out-of-school mathematics* as mathematical literacy is *about the individual's ability to use the mathematics they are supposed to learn at school* (p. 97). Varying with respect to the culture and the context four possible perspectives of mathematical literacy are:

- *The ability to use basic computational and geometrical skills in everyday contexts.*
- *The knowledge and understanding of fundamental mathematical notions.*
- *The ability to develop sophisticated mathematical models.*
- *The capacity for understanding and evaluating another's use of numbers and mathematical models* (p. 76).

With the above-mentioned perspectives as background the different conceptions of mathematical literacy as found in the literature will subsequently be categorised.

### 2.2.1.1 Different conceptions of mathematical literacy

The literature revealed different conceptions of mathematical literacy but the resemblances between mathematical literacy and RME, mathematisation, mathematical modelling, as well as mathematics in action are most evident. As there is opacity as to what each of these conceptions entail and how they differ a clarification of the concepts and notions will be provided.

### *Realistic Mathematics Education*

Hope (2007) expressed the resemblance of mathematical literacy with the theory of RME. RME *uses a theoretical framework that relies on real-world applications and modelling, a didactical belief propagated by Hans Freudenthal* (Gates & Vistro-Yu, 2003, p. 67). According to Van den Heuvel-Panhuizen (1998), Freudenthal and his colleagues laid the foundations of RME in the early seventies to address the world-wide need to reform the teaching and learning of mathematics and to move away from mechanistic mathematics education. Freudenthal's theory of RME rests upon the following five components:

- *Using a real-world context as a starting point for learning.*
- *Bridging the gap between abstract and applied mathematics by using visual models.*
- *Having students develop their own problem-solving strategies rather than memorise rules and procedures.*
- *Making mathematical communication, perhaps in the form of journaling or oral presentations, an integral part of the lesson.*
- *Making connections to other disciplines using meaningful real-world problems* (Hope, 2007, p. 30).

Hope (2007) further believes mathematical literacy is a matter of the appropriate pedagogy that should be used in teaching mathematics. According to these fundamental pedagogical aspects of teaching mathematics, it is comprehensible that the traditional school mathematics instruction is too formal, less intuitive, more abstract, less contextual, more symbolic, and less concrete than the type of instruction that would expand student thinking and develop mathematical literacy (p. 30).

### *Mathematisation*

Freudenthal believed that the focus should not be on *mathematics as a closed system, but on the activity, on the process of mathematisation* (Van den Heuvel-Panhuizen, 1998), and that mathematics should be seen as a human activity that is connected to reality and relevant to society. Treffers (1978) formulated the idea of two types of mathematisation, namely horizontal and vertical mathematisation. He stated that *in horizontal mathematisation the students come up with mathematical tools which can help to organise and solve a problem located in a real-life situation* whereas *vertical mathematisation is the process of reorganization within the mathematical system itself, like, for instance, finding shortcuts and discovering connections between concepts and strategies and then applying these discoveries* (Van den Heuvel-Panhuizen, 1998). Freudenthal (1991) explained horizontal mathematisation as *going from the world of life into the world of symbols, while vertical mathematisation means moving within the world of symbols* (p. 24). It would seem that vertical mathematisation refers to the more formal mathematics while horizontal mathematisation refers to the informal mathematical literacy part.

According to Hope (2007) mathematising is a term used by The Organisation of Economic Co-operation and Development (OECD) which involves five elements:

- *Starting with a problem whose roots are situated in reality.*
- *Organising the information and data according to mathematical concepts.*
- *Transforming a real-world, concrete application to an abstract problem whose roots are situated in mathematics.*
- *Solving the mathematical problem.*
- *Reflecting back from the mathematical solution to the real-world situation to determine whether the answer makes sense* (p. 29).

***Mathematical modelling***

ML bears a strong resemblance to mathematical modelling in that both require an application of Polya's four basic steps in problem-solving namely: a) understanding the problem; b) designing a plan; c) carrying out the plan; and d) looking back on the problem. Mathematical modelling can further be described as *a matter of constructing an idealised, abstract model which may then be compared for its degree of similarity with a real system*. (Giere, 1999, p. 50).

Gellert et al. (2001) use mathematical literacy as a metaphor referring to well-educated and well-informed individuals. According to them different conceptions of mathematical literacy are based on the relationship between mathematics, reality and the society. Their concept of mathematical literacy involves *gaining a level of mathematical understanding that goes beyond the minimal abilities of calculating, estimating, and gaining some number sense, and basic geometrical understanding ... by seeing the power of mathematics in its potential of abstracting from concrete realities by generating concepts and structures for universal application* (p. 59). They further believe these abilities can be developed *by experiencing mathematical modes of thinking, such as searching for patterns, classifying, formalising and symbolising, seeking implications of premises, testing conjectures, arguing, and thinking propositionally* (p. 59) which form the basis of mathematical modelling. Mathematical literacy *requires the mathematical competence to understand the mathematical methods involved and the analytical competence to demystify the justifications for specific mathematical applications as well as to assess their consequences* (p. 66).

***Mathematics in action***

Although some of the above-mentioned conceptions are formal, involving higher-order mathematical skills, there are other researchers who regard mathematical literacy as a fundamental requirement for all people, recognising its essential value to learners in contexts forming part of their everyday living (McCrone & Dossey, 2007; Powell & Anderson, 2007; Skovsmose, 2007). McCrone and Dossey (2007) believe mathematical literacy is not about *studying higher levels of formal mathematics, but about making mathematics relevant and empowering for everyone* (p. 32). They further call for mathematics to play an even greater part in non-mathematics classes where teachers promote the mathematics embedded in their subjects.

Skovsmose (2007) refers to mathematical literacy as mathematics in action and considers the role of mathematical literacy in both mathematicians' and non-mathematicians' lives. He based his study on two types of literacy being either **functional** or **critical**, terms introduced by Apple in 1992. Functional literacy is defined by competencies a person possesses to *fulfil a particular job function* (p. 4) whereas critical literacy addresses themes such as working conditions and political issues. Skovsmose prefers to talk about **reflective** knowledge with respect to mathematics instead of **critical** literacy. Reflective knowledge refers to *a competence in evaluating how mathematics is used or could be used* (p. 4). Critical literacy is associated with the skill to create or design models using mathematics whereas functional literacy is the skill to use and apply those models. To make unambiguous distinctions between these two types is not that simple and it *could have very different interpretations depending on the context of the learner* (p. 4).

***Clarification of basic concepts and notions***

From the discussion above it is clear that the lines between the different concepts such as mathematisation and mathematical modelling are blurred. Blum and Niss (2010) provided a clarification of the different concepts and notions when they described the process of applied problem solving. The process of applied problem solving commences with a **real problem situation** and through a process of simplification, idealisation and structuring of the situation, the process ends with a **real model of the original situation**. Through the process of mathematisation, the real model is translated into mathematics. Mathematisation is therefore the process of converting *the data, concepts, relations, conditions and assumptions* (p. 208) of the real model into a **mathematical model of the original situation**. Mathematical modelling is the entire process leading from the real problem situation to the mathematical model. Then the mathematical model must be processed to obtain certain **mathematical results**. This includes mathematical activities such as *drawing conclusions, calculating and checking concrete examples, applying known mathematical methods and results as well as developing new ones etc.* (p. 208). The next process is to retranslate the results into the real world, *i.e. to be interpreted in relation to the* ***original situation*** (p. 208). The model is then validated and if discrepancies of any kind occur, they may lead to the modification of the model or replacement of the model by going through the process cycle more than once.

***Summary***

In the light of the above discussion of the different conceptions from the literature, mathematical literacy cannot adequately be described in terms of skills only, as it involves mathematical problems in contexts that require attributes such as conceptual understanding of formal mathematical knowledge and problem-solving skills (Gellert et al., 2001). Gellert et al. also believe that the differences between various conceptions of mathematical literacy *consist of the problems to which mathematics is applied* (p. 61). Hope (2007) on the other hand believes mathematical literacy is a matter of the appropriate pedagogy that should be used in teaching mathematics. All mathematics learners should therefore be provided with the opportunity to apply their knowledge and logic to real-world situations that form part of their daily lives. Mathematical literacy implies bridging the gap between abstract and applied mathematics where the contexts and degree of complexity differ. Jablonka's (2003) question of: mathematical literacy for what?, further calls attention to the need for discussing the different contexts in which mathematics could be applied to further explicate the purpose of mathematical literacy.

### 2.2.1.2 Some contexts in which mathematical literacy can be applied

Prescribing the different contexts in which mathematical literacy can be applied is as complicated as conceptualising mathematical literacy. In this section the different categories of contexts are discussed as well as the role technology plays in determining these contexts.

***Context categories according to stakeholders' demands***

Contexts in which mathematical literacy can be applied depend on the stakeholders' philosophy, view or principles and could be guided by, among other things, some socio-economic demands (Jablonka, 2003). Jablonka categorised the different, and in some cases, conflicting contexts in which mathematics can be applied as mathematical literacy for:

- Developing Human Capital – looking at the world through mathematical eyes where higher-order mathematical skills are applicable, where mathematics is not regarded as culture-bound and value-driven.
- Cultural Identity – incorporating ethno-mathematical practices to avoid privileging of Western academic mathematical knowledge.
- Social Change – to uncover and communicate aspects of social or political nature (such as unemployment, life expectancy, national income) in an attempt to overcome the dominance of academic mathematics in the curriculum.
- Environmental Awareness – the mathematical content comprises arguments underpinned by mathematical visualisations, qualitative mathematics that is characterised as not aiming at an analytical solution but serving as thought experiments and computational mathematics, which include the use of simulation packages, graphing calculators and spreadsheets.

- Evaluating Mathematics - includes reasoning with condensed measures and indexes, formalising transactions, reasoning with platonic models, constructing surface-models and numerology.

An example of conflict between some of these conceptions Jablonka referred to is where the application of mathematics in the Cultural Identity is restricted to contexts situated in a specific culture, where in the Environmental Awareness the focus is on applying mathematics to contexts of global nature. Then there is the complex problem of Cultural Identity, where Ethno-mathematics is suggested as a necessity for addressing cultural conflicts in the classroom. Knoblauch's (1990) categories are similar to Jablonka's, speaking of literacy for *professional competence in a technological world, for civic responsibility and the preservation of heritage, for personal growth and self-fulfilment, and for social and political change* (p. 76). Different contexts in which mathematical literacy can be applied can also be categorised according to some processes involved in applying mathematics in real-world contexts.

***Context categories according to processes***

By categorising the content in four processes, Skovsmose (2007) illustrated *how mathematics in action can operate in powerful ways, and power can be exercised through mathematics in action* (p. 8). The following categories show the different conceptions of mathematical literacy categorised as critical and functional literacy as discussed in par. 2.2.1.1.

- Construction – *includes systems of knowledge and techniques, by means of which technology, in the broadest interpretation of the term, is maintained and further developed* (p. 8) for example in the construction of the computer.
- Operating – *bringing technology into operation in work practices and job functions. The operator may not be aware of the mathematical content of the procedures he or she performs* (p. 11) for example, ticket reservations in the travel industry and procedures for buying and selling houses.
- Consuming – as citizens we are the consumers who need to listen to statements from experts that are expressed everyday on television and in the newspapers, for example *numbers and figures concerning elections, the economy, exchange rates and investments are mixed with advertising of any number of special offers* (p. 13).
- Marginalising – *a steady growth of favela-like neighbourhoods gloomily testifies that free-growing globalised capitalism is not an inclusive economy. Instead it marginalises in great measures people as being disposables.* Examples include drugdealing, *selling of sunglasses, lighters and other items possible to carry around along the streets where cars come to a stop* (p. 14).

Skovsmose (2007) concluded that mathematical literacy *could be either functional or critical* but that the *distinction is difficult to maintain, is vague, maybe illusive* (p. 17). In many of these processes technology plays a significant role in the process of context selection.

***The role of technology***

Gellert et al. (2001) pointed out that technology has taken over many processes in society where highly skilled mathematicians develop mathematical models and processes which people generally do not need to understand or even be aware of. There is little need for the majority of people *to learn more mathematics in a more successful way as it is based more on common sense than on rational reason or on justifiable evidence* (p. 58). The result is then *an increasing mathematisation of our society [which] is complemented by an increasing demathematisation of its individual members* (p. 58). In comparing their view with Skovsmose's (2007) categories stated in the preceding paragraph, it is a minority of people who use their advanced mathematical literacy skills to construct or bring technology into operation while the majority of people use their basic mathematical skills to operate and consume.

### 2.2.1.3 Studies measuring learners' mathematical literacy skills

There are various studies measuring learners' mathematical knowledge and skills, but in this study only two international comparative studies will be considered, namely the Organisation for Economic Co-operation and Development's (OECD) Programme for International Student Assessment's (PISA) as well as TIMSS.

***The foci of PISA and TIMSS***

Every three years PISA assesses 15-year-olds' reading, mathematical and scientific literacy in different countries. Its purpose is to measure *how well students can apply their knowledge and skills to problems within real-life contexts. PISA is designed to represent a 'yield' of learning at age 15, rather than a direct measure of attained curriculum knowledge* (National Centre for Education Statistics, 2008b, p. 3). In 2003, when 45 countries participated, the focus was on mathematics. Every four years TIMSS assesses fourth- and eighth-graders' mathematics and science performance in different countries (58 in 2007) to compare U.S. learners' performance with that of their peers in other countries. This study's purpose is to *measure the mathematics and science knowledge and skills broadly aligned with curricula of the participating countries* (National Centre for Education Statistics, 2008a, p. 5). The two studies differ in focus as *TIMSS seeks to find out how well students have mastered curriculum-based scientific and mathematical knowledge and skills* whereby the purpose of PISA is *to assess students' scientific and mathematical literacy, that is, their ability to apply scientific and mathematical concepts and thinking skills to everyday, non-school situations* (Nohara, 2001, p. 11). In the next section attention is given to PISA's definition of mathematical literacy and the criteria used in assessing learners' mathematical literacy skills.

***PISA***

According to the OECD (2004, p. 37), the content of school mathematics and science in the last decade was chosen *to provide the foundations for the professional training of a small number of mathematicians, scientists and engineers*. With the increased emphasis on the application value of science, mathematics and technology in modern life to all adults, the objectives of these three subjects changed to *personal fulfillment, employment and full participation in society*. Mathematical literacy is *concerned with the capacity of students to analyse, reason and communicate effectively as they pose, solve and interpret mathematical problems in a variety of situations involving quantitative, spatial, probabilistic or other mathematical concepts*. PISA's (OECD, 2003) definition of mathematical literacy is:

> *the capacity to identify, to understand and to engage in mathematics and make well-founded judgement about the role that mathematics plays, as needed for an individual's current and future life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen* (p. 20).

PISA was originally designed to measure the extent to which learners can apply their mathematical knowledge in realistic, everyday life situations. It involves the ability to analyse situations in *content areas involving quantity, shape and space, change and relationships and uncertainty* (McCrone et al., 2008, p. 35). In order to be able to assess mathematical literacy, PISA (OECD, 2003) identified three broad criteria to be used, namely:

- The content of mathematics – in terms of clusters of relevant, connected mathematical concepts that appear in real situations and contexts. These include quantity, space and shape, change and relationships, and uncertainty.
- The process of mathematics – different skills needed for mathematics such as **reproduction** – simple computations; **connections** – using of ideas and procedures to solve straightforward and familiar problems; and **reflection** – using of mathematical thinking, generalisations and insight to engage in analysis, identify mathematical elements in a situation, formulate questions and search for solutions.
- The contexts in which mathematics is used – the kinds of problems encountered in real life vary in terms of distance from individual, from effecting one directly regarding private life, school life, work and sports, local community and society and scientific, to scientific problems of more general interest.

From PISA's definition and assessment criteria it is clear that the focus is not just on applying routine procedures but to become cognitively involved in mathematical thought, using and applying formal mathematics to solve real-life problems. The contexts should involve realistic day-to-day situations involving people's personal, occupational and social lives enabling them to become reflective citizens.

Bearing in mind the conceptions, contexts and meanings of the concept mathematical literacy discussed above, I will subsequently discuss the role mathematical literacy plays in the FET school curricula in Australia and briefly mention the situations in the United Kingdom (UK) and United States (US).

#### 2.2.1.4 Defining mathematical literacy

When referring to mathematical literacy or numeracy, many people in and outside the academic field tend to believe that only basic mathematical skills are involved or that numeracy refers only to primary school learners' mathematics. To conceptualise or define mathematical literacy is unfortunately not that simple and is far from being well-defined (Gellert et al., 2001; Jablonka, 2003; Skovsmose, 2007).

Different conceptions or definitions are held by researchers ranging from informal mathematics requiring basic mathematical skills (McCrone & Dossey, 2007; McCrone et al., 2008; Powell & Anderson, 2007; Skovsmose, 2007) to formal mathematics involving higher-order thinking skills (Gellert et al., 2001; Hope, 2007; Jablonka, 2003; Skovsmose, 2007). Gellert et al. (2001) pointed to especially primary school teachers who regard mathematical literacy as informal, defining mathematical literacy as *survival mathematics for all* (p. 68) with the exact purpose to propagate that mathematics is not just formal and complicated, but can be useful and beautiful to all people. Skovsmose (2007) believes that mathematical literacy can be related to notions such as autonomy, empowerment and globalisation, whereas Hope (2007) presumes it implies *that a person is able to reason, analyse, formulate, and solve problems in a real-world setting* (p. 29).

Gellert et al. (2001) and Jablonka (2003) perceive mathematical literacy in terms of higher-order mathematical skills. Jablonka is of the opinion that any attempt to define mathematical literacy *faces the problem that it cannot be conceptualised exclusively in terms of mathematical knowledge, because it is about an individual's capacity to* ***use*** *and* ***apply*** *this knowledge* (p. 78). She defined mathematical literacy as *a bundle of knowledge, skills and values that transcend the difficulties arising from cultural differences and economic inequalities because mathematics and mathematics education themselves are not seen as culture-bound and value-driven* (p. 81). She conceptualises mathematical literacy in terms of *higher-order mathematical skills* (p. 97) that are applicable to all kinds of contexts.

Although a definition of mathematical literacy is elusive, a golden thread running through all attempts to define mathematical literacy is that mathematical literacy is a valuable competence or skill a person possesses to put mathematics to work in solving real-life contextual problems. With the emphasis on globalisation and the information explosion in mind, mathematical literacy should imply the empowerment of learners to meet the demands of living in a 21st century (Gellert et al., 2001;

Queensland Government, 2007b; Skovsmose, 2007). It is informative to investigate the current situation regarding mathematical literacy in some international school curricula.

#### 2.2.1.5 The role of mathematical literacy in some international school curricula

Instead of using the term "mathematical literacy" when referring to the competency of applying mathematical knowledge to life-related problems, Australia and the UK generally refer to the terms "numeracy", while the US refers to "quantitative literacy". A discussion regarding the role mathematical literacy plays in Australia's school curricula subsequently follows. I then briefly mention the situation in the UK and US.

***AUSTRALIA***

In 2008 all Australian regional governments agreed that instead of the eight different arrangements, only one national curriculum is to be implemented in 2013, which should play a key role in delivering quality education (Australian Curriculum, Assessment and Reporting Authority, n.d). Queensland is the second largest region and a study of the Education Department of Queensland provided insight into the role numeracy plays in the education system of Australia.

According to the PISA 2003 results when the focus of the study was on mathematics, Australia came 12th out of 41 countries (OECD, 2004). In the TIMSS 2007 they came 14th out of the 58 participating countries for both Grade 4 and Grade 8 learners (National Center for Education Statistics, 2008). For the past few years the raising of the numeracy levels of Australian learners received serious attention. There are numerous documents and guidelines available to teachers on how to develop learners' numeracy skills in the Mathematics classroom. There are also fact sheets available to parents with information on numeracy, providing some guiding principles on how to support their children in their numeracy development.

In a document called **Numeracy: Lifelong Confidence with Mathematics - Framework for Action 2007 – 2010**, which serves as an action plan to improve numeracy education, the Minister for Education and Training declared that the Queensland Government (QG) recognises numeracy as *a key pillar of learning and an essential component* (QG, 2007b, p. 1) of their curriculum. He also said that teachers have an important role to play in helping learners to become confident appliers of mathematics in their everyday lives. A Queensland Certificate in Education (QCE) is awarded at the end of Year 12 to a person who, in addition to achieving 20 credits in the required pattern of learning has met the requirements for literacy and numeracy. Learners can meet QCE numeracy requirements by satisfying a number of possible options including *a sound achievement* in one of their three Mathematics subjects in school or passing *a short course in numeracy developed by the Queensland Studies Authority* (QSA, 2009b, p. 1). Numeracy is clearly an important component in the Queensland school curriculum, but there is no indication or description of a connection between Mathematics and numeracy in this curriculum.

### *Mathematics and numeracy*

The QSA (2009a) provided a clear explanation of Mathematics and numeracy and said the focus of Mathematics is on the development of learners' *knowledge and ways of working in a range of situations from real life to the purely mathematical* where *numeracy refers to the confident use of mathematical knowledge and problem-solving skills not only in the Mathematics classroom, but across the school curriculum and in everyday life, work or further learning* (p. 9). In the Queensland Government's (QG, 2007b) definition of numeracy it is stated that *to be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in the community and civic life* (p. 2). Mathematics and numeracy are interrelated and it is *the responsibility of the Mathematics curriculum to introduce and develop the mathematics which underpins the numeracy* (QSA, 2009a, p. 9). As numeracy refers to the ability to use mathematics in solving life-related problems, it is essential to determine the contexts in which mathematics could be applied and what the role of the teacher is in developing learners' skills in this regard.

### *The context and teaching of numeracy*

In Year 10 to 12 the numeracy work learners *relate[s] to a specific context across a broad range of work and study options* (QG, 2007a) and involve:

- *Applying mathematical skills in new contexts such as: 1) analysing data to inform decision making; 2) deciding to estimate or calculate an answer depending on the purpose; 3) calculating dimensions and quantities of materials in vocational tasks such as construction or hospitality.*
- *Selecting, sequencing and evaluating information to understand texts and to communicate with other people.*
- *Using particular communication skills needed to effectively participate in the workplace such as industry terms and customer services* (QG, p. 1).

Teachers are the key role players in selecting contexts relevant to the learner. They need to *recognise numeracy demands and opportunities within the curriculum* (QG, 2007b, p. 10) enabling learners to develop their numerical knowledge, skills and confidence. Teachers should intentionally create opportunities in which learners can, among other things, explore mathematical ideas with concrete or visual representations and hands-on activities; experience practical and contextualised learning; communicate about mathematical issues; develop calculator and computer skills and use multiple solution strategies (QSA, 2006). According to the Queensland Government (QG, 2007b) teachers' understanding of mathematics content needs to be developed with respect to *the nature of mathematics as a discipline; the* *mathematics topics they teach; the relationship of those topics to further learning and everyday life; the impact of information and communication technologies on the teaching and learning of mathematics* (p. 4).

### *UNITED KINGDOM (UK)*

England did not participate in the 2003 or 2006 PISA study, but performed very well in TIMSS 2007, taking the 7th position for both the 4th and 8th graders out of the 58 participating countries (National Center for Education Statistics, 2008a). By law all children between ages 5 and 16 must receive a full-time education. The UK introduced a National Curriculum in 1992 to which state schools need to adhere until learners reach the age of 16. National Curriculum core subjects are: English, Mathematics and Science which are offered at different levels.

### *The UK national curriculum*

Within the framework of the National Curriculum, schools are free to plan and organise teaching and learning in the way that best meets the demands of their pupils. The Qualifications and Curriculum Development Agency (QCDA) provides guidelines and assistance in this regard. The National Curriculum is organised in four key stages: Key Stage 1 (5-7 years) and Key Stage 2 (7-11 years) form part of the Primary curriculum while Key Stage 3 (11-14 years) and Key Stage 4 (14-16 years) form part of the Secondary curriculum (Government of United Kingdom, 2010a). The aim of the Government is to address the literacy and numeracy levels of children in the first two Key Stages (5-11 years) in order to develop pupils' mathematical thinking and number skills, with a focus on understanding and application. A document addressed to learners, schools and families, called the **The Primary Framework for literacy and mathematics** makes recommendations on how literacy should be incorporated in daily mathematics lessons (Government of United Kingdom, 2010b). The secondary curriculum focuses on developing the skills and qualities that learners need not only to succeed in school, but also in the broader community.

### *Functional mathematical skills*

Numeracy appears in the Early Year Foundation Stage (birth to 5 years) as part of the learning area: Problem solving, Reasoning and Numeracy. In the Primary (5-11 years) and Secondary (11-16 years) curricula Mathematics, and no longer numeracy, appears as one of the ten compulsory school subjects. **Functional mathematics** frequently appears in the Secondary curriculum referring to functional mathematical skills the learners should acquire. Learners need these skills and abilities to play an active and responsible role in their communities, in their everyday life, workplace and in the educational settings (QCDA, 2010a). Functional mathematical skills are a subset of the key processes set out in the programme of study. These key processes are representing, analysing, interpreting, evaluating, communicating and reflecting. All teaching needs to contribute to the development of these key processes. It requires pupils to be introduced to a range of real-life uses of mathematics, including its role in the modern workplace (QCDA, 2010b). These functional skills need to be developed in the five strands of Mathematics, namely Mathematical processes and applications; Number; Algebra; Geometry and measures; and Statistics. Individuals with functional mathematical skills understand a range of mathematical concepts and know how and when to use these concepts. They have the *confidence and capability to use mathematics to solve increasing complex problems; are able to use a range of tools, including integrated computer technologies as appropriate; possess the analytical and reasoning skills needed to draw conclusions, justify how these conclusions are reached and identify errors or inconsistencies; are able to validate and interpret results, judging the limits of the validity and using the results effectively and efficiently* (QCDA, 2010c).

***UNITED STATES (US)***

In the United States learners take part in both the PISA study and TIMSS. In the PISA 2003 when the focus of the study was on mathematics, they came 31st out of 41 countries (OECD, 2004). In the TIMSS 2007 results they took the 11th position for the 4th graders and the 9th position for the 8th graders out of the 58 countries participating (National Centre for Education Statistics, 2008a).

Although the term "quantitative literacy" is common in the discourse of US mathematics educators, it does not appear often in their curricula (J. Kilpatrick, personal communication, May 24, 2010). Kilpatrick is a mathematics expert, advisor, consultant and professor in Mathematics Education, University Georgia who serves on various mathematical boards and councils. The US does not have a single national curriculum in Mathematics. In search of the term "quantitative literacy" in National Curricula in the Departments of Education of Ohio and North Carolina, as suggested by Kilpatrick, a reference was eventually found on the webpage of the National Council of Teachers of Mathematics (2010) where it was stated that *consumer mathematics should develop a broader quantitative literacy and should consist primarily of work in informal statistics, such as organizing and interpreting quantitative information*.

Comparing the national curriculum documents of Australia, the UK as well as Ohio and North Carolina in the US, it is evident that Australia accentuates the importance of mathematical literacy in an education system. Through their national documents for learners, schools and parents they drive an intensive awareness campaign regarding the raising of the numeracy levels of their learners. Regardless of the terminology used, numeracy, functional mathematical skills and quantitative literacy are embedded in Mathematics and involve the competency or skill to use and apply mathematics to solve contextualised problems.

#### 2.2.1.6 Summary

To define, value, position or conceptualise mathematical literacy is a daunting task. Different views exist but the most common descriptions of mathematical literacy are mathematics in action (Skovsmose, 2007); mathematics in context (McCrone & Dossey, 2007; Powell & Anderson, 2007); realistic mathematics education (Hope, 2007); and mathematising (Gellert et al., 2001; Hope, 2007). The different perspectives of mathematical literacy undoubtedly illustrate how the different conceptions vary in degree of complexity regarding the required mathematical knowledge and skills where in some notions advanced and expert mathematical knowledge and higher order cognitive skills are required. It is however PISA's definition and criteria for assessment that best describe the requirements of this study.

Although some researchers accentuate the formal application of mathematics by mathematicians to real-world contexts demanding a high level of mathematical knowledge and the competence to use and apply it (Gellert et al., 2001; Hope, 2007; Jablonka, 2003; Skovsmose, 2007), other researchers remain convinced that all people need some basic level of literacy to empower them to make well informed decisions in their daily lives, whether personally, to care for their families or to contribute in their workplace or society (McCrone & Dossey, 2007; McCrone et al., 2008; Powell & Anderson, 2007; Skovsmose, 2007). The value of being mathematically literate is evident but it remains uncertain to what extent mathematical literacy could address educational practices and contribute to an individual's quality of life or even the development of the country (Gellert et al., 2001; Jablonka, 2003; Skovsmose, 2007).

Nowhere in the literature has mathematical literacy been referred to as a specialised subject. It is rather regarded as specialised knowledge or a competency or skill embedded in the subject Mathematics. According to Hope (2007) mathematical literacy is a matter of the appropriate pedagogy that should be used in teaching mathematics. As mathematical literacy with its focus on the skill of using and applying mathematical knowledge forms part of Mathematics, the focus of Mathematics teaching should be on knowledge and the development of skills enabling learners to solve real-life application problems. The aforementioned perspectives and conceptions are wide and theoretical and to provide only one international definition of mathematical literacy is not viable as it depends primarily on a particular social practice and the context involved. With these international perspectives in mind, the South African perspective on mathematical literacy is discussed below.

## 2.2.2 An overview of ML

In this overview of ML in South Africa the history and principles of the subject are discussed. An overview of the two subjects ML and Mathematics is given, some general concerns about ML are discussed and lastly a comparison is made between the national and international perspectives on mathematical literacy.

### 2.2.2.1 The history of ML

***Background information to ML***

One of the reasons behind the implementation of ML as an alternative subject to Mathematics in the FET band was the low level of learners' mathematical knowledge and mathematical literacy skills as shown in the results of international studies (DoE, 2003a). The last time South Africa took part in TIMSS, an international study, was in 2003 when Grade 8 learners participated and came last out of 46 countries (National Centre for Education Statistics, 2008b). Recently The World Economic Forum ranked South Africa 120th for Mathematics and Science education, well behind our troubled neighbour Zimbabwe which was ranked 71st (Maths Excellence, 2009). At present the country's GET learners are not participating in any such studies as a four-year Foundations for Learning campaign was introduced in 2008 in the Foundation and Intermediate phases to improve the reading, writing and numeracy abilities of all South African children (DoE, 2008a). Another reason for implementing ML was to address the concern that *Mathematics is too abstract, catering primarily to prepare students to proceed to further mathematically or scientifically oriented studies* (Graven & Venkat, 2007, p. 340). They believe ML now offers an alternative to learners who do not need it for this purpose.

***The ML curriculum reform***

Curriculum 2005 was introduced in 1998, coinciding with the birth of a new democracy in South Africa's post-apartheid era, and was based on the principles of outcomes-based education (OBE) (DoE, 2009). This curriculum was revised in 2000 and in 2002 the NCS for the FET phase was developed. In 2009 a task team reviewed the curriculum and apart from problems related to learning materials and teacher training, the curriculum documents were deemed to be in need of streamlining (DoE, 2009). The task team found that some of these documents contradicted each other while at other times there were repetitions. The review supports the DoE's current move away from OBE and learning outcomes, which are now replaced with clear content, concept and skill standards as well as clear and concise assessment requirements (DoE, 2009). The various subject specific documents will be replaced with a single document called the Curriculum and Assessment Policy Statement (CAPS) (DoE, 2009). The date of implementation should be in 2012.

#### 2.2.2.2 ML principles

This section examines the principles of ML such as the purpose, aims, definition, key elements, composition of the subject as well as the assessment taxonomy on which ML is based.

***The purpose of ML***

The purpose of ML is to ensure that all learners develop an understanding of mathematics and how it relates to the world in order to use mathematical information to make valuable decisions affecting their life, work and society. It is important that learners are able to interpret and critically analyse everyday situations and solve problems. With this purpose in mind, ML aims to ensure a broadening of the education of learners, preparing them to meet the demands of a modern world (DoE, 2003a). According to the DoE (2008b, 2011a), the purpose of the subject includes the ability of a learner to become:

- **A self-managing person** where the focus is on problems that relate to financial issues such as mortgage bonds, hire-purchase and investments, other personal issues such as the ability to estimate and calculate length, areas and volumes, to read a map and follow timetables and to understand house plans, sewing patterns and converting recipes.
- **A contributing worker** at the workplace requires the use of fundamental numerical and spatial skills with understanding in order to deal with work-related formulae, statistical charts and schedules and to understand instructions involving numerical components.
- **A participating citizen** where learners need to acquire a critical stance to mathematical arguments presented to them in the media or other platforms.

These three abilities as part of ML's purpose correspond with the international purpose for learner competence, stating that for learners to be competent means having more than just knowledge, they must know how to use and apply their mathematical knowledge (Gellert et al., 2001; Hope, 2007; Jablonka, 2003; Skovsmose, 2007).

***The aims of ML***

The main aim of ML is to equip learners to be skilled citizens, meeting the demands they will encounter in their future lives. The process to achieve this aim involves the mastering of mathematical content through solving contextualized problems. ML aims to develop the following learner abilities (DoE, 2008b):

- *The ability to use basic mathematics to solve problems encountered in everyday life and in work situations.*
- *The ability to understand information represented in mathematical ways.*
- *The ability to engage critically with mathematically based arguments encountered in daily life.*
- *The ability to communicate mathematically* (p. 8).

### ***The definition of ML***

The DoE's (2003a) national definition of ML reads as follows:

> *Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems* (p. 9).

There are, according to this definition, three key elements of ML namely 1) the mathematical content, 2) the contexts that should involve everyday life-related problems and 3) the abilities and behaviours that a mathematically literate person needs to possess which include problem solving through interpreting and analysing the problem with confidence (Bowie & Frith, 2006). In the new CAPS document (DoE, 2011a) these three key elements of ML have been extended to five key elements[7].

Comparing national and international purposes and definitions of ML, the national purpose and definition closely relate to that of PISA (National Centre for Education Statistics, 2008b; OECD, 2003). The purpose of PISA is *to measure the extent to which students can make use of their mathematical knowledge in realistic and day-to-day situations* (McCrone et al., 2008, p. 35). An international definition of mathematical literacy according to PISA (OECD, 2003) is:

> *the capacity to identify, to understand and to engage in mathematics and make well-founded judgements about the role that mathematics plays, as needed for an individual's current and future life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen* (p. 20).

The interface between the international and national views is the emphasis being placed on the role mathematics plays in the world and the value of applying mathematics in people's personal lives, at the workplace and as participating citizens. Further shared objectives are guiding learners to become engaged in mathematics and to understand and appreciate how it is embedded in everyday life situations. A point of difference however is that nationally mathematical literacy refers to both a subject and competence while internationally it refers to a competence (Christiansen, 2007).

### ***The five key elements of ML***

From the purpose, aims and definition of ML the DoE (2011a) lists five key elements involved in ML namely:

- **The use of elementary mathematical content:** The general idea is that the focus is not on formal abstract mathematical concepts and mathematical content should not be taught in the absence of context.

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[7] The five elements of ML are discussed under the next subheading: *The five key elements of ML.*

- **Real-life contexts:** These contexts should be authentic and relevant, and should relate to learners' daily lives, their future workplace and the wider social, political and global environments.
- **Solving familiar and unfamiliar problems:** Learners should have the ability and skills to interpret both familiar and unfamiliar real-life contextual problems they encounter in the world. They should have the ability to apply both mathematical and non-mathematical techniques and considerations in order to explore and make sense of the context. The interplay between content, context and solving problems is illustrated in the following figure:

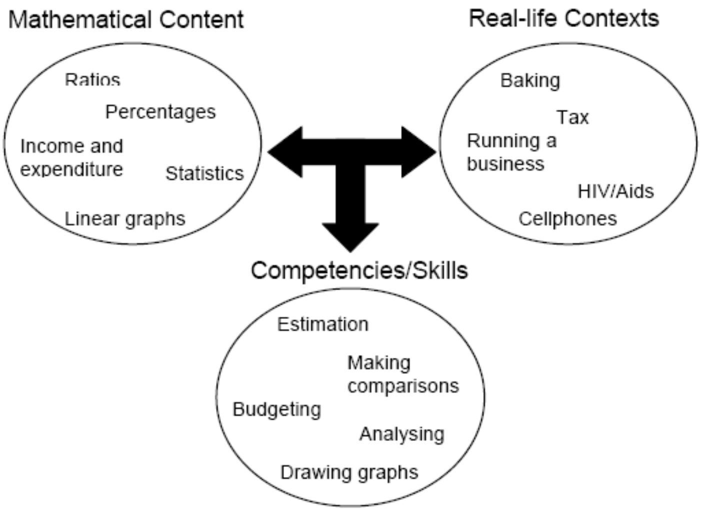


**Figure 2.1: Interplay between content, context and problem-solving skills in ML** (DoE, 2011a, p. 10)

- **Decision-making and communication:** A mathematically literate person should be able to compare solutions, make decisions regarding the most appropriate choice for a given set of conditions and communicate their decisions through the use of appropriate terminology.
- **The use of integrated content and/or skills in solving problems:** Since most real-life problems consist of a range of mathematical topics, learners need to use mathematical content and/or skills drawn from a range of topics and need to identify and use a range of techniques and skills integrated from a range of content topics.

***The composition of ML***

The topics in the new CAPS (DoE, 2011a) replace the learning outcomes from the current NCS for ML. The content, contexts and problem solving skills appropriate to ML are offered in topics and divided into two sets of topics, namely (DoE, 2011a):

- **Basic skills topics:** Much of the content in these topics includes the mathematical content and skills that learners have already been exposed to in Grade 9. Teachers therefore have the opportunity to revise important mathematical concepts and to provide learners now with the opportunity to explore and use these concepts in various contexts.
- **Application topics:** These topics contain contexts that can be related to situations from everyday life, the workplace and business environments as well as wider social, national and global issues that learners are expected to make sense of. A profound understanding of the content and skills from the **Basic skills topics** are required to make sense of the contexts and content from the **Application topics**. Figure 2.2 below shows an overview and weighting of the topics according to which the ML curriculum has been organized for Grades 10, 11 and 12.

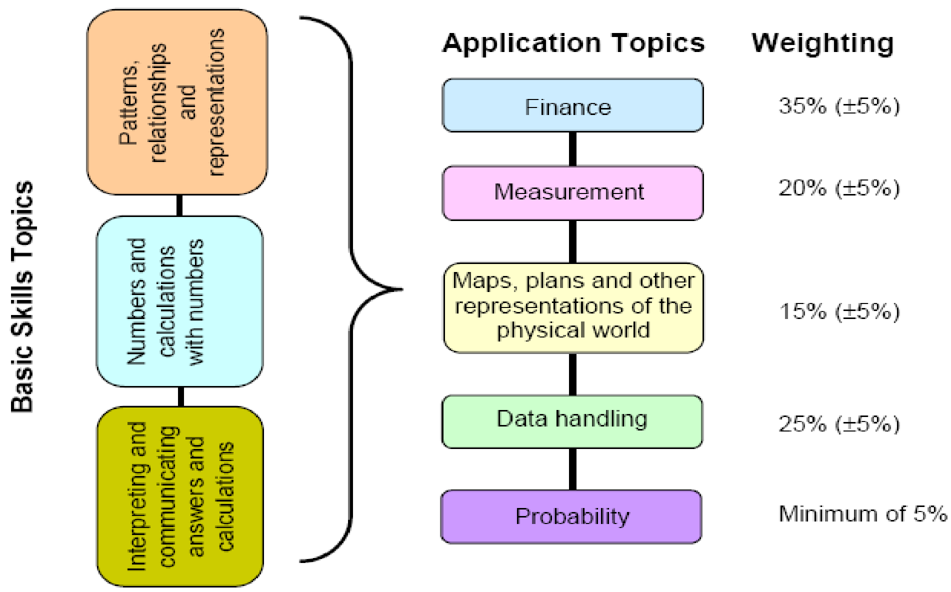


**Figure 2.2: Overview and weighting of topics in Grades 10, 11 and 12** (DoE, 2011a, p. 14)

***The ML assessment taxonomy***

Assessment should be done at different levels of cognitive demand, from *simple reproduction of facts* to *detailed analysis and the use of varied and complex methods and approaches* (DoE, 2011a, p. 91). The following assessment taxonomy framework is used:

- Level 1: Knowing
- Level 2: Applying routine procedures in familiar contexts
- Level 3: Applying multi-step procedures in a variety of contexts
- Level 4: Reasoning and reflecting (p. 84).

According to Venkat, Graven, Lampen and Nalube (2009) the emphasis in Levels 1 and 2 is on routine calculations whereas the key aims of ML are located primarily in Levels 3 and 4. Level 3 refers to the ability of learners to think numerically and spatially whereas Level 4 refers to critically analysing everyday life situations.

The DoE (2011a) explicitly states that since ML *involves the use of both mathematical and non-mathematical techniques and considerations in exploring and making sense of authentic real-life scenarios* (p. 92), the taxonomy should be regarded as follows:

> *This taxonomy should not be seen as being associated exclusively with different levels of mathematical calculations and/or complexity. In determining the level of complexity and cognitive demand of a task, consideration should also be given to the extent to which the task requires the use of integrated content and skills drawn from different topics, the complexity of the context in which the problem is posed, the influence of non-mathematical considerations on the problem, and the extent to which the learner is required to make sense of the problem without guidance or assistance* (DoE, 2011a, p. 92).

### 2.2.2.3 Pedagogical approaches for teaching ML

The DoE (2011a) suggests that the focus of ML teaching is the integration of content and skills in real-life contexts. Teachers should provide learners with opportunities to *analyse problems and devise ways to work mathematically in solving them* (p. 9) and *develop and practice decision-making and communication skills* (p. 10). According to Brown and Schäfer (2006) the emphasis in the ML curriculum is on contextualised mathematics. These contexts should be realistic and demand real-life authenticity to provide learners with opportunities to apply and use mathematics in order to make sense of the world, instead of letting learners do more mathematical content (Bansilal, Mkhwanazl & Mahlaboratoryela, 2010). These problems should relate to a learner's daily life, the workplace and the wider social, political and global environment (DoE, 2011a, p. 12). Brown and Schäfer (2006) found *many similarities to that of mathematical modelling* but the *differences appeared to be that mathematical modelling is generally described using more advanced mathematics, in more technical contexts* (p. 46) but that the basic principles of modelling can be applied on elementary mathematics too. ML further focuses on *de-compartmentalisation, where mathematical topics are no longer taught in isolation of each other* (North, 2005, p. 35). All ML textbooks are written accordingly with the initial four learning outcomes being integrated to enable ML teachers to teach ML in a de-compartmentalised way.

In a longitudinal study performed by the Marang Centre at the University of the Witwatersrand, Graven and Venkat (2009) report that the learners who are part of the study, are for the most part positive about ML, which the researchers attribute to the teachers who substantially changed their pedagogic practices. This differs from traditional Mathematics teaching in that *the nature of tasks in ML (engagement* *with a scenario rather than application of maths in 'word problems') and the nature of interaction in ML (much slower pace, more discussion and group work)* (p. 2). Venkat (2007) argues that if learners are engaged in problems situated in real-life situations, they will develop valuable skills such as mathematical reasoning, sense-making, applying different procedures and decision-making.

In Table 2.1 given below, Graven and Venkat (2007) identify a spectrum of pedagogic agendas that *traverses across the question of the nature and degree of integration of context with mathematics within pedagogic situations* (p. 74). This spectrum of agendas is a tool for the ML teachers to think about the nature of their lessons and may assist the teachers to navigate their teaching along a whole spectrum of pedagogic agendas.

**Table 2.1: A spectrum of pedagogic agendas** (Graven & Venkat, 2007, p. 74-75)

| **1. Context driven (by learners' needs)** | **2. Content and context driven** | **3. Mainly content driven** | **4. Content driven** |
|---|---|---|---|
| Driving agenda:<br><br>To explore contexts that learners need to interact and engage with in their lives (current, future, citizenship) and to use maths to achieve this. | Driving agenda:<br><br>To explore a context so as to deepen maths understanding and to learn maths (new or GET) and to deepen understanding of that context. | Driving agenda:<br><br>To learn maths and then apply it to various contexts. | Driving agenda:<br><br>To give learners a second chance to learn the basics of maths from the GET band. |
| Pedagogic demands:<br><br>• Involves identifying contexts/scenarios needed for the above agenda.<br>• Teaching needs increased discussion of contexts and critical engagement with them and the mathematics embedded in them.<br>• Teaching might require revisiting or learning new maths but largely insofar as it will service critical engagement with and understanding of the context. | Pedagogic demands:<br><br>• Involves selecting real contexts (possibly edited or adapted) that enable the above agenda.<br>• Teaching needs discussion about contexts but this must be balanced with revising maths and learning new maths in new ways. Contextual and mathematical learning need to be balanced and connected in a dialectical relationship that enables the agenda. | Pedagogic demands:<br><br>• Involves selecting contexts that GET maths can be applied to (contrived or more real) and editing these to enable application appropriate to the level of learning.<br>• Teaching focuses on mathematical learning and its use in applications and does not necessarily require much discussion of context. | Pedagogic demands:<br><br>• Involves revision of GET maths without the need for pedagogic change except in relation to slower pacing.<br>• Contexts do not feature much except in relation to their use in teaching GET basics (e.g. in the case of fractions – using cakes for understanding fractions). |

Graven and Venkat (2007) analysed ML's definition and purpose on context as stated by the DoE (2003a) and propose Agenda 2 to be the core business of ML. They call these four agendas a **spectrum** and not a **continuum** which *might imply that teachers move along it in one direction* (Graven & Venkat, 2007, p. 77). The idea is that teachers may use different agendas at different times as required. Although Agenda 2 is the *primary driving agenda,* a teacher can *adopt other agendas at different points in order to support this agenda and also to assist in meeting curricula demands* (p. 77).

### 2.2.2.4 The ML learner profile

Although there are positive and enthusiastic ML learners, the majority of learners are less interested and enthusiastic about mathematics and mathematical activities and many negative feelings result in fear of anything mathematical (Vermeulen, 2007). According to Vermeulen learners could avoid these negative experiences and feelings of anxiety in the past by not choosing mathematics, but now they need to confront them. He argues that it is the parents' and society's incorrect beliefs, teachers' teaching methods based on their beliefs and attitudes, teachers' attitudes towards the learners, and teachers' classroom culture that contribute to learners' negative feelings. According to Mbekwa (2007) it is a challenge to teach ML as learners lack understanding and motivation because ML is seen as the *dumping ground for mathematics underperformers* (p. 227).

### 2.2.2.5 Some general concerns about ML

***The ML teachers***

As ML is a relatively new subject and different to Mathematics, clear guidelines from the DoE regarding issues such as pedagogical approaches to teach ML should have been a given, but instead *the absence of precedents of what pedagogy and assessment should be like* (Graven & Venkat, 2007, p. 67) caused multifarious interpretations of the curriculum aims. Bowie and Frith (2006) were concerned about a perception that ML could be interpreted as *a slightly toned-downed standard grade Mathematics with word sums* (Bowie & Frith, 2006, p. 32). Experience and research have indicated that Mathematics learners and in many cases teachers too, find word or application problems requiring conceptual understanding more difficult than routine problems which require factual recall or the use of routine procedures (Abedi & Lord, 2001; Grobler, Grobler & Esterhuyse, 2001; Johari, 2003; Schoenfeld, 1988; White & Mitchelmore, 2002). My own experience like that of De Villiers (2007) confirms that Mathematics learners cope well with the theory of linear functions, but when it is put in real-life contexts they cannot solve such problems. Even in ML both teachers and learners find the process of mathematising contexts complex as a good understanding of both the context and the mathematical content is required (Bowie & Frith, 2006). A further concern is the number of ML teachers with other specialisations who also teach the subject (Mbekwa, 2007). It is known that in the past, before ML was introduced, there existed a shortage of appropriately qualified Mathematics teachers (Sidiropoulos, 2008) and the question arises as to the provenance of all the teachers who now teach both ML and Mathematics. Sidiropolous (2008) further suggested that *a change is required not only in pedagogical content knowledge, but also in understanding the nature and value of Mathematical Literacy* (p. 205).

***The choice of context***

Apart from the complexity of solving contextual problems, the contexts to which mathematics should be applied in ML are not clear to teachers. A further concern is how mathematical progression is made through the years regarding the complexity of contexts. A good understanding of the context is required by both teachers and learners in order to mathematise a context (Bowie & Frith, 2006). For example when working on personal finances, topics such as budgeting, compound interest, mortgage payments, and retirement options are not part of all teachers' and learners' life experiences. They have inadequate experiences of banks, interests, risks and return on investments. To teach one mathematical content topic requires several periods to first explain the context involved. What further complicates the situation is that in reality banks normally use their own formulae programmes and do not calculate interest as learners are taught to do (Christiansen, 2007). In choosing contexts, teachers may use the principle from PISA (OECD, 2003) that categorises contexts according to their distance from the learner. ML teachers can therefore include contexts from the learner's private life, school life, work and sport, and local community and society. It is crucial that contexts be authentic and applicable to the learners' environment.

***The language issue***

In South Africa the majority of learners are taught in English, which is often not their mother tongue. According to Graven and Venkat (2009) integration with the above-mentioned contexts could be problematic due to the increased English language demands. Many researchers reported on difficulties learners experience regarding the contextualised problems and the role language plays in conceptual understanding (Mbekwa, 2007; Setati, 2005). Maree (2000) posits that insufficient language skills and language usage play an important role in under-achievement of learners in Mathematics. In his classification of learners' mistakes in mathematics, language problems were the most significant problem identified. He expressed his concern about learners having to unravel problems in mathematics that require more sophisticated language skills while they actually lack the minimum language skills to even understand what is being asked.

Debate regarding ML as only alternative to Mathematics exists and many teachers expressed their concern about the existence of two extreme levels of mathematics, especially considering South Africa's diverse population (Maths Excellence, 2009). This group of teachers advises a three-level system consisting of two formal Mathematics subjects and ML as the third option to accommodate the diverse skills and needs of our learners. Their idea is that ML should then be taken by learners who do not wish to take either of the formal Mathematics courses. The ML learners can then be equipped with basic numeracy skills. According to them the current lack of curriculum flexibility could result in the downgrading of mathematical skills. On the other hand a number of people in the school education system are against a two-level system as was applicable in the South African schools up to 2007. According to Kitto (personal communication, February 23, 2011) a reason is that *very few township and rural schools offered higher grade under the old system, so an overwhelming percentage of the higher grade candidates were white. A huge number of competent black students were denied the chance to demonstrate their ability and get into engineering and other faculties*. She reasons that the policy makers are trying to make sure that everyone who has the ability to continue with careers in science and engineering has access to the mathematics that is needed.

### 2.2.2.6 Comparison between the national and international perspectives on mathematical literacy

In South Africa the term "mathematical literacy" refers *both to a school subject and to the competency of individuals,* where *internationally it is mainly the latter* (Christiansen, 2007, p. 91). The original NCS Grades 10-12 General (DoE, 2003a) is based on OBE, social transformation and integration, and applied competence. These principles encourage a learner-centred and activity-based approach.

With the increased international emphasis on the application value of mathematics, science and technology, the objectives of subjects such as ML changed to *personal fulfilment, employment and full participation in society* (OECD, 2004, p. 37). Internationally mathematical literacy as application skills is embedded in the subject Mathematics. For this purpose real-life contexts are used to re-contextualise mathematical concepts. From the literature it is evident that mathematical literacy varies in width and depth and that one needs to interpret it according to the purpose and context being used (Gellert et al., 2001; Hope, 2007; Jablonka, 2003; McCrone & Dossey, 2007; Powell & Anderson, 2007; Skovsmose, 2007). Jablonka (2003) states that the context in which mathematical literacy is applied, sometimes demands higher-order mathematical skills, whereas McCrone and Dossey (2007) believe mathematical literacy should be promoted even in non-mathematics classes to make mathematics relevant and to empower all learners. Nationally the subject ML focuses on *making sense of real-life contexts and scenarios*

(DoE, 2011a, p. 9) and *requires an understanding of only basic mathematical concepts and calculations, and does not require an understanding of complex and/or abstract mathematical principles* (DoE, 2011a, p. 11).

### 2.2.2.7 An overview of ML and Mathematics

Since ML is a compulsory subject for Grade 10 to 12 learners who do not choose Mathematics as subject, parents and learners should know what each subject entails and what the implications are for further studies. For example the DoE (2003a) states that learners who wish to proceed to tertiary studies of a mathematical nature such as engineering, architecture, natural sciences at tertiary institutions should not take ML. Issues dealt with in this overview are the subjects' premises, learning outcomes and topics to be covered as well as the pedagogical approach for teaching ML and the ML learner profile.

***The premises of ML and Mathematics***

In Table 2.2 below the premises of Mathematics and ML are discussed according to the subjects' purposes, aims, definitions and their educational and career links as set out by the DoE (2003a, 2003b, 2008b, 2011a, 2011b).

**Table 2.2: The premises of ML and Mathematics**

| | ML | Mathematics |
|---|---|---|
| **Purpose** | Provide learner with an awareness and understanding of the role mathematics plays in the modern world, enabling learners to become self-managing people, contributing workers and participating citizens (DoE, 2003a, 2011a). | To create an appreciation of the discipline itself and a deeper understanding and successful application of knowledge and skills. This competence contributes not only to personal and social, but also to learners' scientific and economic development (DoE, 2003b). |
| **Aim** | To equip learners to understand information represented in mathematical ways and to solve problems encountered in everyday life and work situations (DoE, 2008b). ML learners should have the ability or skills to think mathematically, interpret, analyse and solve problems (DoE, 2003a). | To allow learners to develop into citizens who are able to deal with the mathematics that forms part of the society they live in and on their daily lives. It is more important for learners to acquire skills such as investigating, generalising and proving instead of only acquiring content knowledge for its own sake (DoE, 2003b). |

| | | |
|---|---|---|
| **Definition** | *Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems* (DoE, 2003a). | *Mathematics enables creative and logical reasoning about problems in the psychical and social world and in the context of mathematics itself … is based on observing patterns, with rigorous logical thinking, this leads to theories of abstract relations … enables us to understand the world and make use of that understanding in our daily lives* (DoE, 2003b). According to CAPS mathematics is *a language that makes use of symbols and notations for describing numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves* (DoE, 2011b, p. 10). |
| **Career links** | ML should not be taken by learners who intend to study mathematically based disciplines such as natural sciences and engineering. ML learners proceeding to Higher Education institutions will have developed the skills needed to deal effectively with mathematically related requirements in disciplines such as the social and life sciences (DoE, 2003a). | The subject provides a platform for linkages to Mathematics in Higher Education institutions. Mathematics is essential for learners who intend to pursue a career in the psychical, mathematical, computer, life, earth, space and environmental sciences or in technology. Mathematics also plays an important role in the social, management and economic sciences (DoE, 2003b). |

Studying these premises, it is clear that the two subjects are different in kind and should not be compared. Sidiropoulos (2008) is also of the opinion that the *distinction between ML and Mathematics is principally not a distinction in level, but a distinction in kind* (p. 208). Mathematics on the one hand is regarded as a purely academic subject with a reputation of being an abstract science, involving mathematical rigour and a high level of cognitive thinking and reasoning based on sound conceptual understanding of the content. ML on the other hand does not focus on abstract mathematical concepts but, instead, primarily on developing practical skills to use elementary mathematical content to find concrete solutions to numeric, spatial and statistical problems associated with everyday life experiences (DoE, 2011a; Maffessanti, 2009). A shared aim however is the development of competent learners who are able to use their mathematical knowledge to solve personal and social real-life problems.

### *The learning outcomes of ML and Mathematics*

With the introduction of ML in 2008, the similarities between the learning outcomes for ML and Mathematics, as seen in the table below, were a major concern as some people thought of ML as a lower grade Mathematics subject. The learning outcomes as they were applied from 2008 to 2011 for the Senior Phase in the GET band (Grades 8-9), ML and Mathematics, both in the FET band, are listed in Table 2.3 below (DoE, 2003a, 2003b; 2010):

**Table 2.3: Learning outcomes for ML and Mathematics**

| | **Mathematics (GET: Senior Phase)** | **ML (FET)** | **Mathematics (FET)** |
|---|---|---|---|
| **Learning outcome 1** | Numbers, Operations and Relationships | Number and Operations in Context | Number and Number Relationships |
| **Learning outcome 2** | Patterns, Functions and Algebra | Functional Relationships | Functions and Algebra |
| **Learning outcome 3** | Measurement | Space, Shape and Measurement | Space, Shape and Measurement |
| **Learning outcome 4** | Data Handling | Data Handling | Data Handling and Probability |

The first two columns show how the learning outcomes for ML build on the learning outcomes for Mathematics in the GET band (DoE, 2005). Some researchers are of the opinion that ML, being a new subject with a different focus, should not have used the same content-based learning outcomes as Mathematics as this scenario ended up being stumbling blocks to the teachers (Bowie & Frith, 2006; Christiansen, 2007; North, 2005). This concern about similar learning outcomes has been addressed in the new CAPS (DoE, 2011a) and is discussed in the paragraph below.

***Topics covered in ML and Mathematics***

Different concerns regarding the content-context issue have been expressed by academics prior to the new CAPS. There were questions about what content knowledge should be taught by the ML teachers, which contexts should they use (Geldenhuys, Kruger & Moss, 2009; Julie, 2006; Vithal & Bishop, 2006), and whether the content should determine the context or vice versa (Bowie & Frith, 2006; Graven & Venkat, 2007). Although these issues were not elucidated clearly in the original NCS for ML (DoE, 2003a), the new CAPS (DoE, 2011a) addresses these issues.

In ML the topics are divided into two groups, namely the **Basic Skills Topics** which *comprise elementary mathematical content and skills that learners have already been exposed to in Grade 9* and the **Application Topics** which *contain the contexts related to scenarios involving daily life, workplace and business environments, and wider social, national and global issues* (DoE, 2011a, p. 13). For this purpose it is necessary to list the content areas and topics covered in the Mathematics Senior Phase[8] as well as the ML and Mathematics in the FET Phase. Different terminology for content areas and topics is used across the different bands (DoE, 2011a; DoE, 2011b; DoE, 2010) as indicated in Table 2.4 below:

[8] The Senior Phase band includes Grade 7 through to Grade 9.

**Table 2.4: Comparison of the composition of ML and Mathematics across the different bands**

| COMPOSITION OF MATHEMATICS AND ML | | |
|---|---|---|
| **Senior Phase Mathematics** | **FET ML** | **FET Mathematics** |
| **Content areas:**<br>1. Number, Operations and Relations<br>2. Patterns, Functions and Algebra<br>3. Space and Shape (Geometry)<br>4. Measurement<br>5. Data Handling (Statistics) | **Basic Skills Topics:**<br>1. Interpreting and communicating answers and calculations<br>2. Numbers and calculations with numbers<br>3. Patterns, relationships and representations<br>**Application Topics:**<br>1. Finance<br>2. Measurement<br>3. Maps, plans and other representations of the physical world<br>4. Data handling<br>5. Probability | **Main content topics:**<br>1. Functions<br>2. Number patterns, sequences, series<br>3. Finance, growth and decay<br>4. Algebra<br>5. Differential calculus<br>6. Probability<br>7. Euclidean Geometry and measurement<br>8. Analytical geometry<br>9. Trigonometry<br>10. Statistics |
| **Content topics:**<br>Example: Exponents, Integers, Fractions etc. under number 1 above are called the content topics. | **Content topics:**<br>A range of content topics based on Senior Phase content topics only. | **Curriculum statement:**<br>Instead of using *Content topics*, *Descriptions* is used to explain the content under each main topic. Example: *Practical problems involving optimisation and rates of change* (DoE, 2011b, p. 11) under number 5 above. |

### 2.2.2.8 Summary

South Africa was the first country in the world to introduce ML as a school subject in 2006 in the FET band (Grades 10 to 12) (Christiansen, 2007). A major reason behind the implementation of a compulsory mathematics subject in the FET band is to improve the low level of learners' mathematical knowledge and mathematical literacy skills. One of ML's purposes is to provide the opportunity for each learner to become mathematically literate in order to effectively deal with *mathematically related requirements in disciplines such as the social and life sciences* (DoE, 2003, p. 11). In comparing the national and international perspectives the latter refers to various levels of specialised knowledge, skills and understanding that are required to apply formal mathematics to solve application problems in various contexts.

The approach to the teaching and learning of ML should provide opportunities to engage with mathematics in diverse contexts at a level that learners can access logically (DoE, 2003c). However, the teaching of ML in a contextualised and de-compartmentalised manner where the content topics are integrated, complicates the teaching of the subject as teachers lack the knowledge and skills to do so.

The *distinction between ML and Mathematics is principally not a distinction in level, but a distinction in kind* (Sidiropoulos, 2008, p. 208). Mathematics is regarded as a purely academic subject, an abstract science involving a high level of cognition. ML on the other hand does not focus on abstract mathematical concepts but primarily on developing practical skills to deal with everyday life experiences (DoE, 2011a; Maffessanti, 2009).

## 2.3 Teachers' instructional practices

The process of teaching and learning is extensive and involves many pedagogical concerns and influences. Teaching in general involves more than the activities in the classroom and includes activities such as working with parents, colleagues and engaging in professional development (Franke et al., 2007). However the instructional practice of the teacher occurs in the classroom where teachers' goals, knowledge and beliefs serve as driving forces behind their instructional efforts to guide and mentor learners in their search of knowledge (Artzt et al., 2008). Different terminology is used in the literature when referring to teachers' performances or the act of teaching in the classroom. Terminology such as teachers' behaviour, instructional behaviour, instructional practices, classroom practices, classroom processes, and classroom instruction are frequently used. Table 2.5 below provides a short definition of four of the frequently used terms when referring to teachers' practices:

**Table 2.5: Different terminology used for teachers' practices**

| | |
|---|---|
| **Classroom practice** | Focus is on three features, namely discourse, norms and building relationships (Franke et al., 2007). |
| **Classroom instruction** | *Involves interactions among teachers and students around mathematical subject matter* (Kilpatrick, 2001, p. 107). |
| **Classroom processes** | Interaction taking place between the teacher and learner and all the factors influencing this interaction (Koehler & Grouws, 1992). |
| **Instructional practice** | Refers to the qualitative dimensions of teacher behaviour regarding their teaching (Englert et al., 1992). |

The term "instructional practice" best portrays the focus of this study being the ML teachers' classroom behaviour. Englert et al. (1992) refer to teachers' instructional practices as teachers' qualitative dimensions in the teaching and learning process. Qualitative dimensions involve teachers' abilities to apply appropriate cognitive strategies in meaningful and purposive activities, promote classroom dialogues and adjust instruction as required, and establish classroom environments in which students cooperatively and collaboratively participate in enquiry-related activities. To examine teachers' instructional practices, Artzt et al. (2008) use a phase dimension framework that is built on three observable aspects of mathematical lessons, namely tasks, discourse and the learning environment[9] (Figure 2.3).

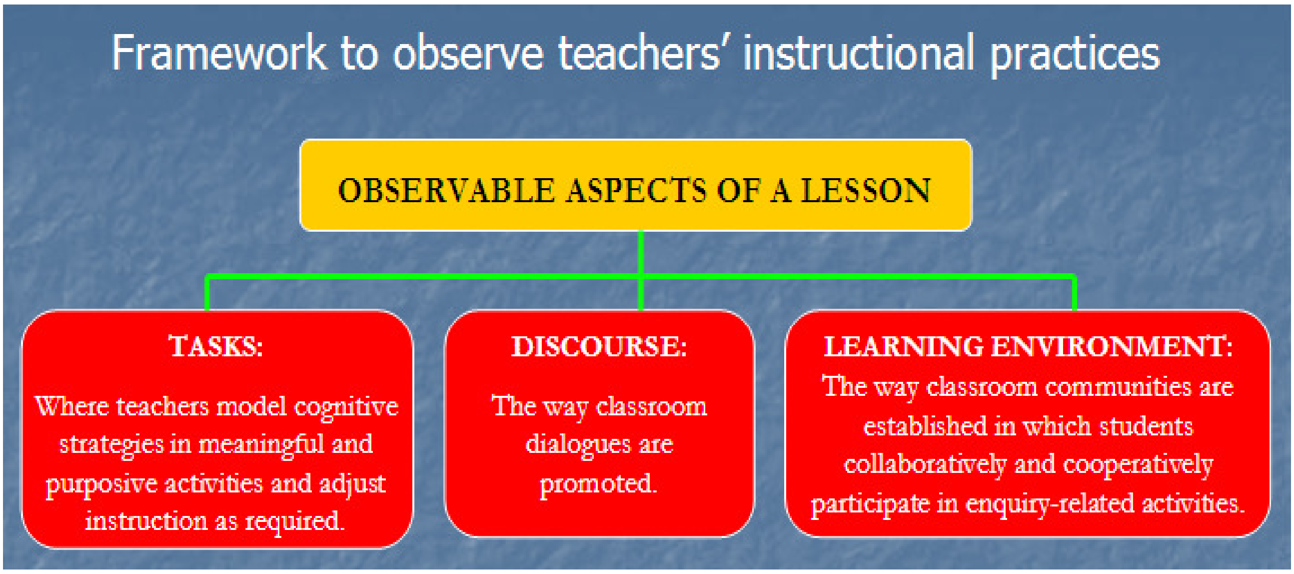


**Figure 2.3: Framework to observe teachers' instructional practices** (Adapted from Artzt et al., 2008; Englert et al., 1992)

In the light of my research paradigm of social constructivism which suggests that all knowledge is constructed and based upon not only prior knowledge, but also the cultural and social context (Ollerton, 2009), the participants' instructional practices are subsequently discussed. Franke et al. (2007) recognise a productive instructional practice as a practice creating ongoing opportunities for learning. There are different perceptions regarding the components of a teacher's instructional practice. Artzt et al.'s (2008) dimensions of instructional practices are tasks, discourse and learning environment whereas Franke et al. (2007) speak of discourse, norms and building relationships as the three features of classroom practices. From these two views the teachers' practices could be described as a social environment where all people in the classroom are in a relationship with one another, have the opportunity to construct and enhance their knowledge through communicating while solving and pursuing their conjectures of challenging tasks. For the purpose of my study the dimensions discussed by Artzt are most appropriate, since they address the practical issues of classroom practice which is fundamental in ML teaching.

[9] The characteristics of the observable aspects of a lesson are further discussed in par. 2.5.5.

I will now briefly discuss researchers' views on tasks, discourse and the learning environment and report on findings in the literature regarding the three aspects of ML teachers' lessons.

### 2.3.1 Tasks

Since knowledge is constructed and based upon, among other things, prior knowledge, the purpose of tasks is to *provide opportunities for learners to connect their knowledge to new information and to build on their knowledge and interest through active engagement in meaningful problem solving* (Artzt et al., 2008, p. 10).

#### *Modes of representation*

Franke et al. (2007) believe teaching *involves orchestrating the content*, that teachers' planning of their actions is crucial to enable learners to progress in their cumulative understanding of a particular content area (p. 228). According to Artzt et al. (2008) modes of representation are the forms for representing mathematical concepts *through the use of oral or written language, diagrams, manipulatives, computers, or calculators* (p. 12). Geldenhuys et al. (2009) recommended that teachers should increase the use of resources such as computers. Bransford, Brown and Cocking (2000) mentioned some people believe technology is money and time wasted whereas others regard the mere presence of computer technology in schools as enhancing the learning in the school. When computer technology is used correctly, Bransford et al. believe it has great potential to enhance student achievement. Since ML is related to real-life situations such as interest rates of home loans or personal income tax, computer technology could enhance the learners' understanding and interest in the subject and its application value as they could find the specific day's interest rates or even general information regarding income taxes.

#### *Motivational strategies*

The tasks teachers use in their lessons should *possess attributes that attract and sustain [the learners'] attention and emotional investment over time* (Artzt et al., 2008, p. 13). Dewey (as cited in Bransford et al., 2000) noted the following:

> *From the standpoint of the child, the great waste in school comes from his inability to utilize the experience he gets outside … while on the other hand, he is unable to apply in daily life what he is learning in school. That is the isolation of the school – its isolation from life* (p. 147).

In my view Dewey's concern is addressed by the DoE (2003a) when ML was implemented with its purpose of providing opportunities for learners to experience how mathematics relates to the world, enabling the learners to use mathematical information to make valuable decisions affecting their life, work and society (DoE, 2003a). The idea of connecting the school and home environments is consistent with Moll and Gonzalez (2004) who argued that teachers need to know and understand their learners' home environments which could be used to understand the learners' participation in the classroom. Especially in ML where the emphasis is on content being taught in context and making the subject applicable to real-life situations (DoE, 2003a), teachers need to take into consideration the knowledge their learners bring to their classrooms.

***Sequencing and difficulty levels***

The difficulty levels and sequencing of tasks *must allow students to use their past knowledge and experience to help them understand the requirements of the task* (Artzt et al., 2008, p. 13). Bransford et al. (2000) mentioned that tasks must be at the appropriate level of difficulty in order for learners to remain motivated. They stated too easy tasks cause learners to become bored while too difficult tasks cause frustration. Hechter (2011b) reported that the cognitive levels of the assessment tasks set by both teachers in her study were on a relatively low level. Bansilal (2008), whose study consisted of an analysis of the answers given by 38 ML teachers to various questions taken from a test and the final examination in a module of their Advanced Certificate in Education (ACE) (ML) programme, revealed that teachers found questions which had multi-steps, difficult.

## 2.3.2 Discourse

To contribute to learner understanding, the discourse in class should provide opportunities for learners to express themselves, to listen to, to question, to respond and to reflect on their thinking (Artzt, et al., 2008). Franke et al. (2007) believed classrooms involve *people who work in social, cultural and political contexts that shape how they do their work and how that work gets interpreted* (p. 227).

***Teacher-learner interaction***

The teacher plays a critical role in orchestrating discourse in class and should know how to use verbal and non-verbal strategies to communicate effectively (Artzt, et al., 2008). According to Franke et al. (2007) teaching is multifaceted and teaching should be seen as *deliberate work*, where the teacher should orchestrate the content, the representations of the content, as well as all people in the classroom in relation to one another. They mentioned that teachers need to have the ability to elicit and interpret what learners do and know, to act appropriately on that and be able to make decisions emerging from complex interactions. They do not regard learning as receiving information but rather as engaging in sense-making as the teacher and learners participate together. Although my study was not concerned with the influence of teacher-learner interaction on learners' performance, it is worth noting that Bansilal et al. (2010) found in their study that the continuous support and feedback the tutors provided to the practising ML teachers (students) in the ACE (ML) programme improved the students' performances over the semester. Bansilal et al. (2010) regarded this increasing interaction in communities of practice as providing positive learning opportunities to the students.

***Learner-learner interaction***

Contributing to learners' development of conceptual understanding are the opportunities learners have to interact with each other in such ways that they can support, strengthen and challenge each others' ideas (Artzt, et al., 2008). Lampert (2004) mentioned that the practice of teaching is not only about the *actions* of the teacher but the *evolution of relationships* between the teacher and learners and among learners themselves around mathematics *and engaging together in constructing mathematical meaning* (p. 2). Franke et al. (2007) expressed their concern that many mathematics classrooms do not provide sufficient opportunities for learners to develop mathematical understanding. They believe learners must have the opportunity to become encouraged and curious and *talk about and with mathematical expertise* (p. 229). National researchers emphasise the importance of learner-centred approaches where learners are involved in the lesson, taking part in discussions and group work (Brown & Schäfer, 2006; Venkat, 2007; Venkat & Graven, 2008).

***Questioning***

The value of proficient oral questioning is that *the teacher encourages students to make public their knowledge, skills, and attitudes in relation to the problem under consideration* (Artzt, et al., p. 16). Knowledge of learners' mathematical thinking will support the teachers to provide opportunities for asking questions which are linked to the learners' thinking, will elicit discussion and will draw on connections learners need to make to comprehend the work (Franke et al., 2007).

## 2.3.3 Learning environment

In my study I based my rationale for a learning environment on the work of Artzt et al. (2008) who state that a learning environment comprises a particular social and intellectual climate, the use of effective modes of instruction and pacing of the content and attending to certain administrative routines. Bransford et al. (2000) on the other hand, regard a learning environment as involving the rethinking of what should be taught, how it should be taught and how it should be assessed. When these two views are compared, a common aspect is **how** the content should be taught and **what** should be taught which forms part of the tasks, and how learners should be **assessed** does not form part of Artzt et al.'s learning environment.

***Social and intellectual climate***

*The social and intellectual climate defines the tone, style, and manner of the interpersonal interactions in the classroom* and contributes to learners' *social and cognitive growth and development* (Artzt et al., 2008, p. 14). Franke et al. (2007) stated that productive practices occur where learners *see themselves as comfortable, confident, and knowledgeable in their abilities to engage in mathematics* (p. 227). Silver, Smith and Nelson (1995) found that creating an atmosphere of trust and mutual respect was critical for the development of valuable discourse between the teacher and learners and among learners themselves.

***Modes of strategies and pacing***

Modes of strategies and pacing are the strategies teachers use in the classroom to help learners attain the objectives of the lesson and teachers should properly pace the activities so that learners have enough time to participate and construct new knowledge (Artzt et al., 2008). The use of cognitively guided instruction is suggested by researchers to support the development of learners' mathematical understanding (Carpenter et al., 2000; Bransford et al., 2000; Franke et al., 2007). This approach to teaching assists learners to overcome their misunderstandings and effectively change conceptual misconceptions. Another effective strategy mentioned is interactive lecture demonstrations (Franke et al., 2007). Nationally some researchers proposed effective strategies for teaching ML, namely mathematical modelling (Brown & Schäfer, 2006); discussions and group work (Venkat & Graven, 2008); co-operative learning (Frith & Prince, 2006) and project work (Vithal, 2006).

***Administrative routines***

According to Artzt et al. (2008) administrative routines are procedures or activities in classroom organisation and management. Kounin and Gump (1974) regard these routines as providing an ongoing sign of organisational and interpersonal behaviour in class.

## 2.4 Mathematics teachers' knowledge and beliefs about mathematics and the teaching thereof

In this section I mention the relationship between knowledge and beliefs, give an overview of different domains of teachers' knowledge, and discuss what is meant by teachers' belief systems. Lastly, I point out what the influence of teachers' knowledge and beliefs is on their instructional practices.

### 2.4.1 Relationship between knowledge and beliefs

There is no agreement on the definitions of knowledge and beliefs, their relationship or even their influence on teaching (Gess-Newsome, Lederman & Gess-Newsome, 2002). She points out some differences and relationships between knowledge and beliefs (Table 2.6) and emphasises that in practice the lines between knowledge and beliefs can easily become blurred.

**Table 2.6: Relationship between knowledge and beliefs**

| | KNOWLEDGE | BELIEFS |
|---|---|---|
| Described as: | Evident, dynamic, emotionally neutral, internally structured. | Both evidential and non-evidential, static, emotionally bound, organised into systems. |
| Develops with: | Age and experience | Episodically |
| Functions: | • Conceptual knowledge (knowledge that is rich in relationships) is used in problem solving situations.<br>• The amount, accessibility and organisation thereof distinguish experts from novices. | • Have both affective and evaluative functions;<br>• Act as information filters;<br>• Have an impact on how knowledge is used, organised and retrieved;<br>• Are powerful predictors of behaviour which can either be consistent or inconsistent with beliefs. |

Artzt et al. (2008) define teacher knowledge *as an integrated system of internalised information acquired over time about pupils, content and pedagogy* and beliefs are defined *as an integrated system of internalised assumptions about the subject, the students, the learning, and teaching* (p. 20). They further believe that *beliefs function as an interpretative filter for teachers' goals and knowledge and strongly affect classroom practice* (p. 20). Their views on knowledge and beliefs correspond with Gess-Newsome et al.'s (2002) except that Artzt et al. (2008) also describe knowledge as organised into systems.

Liljedahl (2008) strongly believes that any discussion on a teacher's knowledge cannot be restricted to knowledge of mathematics and knowledge of teaching mathematics but needs to include a discussion on teacher's beliefs. He believes teachers' actions in the classroom are strongly guided by what they believe about mathematics and the teaching thereof. He further states that it is a false dichotomy to distinguish between knowledge and beliefs, as a belief becomes knowledge once the *truth criterion is satisfied* (p. 2). Leatham (2006, p. 92) explains this argument as follows:

> *Of all the things we believe, there are some things that we 'just believe' and other things we 'more than believe – we know'. Those things we 'more than believe' we refer to as knowledge and those things we 'just believe' we refer to as beliefs. Thus beliefs and knowledge can profitably be viewed as complementary subsets of the things we believe.*

Borko and Putnam (1996) focus on two interrelated aspects of knowledge and beliefs. They argue that prospective and experienced teachers' knowledge and beliefs serve as filters through which their learning takes place and on the other hand knowledge and beliefs themselves are critical targets of change.

In student teacher training it is important that both students' mathematical knowledge and beliefs need to be developed and restructured. In my experience once a student's mathematical knowledge base is enhanced, the new or enriched knowledge influences the student's beliefs about mathematics, reorganising and broadening the student's existing belief system[10]. On the other hand when a student's beliefs about mathematics are restructured, they sometimes become more receptive to new mathematical knowledge.

### 2.4.2 Overview of the different domains of teachers' knowledge

The most fundamental aspect in effective and proficient teaching of mathematics is a high level of knowledge (Kilpatrick, 2001; Taylor, 2008). A teacher needs proper subject matter knowledge and a high level of PCK to assure effective teaching (Shulman, 1986; Ma, 1999). In Taylor's (2008) study short tests in literacy and mathematics amongs others were conducted in primary and secondary schools throughout South Africa and his finding was that teachers clearly do not have the knowledge that the curricula require to proficiently teach the learners. To address this problem of teachers' inadequacy, the school system has to re-establish the emphasis on expert knowledge (Taylor, 2008). Mathematics teaching is a specialised profession, requiring content knowledge, knowledge of the curriculum, knowledge about how to teach mathematics and knowledge about how learners learn mathematics. The question is how these different categories of mathematical knowledge are organised. Some of the leading mathematics researchers' categories or domains of mathematical knowledge are given in Table 2.7 below with a brief summary of each.

[10] Beliefs systems are discussed in Section 2.4.3.2.

**Table 2.7: Overview of different domains of mathematical knowledge**

| OVERVIEW OF DIFFERENT DOMAINS OF MATHEMATICAL KNOWLEDGE | | | | |
|---|---|---|---|---|
| **SHULMAN**<br>**Categories of knowledge**<br>**1986** | **GROSSMAN**<br>**Components of PCK**<br>**1990** | **BORKO AND PUTNAM**<br>**Domains of knowledge**<br>**1996** | **BALL, THAMES AND PHELPS**<br>**Domains of knowledge of teaching**<br>**2005** | **HILL, BALL AND SCHILLING**<br>**Domain map for mathematical knowledge for teaching**<br>**2008** |
| 1. Subject matter content knowledge | 1. Purposes for teaching mathematics | 1. General pedagogical knowledge and beliefs | 1. Subject matter knowledge<br>• Common knowledge of mathematics content<br>• Specialised knowledge of mathematics content | 1. Subject matter knowledge<br>• Common content knowledge<br>• Specialised content knowledge<br>• Knowledge at the mathematical horizon |
| 2. PCK | 2. Learners' understanding, conceptions and misunderstandings | 2. Subject matter knowledge and beliefs | | |
| 3. Curricular knowledge | 3. Curriculum and curricular materials | 3. PCK and beliefs | 2. PCK<br>• Knowledge of content and students<br>• Knowledge of content and teaching | 2. PCK<br>• Knowledge of content and students<br>• Knowledge of content and teaching<br>• Knowledge of curriculum |
| | 4. Instructional strategies and representations for teaching topics | | | |

### 2.4.2.1 Shulman's (1986) categories of content knowledge

Shulman (1986) initiated the debate on different categories of knowledge a mathematics teacher needs. Figure 2.4 indicates his three categories of content knowledge, namely 1) subject matter content knowledge; 2) PCK; and 3) curricular knowledge.

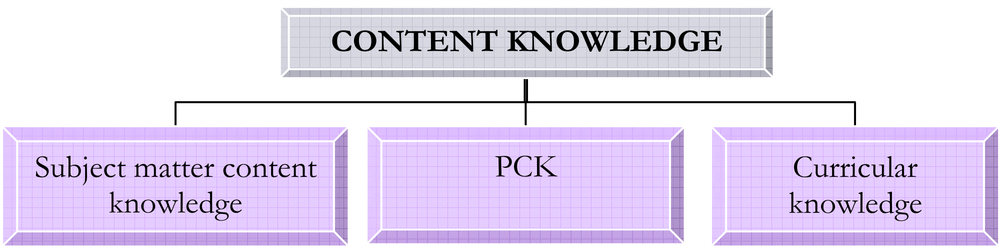


**Figure 2.4: Shulman's (1986) three categories of content knowledge**

**Subject matter content knowledge** as one of the categories of content knowledge goes beyond knowledge of the facts or concepts of a domain to understand the structures of the subject matter. The second category **PCK** refers to pedagogical knowledge that goes beyond subject matter knowledge to subject matter knowledge for teaching, also called teachers' professional knowledge. This knowledge includes *the most useful forms of representation of ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that makes it comprehensible to others* (Shulman, 1986, p. 9). The ability to use different representations may be derived from research or from years of experience in practice. This knowledge further includes a teacher's understanding of why certain topics are comprehensible and others not, and what preconceptions learners have that may be misconceptions that could actually be rectified and reorganised by the teacher through the use of different strategies. Shulman (1987) further describes PCK as *the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students* (p. 15), in other words, the knowledge of how to make the subject comprehensible to others. The third category of knowledge, **curricular knowledge**, refers to the knowledge about the full range of programmes designed for the teaching of different topics at given levels in a subject area. It further includes knowledge regarding the variety of instructional materials available to teach particular curriculum components. It is imperative for teachers to be familiar with the topics and their levels being taught in the same subject during the preceding and subsequent years in school. Teachers also need to be familiar with the curriculum materials studied by learners in other subjects at the same time (Shulman, 1986). Whereas Shulman's work is foundational in this area, other mathematics researchers' categorisations of mathematical knowledge needed for teaching are discussed below.

### 2.4.2.2 Grossman's (1990) components of PCK

Grossman (as cited in Sowder, 2007) (Figure 2.5) distinguishes between four components of PCK, namely 1) purposes for teaching mathematics; 2) learners' understandings, conceptions and potential misunderstandings; 3) curriculum and curricular materials; and 4) instructional strategies and representations for teaching particular topics.

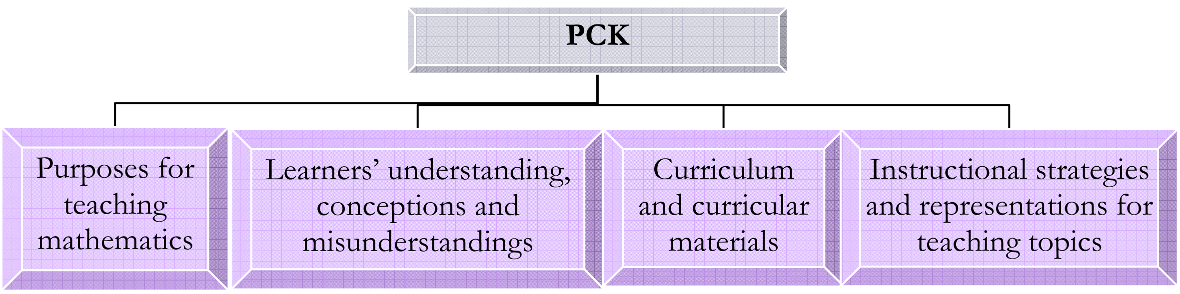


**Figure 2.5: Grossman's (1990) four components of PCK**

Borko and Putnam (1996) believe the **first component** serves as a conceptual map for the teacher's instructional decision-making, and as a basis for making decisions regarding classroom objectives, instructional strategies, student assignments, textbooks, curricular materials and the evaluation of student learning. This is a salient component of the professional knowledge base of teachers as it concerns teachers' knowledge about the nature of the subject and what is important for students to learn. The **second component** is knowledge a proficient teacher has to predict what mathematics learners will understand, how they will understand it, and what their potential misunderstandings will be. This knowledge enables a teacher to (Sowder, 2007):

> … *plan more effectively because they can anticipate learners' difficulties. They know what prior knowledge must be present to understand something new. They know how to scaffold knowledge to assist students in developing understanding. They know how to listen to students. Much of this knowledge comes from practice, but teachers with poor understanding of mathematics are unlikely to develop this type of knowledge, particularly when the mathematics in the curriculum becomes more sophisticated* (p. 165).

Teachers need to have an understanding of learners' preconceptions, misconceptions, and alternative conceptions of specific topics (Borko & Putnam, 1996). The **third component** includes the ability of teachers to recognise the particular strengths and weaknesses of textbooks and materials they use. Competent teachers normally have a collection of materials they use when teaching mathematics. This component also includes knowledge of how the topics are organised and structured both horizontally and vertically, i.e. within a grade level and across grades. The **fourth component** is characterised as a wide selection of significant representations and the ability to adapt these representations in various ways in order to meet specific goals for specific learners (Borko & Putnam, 1996).

#### 2.4.2.3 Borko and Putnam's (1996) domains of knowledge

The framework (Figure 2.6) used by Borko and Putnam (1996) in their study "Learning to teach" was loosely based on Shulman's categories of knowledge. The proposed domains are 1) general pedagogical knowledge and beliefs; 2) subject matter knowledge and beliefs; and 3) PCK and beliefs. These three domains encompass teachers' knowledge of teaching, subject matter and learners, the three major determinants of what teachers do in their classrooms.

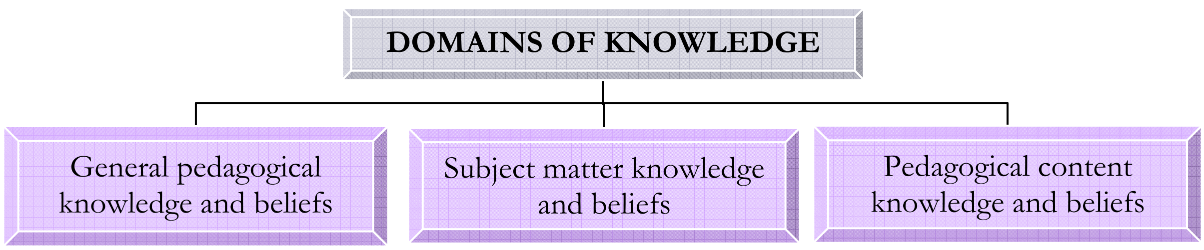


**Figure 2.6: Borko and Putnam's (1996) three domains of knowledge**

The **first domain** does not form part of this study's focus but refers to a teacher's knowledge and beliefs about teaching and learning in general, which include knowledge of strategies for effective classroom management, various instructional strategies for specific lesson topics, how to create a positive learning environment and most fundamental a thorough knowledge of learners, of how they learn and how learning can be fostered by teaching. Borko and Putnam (1996) do not prescribe a specific model or set of categories to be used regarding the **second domain** of knowledge, as long as the need to know more than just facts, terms and concepts of a discipline is recognised. Key aspects in this domain include knowledge of how to organise ideas, how to make connections among ideas and knowing different ways of thinking and argumentation. They briefly refer to the work of Shulman in 1986 as well as Ball in 1990 and 1991. Regarding their **third domain**, they again discuss the work of Shulman in 1986 and that of Grossman in 1990 emphasising the importance of PCK for teachers who want to teach for understanding.

### 2.4.2.4 Ball, Thames and Phelps' (2005) domains of knowledge for teaching

According to Silverman and Thompson (2008), Shulman invented the term "PCK" referring to specific content knowledge as applied to teaching. Since then many researchers within the field of mathematics teacher education have been developing this notion with special reference to the work of Ball in 1990 as well as Ball and Bass in 2000. Ball et al. (2005) use the term "knowledge of mathematics for teaching" when referring to the special knowledge needed to teach mathematics for understanding. *Their pioneering work has succeeded in identifying various examples of special ways in which one must know mathematical procedures and representations to interact productively with students in the context of teaching* (Thompson, 1992, p. 500). Ball et al. (2005) divide knowledge of mathematics for teaching in two domains, namely 1) subject matter knowledge and 2) PCK where each domain is divided in two sub-domains as indicated in Figure 2.7 below.

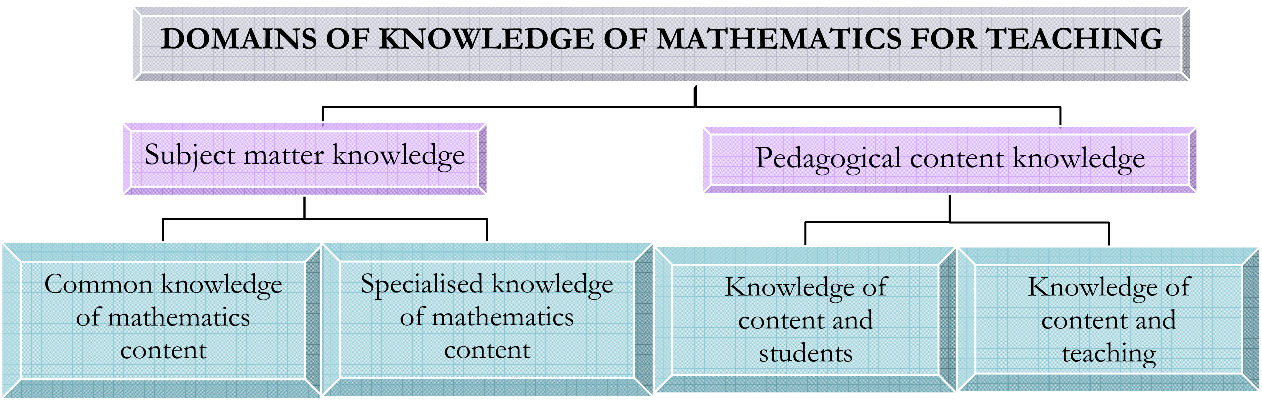


**Figure 2.7: Ball, Thames and Phelps' (2005) domains of knowledge of mathematics for teaching**

#### 2.4.2.5 Hill, Ball and Schilling's (2008) domain map for mathematical knowledge for teaching

This overview concludes with the domain map for mathematical knowledge for teaching of Hill et al. (2008) as indicated in Figure 2.8. Similar to Ball et al. (2005), they also divide knowledge into two domains, namely 1) subject matter knowledge and 2) PCK, but included an additional subdomain under each domain. **Subject matter knowledge** now consists of 1) common content knowledge; 2) specialised content knowledge and 3) knowledge at the mathematical horizon. Common content knowledge involves knowing central facts, concepts and principles within a relationship while specialised content knowledge goes beyond common content knowledge. Teachers need to have specialised knowledge to know more than just explaining the content, but must be able to explain why it is so, why it is worth knowing and how to relate it to other learning outcomes and other disciplines, both in theory and practice. Knowledge at the mathematical horizon refers to having knowledge of the subject beyond the years for which a teacher is responsible for. **PCK** is now divided into 1) knowledge of content and students; 2) knowledge of content and teaching and 3) knowledge of the curriculum.

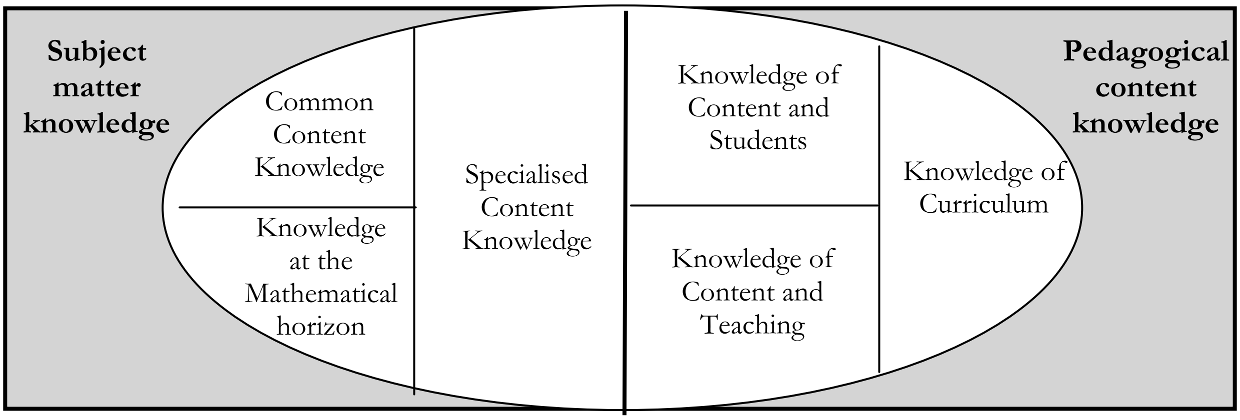


**Figure 2.8: Hill, Ball and Schilling's (2008) domain map for mathematical knowledge for teaching**

My study is based on the PCK domain and is discussed under the heading "Conceptual framework"[11]. Although the focus of this study is not on ML teachers' subject matter knowledge, the value of teachers having a deep knowledge base is still recognised as part of their complete cognitive knowledge base.

#### 2.4.2.6 Summary

Borko and Putnam (1996) argue that the increased attention to teachers' knowledge in recent years has led to multiple schemes for categorising teachers' mathematical knowledge. One must bear in mind that any categorisation of teachers' knowledge is somewhat arbitrary, that there is no single definite system of categorisation of knowledge and that the boundaries between these categorisations are very vague

[11] The conceptual framework is discussed in Section 2.5.

(Borko & Putnam, 1996; Hill et al., 2008; Shulman, 1986; Sowder, 2007). Although knowledge is categorised in different domains, these domains are interwoven in teachers' instructional practices and teachers continually draw on all aspects of their knowledge (Koellner et al., 2007).

## 2.4.3 An overview of mathematics teachers' beliefs about mathematics and the teaching thereof

Thompson (1992) emphasises the complexity involved in distinguishing between beliefs and knowledge and found that in many cases teachers treat their beliefs as knowledge. There is no agreement on how beliefs are to be evaluated, as beliefs cannot be directly observed or measured, but must be inferred from what people say, intend and do (Pajares, 1992; Thompson, 1992). In this section I discuss the nature of beliefs and what is meant by teachers' belief systems.

### 2.4.3.1 The nature of beliefs

According to Pajares (1992, p. 316) beliefs are formed *through a process of enculturation*[12] *and social construction* and influence a person's perceptions, behaviour and the processing of new information. He suggests that beliefs created by individuals years ago are fixed and difficult to change whereas newly formed beliefs are most vulnerable. Listed below are some of the inferences and generalisations Pajares made regarding teachers' educational beliefs:

- *Beliefs are formed early and tend to self-perpetuate, persevering even against contradictions caused by reason, time, schooling, or experience.*
- *The belief system has an adaptive function in helping individuals define and understand the world and themselves.*
- *Epistemological beliefs play a key role in knowledge interpretation and cognitive monitoring.*
- *Beliefs are prioritised according to their connections or relationship to other beliefs or other cognitive and affective structures. Apparent inconsistencies may be explained by exploring the functional connections and centrality of the beliefs.*
- *By their very nature and origin, some beliefs are more incontrovertible than others.*
- *Belief change during adulthood is a relatively rare phenomenon, the most common cause being a conversion from one authority to another or a gestalt shift. Individuals tend to hold on to beliefs based on incorrect or incomplete knowledge, even after scientifically correct explanations are presented to them.*
- *Beliefs must be inferred, and this inference must take into account the congruence among individuals' belief statements, the intentionality to behave in a predisposed manner, and the behaviour related to the belief in question.*
- *Beliefs about teaching are well established by the time a student gets to college* (p. 324-326).

In Table 2.8 below, Schoenfeld (1988, p. 151) mentions four general beliefs held by learners and their effects in practice.

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[12] Enculturation involves the incidental learning process individuals undergo through their lives and includes their assimilation through individual observation, participation and imitation (Pajares, 1992).

**Table 2.8: Some beliefs held by learners and their effects in practice**

| BELIEF | EFFECTS IN PRACTICE |
|---|---|
| *The processes of formal mathematics (e.g. 'proof') have little or nothing to do with discovery or invention.* | *Students fail to use information from formal mathematics when they are in 'problem solving mode'.* |
| *Students who understand the subject matter can solve assigned mathematics problems in five minutes or less.* | *Students stop working on a problem after just a few minutes since, if they haven't solved it, they didn't understand the material (and therefore will not solve it).* |
| *Only geniuses are capable of discovering, creating, or really understanding mathematics.* | *Mathematics is studied passively, with students accepting what is passed down 'from above' without the expectation that they can make sense of it for themselves.* |
| *One succeeds in school by performing the tasks, to the letter, as described by the teacher.* | *Learning is an incidental by-product to 'getting the work done'.* |

Learners cannot be blamed for holding such beliefs as many teachers and parents also hold these or similar beliefs. Beliefs play an important role in how people view ML. This is an influential factor in the success of ML as some teachers and learners have their view of ML influenced by the comments from people outside the mathematics field, and thus see ML as a worthless and insignificant subject.

#### 2.4.3.2 Teachers' belief systems

Leatham (2006) argues that the way an individual's various beliefs are related to each other is just as important as what the individual believes. A belief system according to Thompson (1992) consists of conscious and subconscious beliefs, preferences concerning mathematics as discipline, concepts, meaning, rules, and mental images. In Thompson's (1992) study on teachers' conceptions consisting of beliefs, views and preferences, she points out that some people's actions and behaviours are influenced by the nature of their beliefs. She further describes a belief system as *a metaphor for examining and describing how an individual's beliefs are organised* (p. 130). She also typifies such systems as being dynamic in nature because they undergo change and restructuring as individuals evaluate their beliefs against their experience. According to Ball (1988) and Thompson (1992) preservice teachers' beliefs are formed through the development of a network of interrelated ideas about mathematics, the teaching and learning thereof and also through their experiences at schools.

### 2.4.4 The influence of teachers' knowledge and beliefs on their instructional practices

Pajares (1992) acknowledges the complexity of a psychological construct such as beliefs, but through his extensive study of numerous researchers' findings, he found *a strong relationship between teachers' educational beliefs and their planning, instructional decisions, and classroom practices* (p. 326), although the link to learner outcomes has not been explored extensively. Artzt et al. (2008) refer to teachers' goals, knowledge and beliefs as teachers' cognitions and describe them as *the driving forces* (p. 17) behind teachers' instructional practices. In this section I mention the influence of mathematics teachers' knowledge and beliefs on their learners as well as their teaching of the subject. I also report some findings from South African studies regarding the influence of ML teachers' knowledge and beliefs on their instructional practices.

#### 2.4.4.1 The influence of teachers' knowledge and beliefs on the learners

Learners' beliefs were for the most part consistent with the beliefs and views held by their teachers (Thompson, 1992; Ford, 1994). Ford refers to a study he conducted in which teachers regard good problem solvers as the smarter learners. He found that this belief was then adopted by learners who claim that you need to be smart to be able to solve problems. This finding is supported by Mason's (2003) study in which learners with low achievement comment that in mathematics, intelligence counts 90% and effort 10% and the intelligence a person is born with, can be exploited but not improved, so a person either can or cannot do mathematics.

Teachers need to accept and acknowledge their responsibility towards learners and need to provide learners with opportunities for positive learning experiences. The teacher's attitude towards the subject is also significant. The teacher has the responsibility to ensure that mathematics comes alive, that learners find it constructive and develop a passion for the subject. Ollerton (2009) argues that teachers cannot force learners to have a positive relationship with their subject but they need to realise that they have a *massive impact* (p. 2) on their learners. The teacher has the knowledge and skills to create a positive learning atmosphere where sufficient opportunities are provided to build this relationship. In order to do this, teachers need a positive attitude towards the subject and its learners.

#### 2.4.4.2 The influence of teachers' knowledge and beliefs on their teaching

In practice teachers spontaneously convey their ideas on mathematics to their learners (Ball, 1991). Teachers' beliefs about mathematics and the teaching thereof often serve as a foundation on which their instructional practices are built (Liljedahl, 2008; Pajares, 1992). Liljedahl mentions four researchers' notions of teachers' beliefs which in principle are very similar, each notion consisting of three different perspectives. Dionne's (1984) notion is divided into the traditional, formalist and constructivist perspective. Ernest's (1988) notion describes three philosophies of mathematics, namely instrumentalist, Platonist and problem solving while Törner and Grigutsch (1994) name their three perspectives the toolbox aspect, system aspect and process aspect, which are described as follows:

> *In the toolbox aspect mathematics is seen as a set of rules, formulae, skills and procedures while mathematical activity means calculating as well as using rules, procedures and formulae. The system aspect refers to teachers who*

> *believe mathematics is characterised by logic, rigorous proofs, exact definitions and a precise mathematical language and doing mathematics consists of accurate proofs as well as of the use of a precise and rigorous language. The process aspect refers to teachers who believe mathematics is a constructive process where relations between different notions and sentences play an important role. Here the mathematical activity involves creative steps, such as generating rules and formulae, thereby inventing or re-inventing the mathematics.* (Liljedahl, 2008, p. 2-3)

Beliefs regarding the nature of mathematics influence a teacher's choice of teaching approach. Teachers holding a traditional belief most probably believe that mathematics is an abstract phenomenon that is far distant from reality. These teachers will then struggle to relate mathematics to real-life situations and tend to believe mathematics consists of a set of rules and procedures that must be learned mechanically with little or no connection to each other and hardly any relevance to their everyday lives. They also tend to separate mathematics from the discipline of discovery and creativity (White & Mitchelmore, 2002; Mason, 2003; Schoenfeld, 1988). Thom (as cited in Golafshani, 2002), also claims that *all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics* (p. 204). He comments on disparities that do occur where teachers' conceptions are not reflected in their instructional practices due to constraints such as fixed curricula, time pressure and other external factors.

***Findings regarding the influence of ML teachers' knowledge and beliefs on their teaching of ML***

Sidiropoulos (2008) found that ML teachers' instructional practices were neither aligned to the ML curriculum nor to their alleged beliefs and understanding. She states that external strategies and interventions that promote the required depth of ML teachers' understanding are required to change their instructional practices. She further found that the negative and low expectations those teachers have of their learners negatively affected their implementation of the curriculum in class. One of the two teachers believed that everyone could do ML if taught properly, but when that teacher was asked about his learners' poor performance, the blame was put on learners' past history with mathematics. Mhlolo (2008) believes that ML teachers were not equipped with conceptual skills required for the implementation of the subject and that they need to re-conceptualise their knowledge and beliefs about the subject. He further states that there is a problematic relationship between the idealised teacher in policy documents and teachers' personal identities. He calls it a mismatch, dislocation or disjuncture between espoused policy images and the personal identities of teachers. Although there are many ML teachers who do not meet the requirements as set out by the DoE, there are research studies such as those of Venkat and Graven (2007), telling stories of successful ML teachers.

Venkat and Graven (2007) report on their longitudinal study performed at an inner city school in Johannesburg on the difference positive and knowledgeable ML teachers make to learners' experience of the subject. They suggest learner negativity is associated with a lack of substantive change in teachers' pedagogic practice, that is where teachers still *incorporate the kinds of tasks and pedagogic practice that have predominated within learners' earlier experiences with Mathematics* (p. 81).

### 2.4.5 Summary

Liljedahl (2008) strongly believes that any discussion on a teacher's knowledge cannot be restricted to knowledge of mathematics and knowledge of teaching mathematics, but needs to include a discussion on teacher's beliefs. Different categories or domains of mathematical knowledge exist but any categorisation of teachers' knowledge is somewhat arbitrary as there is no single true system of categories and the boundaries between these categorisations are usually very vague (Borko & Putnam, 1996; Hill et al., 2008; Shulman, 1986; Sowder, 2007). The different categories of a teacher's knowledge are also interwoven in their instructional practices and teachers continually draw on all aspects of their knowledge (Koellner et al., 2007).

Beliefs consist of conscious and subconscious beliefs as well as preferences concerning mathematics as a discipline. Beliefs can further be defined as *convictions or opinions that are formed either by experience or by the intervention of ideas through the learning process* (Ford, 1994, p. 315). Teachers' beliefs about mathematics can be located on a perspective continuum from a traditional to a formalist, to a constructivist perspective (Dionne 1984).

Knowledge and beliefs are closely related and there is a constant interplay between the two, both influencing teachers' instructional practices. Borko and Putnam (1996) argue that on the one hand prospective and experienced teachers' knowledge and beliefs serve as filters through which their learning takes place and on the other hand knowledge and beliefs themselves are critical targets of change.

## 2.5 Conceptual framework

The focus of my study is to determine the relationship between ML teachers' knowledge and beliefs and their instructional practices. My conceptual framework (Figure 2.9) is based on an amalgamation of Artzt et al.'s (2008) phase dimension framework[13], Franke et al.'s (2007) view of a productive practice and Hill et al.'s (2008) domain map for mathematical knowledge for teaching[14].

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13 See Section 2.3: Teachers' instructional practices.
14 See Section 2.4.2.5: Hill, Ball and Schilling's (2008) domain map for mathematical knowledge for teaching.

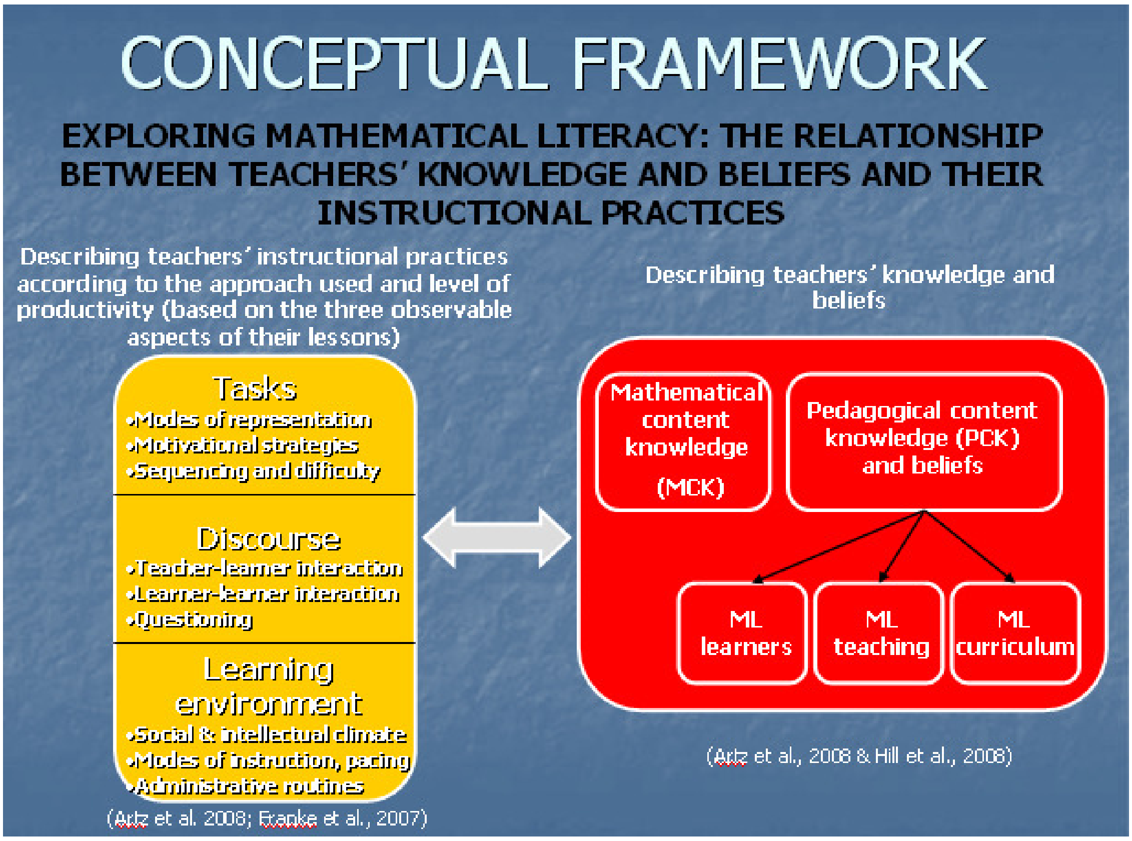


**Figure 2.9: Conceptual framework: Instructional practice, knowledge and beliefs framework of analysis** (adapted from Artzt et al., 2008; Franke et al., 2007; Hill et al., 2008)

### *Explanation of my conceptual framework*

I believe teachers need to apply appropriate instructional strategies to provide learners with opportunities to develop their critical thinking and problem solving skills. Figure 2.9 illustrates the components of, and logic behind my framework. To enable me to determine the relationship between ML teachers' instructional practices and their knowledge and beliefs, their practices can be observed in terms of **tasks**, **discourse** and the **learning environment**[15]. From these observations the ML teachers' instructional practices can be described according to the instructional approach used and level of productivity of their practices. Teacher's instructional approaches will be described as either teacher-centred, learner-centred or a combination of teacher- and learner-centred (Artzt et al., 2008). The level of productivity of the teachers' instructional practices will be described based on Franke et al.'s (2007) view of a productive practice: A practice where the teacher listens to learners' mathematical thinking and aims to use it to encourage conversation that revolves around the mathematical ideas in the sequenced problems. Subsequently I will deal with some of the driving forces behind their lessons, namely teachers' knowledge and beliefs concerning content and learners[16], content and teaching[17] and

[15] See Section 2.5.5: Teachers' instructional practices.
[16] See Section 2.5.2: PCK and beliefs regarding content and learners.

the curriculum[18]. The three segments of PCK and beliefs, namely knowledge and beliefs of the ML learners, ML teaching and the ML curriculum, are strongly influenced by teachers' idiosyncratic beliefs about the nature of mathematics as a discipline and ML as subject. These idiosyncratic beliefs can typically be located on a perspective continuum from traditional to formalist to constructivist[19].

Included in my framework is Hill et al.'s (2008) PCK domain (learners, teaching and curriculum in Figure 2.9) and teachers' MCK which is similar to Hill et al.'s (2008) common content knowledge as part of their subject matter knowledge domain. The rationale for this decision is that ML focuses on solving contextualised problems using only basic mathematics. Notwithstanding the fact that ML teachers need to have MCK, the focus of my study is not on the assessment of their subject matter knowledge per se. PCK is defined by Hill et al. (2008) as teachers' *content knowledge intertwined with* (p. 375) knowledge of students; knowledge of teaching; and knowledge of the curriculum. Teachers' beliefs are integrated in my framework as I believe they are inseparable from teachers' knowledge. Incidentally, I excluded teachers' goals as part of teachers' cognitions in order to keep the study focused (Artzt et al., 2008).

Even though some of the headings in this section may come across as repetitive, the previous two sections focussed on some general views and background from the literature regarding mathematical knowledge, beliefs and instructional practices. In this section I relate the literature to my study concerning 1) a general view on mathematics teachers' PCK and beliefs; 2) PCK and beliefs regarding the learners, teaching and the curriculum; and 3) instructional practices.

## 2.5.1 General view on mathematics teachers' knowledge and beliefs

### 2.5.1.1 Mathematics teachers' MCK

Hill et al.'s (2008), domain map for mathematical knowledge for teaching (Figure 2.8) is used in an attempt to *conceptualise and develop measures of teachers' combined knowledge of content and students* (p. 372). Mathematical knowledge for teaching is divided into two domains, namely subject matter knowledge and PCK. The subject matter knowledge category consists of three strands, namely common content knowledge, specialised content knowledge, and knowledge at the horizon. For the purpose of this study I base ML teachers' MCK on common content knowledge that can be defined as a basic understanding

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[17] See Section 2.5.3: PCK and beliefs regarding content and teaching.
[18] See Section 2.5.4: PCK and beliefs regarding curriculum.
[19] See Section 2.5.1.2: Mathematics teachers' beliefs.

of mathematical skills, procedures, and concepts acquired by any well-educated adult enabling a teacher to solve mathematical problems in the prescribed curriculum (Ball et al., 2005).

#### 2.5.1.2 Mathematics teachers' PCK

The conceptual knowledge demanded of teachers to teach school mathematics is different from the mathematical knowledge mathematicians[20] might have of advanced topics (Ball, 1990; Leinhardt et al., 1991). Dewey (1902) also addressed this issue when he wrote:

> *Every study or subject thus has two aspects: one for the scientist as a scientist; the other for the teacher as a teacher. … For the scientist, the subject matter represents simply a given body of truth to be employed in locating new problems, instituting new researches, and carrying them through to a verified outcome… The problem of the teacher is a different one… What concerns him as teacher is the ways in which that subject may become part of experience, what there is in the child's present that is usable with reference to it; how such elements are to be used; how his own knowledge of the subject-matter may assist in interpreting the child's needs and doings, and determine the medium in which the child should be placed in order that his growth may be properly directed. He is concerned, not with the subject-matter as such, but with the subject-matter as a related factor in a total and growing experience* (p. 162-163).

PCK is regarded as knowledge that is unique to teachers; knowledge that can only be developed over time through experience in the classroom or practice and can therefore not be taught (Ball, 1988; Ball et al., 2005; Koellner et al., 2007; Ma, 1999; Shulman, 1986; Sowder, 2007). Having profound understanding and knowledge of mathematical subject matter is a prerequisite to develop PCK (Ball, 1990; Van Driel, Verloop & De Vos, 1998). Sowder (2007) is of the opinion that it is only as mathematics increases in sophistication, that a deep content knowledge base becomes a prerequisite in developing PCK. Although ML teachers need to have mathematical content knowledge, the DoE (2003a) stated that the content in ML must not be an end in itself, but must serve the learning outcome of applying content to certain contexts. Since the emphasis in ML is on solving real-life contextualised problems using **basic** mathematics, it is debatable whether a high level of subject matter knowledge is required by the ML teacher. My belief is nevertheless that ML teachers do need to have conceptual knowledge of the subject matter involved in the curriculum to enable them to use their knowledge efficiently in preparing their learners for their future lives.

#### 2.5.1.3 Mathematics teachers' beliefs

Teachers' beliefs about mathematics are powerful as they influence their representations of mathematics (Ball, 1990). She mentions a few beliefs of mathematics teachers that need to be examined such as their:

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[20] Mathematicians refer to people using higher levels of formal mathematics in their professions such as engineers and scientists.

> *understandings about the nature of mathematical knowledge and of mathematics as a field and the substance of mathematics. What counts as an answer in mathematics? What establishes the validity of an answer? What is involved in doing mathematics? What do mathematicians do? … What is the origin of some of the mathematics we use today and how does mathematics change? (p. 458) What do they think an explanation is? How do they sort out convention from logic with respect to particular principles or ideas? What do they think it means to 'know' or to 'do' mathematics?* (p. 459)

The saying 'we teach what we believe' emphasises the importance and far-reaching effects of teachers' beliefs on their instructional practice (Leatham, 2006). Ollerton (2009) believes that once teachers have articulated what their pedagogy is and obtain clarity on the beliefs and values they hold and which drive them, it will help them to strengthen effective practice. Mathematics teachers' beliefs about mathematics are located on a perspective continuum from traditional to formalist to constructivist (Dionne 1984). In my study I regard ML teachers' knowledge and beliefs as inseparable driving forces behind their instructional practices. In many cases teachers' beliefs are established by their knowledge and changing their knowledge base will change their belief system.

## 2.5.2 The three domains of PCK and beliefs

In this section I discuss my study's view on mathematics teachers' PCK and beliefs regarding the three domains, namely 1) content and learners; 2) content and teaching; and 3) the curriculum. In my discussion of each domain, I firstly mention some views from the literature regarding mathematics teachers in general and then discuss ML teachers' PCK and beliefs concerning that specific domain.

### 2.5.2.1 PCK and beliefs regarding content and learners

Knowledge of content and learners includes a teacher's ability to predict what mathematics learners will understand and how they will understand it, how learners will probably approach a task, understanding why certain topics are comprehensible and others not, what alternative conceptions and preconceptions learners have that could be misconceptions and that should be rectified and reorganised by the teacher through the use of different strategies (Ball, 1990; Borko & Putnam, 1996; Hill et al., 2008; Shulman, 1986; Sowder 2007). According to Sowder (2007), having this knowledge enables a teacher to:

> *plan more effectively because they can anticipate learners' difficulties. They know what prior knowledge must be present to understand something new. They know how to listen to students. Much of this knowledge comes from practice, but teachers who have poor understanding of mathematics themselves are unlikely to develop this type of knowledge, particularly when the mathematics in the curriculum becomes more sophisticated …* (p. 165)

This knowledge of content and learners should be taken into account when planning lessons (Koellner et al., 2007). Teachers must be able to **see** what learners do, **hear** what they think and then be able to **act** appropriately as mentors to facilitate the learning process (Hill et al., 2008).

***ML teachers' knowledge and beliefs regarding their learners***

Capturing learners' attention is particularly significant in the ML classrooms as many ML learners lack motivation, have negative attitudes and experience anxiety, causing teachers to be discouraged in teaching this subject (Mbekwa, 2007; Venkat, 2007; Venkat & Graven, 2007; Vermeulen, 2007). A shortcoming in the implementation process of ML is that teachers were not empowered to *deal with and assist learners with a past history of low attainment in mathematics* (Sidiropoulos, 2008, p. 250). To meet these challenges teachers need a firm knowledge base of the purpose and goal of the subject, its content, the teaching thereof, its learners and classroom management skills. Some teachers, for instance, only listen for correct answers and do not use incorrect answers to engage learners in mathematical thinking. Hill et al. (2007) emphasise the necessity for teachers to rephrase learners' questions to help them unravel the problem themselves.

### 2.5.2.2 Knowledge and beliefs regarding content and teaching

Knowledge regarding content and teaching includes *the most useful forms of representation of ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others* (Shulman, 1986, p. 9). He further describes this knowledge as *the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students* (Shulman, 1987, p. 15).

Teachers should have the ability to recognise the instructional advantages and constraints of using and adapting various representations depending on the content and needs of the learners and also have the ability to sequence content to facilitate student learning (Ball, 1990; Borko & Putnam, 1996; Koellner et al., 2007). Teachers furthermore need to be able to present subject matter in multiple ways like using story problems, pictures, situations and concrete materials. This knowledge is required to choose the appropriate pedagogical strategy and instructional material for a lesson, to consider which tasks to set and which assessment techniques to use. Knowledge of content and teaching further assists teachers to reflect on their own practice for the purpose of improvement (Koellner et al., 2007). Sowder (2007) feels teachers need to know how to scaffold knowledge to assist learners in developing understanding. Hill et al. (2008) concur with this view and assert that teachers need to know different ways of how to build on student mathematical thinking or how to remedy student errors.

***Approaches to teaching ML***

Approaches to the teaching and learning of ML should provide *extended opportunities to engage with ML in diverse contexts at a level that learners can access logically* (DoE, 2003c, p. 5). The teaching of mathematics in a contextualised and de-compartmentalised way however complicates the teaching of ML as some teachers lack the knowledge and skills to do so. Sidiropoulos (2008) found that teachers use ML textbooks where content is embedded in context, but *predominantly deliver the algorithmic content to the learners*, and afterwards *dress it up* with an artificial level of context, maybe using a picture (p. 227). Principles that guided Frith and Prince's (2006) curriculum in preparing in-service teachers to teach ML are the following:

- *That material should be context-based and make use of real relevant intrinsically motivating contexts, wherever possible.*
- *That curriculum tasks should require the exercise of several related competencies, such as writing and using computers, not just mathematical skills.*
- *That the production of a (mainly verbal) product as an outcome of mathematically literate practice is important (as well as the understanding and interpretation of existing information).*
- *That students' confidence should be promoted.*
- *That co-operative learning should be emphasised* (p. 55).

***ML teachers' knowledge of different teaching approaches***

Sidiropoulos (2008) believes ML teachers' PCK regarding different teaching strategies or approaches is inadequate. The success of ML depends largely on the skills of the teachers to apply appropriate teaching approaches such as discussions and problem solving (Brown & Schäfer, 2006; Venkat, 2007). Venkat (2007) reports that learners became positive about ML, enjoying the subject and finding it practical, useful and challenging when teachers changed the nature of tasks and interactions they used in the ML classroom. *Both these shifts provided openings for learners to communicate and participate in classroom activities, in addition to gaining understandings and make sense of the mathematics being used* (Venkat, 2007, p. 30). They enjoyed being active and focused and coming up with solutions to everyday problems, even sharing them with their parents at home. Vithal (2006) proposes project work as an approach since the purpose of project work is to improve effective participation and to provide learners with the opportunity to 'read the world' using mathematics, to develop mathematical power and to change their orientation towards mathematics. Project work is based on six conceptual principles, namely problem orientation, participant-directed, inter-disciplinarily, exemplarity, assessment, and practical organisation (Venkat, 2007). The notion of ML being inter-disciplinary implies that teachers from different disciplines should work together, drawing on their different disciplines to solve various problems.

***ML teachers' beliefs regarding the teaching of ML***

Learners' positive experiences normally stem from situations in which the teacher has a positive attitude, believes in the subject and uses approaches applicable to the requirements of the subject. Mathematics teachers hold a strong belief that teaching ML is a *major threat to their Mathematics teacher* *status-identity* (Sidiropoulos, 2008, p. 251). Labels teachers put on ML are *lesser maths; it is not real maths; it is the beginning of maths; it is a maths only better than nothing; it is the maths of oranges and bananas; it is a subject for the doffies [dim ones]* (p. 225). Even the learners and broader community *held a similar impoverished view* (p. 222) of teachers who teach ML as learners directly asked ML teachers if they are not as bright as the other teachers or if they are being punished for something they did wrong. Unless teachers undergo appropriate development programmes to seek a change in their behaviour, PCK and beliefs about the nature and value of ML, they will continue to fall back on *knowledge and beliefs already entrenched in their instructional practice* (p. 205-206). *The reality is that deep change is even more difficult to attain on an emotional level* (p. 225), is complex and also personal as *new teacher identities will require time to develop and unfold even under optimal conditions of reform* (p. 205). She further found that the ML teachers in her study do not want to change and they do not want to lower their status in society as Mathematics teachers. She is of the opinion that the best way to solve this problem is to recruit new ML teachers who do not need to undergo change in status-identity instead of trying to change the *qualified and experienced mathematics educators* (p. 226).

### 2.5.2.3 Knowledge and beliefs regarding the curriculum

Curricular knowledge refers to the knowledge of the full range of programmes designed for the teaching of different topics at given levels in a subject area. Teachers need to be familiar with the topics and level thereof being taught in the same subject during the preceding and later years in school, in other words how topics are organised horizontally and vertically. Curricular knowledge further includes knowledge regarding the variety of instructional materials available to teach particular curriculum components. Teachers need to recognise the particular strengths and weaknesses of textbooks and materials they are using. Competent teachers normally have a collection of materials they use when teaching mathematics. They also need to be familiar with the curriculum materials studied by learners in other subjects at the same time (Borko & Putnam, 1996; Shulman, 1986).

***ML teachers and the curriculum***

ML teachers need to be informed not only about the ML subject curriculum but all relevant departmental documents in order to understand what is expected of them to teach this relatively new subject. For example the DoE (2006) provided a list of resources needed to teach ML such as advertisements from the media containing contextual problems on percentage and interest rate, graphs and tables, etcetera. The new CAPS (DoE, 2011a) for ML will hopefully assist teachers concerning the issue of how to progress from one year to the next.

In Sidiropoulos' (2008) study on the implementation of the ML curriculum, she found that the purpose of the ML curriculum had not been well understood by the teachers and consequently they did not value the curriculum and the possibilities it provided for. Negative labels teachers put on ML stem from the fact that the ML curriculum, which is *distinctly different from curricula of the past was diktat on educators without due consideration on how substantial the required change would be in terms of understanding the purpose and possibilities of this new curriculum* (p. 249). She believes that if the broader purpose and value of the ML curriculum is well understood by teachers and all stakeholders, *this threat to identity may not have been as prominent as it was* (p. 225). She further found that the teachers' disjointed understandings of the ML curriculum put emphasis on the *complexity of bridging the gap between curriculum as intended and curriculum as implemented in the context of actual classrooms* (p. 225). Other problems are the fact that the curriculum assumes that all learners can be taught to become mathematically literate and that all *educators understood the concept of mathematical literacy that by its very own nature is distinctly dissimilar from that of mathematics or numeracy* (p. 250), the only known mathematical subjects taught by teachers in South Africa.

### 2.5.3 Teachers' instructional practices

In defining instructional practice, Englert et al. (1992) refer to the qualitative dimensions of teachers' behaviour in their practices. These dimensions involve teachers' abilities to model cognitive strategies in meaningful and purposive activities, adjust instruction as required, promote classroom dialogues, and establish classroom communities in which learners collaboratively and cooperatively participate in enquiry-related activities. A framework used to observe and describe teachers' instructional practices is built on three observable aspects of mathematics lessons, namely tasks, discourse and the learning environment (Artzt et al., 2008). The characteristics of tasks, discourse and the learning environment are provided in Table 2.9 below (Artzt et al., 2008, p. 10-12).

**Table 2.9: The observable aspects of a lesson**

| **TASKS**<br>Provide opportunities for learners to connect new knowledge to existing knowledge through active engagement in problem solving activities. Tasks should be motivational, at an appropriate level of difficulty and sequenced in a meaningful way to help learners clarifying their ideas. | |
|---|---|
| **Modes of representation** | Uses different representations such as symbols, diagrams, manipulatives, and computer representations to facilitate content clarity, enabling learners to connect new knowledge to prior knowledge and skills. |
| **Motivational strategies** | Uses tasks that capture learners' curiosity, inspiring them to reflect on their conjectures. The diversity of learners' interest and experiences should be taken into account. |

| | |
|---|---|
| **Sequencing and difficulty levels** | Sequences tasks in assisting learners to make connections between ideas and develop conceptual understanding. Uses tasks suitable to what learners already know and can do and what they need to learn. |
| **DISCOURSE**<br>Describes the verbal exchange among members of the community in the classroom, both teachers and learners. | |
| **Teacher-learner interaction** | Communicates with learners in an accepting, non-judgmental manner, encouraging learner participation. Requires learners to explain and demonstrate their thinking while carefully listening to provide clarification. |
| **Learner-learner interaction** | Encourages learners to listen to, respond to and question one another in order to assess each other's ideas or solutions, and if necessary to rectify or adjust. |
| **Questioning** | Poses a variety of types and levels of questions and allowing enough time to elicit thinking and to follow their reasoning through. |
| **LEARNING ENVIRONMENT**<br>Describe the conditions under which the teaching and learning process unfolds in the classroom and refer to the circumstances that affect the flow of action in the classroom. This should promote the development of learners' conceptual understanding. | |
| **Social and intellectual climate** | Establishes and maintains a positive culture with and among learners by valuing their ideas and showing respect. Enforces classroom rules to ensure positive learner behaviour. |
| **Modes of instruction and pacing** | Uses instructional strategies that encourage and support student involvement and purposefulness. Attends to time management to ensure learners have the opportunity to explore mathematical ideas and to express them. |
| **Administrative routines** | Uses effective procedures in organising and managing class activities to maximise learners' active involvement in the discourse and tasks. |

Flowing from observing the teachers, their instructional practices will be described according to their instructional approaches and general level of productivity. Table 2.10 below indicates the patterns being identified in teachers' instructional practices (Artzt et al., 2008).

**Table 2.10: Teacher-centred versus learner-centred instructional practices**

| | Teacher-centred | Learner-centred |
|---|---|---|
| **Tasks** | Impede learners' efforts to build on prior knowledge; unrelated to learners' interest; often too easy or too difficult; illogically sequenced. | Multiple accurate representations to facilitate content clarity; connect to learners' prior knowledge; relevant and interesting tasks; challenging and sequenced. |
| **Discourse** | Teacher judges learners' responses and resolves questions without learner input; learners give short responses, lacking explanation and justification; no interaction among learners; low-level, | Teacher has accepting attitude toward learners' ideas and encourage learners to think and reason; learners explain and justify their responses; learners listen to and respond to one another's |

| | | |
|---|---|---|
| | leading questions are asked. | ideas; variety of levels and types of questions. |
| **Learning environment** | Tense and awkward atmosphere; superficial requests for and use of learners' input; use of strategies that discourage learner participation; pace too fast or too slow; learners uninvolved; disorder in class. | Relaxed yet businesslike atmosphere; focus on learner input; strategies focus on learner involvement; effective organising and managing of class; learners actively involved. |

To describe the productivity of the teachers' instructional practices is complex as *it is consistently controversial and will remain controversial* to what constitutes good teaching (Franke et al., 2007, p. 226).

### 2.5.4 Summary

The conceptual framework for my study is based on the domain map for mathematical knowledge for teaching (Hill et al., 2008) and the categories of an instructional practice, namely tasks, discourse and learning environment (Artzt et al., 2008). MCK is based on the category common content knowledge as part of Hill et al's (2008) subject matter domain. PCK consists of knowledge of content and learners; knowledge of content and teaching; and knowledge of the curriculum. Knowledge of content and learners includes teachers' ability to understand and predict what learners will understand, how they will understand it, what their preconceptions are, what prior knowledge they need, what possible misconceptions and alternative conceptions they could have and why some topics are more comprehensible than others. Knowledge of content and teaching includes different pedagogical approaches, strategies and representations, use of meaningful sequencing of content and appropriate instructional material, all depending on the content and learners to make the subject comprehensible. Curriculum knowledge includes knowledge of the purpose, aim, learning outcomes and assessment criteria of the subject; the topics to be covered during the preceding, current and later years as well as the level thereof; the teaching strategies applicable to the subject; and the strengths and weaknesses of instructional materials.

## 2.6 Conclusion

Chapter 2 explored the international and national perspectives of mathematical literacy. Although the emphasis and terminology differ between different countries, researchers unanimously believe in the importance and value of learners' mathematical literacy skills that should be developed and enhanced (Gellert et al., 2001; Jablonka, 2003; Knoblauch, 1990; McCrone & Dossey, 2007; Queensland Government, 2007b; Skovsmose, 2007). In South Africa ML refers to both a subject and a competency whereas in other countries it is mainly the latter (Christiansen, 2007). ML may have become stigmatised as a subject having virtually no meaning when it comes to career opportunities, but a closer investigation showed that this subject has its own demands and requires specialised PCK and a positive belief system towards ML as a specialised subject. Teachers' instructional practices are strongly influenced not only by their MCK, but by their beliefs about the nature of ML as well as their PCK and beliefs about the learners, the teaching of the subject as well as the curriculum. Beliefs are powerful and many times the beliefs learners have are for the most part consistent with those of the teachers. The beliefs teachers hold regarding mathematics as discipline normally varies from a traditional perspective to a formalist perspective through to a constructivist perspective (Liljedahl, 2007). The next chapter provides a lay-out of the study's methodology.

# Chapter 3

# Methodology

## 3.1 Introduction

This chapter provides a description of the methodology used in this study. In my attempt to understand the phenomena being studied, I firstly discuss my research paradigm and assumptions as the lenses through which I view the world. I then explain the rationale for choosing qualitative research as my approach and case studies as the design used for my study. The research site, sample selection and data collection techniques are carefully described followed by the data analysis strategies. Lastly I discuss critical issues such as the trustworthiness of the study and ethical considerations applicable to the study.

## 3.2 Research paradigm and assumptions

In my research endeavour to obtain knowledge and understanding of the phenomenon being studied, I need to mention *the way in which [I] view the world, by what [I] view understanding to be and by what [I] regard as the purpose of understanding* (Cohen et al., 2001, p. 3). This study's research paradigm and ontological, epistemological and methodological assumptions are discussed below.

### 3.2.1 Research paradigm

My research paradigm is social constructivism which suggests that all knowledge is constructed and based upon not only prior knowledge, but also the cultural and social context. According to Nieuwenhuis (2007) the origins of mathematics are social or cultural and the justification of mathematical knowledge rests on its quasi-empirical basis. Constructivism implies a subjective approach that is *concerned with the uniqueness of each particular situation (idiographic)* (Nieuwenhuis, 2007, p. 51). The focus is *on the social construction of people's ideas and concepts*, on *how and why they interact with each other, and their motives and relationships* (Nieuwenhuis, 2007, p. 54). What forms the basis for a social constructivist philosophy of mathematics are the facts that *knowledge is not passively received but actively built up by the cognizing subject* and *the function of cognition is adaptive and serves the organization of the experiential world, not the* *discovery of ontological reality* (Ollerton, 2009, p. 78). Ernest (1988) also believes knowledge is acquired by oneself and that it cannot be transferred from one person to another. Ollerton's (2009) opinion is that people working individually *is not the way most of us operate for the vast majority of the time* and that *students might be encouraged to work individually in the first instance and later share and compare their information* (p. 77-78), a view with which I strongly agree. These stated principles of social constructivism have certain implications on teachers' approaches to teaching.

According to Koehler and Grouws (1992) teaching based on constructivism as learning theory is viewed *on a continuum between negotiation and imposition, and the teacher's role is to find and adjust activities for students* (p. 123). Social interaction where learners have the opportunity to communicate and work in collaboration with their peers is a critical part of knowledge construction (Koehler & Grouws, 1992; Ollerton, 2009). Social interaction, group work, problem solving and learner-centred approaches play significant roles in learners' construction of their own knowledge. I believe that although formal instruction has some influence on learners' understanding, learners do not directly assimilate knowledge or understanding from the teacher, but build their own understanding in the ML classroom through experience and maturation. The role of the teacher is therefore to guide and mentor the learners in developing understanding.

### 3.2.2 Paradigmatic assumptions

The nature of my study is based on three assumptions, namely the ontological, epistemological and methodological assumptions. Ontology and epistemology have direct implications for the methodological assumption as it demands different research methods (Cohen et al., 2001). The nature of my study is subjective as I am personally involved in the process of making sense of the uniqueness of the situation being studied (Nieuwenhuis, 2007). I hold the nominalist position as ontological assumption where I understand reality through words and regard reality as the product of individual consciousness (Cohen et al., 2001). Regarding the epistemological assumption, my study holds an interpretive position where knowledge is of a softer or transcendental kind and based on experience and insight of a personal nature. This nominalist and interpretive position demands an idiographic methodological preference where the focus is on the subjective experience of individuals who create, modify and interpret the world they are in (Cohen et al., 2001).

## 3.3 Research approach and design

The table below provides a synopsis of the research methodology components of my research.

**Table 3.1: Synopsis of methodology**

| | | | |
|---|---|---|---|
| **Research approach** | **QUALITATIVE** | | |
| **Research design** | **Case study: Exploratory**<br>A case can be a unit or group of people that are analysed and can also consist of another group(s) to enhance the trustworthiness of a study. This case study consists of ML teachers as a group. I observe their instructional practices and determine the nature of their knowledge and beliefs in order to explore the relationship between them. The nature of the data gathered is qualitative and the nature of the case study is exploratory (Cohen et al., 2001; Edwards & Talbot 1999; Nieuwenhuis, 2007). | | |
| **Main question** | **What is the relationship between ML teachers' knowledge and beliefs and their instructional practices?** | | |
| **Research sub-questions** | **Question 1**<br>How can ML teachers' instructional practices be described? | **Question 2**<br>What is the nature of ML teachers' knowledge and beliefs? | **Question 3**<br>How do ML teachers' knowledge and beliefs relate to their instructional practices? |
| **Objectives of the sub-questions** | • To determine what teachers do in their classrooms with respect to tasks given, discourse that takes place and the learning environment which is established. | • To comment on the teachers' level of MCK.<br>• To further explore teachers' beliefs regarding ML learners, the teaching of ML and the ML curriculum. | • To explore what the relationship is between teachers' instructional practices and their knowledge and beliefs.<br>• To consider the extent to which teachers use PCK in their lessons.<br>• To determine why teachers do what they do in their ML classrooms. |
| **Participants** | One Grade 11 ML teacher from five different secondary schools | | |
| **Data collection techniques** | • Three observations per teacher<br>• Three semi-structured interviews per teacher: one each before the second and third observed lessons and one after the observations. | | |
| **Techniques per question** | Observations | Observations<br>Interviews | Observations<br>Interviews |
| **Data analysis** | **DEDUCTIVE-inductive approach for data analysis (uppercase denotes the preference given to the style of analysis)**<br>• Establish units of analysis of the data<br>• Create a 'domain analysis'<br>• Use ATLAS.ti 6 to analyse the video and audio data<br>• Establish relationships and links between the domains<br>• Making speculative inferences<br>• Summarising | | |

### 3.3.1 Research approach

The research approach for this study is qualitative. Qualitative research seeks … *to gain better understanding of intentionality (from the speech response of the researchee) and meaning (why did this person/group say something and what did it mean to them?) … to describe and to understand, rather than to explain and predict* (Babbie & Mouton 2001, p. 49). Hogan et al. (2009) point out that qualitative research is about researching *specific meanings, emotions and practices that emerge through the interactions and interdependencies between people* (p. 4). Similarly White (2005) emphasises that qualitative research is concerned with conditions or relationships that exist, beliefs and attitudes that are held, effects that are being felt and trends that are developed. It also provides opportunities for marginalised groups to voice their opinions on matters that are of concern to them and which may have been overlooked in conventional research. The focus of my study is to describe ML teachers' instructional practices, their knowledge and beliefs and the relationship between their knowledge and beliefs and their practices *within their naturally occurring context with the intention of developing an understanding of the meanings imparted by the [ML teachers] – so that the phenomena can be described in terms of the meaning that they have for the [ML learners]* (Nieuwenhuis, 2007).

### 3.3.2 Research design

This is a case study. According to Cohen et al. (2001) *case studies can establish cause and effect* and observe *effects in real contexts, recognising that context is a powerful determinant of both causes and effects* (p. 181). Edwards and Talbot (1999) define a case study as a unit of analysis such as an individual or work team where *each case has within it a set of inter-relationships which both bind it together and shape it, but also interact with the world.* The idea of a case study is to *allow a fine-tuned exploration of complex sets of inter-relationships* (p. 51). Edwards and Talbot (1999) further distinguish between three uses of case studies, namely explanatory, descriptive and exploratory cases. This is an exploratory case study where the focus of the study has already been decided on and explained in the conceptual framework. The focus is *the case itself and its own very particular features, therefore was used to examine complex phenomena* (Edwards & Talbot, 1999, p. 53). The ML teachers are regarded as the 'unit' that is studied in order to explore the relationship between ML teachers' instructional practices and the driving forces behind their practices. My involvement in the case study gives *a sense of being there* (Cohen et al., 2001, p. 79). The analysis of the data enhances my understanding of the phenomena which will be reflected in an improved ML teacher preparation programme and will hopefully add value to theory building.

## 3.4 Research site and sampling

A case study requires intensive data collection as well as high quality data and it is preferable to work in-depth with a small number of teachers. The inductive approach also requires sampling to be small and information-orientated, but representative (Edwards & Talbot, 1999). The population consists of the ML teachers in South Africa which include Mathematics and non-Mathematics teachers from urban and rural government and private schools. Due to this wide variety of teachers it is not possible to choose a representative sample. Convenience and purposive sampling were implemented to select five different secondary schools in Tshwane. Listed below in Table 3.2 are the criteria that justify the inclusion and exclusion of schools and teachers in the sample:

**Table 3.2: Criteria justifying inclusion and exclusion in the sample**

| | Inclusion | Exclusion |
|---|---|---|
| Criteria | • Mathematics teachers<br>• Non-Mathematics teachers<br>• Male and female teachers<br>• Teachers with at least one year's experience of teaching ML<br>• Different races<br>• Schools with different performance levels<br>• Section 21, formerly disadvantaged and independent schools | • Private schools<br>• Schools situated far from my work<br>• Not more than one poor performance school |

The sampling is partly convenient as the five schools were chosen from schools in Tshwane that were easily accessible. Through purposive sampling three traditional black (formerly disadvantaged and independent), one predominantly white (Section 21) and one predominantly black (Section 21) schools were chosen. From each school only the Grade 11 teacher participated with the prerequisite that the teacher had taught ML for at least one year. My rationale for using the Grade 11 teachers in my study is that I presume some problems or challenges experienced by both teachers and learners either diminish or increase from Grade 10 to Grade 12. This allows for a kind of 'middle of the road' scenario where the impediment of data, due to possible problems experienced by Grade 10 and 12 teachers and learners, is reduced. From this sample valuable information was collected regarding the ML teachers' instructional practices, their knowledge and beliefs.

## 3.5 Data collection techniques

Case studies are normally time-consuming and not an easy option as the focus is on meanings and the complexity of interrelations which demand high quality data (Edwards & Talbot, 1999). Cohen et al.

(2001) argue that the purposes of case studies are *to portray, analyse and interpret the uniqueness of real individuals and situations … and to catch the complexity and situatedness of behaviour* (p. 79). The use of observations and interviews as data collection techniques improve the quality of this study's data and increase the trustworthiness of the study. The classroom observations and personal interviews allowed me to explore reality by becoming part of the participants' lives. These data collection techniques were informed by predetermined categories derived from the study's conceptual framework.

The process of data collection for each teacher consisted of three observations in an effort to obtain a relatively true account of the teachers' instructional practices. Interviews were conducted with the teachers the period before the second and third observations were made. These interviews were based on the teachers' planning of the lessons in order for me to obtain information regarding their PCK. This was followed by one in-depth interview, based on their lessons presented, as well as their PCK and beliefs. Following Figure 3.1 below on the data collection process, the observations and interviews are discussed.

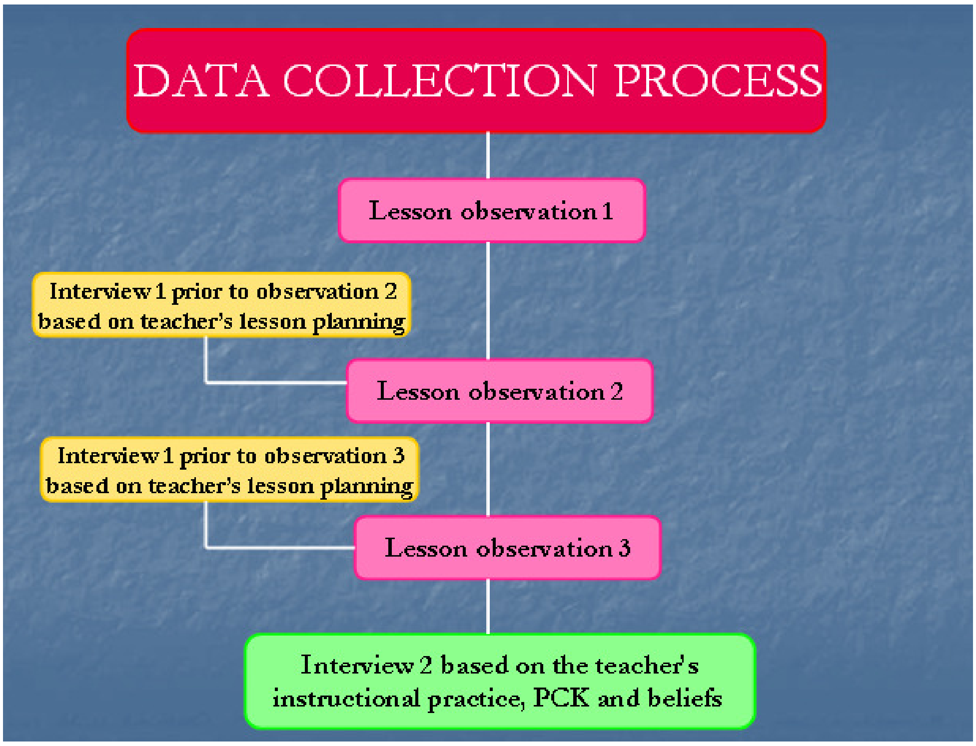


**Figure 3.1: The data collection process**

## 3.5.1 Observations

Cohen et al. (2001) believe that case studies are typified by observations as the purpose of observations is *to probe deeply and to analyse intensively the multifarious phenomena that constitute the life cycle of the unit with a view* *to establishing generalisations about the wider population to which that unit belongs* (p. 185). The type of observation I used was that of the observer as participant, not directly influencing the teaching process in the class situation (Nieuwenhuis, 2007). The purpose of the classroom observations was to describe the ML teachers' instructional practices according to three different dimensions of their lessons, namely tasks given, discourse and the learning environment[21]. I also observed the teachers' classroom performances with a view to studying demonstrations of their knowledge regarding the ML learners, the teaching of ML and the ML curriculum.

I decided to undertake three observations, preferably of different classes, to obtain a general impression of the teacher's instructional practice. The first observation was before any interviews were conducted so that the teacher could not be influenced by the questions from the interviews. An observation sheet was compiled in advance to cover the predetermined categories (Cohen et al., 2001). Figure 3.2 below provides a clarification of the lesson observations in terms of the teachers' instructional practices[22] and knowledge and beliefs.

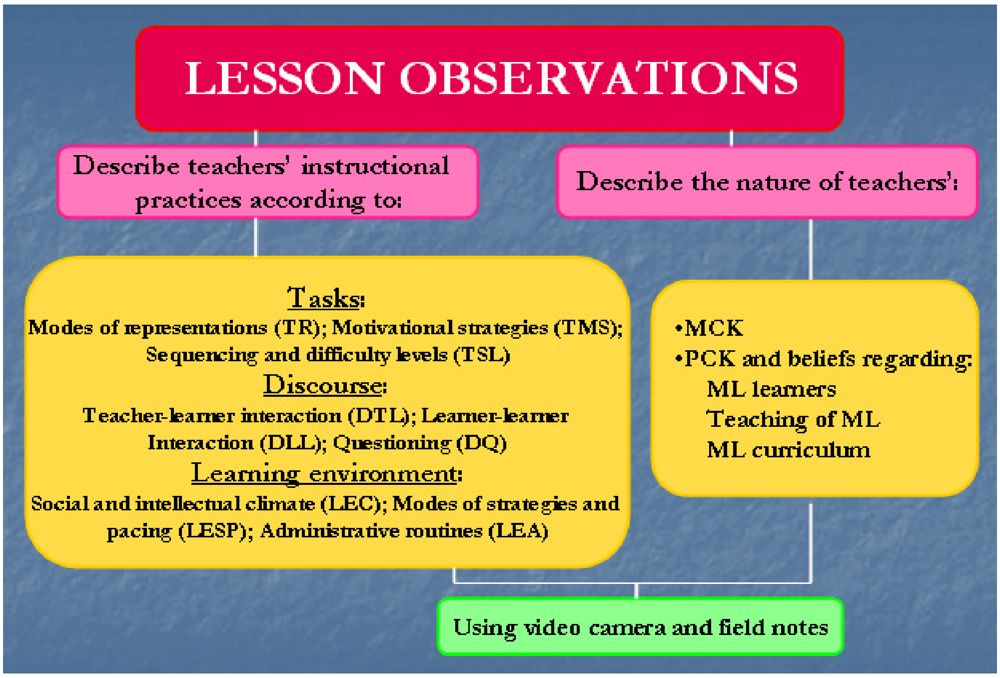


**Figure 3.2: Elucidation of the character of the lesson observations**

The lessons were video-taped and transcribed afterwards. Field notes were made regarding any unexpected valuable data that had emerged. Classroom observations are essential since lesson preparations can provide direction to a lesson, but can never predict exactly what will happen in class,

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21 See Section 2.5.5: Teachers' instructional practices.
22 See Section 2.5.5: Teachers' instructional practices for complete discussion.

as learners' participation, contribution and interaction with the content, teacher and peers allow for that dynamic aspect in class from which valuable data can be collected.

### 3.5.2 Interviews

According to Nieuwenhuis (2007) *[t]he aim of qualitative interviews is to see the world through the eyes of the participant* (p. 87) and to learn more about the participants' behaviours, beliefs and views. In these two-way conversations I tried to remain sensitive to responses of the participants and to identify new aspects to be discussed (Nieuwenhuis, 2007). Table 3.3 below provides an elucidation of the character of the interviews.

**Table 3.3: Elucidation of the character of the interviews**

| | |
|---|---|
| **INTERVIEW 1**<br>A semi-structured interview conducted prior to the 2nd and 3rd lesson observations | **PURPOSE OF INTERVIEW**<br>To gain insight into the participants' planning of their lessons and providing evidence of the teachers' PCK and beliefs regarding the ML learners, the teaching of ML and the ML curriculum. |
| | **EXAMPLES OF INTERVIEW QUESTION CONTENT**<br>• Teachers' predictions on which content the learners would and would not understand and the reasons for their understanding or not,<br>• What possible misunderstandings could occur,<br>• How they planned their lessons in order to bring learners to understanding the content and context, and<br>• What prior-knowledge should be present in the lesson. |
| **INTERVIEW 2**<br>A semi-structured and structured interview conducted at the end of the data collection phase | **PURPOSE OF SECTION A**<br>I used clips from the video recordings from the three lessons presented to guide a discussion with the participants in order to obtain a better understanding of their practices. The discussion focussed on the three dimensions of their lessons, namely tasks, discourse and the learning environment. |
| | **PURPOSE OF SECTION B**<br>The questions in this section were divided into three subsections, probing for teachers':<br>• Beliefs about the nature and value of mathematics as discipline and ML as subject,<br>• Knowledge and beliefs regarding the ML learners, and<br>• Knowledge and beliefs regarding the teaching of ML. |
| | **PURPOSE OF SECTION C**<br>• This section consisted of a set of predetermined questions and allows for clarification of answers in writing. The questions were based on some of the official documents such as the NCS Grades 10-12 (General) Mathematical Literacy (DoE, 2003) as well as the new CAPS (DoE, 2011a).<br>• The reason for including this section in the interview and not treating it separately in a questionnaire was to ensure that true data were captured as it was impossible for the teachers to discuss it with other teachers or to consult the relevant documents. |

All interviews were audio-taped and the tape-recordings were transcribed verbatim and coded afterwards by me. The final aim was to integrate the findings from the observations and interviews to make sense of the reality and the complexity of the phenomenon, in other words to determine the relationship between ML teachers' knowledge and beliefs and their instructional practices.

## 3.6 Data analysis strategies

According to Cohen et al. (2001) data analysis *involves organising, accounting for, and explaining the data; in short, making sense of the data ….., noting patterns, themes, categories and regularities* (p. 147). They further suggest that early analysis will reduce the problem of data overload as huge volumes of data rapidly accumulate in qualitative research. Edwards and Talbot (1999) agree to this practice as they believe continuous analysis of data keeps control of the project and reflects on the approach and design of the project as well as informing the next data gathering process. To analyse interviews as qualitative data, one has to realise it is *more of a reflexive, reactive interaction between the researcher and the de-contextualised data that are already interpretations of a social encounter* (Cohen et al., 2001, p. 282).

In my study I use DEDUCTIVE-inductive (uppercase denotes the preference given to the style of analysis) qualitative data analysis as my analysis will initially be deductive and then inductive. My raw data were analysed according to the categories that have been identified in my study's conceptual framework (Figure 2.9). After this deductive phase of analysis, inductive analysis was done where I studied the organised data in order to explore *undiscovered patterns and emergent understandings* (Patton, 2002, p. 454). Edwards and Talbot (1999) believe that although case studies need a theoretical framework, *their strengths are their capacity to reveal new ways of seeing familiar and complex situations* (p. 131). Through inductive analysis new patterns, themes and categories in the data were discovered which contributed towards possible implications for teacher training and theory building. The inductive approach allows for correlating the study's purpose with the findings.

The following research questions guided my analysis process:

1. How can ML teachers' instructional practices be described?
2. What is the nature of ML teachers' knowledge and beliefs?
3. How do ML teachers' knowledge and beliefs relate to their instructional practices?
4. What are the possible implications of the findings from Questions 1, 2 and 3 for teacher training?
5. What is the value of the study's findings for theory building in teaching and learning ML?

For this purpose I adapted Cohen et al.'s (2001, p.148) seven-step analytic strategy. The purpose is to move from thematically describing the cases to explaining the phenomena to eventually generating theory:

Step 1: *Establish units of analysis of the data, indicating how they are similar and different* – ascribing codes to the data.

Step 2: *Create a domain analysis* – dividing my data into groups, patterns and themes according to my conceptual framework.

Step 3: *Writing a case study narrative* – giving a description of each case, thus providing the reader with *all information needed to understand the case in all its uniqueness* (Patton, 2002, p. 450) **(Research question 1 and 2)**.

Steps 1 to 3 are indicated in Table 3.4 below:

**Table 3.4: Collection, analysis and reporting data**

| **OBSERVATIONS**<br>Three observations per teacher to obtain a general impression of their instructional practices | | |
|---|---|---|
| **DATA COLLECTION MODE** | **DATA ANALYSIS MODE** | **REPORTING DATA** |
| Observe lessons <u>focussing on</u>:<br>• Tasks<br>• Discourse<br>• Learning environment<br><u>using</u>:<br>• Video recordings<br>• Field notes | • Transcribe video data verbatim to text data<br>• Add field notes to above transcripts<br>• ATLAS.ti 6 to code video data:<br><u>CODES</u>:<br>I used the codes from Artzt et al.'s (2008) Framework for the examination of instructional practices. They referred to these codes as lesson dimensions.<br>Tasks: Representations (TR) →<br>Tasks: Motivational strategies (TMS) →<br>Tasks: Sequence and level (TSL) →<br>Discourse: Teacher-learner (DTL) →<br>Discourse: Learner-learner (DLL) →<br>Discourse: Questioning (DQ) →<br>Learning environment: Climate (LEC) →<br>Learning environment: Strategies, pace (LESP) →<br>Learning environment: Administrative (LEA) → | • Describe teachers' instructional practices according to tasks, discourse and learning environment.<br>**(Research question 1)**<br>• Describe nature of teachers' knowledge and beliefs as observed during lesson presentations.<br>**(Research question 2)**<br>Teaching<br>Learners<br>Teaching<br>Teaching and learners<br>Teaching and learners<br>Teaching and learners<br>Teaching and learners<br>Teaching<br>Teaching |
| **INTERVIEW 1**<br>Semi-structured interview conducted prior to observations 2 and 3 and based on the teacher's planning of that day's lesson. | | |

| DATA COLLECTION MODE | DATA ANALYSIS MODE | REPORTING DATA |
|---|---|---|
| Finding evidence of PCK and beliefs regarding the:<br>• ML learners<br>• Teaching of ML<br>• ML curriculum<br>using:<br>• Clips from the video recordings<br>• Tape recordings | • Transcribe audio data verbatim to text data<br>• ATLAS.ti 6 to code audio data:<br>CODES for PCK and beliefs:<br>ML learners (L)<br>Teaching of ML (T)<br>ML curriculum (C) | Describe nature of teachers' PCK and beliefs as new information from interviews could be compared with findings from observations.<br>**(Research question 2)** |
| **INTERVIEW 2**<br>Semi-structured interview conducted after all three observations and based on teachers' personal experiences to gain deeper insight in their practices | | |
| **DATA COLLECTION MODE** | **DATA ANALYSIS MODE** | **REPORTING DATA** |
| **Section A:**<br>Discuss outstanding incidents from their lessons to obtain a better understanding thereof.<br>using:<br>• Tape recordings<br><br>**Section B:**<br>Discussion according to a set of predetermined questions<br>based on:<br>• Beliefs about nature of mathematics as discipline and ML as subject<br>• PCK and beliefs regarding the ML learners<br>• PCK and beliefs regarding ML teaching<br>using:<br>• Tape recordings<br><br>**Section C:**<br>A set of predetermined question answered in writing based on the NCS and CAPS to determine teachers' knowledge of the curriculum. | **For all three sections:**<br>• Transcribe audio data verbatim to text data<br>• Use ATLAS.ti 6 to code audio and written data (using same codes as with observations and interview 1) | Further describe nature of teachers' PCK and beliefs as new information emerges from the second interview in addition to observations and first interview.<br>**(Research question 2)** |

After the process discussed above, I continued with Cohen et al.'s (2001, p. 149) steps of data analysis, namely:

Step 4: *Establish relationships and linkages between the domains* – the data were put in context by establishing relationships and links between the domains and also between the sets of data from the observations and interviews. This was done by *identifying confirming cases, by seeking 'underlying associations' and connections between data subsets.* **(Research question 3)**

Step 5: *Summarising* – reporting on the main features of the research so far indicating the major themes, issues and problems that have arisen from the data, also seeking negative and discrepant cases.

Step 6: *Making speculative inferences* – from the analysis I could draw certain conclusions and could consider the implications of those findings for teacher training. **(Research question 4 and 5)**

All video and audio data were transcribed verbatim to text data immediately after the data were collected. Following the transcribing process, I coded the transcriptions by using ATLAS.ti 6 which allows for codes to be easily accessed, sorted and merged. My transcripts are synchronised with associated files in order to jump from a particular part in the transcript to the original recording.

## 3.7 Quality assurance criteria

To conform to the quality assurance criteria for qualitative research, I considered aspects such as the trustworthiness, validity and reliability of my study and also bore in mind the Hawthorne and Halo effect. Being aware of the use of different terminology (trustworthiness, validity and reliability) by different researchers, I use the terms interchangeably as all these terms are referring to valuable aspects of quality assurance applicable to my qualitative study.

### 3.7.1 Trustworthiness of the study

Nieuwenhuis (2007) uses the term trustworthiness and states that *when qualitative researchers speak of research 'validity and reliability' they are usually referring to research that is credible and trustworthy* (p. 80). By using multiple data collection strategies such as multiple observations and interviews, the researcher as data gathering instrument, enhances the trustworthiness of the study. I acquired the services of a peer researcher with years of experience to assist me with the coding and interpretation of the data to further enhance trustworthiness (Nieuwenhuis, 2007).

Two factors affecting the trustworthiness of the study are the small sample and number of lessons observed influencing the extent to which the sample is representative. There is also no agreement on how PCK and beliefs are to be evaluated. Nespor (1987) reasons that *belief systems often include affective feelings and evaluations, vivid memories of personal experiences, and assumptions about the existence of entities and alternative worlds, all of which are simply not open to outside evaluation or critical examination in the same sense that the components of knowledge systems are* (p. 321).

## 3.7.2 Validity and reliability of the study

Cohen et al. (2001) refer to validity and reliability in qualitative research and do not use the terms "credibility and trustworthiness". They regard validity as an important aspect of both quantitative and qualitative research to ensure that a particular instrument measures what it is supposed to measure. A study may be declared reliable if findings from a particular group are replicated when a similar group in a similar context is investigated. Reliability then refers to the *consistency and re-applicability over time, over instruments and over groups of respondents. It is concerned with precision and accuracy* (Cohen et al., 2001, p. 117). Prompted by these views I came to the conclusion that the validity of my qualitative study was addressed through the … *honesty, depth, richness and scope of [my study's] data* … (Cohen et al., 2001, p. 105). Factors that contributed to a degree of bias were the subjectivity of respondents, their opinions, attitudes and perspectives. I enhanced the reliability of my study by the *stability of observations* meaning that I *would have made the same observations and interpretations* [*if the ML teachers*] *had been observed at a different time or in a different place* (Cohen et al., 2001, p. 119). I facilitated inter-rater reliability by inviting a researcher with many years' experience in analysing qualitative data to act as my external coder.

### 3.7.2.1 The Hawthorne effect

During the data collection stage I took the Hawthorne effect (Cohen et al., 2001) into consideration: the credibility of my data may be influenced due to my presence in class possibly influencing teachers' behaviour during observations. To reduce this effect, the first observation was done without a prior interview or discussion as the interview questions prior to the second and third observations could influence teachers' behaviour in the classroom. I emphasised the fact that I was interested in the uniqueness of each teacher and my purpose was not to report their performances in class to their superiors. To further enhance the trustworthiness of the observations the lessons were video-taped, field notes were taken and after each observation the teacher had to verify my field notes (Nieuwenhuis, 2007).

To enhance the trustworthiness[23] of the interviews, it was important that the interviewees be honest and open in their responses. The data from the two interviews prior to the lessons were compared with the classroom observations. The same interview schedules, including the same questions and sequence thereof were used for all interviewees. The questions were short and concise in order to avoid confusion or misunderstanding. Section C of the last interview where teachers answered the questions in writing was completed in my presence as part of the interview. This was to ensure that the data

[23] Note that for the purpose of my study I use trustworthiness, validity and reliability interchangeably as discussed under Section 3.7.

obtained were credible as the teachers were not able to consult another teacher or the relevant documents. I considered the fact that some teachers might have preferred to complete it in writing instead of orally as they might have felt less threatened or pressured. This allowed for more time to think about the questions and to provide valuable responses.

#### 3.7.2.2 The Halo effect

The Halo effect also needs to be considered during the data collection stage: *where the researcher's knowledge of the person or knowledge of other data about the person or situation exerts an influence on subsequent judgements* (Nieuwenhuis, 2007, p. 116). To ensure trustworthiness of the observations, a pilot study was conducted during the assessment period of one of my internship ML students *to ensure that the observational categories themselves are appropriate, exhaustive, discrete, unambiguous and effectively operationalise the purposes of the research* (Nieuwenhuis, 2007, p. 129). To enhance the trustworthiness of the interviews, I avoided the tendency to seek answers that would have supported my preconceived ideas. The peer researcher who assisted me with the coding and transcribing of the data pre-empted this problem. The interviewees were asked exactly the same questions and after each interview I gave a summary of my interpretation of the interview for them to verify or modify. The interviews were also piloted to refine *contents, wording, length etc.*, to ensure that questions were interpreted the same way by different teachers and that it did not take too much time, as the teachers' demanding schedules had to be taken into account (Nieuwenhuis, 2007, p. 129).

## 3.8 Ethical considerations

Ethics involves the moral issues implicit in the research work with respect to people directly involved in or affected by the project. To ensure that the study adhered to the research ethics requirements, application for ethical clearance was requested from the Ethics Department at the University of Pretoria as well as the Gauteng Department of Education. These applications were submitted after the proposal was successfully defended at faculty level and before fieldwork was conducted. Issues addressed in the application involve the sensitivity level of the research activities, the research approach, design and methodology, including full detail regarding the participants, voluntary participation, informed consent, confidentiality, anonymity and risk.

The participants were invited to take part in the study and were informed of the purpose of the study and their participative roles. They were not obliged to take part in the study but instead had a choice to participate knowing that they could withdraw at any stage. After joining the study, the participants signed a letter of informed consent. The letter explained the purpose of the study, the procedures to be followed during the investigation, the possible advantages and disadvantages as well as information regarding confidentiality, anonymity and possible risks involved in taking part in the study.

This study has a medium level of sensitivity as the participants were video-taped during their lesson presentations and the interviews were audio-taped in order to have a clear and accurate record of all events and verbal communication. It is highly unlikely that any of the participants was physically or psychologically harmed during this research. The only possible harm participants might have experienced is the invasion of their privacy by video-taping the lessons they presented or feelings of anxiety and discomfort in sharing their knowledge and beliefs during the interviews that were audio-taped. To lower the level of discomfort when questions were asked about their PCK of the curriculum, I gave them the option to rather answer the questions in writing. In this manner they had sufficient time to think and to avoid a situation where they could have felt threatened or even embarrassed if they could not answer a question. I am also aware that teachers are very busy and seldom have free periods, and time used for the interviews was also part of the intrusion into their lives.

It is important to notice that *a respondent may be considered anonymous when the researcher cannot identify a given response with a given respondent* (White, 2005). Confidentiality means *that although researchers know who has provided the information or are able to identify participants from the information given, they will in no way make the connection known publicly; the boundaries surrounding the shared secret will be protected* (Cohen et al., 2001, p. 62). To ensure anonymity and confidentiality the participants were not expected to identify themselves publicly and if their names were known, it was kept confidential at all times. No names were mentioned of any school or participant during the dissemination phase of the study when the research report was written, but instead the schools were numbered and pseudonyms were used. The signed consent letters served as a further guarantee to the participants regarding the anonymity and confidentiality of the study. The interviews took place in a private environment. The video-tapes of the observations and audio-tapes of the interviews by means of which the participants can be identified are accessible only to me. Participants were asked to review the draft report before it was finalised.

## 3.9 Conclusion

In this chapter I discussed social constructivism as my research paradigm, stated the nature of my study being subjective and taking up an interpretive position. A qualitative research approach is used and the research design is an exploratory case study. The research site is secondary schools in Tshwane and the sample comprises five grade 11 ML teachers. Observations were used to access teachers' instructional practices and to determine the nature of their MCK and the extent to which they apply their PCK during their instructional practice. Interviews were used to describe the ML teachers' beliefs about mathematics as a discipline as well as their PCK and beliefs about the ML learners, the teaching of the subject and the ML curriculum. ATLAS.ti 6 was used to analyse the video and audio data in order to establish a relationship between teachers' instructional practices and their knowledge and beliefs. I lastly discussed the trustworthiness of the study and ethical considerations that were taken into consideration. In the next chapter the results of the study are presented and discussed.

# Chapter 4

# Presentation and discussion of the findings

## 4.1 Introduction

Since the data collection process and data analysis strategies are discussed in Chapter 3, I briefly rapport on the data collection process[24] and the data analysis strategies[25] while providing a comprehensive elucidation regarding the coding of the data. Based on my conceptual framework (Figure 4.1) I thematically present and discuss the findings from each participant, relate the findings to the literature and lastly explain the identified trends. The two themes are: 1) ML teachers' instructional practices and 2) ML teachers' knowledge and beliefs.

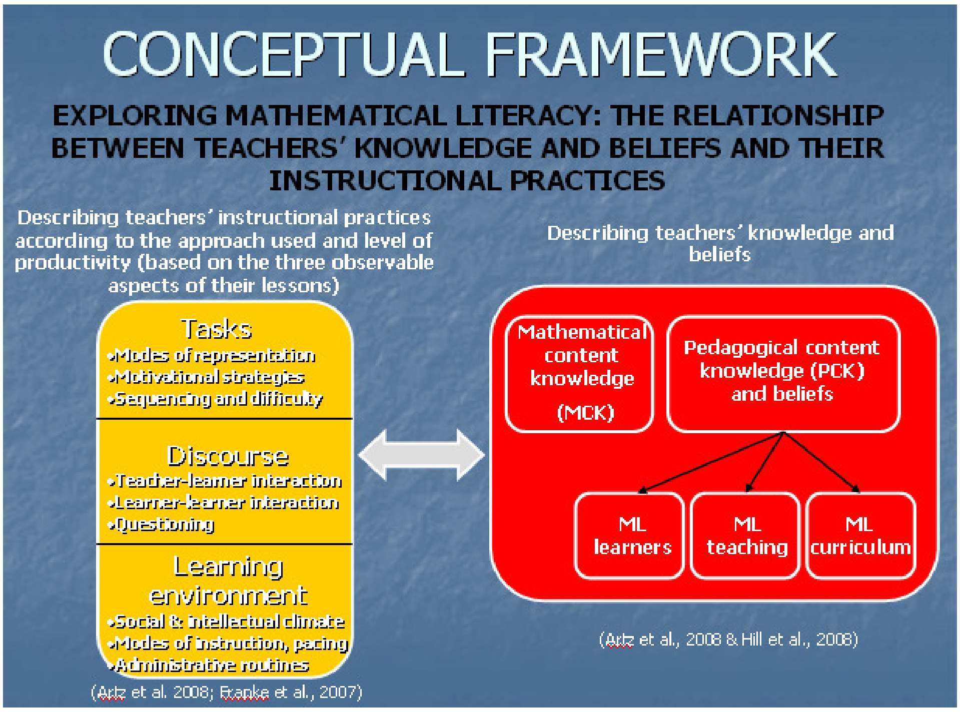


**Figure 4.1: Conceptual framework: Instructional practice, knowledge and beliefs framework of analysis** (adapted from Artzt et al., 2008; Franke et al., 2007; Hill et al., 2008)

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24 A detailed description of the process is provided in Section 3.5: Data collection techniques.
25 For more detail, see Section 3.6: Data analysis strategies.

The research questions are:

**Main question:**

What is the relationship between Mathematical Literacy teachers' knowledge and beliefs and their instructional practices?

**Subquestions:**

1. How can ML teachers' instructional practices be described?
2. What is the nature of ML teachers' knowledge and beliefs?
3. How do ML teachers' knowledge and beliefs relate to their instructional practices?
4. What are the possible implications of the findings from Questions 1, 2 and 3 for teacher training?
5. What is the value of the study's findings for theory building in teaching and learning ML?

## 4.2 The data collection process

The data collection took place in Pretoria during the second quarter (May and June) of 2011. I initially contacted the principals of five schools telephonically to discuss my study and request their participation in my study. Two letters of invitation were sent to the schools, one addressed to the principal and the other to the Grade 11 ML teacher and a meeting was scheduled at each school between the principal, teacher and me. The initial participants[26] were Monty, Elaine, Alice, Edith and Denise. During the data analysis process I realised that Edith's case did not add value to my study since her practice was similar to three of the four teachers' practices. I also reached a point of data saturation and decided to continue with the other four cases only. During the data collection period all communication and arrangements were made directly with the participants except for Alice's school where I worked through the principal.

I kept to the data collection process[27] of three observations with an interview conducted prior to the second and third observations, followed by a last interview some time after the last observation. The duration of the two interviews prior to the observations was approximately half an hour each and was conducted during the period or break before the specific lesson. These interviews were based on the teachers' planning of their lessons. The duration of the last in-depth interview was approximately 50 minutes per interview. I only observed Grade 11 ML lessons and all participants had had at least one year experience of teaching ML, one of the selection criteria[28].

[26] Pseudonyms were used for ethical purposes.
[27] The data collection process is discussed in Section 3.5: Data collection techniques.
[28] The other selection criteria are discussed in Section 3.4: Research site and sampling.

In Table 4.1 a timeline is given indicating the dates all five participant's lessons were observed and interviews conducted.

**Table 4.1: Timeline of the data collection process**

| Data gathering instrument | Participants[29] | Date in 2011 |
|---|---|---|
| Observation 1 | Monty | 3 May |
| Interview & Observation 2 | Monty | 4 May |
| Observation 1 | Elaine | 6 May |
| Interview & Observation 3 | Monty | 9 May |
| Observation 1 | Alice | 9 May |
| Interview & Observation 2 | Alice | 10 May |
| Interview & Observation 2 | Elaine | 12 May |
| Interview & Observation 3 | Alice | 16 May |
| Observation 1 | Edith | 16 May |
| Observation 1 | Denise | 17 May |
| Interview & Observation 3 | Elaine | 19 May |
| Interview & Observation 2 | Edith | 19 May |
| Interview & Observation 2 | Denise | 20 May |
| Interview & Observation 3 | Edith | 25 May |
| Interview & Observation 3 | Denise | 26 May |
| Last interview | Edith | 27 May |
| Last interview | Denise | 30 May |
| Last interview | Alice | 1 June |
| Last interview | Monty | 1 June |
| Last interview | Elaine | 2 June |

## 4.3 Data analysis strategies

The study's DEDUCTIVE-inductive approach and analytic strategies used in analysing the data are discussed in Chapter 3[30]. In this section I only discuss the transcribing and coding of the data. The inclusion and exclusion criteria for coding the data are also discussed and presented in table form.

### 4.3.1 Transcribing the data

I transcribed my video and audio-taped data verbatim to text data immediately after the data had been collected. Care was taken not to interpret the data already during the transcribing phase. After each

[29] Pseudonyms were used to protect the participants' true identities.
[30] See Section 3.6: Data analysis strategies.

observation all hand-written field notes made during the observations as well as insights that were thought of afterwards which had not been noted were typed on a template form. Uncertainties that emerged were cleared by watching the video-tapes of the lessons or listening to the audio-tapes again. Transcripts were read afterwards to ensure the transcripts were true accounts of the actual observations and interviews.

## 4.3.2 Coding of the data

In coding the data, I used a deductive approach based on my conceptual framework. According to the conceptual framework two themes, namely 1) the ML teachers' instructional practices and 2) their knowledge and beliefs were identified while the subthemes for each theme were chosen according to the work of Artzt et al. (2008) and Hill et al. (2008). Codes have been ascribed to the different lesson dimension indicators of each subtheme according to which the raw data were analysed. By using the software programme ATLAS.ti 6, I coded the transcripts according to a set of pre-determined lesson dimension indicators and their associated codes as given in Table 4.2[31] and Table 4.3[32].

After the data were coded I created *coding families* (Archer, 2009) which are clusters comprising codes related to one other. According to my conceptual framework, families were created by selecting from the list of all codes those codes that were related to one another. A specific code could belong to more than one family and families were therefore not exclusive. The data were analysed according to two themes, namely 1) ML teachers' instructional practices and 2) ML teachers' knowledge and beliefs. Sub-themes for each of these themes were created using Atlas.ti 6. Networks for these sub-themes were created afterwards where the connections between the different codes assigned to the families were indicated.

### 4.3.2.1 Theme 1: ML teachers' instructional practices

The three subthemes (also called the lesson dimensions) which could best describe the teachers' practices are 1) tasks; 2) discourse; and 3) learning environment (Artzt et al., 2008). The first column in Table 4.2 below indicates the three subthemes or lesson dimensions with their different categories. In the second column are the descriptions of the lesson dimension indicators with the codes created for them. All data were collected from the observations only.

[31] Table 4.2 is given under Section 4.3.2.1.
[32] Table 4.3 is given under Section 4.3.2.2.

**Table 4.2: Lesson dimensions and dimension indicators as inclusion criteria for coding the data** (Adapted from Artzt et al., 2008)

| LESSON DIMENSIONS | DESCRIPTION OF LESSON DIMENSION INDICATORS (CODES) |
|---|---|
| **TASKS** | |
| Modes of representation (TR) | TR1. Uses representations such as oral or written language, symbols, diagrams, graphs, tables, manipulatives, and computer or calculator representations to accurately facilitate content clarity.<br>TR2. Provides multiple representations that enable learners to connect their prior knowledge and skills to the new mathematical situation such as graphs, tables, formulae. |
| Motivational strategies (TMS) | TMS1. Uses tasks that capture learners' curiosity and inspires them to participate in the lesson, but also to speculate on and pursue their conjectures, such as tasks which elicit a class discussion or an interesting context used.<br>TMS2. Takes into account the diversity of student interests, experiences and abilities, such as when the teacher provides additional tasks for the more advanced learners.<br>TMS3. Points out the value of the mathematics being learned so that learners will appreciate and understand the value of mathematics, such as informing them about real-life situations or even other subjects where the mathematical content is used. |
| Sequencing and difficulty levels (TSL) | TSL1. Sequences tasks and learning activities so that learners can progress in their cumulative understanding of a particular content area and can make connections between ideas learned in the past and those they will learn in the future such as working from easy to difficult and known to unknown tasks.<br>TSL2. Uses tasks, including homework that is suitable to what the learners already know and can do and what they need to learn or improve on. Tasks should involve past work, reinforce current work and set the stage for future work such as tasks where opportunity is given to practice identified or predicted learners' misunderstandings.<br>TSL3. Tasks should reflect quality, not quantity. Should be appropriate and on the learners' level such as increasing the level of difficulty from Grade 10 and applying it to more complex contexts. |
| **DISCOURSE** | |
| Teacher-learner interaction (DTL) | DTL1.Communicates with learners in a non-judgmental manner and encourages the participation of each student, such as addressing a large number of learners or working on a learner's incorrect answer to lead the learner to understanding.<br>DTL2. Requires learners to give full explanations and justifications or demonstrations orally and/or in writing such as |

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| | learners explaining their work on the board.<br>DTL3. Listens carefully to learners' ideas and makes appropriate decisions regarding when to offer information, provide clarification, model, lead, and let learners grapple with difficulties. In response to a learner's question, instead of telling how or doing it for the learner, rather provides scaffolding to support solving a problem or encourage learners to share ideas for carrying out a task.<br>DTL4. When giving feedback to learners' answers, rather accept and praise instead of criticising and rejecting answers.<br>DTL5. Recognises and clarifies learners' misunderstandings and misconceptions. |
| Learner-learner interaction (DLL) | DLL1. Encourages learners to listen to, respond to, and question each other so that they can evaluate and, if necessary, discard or revise ideas and take full responsibility for arriving at mathematical conjectures and/or conclusions.<br>DLL2. Avoids situations in which a group of learners dominate in the verbal communication in class. |
| Questioning (DQ) | DQ1. Teacher needs to pose a variety of levels and types of questions using appropriate wait times that elicit, engage and challenge learners' thinking. Cognitive levels:<br>i) Memory: factual questions such as: What is the mean?<br>ii) Convergent: narrow questions such as: What does it mean to write 12,5% as a decimal? Also complete the word/sentence questions such as: We call it the co? and learners then need to complete the word: coefficient.<br>iii) Divergent: broad and open-ended questions such as: Why did you decide to use the compound interest formula?<br>DQ2. The teacher should listen to learners' ideas and ask them to clarify and justify their ideas such as asking: Why do you say that? or How did you solve that?<br>DQ3. Should contribute to the verbal communication and participation of learners creating opportunities where learners listen to, respond to, question and answer the teacher and one another.<br>DQ4. Learners' responses: i) chorus; ii) volunteered; iii) teacher-selected. |
| **LEARNING ENVIRONMENT** | |
| Social and intellectual climate (LEC) | LEC1. Establishes and maintains a positive rapport with and among learners by showing respect for and valuing learners' ideas and ways of thinking such as no one laughing when an incorrect answer is given.<br>LEC2. Enforces classroom rules and procedures to ensure appropriate classroom behaviour such as one person talking at a time.<br>LEC3. Should have a positive attitude towards the learners and the subject, such as praising learners' attempts and being proud and well-prepared ML teachers. |
| Modes of strategies and pacing | LESP1. Uses various instructional strategies that encourage and support student involvement as well as facilitate goal attainment such as cooperative learning, learners explaining work at the board, direct instruction (lecturing), |

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| (LESP) | abstract procedural, group work, active learning, discussion, problem solving, inquiry and team-teaching.<br>LESP2. Provides and structures the time necessary for learners to express themselves and explore mathematical ideas and problems such as enough opportunities to discuss or do group work.<br>LESP3. Effective use of class time to accommodate all three phases of a lesson: initiation, development and closure.<br>LESP4. There should be a logical flow in the lesson such as revising prior knowledge before introducing new content and assess whether learning occurred. |
| Administrative routines (LEA) | LEA1. Uses effective procedures for organization and management of the classroom so that time is maximized for learners' active involvement in the discourse and tasks such as allowing time for learners to practice what has been explained by the teacher and not to rush them while working on a problem.<br>LEA2. Classroom arrangement should be appropriate to the lesson style used such as learners sitting in groups if group work is applied.<br>LEA3. The position of the teacher in the classroom should contribute to a positive learning atmosphere such as working between the learners, having eye contact with individual learners.<br>LEA4. The written information on the board/transparencies should be correct and ordered in order to contribute to learners' conceptual understanding. |

Source: Adapted from: A Cognitive Model for Examining Teachers' Instructional Practice in Mathematics: A Guide for Facilitating Teacher Reflection, by A.F. Artzt and E. Armour-Thomas, 1999, *Educational Studies in Mathematics*, 40(3), p. 217. Copyright © 1999 by Kluwer Academic Publishers. Adapted with kind permission from Kluwer Academic Publishers.

Using Atlas.ti 6, networks of the *code families* (Archer, 2009) are now illustrated and explained. As I have mentioned, the data for the three *code families* under Theme 1 were collected from the lesson observations only. The *code family* created for the first subtheme **Tasks** appear in Figure 4.2 below. The broken line arrows indicate the three different lesson dimensions being linked to the code family **Instructional practices: Tasks**. Atlas.ti 6 uses solid line arrows with double equal signs to indicate the codes *associated with* the different lesson dimensions (Archer, 2009). For example codes TR1 and TR2 are associated with lesson dimension **Tasks: Modes of representation (TR)**. A full description of each code such as TR1 and TR2 is provided in Table 4.2 above.

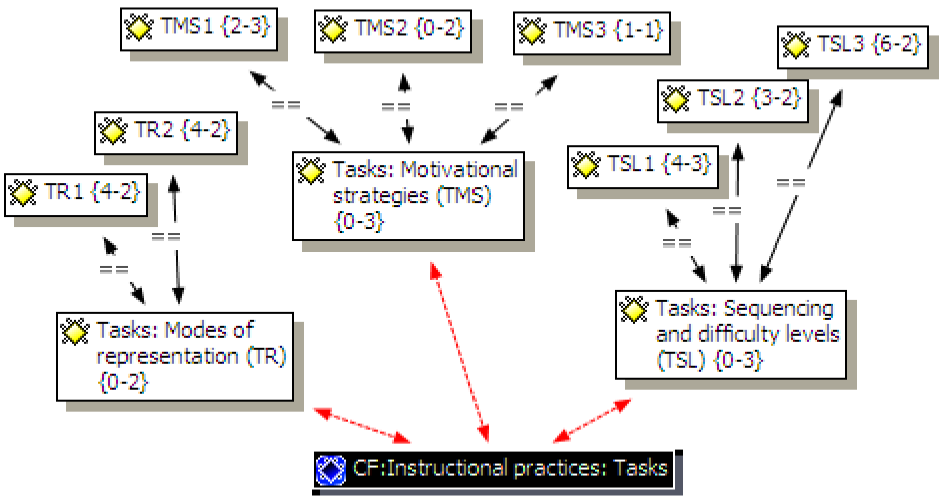


**Figure 4.2: ML instructional practices: Tasks**

At the end of each code, for example **TMS1**[33], there is a pair of numbers in parentheses {2-3}. The **2** refers to the groundedness, in other words the frequency with which the code was attached to quotations in the observation transcripts for a specific participant. This means that there were two incidents during the three lessons observed from a specific participant where there was evidence of the teacher capturing the learners' curiosity. The **3** is the density, indicating the number of times a code has been linked to codes in all the networks that were created. In this example it means the code TMS1 was also associated with two other codes T2 and T5 (see Table 4.3) in the subtheme **ML teaching** under Theme 2: ML teachers' PCK and beliefs. Notice that at the end of the three categories in Figure 4.2, namely: Tasks: Modes of representation (TR), Tasks: Motivational strategies (TMS) and Tasks: Sequencing and difficulty levels (TSL) the numbers in parentheses are {0-2}, {0-3} and {0-2}. This indicates that the codes TR, TMS and TSL were not associated with quotations in the transcripts. These

[33] According to Table 4.2 **TMS1 refers to** Tasks: Motivational strategies: Teacher uses tasks that capture learners' curiosity and inspire them to participate.

are subthemes that were not coded as such. Instead their different lesson dimension indicators were coded.

The *code family* created for the second subtheme **Discourse** is given in Figure 4.3 below. The broken line arrows again indicate the three different lesson dimensions being linked to the code family **Instructional practices: Discourse**. The solid line arrows with double equal signs indicate the codes *associated with* the different lesson dimensions. For example codes DTL1, DTL2, DTL3, DTL4 and DTL5 are associated with the lesson dimension **Discourse: Teacher-learner interaction (DTL)**. The full descriptions of the codes are in Table 4.2.

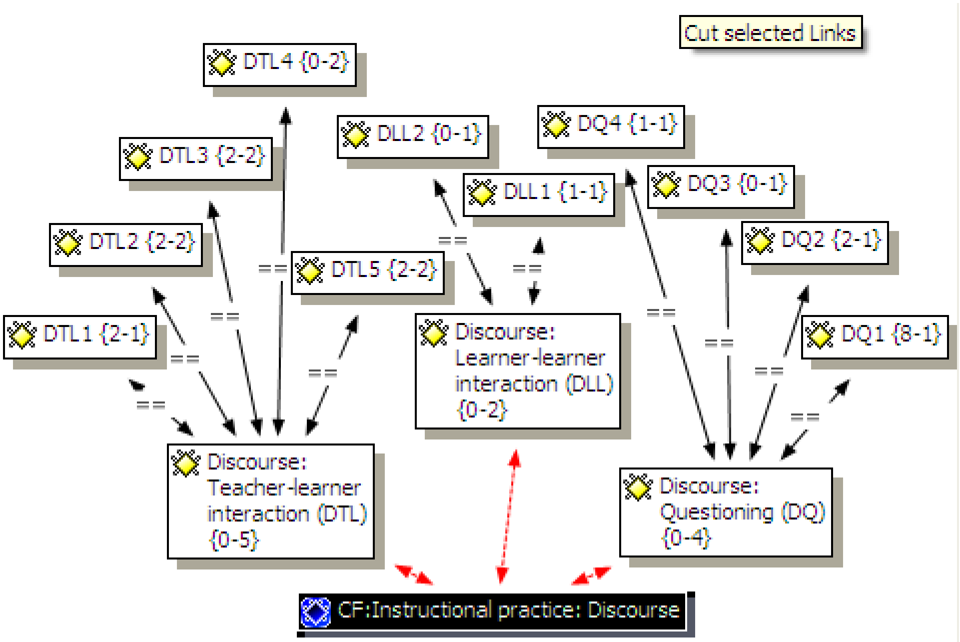


**Figure 4.3: ML teachers' instructional practices: Discourse**

The *code family* created for the third subtheme **Learning environment** appears in Figure 4.4 below. The broken line arrows again indicate the three different lesson dimensions being linked to the code family **Instructional practices: Learning environment**. The solid line arrows with double equal signs indicate the codes *associated with* the different lesson dimensions. For example codes LEC1, LEC2 and LEC3 are associated with the lesson dimension **Learning environment: Social and intellectual climate (LEC)**. The full descriptions of the codes are in Table 4.2.

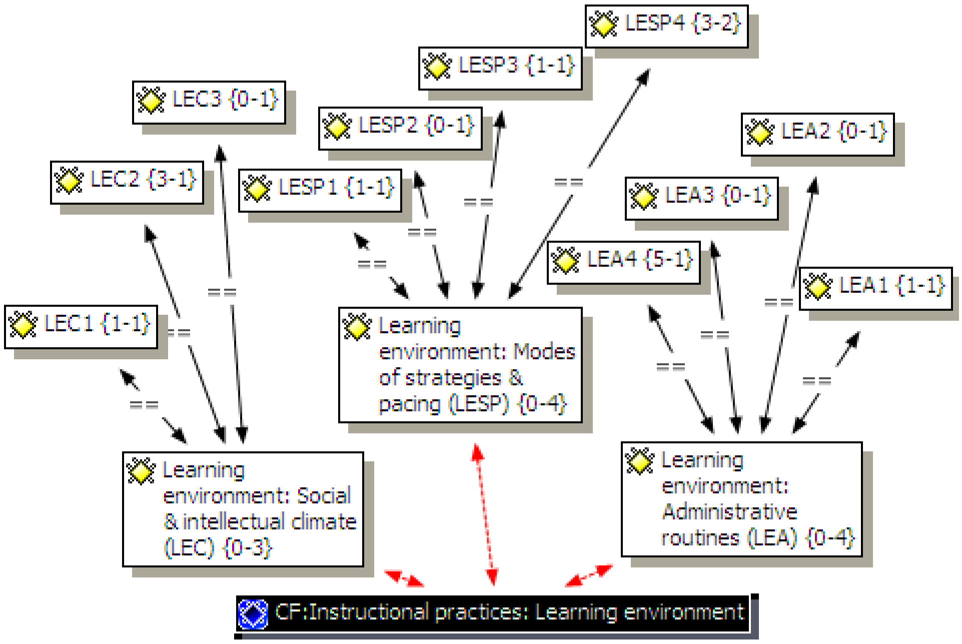


**Figure 4.4: ML teachers' instructional practices: Learning environment**

### 4.3.2.2 Theme 2: ML teachers' knowledge and beliefs

The four subthemes of the ML teachers' knowledge and beliefs are 1) MCK; 2) PCK regarding ML learners; 3) PCK regarding ML teaching; and 4) PCK regarding the ML curriculum (Hill et al., 2008). The first column in Table 4.3 below indicates the four subthemes or dimensions. In the second column are the different descriptions of the dimensions with the codes created for them. These are the codes that appear in the *coding families* on 1) ML learners (Figure 4.5), 2) ML teaching (Figure 4.6); and 3) ML curriculum (Figure 4.7). The data were collected from both the interviews and the lesson observations.

**Table 4.3: Knowledge and beliefs dimensions and its indicators as inclusion criteria for coding the data** (Adapted from Artzt et al., 2008; Ball, 1990; Borko & Putnam, 1996; Hill et al., 2008; Shulman, 1986; Shulman, 1987)

| PCK AND BELIEFS DIMENSIONS | INSTRUMENT: Interview (Int) Observation (Obs) | Correspond with indicators in Table B | Interview question numbers | DESCRIPTION OF TEACHERS' PCK AND BELIEFS INDICATORS (CODES) |
|---|---|---|---|---|
| Mathematical content knowledge (MCK) | Observation | | | Report on teachers' mathematical content knowledge. Record on the accuracy of teachers' content, mathematical errors made by teachers, teacher's misconceptions or misrepresentations. |
| ML learners (L) | | | | Teacher's ability to: |
| | L1. 1st Interview | | 2a, 2b | L1. predict what mathematics learners will understand; but also understand why that mathematics is comprehensible to the learners; |
| | L2. 1st Interview | | 3a, 3b | L2. predict what mathematics learners will not understand; but also understand why that mathematics is incomprehensible to the learners; |
| | L3. 1st Interview | | 4c; 5 | L3. predict how they will come to understand it; |
| | L4. 1st Interview | | 6 | L4. predict how learners will probably approach a task; |
| | L5. 1st Interview | | 8 | L5. understand what alternative conceptions and preconceptions learners have that could be misconceptions and that should be rectified and reorganised by the teacher through the use of different strategies; |
| | L6. Observation | DTL2/3/4/5 | | L6. **see** what learners do, know how to listen and **hear** what they think and then be able to **act** appropriately as mentors to facilitate the learning process. |
| ML teaching (T) | | | | Teachers should: |
| | T1. 1st Int & Obs | TR2\|TSL1 | 7 | T1. know what prior knowledge must be present to understand new work; |
| | T2. Observation | TR1\|TMS1 | | T2. know useful forms of representation of ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject depending on the content and learners' needs, in order to make it comprehensible to them; |
| | T3. Observation | TMS2 | | T3. have the capacity to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the learners; |

| | | | | |
|---|---|---|---|---|
| | T4. 1st Int & Obs<br>T5. 1st Int & Obs<br><br>T6. 2nd Int | TSL1\|LESP4<br>TR3\|TSL2\|<br>TMS1\|TSL3 | 5<br>6a<br><br>Section A (1-4)<br>Section B 1(6);2(2-4,6) 3(1-4) | T4. have the ability to sequence content to facilitate student learning;<br>T5. to choose the appropriate instructional strategy and instructional material for a lesson, consider tasks to set and assessment techniques to use;<br>T6. reflect on their own practices for the purpose of improvement. |
| ML Curriculum (C) | C1-C6: 2nd Int<br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br>C7. 1st & 2nd Int & Obs<br><br><br>C8. 2nd Interview | <br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br>Context<br><br><br>Nature 1<br>Nature 2 | <br>Section C (5)<br>Section C (6)<br><br>Section C (7)<br>Section C (8)<br><br>Section C (1,2,4,9)<br>Section C (10)<br><br>Int.1 (4a,4b)<br>Int. 2(3)<br>Section B (1,3) | Teachers:<br>C1. should have knowledge regarding the variety of resources/instructional materials available to teach particular curriculum components;<br>C2. need to recognise the particular strengths and weaknesses of textbooks and materials they are using and should have a collection of materials they use when teaching mathematics;<br>C3. need to be familiar with the curriculum materials studied by learners in other subjects at the same time and how it integrates with ML;<br>C4. should be informed of the various departmental ML documents, providing info regarding the purpose and value of ML, resources to use and how to progress from one year to the next;<br>C5. should have knowledge about the definition, purpose, learning outcomes and the new CAPS;<br>C6. need to be familiar with the topics and level of different topics being taught in the same subject during the preceding and later years in school, in other words how topics are organised horizontally and vertically.<br>C7. should teach content in context. The context should be applicable to the content and the teacher needs to know the context and be able to apply meaningfully.<br><br>C8. View of mathematics as discipline: From traditional to formalist to constructivist (Includes the role of the teacher in his/her instructional practice)<br>• A traditional view refers to teachers who belief that mathematics is an abstract phenomenon unrelated to reality. These teachers will then struggle to relate mathematics to real-life situations and tend to believe mathematics consists of a set of rules and procedures that must be learned mechanically with little or no |

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| | | | | connection to one another and hardly any relevance to their everyday lives. They also tend to separate mathematics from the discipline of discovery and creativity and an abstract procedural approach is used.<br>• The formalist view refers to teachers who believe mathematics is characterised by logic, rigorous proofs, exact definitions and a precise mathematical language and doing mathematics consists of accurate proofs as well as of the use of a precise and rigorous language.<br>• The constructivist view refers to teachers who believe mathematics is a constructive process where relations between different notions and sentences play an important role. Here the mathematical activity involves creative steps, such as generating rules and formulae, thereby inventing or re-inventing the mathematics.<br>• View of ML as subject being a lower grade Mathematics and/or a life skill.<br>• Possible definitions of the nature of mathematics: i) mathematics is the language of science; ii) is the study of patterns; iii) is a system of abstract ideas. |
| | C9. 2nd Interview | Nature 3 | Section B (2)<br>Section B (4,5 | C9. The value of mathematics and ML for people in their daily lives. |

Source: Adapted from: A Cognitive Model for Examining Teachers' Instructional Practice in Mathematics: A Guide for Facilitating Teacher Reflection, by A.F. Artzt and E. Armour-Thomas, 1999, *Educational Studies in Mathematics*, 40(3), p. 217. Copyright © 1999 by Kluwer Academic Publishers. Adapted with kind permission from Kluwer Academic Publishers.

Using Atlas.ti 6 (Archer, 2009), networks of the *code families* are now illustrated and explained. The data for the three *code families* under Theme 2 were collected from both the teachers' interviews and the lesson observations. The *code family* created for the first subtheme **ML learners** appears in Figure 4.5 below. The broken line arrows indicate the six different codes (indicators) being linked to the code family **ML learners**, namely L1, L2, L3, L4, L5, L6. A full description of each code is provided in Table 4.3 above. The data linked to these six codes were collected from the interviews conducted with the teachers. The solid line arrows with double equal signs indicate other codes from the observations that are *associated with* the codes from the interviews (Archer, 2009). For example, codes DTL2, DTL3, DTL4 and DTL5 (see Table 4.2) are associated with L6 (see Table 4.3).

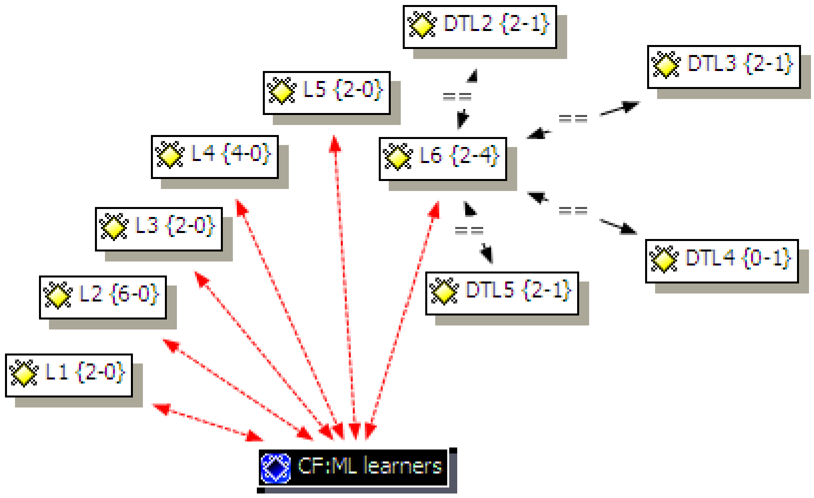



**Figure 4.5: ML teachers' PCK and beliefs: ML learners**

The *code family* created for the second subtheme **ML teaching** appears in Figure 4.6 below. The broken line arrows indicate the six different codes (indicators) being linked to the code family **ML teaching**, namely T1, T2, T3, T4, T5, T6. A full description of each code is provided in Table 4.3 above. The data linked to these codes were collected from the interviews conducted with the teachers. The solid line arrows with double equal signs indicate other codes from the observations that are *associated with* the codes from the interviews. For example, codes TMS1, TR3, TSL2 and TSL3 (see Table 4.2) are associated with T5 (see Table 4.3).

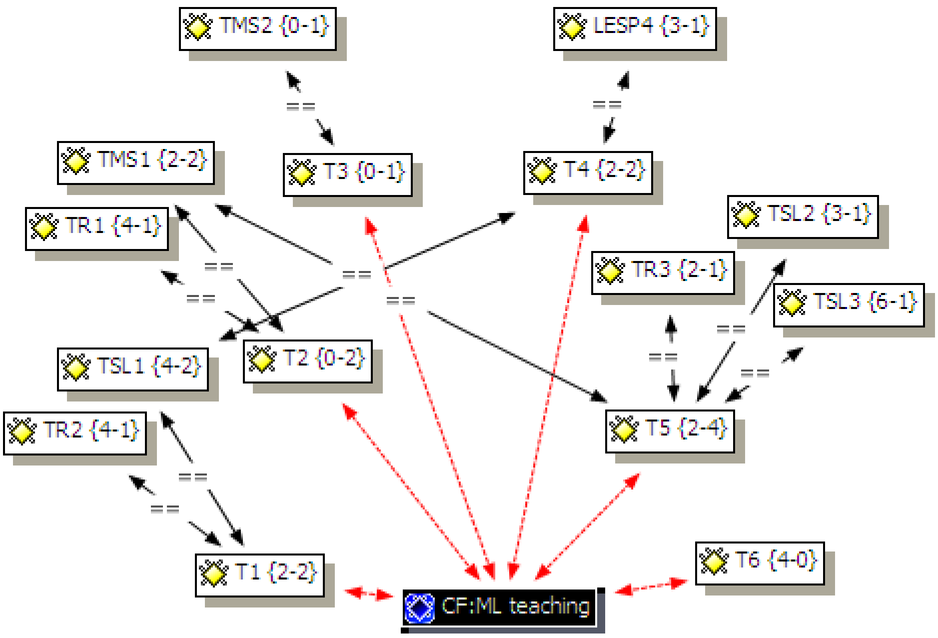



**Figure 4.6: ML teachers' PCK and beliefs: ML teaching**

The *code family* created for the third sub-theme **ML curriculum** is given in Figure 4.7 below. The broken line arrows indicate the nine different codes (indicators) being linked to the code family **ML curriculum**, namely C1, C2, C3, C4, C5, C6, C7, C8 and C9. A full description of each code is provided in Table 4.3 above. The data linked to these nine codes were collected from the interviews conducted with the teachers. The solid line arrows with double equal signs indicate other codes from the interviews too that are *associated with* the initial codes. For example, codes Nature 1 and Nature 2 (see Table 4.3) are associated with C8 (see Table 4.3).

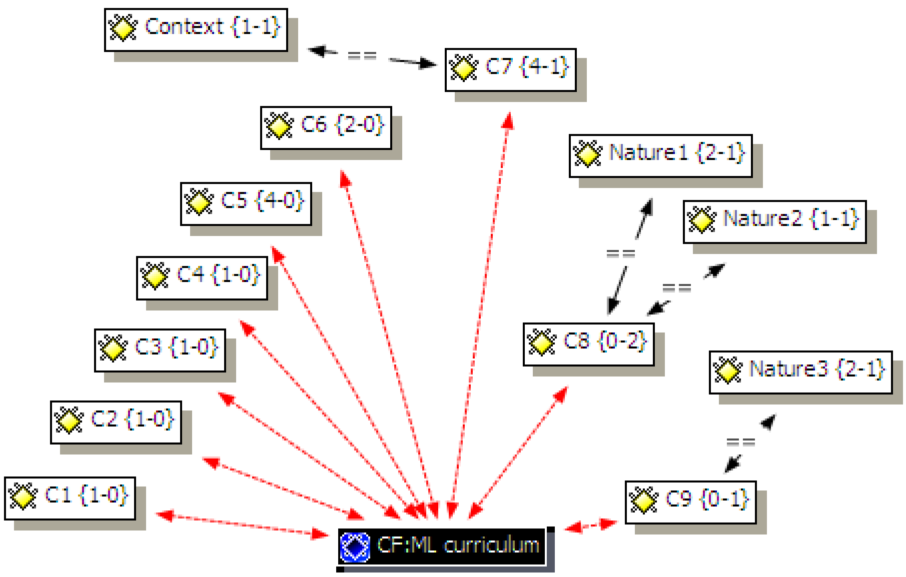



**Figure 4.7: ML teachers' PCK and beliefs: ML curriculum**

#### 4.3.2.3 Inclusion criteria for coding the data

There are two tables indicating the inclusion criteria for coding the data. Table 4.2 was used to analyse the ML teachers' instructional practices. The table consists of the different lesson dimensions, namely tasks, discourse and learning environment and the respective lesson dimension indicators. The descriptions of the lesson dimension indicators serve as inclusion criteria for coding the data from the observations. Examples of each code are provided. These codes were used to analyse the raw data and reporting on the data.

Table 4.3 was used to analyse the ML teachers' knowledge and beliefs. The codes have been assigned according to MCK as well as the three PCK and beliefs regarding the ML learners, ML teaching and the ML curriculum. In Table 4.3 the descriptions of teachers' knowledge and beliefs indicators serve as inclusion criteria for coding the data. Each code has been linked with the corresponding interview question(s) as well as cross referencing to specific code(s) in Table 4.2 of the observations.

#### 4.3.2.4 Exclusion criteria for coding the data

In the process of coding the observations, some of the activities and discourse were not relevant to my study and did not form part of my predescribed lesson indicators whereas others were inaudible. These were excluded when the data were coded. During the interviews some participants did not always keep to the question asked or sometimes used the chance to raise personal points of concern. In Table 4.4 below I listed these exclusion criteria as well as examples of text that were excluded from coding.

**Table 4.4: Exclusion criteria for coding of the data**

| Exclusion criteria | Examples of text excluded from coding |
|---|---|
| Incidents during class observations when I could not hear what was said | This occurred when the teacher attended to individual learners' at their desks or when they had private conversations at the board or at the teacher's desk. Some of the data were inaudible when I did the transcribing. |
| Interruptions | Teachers that needed to attend to people who knocked on the door. |
| The question in the second interview regarding the ML learners' abilities and motivation as the question was included for personal interest only | *Describe your Grade 11 ML learners in terms of their a) mathematical abilities and b) motivation. How does their motivation compare with that of the Mathematics learners?* |
| Elaborations when questions have been misunderstood (The misunderstanding is included in the coding but not the elaboration part of the answer) | ***How do you think will the learners approach these tasks?*** *All they have to do is most times when I try to give them the tasks on data handling and try to make them set the questionnaire and then try to see make it look real-life, you try to make it what people think about the* |

| | |
|---|---|
| | *like a … and then you get a questionnaire and then would give that to your friends and then you collect those data and then you try to sort them out and then present them using a bar graph or line graph.* |
| Providing detailed examples to illustrate their answers. | **Describe the ideal ML classroom in terms of instructional strategies used.** *I try to involve them as much as I can, that they understand that. They must learn through doing it, trying on their own. Try doing it, drawing the Cartesian plane, those are the points, plot them, OK join the points and what do you see? Oh OK, its curved, it's called parabolic function, how do we use it in everyday life. Because if I just draw and plot it myself they look at it but the constructing, they construct while they are doing.* |

## 4.4 Information regarding the four participants

In the next section biographical information regarding the four participants Monty, Alice, Denise and Elaine is provided as well as some background information regarding the observed lessons. Pseudonyms were used to protect their identities.

### 4.4.1 Monty

Monty is a novice teacher in his second year of teaching Grades 10, 11 and 12 ML and one year of teaching Grade 10 Mathematics. He is 24 years old and completed his Baccalaureus Educationis (BEd) degree with Mathematics as major in 2010. Apart from the six ML courses he attended during 2010, organised and presented by the DoE and the District Office, he had had no formal training for teaching ML. He teaches at an inner city school of 500 learners with 18 and 35 learners in the two Grade 11 classes respectively.

The topic of Monty's first two lessons I observed was solving simultaneous equations using the substitution method, only mentioning the elimination method (Learning Outcome 2). The first two observations were done on 3 and 4 May 2011. The third lesson was on data handling where the mean, mode, median and range as measures of dispersion were discussed (Learning Outcome 4). The last lesson was observed on 9 May 2011.

### 4.4.2 Alice

Alice grew up in Nigeria, is 30 years old and in 1995 she obtained a Baccalaureus Technologiae (BTech) Management Accounting degree at a University of Technology with second year Financial Mathematics. She did not take any Mathematics Education or Mathematics Methodology courses. She has no experience of teaching Mathematics and it is her second year of teaching ML. She is teaching at an independent inner city school with 350 learners where the number of learners in her Grade 11 ML classes ranges from 20 to 30.

The first lesson I observed with Alice was based on the use of the quadratic formula to solve quadratic equations (Learning Outcome 1), a sequential or follow-on lesson to reinforce the work done the previous period. A student teacher was responsible for the previous lesson[34]. I observed this lesson on 9 May 2011. The second lesson was on graphing the parabola (Learning Outcome 2) using a table method and finding the intercepts but without determining the turning point. This lesson was observed on 10 May 2011. The third lesson was on data handling: the cumulative frequency, relative frequency, standard deviation and the ogive (Learning Outcome 4). This lesson was observed on 16 May 2011.

### 4.4.3 Denise

Denise is 42 years old and completed a BEd degree in 2003 with Mathematics and Methodology of Mathematics as two of her major subjects and also completed her BEd Honours in 2009. She obtained both degrees from the University of Witwatersrand. She completed a 40-hour course based on ML and the teaching thereof at the University of South Africa. She has seven years' experience of teaching Mathematics and it is her fourth year of teaching ML. She is teaching at a Section 21 (former model C) school in Pretoria with 908 learners where 92% of the learners are black while the other 8% consist of White, Indian, Coloured and Asian learners. The number of learners in her Grade 11 classes ranges from 20 to 32.

The first two lessons I observed were the same lessons presented to two different classes and concerned conversions from metric units to imperial units, involving capacity and mass problems (Learning Outcome 3). These two lessons were observed on 17 May 2011 and 20 May 2011 respectively. The third lesson was a follow-on lesson where more advanced conversions within the metric system only were done on capacity, mass, length, area and volume. This lesson was observed on 26 May 2011. Denise has experience and knowledge of her subject and her lessons were coloured with

[34] The situation that occurred when the student teacher introduced the topic and Alice took over the next period is discussed under Section 4.4.2.3: Mathematical content knowledge.

humorous comments so that learners participated and enjoyed her classes. Learners were involved by solving problems in class, writing on the board and answering questions.

### 4.4.4 Elaine

Elaine is 44 years old and completed her Higher Education Diploma (HED): Senior Primary with Mathematics and Mathematics Didactics as two of her major subjects in 1989 at the Normaal College of Education. During those years the Mathematics I-IV, offered as a major subject at colleges, was equivalent to a first year Mathematics offered at universities. She did not attend any courses on ML or the teaching thereof. She has eight years experience of teaching Mathematics and it is her third year of teaching ML. She is teaching at a Section 21 (former model C) school in Pretoria with 1 300 learners where 95% of the learners are white whereas the other 5% are black. She is responsible for only one Grade 11 ML class and there are 25 learners in the class.

During the period of observation, Elaine was busy with her revision programme. The first and third lessons I observed were based on calculating circumference, area, volume and surface-area of two and three-dimensional shapes (Learning Outcome 3). These lessons were observed on 6 May 2011 and 19 May 2011. The second lesson was based on time (Learning Outcome 3) and interest (Learning Outcome 1) and was observed on 12 May 2011. Elaine is the ML coordinator at her school and her goal is not only to equip her ML learners with knowledge and skills they can use in their lives, but also to promote the purpose and value of ML everywhere she goes. Elaine believes that the notion of contextual mathematics makes ML a valuable and interesting subject as learners' general knowledge is enriched through their experiences with contextual mathematical problems. She mentioned that her ML learners are often envied by the Mathematics learners because of their interesting lessons.

To summarise: The most relevant information appears in the Table 4.5 below:

**Table 4.5: Biographical information of the four participants**

| | Monty | Alice | Denise | Elaine |
|---|---|---|---|---|
| **Age (years)** | 24 | 30 | 42 | 44 |
| **Highest qualification** | BEd: FET | BTech: Management Accounting | BEd Honours | HED: Senior Primary |
| **Mathematics teaching experience (years)** | 1 | 0 | 7 | 8 |
| **ML teaching experience (years)** | 2 | 2 | 4 | 3 |

# 4.5 Theme 1: The ML teachers' instructional practices

In this section I present and discuss the findings from the observations of Monty, Alice, Denise and Elaine. All discussions on the sub-themes **Tasks, Discourse and Learning environment** are structured strictly according to the specific order of the different lesson dimension descriptors (codes) as indicated in Table 4.2[35] (Artzt, et al., 2008). The language of all quotes from Monty, Alice and Denise has not been edited. Since Elaine's classes were conducted in Afrikaans I translated her quotations into English. Background information regarding the observed lessons of the participants is given. A summary is provided at the end of this section in table form analogous to Table 4.2.

## 4.5.1 Monty's instructional practice

### 4.5.1.1Tasks

***Tasks: Modes of representation (TR)***

In the first two lessons on solving simultaneous equations, Monty represented the mathematical concepts by means of written examples on the board using the variables $x$ and $y$ as well as calculators for basic calculations (TR1). For example Monty asked the learners to write an answer of $-\frac{12}{10}$ as a decimal (TR1). He told the learners they could also use their calculators to change a decimal answer to a fraction form and taught them how to operate their Sharp and Casio calculators (TR1). In the introduction of the third lesson on measures of dispersion, Monty called a learner to the front and asked the class to name characteristics that describe him as a boy and not a girl. He used a manipulative (a learner) to demonstrate that even in mathematics concepts have certain characteristics and said:

> *So even if you go to this thing of Maths or ML, you need to have a picture of it, what are the characteristics of it? What must I identify to be able to say this is that thing?*[36] (TR1).

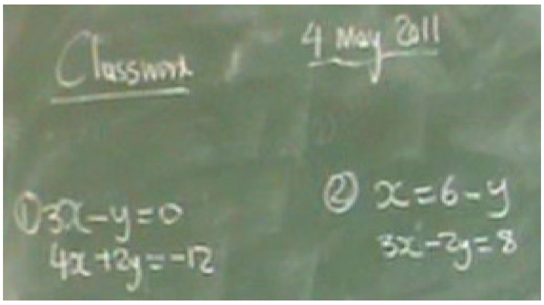
Monty followed through with the *characteristics* idea by asking the learners to provide another word for some of the concepts, for example that average is another word for mean and middle number is another word for median (TR1). He told them how to calculate the measures of dispersion but did not tell them or ask them why and when we use these measures of dispersion and did not apply these concepts to a real-life situation (TR1). He did not use multiple representations to enable learners to connect prior knowledge to the new content, but only verbally referred to prior knowledge when he said:

[35] Table 4.2 is discussed under Section 4.3.2.1: Inclusion criteria for coding the data.
[36] The language of all quotes from Monty, Alice and Denise has not been edited. I translated Elaine's quotes from Afrikaans to English.

*We learned how to solve an unknown. We learned how to draw graphs of linear function, how to draw the graph of linear function* (TR2).

***Tasks: Motivational strategies (TMS)***

The tasks in both the first and second lessons (Picture 4.1) were based on pure mathematics and not on interesting and applicable real-life situations in which learners could do problem solving in groups or discuss the meaning of the solutions (TMS1).



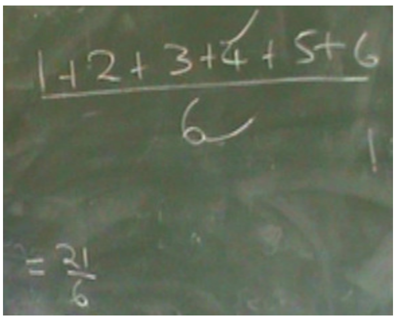

**Picture 4.1: The two tasks from Lesson 1**

The tasks did not capture the learners' curiosity nor inspire them to speculate on their conjectures (TMS1). The learners did not appear interested while they were listening, looking and copying work from the board (TMS1). After the first example ($\begin{matrix} x+2y=4 \\ 7x-5y=9 \end{matrix}$) of solving simultaneous equations during the first lesson, Monty said:

*OK, now you try this one:* $\begin{matrix} y=x+3 \\ y=3x-7 \end{matrix}$ *(pause). OK, you tell me, I write for you.*

But at the end he treated this second example the same way as the first one, telling them what to do and not allowing the learners to solve the problem themselves. This second example was an easier problem since *y* was already the subject (TMS1). The examples used in the data handling lesson were not motivating to the learners (TMS1). These examples based on measures of dispersion were meant for learners with low ability and no experience (TMS2). To illustrate the mean (Picture 4.2), Monty said:

*For example we have numbers 1, 2, 3, 4, 5, 6 and you want to calculate the mean* (TMS2).




**Picture 4.2: Monty example of how to calculate the mean**

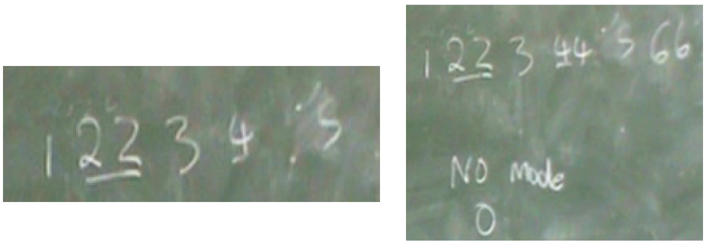
The example for the median (Picture 4.3) was where numbers from 3 to 13 were given in random order and he said:

> *The median. What's a median? A middle number. So for you to find a correct middle number you have to arrange the numbers in an ascending order…* (TMS2).

**Picture 4.3: Monty's example of how to calculate the median**

The example for the mode (Picture 4.4) was:

> *OK, let's move on to mode. Please, if you don't understand something, please ask. OK, the mode (erases work on board). You are given this data: 1 2 2 3 4 5. What's the mode? OK first, what's a mode? A mode is a number that appears? Girl: more than once* (TMS2).




**Picture 4.4: Monty's example of how to calculate the mode**

His motivation is examination driven, preparing the learners for the examination by frequently telling them what is expected of them in the examination, how to use their time sensibly and that all steps should be shown otherwise they will lose marks (TMS3). He once asked in a lesson:

> *What was difficult? Learner: Nothing. Teacher: So what I am saying to you, when you say to yourself something is difficult, it is, but once you did it right you see nothing is impossible* (TMS3).

In his introduction to the data lesson when he had a comprehensive description of how research is done, he said:

> *Why am I telling you this? But in Grade 11 I must just tell you, remember guys, life doesn't end here at school, life doesn't end at school. Outside you will need that information to use it.*

He wanted to emphasise the value of ML for learners' future lives, but did not discuss how the content is applicable to real-life situations and no such example or homework was done (TMS3).

***Tasks: Sequencing and difficulty level (TSL)***

Not much attention was given to sequence the tasks in order for the learners to obtain cumulative understanding of the content (TSL1). He worked from a more difficult example $\begin{matrix} x+2y=4 \\ 7x-5y=9 \end{matrix}$, where $x$ or $y$ first needed to be made the subject of the equation in order to continue with the solution of the problem, to an easier example $\begin{matrix} y=x+3 \\ y=3x-7 \end{matrix}$, where $y$ was already the subject of the equations (TSL1). He explained the elimination method prior to the two lessons I observed in which he explained the substitution method, but he did not expect them to use both methods in order to practise them simultaneously (TSL1). In the data handling lesson the examples discussed in the above paragraph were not sequenced but connections were made with ideas learned in the past (TSL1). The tasks given in the lessons on simultaneous equations provided opportunities where learners could reinforce current work whereas the tasks in the data handling lesson were meant to revise basic Grade 10 work (TSL2). The homework for the data handling lesson was not carefully selected, instead the first four problems from the exercise were chosen, questions that were basic and easy. (TSL2) The questions based on simultaneous equations were appropriate but not set in context and therefore on Level 1 (Knowing) of the ML assessment taxonomy[37] (TSL3). These examples were basic, did not reflect quality and were on a level far below Grade 11, even for a revision lesson[38] (TSL3). The homework was suitable for what Monty did in class, but did not require the learners to do proper revision of their Grade 10 work (TSL3).

#### 4.5.1.2 Discourse

***Discourse: Teacher-learner interaction (DTL)***

Monty ensured that all learners were quiet while listening, attending and copying the work from the board during his presentations (DTL1). He was non-judgmental but did not encourage learner participation, with the result that there was no evidence of the learners' ideas (DTL1). He never asked them to explain or justify themselves, instead asking lower order and basic calculation questions throughout the lessons (DTL2). There was no evidence of the teacher listening to learners' ideas and providing scaffolding to support their attempts (DTL3). Some learners mumbled answers to Monty's questions (DTL3). After the incident in which a learner wrote her solution on the board Monty asked:

> *Anyone who can do the second one? (Silence). Anyone? …… have you tried it? Anyone? ……. (All just look at him). OK first, let's go to our notes. Firstly we said we need to label them…*

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[37] Discussed in Section 2.2.2.2: ML principles.

[38] Examples have already been given under Tasks: Motivational strategies.

He continued to explain in exactly the same way the solution to the first problem (DTL3). When learners answered his questions, he did not comment on their answers. He either repeated the answer or if the answer was wrong, provided the correct answer for example:

> *The mode is 2 and? Girl: 4. Teacher: 4* (DTL3).

In another example $\begin{matrix} 3x - y = 0...............1 \\ 4x + 2y = -12.........2 \end{matrix}$, Monty asked:

> *Is x or y the subject of the formula? Is x or y the subject of the 2 given equations? Boy: y. Teacher: Is x the subject of the formula in the 2 equations? Another boy: No. Teacher: What you do? What do you have to do? Some learners: Third equation. Teacher: A third equation* (DTL3).

When he did the example $\begin{matrix} x + 2y = 4 \\ 7x - 5y = 9 \end{matrix}$, he asked: *Is x or y a subject of an equation?*

He initially got no response and later some learners told him that *x* was the subject (DTL4). In the discourse that followed Monty became irritated, looked troubled and laughed as he could not understand why the learners did not know the answer (DTL4). There was no evidence of the teacher recognising or clarifying learners' misunderstandings (DTL5). He assisted most of the learners while they were busy with classwork (DTL5).

***Discourse: Learner-learner interaction (DLL)***

He did not encourage learners to listen to, respond to or question one another although there were opportunities during classwork where they could do so (DLL1). On one occasion he said:

> *If you did not understand it well yesterday you can now work with your friend. Don't allow your friend to just copy, he must ask and talk.*

Only three groups were formed with two learners per group (DLL1). One group just worked from the same textbook while the other two groups discussed the work. The rest of the learners worked individually (DLL1). There was not a learner or group of learners who dominated the verbal communication in class (DLL2).

***Discourse: Questioning (DQ)***

Most of the questions Monty asked were factual and of lower order, such as complete the word/sentence and calculation type of questions and in many cases he answered the questions himself (DQ1). In one lesson there were 97 such questions. Examples of such questions are:

- When $2x^7$ was written on the board during lesson 1, the following was asked:

*T*[39]*: OK we call this one a co…? (Referring to 2) efficient (and completed the word while some learners mumbled an answer)*
*T: This one we call it…? (Indicating to the x)*
*L: Variable.*
*T: The variable (and writes it on the board).*

- An example of answering his own questions was during the introduction phase of lesson 1:

  *T: What are simultaneous equations? It's a combination of two linear functions.*

- Factual questions such as the following were asked during the lesson on data handling:

  *T: What is meant by mean? What comes to your mind? When you see mean, what comes to your mind?*
  *L: Bigger, smaller.*
  *T: Average. It's average. And once you see that average, you see lots of numbers adding each other and dividing by the number of them, that is the picture you must have*

- Lower order questions during simultaneous equations :

  *T: OK now we know x=-1,2. We go and substitute x=-1,2 in…?*
  *L: 2.*
  *T: Into…?*
  *L: 2.*
  *T: Into equation…?*
  *L: 2.*
  *T: 2. (Teacher did it.) What is the answer there of 4(-1,2)? Huh? What's the answer?*
  *L: -4,8.*
  *T: -4?*
  *L: -4.8.*
  *T: -4.8* (DQ1).

On only one occasion Monty asked a learner to explain her work but did not follow it through:

> *T: Explain your answer, here use this (He gives her a large triangle. She is shy and cannot look at the class and put her head in her hand.) OK people (and he takes it out of her hand) if you are given these 2 equations: Equation 1, x is the subject of the formula …. (and he continues to explain her answer)* (DQ2).

The questions did not contribute to the verbal communication and participation of learners and did not create opportunities when they could listen to and respond to each other's answers (DQ3). In general his questions were addressed to the whole class, who mumbled answers or sometimes responded in chorus (DQ4). A few times some learners volunteered to answer and on only three occasions he called on particular learners to answer his questions (DQ4).

#### 4.5.1.3 Learning environment

***Learning Environment: Climate (LEC)***

To comment on Monty's relationship with and among the learners based on how they valued each other's ideas and ways of thinking is difficult as his lessons were teacher-centred with minimum interaction (LEC1). He generally did not value or seek their ideas (LEC1). When I commented in the

[39] When discourse was quoted, I used the following abbreviations: Teacher (T); Learner (L); and Researcher (R).

last interview on the class discipline I observed, Monty replied that he has classroom rules and values discipline in order for learning to take place (LEC2). He further mentioned that the principal highly values discipline in their classes (LEC2). The learners respected Monty and were well behaved, sat quietly in class listening to him and copying the work from the board. Monty only needed to discipline them once during the three lessons when he said:

> *OK hey hey hey, listen (pause), listen, the break is over. Please don't disturb my class. Shhh. So, if I see you talking, I am going to chase you out and then you will come back next term* (LEC2).

On more than one occasion he reminded them that they are not *Mathematics people* and this may either be comforting to the learners or a matter of degrading them (LEC3). Monty is confident and enthusiastic about ML and has a positive attitude towards the subject and the learners (LEC3).

***Learning Environment: Strategies and pacing (LESP)***

Monty used direct instruction (lecturing) as instructional strategy and his teaching style varied between a traditional and formal authority style (LESP1). Learners were involved copying work from the board, listening to his explanations and answering basic low level questions (LESP1). During the second lesson he gave the learners two problems to complete in class, but allowed only a few minutes to solve the problems before he asked a learner to write her solution on the board. Since she was too shy to talk, he again explained her solution in detail as he had done with the previous examples (LESP1). Near the end of this lesson he was running out of time and hurried through the last example by saying: *OK let's go faster there is no more time* (LESP2). I observed only two phases of a lesson in the first two lessons, namely the initiation and development phases (LESP3). Since there was no closure phase there was no opportunity to summarise or assess learners' knowledge and understanding (LESP3). Only the last six minutes of the first lesson were allocated for a class activity of which the learners used five minutes to get settled (LESP3). His second and third lessons flowed logically (LESP4). For example, in his third lesson he used the following discussion as part of his introduction:

> *T: Where can we solve this simultaneous? OK, remember we are approaching the Election Day and we need to support the campaign. Don't you think the results can be solved simultaneous, how? Remember he has to sit on the parliament and the province and we have 9 provinces né? Remember for the vote of the 18th they are going to take the result because remember people voted for this particular party or this party or organisation. They are going to add all those results and what information do you think we can get out of that? We can convert it into equations and solve simultaneous. That will tell us how many positions that party is going to get in…? Parliament, né? So you see we solve it simultaneous. Once you know the number of how many people voted for the party you know how many will sit in parliament.*

This was followed by more examples of the work they had done the previous day and he again referred to contexts at the end of the lesson (LESP4). It was unclear how the learners were supposed to connect the context to the content (LESP4).

***Learning Environment: Administrative routines (LEA)***

Time was not used efficiently to maximise learners' involvement as Monty spent most of the period using direct instruction (LEA1). Monty did not involve the learners in his explanation of how research is done and did all the explaining of the examples on his own; meaning no active learner involvement (LEA1). There was no opportunity for the learners to be part of an active learning process in which they could learn from each other and improve their understanding (LEA1). The time at the end of the lesson when learners were supposed to start doing classwork, was not well spent as they used five of the six minutes to settle down (LEA1). They were seated at their individual desks during content presentation, but could work in groups during classwork time if they preferred (LEA2). When Monty explained work he was always in front of the class but when the learners were working at their desks, he walked up and down the isles assisting learners with their work which contributed to a positive learning atmosphere (LEA3). The written information on the board was correct but not always well-organised (LEA4).

***Summary***

**Table 4.6: Summary of Monty's instructional practice**

| LESSON DIMENSIONS | DESCRIPTION OF LESSON DIMENSION INDICATORS |
|---|---|
| **Tasks** | |
| Modes of representation (TR) | Monty used representations such as written examples on the board, variables, calculators and a manipulative. He seldom connected learners' prior knowledge to the new mathematical situation. |
| Motivational strategies (TMS) | The tasks he chose were not motivational to the learners. He talked about contexts during two of his lessons but the contexts were not applicable and valuable. He attempted to point out the value of mathematics so that learners would value the work they were doing but his explanation was vague and learners could not relate to it. |
| Sequencing and difficulty levels (TSL) | Not much attention was given to sequencing the tasks and no connections were made with ideas learned in the past. The tasks on simultaneous equations were on a Grade 11 level (Level 1) but the tasks in the data handling lesson were on Grade 10 level (Level 1). |
| **Discourse** | |
| Teacher-learner interaction (DTL) | He was non-judgmental but did not encourage learner participation with the result that there was no evidence of the learners' ideas, occasions when learners' thinking was challenged or even when he recognised or clarified learners' misunderstandings. He assisted most of the learners while they were busy with classwork |
| Learner-learner interaction (DLL) | There was minimum learner-learner interaction. |
| Questioning (DQ) | The questions were of lower order. The type of questions was mainly complete the word/sentence and calculation questions. The questions were addressed to the class and the responses were |

| | |
|---|---|
| | volunteered. |
| **Learning environments** | |
| Social and intellectual climate (LEC) | There was a positive rapport between the teacher and the learners. He maintained good discipline throughout the lessons in order for learning to take place and had a good relationship with his learners. |
| Modes of strategies and pacing (LESP) | His teaching style varied between a traditional and formal authority style. He used direct instruction (lecturing) as instructional strategy and once allowed a learner to work on the board. Typical of teacher-directed lessons, learners were involved copying work from the board, listening to explanations of the teacher and answering basic low level questions. Monty did not get to the closure phase of a lesson where learners' knowledge could be assessed. There was a logical flow to his lessons. |
| Administrative routines (LEA) | Time was not used efficiently to maximise learners' involvement as Monty used most of the period lecturing to the learners. He stayed in contact with the learners as he moved between them which contributed to a positive learning atmosphere. |

## 4.5.2 Alice's instructional practice

### 4.5.2.1 Tasks

***Tasks: Modes of representation (TR)***

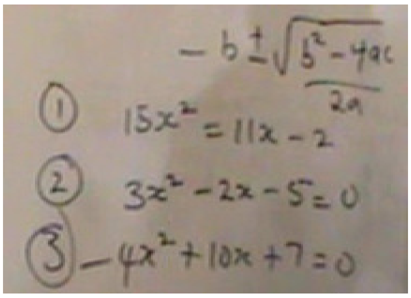
In the first lesson on the use of the quadratic formula, Alice used representations[40] such as written examples on the board, symbols, the formula, tables, graphs and calculators (TR1). In her next lesson on graphing parabolas, she used a table as well as the formula to calculate the $x$-intercepts (TR1). In the third lesson on data handling the following was taught within a single 35 minute period: 1) the mean, mode, median and range (when a set of data were given and when instead of a set of data a table with frequencies were given); 2) pie charts (interpreting a given pie chart and drawing a pie chart), bar graphs and histograms; and 3) tables with tallies, frequencies, $\sum$ symbol, cumulative frequencies (TR1). She did not do the standard deviation or the ogive that she also planned to do (TR1). She started the lesson by revising the bar graph and histogram, continued with another example on how to set up a table with marks of learners, tallies and frequencies when a set of data were given, then mentioned they were going to do the pie chart, but when she turned around to clean the board she continued with different elementary examples on the mode, mean and median. Afterwards she did a comprehensive example on a pie chart. She then introduced the symbol $\sum$ to the learners and continued her lecture on cumulative frequencies. When the learners did not understand her explanation, she drew a table on the board with three columns for the marks, frequency and cumulative frequency. She then mentioned the drawing of

[40] Examples of all these representations are given in the text to follow.

the ogive but told them they would get back to this as she first wanted to explain how a table could be constructed when only the frequencies rather than a set of data were given. She then explained how to find the range and median from the above table (TR1). The learners complained throughout the lesson that they did not understand the work (TR1). She used these various representations in an attempt to have the learners connect their prior knowledge with the new content, but the extent of the content and the way she presented the work was too much for the learners to absorb and led to confusion (TR2). Some learners just withdrew from all activities during the lesson (TR2).

***Tasks: Motivational strategies (TMS)***

As I have said, on the day before the first observation, a student teacher taught the learners how to use the quadratic formula to solve quadratic equations. In the first lesson I observed, Alice decided to do three more such examples (Picture 4.5) with the learners.




**Picture 4.5: The three tasks Alice gave the learners during lesson 1**

She wrote three quadratic equations on the board of which only the first equation was not given in standard form (TMS1). They only did the first one ($15x^2 = 11x - 2$) and much later during the lesson took another example from the textbook ($x^2 - 10x + 25 = 0$). The tasks provided the learners with an opportunity to pursue their conjectures (TMS1). A group of learners were motivated to take part in the lesson and to pursue their conjectures as the teacher was making many mistakes[41] on the board, but neither the teacher nor the learners could correct the errors in the teacher's work (TMS1). Only a few learners in front of the class participated in the lessons by answering questions or asking questions that Alice most of the times could either not hear or understand. The rest of the learners only copied all written work from the board and some were lying on their arms or talking to each other (TMS1). In the data handling lesson some of the learners were inspired to take part in searching for ways to correct their work or to try and find meaning and understanding (TMS1). There was no evidence that Alice took into account the diversity of learners' interests, abilities and experiences (TMS2). She generally presented her lessons on a level suitable for Mathematics and not ML learners as many of the tasks

[41] Examples of these mistakes are provided later on under Mathematical content knowledge.

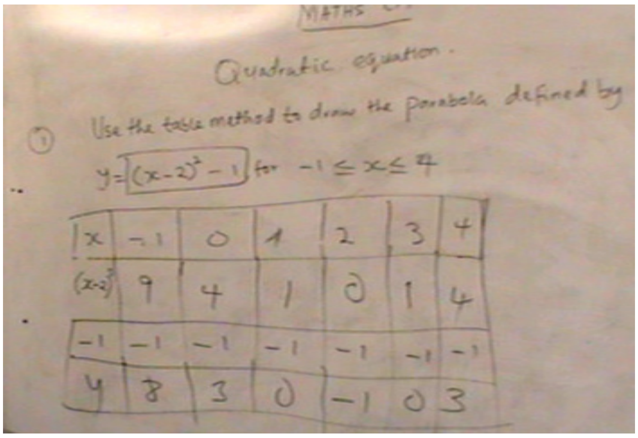
were on Mathematics level as will be discussed under the next subheading (TMS2). Alice did not point out the value of the mathematics being learned, which could have contributed to learners' appreciation of the subject (TMS3).

***Tasks: Sequencing and difficulty levels (TSL)***

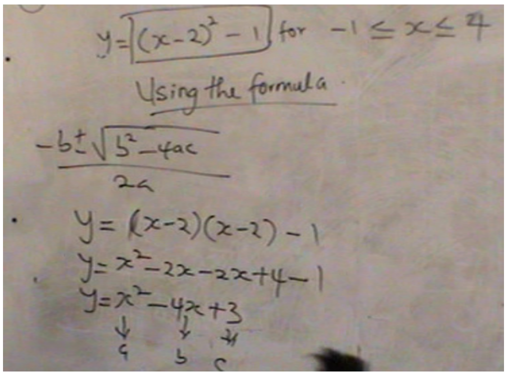
There were incidents for which Alice sequenced her activities. For example, during the first lesson she revised the standard form of quadratic equations and the meaning of the variables *a*, *b*, and *c* (the coefficients) before using the formula (TSL1). For most parts of the lessons Alice tried to link the content with other relevant content or even prior knowledge, but it was not done in a sequential and meaningful way. This resulted in the learners being confused (TSL2). For instance, after she finished the first example of using the formula to solve a quadratic equation, the following dialogue followed:

> *T: You remember when we draw the graph (and she erases part of the board and draw a table for x and y coordinates) Quiet guys! (She writes:* $y = x^2$ *) Guys! Now you are not given any formula. We need to start at a negative.*
> *L: Why do you start with -2?*
> *T: Because they don't give it. I am just assuming this is a problem. (She completes the table). This is now where you draw your graph. (She draws two graphs below the table). This is your positive and this is your negative (indicating to the first and second graph). I am not drawing this one (Pointing to example they did). Quiet!*
> *L: Shhhh.*
> *T: Let's draw (and she draws a set of axes and labels them. Learners talk and teacher looks at example and erases the set of axes before she could even draw something on the axes) Shhh shhh. OK.*
> *L: Mam, where's my textbook?*
> *T: OK, we have* $6x^2 + x = 12$*. Quiet please! If you don't want to learn, you can leave the class (and she continues to solve* $6x^2 + x = 12$ *).*

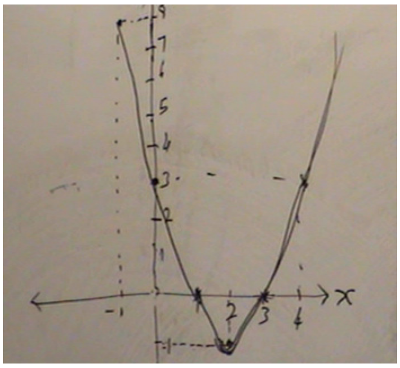
This is but one example of Alice jumping between examples and incomplete explanations (TSL2). In the second lesson when learners had to draw the parabola, they started to draw $y = (x-2)^2 - 1$ for $-1 \le x \le 4$ using a table method (Picture 4.6) followed by the intercept method using the quadratic formula (Picture 4.7). Here she stated that they did not need to calculate the turning point as the graph (Picture 4.8) would automatically go through the correct turning point if they worked accurately (TSL2).




**Picture 4.6: Using a table method to draw the graph of a parabola**



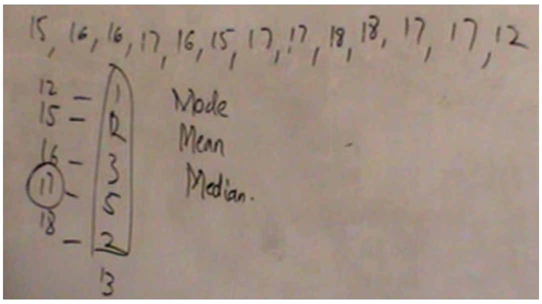

**Picture 4.7: Using the formula to draw the graph of the same parabola above**




**Picture 4.8: The graph of the parabola**

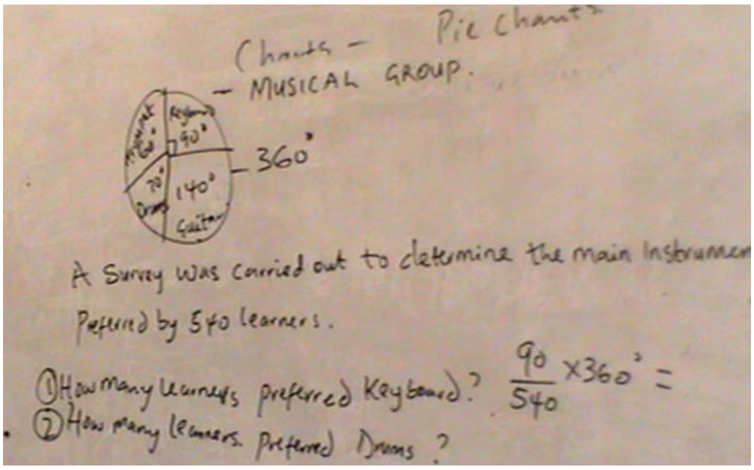
The tasks were on Grade 11 level and suitable for what the learners were supposed to know and be able to do, but would only need to practise (TSL3).

In the data handling lesson she sequenced the concepts and content to be covered during the lesson as she revised, in this order, the mode, mean, median and range (Picture 4.9) before she introduced the cumulative frequencies and ogives (TSL1).




**Picture 4.9: Calculating the measures of dispersion from a list of data**

Although the lesson was well sequenced, the tasks within the presentation of a concept were not sequenced so that learners were not able to progress in their cumulative understanding of the content (TSL1). She gave an appropriate and interesting example using a pie chart was given (Picture 4.10) and learners had to answer questions based on the pie chart.

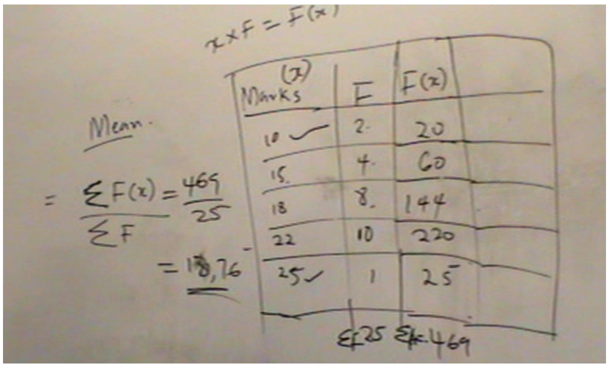



**Picture 4.10: Task given based on the pie chart**

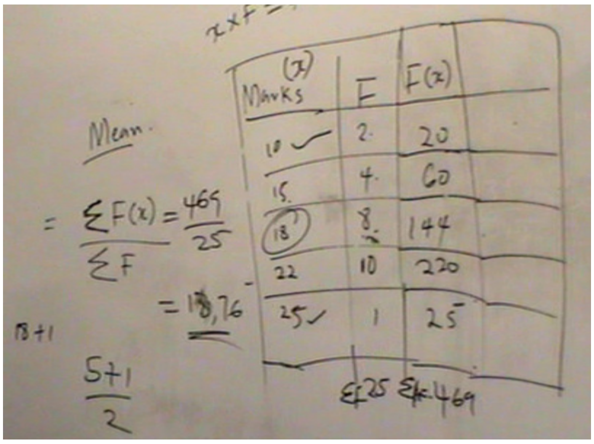
While the learners were still struggling with the task the following discourse took place:

> *T: But sometimes they don't give you this. You are given this information (pointing towards answers) and then you are asked to represent it like this (pointing to the pie chart). Now take for example (erases the pie chart) you are given this, you are only given the number of learners, the answers and then they ask you to represent this information in a pie chart. What do you do? (Silence.)*
> *L1: divide by 540.*
> *L2: 135 over 540.*
> *T: Remember it's a fraction. This is a fraction of this (pointing to 135 and 540), so it's this divided by this times what? 360, which gives you 90 degrees which was given here. So sometimes you are given data to represent in a pie chart, sometimes you are given the pie chart and you have to try and get the number of learners. OK, I am sure you are OK now with your pie graph and the frequency.*

After this verbal explanation she immediately continued with another example of using a table and frequencies to calculate the mean (Picture 4.11) and median (Picture 4.12) (TSL1).




**Picture 4.11: Using a table and frequencies to calculate the mean**




**Picture 4.12: Using a table and frequencies to calculate the median**

The extract from the observation given below and based on Picture 4.12 above is another example where learners could not progress in their cumulative understanding of different methods for finding the mean and median (TSL1.

*T: OK, what about if you want to find your median using this table, what do you do?*
*L1: You write it in ascending order.*
*T: But now your marks are in ascending order and you have your frequency and your total frequency is what?*
*L2: 25.*
*T: 25 and you know that you have 25 data's. So it's an odd number. So you have to get the middle number, so how do you do that?*
*L3: You say 18+1 over …*
*L4: No.*
*L3: You said 18+1. (The boy takes 18 as middle number in the table).*
*L4: Mam, you said there are 5 numbers, so it is 5+1 over 2.*
*T: Remember this is the data, this is the frequency (pointing to the table), so it means you have two 10's, four 15's, eight 18's; ten 22's and one 25. The frequency is 25. So its 25 plus 1 divided by 2 its 13. So your 13th data is going to be your median. So this plus this gives you 6 and this plus this gives you 14 so your 13th data is 18.*
*L3: But that is what I said* (TSL1).

The learners did not appear confident about finding the measures of dispersion using two methods as she never applied both methods using the same example and they also needed clarity concerning aspects of the pie chart (TSL1). The learners were not involved in individual class work as the teacher did several examples either verbally or in writing on the board throughout the lesson (TSL2). Many of these examples were not relevant to the new content she actually planned to introduce and by the time she got to the new content the learners were exhausted and confused (TSL2). The curriculum does not require learners to use the formula $\frac{n+1}{2}$ to find the median and the symbol $\sum F$ to find the sum of the frequencies (TSL3). All tasks were on Level 1 (Knowing) of the ML assessment taxonomy except for the only contextual example, the one on the pie chart that was on Level 2 (Applying routine procedures in familiar contexts) (TSL3).

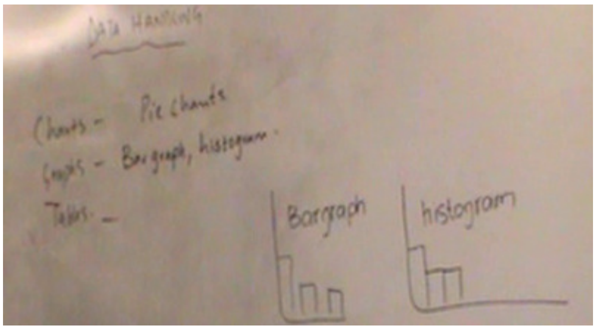
### 4.5.2.2 Discourse

***Discourse: Teacher-learner interactions (DTL)***

During the first lesson Alice used the quadratic formula incorrectly ($x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$) and also omitted brackets during the substitution, causing confusion and chaos in class as several learners talked at once to Alice and one another in attempts to clarify the problems. At first Alice ordered them to keep quiet and later shouted at them as she tried to find the problems herself. Since she could not identify her mistakes, she allowed participation from the group of learners sitting in front while the rest of the class was ignored (DTL1). During the second lesson she did not involve any of the learners and seldom looked back at the learners in the class while working on the board (DTL1). In the third lesson she encouraged a little participation from the learners in front by asking questions. The rest of the boys at the back were still not involved (DTL1). A girl once attempted to rectify the mistakes on the board but Alice ignored her and she went back to her desk (DTL2). On two other occasions Alice asked a boy and later a girl to come and write on the board but she did not require the learners to explain their work. Instead Alice corrected the boy's work by telling him what to write (DTL2).

Although Alice allowed the learners to become involved in the first lesson, she was too anxious to maintain control of the lesson and the learners and as a result she could not listen carefully to their ideas in order to support their thinking (DTL3). Instead of listening to the learners' ideas to direct her instruction, she used a formal authoritative style and told the learners what to do while demonstrating on the board. In the second lesson she kept strictly to this style to avoid the chaos of the first lesson. Alice said to the learners: *Don't ask me how to get this; you have to look at me* (DTL3). When she was busy with the example discussed above, she did not prompt the learners to justify their answers, instead she re-explained the content the same way she did before. Many times she would not comment on their answers, would make a face and re-explain (DTL3). The girl who drew the bar graph and histogram (Picture 4.13) on the board during the third lesson asked the teacher:

*L1: Mam, can I draw it? (Girl comes to the board and draws both graphs on the board).*
*T: Shh, quiet.*
*L1: It's a bar graph (draws the one on the right saying).*
*L2 No, it's a histogram. (L1 writes 'histogram' and then draws the graph on the left). It's the bar graph.*
*T: OK, you know sometimes you get information that you can represent on the bar graph or histogram. OK, then it's your tables, your table always consist of the marks, the tallies, you still remember your frequency.*




**Picture 4.13: Learner's work on the board during the introduction of the third lesson**

The teacher did not follow up on the girl's sketches by discussing the difference between the bar graph and histogram or the use of these graphs to provide scaffolding to support the learners in their conceptual understanding (DTL3).

In the data handling lesson where Alice was more comfortable with the content, she frowned and made a face when the learners gave incorrect answers to her questions and also when they mentioned that they did not understand the work (DTL4). She became irritated when she could not understand their misunderstandings or misconceptions (DTL5). She said it was impossible not to understand data handling especially since all content of the lesson was supposed to be easy and well-known to the learners (DTL5). There was no evidence that she recognised learners' misunderstandings. For example when she had discussed the pie chart and learners complained that it was complicated and she just replied that it was not and continued with the solution of the problem. When she completed the pie chart she said*: OK, I am sure you are OK now with your pie graph and the frequency. Ok, turn to p. 37 (and she starts writing:* $\sum$ …. (DTL5).

### *Discourse: Learner-learner interactions (DLL)*

ML classrooms are supposed to be learner-centred. Instead of applying this approach, Alice wanted the learners to keep quiet (DLL1). The learner-learner interactions were learners talking to each other about non-mathematics issues and discussions of possible ways of correcting the teacher's mistakes on the board (DLL1). The discourse between learners was therefore not as a result of opportunities Alice created enabling them to discuss the work (DLL1). She allowed a group of learners to dominate the verbal communication in the class and it seemed that she depended on their assistance (DLL2). She ignored the boys at the back who did not participate at all (DLL2).

***Discourse: Questioning (DQ)***

The questions Alice asked while writing on the board were generally memory questions and of a low level such as basic calculation, and complete the word/sentence type of questions (DQ1). Examples taken from the different lessons are:

- *T: What do you have to do? You have to get it in standard form so you need to take it over to this side and she writes and the sign is going to change when you take it over.*
- *T: So with this, all you have to do is substitute into your formula, a is? 15, b is? -11 and c is? 2.*
- *T: How do you find the minimum?*
  *L: You just see it.*
  *T: Where do you get it?*
  *L: It's at -1.*
  *T: This is the minimum value (showing at turning point and continues with other work).*
- *T: Do you understand this?*
- *T: Your frequency is what?*
  *L: It's the number.*
  *T: OK, it's the number (and she continues to do a new example on the board)* (DQ1).

When Alice listened to learners' answers she did not ask them to clarify their answers. Instead she would provide the answer:

*T: To find your mean, your mean is always the?*
*L: Middle number.*
*T: No, mean it's the sum of the data divided by the number of data* (DQ2).

Although she did not create many opportunities for learners to contribute to the verbal communication, the group of girls who did take part in the lessons had the opportunity to question the teacher and their peers (DQ3). Learners' responses were mostly volunteered or chorus. Once or twice she did call on a specific learner (DQ4).

### 4.5.2.3 Learning environment

***Learning Environment: Social and intellectual climate (LEC)***

From what I have observed Alice did not maintain a positive relationship with and among the learners (LEC1). She seems to care about them as she claims to sacrifice her breaks to be available for learners to come to her for help (LEC1). Not much evidence could be found when Alice valued her learners' ideas or even the student teacher's ideas and ways of thinking (LEC1). Alice struggled to control the learners because of the mistakes she made in the first lesson and the overload of content she covered in the third lesson and frequently had to shout at them: *Quiet please! If you don't want to learn, you can leave the class*. The reason for the learners' misbehaviour was that learners were confused and discouraged and Alice could not handle the situation (LEC2). Generally she did not create enough opportunities for learner participation which is typical of a teacher-centred strategy (LEC2). From the beginning of the second lesson she was strict and enforced discipline ensuring they kept quiet, sat at their own tables and copied the work from the board and therefore needed to discipline the learners only five times (LEC2). If Alice had had a positive attitude towards the learners and the subject as she stated in an interview, it was not evident in her lessons (LEC3). She rather appeared bored, irritated and un-enthusiastic, never giving the learners any accolades (LEC3).

***Learning Environment: Modes of instruction and pacing (LESP)***

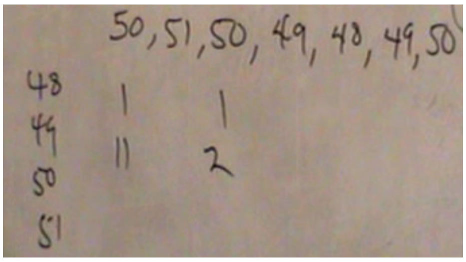
Alice's teaching style varied between a traditional and demonstrative style (LESP1). She used direct instruction (lecturing), a teacher-centred approach in her lessons as well as a little discussion with a few learners in front of the class (LESP1). On two occasions learners worked on the board: the one learner drew the bar graph and histogram and the other wrote a solution on the board (LESP1). The direct instruction strategy Alice used did not always support learner involvement and goal attainment and she was not aware of the learners' lack of knowledge and skills regarding the two topics she covered. Alice assumed that if she understood the work, the learners would understand it too (LESP1). No assessment of the learners' knowledge was done and there was no evidence that Alice's goals had been reached (LESP1). The only time Alice provided time for the learners to express themselves was during the first lesson when the learners tried to rectify Alice's work and to discuss the problems and the nature of the solutions with one another (LESP2). There was no other occasion when Alice structured the time necessary for learners to express themselves and to explore the mathematical content (LESP2). In most of the lessons she did not use her time effectively to accommodate all three phases of the lesson (LESP3). She did not have time for closure at the end of the lesson when she could have summarised or assessed the learners' knowledge and understanding (LESP3). As far as the logical flow of the lessons is concerned, Alice could sequence the content in the lesson, but failed to have a logical flow in her explanations of specific concepts as she attempted to provide the learners with too much disorganised information and incomplete explanations (LESP4).

***Learning Environment: Administrative routines (LEA)***

During the first lesson Alice did allow time for learners' involvement in discourse, but it was not the result of effective procedures and management of her classroom (LEA1). There was not enough time allocated for learners to be actively involved in working on tasks in class. Most of the time she ordered them to keep up with her as she worked on the board and would then say:

> *T: Can I erase this?*
> *L: No.*
> *T: Why? What is taking you so long to write (and she erased it and wrote the following example) Quiet!* (LEA1).

The classroom arrangement was appropriate to the lesson style as learners were seated in three long rows in this very wide classroom, seated at desks with no space between them (LEA2). Alice's position in class during the first and third lesson did not contribute to learners' conceptual understanding as she was in front of the class the whole period, busy working on the board, frequently looking things up in her textbook and talking only to the learners in the front (LEA3). During the second lesson she once walked through the class (LEA3). The work she did on the chalkboard was not organised, she cleaned wherever she needed space to write, causing learners to be confused as they needed to listen and copy the work from the board (LEA4). Some of the examples on the board were incomplete since she often completed her explanations verbally. Sometimes information was missing or unrealistic, like a table without a heading (Picture 4.14) or the angle measurements of the pie chart[42].




**Picture 4.14: Example of no headings given in a table**

***Summary***

**Table 4.7: Summary of Alice's instructional practice**

| **LESSON DIMENSIONS** | **DESCRIPTION OF LESSON DIMENSION INDICATORS** |
|---|---|
| **Tasks** | |
| Modes of representation (TR) | Alice used representations such as written examples on the board, symbols, the formula, tables, graphs and calculators. She could not proficiently use the various representations to connect learners' prior knowledge with the new mathematical situation. |
| Motivational strategies (TMS) | Alice treated the ML learners as if they were Mathematics learners, not taking into account that they were of lesser ability. The only time Alice did an example that was set in context was in the third lesson. Although a small group of learners took part in the lessons, the majority of the learners did not. She did not point out the value of the mathematics being learned. |

[42] See Figure 4.3 discussed under Tasks: Sequencing and difficulty levels.

| | |
|---|---|
| Sequencing and difficulty levels (TSL) | The tasks she chose were appropriate and on Grade 11 level (Level 1) but were not presented logically or in context to ensure that learners were motivated. |
| **Discourse** | |
| Teacher-learner interaction (DTL) | Her interaction with the learners was at times judgmental and it could not be said that she encouraged participation or even created opportunities where learners' thinking was challenged. She did not recognise or acknowledge her own and the learners' mistakes and misunderstandings. |
| Learner-learner interaction (DLL) | Learner-learner interaction was observed during the first lesson but this was not as a result of opportunities Alice created for learners to take part in discussing the work. Instead the learners discussed possible ways to correct Alice's mistakes on the board while others talked about non-mathematics issues. |
| Questioning (DQ) | The types of questions asked were memory, calculation, and complete the sentence questions. Learners' responses were volunteered or chorus. |
| **Learning environments** | |
| Social and intellectual climate (LEC) | Alice did not establish a positive relationship with and among the learners by valuing the learners' ideas and ways of thinking. She at times appeared bored, irritated and unenthusiastic, not praising the learners' work. |
| Modes of strategies and pacing (LESP) | Since Alice used direct instruction (lecturing) as instructional strategy, a large amount of information was shared verbally. Some of the explanations were done incompletely on the board. This strategy did not always support learner involvement and goal attainment. Generally she planned too much content per period causing her to lose the logical flow of her lesson. |
| Administrative routines (LEA) | During the first lesson Alice did allow time for learners' involvement in discourse but it was not the result of effective procedures and management of her classroom. She needed their input to correct her mistakes on the board. The classroom arrangement was appropriate to the lesson style but Alice's position in class did not contribute to learners' conceptual understanding as she was in front of the class most of the time. |

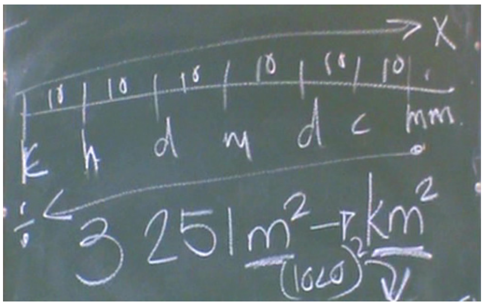
## 4.5.3 Denise's instructional practice

### 4.5.3.1 Tasks

***Tasks: Modes of representation (TR)***

In the first two lessons on conversions from metric to imperial units, Denise used representations[43] such as written work on the board, conversion tables, the variable $x$ to find the unknown values, calculators and during the third lesson she used a diagram (Picture 4.15 below) to explain conversions between different units of length (TR1).

[43] Examples of these representations are discussed under Discourse.

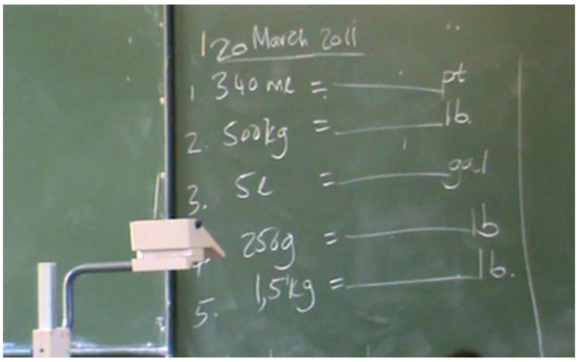



**Picture 4.15: Diagram used for conversions between different units of length**

She used this representation to enable the learners to connect their prior knowledge of different units of length to the conversions between the different units of length (TR2). Most of the learners used equations and cross multiplication to solve the unknown value and she reminded them that ratios could also be used to solve the unknown value (TR2).

***Tasks: Motivational strategies (TMS)***

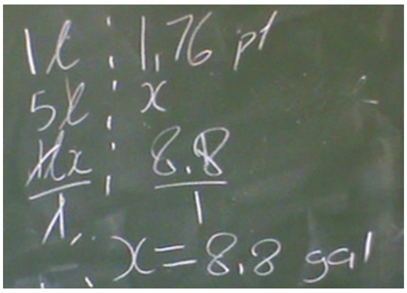
Denise treated the lessons as Mathematics and not ML lessons where the tasks were not set in a context as prescribed by the DoE (2003a), but were instead asked directly (TMS1). The following tasks (Picture 4.16) were given in the first and second lessons of which only the first three were completed during the first period (TMS1):




**Picture 4.16: The tasks during lesson 1 and lesson 2**

Denise's learners were inspired to participate in the lesson which might be due to Denise's teaching style and not necessarily as a result of the nature of the tasks (TMS1). From the following it appeared as if Denise took account of the diversity of learners' abilities (TMS2). She knew all her learners by their names and randomly called on learners to come and work on the board. On one occasion when a learner made mistakes, she called on another learner to come and rectify the work. Another time she also called a specific learner as she knew he used a method different from the rest of the learners who worked on the board (TMS2). All learners who had worked on the board used equations to solve the problems. She then asked another learner and when he started to use ratios instead of equations

(Picture 4.17), she said: *[y]ou know why you are doing the problem, you can see I have purposed it, my boy (indicating to the ratio he uses)* (TMS2).




**Picture 4.17: Learner using ratios instead of equations**

These given tasks were set in context in the textbook: a South African company exporting food products to the UK (TMS3). Denise did not mention this context at all, not even to elicit a class discussion on export in order to increase the learners' interest in the lesson and their appreciation of the value of mathematics in everyday life situations (TMS3). The only time she pointed out the value of the mathematics they were learning was when she told them: *you must understand so that if somebody is absent from class you can explain it to him* (TMS3). Her motivation is examination driven, preparing the learners for examination through practice (TMS3). She further advised them to work faster as they would not be able to finish the coming examination paper in time at their current working pace. She timed the learners while working on their classwork and after one problem she said: *you must take three minutes and you used 10 minutes,* using the mark allocation of the problem as a time guide (TMS3).

***Tasks: Sequencing and difficulty levels (TSL)***

Denise sequenced the tasks over the two different lessons she did on conversions which enabled the learners to progress in their cumulative understanding of the content (TSL1). The prior knowledge of these conversion lessons was the different units of measurement as well as the meaning of the different concepts (TSL2). It was only at the end of the third lesson that Denise asked the learners to identify the different concepts according to the unit of measurement. She did not ask them to explain or define the concepts:

*T: Number 1? Length, mass or capacity?*
*L: Capacity.*
*T: Can you see it? Right. and number 2? What is it?*
*L: Mass.*
*T: Mass and number 3 Jenny is doing now?*
*L: Capacity. (Learners complained that they wanted to leave since the bell had rung)* (TSL2).

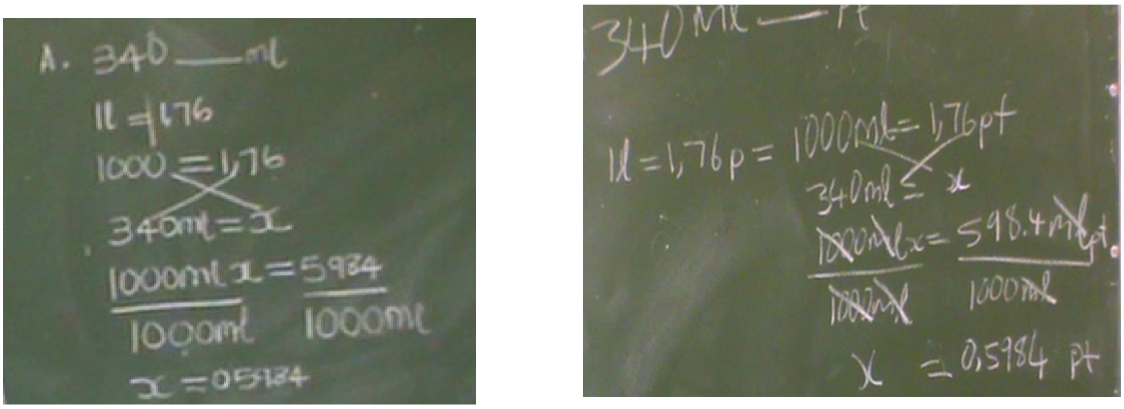
These two lessons were revision lessons so learners were supposed to know how to perform the conversions but still needed to improve on their skills (TSL2). Denise used this opportunity to identify and correct learners' common errors and misunderstandings (TSL2).

The content of the tasks were appropriate and on Grade 11 level but since no task was set in a context, her tasks were only on Level 1 (Knowing) of the ML assessment taxonomy (TSL3). In the first lesson she gave the learners conversions within the metric system such as *change 340ml to litres* as well as conversions from metric to imperial units for example: complete *500kg* = ____ *lb*. The tasks were based on capacity and mass only. In the second lesson the tasks were based on conversions within the metric system but included not only mass and capacity but length, area and volume too. Some of the learners complained about the complexity of the area and volume tasks (TSL3).

### 4.5.3.2 Discourse

***Discourse: Teacher-learner interactions (DTL)***

Denise encouraged participation from the learners during all three lessons as she walked through the class attending to learners' work, questioning and explaining to them (DTL1). She further called specific learners to work on the board and involved the rest of the learners by asking them to comment on the work on the board (DTL1). Learners demonstrated their work in writing on the board (Picture 4.18) but were not asked to explain or justify their work (DTL2).



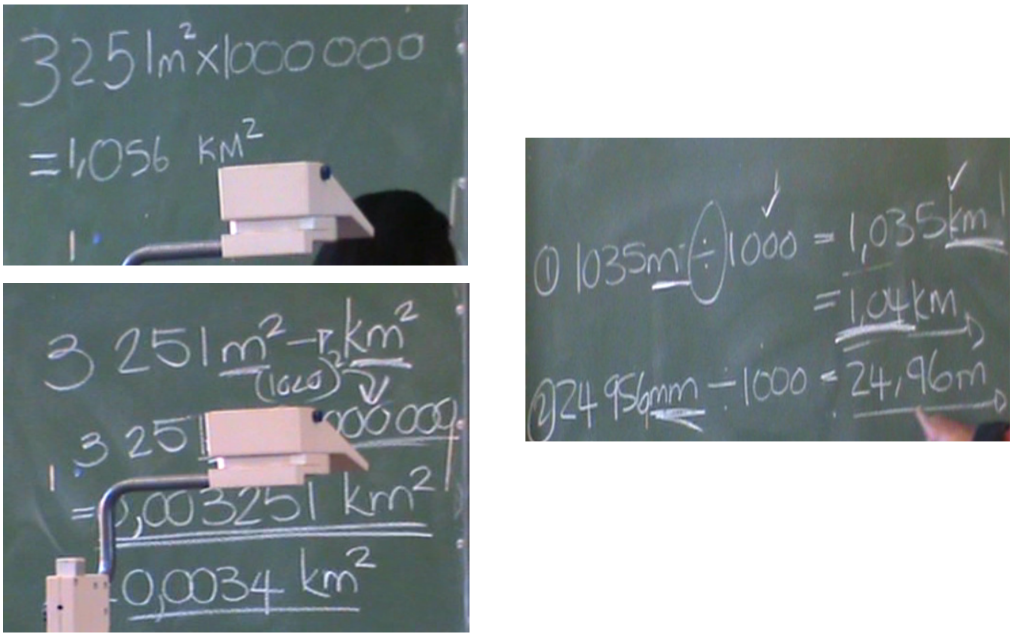

**Picture 4.18: The work on the board of two learners**

Denise based her instruction on what she saw the learners wrote on the board but did not listen to their explanations. She pointed out the errors and through questioning she involved the learners to take part in doing the corrections (DTL3). Once Denise asked a learner to come and correct another learner's work on the board and when he started to do that she said: H*ey David*[44]*, leave Cindy's business, go and write your own stuff so that we can compare* (DTL3). There was little evidence of Denise listening to learners' ideas. Instead she looked at their written work in order to provide scaffolding to support their thinking (DTL3). When they converted an area problem, the following discussion ensued:

> *L: Must I say the amount to the power of 2?*
> *T: (Denise immediately starts writing on the board and sings the following song:) King Henry died a miserable death called measles. This should be in your computer all the time (pointing to their heads), when to multiply and when to divide.*
> *… (Denise continues to explain Picture 4.15 to the class).*
> *L: No mam, I want to know do you square the amount?*
> *T: No!! But OK, it's a good question (and Denise continues with next task)* (DTL3).

Denise applauded the learners' answers and made comments such as: *You did excellent so far, guys* and *I am happy the way she is doing it, not looking at her textbook, because that is what we assess, understanding* (DTL4).

Denise marked some of the learners' work on the board as she would have marked a problem in a test (DTL5). Based on the following work from three different learners (Picture 4.19) the following discourse took place serving as proof of her recognition and clarification of learners' misunderstandings and misconceptions:



**Picture 4.19: Work from three different learners**

> *T: So now your conversion, it's squared metre to squared km. What's wrong there?*
> *L: You're supposed to divide.*
> *T: You're supposed to divide because it's from metre to kilometre. And you have to?*

[44] Pseudonyms were used instead of learners' true names.

*L: Multiply.*

*T: Multiply. OK? So that is where your problem is. So when you move from m to km, what do you do? Multiply or divide? So come and correct it. (She erased the step from the board and moved to the next problem on the board.) So when it's m to km, it's correct? (She encircled the division sign.) So from metre to km it's a 1000, you divide by a 1000, so it's correct and this is correct. Those are the basics. Now the problem is here (underlying the 1,04) two significant figures. So two significant figures, it gives you 1,0cm, right? (She went to the next problem.) So here it's mm to what? m, so we divide by what? a 1000 (and marks it right), gives you 24,96m. Two significant figures, you did not answer that. 24,9 what? 24,96 I said, but now 2 significant figures? Anyone to help? (She called on a learner.) What is the answer?*

*L: 25.*

*T: 25. Don't forget to round off. (She goes to Learner 3' work.) You said divide by 1000000 why? Because moving from m to km you divide by 1000, now it's 1000 to the power of 2 (and she wrote* $(1000)^2$*, so you get 1 million. So you divide by million and this is the answer. It's correct. Now you're answer to 2 significant figures? (The learner did not do that yet). Rectify your results. (Learner corrected her work on the board)* (DTL5).

***Discourse: Learner-learner interactions (DLL)***

She did not encourage the learners to listen to or question each other but instead most of the discourse was between Denise and the learners (DLL1). There was one incident during the second lesson when Denise left the class for a few minutes. Suddenly most of the learners wanted to help the learner who was at that stage making a mistake on the board, but he just became more confused. By the time Denise entered the classroom the learners became quiet and she continued to correct his work (DLL1). There was no learner or group of learners who dominated the verbal communication in class (DLL2).

***Discourse: Questioning (DQ)***

The lessons were characterised by questions directed at the learners (DQ1). Most of the questions were calculation, memory and convergent questions (DQ1). Many questions required the learners to look up conversions from the table in the textbook such as: *1kg equals to how many pounds?* (DQ1). Generally Denise did not allow enough time for the learners to become engaged in the discourse. Instead she provided the answers herself (DQ1). Denise did not ask learners to clarify or justify their ideas and two such examples are given below (DQ2):

- *T: Is the area one now correct? Is this one correct? (She looked at specific learners.) Is this now correct?*
  *L: (mumbles something no one could hear).*
  *T: If you think it's not, then you just say NO, because it is as if you've got doubts. Is it correct that one? (She pointed to another learner). 3 251 squared meter is how many squared km? So now your conversion, its squared metre to squared km. What's wrong there?*
  *L: You're supposed to divide.*
  *T: You're supposed to divide because it's from metre to kilometre* (DQ2).
- *T: So here it's mm to what? m, so we divide by what? A 1000 (Denise marked it right), gives you 24,96m. Two significant figures, you did not answer that. 24,9 what? 24,96 I said, but now 2 significant figures? Anyone to help? (She called on a learner.) What is the answer?*
  *L: 25.*
  *T: 25* (DQ2).

Denise created opportunities for learners to communicate and participate by answering questions she posed throughout all three lessons but she did not create such opportunities among the learners (DQ3). Learners' responses were mostly teacher-selected and at times volunteered (DQ4).

#### 4.5.3.3 Learning environment

***Learning environment: Social and intellectual climate (LEC)***

Denise maintained a positive rapport with the students as she valued their attempts (LEC1). There was one incident when the learners objected to a learner's answer who then appeared puzzled. Denise then told the class to calm down and to notice that the learner had worked accurately but only misread the value from the conversion table (LEC1). Although Denise could be humorous at times which the learners absolutely enjoyed, she was also very strict and did not hesitate to remind the learners of the appropriate classroom behaviour (LEC2). Comments such as the following were frequently heard:

- *Take your hands out of your pocket*
- *Stop talking*
- *T: Is Nelius correct?*
  *L: No.*
  *T: Don't say no under the table please, speak up*
- *Shhh, we are just checking, don't fight. We are not fighting* (LEC2).

Denise had a positive attitude towards the subject and learners and made comments such as:

- *You are just writing like a professor there né? (Everyone laughed)*
- *Keep on practising till we don't see that minor mistakes. Keep on practising.*
- *So it's nice when you say you are ready, it's nice* (LEC3).

A more negative incident occurred when she said: *Anyone who does not have a calculator, you sit on the floor and then you complain to your parents, so that they can give you a calculator. (One boy sat on the floor)* and she said: *Oh, we have one customer today (and she let him sit there)* (LEC3).

***Learning environment: Modes of strategies and pacing (LESP)***

Denise's style of teaching was that of a facilitator. She had a learner-centred approach where discussion and learners writing on the board were used as instructional strategies (LESP1). These strategies supported learner involvement throughout all three lessons and learners had sufficient time to express themselves when answering questions either in their books or on the board (LESP1). In the first lesson Denise did one example on the board to demonstrate and explain what was expected of them (LESP1). She then gave the learners problems to solve individually and after each problem they did the corrections together. In the other two lessons the learners had already completed the tasks at home, so

Denise used the entire period to do corrections (LESP1). The corrections were done by asking learners to write their solutions on the board which Denise then used to guide her instruction (LESP1). She pointed out the learners' errors and misunderstandings and involved the learners by asking questions in order for that the learners could understand (LESP1). After a problem had been discussed, the learners had to assess their own solutions (LESP1).

In the first lesson Denise gave the learners five problems to solve individually in class and after each problem they did the corrections together (LESP2). In the other two lessons the learners already completed the tasks at home, so Denise used the entire period for corrections (LESP2). The learners also knew that anyone of them could be asked at any time and they needed to be prepared at all times (LESP2). She ensured participation of the learners through continual questioning and ensuring that they were doing their own corrections (LESP2). She encouraged the learners to explore and use their textbooks efficiently by reminding them that the textbook consisted of activities, worksheets, assessments, projects and reviews and that they were currently busy with a worksheet (LESP2). Her lessons were not typical lessons consisting of the initial, development and closure phases as she was busy with revision when learners needed to practice their skills (LESP3).

***Learning environment: Administrative routines (LEA)***

She organised and managed the class effectively to ensure that time was maximised for the learners to develop conceptual understanding (LEA1). The learners were seated at individual desks which were appropriate for the lesson style (LEA2). Denise was in contact with her learners as she continually moved between the desks when she was not explaining and demonstrating in front of the class (LEA3). She also attended to learners individually at their desks assisting them with the work (LEA3). The work Denise did on the board was correct and all corrections to the learners' work were indicated so that learners could do their own corrections (LEA4).

***Summary***

**Table 4.8: Summary of Denise's instructional practice**

| **LESSON DIMENSIONS** | **DESCRIPTION OF LESSON DIMENSION INDICATORS** |
|---|---|
| **Tasks** | |
| Modes of representation (TR) | She made use of representations such as written work on the board, conversion tables, calculators and a diagram to illustrate the different units of measurement of length. These representations allowed her to link learners' prior knowledge with the new content of the day. |
| Motivational strategies (TMS) | The learners were motivated and inspired to take part in the lesson, not necessarily due to the nature of the tasks but to the fact that they wanted to show their work on the board. She did not point out the |

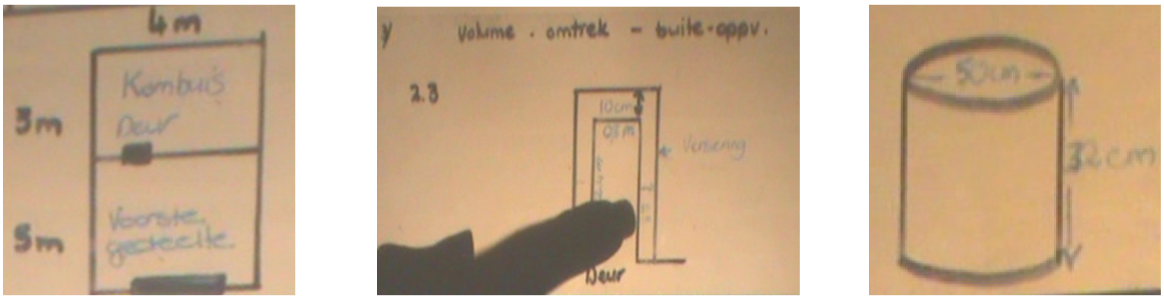
| | |
|---|---|
| | value of mathematics in everyday life. |
| Sequencing and difficulty levels (TSL) | The given tasks were sequenced over the different lessons and were appropriate, although not set in context and the content was on Grade 11 level (Level 1). |
| **Discourse** | |
| Teacher-learner interaction (DTL) | Denise verbally encouraged the learners as she praised their efforts that were written on the board. Most of the times she did not expect the learners to explain their thinking. The lessons were characterised by the discourse between Denise and the learners and the number of questions she posed to the learners. |
| Learner-learner interaction (DLL) | No discourse based on the content was observed among the learners. |
| Questioning (DQ) | Most of the questions were calculation, memory and convergent questions. Many times Denise did not allow enough time for the learners to become engaged in the discourse and just provided the answers herself. |
| **Learning environments** | |
| Social and intellectual climate (LEC) | The social and intellectual climate in the class can be described as positive as Denise had a positive rapport with the learners valuing their ideas and praising their efforts. |
| Modes of strategies and pacing (LESP) | Denise used a teacher-learner-centred approach with discussion, and learners' writing on the board as instructional strategies. These strategies were effective to ensure learner participation. She worked at a manageable pace throughout. |
| Administrative routines (LEA) | The administrative routines such as management of time to maximise learner involvement, classroom arrangement and the information on the board were effective. |

## 4.5.4 Elaine's instructional practice

### 4.5.4.1 Tasks

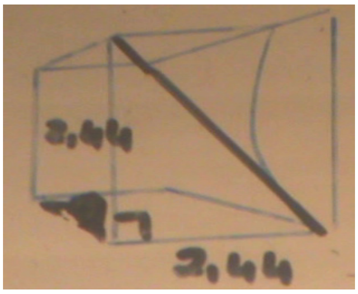
***Tasks: Modes of representation (R)***

To facilitate content clarity Elaine used representations such as written work on transparencies, tables, symbols, formulae, calculators, a demonstration calculator and sketches of manipulatives (Picture 4.20) in all her lessons (TR1).



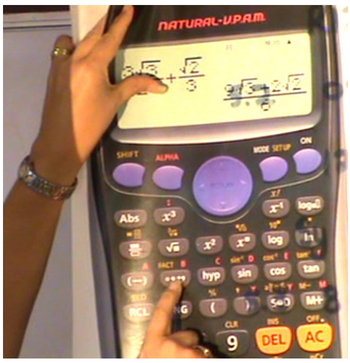

**Picture 4.20: Examples of sketches used in Elaine's discussions of solutions**

In the second lesson on time she used a table (See Question 5A on the next page) for parking tariffs (TR1). One of the formulae the learners used during the second lesson was: $F = \frac{x[(1+i)^n - 1]}{i}$ (TR1). Elaine expected the learners to explain the meaning of each variable in the formula enabling them to proficiently apply their knowledge to other similar unknown formulae (TR1). Below is an example of a sketch of a goal box (Picture 4.21) Elaine gave the learners to assist them in solving a problem based on a soccer field (TR1).




**Picture 4.21: Elaine's drawing of the given goal box to explain the solution**

Initially most of the learners calculated the time problems without the use of their calculators until a learner asked Elaine how she could use her calculator to find the answer. Elaine used a large CASIO demonstration calculator (Picture 4.22) which she put on the board to demonstrate the use of the calculator.




**Picture 4.22: Casio demonstration calculator**

The other learners were eager to master their calculators in calculating the answers to the time problems. Learners then assisted one another while she assisted specific learners who still could not manage their calculators. Since many of the learners were either not aware of or not able to use their calculators, they were very pleased afterwards with their accomplishment. Elaine generally drew the learners' attention to the required prior-knowledge needed to understand the content of the specific tasks (TR2). To connect the learners' prior knowledge with the new knowledge she alternated between discussions with questioning and class tests of which the answers were afterwards discussed in class and self-assessed by learners (TR2).

***Tasks: Motivational strategies (TMS)***

The tasks that captured the learners' curiosity were especially those based on time and interest (TMS1). The learners manually calculated the answers of the following question but enjoyed checking their answers using their calculators (TMS1).

*Question 5A*
*Sam parks her car every day in a parking area at her work. The table below shows the cost of parking for specific time periods:*

| ***Parking tariffs*** | |
|---|---|
| ***Hours*** | ***Cost*** |
| *0 – 1 hour* | *Free* |
| *1 – 3 hours* | *R4,00* |
| *3 – 5 hours* | *R6,00* |
| *5 – 7 hours* | *R8,00* |
| *7 – 9 hours* | *R10,00* |
| *More than 9 hours* | *R12,00* |
| *Saturdays* | *R5,00* |
| *Sundays* | *Free* |

*5.1 On Monday morning Sam arrives at the parking area at 07:50 and leaves the parking area at 17:15. How much does she pay for parking?*
*5.2 On Tuesday she arrives at 08:01 and leaves the parking area again at 08:45. She goes back to work at 12:15 and leaves for home at 17:30. How much does she pay in total for the parking?*
*5.3 On Wednesday Sam parks her car at the parking area. She has to pay R8,00 because she parked there from 08:45 to 13:25. Now use the table to determine whether she paid the correct amount. Show all steps and give a reason for your answer.*

The following interest problems (Question 5B below) were set in a context of buying a house and the impact thereof on an individual's, or even their parents' budgets (TMS1). From the learners' participation in the discussions it seemed as if the learners took an interest in these tasks.

*Question 5B*
*5.1 James wants to buy a house that is in the market for R780 000. If he pays a deposit of R78 000 and then R7 800 per month for 20 years, how much will he pay in total for the house?*
*5.2 James decides to rather first save money to increase his deposit. He invests R7 800 at 15% per year, interest compounded semi-annually. How much money will he have saved after 7 years? ($A = P(1+i)^n$ )*

*5.3 James' parents also invested R450,00 monthly in a savings account for 8 years. The interest rate was 11% per year compounded monthly. How much money does he have in that account? (* $F = \frac{x[(1+i)^n - 1]}{i}$ *)*

*5.4 James can afford to pay R6 500 each month for 18 years on a home loan at 17% per year, interest compounded monthly. How much money can he borrow from the bank? (* $P = \frac{x[1-(1+i)^{-n}]}{i}$ *)* (TMS1).

Elaine had a very demanding learner in class who had, according to her, been diagnosed with attention deficit hyperactivity disorder. She successfully kept him involved and focussed throughout the lessons (TMS2). To a few hard working learners she said: *The people who already completed the work, I will come and assess your work and will then give you the next tasks to be done*[45] (TMS2). These were two examples where Elaine took the diversity of learners' abilities and experiences into account (TMS2). Since all questions were based on realistic everyday life situations the learners were able to relate to the tasks (TMS3). An interesting discussion followed when Elaine asked: *Let us talk a little about why a person would rather wait to buy a house until he increased his deposit.* She then referred to the learners' personal lives where she discussed a typical household's budget and their parents' expenses so that they could understand what their parents sometimes had to tell them: *There is no money for whatever you wanted at that stage* (TMS3). Elaine wanted the learners to gain understanding and to be able to apply their knowledge to other similar problems and situations that might arise in their future lives (TMS3). She emphasised that they needed to show all calculations at all times, also in the examinations since marks are specifically allocated to their calculations and not just the final answers (TMS3).

### *Tasks: Sequencing and difficulty levels (TSL)*

Elaine sequenced the tasks in all three lessons by progressing in a lesson from easier to more complex tasks (TSL1). She also sequenced her class activities: for example during the second lesson on time and interest she first checked their homework and together they did Question 5A[46] on time. Before discussing the next Question 5B[47], she gave them the following class test based on prior-knowledge needed to answer that question.

*Class Test: Banking matters*

$A = P(1+i)^n$

*1. What do I calculate with this formula?*

*2. Write in words the meaning of each of the following:*

*A =    P =    i =    n =*

*3. What does the following mean to you? BODMAS*

---

[45] Since Elaine's classes were not presented in English, her texts were translated by me.

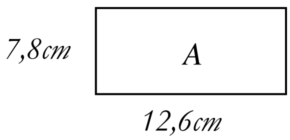
[46] Question 5A is given under Tasks: Motivational strategies.

[47] Question 5B is given under Tasks: Motivational strategies.

After the test they discussed the answers and then proceeded with the questions. In this lesson she also proceeded from the easier task on time to the more complex task on interest. With the task on interest she progressed from discussing the meanings of the unknown values in the formula to discussing the context, the formulae to solve the problems in the given task. She concluded the lesson by interpreting the solutions.

During the third lesson she also first walked through the whole class checking the learners' homework, then introduced the topic for that day's lesson, followed by a discussion of a contextual task the learners completed at home and then the lesson was rounded off by giving the learners the following class test:

*Class Test: Perimeter, area, volume*
1. *What does the following mean to you?*
   a) *Perimeter*
   b) *Area*
   c) *Volume*
2. *If the given figure's measurements are given in centimetres, what will be your unit of measurement for your answer when the following need to be calculated?*
   a) *Perimeter*
   b) *Area*
   c) *Volume*
3. *The following figure is a rectangle.*




   a) *Calculate the perimeter of figure A.*
   b) *What is the area of figure A?*
   c) *Calculate the volume of figure A* (TSL1).

Except for surface area all concepts were already introduced in Grade 10 but she appropriately applied the content to more complex situations (TSL2). Elaine was busy with her revision programme and during the first interview she stated that approximately three quarters of the learners knew the work by then. Half the remaining learners knew half the work while the other half did not have any idea of the work (TSL2). She stated that the purpose of her revision lessons was to either reinforce or enhance learners' current knowledge, to highlight learners' mistakes and to allow them to practise their knowledge and skills in order to prepare them for the coming examination (TSL2). The tasks Elaine covered in class were appropriate, on Grade 11 level, and reflected quality (TSL3). Based on the ML assessment taxonomy, Elaine selected tasks on Level 1 (Knowing) and Level 2 (Applying routine procedures in familiar contexts), but most of the tasks were on Level 3 (Application of multi-step procedures in a variety of contexts) and Level 4 (reasoning and reflecting) which required more advanced levels of thinking skills (TSL3). An example of such a task was a task based on a goal box:

> *The distance from the lower edge of the crossbar to the ground is 2,44 m. The distance from the goal post to the back of the goal is also 2,44 m. An extra pole is to be welded from the top corner of the goal post to the back of the supporting base.*
>
> - *How long must the support pole be (in metres)?*
> - *Convert this length to mm.*
> - *Write this answer in scientific notation.*
> - *If the diameter of the new pole is 4 cm, what will its circumference be?*

### 4.5.4.2 Discourse

***Discourse: Teacher-learner interactions (DTL)***

Elaine involved most of the learners in her class by asking them questions, clarifying their uncertainties and assessing their classwork (DTL1). She knew her learners by name and posed specific questions to specific learners such as:

> *I want to know, Hennie*[48]*, did you do it like that?*
> *Cecil, are you OK now, tell me what does it mean there?*
> *Lindy, what do you have there?*
> *Kevin, what did you say about monthly interest?*
> *Martie, do you agree on this?* (DTL1).

She communicated in a non-judgmental manner especially with the learner who was diagnosed with attention deficit hyperactivity disorder and who at times could be annoying. She remained calm and in control of the situation, attended to his comments or questions and continued with the lesson. An example was the following discourse between her and the specific learner:

> *T: In ML they give you all the formulae and information.*
> *L: It's not always like that. Another time I thought they normally give you everything and they did not. They changed everything just as they wanted, changed this and that!*
> *T: Is that true? Oh, then I must look into what happened there* (DTL1).

Whether learners were doing or discussing classwork or even after writing class tests, Elaine required them to give explanations and justifications orally or in writing (DTL2). She said on several occasions: *You must show me where you get that* and *Yes, but what does it actually mean?* (DTL2).

The following example is one of several incidents during Elaine's lessons where she listened carefully to learners' ideas and provided scaffolding to support their thinking:

[48] Pseudonyms were used instead of learners' true names.

*T: What does it mean there?*
*L1: Compound interest.*
*T: Compound interest. Why did you choose compound interest?*
*L1: It's not simple interest.*
*T: Right, but what tells you that it's not simple interest, but compound interest?*
*L1: The bracket and the part below.*
*L2: But there is no fraction.*
*L3: A is the final amount.*
*T: In simple interest A is also the final amount. I told you earlier that you know it's compound interest when you see 'n' written as a power, then we reason this is more complicated than the normal formula, then it's compound interest. So I don't want you to just guess that it's compound, you must be able to give a reason why you say it is compound interest* (DTL3).

She accepted their answers without criticising their efforts (DTL4). On several occasions Elaine said: *Good* or *Well done* and even thanked them for doing their homework as there were times they did not do their homework, telling her they could pass ML without doing homework (DTL4).

Elaine recognised and clarified the learners' common errors and misunderstandings (DTL5). For example there was a misunderstanding when the discussion was about the initial value and end value when interest was calculated. Instead of saying initial value a learner said present value and Elaine explained how present value could be interpreted as being the value after a certain period or could even mean the end value. She emphasised that the initial value is the value you began with (DTL5).

### *Discourse: Learner-learner interactions (DLL)*

A number of times Elaine encouraged the learners to listen to or respond to other learners' ideas and answers and would say: *Listen, here Simon*[49] *is saying … Ernest, can you respond to Simon's statement?* (DLL1). On another occasion she asked: *Who agrees with her? Who wants to argue with her? I know you sometimes like to argue. So Celeste, do you agree with her? …* (DLL1). Most of the discourse was not among the learners but between Elaine and the learners (DLL1). While doing classwork learners had the opportunity to discuss the work with each other and discourse occurred that I could unfortunately not hear (DLL1). An example where Elaine guided a discussion among learners was when the different variables of the formula were discussed:

*T: If we calculate interest semi-annually, by what must n be divided? How many times will interest be calculated per year?*
*L1: Two.*
*T: Why not six?*
*L2: It is six.*
*L1: Because you will get interest in the middle of the year and then the end of the year.*

[49] Pseudonyms are used in all quotes.

*T: Good, so semi-annually means every half of the year interest will be calculated, so at the end of the sixth month you get your first interest and then up to the twelfth month it will be the second six months period when I will get my next interest. So, if semi-annually I divide by two* (DLL1).

The only learner who tried to dominate the verbal communication was the learner who was diagnosed with attention deficit hyperactivity disorder, but she treated him in a firm and calm manner and once said: *OK, you had your moment* (DLL2).

***Discourse: Questioning (DQ)***

Elaine is a well-prepared and confident teacher who allowed enough time for learners to respond to her questions (DQ1). She posed questions on all three levels, namely memory, convergent and divergent questions (DQ1). Examples of convergent questions were: *What does it mean to write 12,5% as a decimal?*; and

*T: What kind of a triangle is formed?*
*L1: Right-angled triangle.*
*T: Then I can indicate the right angle on my drawing and know that I can work with which theorem?*
*L1: Theorem of Pythagoras*
*T: Who can give me the theorem of Pythagoras?*
*L2:* $r^2 = x^2 + y^2$
*T: Good, also tell me in words what the theorem means …* (DQ1).

Examples of divergent questions were: *Explain in detail to an ignorant person the meaning of each variable in the interest formula*; and when an interest formula was given to the learners and they were asked what kind of interest was represented, she asked: *Why did you decide on compound interest?* (DQ1).

Elaine consistently listened to learners' ideas and in many instances she asked them to clarify and/or justify their answers (DQ2). Her questions contributed to the verbal communication and participation of the learners and she created opportunities which the learners could listen to, respond to and question her as teacher or even their peers (DQ3). Learners' responses were mostly teacher-selected but also volunteered (DQ4).

### 4.5.4.3 Learning environment

***Learning environment: Social and intellectual climate (LEC)***

Elaine was well-prepared, made her lessons interesting and continually involved the learners in class discussions (LEC1). Just as important to her was mutual respect of one another and she maintained a positive rapport with and among learners by emphasising the importance of people valuing each others' ideas and ways of thinking (LEC1).

Discipline and classroom rules played a major role in her classroom ensuring learners' positive behaviour (LEC2). At the beginning of each period she completed the attendance register and then walked through the class to control their homework and made a note of those who did not complete their homework (LEC2). There were only a few times when it was necessary for her to discipline the learners and these were some comments:

> *I again ask you, put your suitcase next to your desk*
> *If I asked you to discuss it with your friend, my choice of words was wrong*
> *Thank you, you had your moment*
> *That was rude* (LEC2).

Elaine appreciates both the subject and her learners and praised them by saying: *I am fond of you;* and *I really appreciate your cooperation* (LEC3).

### *Learning environment: Modes of strategies and pacing (LESP)*

Elaine's teaching style varied between being a facilitator and mediator of learning (LESP1). She proficiently used instructional strategies such as class discussions and direct instruction (LESP1). The use of these strategies provided opportunities for the involvement of the learners and facilitated goal attainment (LESP1). She structured her lessons in such a way that learners had enough time to express themselves and explore their ideas and solutions (LESP2). She never rushed through the work or put pressure on them to work faster (LESP2). Her lessons did not consist of an initial, development and closure stage as they were revision lessons (LESP3). She made valuable use of her class time and completed what she had planned for the day (LESP3). There was a logical flow in her lessons as she worked from easier to more complex tasks and from familiar to less familiar concepts saying:: *Now let's go a little bit further* … (LESP4).

### *Learning environment: Administrative routines (LEA)*

Elaine believed enough time should be allowed for learners to practise their knowledge and skills and therefore made provision for a revision period in her year plan to allow learners to prepare themselves for the examination (LEA1). She allowed a certain amount of time for learners to solve a problem but at the end of that time would still ask: *Who needs more time?* (LEA1). Elaine arranged the class so that learners were seated in pairs, which was appropriate for the particular lesson style (LEA2). When she explained work she was in front of the class facing the learners because she used the overhead projector, but would otherwise move between the learners attending to their needs (LEA3). The written information on the transparencies and blackboard was very neat and organised with no mistakes (LEA4). Permanently visible on the right hand side of the blackboard was the work they already completed so that the learners could take note of their progress (LEA4). The following was written on the board:

*Chapter 4*

- *Units 1,2,4,5,6,7,8,9*
- *Test papers: B1; A2; B2, Class Activity (out of 21)*
- *Taxation: Unit 10*
  *Book: Paper F1; F2*

***Summary***

**Table 4.9: Summary of Elaine's instructional practice**

| LESSON DIMENSIONS | DESCRIPTION OF LESSON DIMENSION INDICATORS |
|---|---|
| **Tasks** | |
| Modes of representation (TR) | To facilitate content clarity Elaine used representations such as written work on the board, tables, symbols, formulae, calculators, a demonstration calculator and sketches of manipulatives in the three lessons I observed. Her class tests consisted of oral or written questions in order to connect learners' prior knowledge to the new mathematical situation. |
| Motivational strategies (TMS) | The learners were interested in the tasks as they spontaneously took part in the class discussions, especially in those tasks that were based on buying a house and working out parking tariffs using their calculators. They enjoyed taking part in the discussions Elaine led. She took learners' diverse abilities into account and accommodated the learner with attention deficit hyperactivity disorder as well as a few hard-working learners. The value of mathematics was frequently emphasised as Elaine discussed real scenarios, also applying the work to their personal lives. |
| Sequencing and difficulty levels (TSL) | Elaine sequenced her tasks to enable the learners to progress in their cumulative understanding of the content and they were able to make connections with ideas learned in the past. The lessons were revision lessons to improve the learners' knowledge and skills. The tasks reflected quality and were on Grade 11 level (Levels 1-4). |
| **Discourse** | |
| Teacher-learner interaction (DTL) | Elaine involved most of the learners, either by attending to their needs at their desks or posing questions. She communicated in a non-judgmental manner at all times. She required learners to give explanations and justifications orally and in writing. She recognised and clarified the learners' common errors and misunderstandings. |
| Learner-learner interaction (DLL) | Elaine encouraged the learners to listen to or respond to other learners' ideas. The discussions were not necessarily among the learners but in most cases between her and the learners. During class work the learners discussed the work with one another. |
| Questioning (DQ) | Elaine is a confident and well-prepared teacher and allowed enough time for learners to respond to her questions. She asked a variety of questions and posed questions on all three levels, namely memory, convergent and divergent. Learners had to clarify and/or justify their answers. Their responses were mostly teacher-selected but also volunteered. |
| **Learning environments** | |
| Social and intellectual | She made her lessons interesting, involved the learners in discussions and |

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| climate<br>(LEC) | maintained a positive rapport with and among learners. Discipline and classroom rules played a major role in her classroom to ensure learners' positive behaviour. Elaine was very fond of the subject and her learners and praised them for their efforts. |
| Modes of strategies and pacing<br>(LESP) | She used instructional strategies such as discussions and direct instruction. The use of discussions provided opportunities for the involvement of learners and facilitated goal attainment. Learners had enough time to express themselves and explore their ideas and solutions and there was a logical flow in her lessons. |
| Administrative routines<br>(LEA) | She allowed enough time before the examination for learners to improve their knowledge and practise their skills. Learners were seated in pairs which was appropriate for the lesson style. Elaine was in front of the class when she explained the work but otherwise moved between the learners. |

### 4.5.5 Summary of participants' instructional practices

Table 4.10 below provides a snapshot of the four participants' background information and their instructional practices.

**Table 4.10: Snapshot of the four participants and their instructional practices**

| PARTICIPANTS | MONTY | ALICE | DENISE | ELAINE |
|---|---|---|---|---|
| **BACKGROUND** | | | | |
| Qualifications and experience | BEd degree with Mathematics and Methodology of Mathematics as major subjects. Novice teacher with one year experience of teaching Mathematics and two years of teaching ML. | BTech Management Accounting degree with no Mathematics Education training. Novice teacher with only one year's teaching experience, teaching ML only. | BEd Honours degree in Mathematics Education with seven years' experience of teaching Mathematics and three years of teaching ML. | HED: Senior Primary with Mathematics and Methodology of Mathematics as major subjects. She had eight years' experience of teaching Mathematics and three years of teaching ML. |
| **TASKS** | | | | |
| Modes of representation (TR) | • Used representations such as written examples on the board, variables, calculators and a manipulative.<br>• Seldom connected learners' prior knowledge to the new mathematical situation. | • Used representations such as written examples on the board, symbols, the formula, tables, graphs and calculators.<br>• The various representations did not contribute to connecting learners' prior knowledge with the new mathematical situation. | • Used representations such as written work on the board, tables, calculators and a diagram.<br>• These representations allowed her to link learners' prior knowledge with the new content of the day. | • Used representations such as written work on the board and transparencies, tables, symbols, formulae, calculators, a demonstration calculator and sketches.<br>• Through tests and oral questioning she connected learners' prior knowledge to the new mathematical situation. |
| Motivational strategies (TMS) | • Only mathematical content was taught. The nature of the tasks did not capture the learners' curiosity or inspire | • Except for the one task being set in a context, the lessons consisted of mathematical content only. When she made | • Pure mathematical content was taught. Learners were motivated and inspired by the teacher and not necessarily by | • The learners were interested in the tasks as they spontaneously took part in the class discussions. She |

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| | them to pursue their conjectures.<br>• He only mentioned contexts to which content could be applied to point out the value of mathematics but the explanations were vague. | mistakes on the board, some of the learners were motivated to pursue their conjectures.<br>• Did not point out the value of mathematics in every-day life. | the nature of the tasks.<br>• Did not point out the value of mathematics in every-day life. | took learners' diverse abilities into account.<br>• She frequently reminded them of the value of mathematics in their lives. |
| Sequencing and difficulty levels (TSL) | • Not much evidence of the sequencing of tasks.<br>• The lessons on simultaneous equations were on Grade 11 level (Level 1) but the data handling lesson on Grade 10 level (Level 1). | • Most of the times the tasks were not successfully sequenced to enable learners to progress in their cumulative understanding of the work.<br>• Tasks were on Grade 11 level (Level 1). | • Tasks were sequenced over the different lessons, were suitable to what the learners already knew but needed to improve on.<br>• Tasks were on Grade 11 level (Level 1). | • Tasks were sequenced in the lessons to enable the learners to progress in their cumulative understanding of the work, set in context.<br>• Tasks were applicable and on Grade 11 level (Levels 1-4). |
| **DISCOURSE** | | | | |
| Teacher-learner interaction (DTL) | • Communicated in a non-judgemental manner but did not encourage learner participation except for posing basic questions.<br>• Did not require learners to give full explanations.<br>• Re-explained the work instead of providing scaffolding to support learners' thinking.<br>• Could not recognise learners' misunderstandings. | • She was judgmental in her communication and did not encourage the participation of learners except where she needed help with her own mistakes and misunderstandings.<br>• Did not require learners to give full explanations.<br>• She did not listen to learners to determine where and how she could provide scaffolding.<br>• Did not recognise learners' misunderstandings as she had misconceptions herself. | • Non-judgmental and verbally encouraged the learners as she praised their efforts.<br>• Required learners to give demonstrations of their work in writing but did not expect them to explain their work.<br>• She provided scaffolding to support learners' understanding.<br>• She recognised and clarified learners' misunderstandings. | • Non-judgmental and all learners were involved through questioning and discussions.<br>• Learners had to give explanations and justifications of their thinking, both orally and in writing.<br>• She provided scaffolding to support learners' understanding.<br>• She recognised and clarified learners' misunderstandings. |

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|---|---|---|---|---|
| Learner-learner interaction (DLL) | • Did not encourage learners to listen to, respond to and question one another. | • The observed interaction during the first lesson only was a result of the mistakes Alice made that were discussed and not because she created positive opportunities for learners to discuss the work | • She did not encourage learners to listen to, respond to or question each other's ideas. | • Elaine encouraged the learners to listen to and respond to other learners' ideas. The discussions were mainly between her and the learners. |
| Questioning (DQ) | • Types of questions were complete the word/sentence and calculation questions.<br>• Did not contribute to learners' participation in discussions.<br>• Responses were volunteered. | • Types of questions asked were memory, rhetoric, calculation, and complete the word/sentence questions.<br>• Did not contribute to learners' participation in discussions.<br>• Learners' responses were volunteered or chorus. | • Types of questions were calculation, memory and convergent questions. Many times Denise did not allow enough time for the learners to become engaged in the discourse and just provided the answers herself.<br>• Contributed to learners' participation in discussions.<br>• Learners' responses were teacher-selected. | • Types of questions were memory, convergent and divergent questions. She allowed enough time for learners to respond to her questions.<br>• Contributed to the verbal communication of learners during discussions.<br>• Learners' responses were mostly teacher-selected but also volunteered. |
| **LEARNER ENVIRONMENT** | | | | |
| Social and intellectual climate (LEC) | • Positive rapport between him and the learners.<br>• Good discipline and in control of the learners and lessons.<br>• Confident and enthusiastic about teaching ML. | • Not a positive relationship with and among learners.<br>• She did not ensure appropriate classroom behaviour.<br>• Seemed bored, irritated and un-enthusiastic at times. | • Had a positive rapport with the learners as she valued their ideas and praised their efforts.<br>• She was confident and strict and applied classroom rules.<br>• She had a positive attitude towards the learners and the subject. | • Positive rapport with and among learners and praised their efforts.<br>• Good discipline.<br>• Confident, well-prepared and enthusiastic. A calm and relaxed atmosphere. |

| | | | | |
|---|---|---|---|---|
| Modes of strategies and pacing (LESP) | • His style varied between traditional and formal authority. He used direct instruction as instructional strategy and once a learner wrote on the board.<br>• Not enough time was provided for learners to explore mathematical ideas.<br>• Good pacing and logical flow. | • Her style varied between traditional and demonstrative. She used direct instruction as instructional strategy.<br>• Not enough learner involvement.<br>• In the third lesson too much content was covered with little logical flow. | • She was a mediator. She used discussions and learners working on the board as instructional strategies.<br>• These strategies ensured learner participation.<br>• Lessons were presented at a manageable pace. | • Her style varied between mediator and facilitator. She used discussion and direct instruction as instructional strategies.<br>• Enough time for learner involvement and goal attainment.<br>• Logical flow in lessons. |
| Administrative routines (LEA) | • Learners' used the time to copy work from the board and listened to the teacher.<br>• Moved between the learners when he was not explaining work on the board.<br>• The written work on the board was disorganised and incomplete at times. | • Not enough time was allocated to learner activities.<br>• Her position in class did not contribute to learners' conceptual understanding as she was mostly standing at a specific desk in front of the class attending to her textbook and the few learners in front of her.<br>• There were mistakes on the board and the work was not always organised. | • Used time effectively to maximise learner involvement.<br>• She moved between the learners to assist them and to ask questions.<br>• Information on the board was correct and ordered. | • Managed time effectively for maximum learner involvement.<br>• Her position in class contributed to a positive learning atmosphere.<br>• Very neat, correct and organised work on transparencies and the board. |

### 4.5.6 Discussion of Theme 1: ML teachers' instructional practices

I again conducted a comprehensive, advanced electronic search after presenting the data in order to establish a basis from which I could execute a literature control of my findings in this chapter. My search covered the period January 2008 to September 2011 as I wanted to correlate my study's findings with the most recent research studies conducted in ML classrooms[50]. Of the 32 studies on ML, 17 were based on discussions, analysis, critiquing, developing frameworks or investigating certain theoretical or curriculum aspects of ML in South Africa. Seven were concerned with in-service ML teachers' experiences and their development through the ACE in the ML programme at different universities. Eight studies investigated ML teachers' instructional practices and the classroom experiences of ML learners. Two of the eight studies were intervention studies conducted with one teacher, and six of the eight studies were conducted at one school only. An experienced academic information specialist at the University of Pretoria, Ms Clarisse Venter, also conducted an advanced electronic search for the period January 2008 to September 2011 but could not add any studies to my existing list. Her reply was: *Most of the studies were discussions, analysis etc., which you do not want* (C. Venter, personal communication, September 12, 2011).

In the next section, I will conduct a literature control where the findings from this study are compared with the findings from other research studies on ML teachers' instructional practices. I base the discussion on Artzt et al.'s (2008) three dimensions of a lesson, namely tasks, discourse and learning environment[51].

#### 4.5.6.1 Tasks

Since I believe knowledge is constructed and based largely on prior knowledge, I support the view that the purpose of tasks such as examples given on the board, problems, activities and projects being given to the learners is to *provide opportunities for learners to connect their knowledge to new information and to build on their knowledge and interest through active engagement in meaningful problem solving* (Artzt et al., 2008, p. 10). The tasks used by ML teachers to facilitate learning in their instructional practices are discussed in this section in terms of Artzt et al.'s (2008) categorisations of such tasks[52]: modes of representation; motivational strategies; sequencing; and difficulty levels of tasks.

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[50] See Addendum H for a list of studies conducted on ML for the period January 2008 to September 2011.
[51] See Table 4.2 under Section 4.3.2.1.
[52] See Table 4.2 under Section 4.3.2.1.

***Modes of representation***

- **Use of various representations by ML teachers**

All four participants in my study used various representations during their classes as was also found with all the participants in the other research studies (Sidiropolous, 2008; Venkat & Graven, 2008; Venkat, 2010). Elaine was the only participant in my study who expressed a need to increase the use of technology such as computers in her ML classroom. As far as I could establish, no previous study has reported this finding.

- **Linking learners' prior knowledge to new situations**

In my study, novice teachers Monty and Alice, unlike experienced teachers Denise and Elaine, neither determined nor used their learners' prior knowledge to facilitate the assimilation of new content knowledge. This finding strongly confirms Sidiropolous' (2008) finding that one of the two teachers in her research group did not determine his learners' prior knowledge or use his learners' prior knowledge to facilitate the assimilation of new content knowledge.

***Motivational strategies***

- **Use of tasks to motivate learners to reflect on and pursue their conjectures**

Only Elaine in my study used contextual tasks that inspired the learners to reflect on their answers. In ensuing class discussions, the learners' conjectures were explored and expanded, enhancing their understanding of the work. Conversely, the other three teachers in my study did not use tasks that would motivate the learners to reflect on or to pursue their conjectures. My finding can therefore probably be regarded as consistent with Sidiropolous' (2008) findings where both teachers in her study did not ask the learners to reflect on or discuss their solutions. My finding that only one of the four teachers in my group used tasks to motivate learners is, however, inconsistent with the research results obtained by Buytenhuys, Graven and Venkatakrishnan (2007) and Venkat (2010) who found that the teachers in their studies used contextual tasks and discussions and succeeded in inspiring the learners to reflect on their answers and explain and justify their arguments. It should, however, be mentioned that the two teachers in the latter two studies were extremely dedicated teachers – not unlike the teacher in my group who used contextual tasks and discussions to inspire the learners to reflect on their answers and to explain and justify their arguments.

- **Pointing out the value of mathematics through the use of life-related tasks**

Part of the definition of ML (DoE, 2003a) is the use of the life-related applications of mathematics to make learners aware of and understand the role of mathematics in the modern world. In my study, only Elaine based her lessons on solving contextual problems on discussions that related the learners' newly acquired knowledge to the outside world and home situations. This finding is inconsistent with the findings of a number of researchers such as Buytenhuys et al. (2007), Hechter (2011a), Venkat and Graven (2008) and Zengela (2008) who found evidence of the successful use of life-related application problems in pointing out the value of mathematical literacy in everyday life. Three of the four teachers in my research group taught mathematical content only and did not once refer to its value in everyday life situations. My finding that only one of the four teachers in my study realised the value of teaching mathematics through the use of life-related tasks in everyday life situations is consistent with the finding of Sidiropolous (2008) who established that only one of the two teachers in her study realised the value of teaching mathematical literacy in real-world contexts and accordingly taught the subject on the basis of expecting the learners to solve real life-related problems.

***Sequencing and difficulty levels***

- **Sequencing of tasks enabling learners to progress in their cumulative understanding**

Three of the four teachers in my research group were not able to sequence their tasks proficiently to enable the learners to progress in their cumulative understanding of a particular task and to make connections between ideas learned in the past and those they will encounter in their future lives. There appears to be a gap in the literature in this regard.

- **Grading of classroom tasks according to the ML Assessment Taxonomy**[53]

My finding in this regard was that three of the four teachers selected tasks only from Level 1 (Knowing) according to the ML Assessment Taxonomy while Elaine selected tasks from all four levels (Knowing; Applying routine procedures in familiar contexts; Applying multi-step procedures in a variety of contexts; and Reasoning and reflecting). My finding is inconsistent with that of Govender (2011) who reported that the only teacher in her study asked the learners to perform tasks on all four levels. Interestingly, whereas the learners in the class of the one teacher (Elaine) who did select tasks from all four levels understood the problems and could solve them, apparently because of the support given by this teacher, in Govender's (2011) study, the learners could not understand the problems and, despite the support given by the teacher, could not solve the problems. Govender stated that the learners found these kinds of problems difficult, were not used to such questions and did not understand the contexts. The difference in the latter set of findings can be explained by the fact that at the time of the study Elaine was teaching in a traditional white school in Pretoria where the learners were familiar with the contexts while the teacher in Govender's study was teaching in a black township school in Port Elizabeth where the contexts were not part of the learners' real-life experiences.

[53] See Section 2.2.2.2: ML principles.

#### 4.5.6.2 Discourse

As a way of contributing to learner understanding, the discourse in class should provide opportunities for learners to express themselves, to listen to, to question, to respond to and to reflect on their thinking (Artzt, et al., 2008). I will now conduct a literature control on the discourse in ML classrooms based on Artzt et al.'s (2008) perspective of discourse[54], namely teacher-learner interaction, learner-learner interaction and the use of questioning to enable learners to build on their existing knowledge.

***Teacher-learner interaction***

- **Nature of teachers' communication and learner participation**

Except for Alice, the other three teachers in my study communicated with the learners in a non-judgmental manner thus contributing to a positive relationship between the teachers and the learners. This finding (only one of the four teachers in my study was judgmental) is inconsistent with that of Sidiropolous (2008) where both teachers in her study judged their learners' abilities and expressed low expectations of the learners.

Monty and Alice did not encourage learner participation apart from posing low-level oral questions where the answers were often provided to the learners before they could try to answer the questions. The finding that two of the four teachers in my group did not encourage learner participation moderately confirms Sidiropolous' (2008) finding that both teachers in her study did not generally encourage learner participation. Apart from one occasion, no time was allowed by the teachers in Sidiropolous' (2008) group for discussion on solutions or any critical engagement with mathematical arguments in their instructional practices. Denise ensured learner participation by requesting the learners to write their solutions on the board and involving them in discussions afterwards whereas Elaine involved the learners only in class discussions. The finding that only two of the four teachers in my group encouraged the use of discussions to enhance learner participation is moderately consistent with the findings of Venkat and Graven (2008) and Venkat (2010). The two experienced teachers in their studies encouraged class discussions, which were used to stimulate enhanced participation and communication.

- **Opportunities for learners to explain and demonstrate their work**

Only Elaine in my study asked the learners to explain their thinking and solutions, a strategy that elicited further discussion between her and the learners in class. Denise did not request the learners to explain their thinking but merely required them to demonstrate their work on the board. The other two teachers did not require the learners to explain or justify their work at all. The finding that two of the four teachers in my group provided opportunities for the learners to explain or demonstrate their work

[54] See Table 4.2 under Section 4.3.2.1.

is moderately consistent with the finding of Venkat (2010) that the (dedicated and experienced) teacher in her group (she had only this person in her research group) asked the learners to explain their thinking and solutions by giving them opportunities to explain and demonstrate their work (this teacher insisted on justification and explanation). Since there is evidence in the literature that collaboration from learners in the ML classroom contributes to the development of positive mathematical identities (Graven, 2011), my findings should be of interest to the Department of Education. Seemingly, during teacher training, more emphasis should be placed on the importance of providing opportunities for learners to express their ideas and thinking and to explain and justify their work.

- **Use of scaffolding to support learner understanding**

Even though Denise's lessons were based on mathematical content only, both she and Elaine provided scaffolding to support learners in solving problems and understanding the tasks instead of merely telling them how to solve the problem or doing the problem for them. In contrast to the instructional practices of Denise and Elaine, Monty either re-explained the work or solved the problem for the learners while Alice was not concerned about the learners' ideas and thinking. My finding that two of the four teachers in my study provided scaffolding is moderately consistent with the results obtained by Hechter (2011a) who found evidence of pedagogical support and scaffolding in the practices of both ML teachers who were part of her study. It should, however, be mentioned that these teachers were students enrolled for the ACE (ML) programme where they had to plan lessons according to certain guidelines (including ways to accommodate scaffolding in their instruction) and then implement those lesson plans in their instructional practices. In other words, they were required to facilitate scaffolding. It is not clear whether they would have done so had they not been required to do so.

***Learner-learner interaction***

Except for the very limited evidence of learner-learner interactions in Elaine's class, interactions between learners where they had the opportunity to support, strengthen and challenge each other's ideas were absent in the other teachers' instructional practices. This finding is inconsistent with that of Venkat and Graven (2008) where the single ML teacher in their research, being experienced and dedicated, used extensive communication and discussion of tasks during her lessons. My finding, however, concurs with the finding of Sidiropolous (2008) where both teachers in her study did not encourage critical engagement by learners in mathematical arguments. Given the belief (Brown & Schäfer, 2006; Venkat, 2007; Venkat & Graven, 2008) that a learner-centred approach is of the utmost importance in teaching ML, that is, where learners are actively involved in the lessons by taking part in discussions and group work but also by using their knowledge outside the classroom, my findings (albeit based on the actions of a limited sample of participants) should be a source of concern to education authorities.

***Oral questioning***

In my study, I found that three of the four teachers asked low-level questions such as complete the sentence and simple calculation and memory questions, which did not allow enough time for the learners to respond, and where in most cases the teachers provided the answers themselves. Elaine, however, asked various types of oral questions on different levels and gave the learners sufficient time to respond. As far as I could establish, these two findings regarding ML teachers' use of oral questioning in their classes have not been reported before and should be a source of concern to education authorities.

### 4.5.6.3 Learning environment

Artzt et al. (2008) use the term learning environment to describe the conditions under which the teaching-learning process unfolds in the classroom. I will now discuss the learning environments of the participants' ML classrooms in terms of Artzt et al.'s (2008) categorisations of a learning environment[55], comprising a social and intellectual climate, modes of strategies and pacing, and administrative routines.

***Social and intellectual climate***

- **Maintaining a positive relationship with and among learners in the classroom**

Monty's formal authoritative style of teaching restrained the building of positive relationships with and among the learners, and Alice focused only on the mathematical content instead of building relationships. However, Denise and especially Elaine created an atmosphere in the class where the learners were comfortable and confident as they engaged in the tasks. My finding that two of the four teachers in my study did not maintain a positive relationship with and among the learners is moderately consistent with that of Venkat and Graven (2008) who found that the teacher in their study was patient and that the learners could therefore work in a relaxed environment. This gave the learners *a sense of exploration, of working without fear or failure or ridicule, and of learning with enjoyment* (p. 40).

- **Use of classroom rules**

All four teachers in my study mentioned that they had to apply classroom rules to ensure appropriate classroom behaviour since learner misbehaviour could be a problem in ML classrooms. Monty and Denise were very strict; Alice at times could not apply her rules effectively while Elaine was more relaxed in applying her classroom rules as the learners seemed to know what was expected of them. As

[55] See Table 4.2 under Section 4.3.2.1.

far as I could establish, findings on ML teachers' application of classroom rules have not been reported before.

- **Teachers' attitudes towards the subject and the learners**

All four of the teachers in my study had a positive attitude towards the subject ML. My finding is strongly consistent with Fransman's (2010) finding where the four ML teachers in her focus group, who were enrolled for the ACE (ML), experienced the training to become *some kind of mathematics teacher, i.e. to be trained as a Mathematical Literacy teacher,* as a challenging experience *in which they were developing some sort of status-embraced identity* (p. 175). My and Fransman's (2010) findings are strongly inconsistent with the finding of Sidiropolous (2008) where both teachers in her study regarded the teaching of ML as a threat to their *'status identity'* (p. 221) as Mathematics teachers. Sidiropolous (2008) surmised that her finding could have been influenced by the fact that her study was conducted only one year after the subject had been introduced. At that stage, negative attitudes towards the teaching of ML were common (Sidiropolous, 2008).

***Modes of strategies and pacing***

- **Use of appropriate instructional strategies**

I found that Monty and Alice, the two novice teachers in my study, seemed to believe that learners learn through direct transfer of information (traditional approach, as defined in Section 2.4.4.2). Both Denise and Elaine, however, based their instruction on their learners' knowledge − Denise by using her learners' written solutions on the board to elicit discussion and Elaine by mainly using class discussions as an instructional strategy. Since two of the four teachers in my study used appropriate instructional strategies (as defined in Section 2.2.2.3), this finding is moderately consistent with that of Graven and Venkat (2009) who reported that all the teachers in their research changed their pedagogical approaches to teaching ML by using discussions and group work. Conversely, since two of the four teachers in my study did not use appropriate instructional strategies, this finding is also moderately consistent with Sidiropolous' (2008) finding − she established that both teachers in her study kept to a traditional teacher-centred approach by not using group work or discussions in their ML classrooms.

On the basis of Graven and Venkat's (2007) proposed spectrum of pedagogic agendas[56] ranging from Context; to Content and context; to Mainly content; to Content driven, only Elaine's pedagogic agenda was Content and context driven. The pedagogic agendas of Monty, Alice and Denise were Content driven. My finding in this regard is consistent with that of Hechter (2011a) who found that the pedagogic agenda of one of the two teachers in her study closely matched the Mainly content driven

[56] See Table 2.1 under Section 2.2.2.3: Pedagogical approaches for teaching ML.

agenda while the other teacher's pedagogic agenda partially matched the Context and Mainly content driven agendas.

- **Effective structuring of available time**

Three of the four teachers in my study worked at a manageable and slower pace compared to the pace in Mathematics classes thus allowing the learners more time to understand the work. This finding is consistent with that of Venkat and Graven (2008) where the only teacher in their study was *willing to 'wait' in ML in contrast to the imperatives to rush ahead in Mathematics* (p. 38).

***Administrative routines***

- **Maximise time for learners' active involvement in tasks and discourse**

I found that too much time was spent in Monty's class on learners who copied work from the board; while in Alice's class, the learners spent most of their time looking at Alice's demonstrations on the board. Denise and Elaine, on the other hand, managed their time to maximise learner involvement. My finding (two of the four teachers maximised the time available for learners to be actively involved in tasks and discourse) is moderately consistent with that of Venkat and Graven (2008) where the teacher in their study waited for the learners to understand before moving on.

- **Classroom arrangement, position of teacher in class, written information on the board**

My finding that the classroom arrangements of the four teachers in my study were appropriate for the lesson styles they used is moderately consistent with the finding of Sidiropolous (2008) that one of the two teachers in her study arranged the desks appropriately in his classroom in groups. Except for Alice, who worked on the board or from her textbook at a table in the front of the class, the other three teachers moved between the learners' desks engaging with the learners and their work. My finding that three of the four teachers in my study moved among the learners is consistent with Sidiropolous' (2008) finding that one of the two teachers in her study moved among his learners. Regarding the written work of the teachers on the board or on transparencies, I found that the work of Denise and Elaine was organised whereas Monty's work was disorganised, and Alice made mistakes or did incomplete work on the board. As far as I could establish, this finding has not been reported before.

## 4.5.6.4 Summary of discussion on Theme 1

To summarise: Denise adopted a teacher-learner approach, Elaine a learner-centred approach while Monty and Alice used a teacher-centred approach. Denise and Elaine used discussions and had the learners working on the board in order to encourage learner participation, allowing the learners to explain and/or justify their thinking. They used various representations and scaffolding to guide the learners to conceptualise new knowledge. Elaine selected tasks on all four levels according to the ML Assessment Taxonomy. The learning environments created by both Denise and Elaine were more relaxed with enough time for learner participation. Monty and Alice used direct instruction and thus discouraged learner participation by not allowing the learners to explain and/or justify their thinking. They used various representations, but not scaffolding, to guide the learners to conceptualise new knowledge. Both these teachers selected tasks on the lowest level of the ML Assessment Taxonomy. Monty's learning environment was formal while Alice's learning environment was relatively tense and awkward, and she did not allow enough time for learner participation.

I agree with Graven and Venkat's (2009) view that ML teachers need to make a substantive change in their instructional (pedagogic) practice in terms of the nature of the educational tasks, the agenda driving their teaching (on the continuum from Content to Context driven) and the way they interpret the subject ML. I also concur with Venkat and Graven's (2008) view that the nature of tasks and the nature of interactions are two key concepts contributing to positive change in ML classrooms that should be considered by ML teachers. In my research study, only Elaine's instructional practice conformed to these two key concepts.

## 4.6 Theme 2: ML teachers' knowledge and beliefs

In this section I present and discuss the findings from the interviews and observations of Monty, Alice, Denise and Elaine. All discussions on the subthemes **MCK,** and **PCK regarding: ML learners, ML teaching and ML curriculum** are structured strictly according to the guidelines in Table 4.3[57]. Background information regarding the observed lessons of the participants is given in Section 4.5. The language of all quotations from Monty, Alice and Denise has not been edited. Since Elaine's classes were conducted in Afrikaans, I translated her quotes from Afrikaans to English. The subthemes of each participant are now discussed.

### 4.6.1 Monty's knowledge and beliefs

#### 4.6.1.1 Mathematical content knowledge[58] (MCK)

I wanted to know from Monty how important it was for a ML teacher to have sufficient MCK and he replied that:

> *ML needs mathematics knowledge because there are many things you need to know from maths, the basics, for example the chapter on calculating angles or areas, surface area or volume. If you have never done this thing before, how are you going to understand it?*

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[57] Table 4.3 is discussed under Section 4.3.2.1: Inclusion criteria for coding the data.

[58] Since there is only one indicator or code in Table 4.3 regarding the teacher's mathematical content knowledge, this whole paragraph's code is: MCK.

Since Monty did not make any mathematical errors in his oral explanations or board work, it appeared as if his MCK regarding the specific content covered in the three lessons is sufficient. Since basic mathematical content was taught, there was no opportunity to observe whether he understood more than just the procedure of solving simultaneous equations. The same applies to the data handling lesson where I could not observe the extent to which he understands why and when we use the different measures of dispersion. Because learners were not expected to explain why different measures of dispersion are used, their conceptual understanding of the concepts also could not be determined. When Monty discussed the median he only said:

> *OK now, a median is a middle number né? I don't have another definition for that, it's a middle number.*

Monty did not use the glossary in the NCS for ML to define the terms properly, so that learners do not just have a synonym but can explain the meaning of the term or as he stated it, know the characteristics of the terms. Some minor mistakes I observed were:

- With simultaneous equations, after finding the solution for $x$ and $y$, he did not put them in an ordered number pair or emphasise that the answer represents a point;
- he should have used parentheses next to the two equations in order to emphasise that it was a system of simultaneous equations;
- in the data handling lesson he was not consequent as he sometimes said *from 3 up, above, no mode* other times he said: *more than 3, no mode.*

#### 4.6.1.2 Knowledge and beliefs regarding ML learners

For the second lesson on simultaneous equations, Monty predicted the learners would understand *how to approach the problem* (L1). For the data handling lesson on the four basic measures of dispersion, he predicted that the learners would understand *how to collect data, organize and summarise them and to present them at the end of the lesson* (L1). He later mentioned that the learners *just need to know the mode, mean, median and range*, but *we don't go in details but I have to give them a definition and how to gather information for future purposes, because they will need it even after they completed the school* (L1). Monty's reason why the learners would have understood those aspects was that all of the mentioned aspects were known to the learners, they were familiar with them and also because *it forms part of living, it is part of their lives* (L1).

What he predicted they would not understand is the variable $x$ *because once they see $x$ they get anxious, because what comes to their mind is once they see $x$ they think it is Maths* (L2). He thought the learners would come to understanding through *many examples whereby I can say look at it, this is how it can be done and everything* (L3). *The more you have examples, the more they can see how to do it,* but also by giving the learners *more sums because the more they practice maths the more they understand it. Especially if they do it individually, that is when they learn*

(L3). Monty predicted learners would approach the tasks by asking a friend or looking it up in their notes (L4). He encouraged the learners to work individually since they were approaching examinations (L4).

According to him, the learners reveal misconceptions when doing substitution. The following is such an example:

> *x=4-2y must be substituted in 7x. They tend to forget 7x means 7 multiplied by x so they just say 7 multiplied by 4 then -2y, which is wrong. That 4-2y is one thing like 7x. They have to multiply 7 by 4-2y (L5).*

In the data handling lesson the only possible misunderstanding according to him is *that they forget to arrange the data in ascending form* (L5). I could not actually determine whether his prediction was correct in this regard, since the learners did not participate in the lesson (L5).

In the following example the learners thought that when the coefficient of a variable (say *x*) is one, it means that *x* is then the subject of the equation. So when Monty did the example $\begin{matrix} x+2y=4 \\ 7x-5y=9 \end{matrix}$, he asked:

> *T: Is x or y a subject of an equation? (No response). Is x or y a subject to the equation?*
> *L: x.*
> *T: Huh?*
> *L: x.*
> *T: Yes?*
> *L's: x.*
> *T: OK. I am asking: Is x or y a subject in the formula? Huh?*
> *L'S: x. (Teacher looks very troubled and learners laugh). OK, give me an example where x is a subject of the equation (wait, no response). OK, people remember it must be? L = something, so L is the subject of the formula. So, do you see x or y is a subject here? (Teacher is irritated).*
> *L's: No.*
> *T: No. What do you do? You get x or y alone.*

Monty was not perceptive as to what the learners were thinking (L6). Most of the learners worked individually and Monty attended to them by looking at their work and talking to them (L6). Unfortunately I could not assess whether Monty acted appropriately to facilitate learning as I could not hear the discourse taking place. I did however, notice that as he looked at their work he did not ask them to explain what they did; instead he was explaining again to them (L6).

#### 4.6.1.3 Knowledge and beliefs regarding ML teaching

Monty regarded the following as prior knowledge for the lessons on simultaneous equations: *the coefficient, variable and index. That's the best knowledge and the sign comes before the number* (T1). During the introduction phase of the lesson, Monty revised the terminology as planned as well as the two methods they used to solve systems of equations (T1). He did not discuss like terms or the multiplicative inverse during the introduction as prior knowledge, but mentioned that later as part of the solution (T1). For the data handling lesson Monty said: *everything is prior knowledge* (T1).

In the examples and explanations on solving systems of simultaneous equations, Monty emphasised the steps to follow which he believed would simplify the work and make it easier for the learners to understand (T2). He did not use graphs as another form of representation to contribute to the learners' conceptual understanding of the work and the meaning of the solutions (T2). At times his explanations confused the learners because they took the form of lectures in which he made careless mistakes such as saying: *dividing with the multiplicative inverse* instead of multiplying with the multiplicative inverse and forgetting a sign in front of the value (T2). In the data handling lesson Monty demonstrated that it is important to be able to know the *characteristics* of mathematical terms, but he did not ask the learners to explain the different measures of dispersion and tell when and why these measures are used. This could have improved learners' understanding of the work (T2). During this lesson he verbally explained most of the examples without illustrating solutions on the board so that learners could see what he was talking about (T2).

There was no evidence that learners' different abilities and backgrounds were taken into account in presenting the content to the learners (T3). It is difficult to comment on his ability to sequence the content in order to facilitate learning since basic examples were used through all the lessons except for the example on simultaneous equations when he proceeded from a more difficult to an easier example (T4). Monty's choice of an instructional strategy to present his lessons was not in line with the purpose of ML as ML learners are supposed to be actively involved in solving contextual problems (T5).

***ML teaching: Reflecting on his practice*** [59] **(T6)**

When Monty reflected on his instructional practice he said he used direct instruction *because our learners are different from other school learners so we need to use the direct instruction*, referring to discipline problems. Commenting on the discipline in his class he said: *I do have classroom rules in my class whereby I say we must respect one another*. He views his role in the ML classroom as being the facilitator where he helps the learners to understand the work. To improve his learners' appreciation of ML, he *will keep on motivating them about real life and what you may do with ML*. To improve the learners' participation in the lessons he gives them questions from previous examination papers to prepare at home and present and explain the next day. When asking him how he feels about teaching ML as this may influence the way he approaches his ML lessons, he said:

[59] Only one code was used to report on the teacher's reflection regarding his own practice namely T6.

> *If I can get a chance I will go for Maths. I like when I am being challenged. I am not really challenged. It is something that I am not enjoying. It is not working with hard working people who always ask questions, who want to learn like the Maths group. Here they must do the subject, it is part of their packages, it is compulsory.*

His goal is *to get 100% pass rate and at least 5 distinctions. Last year I had 97% with 1 distinction and 3 B's and one learner could not make it and most of them got above 40%.*

During the last interview I asked Monty to describe an ideal ML classroom in terms of, among other things, the instructional strategies used. He believes that the teacher should use direct instruction initially when new content is introduced, followed by group work *because the group work it goes with problem solving strategy*. There should be *discussion with writing something down. They must ask me questions*. Regarding the learning environment, he believes *one learner per desk facing the chalkboard. If they need to do group work they can combine… I don't believe too much in rules because I believe the educator can make environment good or nice for learning*. He stated that his classroom is not like this ideal classroom *because it's hard to change things if you are still a new teacher. You find them sitting like that, doing things like that, although you impose all the rules like that, they still do it, for them to cooperate it will take you long*. He mentioned that although he actually prefers group work he encourages individual work now because they are approaching the examinations. *I use the group work just to show them how cooperative work is productive*. He also believes that when the learners communicate in peer groups *they start to understand and they feel free to ask anything*. Monty values the idea of learners writing their solutions on the board and explaining it afterwards because *then it is going to be stored in your memory for always*. He believes the difference in approach used between ML and Mathematics is that fewer examples are done in ML, the pace is slower and the teacher does not need to go the extra mile because *the people you are working with in ML are not like the people you are working with in Maths.*

#### 4.6.1.4 Knowledge and beliefs regarding ML curriculum

The DoE (2006) recommends a list of resources or instructional materials needed to teach ML (C1). The resources Monty used during the three lessons I observed were a textbook and blackboard as (C1). He used a textbook: Mathematical Literacy for the Classroom (Laridon et al., 2006) and previous examination papers. He explained the strengths and weaknesses of the textbook as: *information is clear and understandable, lots of examples and exercises* but *some topics have little information and few examples* (C2). He was not aware of the curriculum content being studied in other school subjects that integrate with ML (C3). Other departmental documents he knew of were the memorandums and circulars of which the latter was useful and valuable to him (C4). As far as the NCS: ML is concerned, he knew that there are four learning outcomes but could not name them (C5). According to Monty the DoE defines ML as *a subject aimed to enhance learners' skills of counting though they are not doing maths* and that the DoE's purpose for the subject is *to give every learner an opportunity to learn how to count because in real-life situation counting is a norm* (C5). Monty did not know which contexts the DoE suggests teachers should use in the teaching of ML (C5). Regarding the new CAPS for ML he only knew that the *teacher is a facilitator not like previously when the learner was a centre of every learning* (C5). At the end of the last interview I provided Monty with a list of concepts and contents to be covered in Learning Outcome 4: Data handling (NCS, 2003a) and Monty could only place seven out of 25 concepts in the correct grade in which they should be introduced (C6).

The lessons were presented as Mathematics lessons whene content was not situated in a context, although he did mention a few examples of contexts where the mathematical content could be applied (C7). During the first interview before the second lesson on simultaneous equations I asked Monty about the context in which the lesson was set (C7). He was startled and took a few seconds to come up with the idea of the elections. There was a 15-minute break between the interview and the class and I assumed he used that time to think about this context. During the introduction he talked about elections and the parties' campaigns but it was not clear how the given information was applicable to that day's lesson (C7). In the data handling lesson he talked about how research is done but did not link this to the learners' experiences or the content of that lesson. At the end of the data handling lesson he said:

*In real-life situation, where can we use data handling? Census or SARS, for SARS to see how many people owe money, they have to get data, they have to have people registered to SARS, those that are only in business, they can see how many people are paying their taxes and so on and so on. Another one, remember guys we have elections of RCL elections ne? We said the class has two representatives ne? But you were able to vote for more than two people. But at the end of the day, they managed to get two representatives per class, ne? How? So that we can say this is our? RCL. So we had many people on the valid paper but at the end of the day we have a certain number of people to represent RCL. You see how we use data.*

The context of elections was appropriate since municipal elections were to be held eight days later, but he could not apply the context appropriately and meaningfully to the content. The same applied to the SARS context he talked about. I doubt whether the learners were able to tell how data handling was applied in SARS and elections (C7). Later in the same lesson he gave this example of where and how statistics could be applied:

> *OK, now another story, when you do athletics, remember we use a stopwatch ne? A stopwatch helps us to record the time. So for example for one particular learner let's say we have athletics, we can record different times and we can calculate that data and you present it using a pie chart or whatever and you can use that data to arrange all things by recording that time during the events. OK now its fine* (C7).

According to Monty mathematics is *a tool used for solving problems* (C8). I asked Monty how he views mathematics as a discipline compared to ML as subject and his answer was:

> *I view it as constructivism because you have to be constructive if whatever you are doing especially in these days so that you can be successful. ML is viewed as a mathematics, but not lower grade and not challenging like Mathematics … ML is like a life skill because you learn how to divide things, how to add things, things like you are always doing when you are going to shopping, more of a life skill than a Mathematics subject* (C8).

He described the value of mathematics and ML as:

> *It's for logical thoughts because you learn to do things step by step and it gives you that strength as a person or individual to reason and think outside the box. The value of ML is that in a few years' time most of them will be doing Mathematics because now they can notice from ML that they can do Mathematics. The learners learn about counting, structures, angles, everything, so they can use that in their working place* (C9).

***Summary***

**Table 4.11: Summary of Monty's knowledge and beliefs**

| KNOWLEDGE AND BELIEFS DIMENSIONS | DESCRIPTION OF TEACHERS' KNOWLEDGE AND BELIEFS' INDICATORS |
|---|---|
| Mathematical content knowledge (MCK) | Monty regarded mathematical knowledge as a prerequisite to teach ML. It appeared as if his MCK regarding the specific content covered is sufficient. |
| ML learners (L) | He believes learners gain understanding by looking at various examples on the board and through much practice. Although he regards group work as important where learners have the opportunity to talk to one another and learn from each other, he did not apply group work in class. He sees individual work as vital before the examinations. |
| ML teaching (T) | He did not always enable learners to connect their prior knowledge to the new content. He chose very basic examples in his data handling lesson and did not take learners' different abilities into account. |
| ML Curriculum (C) | He knew about the value of ML but could not provide the required information from the NCS (2003a). He views Mathematics as logical and constructive, valuable to all people. ML is viewed as a kind of mathematics, but not a lower grade of Mathematics. |

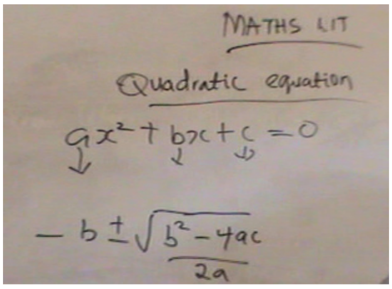
## 4.6.2 Alice's knowledge and beliefs

### 4.6.2.1 Mathematical content knowledge[60] (MCK)

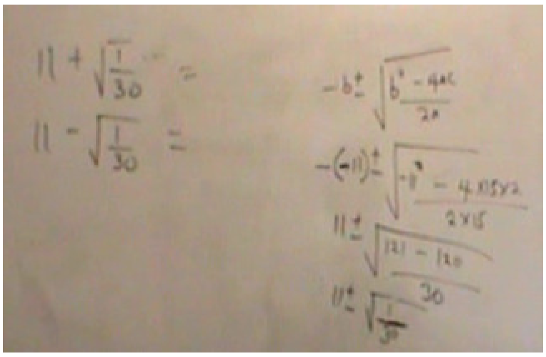
Alice believes that ML teachers need to have sufficient MCK and that no non-mathematics teacher can teach this subject. Regarding her own MCK many mathematical errors were observed. In the first lesson she used the formula incorrectly (See Picture 4.23). Later in the lesson she changed the formula

[60] Since there is only one indicator or code in Table 4.3 regarding the teacher's mathematical content knowledge, this whole paragraph's code is: MCK.

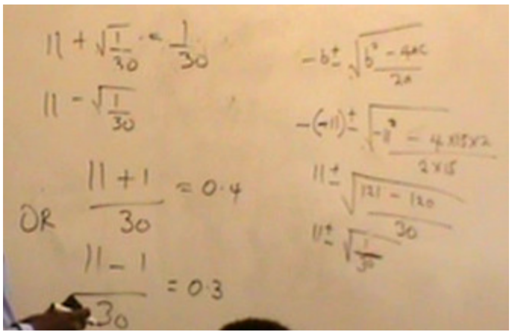
and put the denominator *2a* under the root sign (Picture 4.24). Near the end of the solution she again changed $\sqrt{\frac{1}{30}}$ to $\frac{\sqrt{1}}{30}$ (Picture 4.25).




**Picture 4.23: Formula used incorrectly**

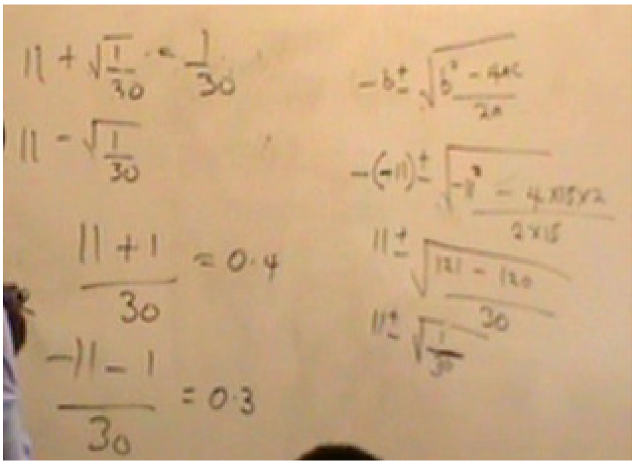


**Picture 4.24: Formula is changed**



**Picture 4.25: Another change in formula during further calculations**

After erasing work a few times from the board, Alice wrote: $11+\frac{\sqrt{1}}{30}=\frac{1}{30}$. A girl corrected the previous step to: $\frac{11+1}{30}$. In many cases Alice omitted to put the values in brackets when she substituted: $-(-10)\frac{\pm\sqrt{-10^2-4\times1\times25}}{2\times1}$. The learners were confused as this formula differed from the formula ($x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$) they used the previous day when the student teacher was responsible for the lesson. Alice believed that the student teacher used a wrong *method* as his work did not correspond to hers. Then there was the issue of having two solutions with the same sign which the teacher and learners believed were not supposed to happen (Picture 4.26).




**Picture 4.26: An attempt to get a positive and negative answer**

The following discourse took place:

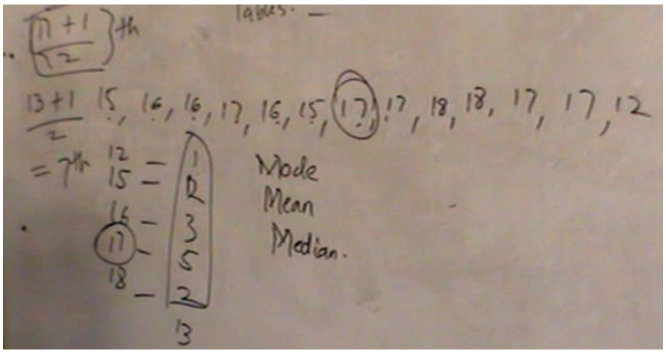
> *T: Am I right? (She checks again her calculations on a calculator.) You are supposed to get a negative and a positive answer. So, what happened here? I am sure there is something wrong because here we have two positive answers. Guys please! (The same girl from earlier who wanted to show the teacher on the board during example one brings her book to the teacher and talks to her but nobody could hear.) Yes but you have a negative and a negative so it should change.*
> *L: Oh OK!*
> *T: Quiet please! We are right, it's OK, as long as you know the method. So that's an exception. This is an exception. Let's do another one.*

In the second lesson where they drew the graph of the parabola, she did not attend to the given restriction in the example $y = (x-2)^2 - 1$ for $-1 \le x \le 4$. She also mentioned that they must have two intercepts with the X-axis, which is not necessarily true. Only at the end of the second lesson did she write the x and y value of a point as an ordered number pair for the first time, saying: *Here you have* $(0,3)$*, this is your x and this is your y.* When using the method of intercepts, they only had three points to plot the parabola with because they did not calculate the turning point and the following discourse took place:

> *T: You now join these three points.*
> *L: How do I know where to go?*
> *T: You can go anywhere, now you don't have this point (pointing to turning point). If you do a proper job, you will find your graph will go exactly through that point (turning point) … If you have to find the minimum, use the graph or the maximum value of y. How do you find the minimum?*
> *L: You just see it.*
> *T: Where do you get it?*
> *L: It's at -1.*
> *T: This is the minimum value (showing at turning point).*
> *L: There are two graphs now.*

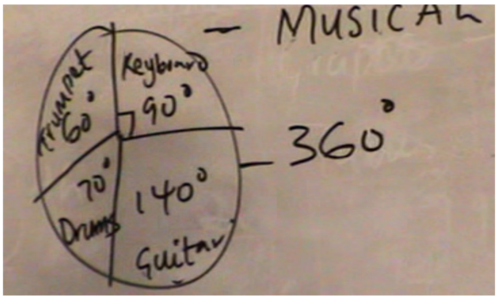
*T: This is the same graph, we used two methods. (Everyone laughed as the graphs did not look the same). It's the same graph. We used the table method and then the formula.*
*L: OK.*
*T: Are you sure you do understand?*
*L: Ja.*
*T: OK.*

During the third lesson she calculated the median without arranging the data (Picture 4.27), then told the learners to arrange the data in ascending order saying: *So it might not be that answer.*




**Picture 4.27: Alice calculating the median incorrectly**

The pie chart (Picture 4.28) was not drawn accurately – even though it was a rough sketch, 60° should have looked like an acute angle.




**Picture 4.28: Unrealistic drawing of the pie chart**

### 4.6.2.2 Knowledge and beliefs regarding ML learners

In the lesson on how to draw the parabola using two methods, Alice predicted that the learners would understand that they *can substitute that equation in that formula* (L1). When I asked her how the learners would understand the work, she told me about the preparation of the lesson and after a prompt she complained about the large classes that needed to be divided in two groups and the 40-minute periods that are not sufficient to do what she planned and therefore never answered the question (L1). She predicted that the learners would not understand *plotting the graph* as they have difficulty doing the following:

> *Maybe your $x = 0$ and then your y, make it -3, so when plotting the graph, where you have the -3, they go plotting it at the point where x is 0 and y is 0. It's always a confusing thing, plotting the graphs* (L2).

On predicting why it is difficult to the learners she replied:

> *It shouldn't be difficult but I don't know …. I cannot say this is why. Sometimes it can be confusing because now when you have 0 as the x and then you're trained to plot the y, you're thinking, you know you have to let both points meet, then you're thinking maybe you should, and this is the -3 and your x is 0 here. They are thinking maybe they put it here or here* (L2).

She predicted that the learners would approach the tasks by coming to her during break so that she could assist them, but offered no other strategy to assist them: *they want individual help but aside from that I don't know* (L4). She could not predict any other misconceptions learners might have (L5). In reality neither the learners nor the teacher understood why there were two positive answers when the quadratic equation was solved and as she predicted the learners made mistakes when point $(0,-3)$ was plotted (L5). When the table was completed in order to draw the graph of $y = (x-2)^2 - 1$ , some learners did not understand how she obtained her answers when she worked directly from $x$ to $(x-2)^2$ because she omitted the steps where $(x-2)$ should have been calculated (L5).

Regarding the data handling lesson, she mentioned that the learners would understand everything as they had done all the concepts in Grade 10 and according to Alice *data handling is one of the simplest aspects of ML so I think they should understand it all* (L1). She stated: *it's going to go smoothly with the chart, the tables and even the graphs* (L1). Regarding the new work, the cumulative frequency and drawing of the ogive, she said *they should understand it* and *they know it but it is just a bit advanced … it is just like a continuation of work they did* (L1). When asked how the learners will understand it she said:

> *I will tell them about the data and the raw information which you have to try to present, make presentable maybe using the graphs which is the pie graph or line graph or the charts which is the pie charts or maybe using the tables where they have to use the tallies or frequency, but there are different ways in which you can present your data and then we are going to try and refresh their memories on the mean, the mode, the median and then we are going to talk about the cumulative frequency, how to plot an ogive, we are going to work out the standard deviation* (L3).

She predicted that the learners would not understand the actual mathematics involved:

> *They used to have a problem with the pie chart, I don't know why. You know the pie chart? I don't know, they find it difficult to allocate those degrees even though from percentage to the grades of the pie chart. It is always a big problem* (L2).

When I asked her why the learners did not understand the pie chart, she said: *I don't know* (L2). Alice stated that the learners would not have a problem doing the tasks and then talked about group tasks set by the department (L4). She could not predict any other misconceptions the learners might have (L5). Although Alice said they would understand everything except the pie chart, they in fact did not understand how she calculated the measures of dispersion using the table method and one learner who was very involved in the lessons mentioned how difficult the pie chart was, but Alice disagreed and continued with the lesson (L5).

There was little evidence of what the learners understood, how they understood it, what they did not understand and how they approached tasks because they were not required to do any tasks in class (L1-L4). She told the learners that they would understand the content if they practised enough and if they watched her. She said: *Don't ask me how to get this, you have to look at me* (L3). The chaos of the first lesson showed that Alice did not have the ability to understand the learners in terms of their thinking (L6). When Alice was asked to comment on the first lesson she said:

> *Because when one person says this, and other person says this, then they get confused and they know it's just everything you tell them, they go the same way, they don't want to think out of the box. If you say it's -(-) they want to know this can just change to plus, so you must do it the sáme way every time. If I am the first person that taught them, then they want to go the same way as me. And I got confused too* (L6).

#### 4.6.2.3 Knowledge and beliefs regarding ML teaching

In the second lesson on drawing parabolas using two methods, she regarded *simplification, factorization and the simultaneous equations* as prior knowledge (T1). She did not revise any prior knowledge during the introduction stage of the lesson, but immediately started with the table method when she reminded them to use both negative and positive values. Thereafter she used the formula which they had learned during the previous periods (T1). In using the formula she revised the values of $a$, $b$ and $c$ in the formula (T1). Regarding the data handling lesson, she said everything she planned was prior knowledge. In her introduction she revised the different kinds of graphs and charts which the learners already knew (T1).

Alice used various useful forms of representing ideas such as graphs, tables, formulae, calculators and symbols, but these were often overrepresented (T2). Although she used various representations, instead of creating opportunities for learners to develop conceptual understanding, the learners were confused (T2). She could not always transform the content knowledge into forms that are pedagogically powerful to attend to learners' diverse needs (T3). She has the ability to sequence the different content of her lessons, but it is when dealing with the particular concepts that too much information is shared using incomplete and varied representations. This happened especially in the data handling lesson (T4). When the learners did not understand or when they gave incorrect answers, she re-explained by using the same words as before, apparently not being capable of reformulation of her explanations (T4). After she did the example in which the pie chart was given, she told them that a question could be asked where they needed to draw the pie chart using given information. Unfortunately she did not follow this through by asking the learners to undertake such a task (T4).

Regarding the class and homework she said: *when we give the class work, we should make it look more practical, not just the theory and the x thing* (T5). About grading of the problems she commented: *They are based on this topic. It's practically the same thing, the same level* (T5). For the data handling lesson, she planned to use exercises from the textbook and at the end of the week a task from the DoE (T5). Alice generally selected too much content for one period so that there was no time for her to assess her learners' knowledge and understanding (T5).

### *ML teaching: Reflecting on her practice*[61] (T6)

Alice claimed that she really loves to teach ML and found it quite interesting. She tries to motivate the learners and invites them to come to her class during break time if they do not understand the work, but still they are not motivated and would rather play pool during break, which distresses her. To improve her learners' appreciation of ML she plans to *get down to their level and make them understand what they really need to know, to make it quite simple*. To improve the learners' participation in the lessons she tries to pose questions to all, but this is not uniformly successful: *The boys at the back are always (pause), they just want to be in class, they don't participate. The ones in front want to take part.* She believes learners master new knowledge as follows: *To learn new things it is all about making up your mind, I want to know this. The learner must go back to what he is taught, or come to the teacher, try to do your own research with books and take it up from there.*

Alice chose direct instruction as strategy because *the first lesson we have you must teach them the basics. I try to introduce them, make sure they understand the basics of what they are doing, or what the topic is about, it is more like formal teaching. Later on the discussion comes*. Her view on calling learners to the board is: *it's all about practising in the presence of everyone and it gives them confidence and then their friends can learn from them and then try to assist them … and it forces them to come to class prepared.* She does not favour discourse between the learners:

> *Actually I don't like to encourage that [discourse between learners] because most times when I am talking I found discussions among themselves and then sometimes they miss out when they do that. Some learners speaking in their dialect and vernacular, it's a bit difficult when you are teaching and when a learner asks you, mam, what is that word? Please can you repeat that and then I am too busy teaching. So, I try to make them listen not to miss out.*

She does not believe that the teaching of ML differs from that of Mathematics. Regarding her role in the classroom she said: *I try to be in charge but give them the chance to talk to me and go and read because ML asks for practising, so I would say I am the mediator.*

[61] Only one code was used to report on the teacher's reflection regarding her own practice namely T6.

During the last interview I asked Alice to describe an ideal ML classroom in terms of, among other things, the instructional strategies used. She described her ideal ML classroom as one where various instructional strategies are used such as *group work and discussion in class and then active learning and learners coming to the front*. In her description of the ideal discourse in her ML class, she mentioned she wants learners *to speak up when they do not understand and to ask intelligent questions. I really want them to ask: why did you do this, how did you do this? Is this how you are supposed to do it?* As far as the learning environment is concerned, she wants all learners to have their own textbooks so that she does not need to waste time by writing the whole question on the board. Her goals in teaching ML are that all learners do well in their examinations and that they understand that *ML is quite simple if you put your mind to it.*

#### 4.6.2.4 Knowledge and beliefs regarding ML curriculum

The DoE (2006) recommends a list of resources or instructional materials needed to teach ML (C1). The resources Alice used in the three lessons I observed were the Oxford Successful ML (Pretorius, Potgieter & Ladewig, 2006) textbook, the whiteboard and calculators. Although the textbooks are available at her school for learners to buy, she said that *the learners want the school to give it to them* and that the learners asked her to give them the money to buy the textbooks, so most learners do not have their own textbooks (C1). According to Alice a point of strength of the textbooks is the large number of *practice questions* but a weakness is that the textbooks do not provide a *step-by-step method of answering questions* (C2). She had no knowledge of the curricula of learners' other subjects and how those curricula integrate with ML (C3). The only departmental document she knew of is the work schedule which she finds useful as it guides her teaching (C4).

The DoE's definition of ML according to Alice is: *The department of education makes it look like ML is a lower substitute for Mathematics* and their stated purpose of ML is *to help learners have a basic knowledge in mathematical related issues* (C5). She knows ML has four learning outcomes but could not state them and she does not know anything about CAPS (C5). At the end of the last interview I provided Alice with a list of concepts and contents to be covered in Learning Outcome 4: Data handling (NCS, 2003a) and Alice placed 16 out of 25 concepts in the correct year that they are to be introduced (C6).

During the last interview I asked Alice to which contexts the content should be applied according to the DoE and Alice's answer was: *Teacher-learners participating* (C7). In the interview before the second lesson on drawing the parabola, I asked about the context she was about to use and she talked about all the mathematical content to be covered. After a prompt she replied that no context is going to be used (C7). Although she mentioned in her interview before the third lesson that elections would be used in the lesson, she did not once refer to elections during the lesson. Pure statistics were done except for the one example that was based on the 'musical group' which the learners found interesting (C7).

To Alice mathematics is a *logical subject and it has to do with constructivism* (C8). She does not see the difference between Mathematics and ML and said:

> *ML is just a little bit easier … it's still part of the Maths, it's just of a lower grade. One of them said I don't see the difference, I said it's just a make believe and then they keep passing the same belief as they come. Everyone comes with the mentality it's difficult, and when they come here they say it is still Maths, it is difficult* (C8).

According to Alice mathematics is *quite important because you find it in everything; it is vital* (C9). The value of mathematics and ML are the same as both cover Financial Mathematics and since all people deal with calculations every day all people need a basic knowledge of mathematics (C9).

***Summary***

**Table 4.12: Summary of Alice's knowledge and beliefs**

| KNOWLEDGE AND BELIEFS DIMENSIONS | DESCRIPTION OF TEACHERS' KNOWLEDGE AND BELIEFS INDICATORS |
|---|---|
| Mathematical content knowledge (MCK) | Alice regards MCK as an important prerequisite in teaching ML. She however made numerous mathematical errors in class and her MCK appeared to be insufficient regarding the specific content covered in the three lessons. |
| ML learners (L) | She did not have sufficient knowledge of learners as she predicted they would understand all content she dealt with them but in reality they did not understand all the work. Apart from her own misunderstandings and misconceptions, she could not understand the learners' misunderstandings. |
| ML Teaching (T) | She struggled to have a logical flow in presenting the different concepts in her lessons and to sequence her activities. Too much content was covered in some of the lessons, which caused learners to become confused and frustrated. |
| ML Curriculum (C) | She does have sufficient curriculum knowledge but needs to know more about the DoE's vision for the subject. Alice views mathematics as a logical subject and believes that it has to do with constructivism. She does not see a difference between Mathematics and ML, ML is just a bit easier. The value of mathematics and ML is that both cover Financial Mathematics. |

## 4.6.3 Denise's knowledge and beliefs

### 4.6.3.1 Mathematical content knowledge[62] (MCK)

Denise believed that MCK is a prerequisite to teach ML. She said:

> *If you don't know the maths, you cannot understand the practicality of this ML. You must have content knowledge, because then at least you can build on that. Especially to take the maths and put it in context form and vice versa. If you don't have maths, I don't know where you will start.*

From the lessons I observed, Denise's MCK is good and no mistakes were made. To prepare the learners for examination, she emphasised the importance of showing all steps and adding the units at all times in their calculations and final answers.

### 4.6.3.2 Knowledge and beliefs regarding ML learners

For the first lesson[63] on conversions from metric to imperial units based on capacity and mass, Denise predicted that the learners would understand *the conversion of metrics to imperials and then straight metrics conversions* because she did *a lot of drilling* in class and they had to practise it at home too (L1). She also mentioned if *they don't keep on doing that [practising conversions], in revision you will see those errors* (L1). For the third lesson on conversions within the metric system based on capacity, mass, length, area and volume, she predicted that they *will easily understand the distance* (L1). Denise's predictions regarding what learners would understand and would not understand were in line with what happened in class (L1). Denise did not predict that the learners would misunderstand any content in the second lesson but concerning the third lesson she predicted that the learners would not understand conversions regarding area and volume because:

> *The distance is easy. Between the conversions from mm to km, the 10, the exponential is to the power of one. Coming to area, now it turns to the power of two, so before they divide or even multiply they start now forgetting, because the 10 is to the power of two. Then the same applies to the volume, the 10 is to the power of three. And then they usually multiply or either divide it without considering that the 10 is to the power of three before they can divide or multiply. There they got difficulties, but with length it is straight forward for them* (L2).

Denise believes that learners come to understand once *they can see their mistakes on the chalkboard … respond to questions … correspond and check what their misunderstanding was* (L3). She also believes that learners develop an understanding through individual practice but also during

> *formal assessment when I am done with new work, the feedback to them is how they will learn and rectify where they don't understand". She believed the learners will approach the tasks all by themselves, not even referring to the textbook as they know how to do these conversions* (L3).

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[62] Since there is only one indicator or code in Table 4.3 regarding the teacher's mathematical content knowledge, this whole paragraph's code is: MCK.

[63] The first lesson was repeated the next day to another class, so only two different lessons were observed.

Denise believes that another possible misunderstanding the learners could have is when 250ml must be converted to pints and they *search for millilitre to pint in the conversion table* not realising millilitre must first be converted to litre and then the conversion from litre to pints is provided in the table (L5). Sometimes, she said, they also confused distance with area (L5). Another example Denise gave of learners' misunderstanding was:

> *I give the measurements of the cube and I say they must calculate the volume in cubic mm, then the second question now I say the answer they got for the cube, they must convert it to cubic metre, but I said to them one cubic metre equals to 1 million cubic mm. Then you know, they can't actually do this, it's in context now, I don't know why … they want it straightforward* (L5).

As Denise discussed the learners' work on the board, it appeared that she understood the learners' alternative conceptions. There was only one incident in which learners had to do a conversion problem based on area and one learner wanted to square the answer too[64]. Denise misunderstood the learner's question and after an elaborate answer, the learner told Denise that was not what she asked and the learner repeated her question. Denise then understood the question but still did not address the learner's problem, just replying *no* to the question (L5). Denise provided opportunities for the learners to express themselves in writing on the board so that she was able to see what they did, but not many opportunities were created for her to listen to their thinking (L6). Based on the work she saw on the board, she acted appropriately to facilitate the learning process by discussing the learners' work with the individual as well as the rest of the class. She even involved learners to correct other learners' work (L6).

#### 4.6.3.3 Knowledge and beliefs regarding ML teaching

According to Denise the prior knowledge needed to be present to understand the work for the second lesson was *metrics [as] they did that in Grade 10,* also *to solve other problems from metrics to imperials and ratios* (T1). Denise did not revise the prior knowledge at the beginning of the lesson but integrated it in and across her lessons (T1). Since the lesson was on conversion from the metric to imperial system, some problems required an initial conversion within the metric system (prior knowledge) before the actual conversion could be made (T1). During the second interview I questioned Denise about the prior knowledge needed for the third lesson to which she replied: *Not applicable. It is a revision lesson.* (T1).

Denise used different forms of representation to make the content comprehensible to the learners such as written demonstrations and oral explanations where the use of either equations or ratios to solve the conversion problems were demonstrated (T2). She also used a diagram to revise prior knowledge based on conversions of length within the metric system (T2). These examples and tasks motivated the

[64] This example is discussed under Discourse: Teacher-learner interactions.

learners to understanding their own solutions and thinking (T2). The way Denise sequenced her tasks and explanations was proof of her ability to transform her own knowledge into forms that are pedagogically powerful (T3). She sequenced the tasks from the first to the third lesson: the tasks became more demanding and also included a wider variety of concepts, but only within the metric system (T4). Within the lessons there was no sequencing of the tasks from easy to difficult (T4).

Denise chose an appropriate instructional strategy for her revision lessons (T5). She used discussions in which she built her instruction on the learners' knowledge, their common errors and misunderstandings (T5). This strategy provided opportunities for informal assessment of the learners' knowledge (T5). The tasks were chosen to include conversions within the metric system but also conversions between the metric and imperial systems (T5). These tasks were based on length, area, volume, mass and capacity (T5). Denise chose the tasks from the learners' textbook but for assignments and assessments she chose tasks from other resources too *so that they can get exposed to other authors questions and approaches* (T5).

***ML teaching: Reflecting on her practice*[65] (T6)**

Denise's experience of teaching ML was:

> *It [ML] is not challenging. I feel a little bit bored, because I have done the pure maths. But for the sake of them [the learners] so that they must understand why we learn maths, I start to be a little bit of motivated. In pure maths you enjoy it throughout.*

It is important for her to motivate the learners by telling them the value of ML:

> *It is going to help you throughout your life where you are able to work out your own things, if you have your own business, your work one day, you are able to work with percentage, unlike not having maths at all. Basically it's personal as well as the reading of the stats, doing the inflation, price increase, petrol increase, you are able to calculate that.*

She ensured learner participation by giving the learners' tasks to complete individually and afterwards allowed some of them to write their solutions on the board so that corrections could be done. She believes that learners learn from the feedback she gave on their work done on the board.

Her reason for choosing the strategy of learners working on the board was: *Because I just want to see what they misunderstood, the content and the context. It then becomes easier for me to rectify any misunderstandings.* When introducing a new topic she believes she needs *to explain the content and concepts that are a little bit new, new words they are not familiar with and explain it thoroughly how its application work. Then from there I start doing the*

[65] Only one code was used to report on the teacher's reflection regarding her own practice namely T6.

*drilling*. Although I did not observe learner-learner interaction, she supported the idea thereof as she said:

> *They feel very comfortable when they talk to one another unlike with me (pause), some they find it very comfortable between peer and peer because they can understand the same group … So when they talk to one another it's not a problem … sometimes you find they talk together and once they argue, they come to me. That's what I like about them.*

Experience has taught her that teaching ML is different from teaching Mathematics as *most of the ML is on contexts whereas the basics are not well aligned like Mathematics is for me. I compare it with natural science and technology, technology is the root of science. Literacy is the root of pure maths.* After two prompts to direct her answer towards the teaching of the two subjects, she still continued to talk about the difference between the two subjects. She sees her role in her ML classroom as being the one who is there *to teach them … and then I facilitate whether they know the content or topics of the contexts are well understood.*

During the last interview I asked Denise to describe an ideal ML classroom in terms of the instructional strategies used. She described an ideal ML classroom as one in which the learners are involved in the lesson, where they *learn through doing it, trying on their own*. She emphasised that it is meaningless if they just look at her doing the work on the board; they should practise it themselves so that they can discover and construct their own meaning. A vital point was to have opportunities where she could communicate with the learners in order to determine learners' misconceptions and errors and have sufficient *time to rectify it through conversations*. Regarding the learning environment she believes:

> *It goes back to motivation, why they are learning this. We must come to a point where we can show them the realistic part and the value of it in everyday life. The lessons are contextual. The learners say where am I going to use ML in tourism? I don't see it. So then I must show them that wherever you are going to work, you are going to use it.*

When I asked her how this ideal classroom compared with her own classroom, she said:

> *Even though the time restricts us, I try. They must also be involved in the learning process. It must not be always me telling them this is how it is done, take it or leave it. They must also contribute, they must come up with example, they sometimes tell you of things from their world. So give them that freedom to participate.*

Her personal goal in teaching ML is *that these learners can be able to use this ML in everyday life; it will be perfect to me.*

#### 4.6.3.4 Knowledge and beliefs regarding ML curriculum

The DoE (2006) recommends a list of resources or instructional materials needed to teach ML (C1). The instructional materials Denise used to teach her lessons on conversions were the Oxford Successful ML (Pretorius et al., 2006) and Classroom ML (Laridon et al., 2006) textbooks (C1). She did not mention any advantages of the textbooks but experienced the textbooks as *not so good and effective* and *the questions are sometimes confusing for the learners and language is difficult for them* (C2). She was not aware of the curricula of other school subjects which integrate with mathematics (C3). Departmental documents she knew of were the *Assessment policy guideline; and Learning outcomes with assessment standards* of which she experienced the *Assessment policy guideline* as useful and valuable (C4).

When I asked Denise in the last interview how the DoE defines ML, she stated: *There is no definition for this subject only the implementation is important* (C5). According to her, the DoE's purpose with ML according to Denise is *for the learners to know how to use maths in their everyday life* (C5). She did not know anything about the new CAPS document as they had had no training yet (C5). She knew ML has four learning outcomes, but could not mention them (C5). At the end of the last interview I provided Denise with a list of concepts and contents to be covered in Learning Outcome 3: space, shape and measurement (NCS, 2003a) and Denise placed seven out of 19 concepts in the correct year that they are to be introduced (C6). In the interview before the second lesson, I asked about the context she was about to use and she replied: *Like the cuboid, to find the volume of a cuboid in mm and then convert this into cubic metres* (C7). For the third lesson she said the context to be used was *a table with matching tables, matching column A and column B, so that they can see one kilogram is how many pounds and so on* (C7). She did not mention any contexts in one of her lessons and also did not mention a cuboid in the second lesson.

Denise views mathematics as follows:

> *It's not formal. If this is the formula, I can also change it as long as I know I can prove it, and make arguments why I change it, as long as I can justify it … For me it's flexible. Then I can construct my own beliefs and my own understandings on what I am working on* (C8).

She summarised her view of mathematics by saying *mathematics is flexible and creative* (C8). Her perception of ML is that it is

> *not a higher grade or standard grade maths … more like a life skill … it's a maths on its own [and learners] must know at least the origin which is in pure maths, the origin of the curved graph, the origin of the parabola, the origin of the straight line graph, not just the application.*

The value of mathematics is endless to her, but among other things, she mentioned the following:

> *Whatever I do, it's maths. Personal, work, everywhere it's found. I used to say to my learners, if you walk, you count the steps you make, its maths, 1,2,3, it's natural numbers. I say go and buy zero bread, I ask what will you bring? Nothing, and zero is a whole number. You see now maths is everything* (C9).

To her the value of ML is that learners who are not capable of doing Mathematics can do ML. She said:

> *These learners don't know how to handle their personal lives and they are now able to work or manipulate with what is outside. For example if they want to read a pie chart, statistics in general, so now unlike before the learners who*

*are not capable to do pure maths or do not have maths, they can do this. So for me this is better than nothing because they can use it somewhere* (C9).

***Summary***

**Table 4.13: Summary of Denise's knowledge and beliefs**

| KNOWLEDGE AND BELIEFS DIMENSIONS | DESCRIPTION OF TEACHERS' KNOWLEDGE AND BELIEFS INDICATORS |
|---|---|
| Mathematical content knowledge (MCK) | Denise regarded MCK as a prerequisite to teach ML. She made no errors in her examples or corrections of the learners' work, but also did not elicit any discussions regarding the conceptual meaning of the different units of measurement or their application value in everyday life situations. |
| ML learners (L) | She correctly predicted what learners would and would not understand. She did not allow learners to explain their thinking so that she could listen to their thinking. She acted appropriately to the work she saw on the board. |
| ML teaching (T) | She correctly identified the prior knowledge for the second lesson. Denise used various appropriate representations to make the content comprehensible to the learners, applied appropriate instructional strategies and sequenced her tasks over the different lessons to enable learners to progress in their cumulative understanding. According to Denise the ideal ML classroom compared well with her own except for allowing the learners more time to discover the content as time is always a restriction. |
| ML Curriculum (C) | Regarding the NCS, Denise could not provide the DoE's definition, purpose and learning outcomes or even place half of the topics in the correct year that they are to be introduced. Denise views mathematics as flexible and creative, constructing one's own understanding and ML as a type of mathematics, but unique. According to her, both mathematics and ML are valuable as people use both in their personal and work environments. |

## 4.6.4 Elaine's knowledge and beliefs

### 4.6.4.1 Mathematical content knowledge[66] (MCK)

When Elaine was asked about the extent to which MCK is a prerequisite to teach ML, she said:

> *A motivated teacher who is prepared to work hard will manage the teaching of ML but there are some details of mathematical knowledge a ML teacher needs to know. You will not be able to just explain rates of change and ratios, you will need to learn about the finer things, how to explain it, what is important. It is only now, after three years of teaching ML that I feel confident in front of my classes, that I know what is important and what should be done in each grade.*

From the lessons I observed, Elaine's MCK is very good. She did not make any mathematical errors and I did not observe any misconceptions.

[66] Since there is only one indicator or code in Table 4.3 regarding the teacher's mathematical content knowledge, this whole paragraph's code is: MCK.

#### 4.6.4.2 Knowledge and beliefs regarding ML learners

Regarding the second revision lesson on time and interest, Elaine predicted during the interview just before the lesson that the learners would recognise the formula and be able to tell that the formula is used for compound interest (L1). Regarding the third lesson on perimeter, area and volume, she predicted that the learners would know the units and how to calculate the perimeter, area and volume as they did that in Grade 10 (L1). She realised some learners would still struggle in the second lesson *to convert the variable in the formula to a decimal as well as with the concept of interest being calculated monthly and semi-annually* (L2). The task she used in the third lesson was more complex than the previous year and *[i]t is possible that not all learners will be able to verbally explain the concepts. Not all learners have conceptual knowledge of the concepts or sometimes they have the concept in their mind but do not have the ability to verbalise that concept* (L2). She believed that the learners would understand the tasks in the second lesson once they *can explore and have that aha feeling*, but she needed to make the lesson *applicable and link the new work with their personal lives such as personal tax, start at home, their parents' salaries …* (L3). Concerning the third lesson, the learners would understand the work once they learnt to *carefully read through the problem, study the drawing and indicate all the information with coloured pens on the drawing as the visual representation simplifies it* (L3). Her predictions about what learners would and would not understand were realised in her lessons.

During the interview just before the lesson she predicted that the learners would approach new tasks by discussing the work with their peers (L4). She frequently noticed that *they do not just ask anyone, they will not ask a friend who knows less than themselves but will ask someone they know is able to explain the work to them* (L4). In her experience *they really listen to and learn from one another*. She said people joke about the *buzz in her class but they [the learners] buzz about the work* (L4). Many times she preferred to *put a weak and strong performer together and then the one learns to explain and the other one learns to understand*, thus using cooperative learning as instructional strategy (L4). At her school all teachers need to remain after school till 14h30 and many learners make appointments with her for individual support (L4). They also use their textbooks and scripts as sources of reference (L4).

Regarding other possible misconceptions learners might have, she mentioned a problem with cubic centimetres: *they do not always have the concept of length x breadth x height, so it is three units multiplied with each other, the same with area* (L5). Elaine required learners to give explanations and justifications orally and in writing (L6). A fundamental part of Elaine's lessons was learners' abilities to explain the meaning of concepts as these explanations provided her with proof of learners' conceptual understanding of these concepts (L6). This enabled Denise to see what individual learners do, listen to what they think and to act appropriately (L6). She recognised and clarified the learners' common errors and misunderstandings (L6).[67]

#### 4.6.4.3 Knowledge and beliefs regarding ML teaching

As far as the prior knowledge of the observed lessons was concerned, Elaine said all mathematical content had been introduced either in Grade 10 or Grade 11 as the lessons were revision lessons (T1). In these three lessons the learners needed to practise their skills and use their existing knowledge by solving unfamiliar and more complex contextual problems (T1). In class there were numerous situations when Elaine revised prior knowledge before presenting a particular concept (T1). Elaine applied representations such as tables, symbols, formulae, calculators, a demonstration calculator and sketches of manipulatives to make the tasks comprehensible to the learners (T2). These representations proved to be useful as learners participated and comprehended the work (T3). The tasks were sequenced and presented in a pedagogically powerful way to facilitate learning (T4).

In preparing her lessons Elaine realised some learners could become bored as they repeated similar work a few times. She also needed to attend to those learners who only knew half of the work or even less (T5). She then planned to discuss the work she knew they struggled with, such as explaining the significance of the different variables in the formula. She involved them in discussions during which their errors and misunderstandings were corrected. Her planning was based on preparing the learners for the examination, so she provided guidance on the type of questions they could expect and how those questions should be approached (T5). Elaine's choice of instructional material was appropriate as she chose tasks from previous examination papers to enable learners to solve typical examination questions in different contexts (T5). She encouraged the learners to use coloured pencils to indicate given information on their sketches (T5).

***ML teaching: Reflecting on her practice*[68] (T6)**

When Elaine was asked two years ago to be the coordinator for ML, she initially felt that she was being demoted, but she claims that ML had grown on her since then. She enjoys being involved in ML and never wants to go back to Mathematics. To improve her learners' appreciation of ML she bases her lessons on real-life situations as the learners need to recognise what the subject is about and where mathematics could be used. She refers her learners to the yearbooks of tertiary institutions to familiarise them with the requirements of possible future studies and how ML can add value to their studies. She

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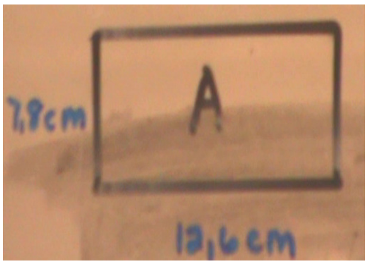
[67] More detail is given under Elaine's Discourse: Teacher-learner interactions.
[68] Only one code was used to report on the teacher's reflection regarding her own practice namely T6.

also mentioned that she wanted to improve her learners' participation in the lessons. She would like to use two of the seven periods a week to do something out of the ordinary like taking them on an excursion or watching a DVD where they can discover the role of mathematics in specific situations or events, which could then be followed by a class discussion. She said she could give them a worksheet before the excursion or DVD which the learners could complete during and/or after such an event. She also wanted to use games and newspapers and magazines to further enrich the learners' appreciation of mathematics. She believes such activities contribute towards proficient learning.

I asked Elaine to reflect on the three lessons I had observed. According to her she chose discussions as one of her instructional strategies since it gave her a platform to work from. She determined the gaps in their knowledge, which became the focus of the lesson. She believes discourse between the learners indicates that the learners are involved and interested in the lesson. To elicit such discussions she enjoys throwing in a question with an impossible answer such as the question in the class test (Picture 4.29) during the third lesson where she asked: *The following figure is a rectangle. Calculate the volume of the figure.*




**Picture 4.29: A question asked in a class test**

According to Elaine her classroom rules are less rigid and structured than other classes in the school. She wants her learners to have fun in a relaxed atmosphere. She said she did not want the subject to have a negative stigma and therefore made an effort to make the subject alive and interesting. She asked the principal and colleagues to respect her subject as she respects their subjects and not to make fun of ML by referring to it as a very low level of Mathematics. Elaine believes ML should not even be compared with Mathematics, but should be regarded as a subject on its own. Evidence that her learners enjoy and value ML lies in the fact that learners who had to change from Mathematics to ML asked to be in her class. One of Elaine's most important rules is that learners should respect one another and value other people's thinking and ideas.

Elaine believed that the teaching approach of ML is totally different to Mathematics because ML is presented in a more relaxed way where the learners are not pressed to finish in time. According to her in ML the high ability learners can continue with additional work and there is enough time to further attend to the slower learners. There is always time to go back to their Grade 9 content and do revision, which is not the case when teaching Mathematics. She wished that a standard grade of Mathematics could be implemented again to cater for those learners who do not actually belong in the ML class but at the same time do not wish to take Mathematics. She regarded her role in class as being the mediator between the content and learners. Often learners had the correct ideas but did not know how to formulate them. She then needed to guide the learners in the process of discovery and provide scaffolding to assist them in moving from their uncertain and disorganised thinking processes to conceptual understanding. She believes that the learners sometimes need to struggle through the process of problem solving and once they experience success, it serves as a lesson in life that there are times one needs to struggle through solving a problem, but it is possible to arrive at a solution.

During the last interview I asked Elaine to describe an ideal ML classroom in terms of the instructional strategies used. She described an ideal ML classroom as one in which a teacher uses instructional strategies that are effective to her as individual. She also stated that apart from the discourse between the teacher and learners there should be enough opportunities for communication between the learners. An ideal is having computers with internet access in such a class to enable learners for example to find the present exchange rate and as such make the tasks more realistic. She believes *information you search for on your own and read it by yourself is more valuable and will be remembered longer*. Elaine planned to have newspapers available in class to enable the learners to work with news of the day. In comparing this ideal ML classroom with her own practice she believes it is the same except for not having computers in her classroom. Her purpose in teaching ML is to equip the learners with life skills and she hoped that one day when they think back they would remember her and what she taught them, but more importantly, the life skills and values she taught them.

#### 4.6.4.4 Knowledge and beliefs regarding ML curriculum

Regarding her curriculum knowledge she knew about the appropriate instructional material that could be used for the lesson she did on perimeter, area and volume as she used textbooks, previous examination papers, models of two-dimensional figures and three-dimensional objects as well as transparencies (C1). She used the Mathematical Literacy for the Classroom (Laridon et al., 2006) textbook, a book containing previous examination papers and the internet (C1). She valued the fact that each learner has his/her own copy and is able to use the textbook at home (C1). She mentioned that a weakness of textbooks is the fact that changes made by the DoE regarding the curriculum cannot be accommodated C2). She found previous examination papers valuable as the questions prepare the learners for examinations (C2). The value of the models was the opportunities for learners to discover knowledge through visual experiences (C2). According to Elaine the ML curriculum should include mathematical content that integrates with the curricula of other school subjects. She provided the following list of subjects that integrate with ML:

*Business economic – graphs*
*Accountancy – salaries*
*Engineering graphical design – scales and drawings*
*Mechanical technology – trigonometry*
*Tourism – map work* (C3).

In her opinion trigonometry that was initially in the curriculum but later omitted should be put back as trigonometry is required by Mechanical Technology (C3). Departmental documents she was aware of were circulars and the CASS document which she found useful (C4).

According to Elaine the DoE defines ML as *equipping all learners with mathematical skills by using problems from real-life situations* (C5) in order to *equip learners with basic mathematical skills. They implemented it based on the recommendation of the private sector that required workers to become mathematically literate* (C5). She had not yet perused the new CAPS document but believed this document will be more distinct, comprehensible and user-friendly (C5). She knew that the four learning outcomes were mathematical concepts, financial mathematics, measurement, data handling and probability (C5). At the end of the last interview I provided Elaine with a list of concepts and contents to be covered in Learning Outcome 3: Space, shape and measurement (NCS, 2003a) and she placed 11 out of 19 concepts in the correct year in which they were to be introduced. The curriculum required that content should be set in context which Elaine appropriately did (C6).

As prescribed by the DoE (2003a), Elaine taught the content in all three her lessons in context (C7). The contexts were applicable to the content and she had sufficient knowledge of the contexts she shared in discussions with the learners (C7). An example of such a discussion was:

*T: Let us quickly discuss, why would somebody rather wait to buy a house and first increase his deposit before doing so?*
*L1: Isn't it to lower his instalment?*
*T: Yes, to decrease his instalment. Did you also know that if one day you have bought your own house you can decrease the period of paying back the money you have borrowed by several years if only you pay all extra money you might have available in a month into your home loan account. By doing that, what would you save on?*
*L2: Money.*
*T: Money yes, but you will save on interest you need to pay on that home loan account.*

Most of the contexts she used in her class were unfamiliar to the learners (C7).

She believes mathematics is about rules and formulae that need to be discovered. Mathematics should not be presented on a blackboard to learners as a rigid discipline with fixed rules (C8). She concluded by saying: *Mathematics is a discovering experience* (C8). Learners should be able to experiment with the mathematics available to them. She believes ML is for everyone, even for Mathematics learners as they too would increase their general knowledge on life-related issues as well as their reading skills (C8). She believes mathematics has an *unbelievable value as mathematics stimulates one's brain and practice higher order thinking … it expands your vision and you generally think better. You don't do rote learning only, you have to reason too* (C9). The value of ML *is incredible as a learner returned from his holiday one day telling her in excitement that he used the mathematics she taught them* (C9). One of her learners told her that recently he had amazed his father by telling him about what he needed to take in account before buying a new car.. These were proof of the value of ML and the life skills ML learners acquire (C9). Elaine explained that she and her husband had bought a house a few years ago and had to learn for the first time in 42 years about all the costs and implications involved in buying a house and arranging finance, whereas her Grade 12 learners had already learnt about this in school (C9).

***Summary***

**Table 4.14: Summary of Elaine's knowledge and beliefs**

| KNOWLEDGE AND BELIEFS DIMENSIONS | DESCRIPTION OF TEACHERS' KNOWLEDGE AND BELIEFS INDICATORS |
|---|---|
| Mathematical content knowledge (MCK) | Elaine has very good MCK and no errors or misconceptions were observed. She believes it is possible, but not ideal, to teach ML without having MCK as long as the teacher is hard working and motivated. |
| ML learners (L) | She had the ability to predict what learners would and would not understand and why and how they would understand the new content. She predicted that they would approach new tasks by discussing the work with peers from which they could learn, but that they would also come to ask her or consult their textbooks. |
| ML teaching (T) | Elaine taught the content in context and knew the prior knowledge needed to explain new concepts. She used relevant examples, illustrations and explanations to make the work comprehensible to the learners. She transformed her content knowledge into forms that were pedagogically powerful. She also efficiently sequenced the content to facilitate learning. The instructional material was chosen appropriately. Elaine likes teaching ML and believes the subject should be taught in an interesting and practical way so that learners can enjoy and value the subject. |
| ML Curriculum (C) | She is well informed regarding the curriculum since she correctly answered most of the curriculum questions I asked during the last interview. She knew about a variety of instructional materials to use in the lessons she presented and was familiar with other school subjects that integrate with ML. She was not aware of all available departmental documents or even the content of CAPS, but knew the definition and purpose of ML according to the NCS. Most of the time the contexts were unknown to the learners. She regarded mathematics as being flexible and that learners should discover and experiment with it. She values |

| | |
|---|---|
| | mathematics as a discipline that stimulates the brain, allowing higher order thinking and reasoning. ML is for everyone, even for Mathematics learners because their general knowledge on life-related issues should also be increased. |

### 4.6.5 Summary of the participants' knowledge and beliefs

Table 4.15 provides a snapshot of the four participants' MCK and PCK and beliefs regarding the ML learners, the teaching of ML and the ML curriculum.

Table 4.15: Snapshot of the four participants' knowledge and beliefs

| PARTICIPANTS | MONTY | ALICE | DENISE | ELAINE |
|---|---|---|---|---|
| **Mathematical content knowledge (MCK)** | He believed MCK is a prerequisite to teach ML. He made no mistakes and it appeared as if his MCK regarding the specific content covered is sufficient. | She believed MCK is a prerequisite to teach ML. She made several mistakes and it appeared as if her MCK is insufficient regarding the specific content covered in the three lessons. | Denise believed MCK is a prerequisite to teach ML. She made no errors in her examples or corrections on the board and it seemed as if she had sufficient MCK regarding the specific content covered in the three lessons. | Elaine believed MCK is a prerequisite to teach ML. No errors or misconceptions were observed and it seemed as if she had sufficient MCK regarding the specific content covered in the three lessons. |
| **PCK and beliefs regarding the ML learners (L)** | • His predictions about the content the learners would and would not understand did not correspond with what happened in class.<br>• Some comprehension of learners' misunderstandings but when he did not understand their misunderstandings he became irritated.<br>• Limited evidence of learners expressing themselves to determine if he could act appropriately on their ideas.<br>• He believed learners gained understanding from looking at examples and practising the work. | • She predicted learners would find all content easy and could not understand their common errors or misunderstandings.<br>• She proved to have certain misconceptions herself so it would be difficult for her to predict possible learner misconceptions.<br>• No evidence of learners expressing themselves to determine if she could act appropriately on their ideas.<br>• She believed learners learn from practising in the presence of someone who gives them confidence. | • She correctly predicted what learners would and would not understand.<br>• She realised learners' possible misconceptions and rectified them in class.<br>• She did not allow learners to explain their thinking so that she could hear their thinking but as they demonstrated their work on the board, she could act appropriately with regard to their written work on the board.<br>• She believed learners learn by explaining the work to others in small groups. | • She had the ability to predict what learners would and would not understand and how they would understand the new content.<br>• She was aware of learners' possible misconceptions and rectified their misunderstandings in class.<br>• She looked at learners' work, listened to their thinking and acted appropriately.<br>• She believed learners learn once the teacher builds on their existing knowledge and they could talk about their thinking. |
| **PCK and beliefs regarding the ML** | • He was aware of prior knowledge needed for | • She did not connect learners' prior knowledge with new | • Prior knowledge was integrated in her revision | • Knew what prior knowledge should have been revised at |

| | | | | |
|---|---|---|---|---|
| **teaching (T)** | learners to gain understanding.<br>• He chose very basic and similar examples and did not use multiple representations.<br>• Not much evidence of sequencing the content.<br>• His choice of direct instruction as instructional strategy and use of textbook mainly were not appropriate to teaching ML.<br>• He believed direct instruction should initially be used to introduce new content, followed by group work and discussions for solving problems.<br>• He believed the difference in approach between ML and Mathematics is the use of fewer examples and working at a slower pace. | situations.<br>• The demonstrations and explanations used did not make the content comprehensible to the learners. She did not use various representations.<br>• She mostly sequenced the content but the amount and pace made it difficult for the learners to comprehend the content.<br>• Her choice of direct instruction as instructional strategy and use of textbook only were not appropriate to teaching ML.<br>• She believed in using formal teaching when introducing the topic and discussions later.<br>• She believed that the teaching of ML does not differ from teaching Mathematics. | lessons.<br>• She used varied and appropriate representations to make the content comprehensible to the learners, applied appropriate instructional strategies and sequenced her tasks.<br>• Some evidence of sequencing the tasks was observed.<br>• Discussions and learners working on the board were appropriate strategies for her revision lessons.<br>• She believed she initially had to explain the content, concepts and application thereof, followed by drilling. The learners should also be involved by sharing their ideas but time is a restriction.<br>• She believed the teaching of ML differs from that of Mathematics as ML is based on contexts. | certain stages of lesson.<br>• She taught the content in context and used powerful examples, illustrations and explanations and various representations to make the work comprehensible to the learners.<br>• She sequenced the content to facilitate learning.<br>• Discussions and using textbooks and previous examination papers were appropriate for her revision lessons.<br>• She believed ML should be taught by basing her class discussions on real-life situations and the learners' prior knowledge.<br>• She believed teaching ML differs from teaching Mathematics as there is enough time for learners to discover new situations through discussions and problem solving. |
| **PCK and beliefs regarding the ML Curriculum (C)** | • He had no knowledge of other subjects' curricula that integrate with ML.<br>• Knew about definition, purpose and learning | • She had no knowledge of other subjects' curricula that integrate with ML.<br>• Knew about definition, purpose and learning | • She had no knowledge of other subjects' curricula that integrate with ML.<br>• She did not know the definition and could not state | • She knew how ML integrates with the curricula of five other subjects in school.<br>• She knew the definition, purpose, learning outcomes |

| | | | | |
|---|---|---|---|---|
| | outcomes but was not aware of all departmental documents.<br>• Did not teach content in context. He referred to real-life scenarios but it was unclear how the content should be applied in those contexts.<br>• He viewed mathematics as constructivism and logical although this view was not implemented in practice. He believed ML is a kind of mathematics, but not a lower grade of Mathematics. | outcomes but were not aware of all departmental documents.<br>• Did not teach content in context except for the one life-related example used.<br>• She viewed mathematics as being logical and having to do with constructivism, the latter not being observed. She believed there was no difference between Mathematics and ML and that ML is just a little easier. | the learning outcomes but knew the purpose of ML and the various departmental documents.<br>• Did not teach content in context.<br>• She viewed mathematics as flexible and creative and regarded ML as a unique form of mathematics. | and relevant departmental documents.<br>• In all her lessons content was taught in context as ML should be taught.<br>• She viewed mathematics as a flexible discipline that should be used to discover and experiment with. She believed ML is a subject on its own which is meant for all learners (Mathematics learners too). |

## 4.6.6. Discussion of Theme 2: ML teachers' knowledge and beliefs

In this section I will conduct a literature control where the findings from this study are compared with the findings from other research studies on ML teachers' knowledge and beliefs regarding the ML learners, ML teaching and the ML curriculum is based on the indicators in Table 4.3[69].

### 4.6.6.1 ML teachers' mathematical content knowledge (MCK)

- **Teachers' belief that MCK is a prerequisite to teach ML**

My finding that all four teachers in my study believed MCK is a prerequisite to teach ML is strongly consistent with Fransman's (2010) finding that all four teachers in her focus group believed ML teachers should know the mathematics content well enough to be able to do the mathematics themselves. The four teachers in my study also believed that all ML teachers should have some form of tertiary training in mathematics, a finding that is moderately consistent with Sidiropolous' (2008) finding that one of the two teachers in her study believed that educators teaching ML should be qualified and have some form of tertiary training in mathematics.

- **ML teachers' level of MCK**

Except for Alice, the teachers in my study appeared to have sufficient MCK regarding the topics they were teaching at the time of the observations. Alice's MCK was not always coherent, and she made several mistakes in the written examples on the board as well as during her verbal explanations. This finding, where one of the four teachers in my study had insufficient MCK of the ML topics taught, is inconsistent with Hechter's (2011a) finding that both the teachers in her study had insufficient MCK of the ML topics taught (*their knowledge was not coherent and some errors were made with respect to the mathematical content dealt with in the classrooms,* p. 149). My finding is also inconsistent with Bansilal's (2008) finding that most of the ML teachers in her study had insufficient knowledge of the ML topics taught.

### 4.6.6.2 ML teachers' knowledge and beliefs regarding their learners

Denise and Elaine (as experienced former Mathematics teachers) demonstrated specific knowledge of the ML learners' prior knowledge and what content should be emphasised and how the content would be understood by the learners. They were able to predict learners' common errors and misconceptions and could act appropriately to facilitate learning. Compared to Denise and Elaine, Monty and Alice (as novice teachers) demonstrated superficial knowledge. As far as I could establish, no study has reported this finding before.

[69]See Table 4.3 under Section 4.3.2.2: Theme 2: ML teachers' PCK and beliefs.

Denise and Elaine once again had similar beliefs on how learners come to understand mathematical content as did Monty and Alice. Denise believed that learners learn by explaining the work to each other while Elaine believed they learn when the teacher builds on their existing knowledge, and they then talk about their thinking. Conversely, Monty believed that learners gain understanding by studying several examples and through a lot practice while Alice believed that learners learn by practising the work in the presence of someone who gives them confidence. My finding that two of the four teachers in my study believed that learners reach understanding through active involvement in the lessons is strongly consistent with the finding of Sidiropolous (2008) that one of the two teachers in her study believed that learners reach understanding through critical and creative engagement in the lessons. However, apart from this observation, my literature control did not yield any other reportable findings, that is, findings that I could realistically compare with my own.

#### 4.6.6.3 ML teachers' knowledge and beliefs regarding the teaching of ML

Knowledge regarding the teaching of ML refers to teachers' ability to know what prior knowledge should be present for learners to understand new work; to use various representations and resources to facilitate learner understanding; to transform their own knowledge into forms that are pedagogically powerful; to sequence content; and to choose appropriate instructional strategies and materials (Artzt, et al., 2008; Ball, 1990; Borko & Putnam, 1996; Hill et al., 2008; Shulman 1986; Shulman, 1987).

In the interviews prior to the observed lessons, only Denise and Elaine could identify the prior knowledge that should have been present for the learners to understand the work. Monty and Alice said *everything* was prior knowledge as they believed all the content had already been done in Grade 10. Regarding the various representations such as tables, figures and graphs that teachers can use to facilitate learners' understanding, both Alice and Elaine conformed to this requirement. Only Alice did not have the ability to transform her own knowledge into forms that were pedagogically powerful. During the interview prior to the lesson on data-handling, it seemed she had sufficient knowledge of the topic, but her lesson presentation was incoherent, and the learners were confused and frustrated. As far as I could establish, these findings have not been reported before.

It is difficult to comment on the teachers' ability to use various resources as three of the four teachers in my study taught content only when only a textbook and calculators were used. Elaine was busy with her revision programme at the time of the study and used various textbooks, previous examination papers and calculators as resources. This finding, namely that three of the four teachers in my study used a textbook and calculators only, is consistent with the finding of Sidiropolous (2008) where both teachers in her study used only a textbook and calculators. The teachers' ability to sequence the content[70] and choose appropriate instructional material[71] as well as their ability to choose appropriate instructional strategies[72] has already been discussed.

#### 4.6.6.4 ML teachers' knowledge and beliefs regarding the ML curriculum

- **Content-context issue**

Denise and Elaine believed that ML teaching differs from teaching Mathematics. This finding is strongly consistent with Sidiropolous' (2008) finding that one of the two teachers in her study believed that ML teaching is different from teaching Mathematics. Elaine demonstrated that her belief and instructional practice conformed to the requirement of the DoE (2003a) of *engaging with contexts rather than applying mathematics already learned to contexts* (p. 43). The other three teachers believed that teachers should initially explain the content and then apply the content to contexts using discussions if time permitted. My finding that only one of the four teachers in my study used relevant contexts in order for the learners to explore the content is inconsistent with the finding of Venkat (2010) where the teacher in her study used contexts but consistent with the finding of Sidiropolous (2008) as one of the teachers in her study did not use contexts. My finding that all four teachers in my study however believed contexts should be used is strongly consistent with both Sidiropolous' (2008) and Hechter's (2011a) finding where both teachers in each study believed that contexts should be used to facilitate learning.

- **Integration of ML with other subjects**

Apart from the fact that ML requires a different teaching approach to that of Mathematics, the ML curriculum also requires ML to be taught in a de-compartmentalised manner and to be integrated with other school subjects (DoE, 2003a; De Villiers, 2007; North, 2005; Venkat & Graven, 2007). All four teachers in my study believed ML should be integrated with other school subjects. My finding is strongly consistent with both Sidiropolous' (2008) and Fransman's (2010) findings where all the teachers in their studies believed ML should be integrated with other disciplines. Only Elaine could identify other school subjects that ML should be integrated with, and only she had knowledge of their curricula. She believed the ML curriculum should address the mathematical needs of those subjects. As far as I could establish, this finding has not been reported before.

- **Other ML curriculum issues**

Only Elaine had sufficient knowledge of other ML curriculum issues such as knowledge of the appropriate use of various instructional materials, the strengths and weaknesses of textbooks, the use of departmental documents as guidelines as well as knowledge of topics that were taught in preceding and would be taught in subsequent years, i.e. Grades 10 and 12. Sidiropolous (2008) reported that both the

[70] See Discussion of Theme 1: Tasks.
[71] See Discussion of Theme 1: Tasks.
[72] See Discussion of Theme 1: Learning environment.

teachers in her study had insufficient knowledge of other curriculum issues (as explained above). My finding that three of the four teachers in my study had insufficient knowledge of other ML curriculum issues is consistent with Sidiropolous' finding.

- **Teachers' beliefs about the nature of mathematics as a discipline**

All four teachers in my study had a constructivist perspective on teaching and learning mathematics. Monty and Alice believed mathematics is a logical discipline while Denise and Elaine considered mathematics as being flexible and creative. As far as I could establish, no study up to now has reported on ML teachers' beliefs about the nature of mathematics as a discipline.

- **Teachers' beliefs about the nature of ML as a subject**

Only Alice considered ML similar to Mathematics but at a lower level − the other three teachers viewed ML as a unique subject. This finding that only one of the four teachers in my study considered ML to be similar to Mathematics but at a lower level provides some (albeit limited) support for Fransman's (2010) finding (only 2 out of the 58 teachers in her study viewed ML as similar to Mathematics but at a lower level) and is consistent with Hechter's (2011a) finding where half of the teachers in her study viewed ML as similar to Mathematics but at a lower level, but is inconsistent with Sidiropolous' (2008) finding that all the teachers in her study viewed ML as similar to Mathematics but at a lower level.

- **Teachers' beliefs about the value of ML as a subject**

All four teachers in my study believed ML has great value as learners obtain knowledge they can use in their everyday lives and work situations in the future. My finding that all four teachers in my study valued ML is strongly inconsistent with Sidiropolous' (2008) finding that neither of the two teachers in her study believed ML has great value in helping learners obtain knowledge they can use in their everyday lives and work situations in the future.

#### 4.6.6.5 Summary of discussion on Theme 2

To summarise: All four teachers in my study believed that MCK is a prerequisite for teaching ML, and three of them appeared to have sufficient MCK of the topics they were teaching at the time of the observations. Compared to Monty and Alice who demonstrated superficial knowledge of the learners' prior knowledge, Denise and Elaine demonstrated specific knowledge of the learners' prior knowledge, which content to emphasise and how it would be understood by learners to the extent that they could accurately predict the learners' problems with the content. Monty and Alice believed learners gain understanding by studying several examples while Denise and Elaine believed a teacher should build on learners' prior knowledge and involve learners in discussions. Only Elaine based her teaching on life-related problems while both she and Denise used appropriate instructional strategies to facilitate learning. Monty and Alice believed the teaching of ML is no different to teaching Mathematics.

Only Elaine knew which subjects are integrated with ML although all four teachers believed that ML should be integrated with other disciplines. Apart from Elaine, the teachers in my study had a superficial understanding of the ML curriculum. All four teachers had a constructivist perspective on teaching and learning mathematics. Only Alice viewed ML as a lower level of Mathematics while the other teachers viewed ML as a unique subject. All four teachers regarded ML as valuable because, through it, learners could obtain knowledge they could use in their daily lives.

## 4.7 Findings, trends and explanations

An analysis of the discussions on Theme 1 and Theme 2 was done and resulted in the following summary in which the findings, trends and explanations are delineated.

- **Experiences and beliefs shared by all four teachers in my study**

I found that all four teachers were positive about teaching ML and all shared the following beliefs: Mathematics is a subject that is best mastered by implementing a constructivist approach to teaching and learning; ML is a valuable subject; ML should be integrated with other school subjects; and MCK is a prerequisite for teaching ML. Many teachers in recent studies share this positive attitude towards teaching ML and attach value to the subject. Sidiropolous (2008), however, found that both the teachers in her study had negative attitudes towards ML. This discrepancy in findings can perhaps be attributed to teachers becoming aware of the uniqueness of the subject and starting to realise its value during the past four years − Sidiropolous conducted her study only a year after ML had been introduced at a time when teachers tended to be negative about this unfamiliar subject. Research indicates that almost all ML teachers believe that MCK is a prerequisite for teaching ML and that all ML teachers should have some form of tertiary mathematics training. With Alice having some form of tertiary mathematics training but still making several errors and not being able to *transform [her] own knowledge into forms that are pedagogical powerful* (Shulman, 1987, p. 15), it seems that not only general training in the content of mathematics but also mathematics teacher training (i.e. training in the teaching and learning of mathematics) is required to teach ML proficiently. A concern is what happens in ML classes where teachers from other disciplines with no formal mathematics teacher training teach ML.

- **Two differing cases**

In my study, two highly differing cases were Alice and Elaine. Alice, as a novice teacher with no mathematics teacher training, was the only teacher who communicated judgmentally with the learners; did not work at a slower pace as stipulated in the ML curriculum (DoE, 2003a); did not have the ability to transform her own knowledge into forms that were pedagogically powerful; and viewed ML as similar but inferior to mathematics. Elaine, a former mathematics teacher with years of experience, was the only teacher who used contextual tasks effectively; pointed out the value of mathematics to the learners; selected tasks from all four levels of the ML Assessment Taxonomy; required the learners to explain their answers; posed a variety of oral questions on different levels; and had sufficient curriculum knowledge. From this comparison, it seems experience and mathematics teacher training play a crucial role in the instructional practice of the ML teachers.

- **Role of teaching experience in ML teachers' instructional practice**

In comparing the instructional practices of experienced and inexperienced teachers from my own and other research studies, I found that not all experienced teachers taught ML in a satisfactory manner. For example, the experienced Denise in my study did not comply with all the requirements regarding the instructional approach to teaching ML as set out by the DoE (2003a). Another example is the study of Sidiropolous (2008) where the instructional practices of the two experienced, but negative, teachers were not aligned with the curriculum or with their claimed beliefs. Conversely, there were inexperienced teachers in the studies (one from each study) of Fransman (2010) and Hechter (2011a) whose practices were aligned with the curriculum and with their claimed beliefs − teachers who had developed a *new status identity* (Fransman, 2010, p. 184) of being a ML teacher. It seems that apart from having sufficient teaching experience, the success of a ML teacher's practice can be improved by being positive about and taking ownership of teaching ML as well as developing a new status identity of being a ML teacher.

- **Value of teacher and ML training**

Apart from Monty and Alice both being novice teachers, a major difference in their instructional practices is that Alice had no teacher training but had completed a mathematics course as part of her Management degree whereas Monty had completed a BEd degree with Mathematics as a major. In contrast to Alice, Monty communicated with the learners in a non-judgmental manner; assisted them individually; worked at a slower pace; demonstrated sufficient MCK; and viewed ML as a unique subject. These findings suggest that teacher training plays a role in teacher-learner interactions and in establishing a manageable pace of work. Only Monty attended short courses for in-service ML teachers. A comparison of the teachers in my study with the teachers who were part of the ACE (ML) programme of Fransman (2010) and Hechter (2011a) revealed that ML teacher training had a positive influence on the instructional practices and beliefs of the teachers who were enrolled for the ACE (ML) course. Examples of this include their ability to provide scaffolding and to teach the mathematical content using contextualised tasks.

- **On a continuum from teacher-centred to learner-centred**

If I could place the teachers in my study on a continuum from teacher-centred on the left to learner-centred on the right, this would be the order from left to right: Alice, Monty, Denise and Elaine. Monty and Alice as novice teachers adopted a teacher-centred approach. Denise as a former Mathematics teacher, with many years' experience of teaching Mathematics, leant more towards a learner-centred approach. Elaine, also a former Mathematics teacher with many years of experience in teaching Mathematics, also adopted a learner-centred approach. It appears that teacher training and experience influenced the teachers' ability to facilitate learning effectively in the ML classroom in my study. Although both teachers in Sidiropolous' (2008) study had mathematics training and teaching experience, they nevertheless still used a teacher-centred approach. This can perhaps be attributed to factors such as their having had no ML teacher training or their beliefs regarding the teaching of ML. Venkat and Graven (2008) and Venkat (2010) reported that the teachers in their studies were trained and experienced, but it was after making an actual decision, as Elaine did, to change their pedagogic practices to a learner-centred and activity-based approach, that the teachers as well as the learners could experience the value of ML.

- **Contradictions between teachers' beliefs and their instructional practices**

The following beliefs held by the teachers in my study were contradicted in their practices.

**Table 4.16: Contradictions between teachers' beliefs and their instructional practices**

| Teacher | Belief | Practice |
|---|---|---|
| Monty, Alice, | Mathematics is a constructivist discipline | Direct instruction (lecturing) was used. |
| Monty, Alice | ML should be integrated with other subjects | No integration was evident in their lessons. Could not explain how ML could be integrated with other subjects. |
| Monty, Alice, Denise | Real-life application problems should be used | With the exception of one real-life example in one of Alice's classes, only content was taught. |
| Monty, Denise | ML is a unique subject | Lessons were typical Mathematics lessons |
| Denise | Learners learn by explaining the work to each other in small groups | She used their solutions to discuss the work with them. |
| Denise | ML teaching differs from Mathematics teaching | Content was lectured and if there was enough time, discussions took place. |

Such contradictions were also evident in Sidiropolous' (2008) study where both teachers believed *ML is a maths only better than nothing* and even *the maths of oranges and bananas* (p. 225) while, in later interviews, they complained about the difficulty level of the subject. They also believed that a teacher could work at a much slower pace, but, during the interviews, they complained about not having enough time to teach the way they knew they were supposed to teach ML. A possible reason for this contradiction is that the theory they espoused was difficult to carry out in practice.

To summarise: The following are some issues that need attention by ML teachers in their instructional practices.

1. Knowledge of the pedagogic approach in which teaching content is integrated with contexts.
2. Knowledge of varied, applicable instructional strategies and material.
3. Selection of tasks according to the ML Assessment Taxonomy.
4. Different types and level of oral questioning in the classroom.
5. Increasing learner participation by giving them the opportunity to verbalise their thinking.
6. How to facilitate learner-learner interactions.
7. Establishing and maintaining a positive learning atmosphere.
8. Having a positive attitude towards the subject and the learners.
9. Choice and efficient use of appropriate instructional strategies.

To ensure proficient ML teaching, it seems that teachers require the following: ML teacher training; a certain level of MCK (it was not the purpose of the study to determine the required level of MCK); experience; and a positive attitude towards and the desire to change their instructional practices.

## 4.8 Conclusion

In this chapter, I discussed the data collection process that took place in Pretoria during the second quarter of 2011. I initially had five participants, but, during the data analysis process, I realised that the one case did not add value to my study, and so I decided to continue only with the other four cases. Data were collected by means of three lesson observations per teacher with interviews conducted prior to the second and third observations and a third and last in-depth interview conducted after the observations based on the observed lessons as well as their knowledge and beliefs. I adopted a deductive approach to coding the data as I had identified two themes: ML teachers' instructional practices and ML teachers' knowledge and beliefs prior to the data collection stage. Different categories for each theme were chosen according to the work of Artzt et al. (2009), Ball (1990), Borko and Putnam (1996), Hill et al. (2008), Shulman (1986) and Shulman (1987) apropos of which the raw data were analysed. In this chapter I also presented the data of the four participants and discussed my findings on the basis of a literature control. I finally identified trends and possible explanations for the trends.

In the next chapter, the research questions are answered, and I reflect on my research study and draw conclusions from the case study. I also discuss the limitations and significance of the study and make recommendations for further research.

# Chapter 5

# Conclusions and implications

## 5.1 Introduction

In this chapter I provide a summary of the previous four chapters, answer the research questions that guided this study and reflect on my research as to what I would have done differently and make provision for the fact that I may have been wrong in my interpretation of the teachers' instructional practices and knowledge and beliefs. This is followed by the conclusions, recommendations and limitations of the study. A final reflection is done on the research study.

## 5.2 Chapter summary

In Chapter 1 I introduced and contextualised the research study. The purpose of this study was to investigate, by means of a case study, the way in which ML is taught with the view to determining the relationship between ML teachers' knowledge and beliefs and their instructional practices. The different meanings attached to mathematical literacy both internationally and nationally have been discussed. I discussed the problem and the rationale for the study, formulated the research questions, and discussed the methodological considerations and the possible contribution and limitations of the study.

Chapter 2 presented an in-depth analysis of the findings in the relevant literature as well as the conceptual framework on which the study is based. Comparisons were made between the different conceptions of mathematical literacy internationally and nationally, followed by a discussion on ML as a compulsory alternative to Mathematics in Grades 10 to 12 in South Africa. Following this was a discussion of the meaning of teachers' instructional practices and the value of various approaches to teaching. Attention was given to the different domains of teachers' knowledge, teachers' belief systems and the relationship between their knowledge and beliefs and their instructional practices. The conceptual framework, which is based on concepts and theories from relevant work in the literature, was then discussed.

A description of the qualitative methodology used in this study was reported in Chapter 3. I discussed social constructivism as my research paradigm, and the nature of my study as subjective and interpretive. This is an exploratory case study. Observations were used to examine teachers' instructional practices and to study demonstrations of their MCK and knowledge regarding the ML learners, the teaching of ML and the ML curriculum. Interviews were used to determine why teachers do what they do in class and to determine how they apply their PCK during their instructional practice. ATLAS.ti 6 was used to analyse the video and audio data. I lastly discussed the trustworthiness of the study and ethical considerations that were taken into consideration.

In Chapter 4 I briefly reported on the data collection process, presented and discussed the findings and lastly identified trends and possible explanations for those trends. A DEDUCTIVE-inductive (uppercase denotes the preference given to the style of analysis) approach to coding the data was used as I identified two themes: ML teachers' instructional practices and ML teachers' knowledge and beliefs prior to the data collection stage. After this deductive phase of analysis, inductive analysis was done when I studied the organised data in order to explore new patterns and trends. I presented the findings from the data obtained through class observations and interviews according to the different categories provided in Table 4.2 and Table 4.3. The findings were then related to the findings in the literature and trends were identified and subsequently explained.

## 5.3 Verification of research questions

Based on the rationale that ML is a significant subject which may positively influence the lives of many learners, and the problem that many teachers have negative views and experiences of the subject, I decided to explore the relationship between ML teachers' instructional practices and their knowledge and beliefs. In order to do so, the following research main question was formulated: What is the relationship between ML teachers' knowledge and beliefs and their instructional practices? To address this main question, the following five subquestions guided the enquiry:

1. How can ML teachers' instructional practices be described?
2. What is the nature of ML teachers' knowledge and beliefs?
3. How do ML teachers' knowledge and beliefs relate to their instructional practices?
4. What are the possible implications of the findings from Questions 1, 2 and 3 for teacher training?
5. What is the value of the study's findings for theory building in teaching and learning ML?

I will subsequently utilize social constructivism as research paradigm (the epistemological approach which guides my own teaching and learning practice and orientation in mathematics) to verify these questions. In short: the social constructivist holds that all knowledge is constructed and based upon not only prior knowledge, but also the cultural and social context (Ollerton, 2009).

The following table (Table 5.1) regarding the four participants' experience, teacher training, instructional approach, productivity of instructional practice, MCK, PCK and beliefs was prepared to facilitate the discussion on the verification of the research questions.

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**Table 5.1: Summary of participants' information**

| | | Monty | Alice | Denise | Elaine |
|---|---|---|---|---|---|
| **Keys used in the table:**<br>MCK en PCK: Sufficient knowledge: ✓ and Insufficient knowledge: ✗<br>Beliefs versus practice: Corresponds: ✓ and Contradicts: ✗<br>Paragraph numbers in the thesis are indicated in brackets. | | | | | |
| **Experience (years)** | | ✗ (3) | ✗ (2) | ✓ (11) | ✓ (11) |
| **Maths teacher training** | | ✓ | ✗ | ✓ | ✓ |
| • Approach used<br>• Productivity of practice | | • Teacher-centred<br>• Somewhat unproductive | • Teacher-centred<br>• Unproductive | • Teacher- and learner-centred<br>• Somewhat productive | • Learner-centred<br>• Productive |
| **MCK** | | ✓ (4.6.1.1) | ✗ (4.6.2.1) | ✓ (4.6.3.1) | ✓ (4.6.4.1) |
| **PCK** | **Learners**<br>**Teaching**<br>**Curriculum** | ✗ (4.6.1.2)<br>✗ (4.6.1.3)<br>✗ (4.6.1.4) | ✗ (4.6.2.2)<br>✗ (4.6.2.3)<br>✗ (4.6.2.4) | ✓ (4.6.3.2)<br>✓ (4.6.3.3)<br>✗ (4.6.3.4) | ✓ (4.6.4.2)<br>✓ (4.6.4.3)<br>✓ (4.6.4.4) |
| **Beliefs versus practice (Only imperative aspects)** | **Learners** | ✓<br>Learn by teacher's examples [4.5.1.3 (LESP1) & 4.6.1.2] | ✓<br>Learn by looking at teacher [4.5.2.3 (LESP1) & 4.6.2.2] | ✓<br>Learn by demonstrating work and teacher building on that [4.5.3.3 (LESP1) & 4.6.3.2] | ✓<br>Learn by discovery and discussions [4.5.4.3 (LESP1) & 4.6.4.2] |
| | **Teaching** | ✗<br>ML teaching is different to Mathematics teaching [4.5.1.1 (TMS3) & 4.6.1.3] | ✓<br>ML is the same as Mathematics teaching [4.5.2.1 (TMS3) & 4.6.2.3] | ✗<br>ML teaching is different to Mathematics teaching [4.5.3.1 (TMS3) & 4.6.3.3] | ✓<br>ML teaching is different to Mathematics teaching [4.5.4.1 (TMS3) & 4.6.4.3] |
| | **Curriculum** | ✗<br>Constructivist perspective; Content in context; ML has application value (4.5.1.3; 4.5.1.1 & 4.6.1.4) | ✗<br>Constructivist perspective; Content in context; ML has application value (4.5.2.3; 4.5.2.1 & 4.6.2.4) | ✗<br>Constructivist perspective; Content in context; ML has application value (4.5.3.3; 4.5.3.1 & 4.6.3.4) | ✓<br>Constructivist perspective; Content in context; ML has application value (4.5.4.3; 4.5.4.1 & 4.6.4.4) |

### 5.3.1 Question 1: How can ML teachers' instructional practices be described?

I used an adapted version of the theoretical framework provided by Artzt et al. (2008) on teachers' instructional practices as well as Franke et al.s' (2007) definition of a productive practice to contextualise and interpret my results. To answer this question, the participants' instructional practices were described according to the lesson dimensions as indicated in this study's conceptual framework, but, to avoid repetition in the thesis, a detailed description is provided in Appendix J.

#### The two novice teachers in my study

Monty and Alice's instructional practices can be described as teacher-centred in that they believed their role as teachers was to transmit mathematical content, demonstrate procedures for solving problems and explain the process of solving sample problems. However, Artzt et al. (2008) suggests that this approach is not ideal as the teacher-centred approach can serve as a mask for teachers who do not fully understand the content, the learners or the pedagogy, as was found in the practices of these two teachers in my study.

Monty and Alice's instructional practices can also not be described as productive (Franke et al. (2007) consider a productive practice as a practice where the teacher creates ongoing opportunities for learning). Alice's instructional practice was less productive than Monty's. In fact, her practice was largely dysfunctional as it was characterised by inattentive learners and ineffective teaching. She also failed to connect the learners' prior knowledge with new mathematical situations. Given that both Monty and Alice were novice teachers, an explanation of the differences between their instructional practices could be that Alice had no formal mathematics education training while Monty had completed a BEd with Mathematics and Methodology of Mathematics as major subjects.

#### The two experienced teachers in my study

Denise's instructional practice can be described as a combination of teacher- and learner-centred, leaning more towards learner-centred, while Elaine's instructional practice can be characterised as learner-centred. Denise and Elaine believed that learners should develop both procedural and a conceptual understanding of the mathematical content. A learner-centred approach to teaching requires the teacher to create opportunities for learners to achieve understanding through active engagement with each other and the problem-solving process (Artzt, et al., 2008). What appears to have made Elaine's practice more productive than Denise's was Elaine's use of contexts to explore the mathematical content; her pointing out the value of mathematics in everyday-life situations; her selection of tasks from Levels 1-4 of the ML Assessment Taxonomy; her allowing her learners to explain their answers; and her asking various types and different levels of oral questions.

## Comparison of the participants' instructional practices

The key differences between the two novice teachers and the two experienced teachers are listed in Table 5.2 below. I found that the instructional practices of the four teachers in my study could be described as predominantly teacher-centred: the practices of two of the four teachers were exclusively teacher-centred; one teacher's practice could be described as a combination of teacher- and learner-centred, leaning more towards learner-centred; and the fourth teacher's practice could be described as exclusively learner-centred.

**Table 5.2: Comparison of the participants' instructional practices**

| | **Two novice teachers (Monty and Alice)** | **Two experienced teachers** | |
|---|---|---|---|
| | | **Denise** | **Elaine** |
| **Approach** | Teacher-centred | Teacher- and learner -centred | Learner-centred |
| **Tasks** | • Content only<br>• On Level 1 only | • Content only<br>• On Level 1 only | • Content in contexts<br>• On Levels 1-4 |
| | | Build lessons on learners' prior knowledge | |
| **Discourse** | • Learners did not express their thinking<br>• No scaffolding<br>• Did not recognise learners' typical misunderstandings | Learners demonstrated their thinking | Learners demonstrated and explained their thinking |
| | | • Provided scaffolding<br>• Recognised learners' typical misunderstandings | |
| **Learning environment** | • Formal atmosphere<br>• Direct instruction<br>• Not enough logical flow<br>• Minimum learner participation | • Discussions<br>• Logical flow in lessons<br>• Positive classroom atmosphere<br>• Maximum learner participation | |

However, certain common trends in all four cases were also observed the most significant of which was their collective failure to encourage learner-learner interaction. Again, this is far from ideal: The DoE (2003a) clearly states that learners need to develop the ability to communicate mathematically and that teachers should create opportunities for classroom dialogue where learners can listen to, respond to and question each other so that they can discard or revise their own ideas (Artzt et al., 2008).

In summary: My study seems to provide evidence that confirms the existence of a relationship between the teaching approach used in ML classrooms and the productivity of the teacher's instructional practice. The four practices observed in my study ranged from Elaine's learner-centred approach, yielding a productive instructional practice, to Alice's teacher-centred approach, yielding an unproductive instructional practice.

According to my findings, it seems that teaching experience as well as mathematics teacher training (Table 5.1) may play a significant role in the productivity of the instructional practices of the four participants: both Denise and Elaine (with 11 years' experience of either teaching Mathematics or ML) had productive practices; and, comparing the practices of the two novice teachers (with three years and less of teaching experience), the teacher with mathematics teacher training had a more productive instructional practice than the teacher without teacher training.

### 5.3.2 Question 2: What is the nature of ML teachers' knowledge and beliefs?

As stated in Question 1, the participants' PCK and beliefs were described according to the study's conceptual framework, but, to avoid repetition in the thesis, a detailed description is provided in Appendix K.

#### Teachers' level of MCK

The purpose of this study was not to assess ML teachers' content knowledge but rather to comment on the accuracy of their mathematical content and the occurrence of their misconceptions. Against this background, I found that the three teachers who had prior teacher training in mathematics appeared to have sufficient MCK regarding the topics they were teaching at the time of the observations as no mathematical errors (except for the two minor omissions and single mistake of one teacher) were made or misconceptions observed. The only teacher with no mathematics teacher training was guilty of several mathematical errors, and some of the mathematical content she taught indicated her own misconceptions regarding the content. All four teachers believed MCK was a prerequisite to teach ML.

#### Teachers' level of PCK

Regarding the nature of the ML teachers' PCK, one of the experienced teachers (11 years' experience) illustrated sufficient knowledge of all three domains of PCK: the ML learners, the teaching of ML and the ML curriculum. The other experienced teacher (also 11 years' experience) had sufficient knowledge of two of the three domains of PCK: the ML learners and the teaching of ML. The other two teachers had only superficial knowledge of all three domains. My finding that the two experienced teachers had developed PCK confirms the findings of Ball (1988), Ball et al. (2005), Koellner et al. (2007), Ma (1999), Shulman (1986) and Sowder, (2007) that PCK can be developed only over time through experience in the classroom and that it cannot be taught. Some researchers (Ball, 1990; Van Driel, Verloop & de Vos, 1998) also believe that solid understanding and knowledge of mathematical subject matter are prerequisites for developing PCK. I also found that the two teachers who had developed a certain level of PCK also had adequate MCK. However, Monty's instructional practice indicates that sufficient MCK (teacher training) does not guarantee PCK (Table 5.1).

## Teachers' beliefs

All four teachers claimed that they had a constructivist perspective on mathematics as a discipline; that ML involves the teaching of mathematics in context; that learners should realise the application value of mathematics; and that learner-learner interaction in the form of group work and discussions is required for learners to develop understanding of mathematics. The main differences between the two novices' and the two experienced teachers' beliefs were in their beliefs on how learners learn: The novice teachers believed that learners learn through information received from the teacher while the experienced teachers believed that learners should be active participants in their own learning. Only one teacher in my study, Alice, believed that ML is similar to Mathematics but on a lower level while the other three teachers believed that ML is a unique subject in its own right.

To summarise (see Table 5.1): Both teachers with sufficient PCK also had sufficient MCK, which suggests that MCK is required to develop PCK. Furthermore, the three teachers with mathematics teacher training (one novice teacher and two experienced teachers) had sufficient MCK, but since the novice teacher still lacked PCK, it could be suggested that although MCK is required to develop PCK, it is teaching experience that plays a crucial role in the development of teachers' PCK. These findings will hopefully contribute to this new field and fill the gap in literature regarding ML teachers' knowledge and beliefs.

Based on the interviews with the four participants in my study, it seems that mathematics teacher training is required to enhance teachers' MCK and that, although MCK is required to develop PCK, it is through teaching experience that teachers develop PCK.

### 5.3.3 Question 3: How do ML teachers' knowledge and beliefs relate to their instructional practices?

#### Teachers' knowledge

Alice was the only teacher in my study who had insufficient MCK, and, because only Alice's instructional practice was described as unproductive, it seems that ML teachers' level of MCK strongly influences the productivity of their practices (Table 5.1). This finding supports Kilpatrick's (2001) view that proficient teaching demands, among other things, teachers' conceptual understanding and procedural fluency. The two novice teachers in my study had insufficient PCK and unproductive instructional practices in contrast to the two experienced teachers who had sufficient PCK and productive instructional practices. This finding suggests that PCK influences the productivity of teachers' practices. Since PCK influences teachers' teaching approach, which, in turn, influences the productivity of teachers' practices, it can be deduced that PCK influences ML teachers' practices.

I realise that other factors also play an important role in the productivity of teachers' instructional practices, but it seems as if MCK and PCK have a definite influence on such practices.

#### Teachers' beliefs

Trends were found in the correspondences and contradictions between the teachers' stated beliefs and their instructional practices. A common belief expressed by all four teachers was how learners learn. The following important contradictions were noted between three of the four ML teachers' stated beliefs about the nature of mathematics and the teaching of ML and their instructional practices. All four teachers claimed that they taught mathematics from a constructivist perspective, but, in practice, it was only Elaine who created opportunities for the learners to discover, experiment and reason in order to achieve understanding. Monty and Alice's perspective was traditional while Denise's was formalist (Dionne, 1984).

- All four teachers believed that ML involved the teaching of mathematics in context and that learners should realise the application value of mathematics, but only Elaine used relevant contexts to enable the learners to explore the mathematical content and to appreciate the value of mathematics.
- All four teachers believed that learner-learner interaction, such as learners explaining their work to each other in small groups, was important in providing opportunities for learners to develop conceptual understanding of mathematics. However, it was only in Elaine's instructional practice that some evidence of learner-learner interaction was observed.

Contradictions between three of the four teachers' stated beliefs and their instructional practices were found in the practices of Monty and Alice, who used a teacher-centred approach, and Denise who used a combination of teacher- and learner-centred approaches. A reason for these inconsistencies could be that the teachers had heard about a constructivist perspective on mathematics or knew how ML should be taught, but their existing cognitive structures were not ready to accommodate the required changes (Artzt et al., 2008).

According to the literature, teachers' knowledge and true beliefs strongly influence their practices (Artzt et al., 2008; Ball, 1990; Liljedahl, 2008; Pajares, 1992). It was only Elaine in my study who adopted a learner-centred approach and whose knowledge and stated beliefs corresponded with her instructional practice. Not only did her knowledge and beliefs influence her instructional practice positively, but conversely her instructional practice (experiences with the subject and its learners) also positively influenced her knowledge and beliefs regarding the ML learners and the teaching of the subject. This was evident from my last interview with Elaine during which she told me that when she had been asked two years ago to be the coordinator for ML, she had initially felt that she was being demoted, but she claimed that ML had grown on her since then. She enjoyed being involved in ML and never wanted to return to Mathematics.

In summary: Based on the findings of this study, it can be tentatively assumed that, except for the one teacher who used a learner-centred approach, the teachers' stated beliefs about teaching ML and the ML curriculum did not influence their instructional practices whereas knowledge had a strong influence. It is possible that the stated beliefs did not reflect the true beliefs of the teachers in this study. In the case where a learner-centred approach was used, not only did the teacher's knowledge and beliefs influence her practice, but her practice also influenced her knowledge and beliefs. These findings will hopefully also contribute to this new field, also filling the gap in literature regarding the relationship between ML teachers' knowledge and beliefs and their instructional practices.

### 5.3.4 Question 4: What are the possible implications of the findings from Questions 1, 2 and 3 for teacher training?

Effective and purposeful training of pre- and in-service ML teachers is of critical importance in South Africa, a finding that was also reported by, among others, Bansilal (2008) and Sidiropolous (2008). The instructional practice of Alice (the only teacher in my study without mathematics teacher training) proved to be unproductive resulting in discouraged and uninvolved learners. Knowledgeable, competent and dedicated ML teachers such as Elaine in my study and the findings from other studies

(Graven & Venkat, 2009; Hechter, 2011a; Venkat & Graven, 2008) reveal that the aims and purposes of ML are realistic and achievable. My findings also indicate that MCK alone is insufficient; ML teachers need knowledge regarding the teaching and learning of ML to teach the subject effectively.

## Teaching ML

Key factors that should be part of all ML instructional practices:

- Having a learner-centred approach and using appropriate instructional strategies.
- Engaging with contexts rather than applying mathematics already learned to contexts.
- Selecting tasks at all four levels of the ML Assessment Taxonomy − the emphasis in Levels 1 and 2 is on routine calculations while the key aims of ML are located primarily in Levels 3 and 4.
- Engaging learners in discussions thereby enabling them to communicate their thinking through the use of appropriate terminology.
- Using various instructional resources to connect learners' knowledge with new situations.
- Teachers having sufficient general knowledge of the contexts in which the lesson is situated, enabling them to engage the learners in meaningful mathematical discourse.

These key factors were also confirmed by Artzt et al. (2008), DoE (2003a, 2011), Graven and Venkat (2007) and Venkat et al. (2009).

Other didactical and methodological issues that were identified during this study and should be addressed include logical sequencing of tasks; efficient oral questioning; managing discipline in ML classrooms; creating a positive learning atmosphere in class; effective board work; valuing teachers who are enthusiastic about ML and its learners; engender an understanding of the theory and practice of PCK; enhancing teachers' curriculum knowledge and their knowledge of how ML is integrated with other subjects.

## ML teachers' knowledge and beliefs

Similar to the training of Mathematics teachers, the focus of ML teacher training should be the development of sufficient MCK to enhance conceptual understanding of various mathematical topics. ML teacher training should however differ from Mathematics teacher training regarding the following aspects: The level of the mathematical content in the ML programme need not be on a second-year BSc level, but should include (apart from a generic component being offered to both Mathematics and ML student teachers) specialised training regarding the teaching and learning of ML. In particular, teachers should learn how to structure mathematical content to enable learners to progress in their cumulative understanding of the content and to link learners' prior knowledge with new content. The theory and practice of PCK should be incorporated in all student teacher training programmes during teaching practice and internship. Only a few tertiary institutions have to date developed such programmes − the teachers who participated in this study, for example, had no such exposure.

ML teachers' instructional practices were strongly positively influenced by their knowledge and not their stated beliefs. The distinction between what teachers say they know and believe and is played out in their classrooms emerged as a pivotal aspect of teaching and learning success. Addressing this distinction during teacher training seems to be important.

In summary: ML teacher training is specialised and implies that teachers should be equipped with MCK and skills to facilitate the learning process. ML teacher training has to a large extent been neglected as the subject was introduced in 2008 yet it was only in 2011 that a ML teacher training programme was introduced at the University of Pretoria. This programme consists of a three-year mathematics content component and a methodology of ML component in the fourth year. It is furthermore recommended that all practising ML teachers should be required to complete an ACE (ML) programme, which is in line with the DoE's (2009) requirement that all intermediate phase teachers should have completed a mathematics course by 2014.

### 5.3.5 Question 5: What is the value of the study's findings for theory building in teaching and learning ML?

Flowing from the finding that ML teachers' knowledge, but not necessarily their beliefs, influences their instructional practices, the following matters warrant the attention of curriculum decision-makers:

- My research revealed that teachers who had formal training or experience in the teaching of mathematics in classrooms also displayed evidence of having adequate MCK. In addition, adequate MCK impacted positively on teaching and learner understanding.
- The ML teachers who had productive instructional practices had sufficient knowledge of both mathematical content and the teaching and learning of ML.
- Competent, dedicated teachers who value the ML curriculum are needed to teach ML. New student-teachers should be recruited to become ML teachers so that they can develop a new status identity.

**To answer my main question:** There is a dynamic but complex relationship between ML teachers' knowledge and beliefs and their instructional practices. Firstly, their knowledge, but not their stated beliefs were reflected in their practices. Secondly, some of the teachers' classroom practices belied their stated beliefs. Conversely, in one case, the teacher's practice also had a positive influence on her knowledge and beliefs.

### 5.3.6 Summary of verification of research questions

In Table 5.3 below I provide a summary of the research questions, data collection techniques used, objectives of the questions and research findings.

**Table 5.3: Summary of verification of research questions**

| Research questions (Data collection techniques) | Objectives of the questions | Research findings |
|---|---|---|
| 1. How can ML teachers' instructional practices be described? (Observations) | • To determine what teachers do in their classrooms with respect to: tasks given, discourse that takes place and the learning environment which is established.<br>• To describe teachers' practices according to the teaching approach used and level of productivity of their practices. | Predominantly teacher-centred: two teachers used a teacher-centred, one teacher a combination of teacher- and learner-centred and one teacher a learner-centred approach. In all practices there is a positive relationship between the approach used and the level of productivity, ranging from a productive practice in which a learner-centred approach was used to an unproductive practice where a teacher-centred approach was used. |
| 2. What is the nature of ML teachers' knowledge and beliefs? (Observations & interviews) | • To comment on the teachers' level of MCK.<br>• To explore teachers' PCK and beliefs regarding ML learners, the teaching of ML and the ML curriculum. | The two more experienced ex-Mathematics teachers proved to have sufficient knowledge while the two novice teachers still lack PCK. Mathematics teacher training is required to enhance teachers' MCK and although MCK is required to develop PCK, it is through teaching experience that teachers develop PCK. |
| 3. How do ML teachers' knowledge and beliefs relate to their instructional practices? (Observations & interviews) | • To explore what the relationship is between teachers' instructional practices and their knowledge and beliefs.<br>• To investigate how teachers use PCK in their lessons. | Teachers' beliefs did not influence their instructional practices, but knowledge strongly influences teachers' instructional practices. In the case where a learner-centred approach was used, not only did the teacher's knowledge and beliefs influence her practice, but her practice also influenced her knowledge and beliefs. |
| 4. What are the possible implications of the findings from Questions 1, 2 and 3 for teacher training? (Observations & interviews) | • To improve my own practice.<br>• To inform current teacher training and development programmes. | ML teacher training is specialised and implies that teachers should be equipped with specific MCK; skills to integrate content and context in their teaching in order to facilitate the learning process; and knowledge of the ML curriculum. An understanding of the theory and practice of PCK and its importance should be engendered in all student teachers training. |

| | | |
|---|---|---|
| 5. What is the value of the study's findings for theory building in teaching and learning ML? (Observations & interviews) | • To add to the body of knowledge regarding the relatively newly introduced subject and to make suggestions to the curriculum stakeholders. | ML teachers' instructional practices are strongly positively influenced by their knowledge, but not their stated beliefs. The distinction between what teachers say they know and believe and is played out in their classrooms emerged as a pivotal aspect of teaching and learning success. Addressing this distinction during teacher training seems to be important. |

## 5.4 What would I have done differently?

During the data presentation stage, I realised that I had missed valuable communications between the teacher and the learners at their desks as I did not want to intrude by moving around in class with a video camera. More information regarding the teachers' PCK would possibly have emerged from this discourse. With the insight of hindsight, I would have employed a research assistant to videotape all my sessions for careful perusal and analysis.

In discussing my findings another aspect I wished I could have done differently was to include ML teachers from other non-mathematics disciplines. Alice's practice already informed this study about teachers who were teaching without having teacher training, but it would have been valuable to investigate the practices of teachers who did have teacher training but no formal mathematics training.

## 5.5 Providing for errors in my conclusion

I engaged with five (although only four actually participated in the research) ML teachers who allowed me in their classrooms and also shared some of their knowledge and beliefs with me. I have made some decisions on their instructional practices and the nature of their knowledge and beliefs, and I have to accept the fact that I may have been wrong in some of my conclusions, albeit unknowingly and unintentionally. I attempted to enhance the credibility and trustworthiness of my study through triangulation by using three observations and three interviews at different stages of the data collection process. Since there is no agreement among academics on how knowledge and beliefs are to be evaluated, I acquired the services of a peer researcher to assist me with the coding and interpretation of the data to further enhance the trustworthiness of my study. I also verified my findings with findings from the literature.

To reduce the Hawthorne effect the first observation was done without a prior interview or discussion because the interview questions prior to the second and third observations could influence teachers' behaviour in the classroom. I emphasised to the teachers the fact that I was interested in the uniqueness of each teacher and my purpose was not to report their performances in class to their superiors. I furthermore used the same interview schedules, including the same questions in the same sequence for all interviewees. Section C of the last interview was based on the teachers' knowledge of the ML curriculum. I gave the teachers the option to answer the questions orally or in writing. I mentioned the advantage of providing the answers in writing: that they might have felt less threatened or pressured and that it also allowed them more time to think about the questions and to provide valuable responses. In choosing to provide the answers in writing, they were requested to complete the section in my presence as part of the interview. This was to ensure that the data obtained were credible, as the teachers were not able to consult another teacher or the relevant documents.

## 5.6 Conclusions

Some conclusions regarding the relationship between ML teachers' knowledge and beliefs and their instructional practices appear below.

### ML teachers' instructional practices:

- Given that the use of contexts should be the focus of ML lessons, the puzzling absence of the use of contexts in three of the four teachers' practices was notable and should be addressed. Not just learners but teachers too need to understand the contexts in order to have informative and enlightening class discussions.
- ML teachers' instructional practices should be predominantly learner-centred, including the use of active learning instructional strategies such as cooperative learning and discussions.
- The following aspects of ML lessons in particular deserve to be emphasized, not only during teacher training but especially in post-teacher training: tasks should be logically sequenced; tasks should not be too easy or too difficult; learners' understanding should be monitored; learner-learner interactions should be optimized; there should be variety in levels and types of oral questioning during instruction; learners' ideas and ways of thinking should be acknowledged and appreciated consistently; and a positive learning atmosphere should be instilled.

### Knowledge and beliefs:

- ML teachers need to attain a certain level of MCK as well as knowledge of the teaching and learning of ML before being allowed to teach the subject.

- ML teachers need to attain an adequate sense of procedural knowledge and conceptual understanding of the ML learners, the teaching of ML and the ML curriculum as well as experience to develop PCK.

**The relationship between ML teachers' knowledge and beliefs and their instructional practices:**

- ML student teachers as well as in-service ML teachers should be afforded ample opportunity to enhance their MCK but also to engender an understanding of the theory and practice of PCK as knowledge strongly influenced the ML teachers' instructional practices.
- ML student teachers (during initial teacher training) as well as practising ML teachers (during in-service training) should be sensitized to the importance of ensuring that their instructional practices are consistent with their true beliefs and not their stated beliefs only.
- Instead of merely claiming to be teaching in a constructivist manner, ML teachers should be taught how to actually teach in a constructivist manner.

## 5.7 Recommendations for further research

Several aspects of the teaching and learning of ML require further research in order for ML to come into its own as a viable and valuable subject in its own right. These include investigation into:

- The knowledge required to engage learners in such a manner as to explore the depths of their prior knowledge during teaching.
- The nature and level of content knowledge that is required to teach ML effectively.
- The ways in which ML teachers can transform their own MCK into learning facilitation strategies that are pedagogically powerful, through the choice of appropriate teaching and learning strategies and supporting materials.
- Identification of authentic and relevant contexts that not only relate to learners' daily lives, their future workplace and the wider social, political and global environment, but how such contexts can be applied effectively to the required lesson content.
- The viability of developing PCK during teacher training.
- ML teachers' true and stated beliefs and the influence thereof on their instructional practices.
- The development of effective questioning techniques and assessment strategies in the ML classroom.

- The guidance of ML teachers to becoming *au fait* with the extent to which language potentially influences ML learners' achievements and acquire the skills needed to deal with these issues in their classrooms adequately.
- ML learners' expectations and experiences of the subject.

## 5.8 Limitations of the study

Data were gathered from a very small number of ML teachers and generalization of the results is impossible. However, generalization was not an aim of the study. Another limitation is the fact that the observations were all done in the second part of Term 2 and two of the four teachers were busy with revision. Furthermore, more data regarding the teachers' knowledge of the learners could have been gathered during the observations if I had been party to the discourse between the teachers and individual learners sitting at their desks. As my presence in class already influenced the teaching process, I did not want to intrude furthermore on the learners' learning process. I am also acutely aware that different researchers may interpret my data differently. My own perspective is bound by space, time and personal experience. Even though my conclusions were carefully scrutinized and confirmed or refuted by my supervisors, my external coder as well as my participants, the possibility that subjectivity may have influenced my findings cannot be ruled out.

## 5.9 Last reflections

It is with mixed feelings that I am making this attempt to bring my 'doctoral journey' to its conclusion. I realize, among many other things that it has been a time of accelerated growth and learning for me, both professionally and personally. I began this journey with a strong assumption that ML has a rightful place in the school curriculum as a valuable subject and that this subject should be taught by a mathematics teacher. Furthermore, I assumed that the success of this relatively new subject depended strongly on the input, training, experience and perceptions of these (mathematics) teachers.

Initially I wanted to explore how ML teachers' PCK influence their instructional practices, but as my study developed, I realised that MCK and teachers' beliefs also play a crucial role in teachers' practices. This study allowed me the opportunity to become part of the lives of four ML teachers whom I have observed and listened to as they shared their knowledge and beliefs with me. However, even though I benefited a great deal from my exchanges with all four teachers, I wish to state that it was especially during the time I worked with Elaine that I gained insight into how a teacher's practice could influence her knowledge and belief. My interactions with her changed the focus of the study from exploring the influence of ML teachers' knowledge and beliefs on their practices to exploring the relationship between teachers' knowledge and beliefs and their practices.

During the data analysis stage I became aware of the complexity involved in analysing teachers' practices, knowledge and beliefs. I realized that the boundaries between these categorisations of knowledge are very vague. More particularly, it dawned on me that not all stated beliefs are true beliefs and that the boundaries between knowledge and beliefs are vague and blurred. I found the process of conducting literature control a most exhausting and emotionally draining exercise. This was mainly the case because of the paucity of research relating to my study, which made it extremely difficult to compare my study with other studies. The vast majority of studies that could be related to mine were conducted on very small samples –, in many cases involving one or two teachers only.

I hope that my findings will contribute to teacher training and theory and that this study will contribute to the building of a mathematically literate nation. Maree (2011) voices my thoughts also when he stated the following:

> *Since what happens in the classroom will eventually determine whether or not lasting change can be effected, the role of the school [teacher] is crucial in creating an optimal learning environment … Learners should leave school better equipped to cope with the challenges of university study and life itself. They need to be empowered to choose appropriate careers, enter society and make meaningful social contributions.*

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# Appendices

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## Appendix A: Letter of consent to the ML teachers

UNIVERSITEIT VAN PRETORIA
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**FACULTY OF EDUCATION**
Mrs. J.J. Botha
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19 April 2011

Dear Ms/Mr ………………..………………

**Letter of consent to the Mathematical Literacy teacher**

You are invited to participate in a research project aimed at investigating the influence of Mathematical Literacy teachers' knowledge and beliefs on their instructional practices. This research will be reported upon in my PhD thesis conducted at the University of Pretoria.

Your participation in this research project is voluntary and confidential. It is proposed that you form part of this study's data collection phase by being observed three times when teaching your Mathematical Literacy class(es) and being individually interviewed twice. The lessons will be video recorded and the interviews will be audio-taped by me in order to have a clear and accurate record of all the activities and communication that took place.

The process will be as follows: during the third term of this year I would like to observe you teaching three Grade 11 Mathematical Literacy lessons during school hours, preferably to different Mathematical Literacy classes. I would like to conduct a short interview with you prior to the second and third lessons and another interview at the end of the three observations. The duration of the interviews prior to the lessons will not be more than 20 minutes and can be conducted during break or a free period you have. The duration of the third and final interview will take a maximum of an hour and will be scheduled at a time convenient to you. The focus of the questions is your knowledge and beliefs regarding Mathematical Literacy as subject, the teaching thereof and the Mathematical Literacy learners. The interviews will be scheduled at a place convenient to you.

Should you declare yourself willing to participate in this study, confidentiality and anonymity will be guaranteed at all times. You may decide to withdraw at any stage should you not wish to continue with your participation. Your decision to accept/decline involvement in this research will not influence your teaching career in any way, nor will your participation be reflected in your performance appraisal.

If you are willing to participate in this study, please sign this letter as a declaration of your consent, i.e. that you participate in this project willingly and that you understand that you may withdraw from the research project at any time.

Yours sincerely

…………………………………………………………. Date: ………………………………

Researcher: Mrs. J.J. Botha

…………………………………………………………. Date: ………………………………

Co-supervisor: Dr. G. Stols

---

**I the undersigned, hereby grant consent to Mrs. J.J. Botha to observe my classes and conduct interviews with me for her PhD research.**

Participant's name ……………..…..………… Participant's signature ………………………Date: ………………

E-mail address ……………………………………………… Contact number ……………………………..

Appendix B: Letter of consent to the principals

**FACULTY OF EDUCATION**

Mrs. J.J. Botha
Natural Science Building 4-13
Groenkloof campus, UP
hanlie.botha@up.ac.za
Tel: 082 475 6096

19 April 2011

Dear Dr/Ms/Mr ………………..………………

**Letter of consent to the Principal**

I hereby request permission to use your school for my research project. I would like to invite a Mathematical Literacy teacher to participate in this research project aimed at investigating the influence of Mathematical Literacy teachers' knowledge and beliefs on their instructional practices. This research will be reported upon in my PhD thesis conducted at the University of Pretoria.

Your participation in this research project is voluntary and confidential. It is proposed that the teacher forms part of this study's data collection phase by being observed three times when teaching Mathematical Literacy class(es) and being individually interviewed three times. The lessons will be video recorded and the interviews will be audio-taped by me in order to have a clear and accurate record of all the activities and communication during the lesson.

The process will be as follows: during the third term of this year, should you look favourably upon my request, I would like to observe the teacher teaching three Grade 11 Mathematical Literacy lessons, preferably to different Mathematical Literacy classes during normal school hours. I would like to conduct a short interview with the teacher prior to the second and third lessons and another interview at the end of the three observations. The duration of the interviews prior to the lessons will not be more than 20 minutes and can be conducted during break or a free period the teacher has. The duration of the third and final interview will take a maximum of an hour and will be scheduled at a time convenient to the teacher. The focus of the questions is on the teachers' knowledge and beliefs regarding Mathematical Literacy as subject, the teaching thereof and the Mathematical Literacy learners. The interviews will be scheduled at a time and place convenient to the teacher.

Confidentiality and anonymity will be guaranteed at all times. Your decision to accept involvement in this research will hopefully contribute to the improvement of Mathematical Literacy teachers' practices. If you are willing to allow a member of your staff to participate in this study, please sign this letter as a declaration of your consent.

Yours sincerely

…………………………………………………………… Date: ………………………………

Researcher: Mrs. J.J. Botha

…………………………………………………………… Date: ………………………………

Co-supervisor: Dr. G. Stols

---

**I the undersigned, hereby grant consent to Mrs. J.J. Botha to conduct her research in this school for her PhD research.**

School principal's name ……………………………………..

School principal's signature............................................. Date:………………………………

E-mail address ……………………………………………… Contact number ……………………………..

UNIVERSITEIT VAN PRETORIA
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YUNIBESITHI YA PRETORIA

# Appendix C: Letter of permission to the department

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

**FACULTY OF EDUCATION**

Mrs JJ Botha
Aldoel Building C04
Groenkloof Campus
hanlie.botha@up.ac.za
Tel: 082 475 6096

15 March 2010

**GAUTENG DEPARTMENT OF EDUCATION**

Dear Sir/ Madam

**Request from GDE for permission to do classroom observations and to conduct interviews**

I am currently enrolled as a doctoral student at the University of Pretoria, where I am also a lecturer in the Department of Science, Mathematics and Technology Education. The title of my proposed thesis is as follows: **The influence of Mathematical Literacy teachers' knowledge, beliefs and attitudes on their instructional practices.** ML is a valuable subject and it is crucial to attain its purpose in our country by addressing problems experienced by both teachers and learners. My research concerns the ML teacher's role in the classroom situation. It is important to determine who the ML teachers are, what knowledge they have regarding the subject and what beliefs and attitudes they hold. Furthermore I want to explore and interpret the influence of those elements on these ML teachers' instructional practices. I hope, at the end of my research, to be able to make a contribution to the improvement of pre-service training in order to perk up ML teachers' instructional practices.

In order to collect data for this project, I would like to observe and interview a purposive sample of Mathematical Literacy teachers, preferably grade 11 teachers at approximately six schools in and around Tshwane. Each teacher will be observed three times and interviewed twice. My observations will be unobtrusive.

I therefore formally request your permission to observe and interview Mathematical Literacy teachers at schools in and around Tshwane in the second term of this year. I trust that my request will meet with a favourable response.

Yours faithfully

| | |
|---|---|
| …………………………………. | …………………………….. |
| Researcher: Mrs JJ Botha | Date |
| …………………………………. | ……………………………… |
| Supervisor: Dr G Stols | Date |

---

**I the undersigned, hereby grant consent to Mrs JJ Botha to conduct research for her PhD at schools in and around Tshwane.**

| | |
|---|---|
| ………………………………… | …………………………….. |
| Departmental officer | Date |

UNIVERSITEIT VAN PRETORIA
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YUNIBESITHI YA PRETORIA

# Appendix D: Ethical clearance certificate

UNIVERSITY OF PRETORIA

FACULTY OF EDUCATION

RESEARCH ETHICS COMMITTEE

| | |
|---|---|
| **CLEARANCE CERTIFICATE** | **CLEARANCE NUMBER :** SM 10/02/01 |
| **DEGREE AND PROJECT** | PhD |
| **INVESTIGATOR(S)** | Johanna Jacoba Botha |
| **DEPARTMENT** | Science, Mathematics and Technology Education |
| **DATE CONSIDERED** | 17 October 2011 |
| **DECISION OF THE COMMITTEE** | APPROVED |

Please note:
*For Masters applications, ethical clearance is valid for 2 years*
*For PhD applications, ethical clearnace is valid for 3 years.*

| | |
|---|---|
| **CHAIRPERSON OF ETHICS COMMITTEE** | Prof L Ebersohn |
| DATE | 17 October 2011 |
| CC | Jeannie Beukes<br>Prof. J.G. Maree<br>Dr G. Stols |

This ethical clearance certificate is issued subject to the following conditions:

1. A signed personal declaration of responsibility
2. If the research question changes significantly so as to alter the nature of the study, a new application for ethical clearance must be submitted
3. It remains the students' responsibility to ensure that all the necessary forms for informed consent are kept for future queries.

Please quote the clearance number in all enquiries.

**Appendix E: Observation sheet for observing ML teachers' lessons**

# OBSERVATION SHEET

(To be used for all three observations per teacher)

| | |
|---|---|
| Name of school | |
| Name of researcher | Mrs. J.J. Botha |
| Subject observed | Mathematical Literacy (ML) |
| Grade observed | |
| Number of learners in class list (present in class) | |
| Topic of the lesson | |
| Name of teacher | |
| Date of observation | |
| Observation number | |

*Table A and Table B are based on the different dimensions of teachers' lessons. Use the indicators in Table B to complete Table A.*

*Table C and Table D are based on the teachers' pedagogical content knowledge (PCK) and beliefs. Use the indicators in Table D to complete Table C.*

**Table A. ASSESSING TEACHERS' INSTRUCTIONAL PRACTICES THROUGH OBSERVATIONS**
(Videotape lesson and make field notes during observations)

| **LESSON DIMENSIONS** | **COMMENTS (Support with examples)** |
|---|---|
| | |
| Tasks | |
| Modes of representation | |
| Motivational strategies | |
| Sequencing/difficulty level | |
| Discourses | |
| Teacher-learner interactions | |
| Learner-learner interactions | |
| Questioning | |
| Learning environments | |
| Social/intellectual climate | |
| Modes of instruction/pacing | |
| Administrative routines | |
| Other | |
| Mathematical content knowledge | |
| Contextual knowledge | |

*Evaluation scale:* Table A: Description of the scale. 3 = commendable (strong presence of indicator); 2 = satisfactory (indicator is somewhat present); 1 = needs attention (there is very little presence of indicator); N/O = not observed or not applicable.

**Table B.** **EVALUATING TEACHERS' PCK AND BELIEFS THROUGH OBSERVATIONS AND INTERVIEWS**
(Videotape lesson and make field notes during observations; audio-tape the interviews)

| **TEACHERS' PCK AND BELIEFS** | **COMMENTS (Support with examples)** |
|---|---|
| | |
| PCK AND BELIEFS | |
| Mathematical content Knowledge | |
| Content and learners | |
| Content and teaching | |
| Curriculum | |
| BELIEFS | |
| Nature of mathematics | |

*Evaluation scale:* Table B: Description of the scale. 3 = commendable (strong presence of indicator); 2 = satisfactory (indicator is somewhat present); 1 = needs attention (there is very little presence of indicator); N/O = not observed or not applicable.

Appendix F: Interview schedule 1 (Prior to lessons 2 and 3)

# INTERVIEW SCHEDULE 1
## Semi-structured interview

**GENERAL INFORMATION**

| | |
|---|---|
| Name of school | |
| Name of researcher | Mrs. J.J. Botha |
| Name of teacher | |
| Date of interview | |
| Teacher's qualification | |
| Level of Mathematics education | |
| Number of years teaching Mathematics | |
| Number of years teaching ML | |
| Courses attended on teaching ML | |

Based on the lesson that you are about to present and your preparation for the lesson, please answer the following questions:

1. What is the topic of the lesson you are going to present?

2. a) What mathematical content do you predict the learners will understand?
   b) Why do you think they will comprehend this content?

3. a) What mathematical content do you predict the learners will not understand?
   b) Why do you think they will not understand this content?

4. a) Tell me about the context to which the mathematical content is applied in today's lesson.
   b) Is the context familiar or unfamiliar to the learners?
   c) If unfamiliar, how do you plan to make it comprehensible to the learners?

5. How did you plan to approach the lesson in order to bring the learners to understand the content and context?

6. a) Tell me about the task(s) you are going to give them.
   b) In which way, in your opinion, will the learners approach these task(s)?

7. What prior knowledge is needed by the learners to enable them to understand today's new work?

8. What alternative or preconceptions do you believe the learners could have that may serve as misconceptions?

**Appendix G: Interview schedule 2 (Final interview)**

# INTERVIEW SCHEDULE 2
## Open-ended and semi-structured interview

**GENERAL INFORMATION**

| | |
|---|---|
| Name of school | |
| Name of researcher | Mrs. J.J. Botha |
| Name of teacher | |
| Date of interview | |

This interview consists of three sections. The **first section** (Section A) is an open discussion based on the lessons presented and focuses on the teacher's demonstrated PCK and beliefs. The purpose is to give the teachers the opportunity to reflect on their lessons and to identify justification for their behaviour in the classroom. The **second section** (Section B) is a discussion according to a set of predetermined questions on the teacher's beliefs regarding the nature of mathematics as discipline, ML as subject, the ML learners, the teaching of ML and the curriculum. The **third section** (Section C) consists of questions regarding the NCS and CAPS and should be answered in writing.

## SECTION A
## Oral questions based on the observed lessons

Questions will be compiled once the observations have been done and will most probably vary from teacher to teacher. The questions will be based on incidents where PCK was identified during the lessons. Clips from the video recordings will be used as probes. Possible questions are the following:

1. Tell me about your positive experiences regarding
   a) the learners
   b) your teaching of the lesson
2. Tell me about your negative experiences regarding
   a) the learners
   b) your teaching of the lesson
3. I noticed that you used … (lecturing, group work, discussion etc.) in today's lesson.
   Why did you choose this teaching strategy for the lesson?

# SECTION B
## Oral questions based on the teacher's beliefs

**The nature and value of mathematics and ML:**

1. How do you view mathematics as discipline?
2. Complete the sentence: Mathematics is ……………
3. How do you view ML as subject?
4. What do you believe is the value of mathematics?
5. What do you believe is the value of ML?
6. What is your role as teacher in your ML classroom?

**ML learners:**

1. Describe your Grade 11 ML learners in terms of their
   a) mathematical abilities
   b) motivation
2. Give me a description/profile of your ML learners.
3. How can you improve your learners' appreciation of the subject ML?
4. How can you improve your learners' participation in the lesson?
5. What is your belief about the way learners proficiently learn new work?

**Teaching of ML:**

1. How do you feel about teaching ML?
2. Describe the ideal ML classroom in terms of
   a) instructional strategies used
   b) discourse
   c) learning environment
3. How does this ideal classroom compare with your own class?
4. What are your goals in teaching ML?
5. To what extent is mathematical content knowledge a prerequisite to teach ML?
6. a) Does the teaching approach of ML differ to that of Mathematics?
   b) If you experience a difference, tell me about your experiences in teaching Mathematics versus ML.

## SECTION C
## Written questions based on the teacher's knowledge regarding the curriculum

1. How does the Department of Education define Mathematical Literacy?

___________________________________________________________

___________________________________________________________

2. What is the purpose of Mathematical Literacy according to the Department of Education?

___________________________________________________________

___________________________________________________________

3. Which contexts does the Department of Education suggest you should use in teaching Mathematical Literacy? ___________________________________________

___________________________________________________________

4. Write down what you know about the new Curriculum Assessment Policy Statement (CAPS) for Mathematical Literacy. _______________________________________

___________________________________________________________

5. a) Which topic are you currently teaching? ______________________________

b) Name the instructional materials you use for your lessons on this topic.

___________________________________________________________

c) Comment on the availability and usefulness of the instructional materials. __________

___________________________________________________________

6. a) Which textbook(s) are you using? ___________________________________

b) Which other material do you use? ___________________________________

c) In your opinion, what are the strengths of these books and materials?

___________________________________________________________

d) In your opinion, what are the weaknesses of these books and materials?

___________________________________________________________

7. Are you aware of the curriculum content being studied by your learners in other subjects that integrate with Mathematics/ML? If yes, tell me about it.

___________________________________________________________

8. a) Which departmental documents exist that you know of?

______________________________________________

______________________________________________

b) Which of these departmental documents do you find useful and valuable? ______________

______________________________________________

9. What are the learning outcomes for Mathematical Literacy?

______________________________________________

______________________________________________

10. A list of concepts and content to be covered in grade 10, 11 or 12 are provided per Learning Outcome in the following table. Indicate in which grade the specific concept/content is introduced. *(Only complete the Learning Outcomes applicable to the observed lessons)*

**Table E: Concepts and content per learning outcome (DoE, 2003a; p. 38-42)**

| **CONCEPTS AND CONTENT PER LEARNING OUTCOME** | **Grade** | | |
|---|---|---|---|
| | **10** | **11** | **12** |
| ***LEARNING OUTCOME 1*** | | | |
| Cost price and selling price | | | |
| Complex formulae | | | |
| Currency fluctuations | | | |
| Direct proportion | | | |
| Financial and other indices | | | |
| Fractions, decimals, percentages | | | |
| Inverse proportion | | | |
| Positive exponents and roots | | | |
| Profit margins | | | |
| Rate | | | |
| Ratio | | | |
| Ratio and proportion | | | |
| Simple and compound growth | | | |
| Simple formulae | | | |
| Square roots and cube roots | | | |
| Scientific notation | | | |

| | | | |
|---|---|---|---|
| Taxation | | | |
| The associative, commutative and distributive laws | | | |
| ***LEARNING OUTCOME 2*** | | | |
| Cartesian co-ordinate system | | | |
| Compound growth | | | |
| Formulae depicting relationships between variables | | | |
| Graphs depicting the relationship between variables | | | |
| Graphs showing the fluctuations of indices over time | | | |
| Inverse proportion | | | |
| Linear functions | | | |
| Maximum and minimum points | | | |
| Simple linear programming (design and planning problems) | | | |
| Simple quadratic functions | | | |
| Solution to linear, quadratic and simple exponential equations | | | |
| Solution to two simultaneous linear equations | | | |
| Rates of change (speed, distance, time) | | | |
| Tables of values | | | |
| ***LEARNING OUTCOME 3*** | | | |
| Angles (0˚-360˚) | | | |
| Basic transformation geometry, symmetry and tessellations | | | |
| Circles | | | |
| Compass directions | | | |
| Conversion of measurements between different scales and systems | | | |
| Conversion of units within the metric system | | | |
| Floor plans | | | |
| Location and position on grids | | | |
| Measurement in 3D (angles included, 0˚-360˚) | | | |
| Measurement of length, distance, volume, area, perimeter | | | |
| Measurement of time (international time zones) | | | |
| Polygons commonly encountered (triangles, squares, rectangles that | | | |
| Properties of plane figures and solids in natural and cultural forms | | | |
| Scale drawings | | | |
| Scale models | | | |
| Sine rule, cosine rule, area rule | | | |

| | | | |
|---|---|---|---|
| Surface areas and volumes of right pyramids and right circular cones and spheres | | | |
| Surface area and volumes of right prisms and right circular cylinders | | | |
| Theorem of Pythagoras | | | |
| Trigonometric ratios: sin $x$, cos $x$, tan $x$ | | | |
| Views | | | |
| ***LEARNING OUTCOME 4*** | | | |
| Bivariate data | | | |
| Compound events | | | |
| Construction of questionnaires | | | |
| Contingency tables | | | |
| Cumulative frequencies | | | |
| Histograms | | | |
| Intuitively-placed lines of best fit | | | |
| Line and broken-line graphs | | | |
| Mean, median, mode | | | |
| Ogives (cumulative frequency graphs) | | | |
| Percentiles | | | |
| Pie charts | | | |
| Populations | | | |
| Probability | | | |
| Quartiles | | | |
| Relative frequency | | | |
| Scatter plots | | | |
| Selection of a sample | | | |
| Selection of samples and bias | | | |
| Single and compound bar graphs | | | |
| Standard deviation (interpretation only) | | | |
| Tables recording data | | | |
| Tally and frequency tables | | | |
| Tree diagrams | | | |
| Variance (interpretation only) | | | |

**END OF INTERVIEW**

## Appendix H: List of research studies for Literature Control

| AUTHOR AND YEAR | TITLE OF ARTICLE | PARTICIPANTS | SOURCE | APPLICABILITY | |
|---|---|---|---|---|---|
| | | | | IP | PCKB |
| Bansilal, S. (2008) | An exploration of teachers' difficulties with certain topics in Mathematical Literacy | In-service teachers in ACE (ML) programme | Proceedings of AMESA 2008 | Tasks: TSL | MCK; PCK Curr: C7 |
| Bansilal, S., Mkhwanazl, T. & Mahlaboratoryela, P. (2010) | Mathematical Literacy teachers' engagement with contexts related to personal finance | In-service teachers in ACE (ML) programme | Proceedings of SAARMSTE 2010 | Discourse: DTL | Curr: C7 |
| Bowie, L. (2009) | Critical issues in school mathematics and science: pathways to progress | Theory | Proceedings of an Academy of Science of South Africa Forum | N/A | |
| Fransman, J.S. (2011) | Exploring the practices of teachers in mathematical literacy training programmes in South Africa and Canada | In-service teachers in ACE (ML) programme | Unpublished dissertation for the degree Master of Education | Learning environment: LEC, LESP | MCK; PCK (T); Curr: C1, C3, C4, C5, C8 |
| Frith, V. (2009) | A framework for understanding the quantitative literacy demands of higher education | Theory | Journal: South African Journal of Higher Education | N/A | |
| Frith, V. (2010) | How to make every graph a straight line (or not!) | Mathematics | Learning and Teaching Mathematics | N/A | |
| Frith, V. (2011) | Towards understanding the quantitative literacy demands of a first year medical curriculum | University students | African Journal of Health Professions Education | N/A | |
| Geldenhuys, J., Kruger, C. & Moss, J. (2009) | Grade 10 learners' experience of Mathematical Literacy | 190 Grade 10 learners from three types of schools | Proceedings of SAARMSTE 2009 | Learning environment: LESP | Curr: C1, C7 |
| Glover, H. & King, L. (2009) | The subject knowledge levels of some Mathematical Literacy teachers | In-service teachers in ACE (ML) programme | Proceedings of SAARMSTE 2009 | | MCK |

| | | | | | |
|---|---|---|---|---|---|
| Govender, V.G. (2008) | Lessons learnt from the 2007 GMSA foundation Mathematical Literacy Olympiad | ML Olympiad | Proceedings of AMESA 2008 | N/A | |
| Govender, V.G. (2011) | An investigation into learners' approaches to solving problems in mathematical literacy | 12 Grade 12 learners from 1 school | Proceedings of AMESA 2011 | Learning environment: LESP | Curr: C7 |
| Govender, V.G. (2011) | University students' experiences of a mathematics service module: Numerical Skills for Nursing | University students | Proceedings of SAARMSTE 2011 | N/A | |
| Graven, M. & Venkat, H. (2009) | Mathematical Literacy | Theory | Book: Chapter 4 in Critical issues in mathematics education | Tasks: TMS; Learning environment: LEC; LESP | Curr: C7 |
| Graven, M. (2011) | Mathematical Literacy in South Africa: Increasing access and quality in learners' mathematical participation both in and beyond the classroom | Theory | Book: Chapter 35 in Mapping equity and quality in mathematics education | Do not have the book | |
| Graven, M. (2011) | Creating new mathematical stories: Exploring potential opportunities within Maths Clubs | Maths Clubs | Proceedings of AMESA 2011 | Discourse; Learning environment: LEC;LESP | |
| Hechter, J. (2011) | Analysing and understanding teacher development on a Mathematical Literacy ACE course | In-service teachers in ACE (ML) programme | Unpublished thesis for Master of Science | Tasks; Discourse: DTL; Learning environment | Beliefs |
| Hechter, J. (2011) | Case studies of teacher development on a mathematical literacy ACE course | In-service teachers in ACE (ML) programme | Proceedings of AMESA 2011 | Tasks: TMS; TSL; Discourse: DQ; Learning environment: LEC | Curr: C7 Question 4 |
| Mthethwa, T.M. (2009) | An analysis of Mathematical Literacy curriculum documents: cohesions, deviations and worries | Theory | Proceedings of AMESA 2009 | N/A | |

| | | | | | |
|---|---|---|---|---|---|
| Nel, B. (2011) | Investigating the transformation of teacher identity of participants in an Advanced Certificate in Education in Mathematical Literacy (Reskilling) programme at a South African University | In-service teachers in ACE (ML) programme | Proceedings from SAARMSTE 2011 | | Beliefs<br>Question 4 |
| North, M. (2008) | The great Mugg and Bean mystery | Theory | Journal: Learning and Teaching Mathematics | Learning environment: LESP | |
| North, M. (2008) | Progression in Mathematical Literacy | Theory | Proceedings of AMESA 2008 | | Curr: C7 |
| North, M. (2010) | How mathematically literate are the matriculants of 2008? | Grade 12 performance | Proceedings from AMESA 2010 | N/A | |
| Rughubar-Reddy, S. (2010) | Beyond Numeracy: Values in the Mathematical Literacy classroom | 5 Grade 10 learners from 1 school | Proceedings of SAARMSTE 2010 | Learning environment: LEC | |
| Sidiropoulos, H. (2008) | The implementation of a mandatory mathematics curriculum in South Africa: The case of mathematical literacy | Two Grade 10 ML teachers from 2 different schools | PhD thesis | Learning environment: LESP | Curr: C1,C7, C8<br>Beliefs<br>PCK (T/C)<br>Q.3 |
| Venkat, H. (2008) | Senior certificate examinations for mathematical literacy: findings from a small study | Grade 12 results | Journal: Learning and Teaching Mathematics | N/A | |
| Venkat, H. & Graven, M. 2008 | Opening up spaces for learning: Learners' perceptions of Mathematical Literacy in Grade 10 | All Grade 10 ML learners in 1 school | Journal: Education as Change | Tasks: TMS; Discourse: DLL; Learning environment: LEC,LESP,LEA | |
| Venkat, H., Graven, M., Lampen, E. & Nalube, P. (2009) | Critiquing the Mathematical Literacy assessment taxonomy: Where is the reasoning and the problem solving? | Theory | Journal: Pythagoras | Learning environment: LESP<br>Problem solving | |

| | | | | | |
|---|---|---|---|---|---|
| Venkat, H., Graven, M., Lampen, E., Nalube, P. & Chitera, N. (2009) | 'Reasoning and reflecting' in Mathematical Literacy | Theory Assessment tasks | Journal: Learning and Teaching Mathematics | Tasks: TSL; Discourse; Scaffolding | |
| Venkat, H. (2010) | Exploring the nature and coherence of mathematical work in South African Mathematical Literacy classrooms | 1 Grade 11 ML teacher | Journal: Research in Mathematics Education | Tasks: TMS; Discourse | Curr: C7 |
| Vithal, R. (2008) | Mathematical power as political power – the politics of mathematics education | Theory | Book: Chapter in Critical issues in mathematics education | N/A | |
| Vithal, R. (2008) | Mathematical Literacy and globalization | Theory | Book: Chapter 1 in Internationalisation and globalization in mathematics and science education | N/A | |
| Zengela, C. (2008) | Turning myself around – Experiences of teaching Mathematical Literacy | 1 Grade 12 teacher | Learning and Teaching Mathematics | | Curr: C1, C7 |

## Appendix I: Analysis of discussions on Theme 1 and Theme 2

An analysis of discussions on Theme 1 and Theme 2 produced the following tables:

**Table: Findings of my study listed according to a Teacher and Learner-centred approach**

| **Teacher-centred (Monty and Alice)** | **Learner-centred (Denise and Elaine)** |
|---|---|
| ***Instructional practices*** | |
| Did not point out the value of mathematics to the learners | Pointed out the value of mathematics to the learners (Not Denise) |
| Did not determine or appropriately use learners' prior knowledge | Lessons were build on learners' prior knowledge |
| Did not encourage learner participation and did not require learners to explain their answers | Involved learners through class discussions and learners working on the board where learners could also explain and/or demonstrate their work |
| Instead of providing scaffolding, either re-explained the work or solved the problem for them | Provided scaffolding to support learner understanding |
| Insufficient knowledge of oral questioning in class | Asked various types of oral questions on different levels (Not Denise) |
| Created a formal atmosphere where focus was on mastering the content | Created a class atmosphere where learners were comfortable and confident |
| Used direct instruction as instructional strategy | Used class discussions and learners working on the board as instructional strategies |
| Board work were incomplete and disorganised | Board and transparency work were organised and no errors were made |
| ***PCK and beliefs*** | |
| Superficial knowledge regarding learners. Believed learners come to understanding by looking at several examples and through much practice | Specific knowledge regarding learners. Believed learners come to understanding by being involved through sharing their ideas and where the teacher build on their prior knowledge |
| Superficial knowledge regarding the teaching of ML. Believed the teaching of ML is the same as that of teaching Mathematics | Specific knowledge regarding the teaching of ML. Believed ML teaching differs from teaching Mathematics |

**Appendix J: Additional information verifying Question 1**

I found that two of the four instructional practices of the ML teachers in my study can be described as being exclusively teacher-centred, one teacher's practice can be described as a combination of learner- and teacher centred, leaning more towards learner-centred, while the fourth teacher's practice could be described as exclusively learner-centred.

## The practices of Monty and Alice

Monty and Alice's instructional practices can be described as teacher-centred where they believed their role as teachers was to transmit mathematical content, demonstrate procedures for solving problems, and explain the process of solving sample problems. This finding is in accordance to the findings of Artzt et al. (2008). From the observations and interviews prior to the observed lessons I realised that their focus was on transmitting mathematical content and not on the needs of the learners to develop conceptual understanding. Their practices are characterised by (according to the three lesson dimensions):

- Tasks: Not pointing out the value of mathematics so that the learners could appreciate the mathematics learned; tasks being illogically sequenced; tasks being too easy or too difficult or excessive; selecting tasks only from Level 1 of the ML Assessment Taxonomy;
- Discourse: An absence of monitoring learners' understanding; Instead of providing scaffolding, solving the problems for the learners; expressing irritation with learners' wrong answers; no constructive learner-learner interaction; low level questioning with inappropriate wait times to engage and challenge learners' thinking;
- Learning environment: Formal atmosphere where the focus was on mastering the content; using direct instruction as instructional strategy; learners being passive recipients of information.

There were differences between Monty and Alice's practices: Alice's practice was largely dysfunctional, with inattentive learners and ineffective teaching. She did not connect the learners' prior knowledge with the new mathematical situation. As both Monty and Alice are novice teachers, a plausible hypothesis seem to be the following: The difference between their practices could be attributed to the fact that Alice had no formal mathematics education training, but Monty completed a BEd with Mathematics and Methodology of Mathematics as major subjects. It is interesting to note that the teacher-centred approach can serve as a mask for teachers who do not possess full knowledge of the content, students and pedagogy (Artzt et al., 2008, p. 35). Compared to Franke et al.'s (2007) view of a productive practice being a practice where the teacher creates ongoing opportunities for learning, the practices of Monty can be described as somewhat unproductive, where Alice's practice was unproductive.

## The practices of Denise and Elaine

Denise's instructional practice can be described as a combination of learner- and teacher-centred, leaning more towards being teacher-centred, while Elaine's instructional practice can be characterised as teacher-centred. Their purpose was that learners should develop both procedural and conceptual understanding of the content. Using a learner-centred approach to teaching requires the teacher to create opportunities for learners to come to understanding by being actively engaged with one another and the problem solving process (Artzt, et al., 2008). Their practices are characterised by (according to the three lesson dimensions):

- Tasks: Lessons being built on learners' prior knowledge; representations contributing to the clarity of the lessons; tasks being logically sequenced and at a suitable level of difficulty;
- Discourse: Encouraging learner participation; meaningful discourse between the teacher and the learners; providing scaffolding to support learner understanding; recognising learners' misunderstandings and misconceptions;
- Learning environment: Having the ability to create learning environments that contributed to proficient learning; having positive attitudes towards the subject and the learners; involving learners through class discussions and learners working on the board; effective managing of time to maximise learners involvement; board and overhead projector work being organised and no errors were made.

There are some differences between the practices of Denise and Elaine. The following are characteristics of only Elaine's practice:

- Tasks: Exploring contexts using mathematical content; pointing out the value of mathematics in everyday-life situations to the learners; selecting tasks from Level 1-4 of the ML Assessment Taxonomy;
- Discourse: Having learners demonstrate and explain their answers; asking various types and different levels of oral questions;

Elaine's practice can therefore be described as a productive instructional practice as she created ongoing opportunities for learning to occur (Franke et al., 2007) while Denise's can be described as somewhat productive.

**Appendix K: Additional information verifying Question 2**

The MCK of the four participants are described in the verification of question 2.

- **PCK and beliefs of two novice teachers**

**Knowledge and beliefs of ML learners:** Monty and Alice believe that learners learn best by receiving clear information transmitted by a knowledgeable teacher, a finding Artzt et al. (2008) also found where teachers used a teacher-centred approach. They could not predict what content the learners would and would not understand; how they would come to understanding; and what possible misconceptions the learners might have.

**Knowledge and beliefs of ML teaching:** Once Alice introduced tasks that caused confusion for her and the learners, she did not know how to adjust - a phenomenon that is according to Artzt et al. (2008) typical of teachers in the initial phase of teaching. Monty and Alice could not predict the prior knowledge that should have been present in the lesson for the learners to understand the new content and could not choose appropriate instructional strategies to use in their teaching of ML. They furthermore used examples too basic or too complex throughout the lesson presentations. They believed the teaching of ML is different to the teaching of Mathematics and that group work and discussions should be used in teaching ML.

**Knowledge and beliefs of the ML curriculum:** Monty and Alice had no knowledge of other subjects integrating with ML, although they did have some knowledge about the definition, purpose and learning outcomes of ML, but not of the various departmental documents. Most importantly they taught content in the absence of contexts and did not adhere to the DoE's (2008b) aim to develop in learners [t]he ability to use basic mathematics to solve problems encountered in everyday life and in work situations (p. 8), although they believe real-life scenarios should be used. They believe mathematics as a constructivist discipline which is logical and that ML is valuable to learners. According to Monty, ML is a unique subject, but Alice believes that ML is a lower level of Mathematics.

- **PCK and beliefs of two experienced teachers**

**Knowledge and beliefs of ML learners**: Denise and Elaine have specific knowledge of learners' prior knowledge, experiences and abilities. They could predict what learners would and would not understand; how they would come to understanding; and what misconceptions learners have and typical errors the learners make. They believe the learners should be active participants in their own learning by explaining the work to each other in small groups.

**Knowledge and beliefs of ML teaching**: Since they understand how learners learn mathematics, they knew how to select appropriate instructional strategies and could adjust their teaching when required. They predicted and integrated the prior knowledge needed to enable the learners to understand the work and chose appropriate instructional strategies. They believed their role as teacher is facilitating learners' learning through selecting appropriate tasks and leading the discussions in class. They furthermore believed that teachers should provide opportunities where learners can discover and construct their own meaning through meaningful communication.

**Knowledge and beliefs of the curriculum**: Only Elaine knew about other subjects that integrate with ML, and she knew the definition and learning outcomes. Denise and Elaine knew the purpose of ML and were familiar with various departmental documents. Only Elaine taught the mathematical content in context where all her tasks were based on applicable real-life scenarios (DoE, 2003a). Denise taught content only although she believes a teacher should use contexts. Both these teachers believe mathematics is a flexible and logical discipline and that ML is a unique subject and valuable to the learners.

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**Appendix L: Declaration: External coder**

28 September 2011

Hiermee verklaar ek, Barbara Posthuma, dat ek as eksterne kodeerder opgetree het by die kodering van data vervat in Hanlie Botha se tesis.

Ek verklaar dat ons na toepaslike beraadslaging ooreengekom het oor temas en subtemas wat gebruik is tydens dataontleding.

Ek verklaar verder dat hierdie temas en subtemas toepaslik en op wetenskaplik-gefundeerde wyse bepaal is en die tendense wat in die data voorkom, na my mening so akkuraat weergegee is as wat moontlik is met die kwalitatiewe wyse van analise wat onderneem is.

Die uwe

B. Posthuma