
Chapter 5

Conclusions and implications

5.1 Introduction

In this chapter I provide a summary of the previous four chapters, answer the research questions that guided this study and reflect on my research as to what I would have done differently and make provision for the fact that I may have been wrong in my interpretation of the teachers' instructional practices and knowledge and beliefs. This is followed by the conclusions, recommendations and limitations of the study. A final reflection is done on the research study.

5.2 Chapter summary

In Chapter 1 I introduced and contextualised the research study. The purpose of this study was to investigate, by means of a case study, the way in which ML is taught with the view to determining the relationship between ML teachers' knowledge and beliefs and their instructional practices. The different meanings attached to mathematical literacy both internationally and nationally have been discussed. I discussed the problem and the rationale for the study, formulated the research questions, and discussed the methodological considerations and the possible contribution and limitations of the study.

Chapter 2 presented an in-depth analysis of the findings in the relevant literature as well as the conceptual framework on which the study is based. Comparisons were made between the different conceptions of mathematical literacy internationally and nationally, followed by a discussion on ML as a compulsory alternative to Mathematics in Grades 10 to 12 in South Africa. Following this was a discussion of the meaning of teachers' instructional practices and the value of various approaches to teaching. Attention was given to the different domains of teachers' knowledge, teachers' belief systems and the relationship between their knowledge and beliefs and their instructional practices. The conceptual framework, which is based on concepts and theories from relevant work in the literature, was then discussed.

A description of the qualitative methodology used in this study was reported in Chapter 3. I discussed social constructivism as my research paradigm, and the nature of my study as subjective and interpretive. This is an exploratory case study. Observations were used to examine teachers' instructional practices and to study demonstrations of their MCK and knowledge regarding the ML learners, the teaching of ML and the ML curriculum. Interviews were used to determine why teachers do what they do in class and to determine how they apply their PCK during their instructional practice. ATLAS.ti 6 was used to analyse the video and audio data. I lastly discussed the trustworthiness of the study and ethical considerations that were taken into consideration.

In Chapter 4 I briefly reported on the data collection process, presented and discussed the findings and lastly identified trends and possible explanations for those trends. A DEDUCTIVE-inductive (uppercase denotes the preference given to the style of analysis) approach to coding the data was used as I identified two themes: ML teachers' instructional practices and ML teachers' knowledge and beliefs prior to the data collection stage. After this deductive phase of analysis, inductive analysis was done when I studied the organised data in order to explore new patterns and trends. I presented the findings from the data obtained through class observations and interviews according to the different categories provided in Table 4.2 and Table 4.3. The findings were then related to the findings in the literature and trends were identified and subsequently explained.

5.3 Verification of research questions

Based on the rationale that ML is a significant subject which may positively influence the lives of many learners, and the problem that many teachers have negative views and experiences of the subject, I decided to explore the relationship between ML teachers' instructional practices and their knowledge and beliefs. In order to do so, the following research main question was formulated: What is the relationship between ML teachers' knowledge and beliefs and their instructional practices? To address this main question, the following five subquestions guided the enquiry:

1. How can ML teachers' instructional practices be described?
2. What is the nature of ML teachers' knowledge and beliefs?
3. How do ML teachers' knowledge and beliefs relate to their instructional practices?
4. What are the possible implications of the findings from Questions 1, 2 and 3 for teacher training?
5. What is the value of the study's findings for theory building in teaching and learning ML?

I will subsequently utilize social constructivism as research paradigm (the epistemological approach which guides my own teaching and learning practice and orientation in mathematics) to verify these

questions. In short: the social constructivist holds that all knowledge is constructed and based upon not only prior knowledge, but also the cultural and social context (Ollerton, 2009).

The following table (Table 5.1) regarding the four participants' experience, teacher training, instructional approach, productivity of instructional practice, MCK, PCK and beliefs was prepared to facilitate the discussion on the verification of the research questions.

Table 5.1: Summary of participants' information

		Monty	Alice	Denise	Elaine
Keys used in the table: <u>MCK en PCK</u> : Sufficient knowledge: ✓ and Insufficient knowledge: ✗ <u>Beliefs versus practice</u> : Corresponds: ✓ and Contradicts: ✗ Paragraph numbers in the thesis are indicated in brackets.					
Experience (years)		✗ (3)	✗ (2)	✓ (11)	✓ (11)
Maths teacher training		✓	✗	✓	✓
<ul style="list-style-type: none"> • Approach used • Productivity of practice 		<ul style="list-style-type: none"> • Teacher-centred • Somewhat unproductive 	<ul style="list-style-type: none"> • Teacher-centred • Unproductive 	<ul style="list-style-type: none"> • Teacher- and learner-centred • Somewhat productive 	<ul style="list-style-type: none"> • Learner-centred • Productive
MCK		✓ (4.6.1.1)	✗ (4.6.2.1)	✓ (4.6.3.1)	✓ (4.6.4.1)
PCK	Learners	✗ (4.6.1.2)	✗ (4.6.2.2)	✓ (4.6.3.2)	✓ (4.6.4.2)
	Teaching	✗ (4.6.1.3)	✗ (4.6.2.3)	✓ (4.6.3.3)	✓ (4.6.4.3)
	Curriculum	✗ (4.6.1.4)	✗ (4.6.2.4)	✗ (4.6.3.4)	✓ (4.6.4.4)
Beliefs versus practice (Only imperative aspects)	Learners	✓ Learn by teacher's examples [4.5.1.3 (LESP1) & 4.6.1.2]	✓ Learn by looking at teacher [4.5.2.3 (LESP1) & 4.6.2.2]	✓ Learn by demonstrating work and teacher building on that [4.5.3.3 (LESP1) & 4.6.3.2]	✓ Learn by discovery and discussions [4.5.4.3 (LESP1) & 4.6.4.2]
	Teaching	✗ ML teaching is different to Mathematics teaching [4.5.1.1 (TMS3) & 4.6.1.3]	✓ ML is the same as Mathematics teaching [4.5.2.1 (TMS3) & 4.6.2.3]	✗ ML teaching is different to Mathematics teaching [4.5.3.1 (TMS3) & 4.6.3.3]	✓ ML teaching is different to Mathematics teaching [4.5.4.1 (TMS3) & 4.6.4.3]
	Curriculum	✗ Constructivist perspective; Content in context; ML has application value (4.5.1.3; 4.5.1.1 & 4.6.1.4)	✗ Constructivist perspective; Content in context; ML has application value (4.5.2.3; 4.5.2.1 & 4.6.2.4)	✗ Constructivist perspective; Content in context; ML has application value (4.5.3.3; 4.5.3.1 & 4.6.3.4)	✓ Constructivist perspective; Content in context; ML has application value (4.5.4.3; 4.5.4.1 & 4.6.4.4)

5.3.1 Question 1: How can ML teachers' instructional practices be described?

I used an adapted version of the theoretical framework provided by Artzt et al. (2008) on teachers' instructional practices as well as Franke et al.'s (2007) definition of a productive practice to contextualise and interpret my results. To answer this question, the participants' instructional practices were described according to the lesson dimensions as indicated in this study's conceptual framework, but, to avoid repetition in the thesis, a detailed description is provided in Appendix J.

The two novice teachers in my study

Monty and Alice's instructional practices can be described as teacher-centred in that they believed their role as teachers was to transmit mathematical content, demonstrate procedures for solving problems and explain the process of solving sample problems. However, Artzt et al. (2008) suggests that this approach is not ideal as the teacher-centred approach can serve as a mask for teachers who do not fully understand the content, the learners or the pedagogy, as was found in the practices of these two teachers in my study.

Monty and Alice's instructional practices can also not be described as productive (Franke et al. (2007) consider a productive practice as a practice where the teacher creates ongoing opportunities for learning). Alice's instructional practice was less productive than Monty's. In fact, her practice was largely dysfunctional as it was characterised by inattentive learners and ineffective teaching. She also failed to connect the learners' prior knowledge with new mathematical situations. Given that both Monty and Alice were novice teachers, an explanation of the differences between their instructional practices could be that Alice had no formal mathematics education training while Monty had completed a BEd with Mathematics and Methodology of Mathematics as major subjects.

The two experienced teachers in my study

Denise's instructional practice can be described as a combination of teacher- and learner-centred, leaning more towards learner-centred, while Elaine's instructional practice can be characterised as learner-centred. Denise and Elaine believed that learners should develop both procedural and a conceptual understanding of the mathematical content. A learner-centred approach to teaching requires the teacher to create opportunities for learners to achieve understanding through active engagement with each other and the problem-solving process (Artzt, et al., 2008). What appears to have made Elaine's practice more productive than Denise's was Elaine's use of contexts to explore the mathematical content; her pointing out the value of mathematics in everyday-life situations; her

selection of tasks from Levels 1-4 of the ML Assessment Taxonomy; her allowing her learners to explain their answers; and her asking various types and different levels of oral questions.

Comparison of the participants' instructional practices

The key differences between the two novice teachers and the two experienced teachers are listed in Table 5.2 below. I found that the instructional practices of the four teachers in my study could be described as predominantly teacher-centred: the practices of two of the four teachers were exclusively teacher-centred; one teacher's practice could be described as a combination of teacher- and learner-centred, leaning more towards learner-centred; and the fourth teacher's practice could be described as exclusively learner-centred.

Table 5.2: Comparison of the participants' instructional practices

	Two novice teachers (Monty and Alice)	Two experienced teachers	
		Denise	Elaine
Approach	Teacher-centred	Teacher- and learner -centred	Learner-centred
Tasks	<ul style="list-style-type: none"> • Content only • On Level 1 only 	<ul style="list-style-type: none"> • Content only • On Level 1 only 	<ul style="list-style-type: none"> • Content in contexts • On Levels 1-4
		Build lessons on learners' prior knowledge	
Discourse	<ul style="list-style-type: none"> • Learners did not express their thinking • No scaffolding • Did not recognise learners' typical misunderstandings 	Learners demonstrated their thinking	Learners demonstrated and explained their thinking
		<ul style="list-style-type: none"> • Provided scaffolding • Recognised learners' typical misunderstandings 	
Learning environment	<ul style="list-style-type: none"> • Formal atmosphere • Direct instruction • Not enough logical flow • Minimum learner participation 	<ul style="list-style-type: none"> • Discussions • Logical flow in lessons • Positive classroom atmosphere • Maximum learner participation 	

However, certain common trends in all four cases were also observed the most significant of which was their collective failure to encourage learner-learner interaction. Again, this is far from ideal: The DoE (2003a) clearly states that learners need to develop the ability to communicate mathematically and that teachers should create opportunities for classroom dialogue where learners can listen to, respond to and question each other so that they can discard or revise their own ideas (Artzt et al., 2008).

In summary: My study seems to provide evidence that confirms the existence of a relationship between the teaching approach used in ML classrooms and the productivity of the teacher's instructional practice. The four practices observed in my study ranged from Elaine's learner-centred approach, yielding a productive instructional practice, to Alice's teacher-centred approach, yielding an unproductive instructional practice.

According to my findings, it seems that teaching experience as well as mathematics teacher training (Table 5.1) may play a significant role in the productivity of the instructional practices of the four participants: both Denise and Elaine (with 11 years' experience of either teaching Mathematics or ML) had productive practices; and, comparing the practices of the two novice teachers (with three years and less of teaching experience), the teacher with mathematics teacher training had a more productive instructional practice than the teacher without teacher training.

5.3.2 Question 2: What is the nature of ML teachers' knowledge and beliefs?

As stated in Question 1, the participants' PCK and beliefs were described according to the study's conceptual framework, but, to avoid repetition in the thesis, a detailed description is provided in Appendix K.

Teachers' level of MCK

The purpose of this study was not to assess ML teachers' content knowledge but rather to comment on the accuracy of their mathematical content and the occurrence of their misconceptions. Against this background, I found that the three teachers who had prior teacher training in mathematics appeared to have sufficient MCK regarding the topics they were teaching at the time of the observations as no mathematical errors (except for the two minor omissions and single mistake of one teacher) were made or misconceptions observed. The only teacher with no mathematics teacher training was guilty of several mathematical errors, and some of the mathematical content she taught indicated her own misconceptions regarding the content. All four teachers believed MCK was a prerequisite to teach ML.

Teachers' level of PCK

Regarding the nature of the ML teachers' PCK, one of the experienced teachers (11 years' experience) illustrated sufficient knowledge of all three domains of PCK: the ML learners, the teaching of ML and the ML curriculum. The other experienced teacher (also 11 years' experience) had sufficient knowledge of two of the three domains of PCK: the ML learners and the teaching of ML. The other two teachers had only superficial knowledge of all three domains. My finding that the two experienced teachers had

developed PCK confirms the findings of Ball (1988), Ball et al. (2005), Koellner et al. (2007), Ma (1999), Shulman (1986) and Sowder, (2007) that PCK can be developed only over time through experience in the classroom and that it cannot be taught. Some researchers (Ball, 1990; Van Driel, Verloop & de Vos, 1998) also believe that solid understanding and knowledge of mathematical subject matter are prerequisites for developing PCK. I also found that the two teachers who had developed a certain level of PCK also had adequate MCK. However, Monty's instructional practice indicates that sufficient MCK (teacher training) does not guarantee PCK (Table 5.1).

Teachers' beliefs

All four teachers claimed that they had a constructivist perspective on mathematics as a discipline; that ML involves the teaching of mathematics in context; that learners should realise the application value of mathematics; and that learner-learner interaction in the form of group work and discussions is required for learners to develop understanding of mathematics. The main differences between the two novices' and the two experienced teachers' beliefs were in their beliefs on how learners learn: The novice teachers believed that learners learn through information received from the teacher while the experienced teachers believed that learners should be active participants in their own learning. Only one teacher in my study, Alice, believed that ML is similar to Mathematics but on a lower level while the other three teachers believed that ML is a unique subject in its own right.

To summarise (see Table 5.1): Both teachers with sufficient PCK also had sufficient MCK, which suggests that MCK is required to develop PCK. Furthermore, the three teachers with mathematics teacher training (one novice teacher and two experienced teachers) had sufficient MCK, but since the novice teacher still lacked PCK, it could be suggested that although MCK is required to develop PCK, it is teaching experience that plays a crucial role in the development of teachers' PCK. These findings will hopefully contribute to this new field and fill the gap in literature regarding ML teachers' knowledge and beliefs.

Based on the interviews with the four participants in my study, it seems that mathematics teacher training is required to enhance teachers' MCK and that, although MCK is required to develop PCK, it is through teaching experience that teachers develop PCK.

5.3.3 Question 3: How do ML teachers' knowledge and beliefs relate to their instructional practices?

Teachers' knowledge

Alice was the only teacher in my study who had insufficient MCK, and, because only Alice's instructional practice was described as unproductive, it seems that ML teachers' level of MCK strongly influences the productivity of their practices (Table 5.1). This finding supports Kilpatrick's (2001) view that proficient teaching demands, among other things, teachers' conceptual understanding and procedural fluency. The two novice teachers in my study had insufficient PCK and unproductive instructional practices in contrast to the two experienced teachers who had sufficient PCK and productive instructional practices. This finding suggests that PCK influences the productivity of teachers' practices. Since PCK influences teachers' teaching approach, which, in turn, influences the productivity of teachers' practices, it can be deduced that PCK influences ML teachers' practices.

I realise that other factors also play an important role in the productivity of teachers' instructional practices, but it seems as if MCK and PCK have a definite influence on such practices.

Teachers' beliefs

Trends were found in the correspondences and contradictions between the teachers' stated beliefs and their instructional practices. A common belief expressed by all four teachers was how learners learn. The following important contradictions were noted between three of the four ML teachers' stated beliefs about the nature of mathematics and the teaching of ML and their instructional practices. All four teachers claimed that they taught mathematics from a constructivist perspective, but, in practice, it was only Elaine who created opportunities for the learners to discover, experiment and reason in order to achieve understanding. Monty and Alice's perspective was traditional while Denise's was formalist (Dionne, 1984).

- All four teachers believed that ML involved the teaching of mathematics in context and that learners should realise the application value of mathematics, but only Elaine used relevant contexts to enable the learners to explore the mathematical content and to appreciate the value of mathematics.
- All four teachers believed that learner-learner interaction, such as learners explaining their work to each other in small groups, was important in providing opportunities for learners to develop conceptual understanding of mathematics. However, it was only in Elaine's instructional practice that some evidence of learner-learner interaction was observed.

Contradictions between three of the four teachers' stated beliefs and their instructional practices were found in the practices of Monty and Alice, who used a teacher-centred approach, and Denise who used a combination of teacher- and learner-centred approaches. A reason for these inconsistencies could be that the teachers had heard about a constructivist perspective on mathematics or knew how ML should be taught, but their existing cognitive structures were not ready to accommodate the required changes (Artzt et al., 2008).

According to the literature, teachers' knowledge and true beliefs strongly influence their practices (Artzt et al., 2008; Ball, 1990; Liljedahl, 2008; Pajares, 1992). It was only Elaine in my study who adopted a learner-centred approach and whose knowledge and stated beliefs corresponded with her instructional practice. Not only did her knowledge and beliefs influence her instructional practice positively, but conversely her instructional practice (experiences with the subject and its learners) also positively influenced her knowledge and beliefs regarding the ML learners and the teaching of the subject. This was evident from my last interview with Elaine during which she told me that when she had been asked two years ago to be the coordinator for ML, she had initially felt that she was being demoted, but she claimed that ML had grown on her since then. She enjoyed being involved in ML and never wanted to return to Mathematics.

In summary: Based on the findings of this study, it can be tentatively assumed that, except for the one teacher who used a learner-centred approach, the teachers' stated beliefs about teaching ML and the ML curriculum did not influence their instructional practices whereas knowledge had a strong influence. It is possible that the stated beliefs did not reflect the true beliefs of the teachers in this study. In the case where a learner-centred approach was used, not only did the teacher's knowledge and beliefs influence her practice, but her practice also influenced her knowledge and beliefs. These findings will hopefully also contribute to this new field, also filling the gap in literature regarding the relationship between ML teachers' knowledge and beliefs and their instructional practices.

5.3.4 Question 4: What are the possible implications of the findings from Questions 1, 2 and 3 for teacher training?

Effective and purposeful training of pre- and in-service ML teachers is of critical importance in South Africa, a finding that was also reported by, among others, Bansilal (2008) and Sidiropolous (2008). The instructional practice of Alice (the only teacher in my study without mathematics teacher training) proved to be unproductive resulting in discouraged and uninvolved learners. Knowledgeable, competent and dedicated ML teachers such as Elaine in my study and the findings from other studies

(Graven & Venkat, 2009; Hechter, 2011a; Venkat & Graven, 2008) reveal that the aims and purposes of ML are realistic and achievable. My findings also indicate that MCK alone is insufficient; ML teachers need knowledge regarding the teaching and learning of ML to teach the subject effectively.

Teaching ML

Key factors that should be part of all ML instructional practices:

- Having a learner-centred approach and using appropriate instructional strategies.
- Engaging with contexts rather than applying mathematics already learned to contexts.
- Selecting tasks at all four levels of the ML Assessment Taxonomy – the emphasis in Levels 1 and 2 is on routine calculations while the key aims of ML are located primarily in Levels 3 and 4.
- Engaging learners in discussions thereby enabling them to communicate their thinking through the use of appropriate terminology.
- Using various instructional resources to connect learners' knowledge with new situations.
- Teachers having sufficient general knowledge of the contexts in which the lesson is situated, enabling them to engage the learners in meaningful mathematical discourse.

These key factors were also confirmed by Artzt et al. (2008), DoE (2003a, 2011), Graven and Venkat (2007) and Venkat et al. (2009).

Other didactical and methodological issues that were identified during this study and should be addressed include logical sequencing of tasks; efficient oral questioning; managing discipline in ML classrooms; creating a positive learning atmosphere in class; effective board work; valuing teachers who are enthusiastic about ML and its learners; engender an understanding of the theory and practice of PCK; enhancing teachers' curriculum knowledge and their knowledge of how ML is integrated with other subjects.

ML teachers' knowledge and beliefs

Similar to the training of Mathematics teachers, the focus of ML teacher training should be the development of sufficient MCK to enhance conceptual understanding of various mathematical topics. ML teacher training should however differ from Mathematics teacher training regarding the following aspects: The level of the mathematical content in the ML programme need not be on a second-year BSc level, but should include (apart from a generic component being offered to both Mathematics and ML student teachers) specialised training regarding the teaching and learning of ML. In particular, teachers should learn how to structure mathematical content to enable learners to progress in their

cumulative understanding of the content and to link learners' prior knowledge with new content. The theory and practice of PCK should be incorporated in all student teacher training programmes during teaching practice and internship. Only a few tertiary institutions have to date developed such programmes – the teachers who participated in this study, for example, had no such exposure.

ML teachers' instructional practices were strongly positively influenced by their knowledge and not their stated beliefs. The distinction between what teachers say they know and believe and is played out in their classrooms emerged as a pivotal aspect of teaching and learning success. Addressing this distinction during teacher training seems to be important.

In summary: ML teacher training is specialised and implies that teachers should be equipped with MCK and skills to facilitate the learning process. ML teacher training has to a large extent been neglected as the subject was introduced in 2008 yet it was only in 2011 that a ML teacher training programme was introduced at the University of Pretoria. This programme consists of a three-year mathematics content component and a methodology of ML component in the fourth year. It is furthermore recommended that all practising ML teachers should be required to complete an ACE (ML) programme, which is in line with the DoE's (2009) requirement that all intermediate phase teachers should have completed a mathematics course by 2014.

5.3.5 Question 5: What is the value of the study's findings for theory building in teaching and learning ML?

Flowing from the finding that ML teachers' knowledge, but not necessarily their beliefs, influences their instructional practices, the following matters warrant the attention of curriculum decision-makers:

- My research revealed that teachers who had formal training or experience in the teaching of mathematics in classrooms also displayed evidence of having adequate MCK. In addition, adequate MCK impacted positively on teaching and learner understanding.
- The ML teachers who had productive instructional practices had sufficient knowledge of both mathematical content and the teaching and learning of ML.
- Competent, dedicated teachers who value the ML curriculum are needed to teach ML. New student-teachers should be recruited to become ML teachers so that they can develop a new status identity.

To answer my main question: There is a dynamic but complex relationship between ML teachers' knowledge and beliefs and their instructional practices. Firstly, their knowledge, but not their stated beliefs were reflected in their practices. Secondly, some of the teachers' classroom practices belied their

stated beliefs. Conversely, in one case, the teacher's practice also had a positive influence on her knowledge and beliefs.

5.3.6 Summary of verification of research questions

In Table 5.3 below I provide a summary of the research questions, data collection techniques used, objectives of the questions and research findings.

Table 5.3: Summary of verification of research questions

Research questions (Data collection techniques)	Objectives of the questions	Research findings
1. How can ML teachers' instructional practices be described? (Observations)	<ul style="list-style-type: none"> To determine what teachers do in their classrooms with respect to: tasks given, discourse that takes place and the learning environment which is established. To describe teachers' practices according to the teaching approach used and level of productivity of their practices. 	Predominantly teacher-centred: two teachers used a teacher-centred, one teacher a combination of teacher- and learner-centred and one teacher a learner-centred approach. In all practices there is a positive relationship between the approach used and the level of productivity, ranging from a productive practice in which a learner-centred approach was used to an unproductive practice where a teacher-centred approach was used.
2. What is the nature of ML teachers' knowledge and beliefs? (Observations & interviews)	<ul style="list-style-type: none"> To comment on the teachers' level of MCK. To explore teachers' PCK and beliefs regarding ML learners, the teaching of ML and the ML curriculum. 	The two more experienced ex-Mathematics teachers proved to have sufficient knowledge while the two novice teachers still lack PCK. Mathematics teacher training is required to enhance teachers' MCK and although MCK is required to develop PCK, it is through teaching experience that teachers develop PCK.
3. How do ML teachers' knowledge and beliefs relate to their instructional practices? (Observations & interviews)	<ul style="list-style-type: none"> To explore what the relationship is between teachers' instructional practices and their knowledge and beliefs. To investigate how teachers use PCK in their lessons. 	Teachers' beliefs did not influence their instructional practices, but knowledge strongly influences teachers' instructional practices. In the case where a learner-centred approach was used, not only did the teacher's knowledge and beliefs influence her practice, but her practice also influenced her knowledge and beliefs.
4. What are the possible implications of the findings from Questions 1, 2 and 3 for teacher training? (Observations & interviews)	<ul style="list-style-type: none"> To improve my own practice. To inform current teacher training and development programmes. 	ML teacher training is specialised and implies that teachers should be equipped with specific MCK; skills to integrate content and context in their teaching in order to facilitate the learning process; and knowledge of the ML curriculum. An understanding of the theory and practice of PCK and its importance should be engendered in all student teachers training.

<p>5. What is the value of the study's findings for theory building in teaching and learning ML? (Observations & interviews)</p>	<ul style="list-style-type: none"> To add to the body of knowledge regarding the relatively newly introduced subject and to make suggestions to the curriculum stakeholders. 	<p>ML teachers' instructional practices are strongly positively influenced by their knowledge, but not their stated beliefs. The distinction between what teachers say they know and believe and is played out in their classrooms emerged as a pivotal aspect of teaching and learning success. Addressing this distinction during teacher training seems to be important.</p>
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5.4 What would I have done differently?

During the data presentation stage, I realised that I had missed valuable communications between the teacher and the learners at their desks as I did not want to intrude by moving around in class with a video camera. More information regarding the teachers' PCK would possibly have emerged from this discourse. With the insight of hindsight, I would have employed a research assistant to videotape all my sessions for careful perusal and analysis.

In discussing my findings another aspect I wished I could have done differently was to include ML teachers from other non-mathematics disciplines. Alice's practice already informed this study about teachers who were teaching without having teacher training, but it would have been valuable to investigate the practices of teachers who did have teacher training but no formal mathematics training.

5.5 Providing for errors in my conclusion

I engaged with five (although only four actually participated in the research) ML teachers who allowed me in their classrooms and also shared some of their knowledge and beliefs with me. I have made some decisions on their instructional practices and the nature of their knowledge and beliefs, and I have to accept the fact that I may have been wrong in some of my conclusions, albeit unknowingly and unintentionally. I attempted to enhance the credibility and trustworthiness of my study through triangulation by using three observations and three interviews at different stages of the data collection process. Since there is no agreement among academics on how knowledge and beliefs are to be evaluated, I acquired the services of a peer researcher to assist me with the coding and interpretation of the data to further enhance the trustworthiness of my study. I also verified my findings with findings from the literature.

To reduce the Hawthorne effect the first observation was done without a prior interview or discussion because the interview questions prior to the second and third observations could influence teachers'

behaviour in the classroom. I emphasised to the teachers the fact that I was interested in the uniqueness of each teacher and my purpose was not to report their performances in class to their superiors. I furthermore used the same interview schedules, including the same questions in the same sequence for all interviewees. Section C of the last interview was based on the teachers' knowledge of the ML curriculum. I gave the teachers the option to answer the questions orally or in writing. I mentioned the advantage of providing the answers in writing: that they might have felt less threatened or pressured and that it also allowed them more time to think about the questions and to provide valuable responses. In choosing to provide the answers in writing, they were requested to complete the section in my presence as part of the interview. This was to ensure that the data obtained were credible, as the teachers were not able to consult another teacher or the relevant documents.

5.6 Conclusions

Some conclusions regarding the relationship between ML teachers' knowledge and beliefs and their instructional practices appear below.

ML teachers' instructional practices:

- Given that the use of contexts should be the focus of ML lessons, the puzzling absence of the use of contexts in three of the four teachers' practices was notable and should be addressed. Not just learners but teachers too need to understand the contexts in order to have informative and enlightening class discussions.
- ML teachers' instructional practices should be predominantly learner-centred, including the use of active learning instructional strategies such as cooperative learning and discussions.
- The following aspects of ML lessons in particular deserve to be emphasized, not only during teacher training but especially in post-teacher training: tasks should be logically sequenced; tasks should not be too easy or too difficult; learners' understanding should be monitored; learner-learner interactions should be optimized; there should be variety in levels and types of oral questioning during instruction; learners' ideas and ways of thinking should be acknowledged and appreciated consistently; and a positive learning atmosphere should be instilled.

Knowledge and beliefs:

- ML teachers need to attain a certain level of MCK as well as knowledge of the teaching and learning of ML before being allowed to teach the subject.

- ML teachers need to attain an adequate sense of procedural knowledge and conceptual understanding of the ML learners, the teaching of ML and the ML curriculum as well as experience to develop PCK.

The relationship between ML teachers' knowledge and beliefs and their instructional practices:

- ML student teachers as well as in-service ML teachers should be afforded ample opportunity to enhance their MCK but also to engender an understanding of the theory and practice of PCK as knowledge strongly influenced the ML teachers' instructional practices.
- ML student teachers (during initial teacher training) as well as practising ML teachers (during in-service training) should be sensitized to the importance of ensuring that their instructional practices are consistent with their true beliefs and not their stated beliefs only.
- Instead of merely claiming to be teaching in a constructivist manner, ML teachers should be taught how to actually teach in a constructivist manner.

5.7 Recommendations for further research

Several aspects of the teaching and learning of ML require further research in order for ML to come into its own as a viable and valuable subject in its own right. These include investigation into:

- The knowledge required to engage learners in such a manner as to explore the depths of their prior knowledge during teaching.
- The nature and level of content knowledge that is required to teach ML effectively.
- The ways in which ML teachers can transform their own MCK into learning facilitation strategies that are pedagogically powerful, through the choice of appropriate teaching and learning strategies and supporting materials.
- Identification of authentic and relevant contexts that not only relate to learners' daily lives, their future workplace and the wider social, political and global environment, but how such contexts can be applied effectively to the required lesson content.
- The viability of developing PCK during teacher training.
- ML teachers' true and stated beliefs and the influence thereof on their instructional practices.
- The development of effective questioning techniques and assessment strategies in the ML classroom.

- The guidance of ML teachers to becoming *au fait* with the extent to which language potentially influences ML learners' achievements and acquire the skills needed to deal with these issues in their classrooms adequately.
- ML learners' expectations and experiences of the subject.

5.8 Limitations of the study

Data were gathered from a very small number of ML teachers and generalization of the results is impossible. However, generalization was not an aim of the study. Another limitation is the fact that the observations were all done in the second part of Term 2 and two of the four teachers were busy with revision. Furthermore, more data regarding the teachers' knowledge of the learners could have been gathered during the observations if I had been party to the discourse between the teachers and individual learners sitting at their desks. As my presence in class already influenced the teaching process, I did not want to intrude furthermore on the learners' learning process. I am also acutely aware that different researchers may interpret my data differently. My own perspective is bound by space, time and personal experience. Even though my conclusions were carefully scrutinized and confirmed or refuted by my supervisors, my external coder as well as my participants, the possibility that subjectivity may have influenced my findings cannot be ruled out.

5.9 Last reflections

It is with mixed feelings that I am making this attempt to bring my 'doctoral journey' to its conclusion. I realize, among many other things that it has been a time of accelerated growth and learning for me, both professionally and personally. I began this journey with a strong assumption that ML has a rightful place in the school curriculum as a valuable subject and that this subject should be taught by a mathematics teacher. Furthermore, I assumed that the success of this relatively new subject depended strongly on the input, training, experience and perceptions of these (mathematics) teachers.

Initially I wanted to explore how ML teachers' PCK influence their instructional practices, but as my study developed, I realised that MCK and teachers' beliefs also play a crucial role in teachers' practices. This study allowed me the opportunity to become part of the lives of four ML teachers whom I have observed and listened to as they shared their knowledge and beliefs with me. However, even though I benefited a great deal from my exchanges with all four teachers, I wish to state that it was especially during the time I worked with Elaine that I gained insight into how a teacher's practice could influence her knowledge and belief. My interactions with her changed the focus of the study

from exploring the influence of ML teachers' knowledge and beliefs on their practices to exploring the relationship between teachers' knowledge and beliefs and their practices.

During the data analysis stage I became aware of the complexity involved in analysing teachers' practices, knowledge and beliefs. I realized that the boundaries between these categorisations of knowledge are very vague. More particularly, it dawned on me that not all stated beliefs are true beliefs and that the boundaries between knowledge and beliefs are vague and blurred. I found the process of conducting literature control a most exhausting and emotionally draining exercise. This was mainly the case because of the paucity of research relating to my study, which made it extremely difficult to compare my study with other studies. The vast majority of studies that could be related to mine were conducted on very small samples –, in many cases involving one or two teachers only.

I hope that my findings will contribute to teacher training and theory and that this study will contribute to the building of a mathematically literate nation. Maree (2011) voices my thoughts also when he stated the following:

Since what happens in the classroom will eventually determine whether or not lasting change can be effected, the role of the school [teacher] is crucial in creating an optimal learning environment ... Learners should leave school better equipped to cope with the challenges of university study and life itself. They need to be empowered to choose appropriate careers, enter society and make meaningful social contributions.