Chapter 4

Presentation and discussion of the findings

4.1 Introduction

Since the data collection process and data analysis strategies are discussed in Chapter 3, I briefly rapport on the data collection process[24] and the data analysis strategies[25] while providing a comprehensive elucidation regarding the coding of the data. Based on my conceptual framework (Figure 4.1) I thematically present and discuss the findings from each participant, relate the findings to the literature and lastly explain the identified trends. The two themes are: 1) ML teachers' instructional practices and 2) ML teachers' knowledge and beliefs.

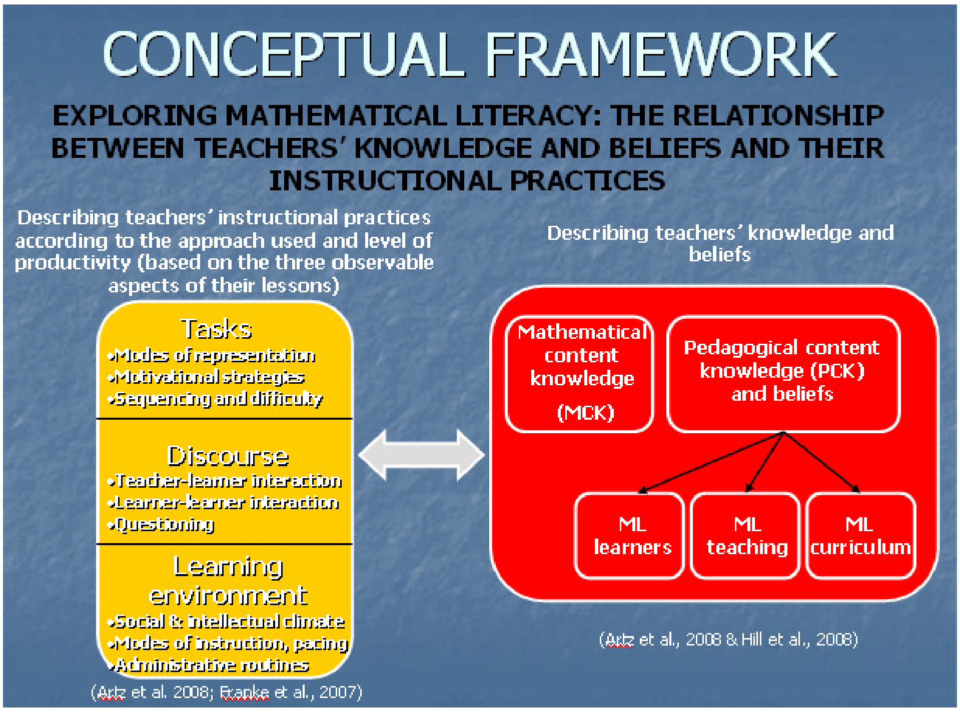

Figure 4.1: Conceptual framework: Instructional practice, knowledge and beliefs framework of analysis (adapted from Artzt et al., 2008; Franke et al., 2007; Hill et al., 2008)

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[24] A detailed description of the process is provided in Section 3.5: Data collection techniques.
[25] For more detail, see Section 3.6: Data analysis strategies.

The research questions are:

Main question:

What is the relationship between Mathematical Literacy teachers' knowledge and beliefs and their instructional practices?

Subquestions:

1. How can ML teachers' instructional practices be described?
2. What is the nature of ML teachers' knowledge and beliefs?
3. How do ML teachers' knowledge and beliefs relate to their instructional practices?
4. What are the possible implications of the findings from Questions 1, 2 and 3 for teacher training?
5. What is the value of the study's findings for theory building in teaching and learning ML?

4.2 The data collection process

The data collection took place in Pretoria during the second quarter (May and June) of 2011. I initially contacted the principals of five schools telephonically to discuss my study and request their participation in my study. Two letters of invitation were sent to the schools, one addressed to the principal and the other to the Grade 11 ML teacher and a meeting was scheduled at each school between the principal, teacher and me. The initial participants[26] were Monty, Elaine, Alice, Edith and Denise. During the data analysis process I realised that Edith's case did not add value to my study since her practice was similar to three of the four teachers' practices. I also reached a point of data saturation and decided to continue with the other four cases only. During the data collection period all communication and arrangements were made directly with the participants except for Alice's school where I worked through the principal.

I kept to the data collection process[27] of three observations with an interview conducted prior to the second and third observations, followed by a last interview some time after the last observation. The duration of the two interviews prior to the observations was approximately half an hour each and was conducted during the period or break before the specific lesson. These interviews were based on the teachers' planning of their lessons. The duration of the last in-depth interview was approximately 50 minutes per interview. I only observed Grade 11 ML lessons and all participants had had at least one year experience of teaching ML, one of the selection criteria[28].

[26] Pseudonyms were used for ethical purposes.
[27] The data collection process is discussed in Section 3.5: Data collection techniques.
[28] The other selection criteria are discussed in Section 3.4: Research site and sampling.

In Table 4.1 a timeline is given indicating the dates all five participant's lessons were observed and interviews conducted.

Table 4.1: Timeline of the data collection process

Data gathering instrument	Participants[29]	Date in 2011
Observation 1	Monty	3 May
Interview & Observation 2	Monty	4 May
Observation 1	Elaine	6 May
Interview & Observation 3	Monty	9 May
Observation 1	Alice	9 May
Interview & Observation 2	Alice	10 May
Interview & Observation 2	Elaine	12 May
Interview & Observation 3	Alice	16 May
Observation 1	Edith	16 May
Observation 1	Denise	17 May
Interview & Observation 3	Elaine	19 May
Interview & Observation 2	Edith	19 May
Interview & Observation 2	Denise	20 May
Interview & Observation 3	Edith	25 May
Interview & Observation 3	Denise	26 May
Last interview	Edith	27 May
Last interview	Denise	30 May
Last interview	Alice	1 June
Last interview	Monty	1 June
Last interview	Elaine	2 June

4.3 Data analysis strategies

The study's DEDUCTIVE-inductive approach and analytic strategies used in analysing the data are discussed in Chapter 3[30]. In this section I only discuss the transcribing and coding of the data. The inclusion and exclusion criteria for coding the data are also discussed and presented in table form.

4.3.1 Transcribing the data

I transcribed my video and audio-taped data verbatim to text data immediately after the data had been collected. Care was taken not to interpret the data already during the transcribing phase. After each

[29] Pseudonyms were used to protect the participants' true identities.
[30] See Section 3.6: Data analysis strategies.

observation all hand-written field notes made during the observations as well as insights that were thought of afterwards which had not been noted were typed on a template form. Uncertainties that emerged were cleared by watching the video-tapes of the lessons or listening to the audio-tapes again. Transcripts were read afterwards to ensure the transcripts were true accounts of the actual observations and interviews.

4.3.2 Coding of the data

In coding the data, I used a deductive approach based on my conceptual framework. According to the conceptual framework two themes, namely 1) the ML teachers' instructional practices and 2) their knowledge and beliefs were identified while the subthemes for each theme were chosen according to the work of Artzt et al. (2008) and Hill et al. (2008). Codes have been ascribed to the different lesson dimension indicators of each subtheme according to which the raw data were analysed. By using the software programme ATLAS.ti 6, I coded the transcripts according to a set of pre-determined lesson dimension indicators and their associated codes as given in Table 4.2[31] and Table 4.3[32].

After the data were coded I created *coding families* (Archer, 2009) which are clusters comprising codes related to one other. According to my conceptual framework, families were created by selecting from the list of all codes those codes that were related to one another. A specific code could belong to more than one family and families were therefore not exclusive. The data were analysed according to two themes, namely 1) ML teachers' instructional practices and 2) ML teachers' knowledge and beliefs. Sub-themes for each of these themes were created using Atlas.ti 6. Networks for these sub-themes were created afterwards where the connections between the different codes assigned to the families were indicated.

4.3.2.1 Theme 1: ML teachers' instructional practices

The three subthemes (also called the lesson dimensions) which could best describe the teachers' practices are 1) tasks; 2) discourse; and 3) learning environment (Artzt et al., 2008). The first column in Table 4.2 below indicates the three subthemes or lesson dimensions with their different categories. In the second column are the descriptions of the lesson dimension indicators with the codes created for them. All data were collected from the observations only.

[31] Table 4.2 is given under Section 4.3.2.1.
[32] Table 4.3 is given under Section 4.3.2.2.

Table 4.2: Lesson dimensions and dimension indicators as inclusion criteria for coding the data (Adapted from Artzt et al., 2008)

LESSON DIMENSIONS	DESCRIPTION OF LESSON DIMENSION INDICATORS (CODES)
TASKS	
Modes of representation (TR)	TR1. Uses representations such as oral or written language, symbols, diagrams, graphs, tables, manipulatives, and computer or calculator representations to accurately facilitate content clarity. TR2. Provides multiple representations that enable learners to connect their prior knowledge and skills to the new mathematical situation such as graphs, tables, formulae.
Motivational strategies (TMS)	TMS1. Uses tasks that capture learners' curiosity and inspires them to participate in the lesson, but also to speculate on and pursue their conjectures, such as tasks which elicit a class discussion or an interesting context used. TMS2. Takes into account the diversity of student interests, experiences and abilities, such as when the teacher provides additional tasks for the more advanced learners. TMS3. Points out the value of the mathematics being learned so that learners will appreciate and understand the value of mathematics, such as informing them about real-life situations or even other subjects where the mathematical content is used.
Sequencing and difficulty levels (TSL)	TSL1. Sequences tasks and learning activities so that learners can progress in their cumulative understanding of a particular content area and can make connections between ideas learned in the past and those they will learn in the future such as working from easy to difficult and known to unknown tasks. TSL2. Uses tasks, including homework that is suitable to what the learners already know and can do and what they need to learn or improve on. Tasks should involve past work, reinforce current work and set the stage for future work such as tasks where opportunity is given to practice identified or predicted learners' misunderstandings. TSL3. Tasks should reflect quality, not quantity. Should be appropriate and on the learners' level such as increasing the level of difficulty from Grade 10 and applying it to more complex contexts.
DISCOURSE	
Teacher-learner interaction (DTL)	DTL1.Communicates with learners in a non-judgmental manner and encourages the participation of each student, such as addressing a large number of learners or working on a learner's incorrect answer to lead the learner to understanding. DTL2. Requires learners to give full explanations and justifications or demonstrations orally and/or in writing such as

	learners explaining their work on the board. DTL3. Listens carefully to learners' ideas and makes appropriate decisions regarding when to offer information, provide clarification, model, lead, and let learners grapple with difficulties. In response to a learner's question, instead of telling how or doing it for the learner, rather provides scaffolding to support solving a problem or encourage learners to share ideas for carrying out a task. DTL4. When giving feedback to learners' answers, rather accept and praise instead of criticising and rejecting answers. DTL5. Recognises and clarifies learners' misunderstandings and misconceptions.
Learner-learner interaction (DLL)	DLL1. Encourages learners to listen to, respond to, and question each other so that they can evaluate and, if necessary, discard or revise ideas and take full responsibility for arriving at mathematical conjectures and/or conclusions. DLL2. Avoids situations in which a group of learners dominate in the verbal communication in class.
Questioning (DQ)	DQ1. Teacher needs to pose a variety of levels and types of questions using appropriate wait times that elicit, engage and challenge learners' thinking. Cognitive levels: i) Memory: factual questions such as: What is the mean? ii) Convergent: narrow questions such as: What does it mean to write 12,5% as a decimal? Also complete the word/sentence questions such as: We call it the co? and learners then need to complete the word: coefficient. iii) Divergent: broad and open-ended questions such as: Why did you decide to use the compound interest formula? DQ2. The teacher should listen to learners' ideas and ask them to clarify and justify their ideas such as asking: Why do you say that? or How did you solve that? DQ3. Should contribute to the verbal communication and participation of learners creating opportunities where learners listen to, respond to, question and answer the teacher and one another. DQ4. Learners' responses: i) chorus; ii) volunteered; iii) teacher-selected.
LEARNING ENVIRONMENT	
Social and intellectual climate (LEC)	LEC1. Establishes and maintains a positive rapport with and among learners by showing respect for and valuing learners' ideas and ways of thinking such as no one laughing when an incorrect answer is given. LEC2. Enforces classroom rules and procedures to ensure appropriate classroom behaviour such as one person talking at a time. LEC3. Should have a positive attitude towards the learners and the subject, such as praising learners' attempts and being proud and well-prepared ML teachers.
Modes of strategies and pacing	LESP1. Uses various instructional strategies that encourage and support student involvement as well as facilitate goal attainment such as cooperative learning, learners explaining work at the board, direct instruction (lecturing),

(LESP)	abstract procedural, group work, active learning, discussion, problem solving, inquiry and team-teaching. LESP2. Provides and structures the time necessary for learners to express themselves and explore mathematical ideas and problems such as enough opportunities to discuss or do group work. LESP3. Effective use of class time to accommodate all three phases of a lesson: initiation, development and closure. LESP4. There should be a logical flow in the lesson such as revising prior knowledge before introducing new content and assess whether learning occurred.
Administrative routines (LEA)	LEA1. Uses effective procedures for organization and management of the classroom so that time is maximized for learners' active involvement in the discourse and tasks such as allowing time for learners to practice what has been explained by the teacher and not to rush them while working on a problem. LEA2. Classroom arrangement should be appropriate to the lesson style used such as learners sitting in groups if group work is applied. LEA3. The position of the teacher in the classroom should contribute to a positive learning atmosphere such as working between the learners, having eye contact with individual learners. LEA4. The written information on the board/transparencies should be correct and ordered in order to contribute to learners' conceptual understanding.

Source: Adapted from: A Cognitive Model for Examining Teachers' Instructional Practice in Mathematics: A Guide for Facilitating Teacher Reflection, by A.F. Artzt and E. Armour-Thomas, 1999, *Educational Studies in Mathematics*, 40(3), p. 217. Copyright © 1999 by Kluwer Academic Publishers. Adapted with kind permission from Kluwer Academic Publishers.

Using Atlas.ti 6, networks of the *code families* (Archer, 2009) are now illustrated and explained. As I have mentioned, the data for the three *code families* under Theme 1 were collected from the lesson observations only. The *code family* created for the first subtheme **Tasks** appear in Figure 4.2 below. The broken line arrows indicate the three different lesson dimensions being linked to the code family **Instructional practices: Tasks**. Atlas.ti 6 uses solid line arrows with double equal signs to indicate the codes *associated with* the different lesson dimensions (Archer, 2009). For example codes TR1 and TR2 are associated with lesson dimension **Tasks: Modes of representation (TR)**. A full description of each code such as TR1 and TR2 is provided in Table 4.2 above.

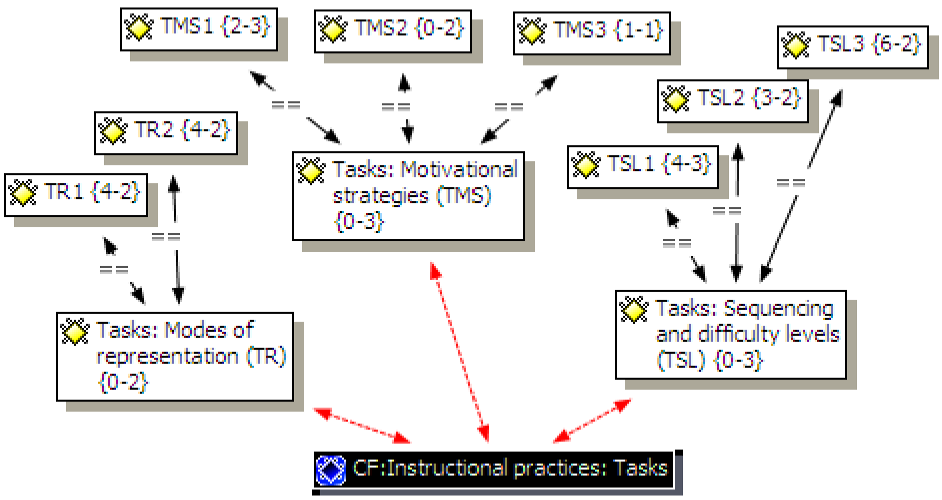

Figure 4.2: ML instructional practices: Tasks

At the end of each code, for example **TMS1**[33], there is a pair of numbers in parentheses {2-3}. The **2** refers to the groundedness, in other words the frequency with which the code was attached to quotations in the observation transcripts for a specific participant. This means that there were two incidents during the three lessons observed from a specific participant where there was evidence of the teacher capturing the learners' curiosity. The **3** is the density, indicating the number of times a code has been linked to codes in all the networks that were created. In this example it means the code TMS1 was also associated with two other codes T2 and T5 (see Table 4.3) in the subtheme **ML teaching** under Theme 2: ML teachers' PCK and beliefs. Notice that at the end of the three categories in Figure 4.2, namely: Tasks: Modes of representation (TR), Tasks: Motivational strategies (TMS) and Tasks: Sequencing and difficulty levels (TSL) the numbers in parentheses are {0-2}, {0-3} and {0-2}. This indicates that the codes TR, TMS and TSL were not associated with quotations in the transcripts. These

[33] According to Table 4.2 **TMS1** refers to Tasks: Motivational strategies: Teacher uses tasks that capture learners' curiosity and inspire them to participate.

are subthemes that were not coded as such. Instead their different lesson dimension indicators were coded.

The *code family* created for the second subtheme **Discourse** is given in Figure 4.3 below. The broken line arrows again indicate the three different lesson dimensions being linked to the code family **Instructional practices: Discourse**. The solid line arrows with double equal signs indicate the codes *associated with* the different lesson dimensions. For example codes DTL1, DTL2, DTL3, DTL4 and DTL5 are associated with the lesson dimension **Discourse: Teacher-learner interaction (DTL)**. The full descriptions of the codes are in Table 4.2.

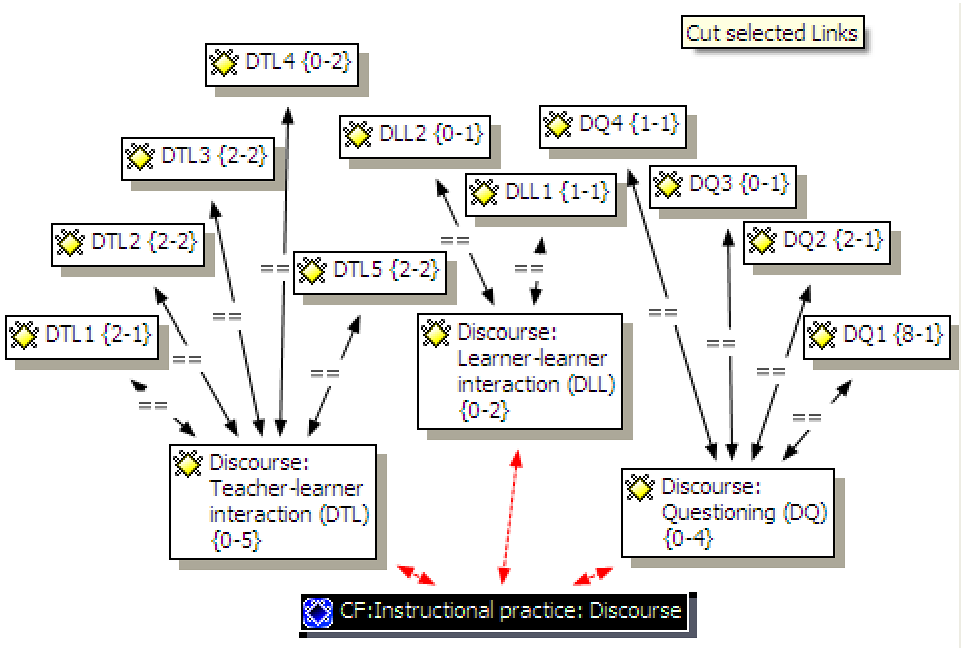

Figure 4.3: ML teachers' instructional practices: Discourse

The *code family* created for the third subtheme **Learning environment** appears in Figure 4.4 below. The broken line arrows again indicate the three different lesson dimensions being linked to the code family **Instructional practices: Learning environment**. The solid line arrows with double equal signs indicate the codes *associated with* the different lesson dimensions. For example codes LEC1, LEC2 and LEC3 are associated with the lesson dimension **Learning environment: Social and intellectual climate (LEC)**. The full descriptions of the codes are in Table 4.2.

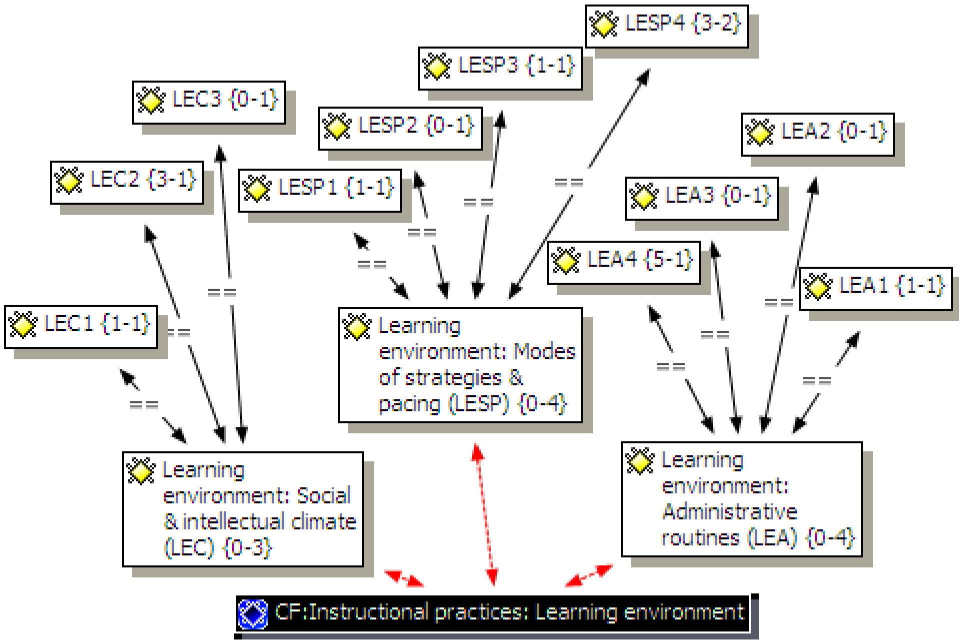

Figure 4.4: ML teachers' instructional practices: Learning environment

4.3.2.2 Theme 2: ML teachers' knowledge and beliefs

The four subthemes of the ML teachers' knowledge and beliefs are 1) MCK; 2) PCK regarding ML learners; 3) PCK regarding ML teaching; and 4) PCK regarding the ML curriculum (Hill et al., 2008). The first column in Table 4.3 below indicates the four subthemes or dimensions. In the second column are the different descriptions of the dimensions with the codes created for them. These are the codes that appear in the *coding families* on 1) ML learners (Figure 4.5), 2) ML teaching (Figure 4.6); and 3) ML curriculum (Figure 4.7). The data were collected from both the interviews and the lesson observations.

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Table 4.3: Knowledge and beliefs dimensions and its indicators as inclusion criteria for coding the data (Adapted from Artzt et al., 2008; Ball, 1990; Borko & Putnam, 1996; Hill et al., 2008; Shulman, 1986; Shulman, 1987)

PCK AND BELIEFS DIMENSIONS	INSTRUMENT: Interview (Int) Observation (Obs)	Correspond with indicators in Table B	Interview question numbers	DESCRIPTION OF TEACHERS' PCK AND BELIEFS INDICATORS (CODES)
Mathematical content knowledge (MCK)	Observation			Report on teachers' mathematical content knowledge. Record on the accuracy of teachers' content, mathematical errors made by teachers, teacher's misconceptions or misrepresentations.
ML learners (L)	L1. 1st Interview L2. 1st Interview L3. 1st Interview L4. 1st Interview L5. 1st Interview L6. Observation	DTL2/3/4/5	2a, 2b 3a, 3b 4c; 5 6 8	Teacher's ability to: L1. predict what mathematics learners will understand; but also understand why that mathematics is comprehensible to the learners; L2. predict what mathematics learners will not understand; but also understand why that mathematics is incomprehensible to the learners; L3. predict how they will come to understand it; L4. predict how learners will probably approach a task; L5. understand what alternative conceptions and preconceptions learners have that could be misconceptions and that should be rectified and reorganised by the teacher through the use of different strategies; L6. **see** what learners do, know how to listen and **hear** what they think and then be able to **act** appropriately as mentors to facilitate the learning process.
ML teaching (T)	T1. 1st Int & Obs T2. Observation T3. Observation	TR2|TSL1 TR1|TMS1 TMS2	7	Teachers should: T1. know what prior knowledge must be present to understand new work; T2. know useful forms of representation of ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject depending on the content and learners' needs, in order to make it comprehensible to them; T3. have the capacity to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the learners;

	T4. 1st Int & Obs T5. 1st Int & Obs T6. 2nd Int	TSL1|LESP4 TR3|TSL2| TMS1|TSL3	5 6a Section A (1-4) Section B 1(6);2(2-4,6) 3(1-4)	T4. have the ability to sequence content to facilitate student learning; T5. to choose the appropriate instructional strategy and instructional material for a lesson, consider tasks to set and assessment techniques to use; T6. reflect on their own practices for the purpose of improvement.
ML Curriculum (C)	C1-C6: 2nd Int C7. 1st & 2nd Int & Obs C8. 2nd Interview	 Context Nature 1 Nature 2	 Section C (5) Section C (6) Section C (7) Section C (8) Section C (1,2,4,9) Section C (10) Int.1 (4a,4b) Int. 2(3) Section B (1,3)	Teachers: C1. should have knowledge regarding the variety of resources/instructional materials available to teach particular curriculum components; C2. need to recognise the particular strengths and weaknesses of textbooks and materials they are using and should have a collection of materials they use when teaching mathematics; C3. need to be familiar with the curriculum materials studied by learners in other subjects at the same time and how it integrates with ML; C4. should be informed of the various departmental ML documents, providing info regarding the purpose and value of ML, resources to use and how to progress from one year to the next; C5. should have knowledge about the definition, purpose, learning outcomes and the new CAPS; C6. need to be familiar with the topics and level of different topics being taught in the same subject during the preceding and later years in school, in other words how topics are organised horizontally and vertically. C7. should teach content in context. The context should be applicable to the content and the teacher needs to know the context and be able to apply meaningfully. C8. View of mathematics as discipline: From traditional to formalist to constructivist (Includes the role of the teacher in his/her instructional practice) • A traditional view refers to teachers who belief that mathematics is an abstract phenomenon unrelated to reality. These teachers will then struggle to relate mathematics to real-life situations and tend to believe mathematics consists of a set of rules and procedures that must be learned mechanically with little or no

				connection to one another and hardly any relevance to their everyday lives. They also tend to separate mathematics from the discipline of discovery and creativity and an abstract procedural approach is used. • The formalist view refers to teachers who believe mathematics is characterised by logic, rigorous proofs, exact definitions and a precise mathematical language and doing mathematics consists of accurate proofs as well as of the use of a precise and rigorous language. • The constructivist view refers to teachers who believe mathematics is a constructive process where relations between different notions and sentences play an important role. Here the mathematical activity involves creative steps, such as generating rules and formulae, thereby inventing or re-inventing the mathematics. • View of ML as subject being a lower grade Mathematics and/or a life skill.
			Section B (2)	• Possible definitions of the nature of mathematics: i) mathematics is the language of science; ii) is the study of patterns; iii) is a system of abstract ideas.
	C9. 2nd Interview	Nature 3	Section B (4,5	C9. The value of mathematics and ML for people in their daily lives.

Source: Adapted from: A Cognitive Model for Examining Teachers' Instructional Practice in Mathematics: A Guide for Facilitating Teacher Reflection, by A.F. Artzt and E. Armour-Thomas, 1999, *Educational Studies in Mathematics*, 40(3), p. 217. Copyright © 1999 by Kluwer Academic Publishers. Adapted with kind permission from Kluwer Academic Publishers.

Using Atlas.ti 6 (Archer, 2009), networks of the *code families* are now illustrated and explained. The data for the three *code families* under Theme 2 were collected from both the teachers' interviews and the lesson observations. The *code family* created for the first subtheme **ML learners** appears in Figure 4.5 below. The broken line arrows indicate the six different codes (indicators) being linked to the code family **ML learners**, namely L1, L2, L3, L4, L5, L6. A full description of each code is provided in Table 4.3 above. The data linked to these six codes were collected from the interviews conducted with the teachers. The solid line arrows with double equal signs indicate other codes from the observations that are *associated with* the codes from the interviews (Archer, 2009). For example, codes DTL2, DTL3, DTL4 and DTL5 (see Table 4.2) are associated with L6 (see Table 4.3).

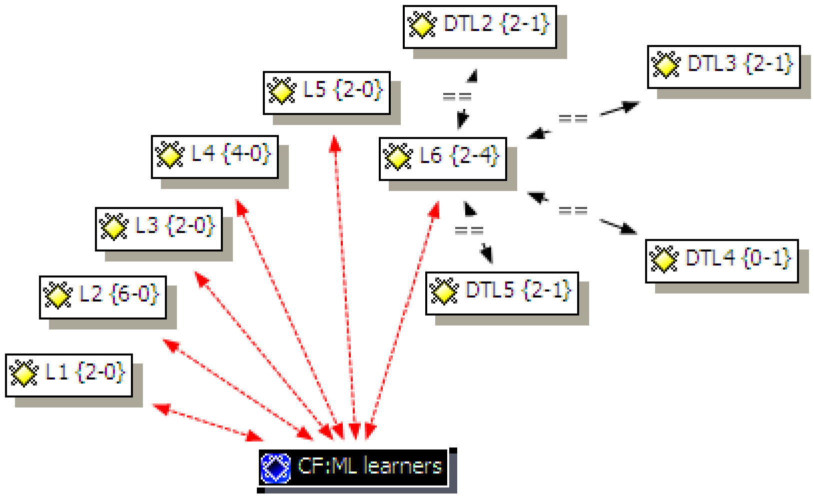


Figure 4.5: ML teachers' PCK and beliefs: ML learners

The *code family* created for the second subtheme **ML teaching** appears in Figure 4.6 below. The broken line arrows indicate the six different codes (indicators) being linked to the code family **ML teaching**, namely T1, T2, T3, T4, T5, T6. A full description of each code is provided in Table 4.3 above. The data linked to these codes were collected from the interviews conducted with the teachers. The solid line arrows with double equal signs indicate other codes from the observations that are *associated with* the codes from the interviews. For example, codes TMS1, TR3, TSL2 and TSL3 (see Table 4.2) are associated with T5 (see Table 4.3).

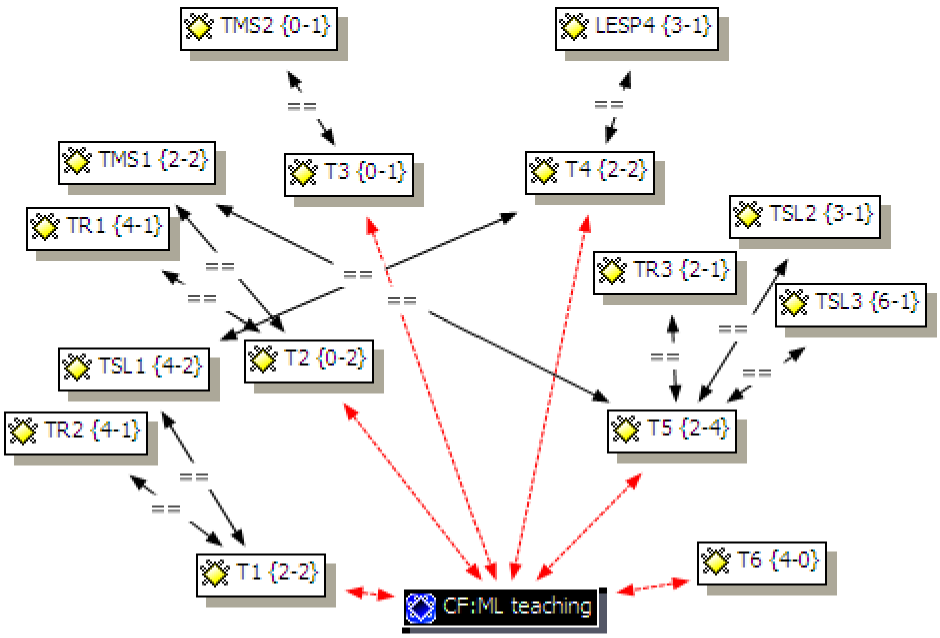


Figure 4.6: ML teachers' PCK and beliefs: ML teaching

The *code family* created for the third sub-theme **ML curriculum** is given in Figure 4.7 below. The broken line arrows indicate the nine different codes (indicators) being linked to the code family **ML curriculum**, namely C1, C2, C3, C4, C5, C6, C7, C8 and C9. A full description of each code is provided in Table 4.3 above. The data linked to these nine codes were collected from the interviews conducted with the teachers. The solid line arrows with double equal signs indicate other codes from the interviews too that are *associated with* the initial codes. For example, codes Nature 1 and Nature 2 (see Table 4.3) are associated with C8 (see Table 4.3).

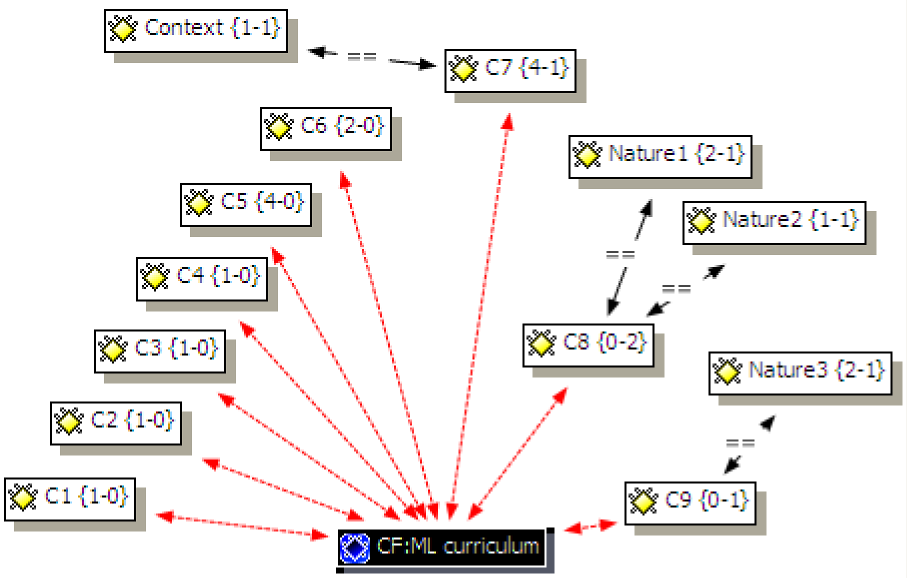


Figure 4.7: ML teachers' PCK and beliefs: ML curriculum

4.3.2.3 Inclusion criteria for coding the data

There are two tables indicating the inclusion criteria for coding the data. Table 4.2 was used to analyse the ML teachers' instructional practices. The table consists of the different lesson dimensions, namely tasks, discourse and learning environment and the respective lesson dimension indicators. The descriptions of the lesson dimension indicators serve as inclusion criteria for coding the data from the observations. Examples of each code are provided. These codes were used to analyse the raw data and reporting on the data.

Table 4.3 was used to analyse the ML teachers' knowledge and beliefs. The codes have been assigned according to MCK as well as the three PCK and beliefs regarding the ML learners, ML teaching and the ML curriculum. In Table 4.3 the descriptions of teachers' knowledge and beliefs indicators serve as inclusion criteria for coding the data. Each code has been linked with the corresponding interview question(s) as well as cross referencing to specific code(s) in Table 4.2 of the observations.

4.3.2.4 Exclusion criteria for coding the data

In the process of coding the observations, some of the activities and discourse were not relevant to my study and did not form part of my predescribed lesson indicators whereas others were inaudible. These were excluded when the data were coded. During the interviews some participants did not always keep to the question asked or sometimes used the chance to raise personal points of concern. In Table 4.4 below I listed these exclusion criteria as well as examples of text that were excluded from coding.

Table 4.4: Exclusion criteria for coding of the data

Exclusion criteria	Examples of text excluded from coding
Incidents during class observations when I could not hear what was said	This occurred when the teacher attended to individual learners' at their desks or when they had private conversations at the board or at the teacher's desk. Some of the data were inaudible when I did the transcribing.
Interruptions	Teachers that needed to attend to people who knocked on the door.
The question in the second interview regarding the ML learners' abilities and motivation as the question was included for personal interest only	*Describe your Grade 11 ML learners in terms of their a) mathematical abilities and b) motivation. How does their motivation compare with that of the Mathematics learners?*
Elaborations when questions have been misunderstood (The misunderstanding is included in the coding but not the elaboration part of the answer)	***How do you think will the learners approach these tasks?*** *All they have to do is most times when I try to give them the tasks on data handling and try to make them set the questionnaire and then try to see make it look real-life, you try to make it what people think about the*

	like a … and then you get a questionnaire and then would give that to your friends and then you collect those data and then you try to sort them out and then present them using a bar graph or line graph.
Providing detailed examples to illustrate their answers.	**Describe the ideal ML classroom in terms of instructional strategies used.** *I try to involve them as much as I can, that they understand that. They must learn through doing it, trying on their own. Try doing it, drawing the Cartesian plane, those are the points, plot them, OK join the points and what do you see? Oh OK, its curved, it's called parabolic function, how do we use it in everyday life. Because if I just draw and plot it myself they look at it but the constructing, they construct while they are doing.*

4.4 Information regarding the four participants

In the next section biographical information regarding the four participants Monty, Alice, Denise and Elaine is provided as well as some background information regarding the observed lessons. Pseudonyms were used to protect their identities.

4.4.1 Monty

Monty is a novice teacher in his second year of teaching Grades 10, 11 and 12 ML and one year of teaching Grade 10 Mathematics. He is 24 years old and completed his Baccalaureus Educationis (BEd) degree with Mathematics as major in 2010. Apart from the six ML courses he attended during 2010, organised and presented by the DoE and the District Office, he had had no formal training for teaching ML. He teaches at an inner city school of 500 learners with 18 and 35 learners in the two Grade 11 classes respectively.

The topic of Monty's first two lessons I observed was solving simultaneous equations using the substitution method, only mentioning the elimination method (Learning Outcome 2). The first two observations were done on 3 and 4 May 2011. The third lesson was on data handling where the mean, mode, median and range as measures of dispersion were discussed (Learning Outcome 4). The last lesson was observed on 9 May 2011.

4.4.2 Alice

Alice grew up in Nigeria, is 30 years old and in 1995 she obtained a Baccalaureus Technologiae (BTech) Management Accounting degree at a University of Technology with second year Financial Mathematics. She did not take any Mathematics Education or Mathematics Methodology courses. She has no experience of teaching Mathematics and it is her second year of teaching ML. She is teaching at an independent inner city school with 350 learners where the number of learners in her Grade 11 ML classes ranges from 20 to 30.

The first lesson I observed with Alice was based on the use of the quadratic formula to solve quadratic equations (Learning Outcome 1), a sequential or follow-on lesson to reinforce the work done the previous period. A student teacher was responsible for the previous lesson[34]. I observed this lesson on 9 May 2011. The second lesson was on graphing the parabola (Learning Outcome 2) using a table method and finding the intercepts but without determining the turning point. This lesson was observed on 10 May 2011. The third lesson was on data handling: the cumulative frequency, relative frequency, standard deviation and the ogive (Learning Outcome 4). This lesson was observed on 16 May 2011.

4.4.3 Denise

Denise is 42 years old and completed a BEd degree in 2003 with Mathematics and Methodology of Mathematics as two of her major subjects and also completed her BEd Honours in 2009. She obtained both degrees from the University of Witwatersrand. She completed a 40-hour course based on ML and the teaching thereof at the University of South Africa. She has seven years' experience of teaching Mathematics and it is her fourth year of teaching ML. She is teaching at a Section 21 (former model C) school in Pretoria with 908 learners where 92% of the learners are black while the other 8% consist of White, Indian, Coloured and Asian learners. The number of learners in her Grade 11 classes ranges from 20 to 32.

The first two lessons I observed were the same lessons presented to two different classes and concerned conversions from metric units to imperial units, involving capacity and mass problems (Learning Outcome 3). These two lessons were observed on 17 May 2011 and 20 May 2011 respectively. The third lesson was a follow-on lesson where more advanced conversions within the metric system only were done on capacity, mass, length, area and volume. This lesson was observed on 26 May 2011. Denise has experience and knowledge of her subject and her lessons were coloured with

[34] The situation that occurred when the student teacher introduced the topic and Alice took over the next period is discussed under Section 4.4.2.3: Mathematical content knowledge.

humorous comments so that learners participated and enjoyed her classes. Learners were involved by solving problems in class, writing on the board and answering questions.

4.4.4 Elaine

Elaine is 44 years old and completed her Higher Education Diploma (HED): Senior Primary with Mathematics and Mathematics Didactics as two of her major subjects in 1989 at the Normaal College of Education. During those years the Mathematics I-IV, offered as a major subject at colleges, was equivalent to a first year Mathematics offered at universities. She did not attend any courses on ML or the teaching thereof. She has eight years experience of teaching Mathematics and it is her third year of teaching ML. She is teaching at a Section 21 (former model C) school in Pretoria with 1 300 learners where 95% of the learners are white whereas the other 5% are black. She is responsible for only one Grade 11 ML class and there are 25 learners in the class.

During the period of observation, Elaine was busy with her revision programme. The first and third lessons I observed were based on calculating circumference, area, volume and surface-area of two and three-dimensional shapes (Learning Outcome 3). These lessons were observed on 6 May 2011 and 19 May 2011. The second lesson was based on time (Learning Outcome 3) and interest (Learning Outcome 1) and was observed on 12 May 2011. Elaine is the ML coordinator at her school and her goal is not only to equip her ML learners with knowledge and skills they can use in their lives, but also to promote the purpose and value of ML everywhere she goes. Elaine believes that the notion of contextual mathematics makes ML a valuable and interesting subject as learners' general knowledge is enriched through their experiences with contextual mathematical problems. She mentioned that her ML learners are often envied by the Mathematics learners because of their interesting lessons.

To summarise: The most relevant information appears in the Table 4.5 below:

Table 4.5: Biographical information of the four participants

	Monty	Alice	Denise	Elaine
Age (years)	24	30	42	44
Highest qualification	BEd: FET	BTech: Management Accounting	BEd Honours	HED: Senior Primary
Mathematics teaching experience (years)	1	0	7	8
ML teaching experience (years)	2	2	4	3

4.5 Theme 1: The ML teachers' instructional practices

In this section I present and discuss the findings from the observations of Monty, Alice, Denise and Elaine. All discussions on the sub-themes **Tasks, Discourse and Learning environment** are structured strictly according to the specific order of the different lesson dimension descriptors (codes) as indicated in Table 4.2[35] (Artzt, et al., 2008). The language of all quotes from Monty, Alice and Denise has not been edited. Since Elaine's classes were conducted in Afrikaans I translated her quotations into English. Background information regarding the observed lessons of the participants is given. A summary is provided at the end of this section in table form analogous to Table 4.2.

4.5.1 Monty's instructional practice

4.5.1.1Tasks

Tasks: Modes of representation (TR)

In the first two lessons on solving simultaneous equations, Monty represented the mathematical concepts by means of written examples on the board using the variables x and y as well as calculators for basic calculations (TR1). For example Monty asked the learners to write an answer of $-\frac{12}{10}$ as a decimal (TR1). He told the learners they could also use their calculators to change a decimal answer to a fraction form and taught them how to operate their Sharp and Casio calculators (TR1). In the introduction of the third lesson on measures of dispersion, Monty called a learner to the front and asked the class to name characteristics that describe him as a boy and not a girl. He used a manipulative (a learner) to demonstrate that even in mathematics concepts have certain characteristics and said:

> *So even if you go to this thing of Maths or ML, you need to have a picture of it, what are the characteristics of it? What must I identify to be able to say this is that thing?*[36] (TR1).

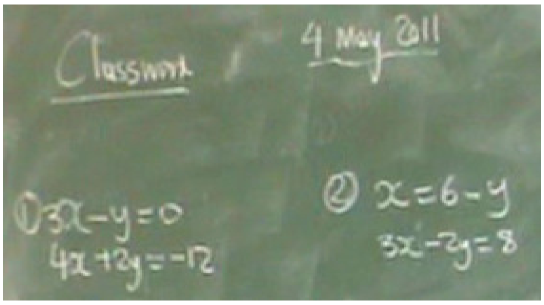
Monty followed through with the *characteristics* idea by asking the learners to provide another word for some of the concepts, for example that average is another word for mean and middle number is another word for median (TR1). He told them how to calculate the measures of dispersion but did not tell them or ask them why and when we use these measures of dispersion and did not apply these concepts to a real-life situation (TR1). He did not use multiple representations to enable learners to connect prior knowledge to the new content, but only verbally referred to prior knowledge when he said:

[35] Table 4.2 is discussed under Section 4.3.2.1: Inclusion criteria for coding the data.
[36] The language of all quotes from Monty, Alice and Denise has not been edited. I translated Elaine's quotes from Afrikaans to English.

> *We learned how to solve an unknown. We learned how to draw graphs of linear function, how to draw the graph of linear function* (TR2).

Tasks: Motivational strategies (TMS)

The tasks in both the first and second lessons (Picture 4.1) were based on pure mathematics and not on interesting and applicable real-life situations in which learners could do problem solving in groups or discuss the meaning of the solutions (TMS1).



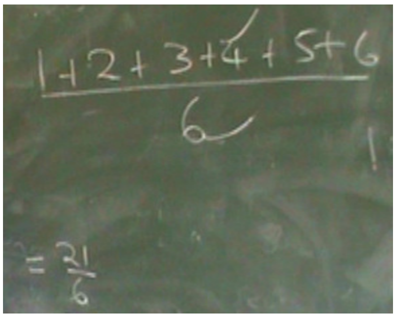
Picture 4.1: The two tasks from Lesson 1

The tasks did not capture the learners' curiosity nor inspire them to speculate on their conjectures (TMS1). The learners did not appear interested while they were listening, looking and copying work from the board (TMS1). After the first example ($\begin{matrix} x+2y=4 \\ 7x-5y=9 \end{matrix}$) of solving simultaneous equations during the first lesson, Monty said:

> *OK, now you try this one:* $\begin{matrix} y=x+3 \\ y=3x-7 \end{matrix}$ *(pause). OK, you tell me, I write for you.*

But at the end he treated this second example the same way as the first one, telling them what to do and not allowing the learners to solve the problem themselves. This second example was an easier problem since *y* was already the subject (TMS1). The examples used in the data handling lesson were not motivating to the learners (TMS1). These examples based on measures of dispersion were meant for learners with low ability and no experience (TMS2). To illustrate the mean (Picture 4.2), Monty said:

> *For example we have numbers 1, 2, 3, 4, 5, 6 and you want to calculate the mean* (TMS2).



Picture 4.2: Monty example of how to calculate the mean

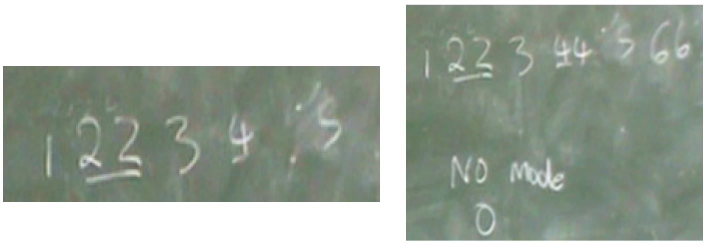
The example for the median (Picture 4.3) was where numbers from 3 to 13 were given in random order and he said:

> *The median. What's a median? A middle number. So for you to find a correct middle number you have to arrange the numbers in an ascending order…* (TMS2).

Picture 4.3: Monty's example of how to calculate the median

The example for the mode (Picture 4.4) was:

> *OK, let's move on to mode. Please, if you don't understand something, please ask. OK, the mode (erases work on board). You are given this data: 1 2 2 3 4 5. What's the mode? OK first, what's a mode? A mode is a number that appears? Girl: more than once* (TMS2).



Picture 4.4: Monty's example of how to calculate the mode

His motivation is examination driven, preparing the learners for the examination by frequently telling them what is expected of them in the examination, how to use their time sensibly and that all steps should be shown otherwise they will lose marks (TMS3). He once asked in a lesson:

> *What was difficult? Learner: Nothing. Teacher: So what I am saying to you, when you say to yourself something is difficult, it is, but once you did it right you see nothing is impossible* (TMS3).

In his introduction to the data lesson when he had a comprehensive description of how research is done, he said:

> *Why am I telling you this? But in Grade 11 I must just tell you, remember guys, life doesn't end here at school, life doesn't end at school. Outside you will need that information to use it.*

He wanted to emphasise the value of ML for learners' future lives, but did not discuss how the content is applicable to real-life situations and no such example or homework was done (TMS3).

Tasks: Sequencing and difficulty level (TSL)

Not much attention was given to sequence the tasks in order for the learners to obtain cumulative understanding of the content (TSL1). He worked from a more difficult example $\begin{matrix} x+2y=4 \\ 7x-5y=9 \end{matrix}$, where *x* or *y* first needed to be made the subject of the equation in order to continue with the solution of the problem, to an easier example $\begin{matrix} y=x+3 \\ y=3x-7 \end{matrix}$, where *y* was already the subject of the equations (TSL1). He explained the elimination method prior to the two lessons I observed in which he explained the substitution method, but he did not expect them to use both methods in order to practise them simultaneously (TSL1). In the data handling lesson the examples discussed in the above paragraph were not sequenced but connections were made with ideas learned in the past (TSL1). The tasks given in the lessons on simultaneous equations provided opportunities where learners could reinforce current work whereas the tasks in the data handling lesson were meant to revise basic Grade 10 work (TSL2). The homework for the data handling lesson was not carefully selected, instead the first four problems from the exercise were chosen, questions that were basic and easy. (TSL2) The questions based on simultaneous equations were appropriate but not set in context and therefore on Level 1 (Knowing) of the ML assessment taxonomy[37] (TSL3). These examples were basic, did not reflect quality and were on a level far below Grade 11, even for a revision lesson[38] (TSL3). The homework was suitable for what Monty did in class, but did not require the learners to do proper revision of their Grade 10 work (TSL3).

4.5.1.2 Discourse

Discourse: Teacher-learner interaction (DTL)

Monty ensured that all learners were quiet while listening, attending and copying the work from the board during his presentations (DTL1). He was non-judgmental but did not encourage learner participation, with the result that there was no evidence of the learners' ideas (DTL1). He never asked them to explain or justify themselves, instead asking lower order and basic calculation questions throughout the lessons (DTL2). There was no evidence of the teacher listening to learners' ideas and providing scaffolding to support their attempts (DTL3). Some learners mumbled answers to Monty's questions (DTL3). After the incident in which a learner wrote her solution on the board Monty asked:

> *Anyone who can do the second one? (Silence). Anyone? …… have you tried it? Anyone? ……. (All just look at him). OK first, let's go to our notes. Firstly we said we need to label them…*

[37] Discussed in Section 2.2.2.2: ML principles.
[38] Examples have already been given under Tasks: Motivational strategies.

He continued to explain in exactly the same way the solution to the first problem (DTL3). When learners answered his questions, he did not comment on their answers. He either repeated the answer or if the answer was wrong, provided the correct answer for example:

> *The mode is 2 and? Girl: 4. Teacher: 4* (DTL3).

In another example $\begin{array}{l} 3x - y = 0...............1 \\ 4x + 2y = -12.........2 \end{array}$, Monty asked:

> *Is x or y the subject of the formula? Is x or y the subject of the 2 given equations? Boy: y. Teacher: Is x the subject of the formula in the 2 equations? Another boy: No. Teacher: What you do? What do you have to do? Some learners: Third equation. Teacher: A third equation* (DTL3).

When he did the example $\begin{array}{c} x + 2y = 4 \\ 7x - 5y = 9 \end{array}$, he asked: *Is x or y a subject of an equation?*

He initially got no response and later some learners told him that *x* was the subject (DTL4). In the discourse that followed Monty became irritated, looked troubled and laughed as he could not understand why the learners did not know the answer (DTL4). There was no evidence of the teacher recognising or clarifying learners' misunderstandings (DTL5). He assisted most of the learners while they were busy with classwork (DTL5).

Discourse: Learner-learner interaction (DLL)

He did not encourage learners to listen to, respond to or question one another although there were opportunities during classwork where they could do so (DLL1). On one occasion he said:

> *If you did not understand it well yesterday you can now work with your friend. Don't allow your friend to just copy, he must ask and talk.*

Only three groups were formed with two learners per group (DLL1). One group just worked from the same textbook while the other two groups discussed the work. The rest of the learners worked individually (DLL1). There was not a learner or group of learners who dominated the verbal communication in class (DLL2).

Discourse: Questioning (DQ)

Most of the questions Monty asked were factual and of lower order, such as complete the word/sentence and calculation type of questions and in many cases he answered the questions himself (DQ1). In one lesson there were 97 such questions. Examples of such questions are:

- When $2x^7$ was written on the board during lesson 1, the following was asked:

T[39]*: OK we call this one a co…? (Referring to 2) efficient (and completed the word while some learners mumbled an answer)*
T: This one we call it…? (Indicating to the x)
L: Variable.
T: The variable (and writes it on the board).

- An example of answering his own questions was during the introduction phase of lesson 1:

  T: What are simultaneous equations? It's a combination of two linear functions.

- Factual questions such as the following were asked during the lesson on data handling:

  T: What is meant by mean? What comes to your mind? When you see mean, what comes to your mind?
  L: Bigger, smaller.
  T: Average. It's average. And once you see that average, you see lots of numbers adding each other and dividing by the number of them, that is the picture you must have

- Lower order questions during simultaneous equations :

  T: OK now we know x=-1,2. We go and substitute x=-1,2 in…?
  L: 2.
  T: Into…?
  L: 2.
  T: Into equation…?
  L: 2.
  T: 2. (Teacher did it.) What is the answer there of 4(-1,2)? Huh? What's the answer?
  L: -4,8.
  T: -4?
  L: -4.8.
  T: -4.8 (DQ1).

On only one occasion Monty asked a learner to explain her work but did not follow it through:

> *T: Explain your answer, here use this (He gives her a large triangle. She is shy and cannot look at the class and put her head in her hand.) OK people (and he takes it out of her hand) if you are given these 2 equations: Equation 1, x is the subject of the formula …. (and he continues to explain her answer)* (DQ2).

The questions did not contribute to the verbal communication and participation of learners and did not create opportunities when they could listen to and respond to each other's answers (DQ3). In general his questions were addressed to the whole class, who mumbled answers or sometimes responded in chorus (DQ4). A few times some learners volunteered to answer and on only three occasions he called on particular learners to answer his questions (DQ4).

4.5.1.3 Learning environment

Learning Environment: Climate (LEC)

To comment on Monty's relationship with and among the learners based on how they valued each other's ideas and ways of thinking is difficult as his lessons were teacher-centred with minimum interaction (LEC1). He generally did not value or seek their ideas (LEC1). When I commented in the

[39] When discourse was quoted, I used the following abbreviations: Teacher (T); Learner (L); and Researcher (R).

last interview on the class discipline I observed, Monty replied that he has classroom rules and values discipline in order for learning to take place (LEC2). He further mentioned that the principal highly values discipline in their classes (LEC2). The learners respected Monty and were well behaved, sat quietly in class listening to him and copying the work from the board. Monty only needed to discipline them once during the three lessons when he said:

> *OK hey hey hey, listen (pause), listen, the break is over. Please don't disturb my class. Shhh. So, if I see you talking, I am going to chase you out and then you will come back next term* (LEC2).

On more than one occasion he reminded them that they are not *Mathematics people* and this may either be comforting to the learners or a matter of degrading them (LEC3). Monty is confident and enthusiastic about ML and has a positive attitude towards the subject and the learners (LEC3).

Learning Environment: Strategies and pacing (LESP)

Monty used direct instruction (lecturing) as instructional strategy and his teaching style varied between a traditional and formal authority style (LESP1). Learners were involved copying work from the board, listening to his explanations and answering basic low level questions (LESP1). During the second lesson he gave the learners two problems to complete in class, but allowed only a few minutes to solve the problems before he asked a learner to write her solution on the board. Since she was too shy to talk, he again explained her solution in detail as he had done with the previous examples (LESP1). Near the end of this lesson he was running out of time and hurried through the last example by saying: *OK let's go faster there is no more time* (LESP2). I observed only two phases of a lesson in the first two lessons, namely the initiation and development phases (LESP3). Since there was no closure phase there was no opportunity to summarise or assess learners' knowledge and understanding (LESP3). Only the last six minutes of the first lesson were allocated for a class activity of which the learners used five minutes to get settled (LESP3). His second and third lessons flowed logically (LESP4). For example, in his third lesson he used the following discussion as part of his introduction:

> *T: Where can we solve this simultaneous? OK, remember we are approaching the Election Day and we need to support the campaign. Don't you think the results can be solved simultaneous, how? Remember he has to sit on the parliament and the province and we have 9 provinces né? Remember for the vote of the 18th they are going to take the result because remember people voted for this particular party or this party or organisation. They are going to add all those results and what information do you think we can get out of that? We can convert it into equations and solve simultaneous. That will tell us how many positions that party is going to get in…? Parliament, né? So you see we solve it simultaneous. Once you know the number of how many people voted for the party you know how many will sit in parliament.*

This was followed by more examples of the work they had done the previous day and he again referred to contexts at the end of the lesson (LESP4). It was unclear how the learners were supposed to connect the context to the content (LESP4).

Learning Environment: Administrative routines (LEA)

Time was not used efficiently to maximise learners' involvement as Monty spent most of the period using direct instruction (LEA1). Monty did not involve the learners in his explanation of how research is done and did all the explaining of the examples on his own; meaning no active learner involvement (LEA1). There was no opportunity for the learners to be part of an active learning process in which they could learn from each other and improve their understanding (LEA1). The time at the end of the lesson when learners were supposed to start doing classwork, was not well spent as they used five of the six minutes to settle down (LEA1). They were seated at their individual desks during content presentation, but could work in groups during classwork time if they preferred (LEA2). When Monty explained work he was always in front of the class but when the learners were working at their desks, he walked up and down the isles assisting learners with their work which contributed to a positive learning atmosphere (LEA3). The written information on the board was correct but not always well-organised (LEA4).

Summary

Table 4.6: Summary of Monty's instructional practice

LESSON DIMENSIONS	DESCRIPTION OF LESSON DIMENSION INDICATORS
Tasks	
Modes of representation (TR)	Monty used representations such as written examples on the board, variables, calculators and a manipulative. He seldom connected learners' prior knowledge to the new mathematical situation.
Motivational strategies (TMS)	The tasks he chose were not motivational to the learners. He talked about contexts during two of his lessons but the contexts were not applicable and valuable. He attempted to point out the value of mathematics so that learners would value the work they were doing but his explanation was vague and learners could not relate to it.
Sequencing and difficulty levels (TSL)	Not much attention was given to sequencing the tasks and no connections were made with ideas learned in the past. The tasks on simultaneous equations were on a Grade 11 level (Level 1) but the tasks in the data handling lesson were on Grade 10 level (Level 1).
Discourse	
Teacher-learner interaction (DTL)	He was non-judgmental but did not encourage learner participation with the result that there was no evidence of the learners' ideas, occasions when learners' thinking was challenged or even when he recognised or clarified learners' misunderstandings. He assisted most of the learners while they were busy with classwork
Learner-learner interaction (DLL)	There was minimum learner-learner interaction.
Questioning (DQ)	The questions were of lower order. The type of questions was mainly complete the word/sentence and calculation questions. The questions were addressed to the class and the responses were

	volunteered.
Learning environments	
Social and intellectual climate (LEC)	There was a positive rapport between the teacher and the learners. He maintained good discipline throughout the lessons in order for learning to take place and had a good relationship with his learners.
Modes of strategies and pacing (LESP)	His teaching style varied between a traditional and formal authority style. He used direct instruction (lecturing) as instructional strategy and once allowed a learner to work on the board. Typical of teacher-directed lessons, learners were involved copying work from the board, listening to explanations of the teacher and answering basic low level questions. Monty did not get to the closure phase of a lesson where learners' knowledge could be assessed. There was a logical flow to his lessons.
Administrative routines (LEA)	Time was not used efficiently to maximise learners' involvement as Monty used most of the period lecturing to the learners. He stayed in contact with the learners as he moved between them which contributed to a positive learning atmosphere.

4.5.2 Alice's instructional practice

4.5.2.1 Tasks

Tasks: Modes of representation (TR)

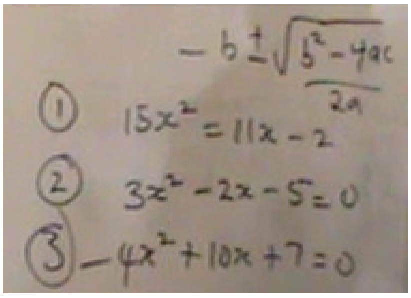
In the first lesson on the use of the quadratic formula, Alice used representations[40] such as written examples on the board, symbols, the formula, tables, graphs and calculators (TR1). In her next lesson on graphing parabolas, she used a table as well as the formula to calculate the x-intercepts (TR1). In the third lesson on data handling the following was taught within a single 35 minute period: 1) the mean, mode, median and range (when a set of data were given and when instead of a set of data a table with frequencies were given); 2) pie charts (interpreting a given pie chart and drawing a pie chart), bar graphs and histograms; and 3) tables with tallies, frequencies, $\sum$ symbol, cumulative frequencies (TR1). She did not do the standard deviation or the ogive that she also planned to do (TR1). She started the lesson by revising the bar graph and histogram, continued with another example on how to set up a table with marks of learners, tallies and frequencies when a set of data were given, then mentioned they were going to do the pie chart, but when she turned around to clean the board she continued with different elementary examples on the mode, mean and median. Afterwards she did a comprehensive example on a pie chart. She then introduced the symbol $\sum$ to the learners and continued her lecture on cumulative frequencies. When the learners did not understand her explanation, she drew a table on the board with three columns for the marks, frequency and cumulative frequency. She then mentioned the drawing of

[40] Examples of all these representations are given in the text to follow.

the ogive but told them they would get back to this as she first wanted to explain how a table could be constructed when only the frequencies rather than a set of data were given. She then explained how to find the range and median from the above table (TR1). The learners complained throughout the lesson that they did not understand the work (TR1). She used these various representations in an attempt to have the learners connect their prior knowledge with the new content, but the extent of the content and the way she presented the work was too much for the learners to absorb and led to confusion (TR2). Some learners just withdrew from all activities during the lesson (TR2).

Tasks: Motivational strategies (TMS)

As I have said, on the day before the first observation, a student teacher taught the learners how to use the quadratic formula to solve quadratic equations. In the first lesson I observed, Alice decided to do three more such examples (Picture 4.5) with the learners.



Picture 4.5: The three tasks Alice gave the learners during lesson 1

She wrote three quadratic equations on the board of which only the first equation was not given in standard form (TMS1). They only did the first one ($15x^2 = 11x - 2$) and much later during the lesson took another example from the textbook ($x^2 - 10x + 25 = 0$). The tasks provided the learners with an opportunity to pursue their conjectures (TMS1). A group of learners were motivated to take part in the lesson and to pursue their conjectures as the teacher was making many mistakes[41] on the board, but neither the teacher nor the learners could correct the errors in the teacher's work (TMS1). Only a few learners in front of the class participated in the lessons by answering questions or asking questions that Alice most of the times could either not hear or understand. The rest of the learners only copied all written work from the board and some were lying on their arms or talking to each other (TMS1). In the data handling lesson some of the learners were inspired to take part in searching for ways to correct their work or to try and find meaning and understanding (TMS1). There was no evidence that Alice took into account the diversity of learners' interests, abilities and experiences (TMS2). She generally presented her lessons on a level suitable for Mathematics and not ML learners as many of the tasks

[41] Examples of these mistakes are provided later on under Mathematical content knowledge.

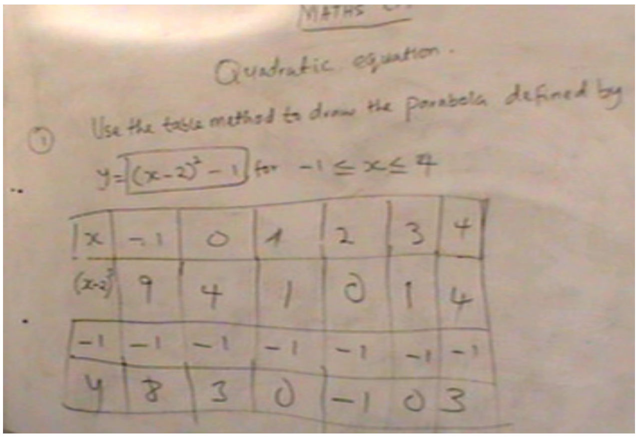
were on Mathematics level as will be discussed under the next subheading (TMS2). Alice did not point out the value of the mathematics being learned, which could have contributed to learners' appreciation of the subject (TMS3).

Tasks: Sequencing and difficulty levels (TSL)

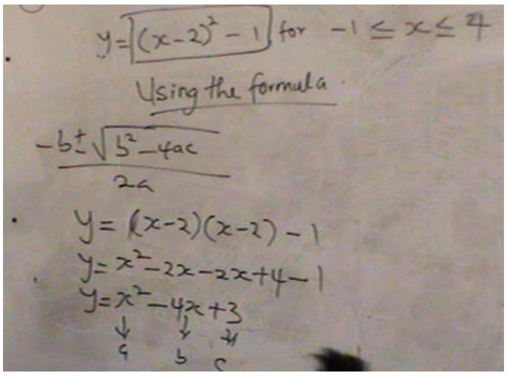
There were incidents for which Alice sequenced her activities. For example, during the first lesson she revised the standard form of quadratic equations and the meaning of the variables *a*, *b*, and *c* (the coefficients) before using the formula (TSL1). For most parts of the lessons Alice tried to link the content with other relevant content or even prior knowledge, but it was not done in a sequential and meaningful way. This resulted in the learners being confused (TSL2). For instance, after she finished the first example of using the formula to solve a quadratic equation, the following dialogue followed:

T: You remember when we draw the graph (and she erases part of the board and draw a table for x and y coordinates) Quiet guys! (She writes: $y = x^2$) Guys! Now you are not given any formula. We need to start at a negative.
L: Why do you start with -2?
T: Because they don't give it. I am just assuming this is a problem. (She completes the table). This is now where you draw your graph. (She draws two graphs below the table). This is your positive and this is your negative (indicating to the first and second graph). I am not drawing this one (Pointing to example they did). Quiet!
L: Shhhh.
T: Let's draw (and she draws a set of axes and labels them. Learners talk and teacher looks at example and erases the set of axes before she could even draw something on the axes) Shhh shhh. OK.
L: Mam, where's my textbook?
T: OK, we have $6x^2 + x = 12$. Quiet please! If you don't want to learn, you can leave the class (and she continues to solve $6x^2 + x = 12$).

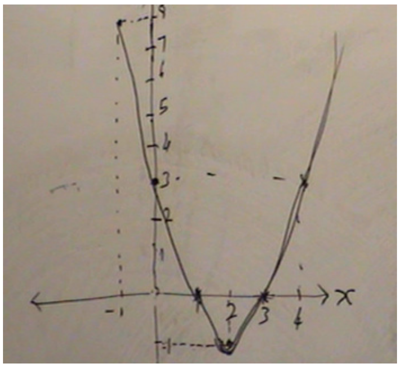
This is but one example of Alice jumping between examples and incomplete explanations (TSL2). In the second lesson when learners had to draw the parabola, they started to draw $y = (x-2)^2 - 1$ for $-1 \le x \le 4$ using a table method (Picture 4.6) followed by the intercept method using the quadratic formula (Picture 4.7). Here she stated that they did not need to calculate the turning point as the graph (Picture 4.8) would automatically go through the correct turning point if they worked accurately (TSL2).



Picture 4.6: Using a table method to draw the graph of a parabola



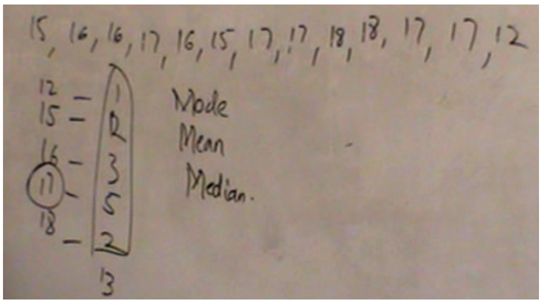
Picture 4.7: Using the formula to draw the graph of the same parabola above



Picture 4.8: The graph of the parabola

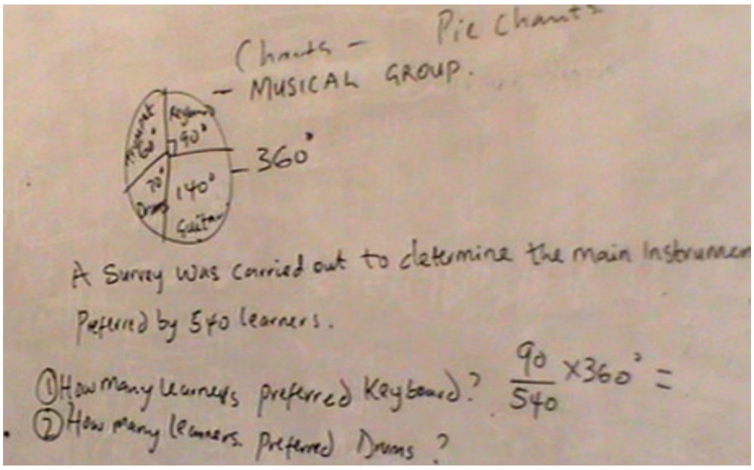
The tasks were on Grade 11 level and suitable for what the learners were supposed to know and be able to do, but would only need to practise (TSL3).

In the data handling lesson she sequenced the concepts and content to be covered during the lesson as she revised, in this order, the mode, mean, median and range (Picture 4.9) before she introduced the cumulative frequencies and ogives (TSL1).



Picture 4.9: Calculating the measures of dispersion from a list of data

Although the lesson was well sequenced, the tasks within the presentation of a concept were not sequenced so that learners were not able to progress in their cumulative understanding of the content (TSL1). She gave an appropriate and interesting example using a pie chart was given (Picture 4.10) and learners had to answer questions based on the pie chart.

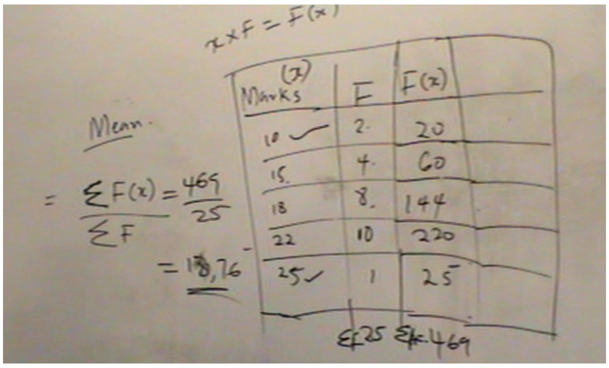


Picture 4.10: Task given based on the pie chart

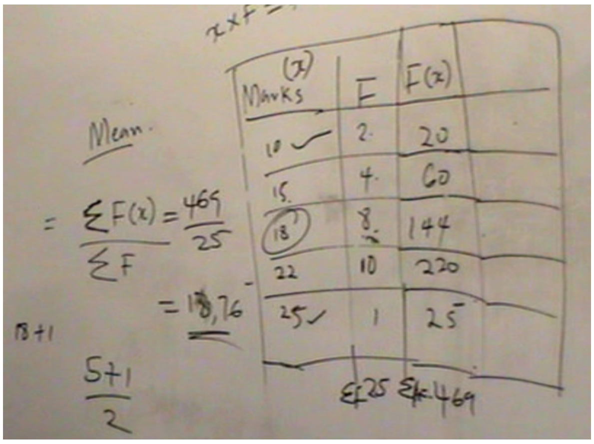
While the learners were still struggling with the task the following discourse took place:

> *T: But sometimes they don't give you this. You are given this information (pointing towards answers) and then you are asked to represent it like this (pointing to the pie chart). Now take for example (erases the pie chart) you are given this, you are only given the number of learners, the answers and then they ask you to represent this information in a pie chart. What do you do? (Silence.)*
> *L1: divide by 540.*
> *L2: 135 over 540.*
> *T: Remember it's a fraction. This is a fraction of this (pointing to 135 and 540), so it's this divided by this times what? 360, which gives you 90 degrees which was given here. So sometimes you are given data to represent in a pie chart, sometimes you are given the pie chart and you have to try and get the number of learners. OK, I am sure you are OK now with your pie graph and the frequency.*

After this verbal explanation she immediately continued with another example of using a table and frequencies to calculate the mean (Picture 4.11) and median (Picture 4.12) (TSL1).



Picture 4.11: Using a table and frequencies to calculate the mean



Picture 4.12: Using a table and frequencies to calculate the median

The extract from the observation given below and based on Picture 4.12 above is another example where learners could not progress in their cumulative understanding of different methods for finding the mean and median (TSL1.

T: OK, what about if you want to find your median using this table, what do you do?
L1: You write it in ascending order.
T: But now your marks are in ascending order and you have your frequency and your total frequency is what?
L2: 25.
T: 25 and you know that you have 25 data's. So it's an odd number. So you have to get the middle number, so how do you do that?
L3: You say 18+1 over …
L4: No.
L3: You said 18+1. (The boy takes 18 as middle number in the table).
L4: Mam, you said there are 5 numbers, so it is 5+1 over 2.
T: Remember this is the data, this is the frequency (pointing to the table), so it means you have two 10's, four 15's, eight 18's; ten 22's and one 25. The frequency is 25. So its 25 plus 1 divided by 2 its 13. So your 13th data is going to be your median. So this plus this gives you 6 and this plus this gives you 14 so your 13th data is 18.
L3: But that is what I said (TSL1).

The learners did not appear confident about finding the measures of dispersion using two methods as she never applied both methods using the same example and they also needed clarity concerning aspects of the pie chart (TSL1). The learners were not involved in individual class work as the teacher did several examples either verbally or in writing on the board throughout the lesson (TSL2). Many of these examples were not relevant to the new content she actually planned to introduce and by the time she got to the new content the learners were exhausted and confused (TSL2). The curriculum does not require learners to use the formula $\frac{n+1}{2}$ to find the median and the symbol $\sum F$ to find the sum of the frequencies (TSL3). All tasks were on Level 1 (Knowing) of the ML assessment taxonomy except for the only contextual example, the one on the pie chart that was on Level 2 (Applying routine procedures in familiar contexts) (TSL3).

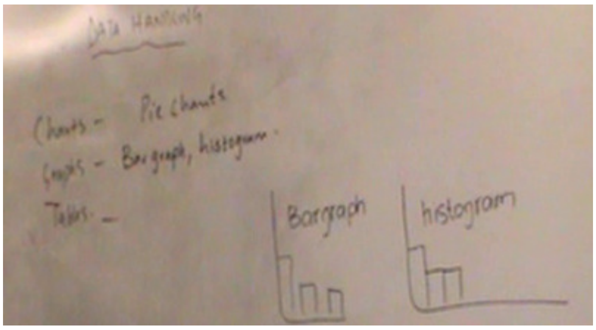
4.5.2.2 Discourse

Discourse: Teacher-learner interactions (DTL)

During the first lesson Alice used the quadratic formula incorrectly ($x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$) and also omitted brackets during the substitution, causing confusion and chaos in class as several learners talked at once to Alice and one another in attempts to clarify the problems. At first Alice ordered them to keep quiet and later shouted at them as she tried to find the problems herself. Since she could not identify her mistakes, she allowed participation from the group of learners sitting in front while the rest of the class was ignored (DTL1). During the second lesson she did not involve any of the learners and seldom looked back at the learners in the class while working on the board (DTL1). In the third lesson she encouraged a little participation from the learners in front by asking questions. The rest of the boys at the back were still not involved (DTL1). A girl once attempted to rectify the mistakes on the board but Alice ignored her and she went back to her desk (DTL2). On two other occasions Alice asked a boy and later a girl to come and write on the board but she did not require the learners to explain their work. Instead Alice corrected the boy's work by telling him what to write (DTL2).

Although Alice allowed the learners to become involved in the first lesson, she was too anxious to maintain control of the lesson and the learners and as a result she could not listen carefully to their ideas in order to support their thinking (DTL3). Instead of listening to the learners' ideas to direct her instruction, she used a formal authoritative style and told the learners what to do while demonstrating on the board. In the second lesson she kept strictly to this style to avoid the chaos of the first lesson. Alice said to the learners: *Don't ask me how to get this; you have to look at me* (DTL3). When she was busy with the example discussed above, she did not prompt the learners to justify their answers, instead she re-explained the content the same way she did before. Many times she would not comment on their answers, would make a face and re-explain (DTL3). The girl who drew the bar graph and histogram (Picture 4.13) on the board during the third lesson asked the teacher:

L1: Mam, can I draw it? (Girl comes to the board and draws both graphs on the board).
T: Shh, quiet.
L1: It's a bar graph (draws the one on the right saying).
L2 No, it's a histogram. (L1 writes 'histogram' and then draws the graph on the left). It's the bar graph.
T: OK, you know sometimes you get information that you can represent on the bar graph or histogram. OK, then it's your tables, your table always consist of the marks, the tallies, you still remember your frequency.



Picture 4.13: Learner's work on the board during the introduction of the third lesson

The teacher did not follow up on the girl's sketches by discussing the difference between the bar graph and histogram or the use of these graphs to provide scaffolding to support the learners in their conceptual understanding (DTL3).

In the data handling lesson where Alice was more comfortable with the content, she frowned and made a face when the learners gave incorrect answers to her questions and also when they mentioned that they did not understand the work (DTL4). She became irritated when she could not understand their misunderstandings or misconceptions (DTL5). She said it was impossible not to understand data handling especially since all content of the lesson was supposed to be easy and well-known to the learners (DTL5). There was no evidence that she recognised learners' misunderstandings. For example when she had discussed the pie chart and learners complained that it was complicated and she just replied that it was not and continued with the solution of the problem. When she completed the pie chart she said*: OK, I am sure you are OK now with your pie graph and the frequency. Ok, turn to p. 37 (and she starts writing:* $\sum$ …. (DTL5).

Discourse: Learner-learner interactions (DLL)

ML classrooms are supposed to be learner-centred. Instead of applying this approach, Alice wanted the learners to keep quiet (DLL1). The learner-learner interactions were learners talking to each other about non-mathematics issues and discussions of possible ways of correcting the teacher's mistakes on the board (DLL1). The discourse between learners was therefore not as a result of opportunities Alice created enabling them to discuss the work (DLL1). She allowed a group of learners to dominate the verbal communication in the class and it seemed that she depended on their assistance (DLL2). She ignored the boys at the back who did not participate at all (DLL2).

Discourse: Questioning (DQ)

The questions Alice asked while writing on the board were generally memory questions and of a low level such as basic calculation, and complete the word/sentence type of questions (DQ1). Examples taken from the different lessons are:

- *T: What do you have to do? You have to get it in standard form so you need to take it over to this side and she writes and the sign is going to change when you take it over.*
- *T: So with this, all you have to do is substitute into your formula, a is? 15, b is? -11 and c is? 2.*
- *T: How do you find the minimum?*
  L: You just see it.
  T: Where do you get it?
  L: It's at -1.
  T: This is the minimum value (showing at turning point and continues with other work).
- *T: Do you understand this?*
- *T: Your frequency is what?*
  L: It's the number.
  T: OK, it's the number (and she continues to do a new example on the board) (DQ1).

When Alice listened to learners' answers she did not ask them to clarify their answers. Instead she would provide the answer:

> *T: To find your mean, your mean is always the?*
> *L: Middle number.*
> *T: No, mean it's the sum of the data divided by the number of data* (DQ2).

Although she did not create many opportunities for learners to contribute to the verbal communication, the group of girls who did take part in the lessons had the opportunity to question the teacher and their peers (DQ3). Learners' responses were mostly volunteered or chorus. Once or twice she did call on a specific learner (DQ4).

4.5.2.3 Learning environment

Learning Environment: Social and intellectual climate (LEC)

From what I have observed Alice did not maintain a positive relationship with and among the learners (LEC1). She seems to care about them as she claims to sacrifice her breaks to be available for learners to come to her for help (LEC1). Not much evidence could be found when Alice valued her learners' ideas or even the student teacher's ideas and ways of thinking (LEC1). Alice struggled to control the learners because of the mistakes she made in the first lesson and the overload of content she covered in the third lesson and frequently had to shout at them: *Quiet please! If you don't want to learn, you can leave the class*. The reason for the learners' misbehaviour was that learners were confused and discouraged and Alice could not handle the situation (LEC2). Generally she did not create enough opportunities for learner participation which is typical of a teacher-centred strategy (LEC2). From the beginning of the second lesson she was strict and enforced discipline ensuring they kept quiet, sat at their own tables and copied the work from the board and therefore needed to discipline the learners only five times (LEC2). If Alice had had a positive attitude towards the learners and the subject as she stated in an interview, it was not evident in her lessons (LEC3). She rather appeared bored, irritated and un-enthusiastic, never giving the learners any accolades (LEC3).

Learning Environment: Modes of instruction and pacing (LESP)

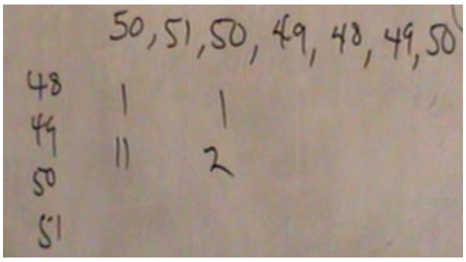
Alice's teaching style varied between a traditional and demonstrative style (LESP1). She used direct instruction (lecturing), a teacher-centred approach in her lessons as well as a little discussion with a few learners in front of the class (LESP1). On two occasions learners worked on the board: the one learner drew the bar graph and histogram and the other wrote a solution on the board (LESP1). The direct instruction strategy Alice used did not always support learner involvement and goal attainment and she was not aware of the learners' lack of knowledge and skills regarding the two topics she covered. Alice assumed that if she understood the work, the learners would understand it too (LESP1). No assessment of the learners' knowledge was done and there was no evidence that Alice's goals had been reached (LESP1). The only time Alice provided time for the learners to express themselves was during the first lesson when the learners tried to rectify Alice's work and to discuss the problems and the nature of the solutions with one another (LESP2). There was no other occasion when Alice structured the time necessary for learners to express themselves and to explore the mathematical content (LESP2). In most of the lessons she did not use her time effectively to accommodate all three phases of the lesson (LESP3). She did not have time for closure at the end of the lesson when she could have summarised or assessed the learners' knowledge and understanding (LESP3). As far as the logical flow of the lessons is concerned, Alice could sequence the content in the lesson, but failed to have a logical flow in her explanations of specific concepts as she attempted to provide the learners with too much disorganised information and incomplete explanations (LESP4).

Learning Environment: Administrative routines (LEA)

During the first lesson Alice did allow time for learners' involvement in discourse, but it was not the result of effective procedures and management of her classroom (LEA1). There was not enough time allocated for learners to be actively involved in working on tasks in class. Most of the time she ordered them to keep up with her as she worked on the board and would then say:

T: Can I erase this?
L: No.
T: Why? What is taking you so long to write (and she erased it and wrote the following example) Quiet! (LEA1).

The classroom arrangement was appropriate to the lesson style as learners were seated in three long rows in this very wide classroom, seated at desks with no space between them (LEA2). Alice's position in class during the first and third lesson did not contribute to learners' conceptual understanding as she was in front of the class the whole period, busy working on the board, frequently looking things up in her textbook and talking only to the learners in the front (LEA3). During the second lesson she once walked through the class (LEA3). The work she did on the chalkboard was not organised, she cleaned wherever she needed space to write, causing learners to be confused as they needed to listen and copy the work from the board (LEA4). Some of the examples on the board were incomplete since she often completed her explanations verbally. Sometimes information was missing or unrealistic, like a table without a heading (Picture 4.14) or the angle measurements of the pie chart[42].



Picture 4.14: Example of no headings given in a table

Summary

Table 4.7: Summary of Alice's instructional practice

LESSON DIMENSIONS	DESCRIPTION OF LESSON DIMENSION INDICATORS
Tasks	
Modes of representation (TR)	Alice used representations such as written examples on the board, symbols, the formula, tables, graphs and calculators. She could not proficiently use the various representations to connect learners' prior knowledge with the new mathematical situation.
Motivational strategies (TMS)	Alice treated the ML learners as if they were Mathematics learners, not taking into account that they were of lesser ability. The only time Alice did an example that was set in context was in the third lesson. Although a small group of learners took part in the lessons, the majority of the learners did not. She did not point out the value of the mathematics being learned.

[42] See Figure 4.3 discussed under Tasks: Sequencing and difficulty levels.

Sequencing and difficulty levels (TSL)	The tasks she chose were appropriate and on Grade 11 level (Level 1) but were not presented logically or in context to ensure that learners were motivated.
Discourse	
Teacher-learner interaction (DTL)	Her interaction with the learners was at times judgmental and it could not be said that she encouraged participation or even created opportunities where learners' thinking was challenged. She did not recognise or acknowledge her own and the learners' mistakes and misunderstandings.
Learner-learner interaction (DLL)	Learner-learner interaction was observed during the first lesson but this was not as a result of opportunities Alice created for learners to take part in discussing the work. Instead the learners discussed possible ways to correct Alice's mistakes on the board while others talked about non-mathematics issues.
Questioning (DQ)	The types of questions asked were memory, calculation, and complete the sentence questions. Learners' responses were volunteered or chorus.
Learning environments	
Social and intellectual climate (LEC)	Alice did not establish a positive relationship with and among the learners by valuing the learners' ideas and ways of thinking. She at times appeared bored, irritated and unenthusiastic, not praising the learners' work.
Modes of strategies and pacing (LESP)	Since Alice used direct instruction (lecturing) as instructional strategy, a large amount of information was shared verbally. Some of the explanations were done incompletely on the board. This strategy did not always support learner involvement and goal attainment. Generally she planned too much content per period causing her to lose the logical flow of her lesson.
Administrative routines (LEA)	During the first lesson Alice did allow time for learners' involvement in discourse but it was not the result of effective procedures and management of her classroom. She needed their input to correct her mistakes on the board. The classroom arrangement was appropriate to the lesson style but Alice's position in class did not contribute to learners' conceptual understanding as she was in front of the class most of the time.

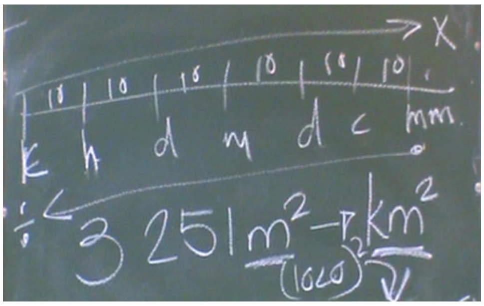
4.5.3 Denise's instructional practice

4.5.3.1 Tasks

Tasks: Modes of representation (TR)

In the first two lessons on conversions from metric to imperial units, Denise used representations[43] such as written work on the board, conversion tables, the variable x to find the unknown values, calculators and during the third lesson she used a diagram (Picture 4.15 below) to explain conversions between different units of length (TR1).

[43] Examples of these representations are discussed under Discourse.

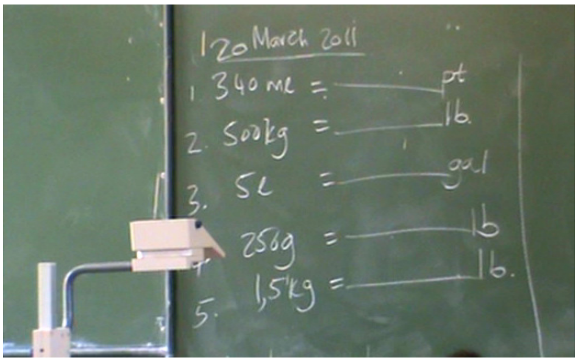


Picture 4.15: Diagram used for conversions between different units of length

She used this representation to enable the learners to connect their prior knowledge of different units of length to the conversions between the different units of length (TR2). Most of the learners used equations and cross multiplication to solve the unknown value and she reminded them that ratios could also be used to solve the unknown value (TR2).

Tasks: Motivational strategies (TMS)

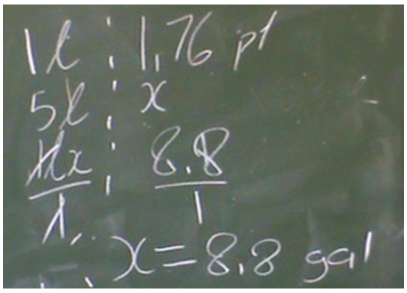
Denise treated the lessons as Mathematics and not ML lessons where the tasks were not set in a context as prescribed by the DoE (2003a), but were instead asked directly (TMS1). The following tasks (Picture 4.16) were given in the first and second lessons of which only the first three were completed during the first period (TMS1):



Picture 4.16: The tasks during lesson 1 and lesson 2

Denise's learners were inspired to participate in the lesson which might be due to Denise's teaching style and not necessarily as a result of the nature of the tasks (TMS1). From the following it appeared as if Denise took account of the diversity of learners' abilities (TMS2). She knew all her learners by their names and randomly called on learners to come and work on the board. On one occasion when a learner made mistakes, she called on another learner to come and rectify the work. Another time she also called a specific learner as she knew he used a method different from the rest of the learners who worked on the board (TMS2). All learners who had worked on the board used equations to solve the problems. She then asked another learner and when he started to use ratios instead of equations

(Picture 4.17), she said: *[y]ou know why you are doing the problem, you can see I have purposed it, my boy (indicating to the ratio he uses)* (TMS2).



Picture 4.17: Learner using ratios instead of equations

These given tasks were set in context in the textbook: a South African company exporting food products to the UK (TMS3). Denise did not mention this context at all, not even to elicit a class discussion on export in order to increase the learners' interest in the lesson and their appreciation of the value of mathematics in everyday life situations (TMS3). The only time she pointed out the value of the mathematics they were learning was when she told them: *you must understand so that if somebody is absent from class you can explain it to him* (TMS3). Her motivation is examination driven, preparing the learners for examination through practice (TMS3). She further advised them to work faster as they would not be able to finish the coming examination paper in time at their current working pace. She timed the learners while working on their classwork and after one problem she said: *you must take three minutes and you used 10 minutes,* using the mark allocation of the problem as a time guide (TMS3).

Tasks: Sequencing and difficulty levels (TSL)

Denise sequenced the tasks over the two different lessons she did on conversions which enabled the learners to progress in their cumulative understanding of the content (TSL1). The prior knowledge of these conversion lessons was the different units of measurement as well as the meaning of the different concepts (TSL2). It was only at the end of the third lesson that Denise asked the learners to identify the different concepts according to the unit of measurement. She did not ask them to explain or define the concepts:

T: Number 1? Length, mass or capacity?
L: Capacity.
T: Can you see it? Right. and number 2? What is it?
L: Mass.
T: Mass and number 3 Jenny is doing now?
L: Capacity. (Learners complained that they wanted to leave since the bell had rung) (TSL2).

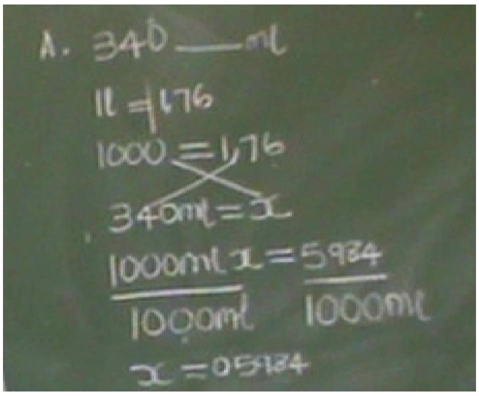
These two lessons were revision lessons so learners were supposed to know how to perform the conversions but still needed to improve on their skills (TSL2). Denise used this opportunity to identify and correct learners' common errors and misunderstandings (TSL2).

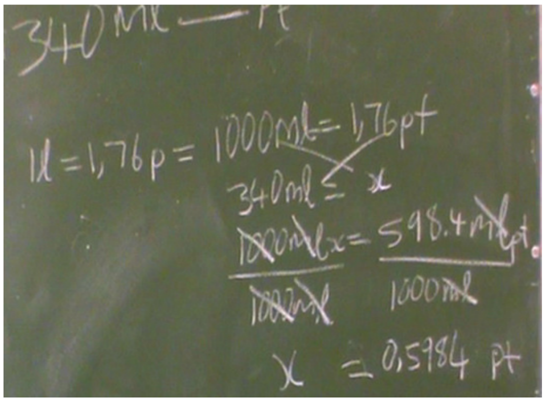
The content of the tasks were appropriate and on Grade 11 level but since no task was set in a context, her tasks were only on Level 1 (Knowing) of the ML assessment taxonomy (TSL3). In the first lesson she gave the learners conversions within the metric system such as *change 340ml to litres* as well as conversions from metric to imperial units for example: complete *500kg* = ____ *lb*. The tasks were based on capacity and mass only. In the second lesson the tasks were based on conversions within the metric system but included not only mass and capacity but length, area and volume too. Some of the learners complained about the complexity of the area and volume tasks (TSL3).

4.5.3.2 Discourse

Discourse: Teacher-learner interactions (DTL)

Denise encouraged participation from the learners during all three lessons as she walked through the class attending to learners' work, questioning and explaining to them (DTL1). She further called specific learners to work on the board and involved the rest of the learners by asking them to comment on the work on the board (DTL1). Learners demonstrated their work in writing on the board (Picture 4.18) but were not asked to explain or justify their work (DTL2).





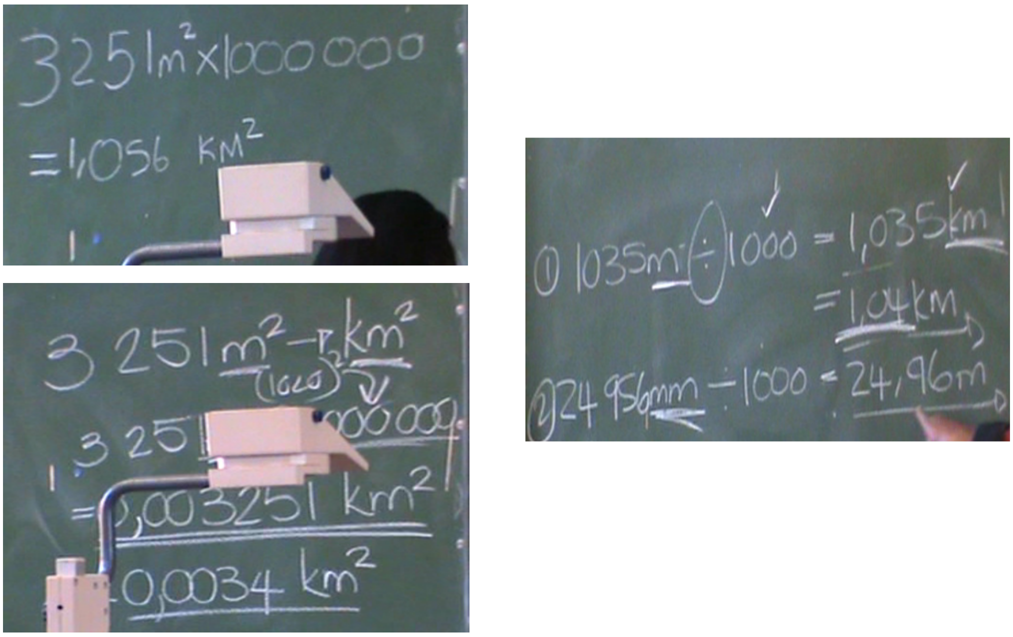
Picture 4.18: The work on the board of two learners

Denise based her instruction on what she saw the learners wrote on the board but did not listen to their explanations. She pointed out the errors and through questioning she involved the learners to take part in doing the corrections (DTL3). Once Denise asked a learner to come and correct another learner's work on the board and when he started to do that she said: H*ey David*[44]*, leave Cindy's business, go and write your own stuff so that we can compare* (DTL3). There was little evidence of Denise listening to learners' ideas. Instead she looked at their written work in order to provide scaffolding to support their thinking (DTL3). When they converted an area problem, the following discussion ensued:

> *L: Must I say the amount to the power of 2?*
> *T: (Denise immediately starts writing on the board and sings the following song:) King Henry died a miserable death called measles. This should be in your computer all the time (pointing to their heads), when to multiply and when to divide.*
> *… (Denise continues to explain Picture 4.15 to the class).*
> *L: No mam, I want to know do you square the amount?*
> *T: No!! But OK, it's a good question (and Denise continues with next task)* (DTL3).

Denise applauded the learners' answers and made comments such as: *You did excellent so far, guys* and *I am happy the way she is doing it, not looking at her textbook, because that is what we assess, understanding* (DTL4).

Denise marked some of the learners' work on the board as she would have marked a problem in a test (DTL5). Based on the following work from three different learners (Picture 4.19) the following discourse took place serving as proof of her recognition and clarification of learners' misunderstandings and misconceptions:



Picture 4.19: Work from three different learners

> *T: So now your conversion, it's squared metre to squared km. What's wrong there?*
> *L: You're supposed to divide.*
> *T: You're supposed to divide because it's from metre to kilometre. And you have to?*

[44] Pseudonyms were used instead of learners' true names.

L: Multiply.

T: Multiply. OK? So that is where your problem is. So when you move from m to km, what do you do? Multiply or divide? So come and correct it. (She erased the step from the board and moved to the next problem on the board.) So when it's m to km, it's correct? (She encircled the division sign.) So from metre to km it's a 1000, you divide by a 1000, so it's correct and this is correct. Those are the basics. Now the problem is here (underlying the 1,04) two significant figures. So two significant figures, it gives you 1,0cm, right? (She went to the next problem.) So here it's mm to what? m, so we divide by what? a 1000 (and marks it right), gives you 24,96m. Two significant figures, you did not answer that. 24,9 what? 24,96 I said, but now 2 significant figures? Anyone to help? (She called on a learner.) What is the answer?

L: 25.

T: 25. Don't forget to round off. (She goes to Learner 3' work.) You said divide by 1000000 why? Because moving from m to km you divide by 1000, now it's 1000 to the power of 2 (and she wrote $(1000)^2$*, so you get 1 million. So you divide by million and this is the answer. It's correct. Now you're answer to 2 significant figures? (The learner did not do that yet). Rectify your results. (Learner corrected her work on the board)* (DTL5).

Discourse: Learner-learner interactions (DLL)

She did not encourage the learners to listen to or question each other but instead most of the discourse was between Denise and the learners (DLL1). There was one incident during the second lesson when Denise left the class for a few minutes. Suddenly most of the learners wanted to help the learner who was at that stage making a mistake on the board, but he just became more confused. By the time Denise entered the classroom the learners became quiet and she continued to correct his work (DLL1). There was no learner or group of learners who dominated the verbal communication in class (DLL2).

Discourse: Questioning (DQ)

The lessons were characterised by questions directed at the learners (DQ1). Most of the questions were calculation, memory and convergent questions (DQ1). Many questions required the learners to look up conversions from the table in the textbook such as: *1kg equals to how many pounds?* (DQ1). Generally Denise did not allow enough time for the learners to become engaged in the discourse. Instead she provided the answers herself (DQ1). Denise did not ask learners to clarify or justify their ideas and two such examples are given below (DQ2):

- *T: Is the area one now correct? Is this one correct? (She looked at specific learners.) Is this now correct?*
  L: (mumbles something no one could hear).
  T: If you think it's not, then you just say NO, because it is as if you've got doubts. Is it correct that one? (She pointed to another learner). 3 251 squared meter is how many squared km? So now your conversion, its squared metre to squared km. What's wrong there?
  L: You're supposed to divide.
  T: You're supposed to divide because it's from metre to kilometre (DQ2).
- *T: So here it's mm to what? m, so we divide by what? A 1000 (Denise marked it right), gives you 24,96m. Two significant figures, you did not answer that. 24,9 what? 24,96 I said, but now 2 significant figures? Anyone to help? (She called on a learner.) What is the answer?*
  L: 25.
  T: 25 (DQ2).

Denise created opportunities for learners to communicate and participate by answering questions she posed throughout all three lessons but she did not create such opportunities among the learners (DQ3). Learners' responses were mostly teacher-selected and at times volunteered (DQ4).

4.5.3.3 Learning environment

Learning environment: Social and intellectual climate (LEC)

Denise maintained a positive rapport with the students as she valued their attempts (LEC1). There was one incident when the learners objected to a learner's answer who then appeared puzzled. Denise then told the class to calm down and to notice that the learner had worked accurately but only misread the value from the conversion table (LEC1). Although Denise could be humorous at times which the learners absolutely enjoyed, she was also very strict and did not hesitate to remind the learners of the appropriate classroom behaviour (LEC2). Comments such as the following were frequently heard:

- *Take your hands out of your pocket*
- *Stop talking*
- *T: Is Nelius correct?*
  L: No.
  T: Don't say no under the table please, speak up
- *Shhh, we are just checking, don't fight. We are not fighting* (LEC2).

Denise had a positive attitude towards the subject and learners and made comments such as:

- *You are just writing like a professor there né? (Everyone laughed)*
- *Keep on practising till we don't see that minor mistakes. Keep on practising.*
- *So it's nice when you say you are ready, it's nice* (LEC3).

A more negative incident occurred when she said: *Anyone who does not have a calculator, you sit on the floor and then you complain to your parents, so that they can give you a calculator. (One boy sat on the floor)* and she said: *Oh, we have one customer today (and she let him sit there)* (LEC3).

Learning environment: Modes of strategies and pacing (LESP)

Denise's style of teaching was that of a facilitator. She had a learner-centred approach where discussion and learners writing on the board were used as instructional strategies (LESP1). These strategies supported learner involvement throughout all three lessons and learners had sufficient time to express themselves when answering questions either in their books or on the board (LESP1). In the first lesson Denise did one example on the board to demonstrate and explain what was expected of them (LESP1). She then gave the learners problems to solve individually and after each problem they did the corrections together. In the other two lessons the learners had already completed the tasks at home, so

Denise used the entire period to do corrections (LESP1). The corrections were done by asking learners to write their solutions on the board which Denise then used to guide her instruction (LESP1). She pointed out the learners' errors and misunderstandings and involved the learners by asking questions in order for that the learners could understand (LESP1). After a problem had been discussed, the learners had to assess their own solutions (LESP1).

In the first lesson Denise gave the learners five problems to solve individually in class and after each problem they did the corrections together (LESP2). In the other two lessons the learners already completed the tasks at home, so Denise used the entire period for corrections (LESP2). The learners also knew that anyone of them could be asked at any time and they needed to be prepared at all times (LESP2). She ensured participation of the learners through continual questioning and ensuring that they were doing their own corrections (LESP2). She encouraged the learners to explore and use their textbooks efficiently by reminding them that the textbook consisted of activities, worksheets, assessments, projects and reviews and that they were currently busy with a worksheet (LESP2). Her lessons were not typical lessons consisting of the initial, development and closure phases as she was busy with revision when learners needed to practice their skills (LESP3).

Learning environment: Administrative routines (LEA)

She organised and managed the class effectively to ensure that time was maximised for the learners to develop conceptual understanding (LEA1). The learners were seated at individual desks which were appropriate for the lesson style (LEA2). Denise was in contact with her learners as she continually moved between the desks when she was not explaining and demonstrating in front of the class (LEA3). She also attended to learners individually at their desks assisting them with the work (LEA3). The work Denise did on the board was correct and all corrections to the learners' work were indicated so that learners could do their own corrections (LEA4).

Summary

Table 4.8: Summary of Denise's instructional practice

LESSON DIMENSIONS	DESCRIPTION OF LESSON DIMENSION INDICATORS
Tasks	
Modes of representation (TR)	She made use of representations such as written work on the board, conversion tables, calculators and a diagram to illustrate the different units of measurement of length. These representations allowed her to link learners' prior knowledge with the new content of the day.
Motivational strategies (TMS)	The learners were motivated and inspired to take part in the lesson, not necessarily due to the nature of the tasks but to the fact that they wanted to show their work on the board. She did not point out the

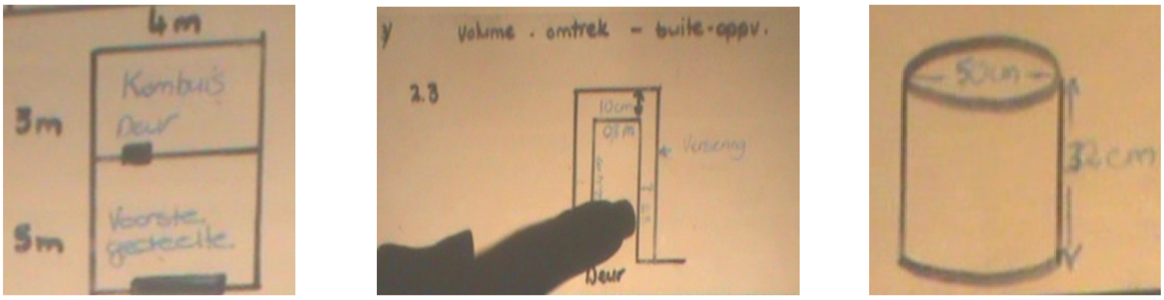
	value of mathematics in everyday life.
Sequencing and difficulty levels (TSL)	The given tasks were sequenced over the different lessons and were appropriate, although not set in context and the content was on Grade 11 level (Level 1).
Discourse	
Teacher-learner interaction (DTL)	Denise verbally encouraged the learners as she praised their efforts that were written on the board. Most of the times she did not expect the learners to explain their thinking. The lessons were characterised by the discourse between Denise and the learners and the number of questions she posed to the learners.
Learner-learner interaction (DLL)	No discourse based on the content was observed among the learners.
Questioning (DQ)	Most of the questions were calculation, memory and convergent questions. Many times Denise did not allow enough time for the learners to become engaged in the discourse and just provided the answers herself.
Learning environments	
Social and intellectual climate (LEC)	The social and intellectual climate in the class can be described as positive as Denise had a positive rapport with the learners valuing their ideas and praising their efforts.
Modes of strategies and pacing (LESP)	Denise used a teacher-learner-centred approach with discussion, and learners' writing on the board as instructional strategies. These strategies were effective to ensure learner participation. She worked at a manageable pace throughout.
Administrative routines (LEA)	The administrative routines such as management of time to maximise learner involvement, classroom arrangement and the information on the board were effective.

4.5.4 Elaine's instructional practice

4.5.4.1 Tasks

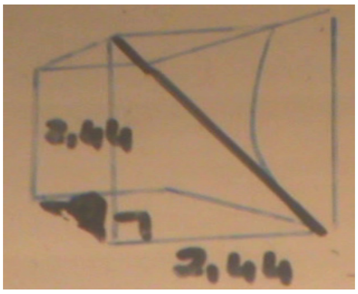
Tasks: Modes of representation (R)

To facilitate content clarity Elaine used representations such as written work on transparencies, tables, symbols, formulae, calculators, a demonstration calculator and sketches of manipulatives (Picture 4.20) in all her lessons (TR1).



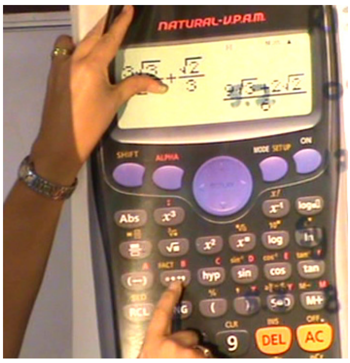
Picture 4.20: Examples of sketches used in Elaine's discussions of solutions

In the second lesson on time she used a table (See Question 5A on the next page) for parking tariffs (TR1). One of the formulae the learners used during the second lesson was: $F = \frac{x[(1+i)^n - 1]}{i}$ (TR1). Elaine expected the learners to explain the meaning of each variable in the formula enabling them to proficiently apply their knowledge to other similar unknown formulae (TR1). Below is an example of a sketch of a goal box (Picture 4.21) Elaine gave the learners to assist them in solving a problem based on a soccer field (TR1).



Picture 4.21: Elaine's drawing of the given goal box to explain the solution

Initially most of the learners calculated the time problems without the use of their calculators until a learner asked Elaine how she could use her calculator to find the answer. Elaine used a large CASIO demonstration calculator (Picture 4.22) which she put on the board to demonstrate the use of the calculator.



Picture 4.22: Casio demonstration calculator

The other learners were eager to master their calculators in calculating the answers to the time problems. Learners then assisted one another while she assisted specific learners who still could not manage their calculators. Since many of the learners were either not aware of or not able to use their calculators, they were very pleased afterwards with their accomplishment. Elaine generally drew the learners' attention to the required prior-knowledge needed to understand the content of the specific tasks (TR2). To connect the learners' prior knowledge with the new knowledge she alternated between discussions with questioning and class tests of which the answers were afterwards discussed in class and self-assessed by learners (TR2).

Tasks: Motivational strategies (TMS)

The tasks that captured the learners' curiosity were especially those based on time and interest (TMS1). The learners manually calculated the answers of the following question but enjoyed checking their answers using their calculators (TMS1).

Question 5A

Sam parks her car every day in a parking area at her work. The table below shows the cost of parking for specific time periods:

Parking tariffs	
Hours	***Cost***
0 – 1 hour	*Free*
1 – 3 hours	*R4,00*
3 – 5 hours	*R6,00*
5 – 7 hours	*R8,00*
7 – 9 hours	*R10,00*
More than 9 hours	*R12,00*
Saturdays	*R5,00*
Sundays	*Free*

5.1 On Monday morning Sam arrives at the parking area at 07:50 and leaves the parking area at 17:15. How much does she pay for parking?

5.2 On Tuesday she arrives at 08:01 and leaves the parking area again at 08:45. She goes back to work at 12:15 and leaves for home at 17:30. How much does she pay in total for the parking?

5.3 On Wednesday Sam parks her car at the parking area. She has to pay R8,00 because she parked there from 08:45 to 13:25. Now use the table to determine whether she paid the correct amount. Show all steps and give a reason for your answer.

The following interest problems (Question 5B below) were set in a context of buying a house and the impact thereof on an individual's, or even their parents' budgets (TMS1). From the learners' participation in the discussions it seemed as if the learners took an interest in these tasks.

Question 5B

5.1 James wants to buy a house that is in the market for R780 000. If he pays a deposit of R78 000 and then R7 800 per month for 20 years, how much will he pay in total for the house?

5.2 James decides to rather first save money to increase his deposit. He invests R7 800 at 15% per year, interest compounded semi-annually. How much money will he have saved after 7 years? ($A = P(1+i)^n$)

5.3 James' parents also invested R450,00 monthly in a savings account for 8 years. The interest rate was 11% per year compounded monthly. How much money does he have in that account? ($F = \frac{x[(1+i)^n - 1]}{i}$ *)*

5.4 James can afford to pay R6 500 each month for 18 years on a home loan at 17% per year, interest compounded monthly. How much money can he borrow from the bank? ($P = \frac{x[1-(1+i)^{-n}]}{i}$ *)* (TMS1).

Elaine had a very demanding learner in class who had, according to her, been diagnosed with attention deficit hyperactivity disorder. She successfully kept him involved and focussed throughout the lessons (TMS2). To a few hard working learners she said: *The people who already completed the work, I will come and assess your work and will then give you the next tasks to be done*[45] (TMS2). These were two examples where Elaine took the diversity of learners' abilities and experiences into account (TMS2). Since all questions were based on realistic everyday life situations the learners were able to relate to the tasks (TMS3). An interesting discussion followed when Elaine asked: *Let us talk a little about why a person would rather wait to buy a house until he increased his deposit.* She then referred to the learners' personal lives where she discussed a typical household's budget and their parents' expenses so that they could understand what their parents sometimes had to tell them: *There is no money for whatever you wanted at that stage* (TMS3). Elaine wanted the learners to gain understanding and to be able to apply their knowledge to other similar problems and situations that might arise in their future lives (TMS3). She emphasised that they needed to show all calculations at all times, also in the examinations since marks are specifically allocated to their calculations and not just the final answers (TMS3).

Tasks: Sequencing and difficulty levels (TSL)

Elaine sequenced the tasks in all three lessons by progressing in a lesson from easier to more complex tasks (TSL1). She also sequenced her class activities: for example during the second lesson on time and interest she first checked their homework and together they did Question 5A[46] on time. Before discussing the next Question 5B[47], she gave them the following class test based on prior-knowledge needed to answer that question.

Class Test: Banking matters

$A = P(1+i)^n$

1. What do I calculate with this formula?

2. Write in words the meaning of each of the following:

A = P = i = n =

3. What does the following mean to you? BODMAS

[45] Since Elaine's classes were not presented in English, her texts were translated by me.

[46] Question 5A is given under Tasks: Motivational strategies.

[47] Question 5B is given under Tasks: Motivational strategies.

After the test they discussed the answers and then proceeded with the questions. In this lesson she also proceeded from the easier task on time to the more complex task on interest. With the task on interest she progressed from discussing the meanings of the unknown values in the formula to discussing the context, the formulae to solve the problems in the given task. She concluded the lesson by interpreting the solutions.

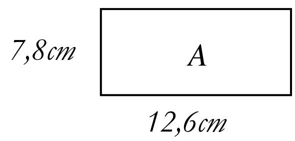
During the third lesson she also first walked through the whole class checking the learners' homework, then introduced the topic for that day's lesson, followed by a discussion of a contextual task the learners completed at home and then the lesson was rounded off by giving the learners the following class test:

<u>Class Test</u>: Perimeter, area, volume

1. What does the following mean to you?
- *a) Perimeter*
- *b) Area*
- *c) Volume*

2. If the given figure's measurements are given in centimetres, what will be your unit of measurement for your answer when the following need to be calculated?
- *a) Perimeter*
- *b) Area*
- *c) Volume*

3. The following figure is a rectangle.



- *a) Calculate the perimeter of figure A.*
- *b) What is the area of figure A?*
- *c) Calculate the volume of figure A* (TSL1).

Except for surface area all concepts were already introduced in Grade 10 but she appropriately applied the content to more complex situations (TSL2). Elaine was busy with her revision programme and during the first interview she stated that approximately three quarters of the learners knew the work by then. Half the remaining learners knew half the work while the other half did not have any idea of the work (TSL2). She stated that the purpose of her revision lessons was to either reinforce or enhance learners' current knowledge, to highlight learners' mistakes and to allow them to practise their knowledge and skills in order to prepare them for the coming examination (TSL2). The tasks Elaine covered in class were appropriate, on Grade 11 level, and reflected quality (TSL3). Based on the ML assessment taxonomy, Elaine selected tasks on Level 1 (Knowing) and Level 2 (Applying routine procedures in familiar contexts), but most of the tasks were on Level 3 (Application of multi-step procedures in a variety of contexts) and Level 4 (reasoning and reflecting) which required more advanced levels of thinking skills (TSL3). An example of such a task was a task based on a goal box:

> *The distance from the lower edge of the crossbar to the ground is 2,44 m. The distance from the goal post to the back of the goal is also 2,44 m. An extra pole is to be welded from the top corner of the goal post to the back of the supporting base.*
>
> - *How long must the support pole be (in metres)?*
> - *Convert this length to mm.*
> - *Write this answer in scientific notation.*
> - *If the diameter of the new pole is 4 cm, what will its circumference be?*

4.5.4.2 Discourse

Discourse: Teacher-learner interactions (DTL)

Elaine involved most of the learners in her class by asking them questions, clarifying their uncertainties and assessing their classwork (DTL1). She knew her learners by name and posed specific questions to specific learners such as:

> *I want to know, Hennie*[48]*, did you do it like that?*
> *Cecil, are you OK now, tell me what does it mean there?*
> *Lindy, what do you have there?*
> *Kevin, what did you say about monthly interest?*
> *Martie, do you agree on this?* (DTL1).

She communicated in a non-judgmental manner especially with the learner who was diagnosed with attention deficit hyperactivity disorder and who at times could be annoying. She remained calm and in control of the situation, attended to his comments or questions and continued with the lesson. An example was the following discourse between her and the specific learner:

> *T: In ML they give you all the formulae and information.*
> *L: It's not always like that. Another time I thought they normally give you everything and they did not. They changed everything just as they wanted, changed this and that!*
> *T: Is that true? Oh, then I must look into what happened there* (DTL1).

Whether learners were doing or discussing classwork or even after writing class tests, Elaine required them to give explanations and justifications orally or in writing (DTL2). She said on several occasions: *You must show me where you get that* and *Yes, but what does it actually mean?* (DTL2).

The following example is one of several incidents during Elaine's lessons where she listened carefully to learners' ideas and provided scaffolding to support their thinking:

---

[48] Pseudonyms were used instead of learners' true names.

T: What does it mean there?
L1: Compound interest.
T: Compound interest. Why did you choose compound interest?
L1: It's not simple interest.
T: Right, but what tells you that it's not simple interest, but compound interest?
L1: The bracket and the part below.
L2: But there is no fraction.
L3: A is the final amount.
T: In simple interest A is also the final amount. I told you earlier that you know it's compound interest when you see 'n' written as a power, then we reason this is more complicated than the normal formula, then it's compound interest. So I don't want you to just guess that it's compound, you must be able to give a reason why you say it is compound interest (DTL3).

She accepted their answers without criticising their efforts (DTL4). On several occasions Elaine said: *Good* or *Well done* and even thanked them for doing their homework as there were times they did not do their homework, telling her they could pass ML without doing homework (DTL4).

Elaine recognised and clarified the learners' common errors and misunderstandings (DTL5). For example there was a misunderstanding when the discussion was about the initial value and end value when interest was calculated. Instead of saying initial value a learner said present value and Elaine explained how present value could be interpreted as being the value after a certain period or could even mean the end value. She emphasised that the initial value is the value you began with (DTL5).

Discourse: Learner-learner interactions (DLL)

A number of times Elaine encouraged the learners to listen to or respond to other learners' ideas and answers and would say: *Listen, here Simon*[49] *is saying … Ernest, can you respond to Simon's statement?* (DLL1). On another occasion she asked: *Who agrees with her? Who wants to argue with her? I know you sometimes like to argue. So Celeste, do you agree with her? …* (DLL1). Most of the discourse was not among the learners but between Elaine and the learners (DLL1). While doing classwork learners had the opportunity to discuss the work with each other and discourse occurred that I could unfortunately not hear (DLL1). An example where Elaine guided a discussion among learners was when the different variables of the formula were discussed:

T: If we calculate interest semi-annually, by what must n be divided? How many times will interest be calculated per year?
L1: Two.
T: Why not six?
L2: It is six.
L1: Because you will get interest in the middle of the year and then the end of the year.

[49] Pseudonyms are used in all quotes.

> *T: Good, so semi-annually means every half of the year interest will be calculated, so at the end of the sixth month you get your first interest and then up to the twelfth month it will be the second six months period when I will get my next interest. So, if semi-annually I divide by two* (DLL1).

The only learner who tried to dominate the verbal communication was the learner who was diagnosed with attention deficit hyperactivity disorder, but she treated him in a firm and calm manner and once said: *OK, you had your moment* (DLL2).

Discourse: Questioning (DQ)

Elaine is a well-prepared and confident teacher who allowed enough time for learners to respond to her questions (DQ1). She posed questions on all three levels, namely memory, convergent and divergent questions (DQ1). Examples of convergent questions were: *What does it mean to write 12,5% as a decimal?*; and

> *T: What kind of a triangle is formed?*
> *L1: Right-angled triangle.*
> *T: Then I can indicate the right angle on my drawing and know that I can work with which theorem?*
> *L1: Theorem of Pythagoras*
> *T: Who can give me the theorem of Pythagoras?*
> *L2:* $r^2 = x^2 + y^2$
> *T: Good, also tell me in words what the theorem means …* (DQ1).

Examples of divergent questions were: *Explain in detail to an ignorant person the meaning of each variable in the interest formula*; and when an interest formula was given to the learners and they were asked what kind of interest was represented, she asked: *Why did you decide on compound interest?* (DQ1).

Elaine consistently listened to learners' ideas and in many instances she asked them to clarify and/or justify their answers (DQ2). Her questions contributed to the verbal communication and participation of the learners and she created opportunities which the learners could listen to, respond to and question her as teacher or even their peers (DQ3). Learners' responses were mostly teacher-selected but also volunteered (DQ4).

4.5.4.3 Learning environment

Learning environment: Social and intellectual climate (LEC)

Elaine was well-prepared, made her lessons interesting and continually involved the learners in class discussions (LEC1). Just as important to her was mutual respect of one another and she maintained a positive rapport with and among learners by emphasising the importance of people valuing each others' ideas and ways of thinking (LEC1).

Discipline and classroom rules played a major role in her classroom ensuring learners' positive behaviour (LEC2). At the beginning of each period she completed the attendance register and then walked through the class to control their homework and made a note of those who did not complete their homework (LEC2). There were only a few times when it was necessary for her to discipline the learners and these were some comments:

> *I again ask you, put your suitcase next to your desk*
> *If I asked you to discuss it with your friend, my choice of words was wrong*
> *Thank you, you had your moment*
> *That was rude* (LEC2).

Elaine appreciates both the subject and her learners and praised them by saying: *I am fond of you;* and *I really appreciate your cooperation* (LEC3).

Learning environment: Modes of strategies and pacing (LESP)

Elaine's teaching style varied between being a facilitator and mediator of learning (LESP1). She proficiently used instructional strategies such as class discussions and direct instruction (LESP1). The use of these strategies provided opportunities for the involvement of the learners and facilitated goal attainment (LESP1). She structured her lessons in such a way that learners had enough time to express themselves and explore their ideas and solutions (LESP2). She never rushed through the work or put pressure on them to work faster (LESP2). Her lessons did not consist of an initial, development and closure stage as they were revision lessons (LESP3). She made valuable use of her class time and completed what she had planned for the day (LESP3). There was a logical flow in her lessons as she worked from easier to more complex tasks and from familiar to less familiar concepts saying:: *Now let's go a little bit further …* (LESP4).

Learning environment: Administrative routines (LEA)

Elaine believed enough time should be allowed for learners to practise their knowledge and skills and therefore made provision for a revision period in her year plan to allow learners to prepare themselves for the examination (LEA1). She allowed a certain amount of time for learners to solve a problem but at the end of that time would still ask: *Who needs more time?* (LEA1). Elaine arranged the class so that learners were seated in pairs, which was appropriate for the particular lesson style (LEA2). When she explained work she was in front of the class facing the learners because she used the overhead projector, but would otherwise move between the learners attending to their needs (LEA3). The written information on the transparencies and blackboard was very neat and organised with no mistakes (LEA4). Permanently visible on the right hand side of the blackboard was the work they already completed so that the learners could take note of their progress (LEA4). The following was written on the board:

Chapter 4

- *Units 1,2,4,5,6,7,8,9*
- *Test papers: B1; A2; B2, Class Activity (out of 21)*
- *Taxation: Unit 10*

Book: Paper F1; F2

Summary

Table 4.9: Summary of Elaine's instructional practice

LESSON DIMENSIONS	DESCRIPTION OF LESSON DIMENSION INDICATORS
Tasks	
Modes of representation (TR)	To facilitate content clarity Elaine used representations such as written work on the board, tables, symbols, formulae, calculators, a demonstration calculator and sketches of manipulatives in the three lessons I observed. Her class tests consisted of oral or written questions in order to connect learners' prior knowledge to the new mathematical situation.
Motivational strategies (TMS)	The learners were interested in the tasks as they spontaneously took part in the class discussions, especially in those tasks that were based on buying a house and working out parking tariffs using their calculators. They enjoyed taking part in the discussions Elaine led. She took learners' diverse abilities into account and accommodated the learner with attention deficit hyperactivity disorder as well as a few hard-working learners. The value of mathematics was frequently emphasised as Elaine discussed real scenarios, also applying the work to their personal lives.
Sequencing and difficulty levels (TSL)	Elaine sequenced her tasks to enable the learners to progress in their cumulative understanding of the content and they were able to make connections with ideas learned in the past. The lessons were revision lessons to improve the learners' knowledge and skills. The tasks reflected quality and were on Grade 11 level (Levels 1-4).
Discourse	
Teacher-learner interaction (DTL)	Elaine involved most of the learners, either by attending to their needs at their desks or posing questions. She communicated in a non-judgmental manner at all times. She required learners to give explanations and justifications orally and in writing. She recognised and clarified the learners' common errors and misunderstandings.
Learner-learner interaction (DLL)	Elaine encouraged the learners to listen to or respond to other learners' ideas. The discussions were not necessarily among the learners but in most cases between her and the learners. During class work the learners discussed the work with one another.
Questioning (DQ)	Elaine is a confident and well-prepared teacher and allowed enough time for learners to respond to her questions. She asked a variety of questions and posed questions on all three levels, namely memory, convergent and divergent. Learners had to clarify and/or justify their answers. Their responses were mostly teacher-selected but also volunteered.
Learning environments	
Social and intellectual	She made her lessons interesting, involved the learners in discussions and

climate (LEC)	maintained a positive rapport with and among learners. Discipline and classroom rules played a major role in her classroom to ensure learners' positive behaviour. Elaine was very fond of the subject and her learners and praised them for their efforts.
Modes of strategies and pacing (LESP)	She used instructional strategies such as discussions and direct instruction. The use of discussions provided opportunities for the involvement of learners and facilitated goal attainment. Learners had enough time to express themselves and explore their ideas and solutions and there was a logical flow in her lessons.
Administrative routines (LEA)	She allowed enough time before the examination for learners to improve their knowledge and practise their skills. Learners were seated in pairs which was appropriate for the lesson style. Elaine was in front of the class when she explained the work but otherwise moved between the learners.

4.5.5 Summary of participants' instructional practices

Table 4.10 below provides a snapshot of the four participants' background information and their instructional practices.

Table 4.10: Snapshot of the four participants and their instructional practices

PARTICIPANTS	MONTY	ALICE	DENISE	ELAINE
BACKGROUND				
Qualifications and experience	BEd degree with Mathematics and Methodology of Mathematics as major subjects. Novice teacher with one year experience of teaching Mathematics and two years of teaching ML.	BTech Management Accounting degree with no Mathematics Education training. Novice teacher with only one year's teaching experience, teaching ML only.	BEd Honours degree in Mathematics Education with seven years' experience of teaching Mathematics and three years of teaching ML.	HED: Senior Primary with Mathematics and Methodology of Mathematics as major subjects. She had eight years' experience of teaching Mathematics and three years of teaching ML.
TASKS				
Modes of representation (TR)	• Used representations such as written examples on the board, variables, calculators and a manipulative. • Seldom connected learners' prior knowledge to the new mathematical situation.	• Used representations such as written examples on the board, symbols, the formula, tables, graphs and calculators. • The various representations did not contribute to connecting learners' prior knowledge with the new mathematical situation.	• Used representations such as written work on the board, tables, calculators and a diagram. • These representations allowed her to link learners' prior knowledge with the new content of the day.	• Used representations such as written work on the board and transparencies, tables, symbols, formulae, calculators, a demonstration calculator and sketches. • Through tests and oral questioning she connected learners' prior knowledge to the new mathematical situation.
Motivational strategies (TMS)	• Only mathematical content was taught. The nature of the tasks did not capture the learners' curiosity or inspire	• Except for the one task being set in a context, the lessons consisted of mathematical content only. When she made	• Pure mathematical content was taught. Learners were motivated and inspired by the teacher and not necessarily by	• The learners were interested in the tasks as they spontaneously took part in the class discussions. She

	them to pursue their conjectures. • He only mentioned contexts to which content could be applied to point out the value of mathematics but the explanations were vague.	mistakes on the board, some of the learners were motivated to pursue their conjectures. • Did not point out the value of mathematics in every-day life.	the nature of the tasks. • Did not point out the value of mathematics in every-day life.	took learners' diverse abilities into account. • She frequently reminded them of the value of mathematics in their lives.
Sequencing and difficulty levels (TSL)	• Not much evidence of the sequencing of tasks. • The lessons on simultaneous equations were on Grade 11 level (Level 1) but the data handling lesson on Grade 10 level (Level 1).	• Most of the times the tasks were not successfully sequenced to enable learners to progress in their cumulative understanding of the work. • Tasks were on Grade 11 level (Level 1).	• Tasks were sequenced over the different lessons, were suitable to what the learners already knew but needed to improve on. • Tasks were on Grade 11 level (Level 1).	• Tasks were sequenced in the lessons to enable the learners to progress in their cumulative understanding of the work, set in context. • Tasks were applicable and on Grade 11 level (Levels 1-4).
DISCOURSE				
Teacher-learner interaction (DTL)	• Communicated in a non-judgemental manner but did not encourage learner participation except for posing basic questions. • Did not require learners to give full explanations. • Re-explained the work instead of providing scaffolding to support learners' thinking. • Could not recognise learners' misunderstandings.	• She was judgmental in her communication and did not encourage the participation of learners except where she needed help with her own mistakes and misunderstandings. • Did not require learners to give full explanations. • She did not listen to learners to determine where and how she could provide scaffolding. • Did not recognise learners' misunderstandings as she had misconceptions herself.	• Non-judgmental and verbally encouraged the learners as she praised their efforts. • Required learners to give demonstrations of their work in writing but did not expect them to explain their work. • She provided scaffolding to support learners' understanding. • She recognised and clarified learners' misunderstandings.	• Non-judgmental and all learners were involved through questioning and discussions. • Learners had to give explanations and justifications of their thinking, both orally and in writing. • She provided scaffolding to support learners' understanding. • She recognised and clarified learners' misunderstandings.

Learner-learner interaction (DLL)	• Did not encourage learners to listen to, respond to and question one another.	• The observed interaction during the first lesson only was a result of the mistakes Alice made that were discussed and not because she created positive opportunities for learners to discuss the work	• She did not encourage learners to listen to, respond to or question each other's ideas.	• Elaine encouraged the learners to listen to and respond to other learners' ideas. The discussions were mainly between her and the learners.
Questioning (DQ)	• Types of questions were complete the word/sentence and calculation questions. • Did not contribute to learners' participation in discussions. • Responses were volunteered.	• Types of questions asked were memory, rhetoric, calculation, and complete the word/sentence questions. • Did not contribute to learners' participation in discussions. • Learners' responses were volunteered or chorus.	• Types of questions were calculation, memory and convergent questions. Many times Denise did not allow enough time for the learners to become engaged in the discourse and just provided the answers herself. • Contributed to learners' participation in discussions. • Learners' responses were teacher-selected.	• Types of questions were memory, convergent and divergent questions. She allowed enough time for learners to respond to her questions. • Contributed to the verbal communication of learners during discussions. • Learners' responses were mostly teacher-selected but also volunteered.
LEARNER ENVIRONMENT				
Social and intellectual climate (LEC)	• Positive rapport between him and the learners. • Good discipline and in control of the learners and lessons. • Confident and enthusiastic about teaching ML.	• Not a positive relationship with and among learners. • She did not ensure appropriate classroom behaviour. • Seemed bored, irritated and un-enthusiastic at times.	• Had a positive rapport with the learners as she valued their ideas and praised their efforts. • She was confident and strict and applied classroom rules. • She had a positive attitude towards the learners and the subject.	• Positive rapport with and among learners and praised their efforts. • Good discipline. • Confident, well-prepared and enthusiastic. A calm and relaxed atmosphere.

Modes of strategies and pacing (LESP)	• His style varied between traditional and formal authority. He used direct instruction as instructional strategy and once a learner wrote on the board. • Not enough time was provided for learners to explore mathematical ideas. • Good pacing and logical flow.	• Her style varied between traditional and demonstrative. She used direct instruction as instructional strategy. • Not enough learner involvement. • In the third lesson too much content was covered with little logical flow.	• She was a mediator. She used discussions and learners working on the board as instructional strategies. • These strategies ensured learner participation. • Lessons were presented at a manageable pace.	• Her style varied between mediator and facilitator. She used discussion and direct instruction as instructional strategies. • Enough time for learner involvement and goal attainment. • Logical flow in lessons.
Administrative routines (LEA)	• Learners' used the time to copy work from the board and listened to the teacher. • Moved between the learners when he was not explaining work on the board. • The written work on the board was disorganised and incomplete at times.	• Not enough time was allocated to learner activities. • Her position in class did not contribute to learners' conceptual understanding as she was mostly standing at a specific desk in front of the class attending to her textbook and the few learners in front of her. • There were mistakes on the board and the work was not always organised.	• Used time effectively to maximise learner involvement. • She moved between the learners to assist them and to ask questions. • Information on the board was correct and ordered.	• Managed time effectively for maximum learner involvement. • Her position in class contributed to a positive learning atmosphere. • Very neat, correct and organised work on transparencies and the board.

4.5.6 Discussion of Theme 1: ML teachers' instructional practices

I again conducted a comprehensive, advanced electronic search after presenting the data in order to establish a basis from which I could execute a literature control of my findings in this chapter. My search covered the period January 2008 to September 2011 as I wanted to correlate my study's findings with the most recent research studies conducted in ML classrooms[50]. Of the 32 studies on ML, 17 were based on discussions, analysis, critiquing, developing frameworks or investigating certain theoretical or curriculum aspects of ML in South Africa. Seven were concerned with in-service ML teachers' experiences and their development through the ACE in the ML programme at different universities. Eight studies investigated ML teachers' instructional practices and the classroom experiences of ML learners. Two of the eight studies were intervention studies conducted with one teacher, and six of the eight studies were conducted at one school only. An experienced academic information specialist at the University of Pretoria, Ms Clarisse Venter, also conducted an advanced electronic search for the period January 2008 to September 2011 but could not add any studies to my existing list. Her reply was: *Most of the studies were discussions, analysis etc., which you do not want* (C. Venter, personal communication, September 12, 2011).

In the next section, I will conduct a literature control where the findings from this study are compared with the findings from other research studies on ML teachers' instructional practices. I base the discussion on Artzt et al.'s (2008) three dimensions of a lesson, namely tasks, discourse and learning environment[51].

4.5.6.1 Tasks

Since I believe knowledge is constructed and based largely on prior knowledge, I support the view that the purpose of tasks such as examples given on the board, problems, activities and projects being given to the learners is to *provide opportunities for learners to connect their knowledge to new information and to build on their knowledge and interest through active engagement in meaningful problem solving* (Artzt et al., 2008, p. 10). The tasks used by ML teachers to facilitate learning in their instructional practices are discussed in this section in terms of Artzt et al.'s (2008) categorisations of such tasks[52]: modes of representation; motivational strategies; sequencing; and difficulty levels of tasks.

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50 See Addendum H for a list of studies conducted on ML for the period January 2008 to September 2011.
51 See Table 4.2 under Section 4.3.2.1.
52 See Table 4.2 under Section 4.3.2.1.

Modes of representation

- **Use of various representations by ML teachers**

All four participants in my study used various representations during their classes as was also found with all the participants in the other research studies (Sidiropolous, 2008; Venkat & Graven, 2008; Venkat, 2010). Elaine was the only participant in my study who expressed a need to increase the use of technology such as computers in her ML classroom. As far as I could establish, no previous study has reported this finding.

- **Linking learners' prior knowledge to new situations**

In my study, novice teachers Monty and Alice, unlike experienced teachers Denise and Elaine, neither determined nor used their learners' prior knowledge to facilitate the assimilation of new content knowledge. This finding strongly confirms Sidiropolous' (2008) finding that one of the two teachers in her research group did not determine his learners' prior knowledge or use his learners' prior knowledge to facilitate the assimilation of new content knowledge.

Motivational strategies

- **Use of tasks to motivate learners to reflect on and pursue their conjectures**

Only Elaine in my study used contextual tasks that inspired the learners to reflect on their answers. In ensuing class discussions, the learners' conjectures were explored and expanded, enhancing their understanding of the work. Conversely, the other three teachers in my study did not use tasks that would motivate the learners to reflect on or to pursue their conjectures. My finding can therefore probably be regarded as consistent with Sidiropolous' (2008) findings where both teachers in her study did not ask the learners to reflect on or discuss their solutions. My finding that only one of the four teachers in my group used tasks to motivate learners is, however, inconsistent with the research results obtained by Buytenhuys, Graven and Venkatakrishnan (2007) and Venkat (2010) who found that the teachers in their studies used contextual tasks and discussions and succeeded in inspiring the learners to reflect on their answers and explain and justify their arguments. It should, however, be mentioned that the two teachers in the latter two studies were extremely dedicated teachers – not unlike the teacher in my group who used contextual tasks and discussions to inspire the learners to reflect on their answers and to explain and justify their arguments.

- **Pointing out the value of mathematics through the use of life-related tasks**

Part of the definition of ML (DoE, 2003a) is the use of the life-related applications of mathematics to make learners aware of and understand the role of mathematics in the modern world. In my study, only Elaine based her lessons on solving contextual problems on discussions that related the learners' newly acquired knowledge to the outside world and home situations. This finding is inconsistent with the findings of a number of researchers such as Buytenhuys et al. (2007), Hechter (2011a), Venkat and Graven (2008) and Zengela (2008) who found evidence of the successful use of life-related application problems in pointing out the value of mathematical literacy in everyday life. Three of the four teachers in my research group taught mathematical content only and did not once refer to its value in everyday life situations. My finding that only one of the four teachers in my study realised the value of teaching mathematics through the use of life-related tasks in everyday life situations is consistent with the finding of Sidiropolous (2008) who established that only one of the two teachers in her study realised the value of teaching mathematical literacy in real-world contexts and accordingly taught the subject on the basis of expecting the learners to solve real life-related problems.

Sequencing and difficulty levels

- **Sequencing of tasks enabling learners to progress in their cumulative understanding**

Three of the four teachers in my research group were not able to sequence their tasks proficiently to enable the learners to progress in their cumulative understanding of a particular task and to make connections between ideas learned in the past and those they will encounter in their future lives. There appears to be a gap in the literature in this regard.

- **Grading of classroom tasks according to the ML Assessment Taxonomy**[53]

My finding in this regard was that three of the four teachers selected tasks only from Level 1 (Knowing) according to the ML Assessment Taxonomy while Elaine selected tasks from all four levels (Knowing; Applying routine procedures in familiar contexts; Applying multi-step procedures in a variety of contexts; and Reasoning and reflecting). My finding is inconsistent with that of Govender (2011) who reported that the only teacher in her study asked the learners to perform tasks on all four levels. Interestingly, whereas the learners in the class of the one teacher (Elaine) who did select tasks from all four levels understood the problems and could solve them, apparently because of the support given by this teacher, in Govender's (2011) study, the learners could not understand the problems and, despite the support given by the teacher, could not solve the problems. Govender stated that the learners found these kinds of problems difficult, were not used to such questions and did not understand the contexts. The difference in the latter set of findings can be explained by the fact that at the time of the study Elaine was teaching in a traditional white school in Pretoria where the learners were familiar with the contexts while the teacher in Govender's study was teaching in a black township school in Port Elizabeth where the contexts were not part of the learners' real-life experiences.

[53] See Section 2.2.2.2: ML principles.

4.5.6.2 Discourse

As a way of contributing to learner understanding, the discourse in class should provide opportunities for learners to express themselves, to listen to, to question, to respond to and to reflect on their thinking (Artzt, et al., 2008). I will now conduct a literature control on the discourse in ML classrooms based on Artzt et al.'s (2008) perspective of discourse[54], namely teacher-learner interaction, learner-learner interaction and the use of questioning to enable learners to build on their existing knowledge.

Teacher-learner interaction

- **Nature of teachers' communication and learner participation**

Except for Alice, the other three teachers in my study communicated with the learners in a non-judgmental manner thus contributing to a positive relationship between the teachers and the learners. This finding (only one of the four teachers in my study was judgmental) is inconsistent with that of Sidiropolous (2008) where both teachers in her study judged their learners' abilities and expressed low expectations of the learners.

Monty and Alice did not encourage learner participation apart from posing low-level oral questions where the answers were often provided to the learners before they could try to answer the questions. The finding that two of the four teachers in my group did not encourage learner participation moderately confirms Sidiropolous' (2008) finding that both teachers in her study did not generally encourage learner participation. Apart from one occasion, no time was allowed by the teachers in Sidiropolous' (2008) group for discussion on solutions or any critical engagement with mathematical arguments in their instructional practices. Denise ensured learner participation by requesting the learners to write their solutions on the board and involving them in discussions afterwards whereas Elaine involved the learners only in class discussions. The finding that only two of the four teachers in my group encouraged the use of discussions to enhance learner participation is moderately consistent with the findings of Venkat and Graven (2008) and Venkat (2010). The two experienced teachers in their studies encouraged class discussions, which were used to stimulate enhanced participation and communication.

- **Opportunities for learners to explain and demonstrate their work**

Only Elaine in my study asked the learners to explain their thinking and solutions, a strategy that elicited further discussion between her and the learners in class. Denise did not request the learners to explain their thinking but merely required them to demonstrate their work on the board. The other two teachers did not require the learners to explain or justify their work at all. The finding that two of the four teachers in my group provided opportunities for the learners to explain or demonstrate their work

[54] See Table 4.2 under Section 4.3.2.1.

is moderately consistent with the finding of Venkat (2010) that the (dedicated and experienced) teacher in her group (she had only this person in her research group) asked the learners to explain their thinking and solutions by giving them opportunities to explain and demonstrate their work (this teacher insisted on justification and explanation). Since there is evidence in the literature that collaboration from learners in the ML classroom contributes to the development of positive mathematical identities (Graven, 2011), my findings should be of interest to the Department of Education. Seemingly, during teacher training, more emphasis should be placed on the importance of providing opportunities for learners to express their ideas and thinking and to explain and justify their work.

- **Use of scaffolding to support learner understanding**

Even though Denise's lessons were based on mathematical content only, both she and Elaine provided scaffolding to support learners in solving problems and understanding the tasks instead of merely telling them how to solve the problem or doing the problem for them. In contrast to the instructional practices of Denise and Elaine, Monty either re-explained the work or solved the problem for the learners while Alice was not concerned about the learners' ideas and thinking. My finding that two of the four teachers in my study provided scaffolding is moderately consistent with the results obtained by Hechter (2011a) who found evidence of pedagogical support and scaffolding in the practices of both ML teachers who were part of her study. It should, however, be mentioned that these teachers were students enrolled for the ACE (ML) programme where they had to plan lessons according to certain guidelines (including ways to accommodate scaffolding in their instruction) and then implement those lesson plans in their instructional practices. In other words, they were required to facilitate scaffolding. It is not clear whether they would have done so had they not been required to do so.

Learner-learner interaction

Except for the very limited evidence of learner-learner interactions in Elaine's class, interactions between learners where they had the opportunity to support, strengthen and challenge each other's ideas were absent in the other teachers' instructional practices. This finding is inconsistent with that of Venkat and Graven (2008) where the single ML teacher in their research, being experienced and dedicated, used extensive communication and discussion of tasks during her lessons. My finding, however, concurs with the finding of Sidiropolous (2008) where both teachers in her study did not encourage critical engagement by learners in mathematical arguments. Given the belief (Brown & Schäfer, 2006; Venkat, 2007; Venkat & Graven, 2008) that a learner-centred approach is of the utmost importance in teaching ML, that is, where learners are actively involved in the lessons by taking part in discussions and group work but also by using their knowledge outside the classroom, my findings (albeit based on the actions of a limited sample of participants) should be a source of concern to education authorities.

Oral questioning

In my study, I found that three of the four teachers asked low-level questions such as complete the sentence and simple calculation and memory questions, which did not allow enough time for the learners to respond, and where in most cases the teachers provided the answers themselves. Elaine, however, asked various types of oral questions on different levels and gave the learners sufficient time to respond. As far as I could establish, these two findings regarding ML teachers' use of oral questioning in their classes have not been reported before and should be a source of concern to education authorities.

4.5.6.3 Learning environment

Artzt et al. (2008) use the term learning environment to describe the conditions under which the teaching-learning process unfolds in the classroom. I will now discuss the learning environments of the participants' ML classrooms in terms of Artzt et al.'s (2008) categorisations of a learning environment[55], comprising a social and intellectual climate, modes of strategies and pacing, and administrative routines.

Social and intellectual climate

- **Maintaining a positive relationship with and among learners in the classroom**

Monty's formal authoritative style of teaching restrained the building of positive relationships with and among the learners, and Alice focused only on the mathematical content instead of building relationships. However, Denise and especially Elaine created an atmosphere in the class where the learners were comfortable and confident as they engaged in the tasks. My finding that two of the four teachers in my study did not maintain a positive relationship with and among the learners is moderately consistent with that of Venkat and Graven (2008) who found that the teacher in their study was patient and that the learners could therefore work in a relaxed environment. This gave the learners *a sense of exploration, of working without fear or failure or ridicule, and of learning with enjoyment* (p. 40).

- **Use of classroom rules**

All four teachers in my study mentioned that they had to apply classroom rules to ensure appropriate classroom behaviour since learner misbehaviour could be a problem in ML classrooms. Monty and Denise were very strict; Alice at times could not apply her rules effectively while Elaine was more relaxed in applying her classroom rules as the learners seemed to know what was expected of them. As

[55] See Table 4.2 under Section 4.3.2.1.

far as I could establish, findings on ML teachers' application of classroom rules have not been reported before.

- **Teachers' attitudes towards the subject and the learners**

All four of the teachers in my study had a positive attitude towards the subject ML. My finding is strongly consistent with Fransman's (2010) finding where the four ML teachers in her focus group, who were enrolled for the ACE (ML), experienced the training to become *some kind of mathematics teacher, i.e. to be trained as a Mathematical Literacy teacher,* as a challenging experience *in which they were developing some sort of status-embraced identity* (p. 175). My and Fransman's (2010) findings are strongly inconsistent with the finding of Sidiropolous (2008) where both teachers in her study regarded the teaching of ML as a threat to their *'status identity'* (p. 221) as Mathematics teachers. Sidiropolous (2008) surmised that her finding could have been influenced by the fact that her study was conducted only one year after the subject had been introduced. At that stage, negative attitudes towards the teaching of ML were common (Sidiropolous, 2008).

Modes of strategies and pacing

- **Use of appropriate instructional strategies**

I found that Monty and Alice, the two novice teachers in my study, seemed to believe that learners learn through direct transfer of information (traditional approach, as defined in Section 2.4.4.2). Both Denise and Elaine, however, based their instruction on their learners' knowledge − Denise by using her learners' written solutions on the board to elicit discussion and Elaine by mainly using class discussions as an instructional strategy. Since two of the four teachers in my study used appropriate instructional strategies (as defined in Section 2.2.2.3), this finding is moderately consistent with that of Graven and Venkat (2009) who reported that all the teachers in their research changed their pedagogical approaches to teaching ML by using discussions and group work. Conversely, since two of the four teachers in my study did not use appropriate instructional strategies, this finding is also moderately consistent with Sidiropolous' (2008) finding − she established that both teachers in her study kept to a traditional teacher-centred approach by not using group work or discussions in their ML classrooms.

On the basis of Graven and Venkat's (2007) proposed spectrum of pedagogic agendas[56] ranging from Context; to Content and context; to Mainly content; to Content driven, only Elaine's pedagogic agenda was Content and context driven. The pedagogic agendas of Monty, Alice and Denise were Content driven. My finding in this regard is consistent with that of Hechter (2011a) who found that the pedagogic agenda of one of the two teachers in her study closely matched the Mainly content driven

[56] See Table 2.1 under Section 2.2.2.3: Pedagogical approaches for teaching ML.

agenda while the other teacher's pedagogic agenda partially matched the Context and Mainly content driven agendas.

- **Effective structuring of available time**

Three of the four teachers in my study worked at a manageable and slower pace compared to the pace in Mathematics classes thus allowing the learners more time to understand the work. This finding is consistent with that of Venkat and Graven (2008) where the only teacher in their study was *willing to 'wait' in ML in contrast to the imperatives to rush ahead in Mathematics* (p. 38).

Administrative routines

- **Maximise time for learners' active involvement in tasks and discourse**

I found that too much time was spent in Monty's class on learners who copied work from the board; while in Alice's class, the learners spent most of their time looking at Alice's demonstrations on the board. Denise and Elaine, on the other hand, managed their time to maximise learner involvement. My finding (two of the four teachers maximised the time available for learners to be actively involved in tasks and discourse) is moderately consistent with that of Venkat and Graven (2008) where the teacher in their study waited for the learners to understand before moving on.

- **Classroom arrangement, position of teacher in class, written information on the board**

My finding that the classroom arrangements of the four teachers in my study were appropriate for the lesson styles they used is moderately consistent with the finding of Sidiropolous (2008) that one of the two teachers in her study arranged the desks appropriately in his classroom in groups. Except for Alice, who worked on the board or from her textbook at a table in the front of the class, the other three teachers moved between the learners' desks engaging with the learners and their work. My finding that three of the four teachers in my study moved among the learners is consistent with Sidiropolous' (2008) finding that one of the two teachers in her study moved among his learners. Regarding the written work of the teachers on the board or on transparencies, I found that the work of Denise and Elaine was organised whereas Monty's work was disorganised, and Alice made mistakes or did incomplete work on the board. As far as I could establish, this finding has not been reported before.

4.5.6.4 Summary of discussion on Theme 1

To summarise: Denise adopted a teacher-learner approach, Elaine a learner-centred approach while Monty and Alice used a teacher-centred approach. Denise and Elaine used discussions and had the learners working on the board in order to encourage learner participation, allowing the learners to explain and/or justify their thinking. They used various representations and scaffolding to guide the learners to conceptualise new knowledge. Elaine selected tasks on all four levels according to the ML Assessment Taxonomy. The learning environments created by both Denise and Elaine were more relaxed with enough time for learner participation. Monty and Alice used direct instruction and thus discouraged learner participation by not allowing the learners to explain and/or justify their thinking. They used various representations, but not scaffolding, to guide the learners to conceptualise new knowledge. Both these teachers selected tasks on the lowest level of the ML Assessment Taxonomy. Monty's learning environment was formal while Alice's learning environment was relatively tense and awkward, and she did not allow enough time for learner participation.

I agree with Graven and Venkat's (2009) view that ML teachers need to make a substantive change in their instructional (pedagogic) practice in terms of the nature of the educational tasks, the agenda driving their teaching (on the continuum from Content to Context driven) and the way they interpret the subject ML. I also concur with Venkat and Graven's (2008) view that the nature of tasks and the nature of interactions are two key concepts contributing to positive change in ML classrooms that should be considered by ML teachers. In my research study, only Elaine's instructional practice conformed to these two key concepts.

4.6 Theme 2: ML teachers' knowledge and beliefs

In this section I present and discuss the findings from the interviews and observations of Monty, Alice, Denise and Elaine. All discussions on the subthemes **MCK,** and **PCK regarding: ML learners, ML teaching and ML curriculum** are structured strictly according to the guidelines in Table 4.3[57]. Background information regarding the observed lessons of the participants is given in Section 4.5. The language of all quotations from Monty, Alice and Denise has not been edited. Since Elaine's classes were conducted in Afrikaans, I translated her quotes from Afrikaans to English. The subthemes of each participant are now discussed.

4.6.1 Monty's knowledge and beliefs

4.6.1.1 Mathematical content knowledge[58] (MCK)

I wanted to know from Monty how important it was for a ML teacher to have sufficient MCK and he replied that:

> *ML needs mathematics knowledge because there are many things you need to know from maths, the basics, for example the chapter on calculating angles or areas, surface area or volume. If you have never done this thing before, how are you going to understand it?*

[57] Table 4.3 is discussed under Section 4.3.2.1: Inclusion criteria for coding the data.

[58] Since there is only one indicator or code in Table 4.3 regarding the teacher's mathematical content knowledge, this whole paragraph's code is: MCK.

Since Monty did not make any mathematical errors in his oral explanations or board work, it appeared as if his MCK regarding the specific content covered in the three lessons is sufficient. Since basic mathematical content was taught, there was no opportunity to observe whether he understood more than just the procedure of solving simultaneous equations. The same applies to the data handling lesson where I could not observe the extent to which he understands why and when we use the different measures of dispersion. Because learners were not expected to explain why different measures of dispersion are used, their conceptual understanding of the concepts also could not be determined. When Monty discussed the median he only said:

> *OK now, a median is a middle number né? I don't have another definition for that, it's a middle number.*

Monty did not use the glossary in the NCS for ML to define the terms properly, so that learners do not just have a synonym but can explain the meaning of the term or as he stated it, know the characteristics of the terms. Some minor mistakes I observed were:

- With simultaneous equations, after finding the solution for x and y, he did not put them in an ordered number pair or emphasise that the answer represents a point;
- he should have used parentheses next to the two equations in order to emphasise that it was a system of simultaneous equations;
- in the data handling lesson he was not consequent as he sometimes said *from 3 up, above, no mode* other times he said: *more than 3, no mode.*

4.6.1.2 Knowledge and beliefs regarding ML learners

For the second lesson on simultaneous equations, Monty predicted the learners would understand *how to approach the problem* (L1). For the data handling lesson on the four basic measures of dispersion, he predicted that the learners would understand *how to collect data, organize and summarise them and to present them at the end of the lesson* (L1). He later mentioned that the learners *just need to know the mode, mean, median and range*, but *we don't go in details but I have to give them a definition and how to gather information for future purposes, because they will need it even after they completed the school* (L1). Monty's reason why the learners would have understood those aspects was that all of the mentioned aspects were known to the learners, they were familiar with them and also because *it forms part of living, it is part of their lives* (L1).

What he predicted they would not understand is the variable x *because once they see x they get anxious, because what comes to their mind is once they see x they think it is Maths* (L2). He thought the learners would come to understanding through *many examples whereby I can say look at it, this is how it can be done and everything* (L3). *The more you have examples, the more they can see how to do it,* but also by giving the learners *more sums because the more they practice maths the more they understand it. Especially if they do it individually, that is when they learn*

(L3). Monty predicted learners would approach the tasks by asking a friend or looking it up in their notes (L4). He encouraged the learners to work individually since they were approaching examinations (L4).

According to him, the learners reveal misconceptions when doing substitution. The following is such an example:

> *x=4-2y must be substituted in 7x. They tend to forget 7x means 7 multiplied by x so they just say 7 multiplied by 4 then -2y, which is wrong. That 4-2y is one thing like 7x. They have to multiply 7 by 4-2y (L5).*

In the data handling lesson the only possible misunderstanding according to him is *that they forget to arrange the data in ascending form* (L5). I could not actually determine whether his prediction was correct in this regard, since the learners did not participate in the lesson (L5).

In the following example the learners thought that when the coefficient of a variable (say *x*) is one, it means that *x* is then the subject of the equation. So when Monty did the example $\begin{matrix} x+2y=4 \\ 7x-5y=9 \end{matrix}$, he asked:

> *T: Is x or y a subject of an equation? (No response). Is x or y a subject to the equation?*
> *L: x.*
> *T: Huh?*
> *L: x.*
> *T: Yes?*
> *L's: x.*
> *T: OK. I am asking: Is x or y a subject in the formula? Huh?*
> *L'S: x. (Teacher looks very troubled and learners laugh). OK, give me an example where x is a subject of the equation (wait, no response). OK, people remember it must be? L = something, so L is the subject of the formula. So, do you see x or y is a subject here? (Teacher is irritated).*
> *L's: No.*
> *T: No. What do you do? You get x or y alone.*

Monty was not perceptive as to what the learners were thinking (L6). Most of the learners worked individually and Monty attended to them by looking at their work and talking to them (L6). Unfortunately I could not assess whether Monty acted appropriately to facilitate learning as I could not hear the discourse taking place. I did however, notice that as he looked at their work he did not ask them to explain what they did; instead he was explaining again to them (L6).

4.6.1.3 Knowledge and beliefs regarding ML teaching

Monty regarded the following as prior knowledge for the lessons on simultaneous equations: *the coefficient, variable and index. That's the best knowledge and the sign comes before the number* (T1). During the introduction phase of the lesson, Monty revised the terminology as planned as well as the two methods they used to solve systems of equations (T1). He did not discuss like terms or the multiplicative inverse during the introduction as prior knowledge, but mentioned that later as part of the solution (T1). For the data handling lesson Monty said: *everything is prior knowledge* (T1).

In the examples and explanations on solving systems of simultaneous equations, Monty emphasised the steps to follow which he believed would simplify the work and make it easier for the learners to understand (T2). He did not use graphs as another form of representation to contribute to the learners' conceptual understanding of the work and the meaning of the solutions (T2). At times his explanations confused the learners because they took the form of lectures in which he made careless mistakes such as saying: *dividing with the multiplicative inverse* instead of multiplying with the multiplicative inverse and forgetting a sign in front of the value (T2). In the data handling lesson Monty demonstrated that it is important to be able to know the *characteristics* of mathematical terms, but he did not ask the learners to explain the different measures of dispersion and tell when and why these measures are used. This could have improved learners' understanding of the work (T2). During this lesson he verbally explained most of the examples without illustrating solutions on the board so that learners could see what he was talking about (T2).

There was no evidence that learners' different abilities and backgrounds were taken into account in presenting the content to the learners (T3). It is difficult to comment on his ability to sequence the content in order to facilitate learning since basic examples were used through all the lessons except for the example on simultaneous equations when he proceeded from a more difficult to an easier example (T4). Monty's choice of an instructional strategy to present his lessons was not in line with the purpose of ML as ML learners are supposed to be actively involved in solving contextual problems (T5).

ML teaching: Reflecting on his practice[59] **(T6)**

When Monty reflected on his instructional practice he said he used direct instruction *because our learners are different from other school learners so we need to use the direct instruction*, referring to discipline problems. Commenting on the discipline in his class he said: *I do have classroom rules in my class whereby I say we must respect one another*. He views his role in the ML classroom as being the facilitator where he helps the learners to understand the work. To improve his learners' appreciation of ML, he *will keep on motivating them about real life and what you may do with ML*. To improve the learners' participation in the lessons he gives them questions from previous examination papers to prepare at home and present and explain the next day. When asking him how he feels about teaching ML as this may influence the way he approaches his ML lessons, he said:

[59] Only one code was used to report on the teacher's reflection regarding his own practice namely T6.

> *If I can get a chance I will go for Maths. I like when I am being challenged. I am not really challenged. It is something that I am not enjoying. It is not working with hard working people who always ask questions, who want to learn like the Maths group. Here they must do the subject, it is part of their packages, it is compulsory.*

His goal is *to get 100% pass rate and at least 5 distinctions. Last year I had 97% with 1 distinction and 3 B's and one learner could not make it and most of them got above 40%.*

During the last interview I asked Monty to describe an ideal ML classroom in terms of, among other things, the instructional strategies used. He believes that the teacher should use direct instruction initially when new content is introduced, followed by group work *because the group work it goes with problem solving strategy*. There should be *discussion with writing something down. They must ask me questions*. Regarding the learning environment, he believes *one learner per desk facing the chalkboard. If they need to do group work they can combine… I don't believe too much in rules because I believe the educator can make environment good or nice for learning*. He stated that his classroom is not like this ideal classroom *because it's hard to change things if you are still a new teacher. You find them sitting like that, doing things like that, although you impose all the rules like that, they still do it, for them to cooperate it will take you long*. He mentioned that although he actually prefers group work he encourages individual work now because they are approaching the examinations. *I use the group work just to show them how cooperative work is productive*. He also believes that when the learners communicate in peer groups *they start to understand and they feel free to ask anything*. Monty values the idea of learners writing their solutions on the board and explaining it afterwards because *then it is going to be stored in your memory for always*. He believes the difference in approach used between ML and Mathematics is that fewer examples are done in ML, the pace is slower and the teacher does not need to go the extra mile because *the people you are working with in ML are not like the people you are working with in Maths.*

4.6.1.4 Knowledge and beliefs regarding ML curriculum

The DoE (2006) recommends a list of resources or instructional materials needed to teach ML (C1). The resources Monty used during the three lessons I observed were a textbook and blackboard as (C1). He used a textbook: Mathematical Literacy for the Classroom (Laridon et al., 2006) and previous examination papers. He explained the strengths and weaknesses of the textbook as: *information is clear and understandable, lots of examples and exercises* but *some topics have little information and few examples* (C2). He was not aware of the curriculum content being studied in other school subjects that integrate with ML (C3). Other departmental documents he knew of were the memorandums and circulars of which the latter was useful and valuable to him (C4). As far as the NCS: ML is concerned, he knew that there are four learning outcomes but could not name them (C5). According to Monty the DoE defines ML as *a subject aimed to enhance learners' skills of counting though they are not doing maths* and that the DoE's purpose for the subject is *to give every learner an opportunity to learn how to count because in real-life situation counting is a norm* (C5). Monty did not know which contexts the DoE suggests teachers should use in the teaching of ML (C5). Regarding the new CAPS for ML he only knew that the *teacher is a facilitator not like previously when the learner was a centre of every learning* (C5). At the end of the last interview I provided Monty with a list of concepts and contents to be covered in Learning Outcome 4: Data handling (NCS, 2003a) and Monty could only place seven out of 25 concepts in the correct grade in which they should be introduced (C6).

The lessons were presented as Mathematics lessons whene content was not situated in a context, although he did mention a few examples of contexts where the mathematical content could be applied (C7). During the first interview before the second lesson on simultaneous equations I asked Monty about the context in which the lesson was set (C7). He was startled and took a few seconds to come up with the idea of the elections. There was a 15-minute break between the interview and the class and I assumed he used that time to think about this context. During the introduction he talked about elections and the parties' campaigns but it was not clear how the given information was applicable to that day's lesson (C7). In the data handling lesson he talked about how research is done but did not link this to the learners' experiences or the content of that lesson. At the end of the data handling lesson he said:

In real-life situation, where can we use data handling? Census or SARS, for SARS to see how many people owe money, they have to get data, they have to have people registered to SARS, those that are only in business, they can see how many people are paying their taxes and so on and so on. Another one, remember guys we have elections of RCL elections ne? We said the class has two representatives ne? But you were able to vote for more than two people. But at the end of the day, they managed to get two representatives per class, ne? How? So that we can say this is our? RCL. So we had many people on the valid paper but at the end of the day we have a certain number of people to represent RCL. You see how we use data.

The context of elections was appropriate since municipal elections were to be held eight days later, but he could not apply the context appropriately and meaningfully to the content. The same applied to the SARS context he talked about. I doubt whether the learners were able to tell how data handling was applied in SARS and elections (C7). Later in the same lesson he gave this example of where and how statistics could be applied:

> *OK, now another story, when you do athletics, remember we use a stopwatch ne? A stopwatch helps us to record the time. So for example for one particular learner let's say we have athletics, we can record different times and we can calculate that data and you present it using a pie chart or whatever and you can use that data to arrange all things by recording that time during the events. OK now its fine* (C7).

According to Monty mathematics is *a tool used for solving problems* (C8). I asked Monty how he views mathematics as a discipline compared to ML as subject and his answer was:

> *I view it as constructivism because you have to be constructive if whatever you are doing especially in these days so that you can be successful. ML is viewed as a mathematics, but not lower grade and not challenging like Mathematics … ML is like a life skill because you learn how to divide things, how to add things, things like you are always doing when you are going to shopping, more of a life skill than a Mathematics subject* (C8).

He described the value of mathematics and ML as:

> *It's for logical thoughts because you learn to do things step by step and it gives you that strength as a person or individual to reason and think outside the box. The value of ML is that in a few years' time most of them will be doing Mathematics because now they can notice from ML that they can do Mathematics. The learners learn about counting, structures, angles, everything, so they can use that in their working place* (C9).

Summary

Table 4.11: Summary of Monty's knowledge and beliefs

KNOWLEDGE AND BELIEFS DIMENSIONS	DESCRIPTION OF TEACHERS' KNOWLEDGE AND BELIEFS' INDICATORS
Mathematical content knowledge (MCK)	Monty regarded mathematical knowledge as a prerequisite to teach ML. It appeared as if his MCK regarding the specific content covered is sufficient.
ML learners (L)	He believes learners gain understanding by looking at various examples on the board and through much practice. Although he regards group work as important where learners have the opportunity to talk to one another and learn from each other, he did not apply group work in class. He sees individual work as vital before the examinations.
ML teaching (T)	He did not always enable learners to connect their prior knowledge to the new content. He chose very basic examples in his data handling lesson and did not take learners' different abilities into account.
ML Curriculum (C)	He knew about the value of ML but could not provide the required information from the NCS (2003a). He views Mathematics as logical and constructive, valuable to all people. ML is viewed as a kind of mathematics, but not a lower grade of Mathematics.

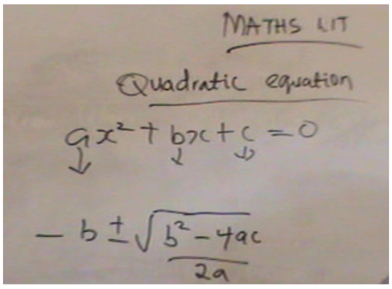
4.6.2 Alice's knowledge and beliefs

4.6.2.1 Mathematical content knowledge[60] (MCK)

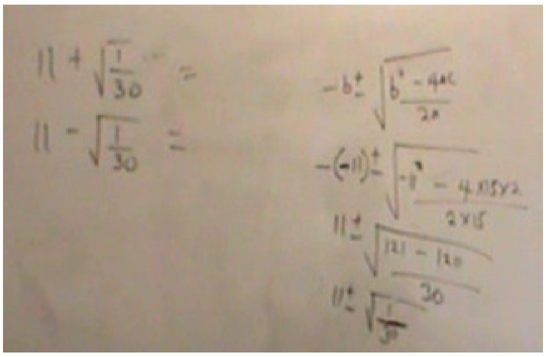
Alice believes that ML teachers need to have sufficient MCK and that no non-mathematics teacher can teach this subject. Regarding her own MCK many mathematical errors were observed. In the first lesson she used the formula incorrectly (See Picture 4.23). Later in the lesson she changed the formula

[60] Since there is only one indicator or code in Table 4.3 regarding the teacher's mathematical content knowledge, this whole paragraph's code is: MCK.

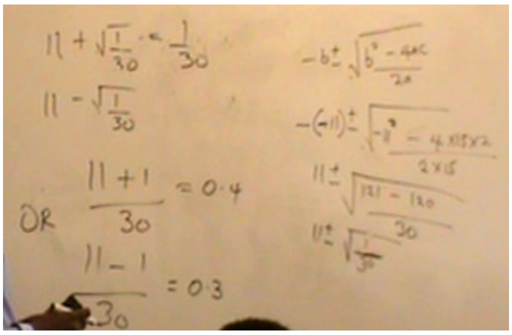
and put the denominator *2a* under the root sign (Picture 4.24). Near the end of the solution she again changed $\sqrt{\frac{1}{30}}$ to $\frac{\sqrt{1}}{30}$ (Picture 4.25).



Picture 4.23: Formula used incorrectly

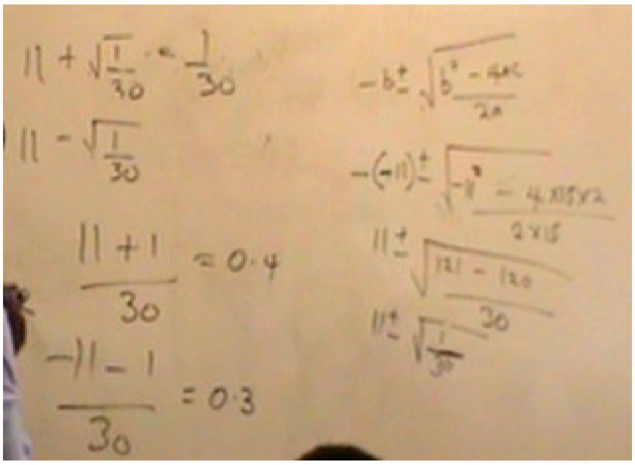


Picture 4.24: Formula is changed



Picture 4.25: Another change in formula during further calculations

After erasing work a few times from the board, Alice wrote: $11+\frac{\sqrt{1}}{30}=\frac{1}{30}$. A girl corrected the previous step to: $\frac{11+1}{30}$. In many cases Alice omitted to put the values in brackets when she substituted: $-(-10)\frac{\pm\sqrt{-10^2-4\times1\times25}}{2\times1}$. The learners were confused as this formula differed from the formula ($x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$) they used the previous day when the student teacher was responsible for the lesson. Alice believed that the student teacher used a wrong *method* as his work did not correspond to hers. Then there was the issue of having two solutions with the same sign which the teacher and learners believed were not supposed to happen (Picture 4.26).



Picture 4.26: An attempt to get a positive and negative answer

The following discourse took place:

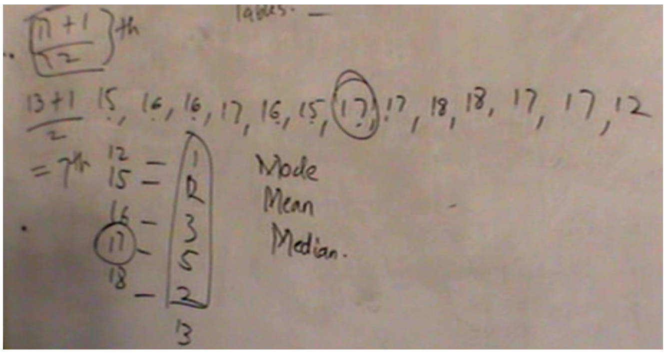
> *T: Am I right? (She checks again her calculations on a calculator.) You are supposed to get a negative and a positive answer. So, what happened here? I am sure there is something wrong because here we have two positive answers. Guys please! (The same girl from earlier who wanted to show the teacher on the board during example one brings her book to the teacher and talks to her but nobody could hear.) Yes but you have a negative and a negative so it should change.*
> *L: Oh OK!*
> *T: Quiet please! We are right, it's OK, as long as you know the method. So that's an exception. This is an exception. Let's do another one.*

In the second lesson where they drew the graph of the parabola, she did not attend to the given restriction in the example $y = (x-2)^2 - 1$ for $-1 \le x \le 4$. She also mentioned that they must have two intercepts with the X-axis, which is not necessarily true. Only at the end of the second lesson did she write the x and y value of a point as an ordered number pair for the first time, saying: *Here you have* $(0,3)$*, this is your x and this is your y.* When using the method of intercepts, they only had three points to plot the parabola with because they did not calculate the turning point and the following discourse took place:

> *T: You now join these three points.*
> *L: How do I know where to go?*
> *T: You can go anywhere, now you don't have this point (pointing to turning point). If you do a proper job, you will find your graph will go exactly through that point (turning point) … If you have to find the minimum, use the graph or the maximum value of y. How do you find the minimum?*
> *L: You just see it.*
> *T: Where do you get it?*
> *L: It's at -1.*
> *T: This is the minimum value (showing at turning point).*
> *L: There are two graphs now.*

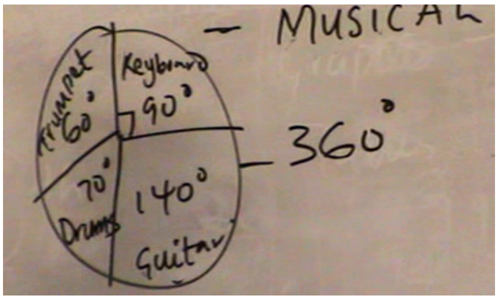
T: This is the same graph, we used two methods. (Everyone laughed as the graphs did not look the same). It's the same graph. We used the table method and then the formula.
L: OK.
T: Are you sure you do understand?
L: Ja.
T: OK.

During the third lesson she calculated the median without arranging the data (Picture 4.27), then told the learners to arrange the data in ascending order saying: *So it might not be that answer.*



Picture 4.27: Alice calculating the median incorrectly

The pie chart (Picture 4.28) was not drawn accurately – even though it was a rough sketch, 60° should have looked like an acute angle.



Picture 4.28: Unrealistic drawing of the pie chart

4.6.2.2 Knowledge and beliefs regarding ML learners

In the lesson on how to draw the parabola using two methods, Alice predicted that the learners would understand that they *can substitute that equation in that formula* (L1). When I asked her how the learners would understand the work, she told me about the preparation of the lesson and after a prompt she complained about the large classes that needed to be divided in two groups and the 40-minute periods that are not sufficient to do what she planned and therefore never answered the question (L1). She predicted that the learners would not understand *plotting the graph* as they have difficulty doing the following:

> *Maybe your $x = 0$ and then your y, make it -3, so when plotting the graph, where you have the -3, they go plotting it at the point where x is 0 and y is 0. It's always a confusing thing, plotting the graphs* (L2).

On predicting why it is difficult to the learners she replied:

> *It shouldn't be difficult but I don't know …. I cannot say this is why. Sometimes it can be confusing because now when you have 0 as the x and then you're trained to plot the y, you're thinking, you know you have to let both points meet, then you're thinking maybe you should, and this is the -3 and your x is 0 here. They are thinking maybe they put it here or here* (L2).

She predicted that the learners would approach the tasks by coming to her during break so that she could assist them, but offered no other strategy to assist them: *they want individual help but aside from that I don't know* (L4). She could not predict any other misconceptions learners might have (L5). In reality neither the learners nor the teacher understood why there were two positive answers when the quadratic equation was solved and as she predicted the learners made mistakes when point $(0,-3)$ was plotted (L5). When the table was completed in order to draw the graph of $y = (x-2)^2 - 1$, some learners did not understand how she obtained her answers when she worked directly from x to $(x-2)^2$ because she omitted the steps where $(x-2)$ should have been calculated (L5).

Regarding the data handling lesson, she mentioned that the learners would understand everything as they had done all the concepts in Grade 10 and according to Alice *data handling is one of the simplest aspects of ML so I think they should understand it all* (L1). She stated: *it's going to go smoothly with the chart, the tables and even the graphs* (L1). Regarding the new work, the cumulative frequency and drawing of the ogive, she said *they should understand it* and *they know it but it is just a bit advanced … it is just like a continuation of work they did* (L1). When asked how the learners will understand it she said:

> *I will tell them about the data and the raw information which you have to try to present, make presentable maybe using the graphs which is the pie graph or line graph or the charts which is the pie charts or maybe using the tables where they have to use the tallies or frequency, but there are different ways in which you can present your data and then we are going to try and refresh their memories on the mean, the mode, the median and then we are going to talk about the cumulative frequency, how to plot an ogive, we are going to work out the standard deviation* (L3).

She predicted that the learners would not understand the actual mathematics involved:

> *They used to have a problem with the pie chart, I don't know why. You know the pie chart? I don't know, they find it difficult to allocate those degrees even though from percentage to the grades of the pie chart. It is always a big problem* (L2).

When I asked her why the learners did not understand the pie chart, she said: *I don't know* (L2). Alice stated that the learners would not have a problem doing the tasks and then talked about group tasks set by the department (L4). She could not predict any other misconceptions the learners might have (L5). Although Alice said they would understand everything except the pie chart, they in fact did not understand how she calculated the measures of dispersion using the table method and one learner who was very involved in the lessons mentioned how difficult the pie chart was, but Alice disagreed and continued with the lesson (L5).

There was little evidence of what the learners understood, how they understood it, what they did not understand and how they approached tasks because they were not required to do any tasks in class (L1-L4). She told the learners that they would understand the content if they practised enough and if they watched her. She said: *Don't ask me how to get this, you have to look at me* (L3). The chaos of the first lesson showed that Alice did not have the ability to understand the learners in terms of their thinking (L6). When Alice was asked to comment on the first lesson she said:

> *Because when one person says this, and other person says this, then they get confused and they know it's just everything you tell them, they go the same way, they don't want to think out of the box. If you say it's -(-) they want to know this can just change to plus, so you must do it the sáme way every time. If I am the first person that taught them, then they want to go the same way as me. And I got confused too* (L6).

4.6.2.3 Knowledge and beliefs regarding ML teaching

In the second lesson on drawing parabolas using two methods, she regarded *simplification, factorization and the simultaneous equations* as prior knowledge (T1). She did not revise any prior knowledge during the introduction stage of the lesson, but immediately started with the table method when she reminded them to use both negative and positive values. Thereafter she used the formula which they had learned during the previous periods (T1). In using the formula she revised the values of a, b and c in the formula (T1). Regarding the data handling lesson, she said everything she planned was prior knowledge. In her introduction she revised the different kinds of graphs and charts which the learners already knew (T1).

Alice used various useful forms of representing ideas such as graphs, tables, formulae, calculators and symbols, but these were often overrepresented (T2). Although she used various representations, instead of creating opportunities for learners to develop conceptual understanding, the learners were confused (T2). She could not always transform the content knowledge into forms that are pedagogically powerful to attend to learners' diverse needs (T3). She has the ability to sequence the different content of her lessons, but it is when dealing with the particular concepts that too much information is shared using incomplete and varied representations. This happened especially in the data handling lesson (T4). When the learners did not understand or when they gave incorrect answers, she re-explained by using the same words as before, apparently not being capable of reformulation of her explanations (T4). After she did the example in which the pie chart was given, she told them that a question could be asked where they needed to draw the pie chart using given information. Unfortunately she did not follow this through by asking the learners to undertake such a task (T4).

Regarding the class and homework she said: *when we give the class work, we should make it look more practical, not just the theory and the x thing* (T5). About grading of the problems she commented: *They are based on this topic. It's practically the same thing, the same level* (T5). For the data handling lesson, she planned to use exercises from the textbook and at the end of the week a task from the DoE (T5). Alice generally selected too much content for one period so that there was no time for her to assess her learners' knowledge and understanding (T5).

ML teaching: Reflecting on her practice[61] (T6)

Alice claimed that she really loves to teach ML and found it quite interesting. She tries to motivate the learners and invites them to come to her class during break time if they do not understand the work, but still they are not motivated and would rather play pool during break, which distresses her. To improve her learners' appreciation of ML she plans to *get down to their level and make them understand what they really need to know, to make it quite simple*. To improve the learners' participation in the lessons she tries to pose questions to all, but this is not uniformly successful: *The boys at the back are always (pause), they just want to be in class, they don't participate. The ones in front want to take part.* She believes learners master new knowledge as follows: *To learn new things it is all about making up your mind, I want to know this. The learner must go back to what he is taught, or come to the teacher, try to do your own research with books and take it up from there.*

Alice chose direct instruction as strategy because *the first lesson we have you must teach them the basics. I try to introduce them, make sure they understand the basics of what they are doing, or what the topic is about, it is more like formal teaching. Later on the discussion comes.* Her view on calling learners to the board is: *it's all about practising in the presence of everyone and it gives them confidence and then their friends can learn from them and then try to assist them … and it forces them to come to class prepared.* She does not favour discourse between the learners:

> *Actually I don't like to encourage that [discourse between learners] because most times when I am talking I found discussions among themselves and then sometimes they miss out when they do that. Some learners speaking in their dialect and vernacular, it's a bit difficult when you are teaching and when a learner asks you, mam, what is that word? Please can you repeat that and then I am too busy teaching. So, I try to make them listen not to miss out.*

She does not believe that the teaching of ML differs from that of Mathematics. Regarding her role in the classroom she said: *I try to be in charge but give them the chance to talk to me and go and read because ML asks for practising, so I would say I am the mediator.*

[61] Only one code was used to report on the teacher's reflection regarding her own practice namely T6.

During the last interview I asked Alice to describe an ideal ML classroom in terms of, among other things, the instructional strategies used. She described her ideal ML classroom as one where various instructional strategies are used such as *group work and discussion in class and then active learning and learners coming to the front*. In her description of the ideal discourse in her ML class, she mentioned she wants learners *to speak up when they do not understand and to ask intelligent questions. I really want them to ask: why did you do this, how did you do this? Is this how you are supposed to do it?* As far as the learning environment is concerned, she wants all learners to have their own textbooks so that she does not need to waste time by writing the whole question on the board. Her goals in teaching ML are that all learners do well in their examinations and that they understand that *ML is quite simple if you put your mind to it.*

4.6.2.4 Knowledge and beliefs regarding ML curriculum

The DoE (2006) recommends a list of resources or instructional materials needed to teach ML (C1). The resources Alice used in the three lessons I observed were the Oxford Successful ML (Pretorius, Potgieter & Ladewig, 2006) textbook, the whiteboard and calculators. Although the textbooks are available at her school for learners to buy, she said that *the learners want the school to give it to them* and that the learners asked her to give them the money to buy the textbooks, so most learners do not have their own textbooks (C1). According to Alice a point of strength of the textbooks is the large number of *practice questions* but a weakness is that the textbooks do not provide a *step-by-step method of answering questions* (C2). She had no knowledge of the curricula of learners' other subjects and how those curricula integrate with ML (C3). The only departmental document she knew of is the work schedule which she finds useful as it guides her teaching (C4).

The DoE's definition of ML according to Alice is: *The department of education makes it look like ML is a lower substitute for Mathematics* and their stated purpose of ML is *to help learners have a basic knowledge in mathematical related issues* (C5). She knows ML has four learning outcomes but could not state them and she does not know anything about CAPS (C5). At the end of the last interview I provided Alice with a list of concepts and contents to be covered in Learning Outcome 4: Data handling (NCS, 2003a) and Alice placed 16 out of 25 concepts in the correct year that they are to be introduced (C6).

During the last interview I asked Alice to which contexts the content should be applied according to the DoE and Alice's answer was: *Teacher-learners participating* (C7). In the interview before the second lesson on drawing the parabola, I asked about the context she was about to use and she talked about all the mathematical content to be covered. After a prompt she replied that no context is going to be used (C7). Although she mentioned in her interview before the third lesson that elections would be used in the lesson, she did not once refer to elections during the lesson. Pure statistics were done except for the one example that was based on the 'musical group' which the learners found interesting (C7).

To Alice mathematics is a *logical subject and it has to do with constructivism* (C8). She does not see the difference between Mathematics and ML and said:

> *ML is just a little bit easier … it's still part of the Maths, it's just of a lower grade. One of them said I don't see the difference, I said it's just a make believe and then they keep passing the same belief as they come. Everyone comes with the mentality it's difficult, and when they come here they say it is still Maths, it is difficult* (C8).

According to Alice mathematics is *quite important because you find it in everything; it is vital* (C9). The value of mathematics and ML are the same as both cover Financial Mathematics and since all people deal with calculations every day all people need a basic knowledge of mathematics (C9).

Summary

Table 4.12: Summary of Alice's knowledge and beliefs

KNOWLEDGE AND BELIEFS DIMENSIONS	DESCRIPTION OF TEACHERS' KNOWLEDGE AND BELIEFS INDICATORS
Mathematical content knowledge (MCK)	Alice regards MCK as an important prerequisite in teaching ML. She however made numerous mathematical errors in class and her MCK appeared to be insufficient regarding the specific content covered in the three lessons.
ML learners (L)	She did not have sufficient knowledge of learners as she predicted they would understand all content she dealt with them but in reality they did not understand all the work. Apart from her own misunderstandings and misconceptions, she could not understand the learners' misunderstandings.
ML Teaching (T)	She struggled to have a logical flow in presenting the different concepts in her lessons and to sequence her activities. Too much content was covered in some of the lessons, which caused learners to become confused and frustrated.
ML Curriculum (C)	She does have sufficient curriculum knowledge but needs to know more about the DoE's vision for the subject. Alice views mathematics as a logical subject and believes that it has to do with constructivism. She does not see a difference between Mathematics and ML, ML is just a bit easier. The value of mathematics and ML is that both cover Financial Mathematics.

4.6.3 Denise's knowledge and beliefs

4.6.3.1 Mathematical content knowledge[62] (MCK)

Denise believed that MCK is a prerequisite to teach ML. She said:

> *If you don't know the maths, you cannot understand the practicality of this ML. You must have content knowledge, because then at least you can build on that. Especially to take the maths and put it in context form and vice versa. If you don't have maths, I don't know where you will start.*

From the lessons I observed, Denise's MCK is good and no mistakes were made. To prepare the learners for examination, she emphasised the importance of showing all steps and adding the units at all times in their calculations and final answers.

4.6.3.2 Knowledge and beliefs regarding ML learners

For the first lesson[63] on conversions from metric to imperial units based on capacity and mass, Denise predicted that the learners would understand *the conversion of metrics to imperials and then straight metrics conversions* because she did *a lot of drilling* in class and they had to practise it at home too (L1). She also mentioned if *they don't keep on doing that [practising conversions], in revision you will see those errors* (L1). For the third lesson on conversions within the metric system based on capacity, mass, length, area and volume, she predicted that they *will easily understand the distance* (L1). Denise's predictions regarding what learners would understand and would not understand were in line with what happened in class (L1). Denise did not predict that the learners would misunderstand any content in the second lesson but concerning the third lesson she predicted that the learners would not understand conversions regarding area and volume because:

> *The distance is easy. Between the conversions from mm to km, the 10, the exponential is to the power of one. Coming to area, now it turns to the power of two, so before they divide or even multiply they start now forgetting, because the 10 is to the power of two. Then the same applies to the volume, the 10 is to the power of three. And then they usually multiply or either divide it without considering that the 10 is to the power of three before they can divide or multiply. There they got difficulties, but with length it is straight forward for them* (L2).

Denise believes that learners come to understand once *they can see their mistakes on the chalkboard … respond to questions … correspond and check what their misunderstanding was* (L3). She also believes that learners develop an understanding through individual practice but also during

> *formal assessment when I am done with new work, the feedback to them is how they will learn and rectify where they don't understand". She believed the learners will approach the tasks all by themselves, not even referring to the textbook as they know how to do these conversions* (L3).

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62 Since there is only one indicator or code in Table 4.3 regarding the teacher's mathematical content knowledge, this whole paragraph's code is: MCK.

63 The first lesson was repeated the next day to another class, so only two different lessons were observed.

Denise believes that another possible misunderstanding the learners could have is when 250ml must be converted to pints and they *search for millilitre to pint in the conversion table* not realising millilitre must first be converted to litre and then the conversion from litre to pints is provided in the table (L5). Sometimes, she said, they also confused distance with area (L5). Another example Denise gave of learners' misunderstanding was:

> *I give the measurements of the cube and I say they must calculate the volume in cubic mm, then the second question now I say the answer they got for the cube, they must convert it to cubic metre, but I said to them one cubic metre equals to 1 million cubic mm. Then you know, they can't actually do this, it's in context now, I don't know why … they want it straightforward* (L5).

As Denise discussed the learners' work on the board, it appeared that she understood the learners' alternative conceptions. There was only one incident in which learners had to do a conversion problem based on area and one learner wanted to square the answer too[64]. Denise misunderstood the learner's question and after an elaborate answer, the learner told Denise that was not what she asked and the learner repeated her question. Denise then understood the question but still did not address the learner's problem, just replying *no* to the question (L5). Denise provided opportunities for the learners to express themselves in writing on the board so that she was able to see what they did, but not many opportunities were created for her to listen to their thinking (L6). Based on the work she saw on the board, she acted appropriately to facilitate the learning process by discussing the learners' work with the individual as well as the rest of the class. She even involved learners to correct other learners' work (L6).

4.6.3.3 Knowledge and beliefs regarding ML teaching

According to Denise the prior knowledge needed to be present to understand the work for the second lesson was *metrics [as] they did that in Grade 10,* also *to solve other problems from metrics to imperials and ratios* (T1). Denise did not revise the prior knowledge at the beginning of the lesson but integrated it in and across her lessons (T1). Since the lesson was on conversion from the metric to imperial system, some problems required an initial conversion within the metric system (prior knowledge) before the actual conversion could be made (T1). During the second interview I questioned Denise about the prior knowledge needed for the third lesson to which she replied: *Not applicable. It is a revision lesson.* (T1).

Denise used different forms of representation to make the content comprehensible to the learners such as written demonstrations and oral explanations where the use of either equations or ratios to solve the conversion problems were demonstrated (T2). She also used a diagram to revise prior knowledge based on conversions of length within the metric system (T2). These examples and tasks motivated the

[64] This example is discussed under Discourse: Teacher-learner interactions.

learners to understanding their own solutions and thinking (T2). The way Denise sequenced her tasks and explanations was proof of her ability to transform her own knowledge into forms that are pedagogically powerful (T3). She sequenced the tasks from the first to the third lesson: the tasks became more demanding and also included a wider variety of concepts, but only within the metric system (T4). Within the lessons there was no sequencing of the tasks from easy to difficult (T4).

Denise chose an appropriate instructional strategy for her revision lessons (T5). She used discussions in which she built her instruction on the learners' knowledge, their common errors and misunderstandings (T5). This strategy provided opportunities for informal assessment of the learners' knowledge (T5). The tasks were chosen to include conversions within the metric system but also conversions between the metric and imperial systems (T5). These tasks were based on length, area, volume, mass and capacity (T5). Denise chose the tasks from the learners' textbook but for assignments and assessments she chose tasks from other resources too *so that they can get exposed to other authors questions and approaches* (T5).

***ML teaching: Reflecting on her practice*[65] (T6)**

Denise's experience of teaching ML was:

> *It [ML] is not challenging. I feel a little bit bored, because I have done the pure maths. But for the sake of them [the learners] so that they must understand why we learn maths, I start to be a little bit of motivated. In pure maths you enjoy it throughout.*

It is important for her to motivate the learners by telling them the value of ML:

> *It is going to help you throughout your life where you are able to work out your own things, if you have your own business, your work one day, you are able to work with percentage, unlike not having maths at all. Basically it's personal as well as the reading of the stats, doing the inflation, price increase, petrol increase, you are able to calculate that.*

She ensured learner participation by giving the learners' tasks to complete individually and afterwards allowed some of them to write their solutions on the board so that corrections could be done. She believes that learners learn from the feedback she gave on their work done on the board.

Her reason for choosing the strategy of learners working on the board was: *Because I just want to see what they misunderstood, the content and the context. It then becomes easier for me to rectify any misunderstandings.* When introducing a new topic she believes she needs *to explain the content and concepts that are a little bit new, new words they are not familiar with and explain it thoroughly how its application work. Then from there I start doing the*

[65] Only one code was used to report on the teacher's reflection regarding her own practice namely T6.

drilling. Although I did not observe learner-learner interaction, she supported the idea thereof as she said:

> *They feel very comfortable when they talk to one another unlike with me (pause), some they find it very comfortable between peer and peer because they can understand the same group … So when they talk to one another it's not a problem … sometimes you find they talk together and once they argue, they come to me. That's what I like about them.*

Experience has taught her that teaching ML is different from teaching Mathematics as *most of the ML is on contexts whereas the basics are not well aligned like Mathematics is for me. I compare it with natural science and technology, technology is the root of science. Literacy is the root of pure maths.* After two prompts to direct her answer towards the teaching of the two subjects, she still continued to talk about the difference between the two subjects. She sees her role in her ML classroom as being the one who is there *to teach them … and then I facilitate whether they know the content or topics of the contexts are well understood.*

During the last interview I asked Denise to describe an ideal ML classroom in terms of the instructional strategies used. She described an ideal ML classroom as one in which the learners are involved in the lesson, where they *learn through doing it, trying on their own*. She emphasised that it is meaningless if they just look at her doing the work on the board; they should practise it themselves so that they can discover and construct their own meaning. A vital point was to have opportunities where she could communicate with the learners in order to determine learners' misconceptions and errors and have sufficient *time to rectify it through conversations*. Regarding the learning environment she believes:

> *It goes back to motivation, why they are learning this. We must come to a point where we can show them the realistic part and the value of it in everyday life. The lessons are contextual. The learners say where am I going to use ML in tourism? I don't see it. So then I must show them that wherever you are going to work, you are going to use it.*

When I asked her how this ideal classroom compared with her own classroom, she said:

> *Even though the time restricts us, I try. They must also be involved in the learning process. It must not be always me telling them this is how it is done, take it or leave it. They must also contribute, they must come up with example, they sometimes tell you of things from their world. So give them that freedom to participate.*

Her personal goal in teaching ML is *that these learners can be able to use this ML in everyday life; it will be perfect to me.*

4.6.3.4 Knowledge and beliefs regarding ML curriculum

The DoE (2006) recommends a list of resources or instructional materials needed to teach ML (C1). The instructional materials Denise used to teach her lessons on conversions were the Oxford Successful ML (Pretorius et al., 2006) and Classroom ML (Laridon et al., 2006) textbooks (C1). She did not mention any advantages of the textbooks but experienced the textbooks as *not so good and effective* and *the questions are sometimes confusing for the learners and language is difficult for them* (C2). She was not aware of the curricula of other school subjects which integrate with mathematics (C3). Departmental documents she knew of were the *Assessment policy guideline; and Learning outcomes with assessment standards* of which she experienced the *Assessment policy guideline* as useful and valuable (C4).

When I asked Denise in the last interview how the DoE defines ML, she stated: *There is no definition for this subject only the implementation is important* (C5). According to her, the DoE's purpose with ML according to Denise is *for the learners to know how to use maths in their everyday life* (C5). She did not know anything about the new CAPS document as they had had no training yet (C5). She knew ML has four learning outcomes, but could not mention them (C5). At the end of the last interview I provided Denise with a list of concepts and contents to be covered in Learning Outcome 3: space, shape and measurement (NCS, 2003a) and Denise placed seven out of 19 concepts in the correct year that they are to be introduced (C6). In the interview before the second lesson, I asked about the context she was about to use and she replied: *Like the cuboid, to find the volume of a cuboid in mm and then convert this into cubic metres* (C7). For the third lesson she said the context to be used was *a table with matching tables, matching column A and column B, so that they can see one kilogram is how many pounds and so on* (C7). She did not mention any contexts in one of her lessons and also did not mention a cuboid in the second lesson.

Denise views mathematics as follows:

> *It's not formal. If this is the formula, I can also change it as long as I know I can prove it, and make arguments why I change it, as long as I can justify it … For me it's flexible. Then I can construct my own beliefs and my own understandings on what I am working on* (C8).

She summarised her view of mathematics by saying *mathematics is flexible and creative* (C8). Her perception of ML is that it is

> *not a higher grade or standard grade maths … more like a life skill … it's a maths on its own [and learners] must know at least the origin which is in pure maths, the origin of the curved graph, the origin of the parabola, the origin of the straight line graph, not just the application.*

The value of mathematics is endless to her, but among other things, she mentioned the following:

> *Whatever I do, it's maths. Personal, work, everywhere it's found. I used to say to my learners, if you walk, you count the steps you make, its maths, 1,2,3, it's natural numbers. I say go and buy zero bread, I ask what will you bring? Nothing, and zero is a whole number. You see now maths is everything* (C9).

To her the value of ML is that learners who are not capable of doing Mathematics can do ML. She said:

> *These learners don't know how to handle their personal lives and they are now able to work or manipulate with what is outside. For example if they want to read a pie chart, statistics in general, so now unlike before the learners who*

are not capable to do pure maths or do not have maths, they can do this. So for me this is better than nothing because they can use it somewhere (C9).

Summary

Table 4.13: Summary of Denise's knowledge and beliefs

KNOWLEDGE AND BELIEFS DIMENSIONS	DESCRIPTION OF TEACHERS' KNOWLEDGE AND BELIEFS INDICATORS
Mathematical content knowledge (MCK)	Denise regarded MCK as a prerequisite to teach ML. She made no errors in her examples or corrections of the learners' work, but also did not elicit any discussions regarding the conceptual meaning of the different units of measurement or their application value in everyday life situations.
ML learners (L)	She correctly predicted what learners would and would not understand. She did not allow learners to explain their thinking so that she could listen to their thinking. She acted appropriately to the work she saw on the board.
ML teaching (T)	She correctly identified the prior knowledge for the second lesson. Denise used various appropriate representations to make the content comprehensible to the learners, applied appropriate instructional strategies and sequenced her tasks over the different lessons to enable learners to progress in their cumulative understanding. According to Denise the ideal ML classroom compared well with her own except for allowing the learners more time to discover the content as time is always a restriction.
ML Curriculum (C)	Regarding the NCS, Denise could not provide the DoE's definition, purpose and learning outcomes or even place half of the topics in the correct year that they are to be introduced. Denise views mathematics as flexible and creative, constructing one's own understanding and ML as a type of mathematics, but unique. According to her, both mathematics and ML are valuable as people use both in their personal and work environments.

4.6.4 Elaine's knowledge and beliefs

4.6.4.1 Mathematical content knowledge[66] (MCK)

When Elaine was asked about the extent to which MCK is a prerequisite to teach ML, she said:

A motivated teacher who is prepared to work hard will manage the teaching of ML but there are some details of mathematical knowledge a ML teacher needs to know. You will not be able to just explain rates of change and ratios, you will need to learn about the finer things, how to explain it, what is important. It is only now, after three years of teaching ML that I feel confident in front of my classes, that I know what is important and what should be done in each grade.

From the lessons I observed, Elaine's MCK is very good. She did not make any mathematical errors and I did not observe any misconceptions.

[66] Since there is only one indicator or code in Table 4.3 regarding the teacher's mathematical content knowledge, this whole paragraph's code is: MCK.

4.6.4.2 Knowledge and beliefs regarding ML learners

Regarding the second revision lesson on time and interest, Elaine predicted during the interview just before the lesson that the learners would recognise the formula and be able to tell that the formula is used for compound interest (L1). Regarding the third lesson on perimeter, area and volume, she predicted that the learners would know the units and how to calculate the perimeter, area and volume as they did that in Grade 10 (L1). She realised some learners would still struggle in the second lesson *to convert the variable in the formula to a decimal as well as with the concept of interest being calculated monthly and semi-annually* (L2). The task she used in the third lesson was more complex than the previous year and *[i]t is possible that not all learners will be able to verbally explain the concepts. Not all learners have conceptual knowledge of the concepts or sometimes they have the concept in their mind but do not have the ability to verbalise that concept* (L2). She believed that the learners would understand the tasks in the second lesson once they *can explore and have that aha feeling*, but she needed to make the lesson *applicable and link the new work with their personal lives such as personal tax, start at home, their parents' salaries …* (L3). Concerning the third lesson, the learners would understand the work once they learnt to *carefully read through the problem, study the drawing and indicate all the information with coloured pens on the drawing as the visual representation simplifies it* (L3). Her predictions about what learners would and would not understand were realised in her lessons.

During the interview just before the lesson she predicted that the learners would approach new tasks by discussing the work with their peers (L4). She frequently noticed that *they do not just ask anyone, they will not ask a friend who knows less than themselves but will ask someone they know is able to explain the work to them* (L4). In her experience *they really listen to and learn from one another*. She said people joke about the *buzz in her class but they [the learners] buzz about the work* (L4). Many times she preferred to *put a weak and strong performer together and then the one learns to explain and the other one learns to understand*, thus using cooperative learning as instructional strategy (L4). At her school all teachers need to remain after school till 14h30 and many learners make appointments with her for individual support (L4). They also use their textbooks and scripts as sources of reference (L4).

Regarding other possible misconceptions learners might have, she mentioned a problem with cubic centimetres: *they do not always have the concept of length x breadth x height, so it is three units multiplied with each other, the same with area* (L5). Elaine required learners to give explanations and justifications orally and in writing (L6). A fundamental part of Elaine's lessons was learners' abilities to explain the meaning of concepts as these explanations provided her with proof of learners' conceptual understanding of these concepts (L6). This enabled Denise to see what individual learners do, listen to what they think and to act appropriately (L6). She recognised and clarified the learners' common errors and misunderstandings (L6).[67]

4.6.4.3 Knowledge and beliefs regarding ML teaching

As far as the prior knowledge of the observed lessons was concerned, Elaine said all mathematical content had been introduced either in Grade 10 or Grade 11 as the lessons were revision lessons (T1). In these three lessons the learners needed to practise their skills and use their existing knowledge by solving unfamiliar and more complex contextual problems (T1). In class there were numerous situations when Elaine revised prior knowledge before presenting a particular concept (T1). Elaine applied representations such as tables, symbols, formulae, calculators, a demonstration calculator and sketches of manipulatives to make the tasks comprehensible to the learners (T2). These representations proved to be useful as learners participated and comprehended the work (T3). The tasks were sequenced and presented in a pedagogically powerful way to facilitate learning (T4).

In preparing her lessons Elaine realised some learners could become bored as they repeated similar work a few times. She also needed to attend to those learners who only knew half of the work or even less (T5). She then planned to discuss the work she knew they struggled with, such as explaining the significance of the different variables in the formula. She involved them in discussions during which their errors and misunderstandings were corrected. Her planning was based on preparing the learners for the examination, so she provided guidance on the type of questions they could expect and how those questions should be approached (T5). Elaine's choice of instructional material was appropriate as she chose tasks from previous examination papers to enable learners to solve typical examination questions in different contexts (T5). She encouraged the learners to use coloured pencils to indicate given information on their sketches (T5).

***ML teaching: Reflecting on her practice*[68] (T6)**

When Elaine was asked two years ago to be the coordinator for ML, she initially felt that she was being demoted, but she claims that ML had grown on her since then. She enjoys being involved in ML and never wants to go back to Mathematics. To improve her learners' appreciation of ML she bases her lessons on real-life situations as the learners need to recognise what the subject is about and where mathematics could be used. She refers her learners to the yearbooks of tertiary institutions to familiarise them with the requirements of possible future studies and how ML can add value to their studies. She

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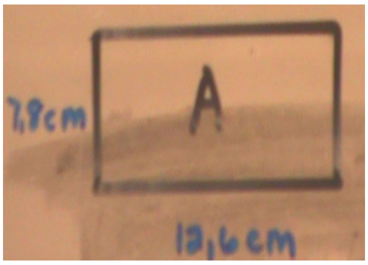
[67] More detail is given under Elaine's Discourse: Teacher-learner interactions.
[68] Only one code was used to report on the teacher's reflection regarding her own practice namely T6.

also mentioned that she wanted to improve her learners' participation in the lessons. She would like to use two of the seven periods a week to do something out of the ordinary like taking them on an excursion or watching a DVD where they can discover the role of mathematics in specific situations or events, which could then be followed by a class discussion. She said she could give them a worksheet before the excursion or DVD which the learners could complete during and/or after such an event. She also wanted to use games and newspapers and magazines to further enrich the learners' appreciation of mathematics. She believes such activities contribute towards proficient learning.

I asked Elaine to reflect on the three lessons I had observed. According to her she chose discussions as one of her instructional strategies since it gave her a platform to work from. She determined the gaps in their knowledge, which became the focus of the lesson. She believes discourse between the learners indicates that the learners are involved and interested in the lesson. To elicit such discussions she enjoys throwing in a question with an impossible answer such as the question in the class test (Picture 4.29) during the third lesson where she asked: *The following figure is a rectangle. Calculate the volume of the figure.*



Picture 4.29: A question asked in a class test

According to Elaine her classroom rules are less rigid and structured than other classes in the school. She wants her learners to have fun in a relaxed atmosphere. She said she did not want the subject to have a negative stigma and therefore made an effort to make the subject alive and interesting. She asked the principal and colleagues to respect her subject as she respects their subjects and not to make fun of ML by referring to it as a very low level of Mathematics. Elaine believes ML should not even be compared with Mathematics, but should be regarded as a subject on its own. Evidence that her learners enjoy and value ML lies in the fact that learners who had to change from Mathematics to ML asked to be in her class. One of Elaine's most important rules is that learners should respect one another and value other people's thinking and ideas.

Elaine believed that the teaching approach of ML is totally different to Mathematics because ML is presented in a more relaxed way where the learners are not pressed to finish in time. According to her in ML the high ability learners can continue with additional work and there is enough time to further attend to the slower learners. There is always time to go back to their Grade 9 content and do revision, which is not the case when teaching Mathematics. She wished that a standard grade of Mathematics could be implemented again to cater for those learners who do not actually belong in the ML class but at the same time do not wish to take Mathematics. She regarded her role in class as being the mediator between the content and learners. Often learners had the correct ideas but did not know how to formulate them. She then needed to guide the learners in the process of discovery and provide scaffolding to assist them in moving from their uncertain and disorganised thinking processes to conceptual understanding. She believes that the learners sometimes need to struggle through the process of problem solving and once they experience success, it serves as a lesson in life that there are times one needs to struggle through solving a problem, but it is possible to arrive at a solution.

During the last interview I asked Elaine to describe an ideal ML classroom in terms of the instructional strategies used. She described an ideal ML classroom as one in which a teacher uses instructional strategies that are effective to her as individual. She also stated that apart from the discourse between the teacher and learners there should be enough opportunities for communication between the learners. An ideal is having computers with internet access in such a class to enable learners for example to find the present exchange rate and as such make the tasks more realistic. She believes *information you search for on your own and read it by yourself is more valuable and will be remembered longer*. Elaine planned to have newspapers available in class to enable the learners to work with news of the day. In comparing this ideal ML classroom with her own practice she believes it is the same except for not having computers in her classroom. Her purpose in teaching ML is to equip the learners with life skills and she hoped that one day when they think back they would remember her and what she taught them, but more importantly, the life skills and values she taught them.

4.6.4.4 Knowledge and beliefs regarding ML curriculum

Regarding her curriculum knowledge she knew about the appropriate instructional material that could be used for the lesson she did on perimeter, area and volume as she used textbooks, previous examination papers, models of two-dimensional figures and three-dimensional objects as well as transparencies (C1). She used the Mathematical Literacy for the Classroom (Laridon et al., 2006) textbook, a book containing previous examination papers and the internet (C1). She valued the fact that each learner has his/her own copy and is able to use the textbook at home (C1). She mentioned that a weakness of textbooks is the fact that changes made by the DoE regarding the curriculum cannot be accommodated C2). She found previous examination papers valuable as the questions prepare the learners for examinations (C2). The value of the models was the opportunities for learners to discover knowledge through visual experiences (C2). According to Elaine the ML curriculum should include mathematical content that integrates with the curricula of other school subjects. She provided the following list of subjects that integrate with ML:

Business economic – graphs
Accountancy – salaries
Engineering graphical design – scales and drawings
Mechanical technology – trigonometry
Tourism – map work (C3).

In her opinion trigonometry that was initially in the curriculum but later omitted should be put back as trigonometry is required by Mechanical Technology (C3). Departmental documents she was aware of were circulars and the CASS document which she found useful (C4).

According to Elaine the DoE defines ML as *equipping all learners with mathematical skills by using problems from real-life situations* (C5) in order to *equip learners with basic mathematical skills. They implemented it based on the recommendation of the private sector that required workers to become mathematically literate* (C5). She had not yet perused the new CAPS document but believed this document will be more distinct, comprehensible and user-friendly (C5). She knew that the four learning outcomes were mathematical concepts, financial mathematics, measurement, data handling and probability (C5). At the end of the last interview I provided Elaine with a list of concepts and contents to be covered in Learning Outcome 3: Space, shape and measurement (NCS, 2003a) and she placed 11 out of 19 concepts in the correct year in which they were to be introduced. The curriculum required that content should be set in context which Elaine appropriately did (C6).

As prescribed by the DoE (2003a), Elaine taught the content in all three her lessons in context (C7). The contexts were applicable to the content and she had sufficient knowledge of the contexts she shared in discussions with the learners (C7). An example of such a discussion was:

T: Let us quickly discuss, why would somebody rather wait to buy a house and first increase his deposit before doing so?
L1: Isn't it to lower his instalment?
T: Yes, to decrease his instalment. Did you also know that if one day you have bought your own house you can decrease the period of paying back the money you have borrowed by several years if only you pay all extra money you might have available in a month into your home loan account. By doing that, what would you save on?
L2: Money.
T: Money yes, but you will save on interest you need to pay on that home loan account.

Most of the contexts she used in her class were unfamiliar to the learners (C7).

She believes mathematics is about rules and formulae that need to be discovered. Mathematics should not be presented on a blackboard to learners as a rigid discipline with fixed rules (C8). She concluded by saying: *Mathematics is a discovering experience* (C8). Learners should be able to experiment with the mathematics available to them. She believes ML is for everyone, even for Mathematics learners as they too would increase their general knowledge on life-related issues as well as their reading skills (C8). She believes mathematics has an *unbelievable value as mathematics stimulates one's brain and practice higher order thinking … it expands your vision and you generally think better. You don't do rote learning only, you have to reason too* (C9). The value of ML *is incredible as a learner returned from his holiday one day telling her in excitement that he used the mathematics she taught them* (C9). One of her learners told her that recently he had amazed his father by telling him about what he needed to take in account before buying a new car.. These were proof of the value of ML and the life skills ML learners acquire (C9). Elaine explained that she and her husband had bought a house a few years ago and had to learn for the first time in 42 years about all the costs and implications involved in buying a house and arranging finance, whereas her Grade 12 learners had already learnt about this in school (C9).

Summary

Table 4.14: Summary of Elaine's knowledge and beliefs

KNOWLEDGE AND BELIEFS DIMENSIONS	DESCRIPTION OF TEACHERS' KNOWLEDGE AND BELIEFS INDICATORS
Mathematical content knowledge (MCK)	Elaine has very good MCK and no errors or misconceptions were observed. She believes it is possible, but not ideal, to teach ML without having MCK as long as the teacher is hard working and motivated.
ML learners (L)	She had the ability to predict what learners would and would not understand and why and how they would understand the new content. She predicted that they would approach new tasks by discussing the work with peers from which they could learn, but that they would also come to ask her or consult their textbooks.
ML teaching (T)	Elaine taught the content in context and knew the prior knowledge needed to explain new concepts. She used relevant examples, illustrations and explanations to make the work comprehensible to the learners. She transformed her content knowledge into forms that were pedagogically powerful. She also efficiently sequenced the content to facilitate learning. The instructional material was chosen appropriately. Elaine likes teaching ML and believes the subject should be taught in an interesting and practical way so that learners can enjoy and value the subject.
ML Curriculum (C)	She is well informed regarding the curriculum since she correctly answered most of the curriculum questions I asked during the last interview. She knew about a variety of instructional materials to use in the lessons she presented and was familiar with other school subjects that integrate with ML. She was not aware of all available departmental documents or even the content of CAPS, but knew the definition and purpose of ML according to the NCS. Most of the time the contexts were unknown to the learners. She regarded mathematics as being flexible and that learners should discover and experiment with it. She values

	mathematics as a discipline that stimulates the brain, allowing higher order thinking and reasoning. ML is for everyone, even for Mathematics learners because their general knowledge on life-related issues should also be increased.

4.6.5 Summary of the participants' knowledge and beliefs

Table 4.15 provides a snapshot of the four participants' MCK and PCK and beliefs regarding the ML learners, the teaching of ML and the ML curriculum.

Table 4.15: Snapshot of the four participants' knowledge and beliefs

PARTICIPANTS	MONTY	ALICE	DENISE	ELAINE
Mathematical content knowledge (MCK)	He believed MCK is a prerequisite to teach ML. He made no mistakes and it appeared as if his MCK regarding the specific content covered is sufficient.	She believed MCK is a prerequisite to teach ML. She made several mistakes and it appeared as if her MCK is insufficient regarding the specific content covered in the three lessons.	Denise believed MCK is a prerequisite to teach ML. She made no errors in her examples or corrections on the board and it seemed as if she had sufficient MCK regarding the specific content covered in the three lessons.	Elaine believed MCK is a prerequisite to teach ML. No errors or misconceptions were observed and it seemed as if she had sufficient MCK regarding the specific content covered in the three lessons.
PCK and beliefs regarding the ML learners (L)	• His predictions about the content the learners would and would not understand did not correspond with what happened in class. • Some comprehension of learners' misunderstandings but when he did not understand their misunderstandings he became irritated. • Limited evidence of learners expressing themselves to determine if he could act appropriately on their ideas. • He believed learners gained understanding from looking at examples and practising the work.	• She predicted learners would find all content easy and could not understand their common errors or misunderstandings. • She proved to have certain misconceptions herself so it would be difficult for her to predict possible learner misconceptions. • No evidence of learners expressing themselves to determine if she could act appropriately on their ideas. • She believed learners learn from practising in the presence of someone who gives them confidence.	• She correctly predicted what learners would and would not understand. • She realised learners' possible misconceptions and rectified them in class. • She did not allow learners to explain their thinking so that she could hear their thinking but as they demonstrated their work on the board, she could act appropriately with regard to their written work on the board. • She believed learners learn by explaining the work to others in small groups.	• She had the ability to predict what learners would and would not understand and how they would understand the new content. • She was aware of learners' possible misconceptions and rectified their misunderstandings in class. • She looked at learners' work, listened to their thinking and acted appropriately. • She believed learners learn once the teacher builds on their existing knowledge and they could talk about their thinking.
PCK and beliefs regarding the ML	• He was aware of prior knowledge needed for	• She did not connect learners' prior knowledge with new	• Prior knowledge was integrated in her revision	• Knew what prior knowledge should have been revised at

teaching (T)	learners to gain understanding. • He chose very basic and similar examples and did not use multiple representations. • Not much evidence of sequencing the content. • His choice of direct instruction as instructional strategy and use of textbook mainly were not appropriate to teaching ML. • He believed direct instruction should initially be used to introduce new content, followed by group work and discussions for solving problems. • He believed the difference in approach between ML and Mathematics is the use of fewer examples and working at a slower pace.	situations. • The demonstrations and explanations used did not make the content comprehensible to the learners. She did not use various representations. • She mostly sequenced the content but the amount and pace made it difficult for the learners to comprehend the content. • Her choice of direct instruction as instructional strategy and use of textbook only were not appropriate to teaching ML. • She believed in using formal teaching when introducing the topic and discussions later. • She believed that the teaching of ML does not differ from teaching Mathematics.	lessons. • She used varied and appropriate representations to make the content comprehensible to the learners, applied appropriate instructional strategies and sequenced her tasks. • Some evidence of sequencing the tasks was observed. • Discussions and learners working on the board were appropriate strategies for her revision lessons. • She believed she initially had to explain the content, concepts and application thereof, followed by drilling. The learners should also be involved by sharing their ideas but time is a restriction. • She believed the teaching of ML differs from that of Mathematics as ML is based on contexts.	certain stages of lesson. • She taught the content in context and used powerful examples, illustrations and explanations and various representations to make the work comprehensible to the learners. • She sequenced the content to facilitate learning. • Discussions and using textbooks and previous examination papers were appropriate for her revision lessons. • She believed ML should be taught by basing her class discussions on real-life situations and the learners' prior knowledge. • She believed teaching ML differs from teaching Mathematics as there is enough time for learners to discover new situations through discussions and problem solving.
PCK and beliefs regarding the ML Curriculum (C)	• He had no knowledge of other subjects' curricula that integrate with ML. • Knew about definition, purpose and learning	• She had no knowledge of other subjects' curricula that integrate with ML. • Knew about definition, purpose and learning	• She had no knowledge of other subjects' curricula that integrate with ML. • She did not know the definition and could not state	• She knew how ML integrates with the curricula of five other subjects in school. • She knew the definition, purpose, learning outcomes

	outcomes but was not aware of all departmental documents. • Did not teach content in context. He referred to real-life scenarios but it was unclear how the content should be applied in those contexts. • He viewed mathematics as constructivism and logical although this view was not implemented in practice. He believed ML is a kind of mathematics, but not a lower grade of Mathematics.	outcomes but were not aware of all departmental documents. • Did not teach content in context except for the one life-related example used. • She viewed mathematics as being logical and having to do with constructivism, the latter not being observed. She believed there was no difference between Mathematics and ML and that ML is just a little easier.	the learning outcomes but knew the purpose of ML and the various departmental documents. • Did not teach content in context. • She viewed mathematics as flexible and creative and regarded ML as a unique form of mathematics.	and relevant departmental documents. • In all her lessons content was taught in context as ML should be taught. • She viewed mathematics as a flexible discipline that should be used to discover and experiment with. She believed ML is a subject on its own which is meant for all learners (Mathematics learners too).

4.6.6. Discussion of Theme 2: ML teachers' knowledge and beliefs

In this section I will conduct a literature control where the findings from this study are compared with the findings from other research studies on ML teachers' knowledge and beliefs regarding the ML learners, ML teaching and the ML curriculum is based on the indicators in Table 4.3[69].

4.6.6.1 ML teachers' mathematical content knowledge (MCK)

- **Teachers' belief that MCK is a prerequisite to teach ML**

My finding that all four teachers in my study believed MCK is a prerequisite to teach ML is strongly consistent with Fransman's (2010) finding that all four teachers in her focus group believed ML teachers should know the mathematics content well enough to be able to do the mathematics themselves. The four teachers in my study also believed that all ML teachers should have some form of tertiary training in mathematics, a finding that is moderately consistent with Sidiropolous' (2008) finding that one of the two teachers in her study believed that educators teaching ML should be qualified and have some form of tertiary training in mathematics.

- **ML teachers' level of MCK**

Except for Alice, the teachers in my study appeared to have sufficient MCK regarding the topics they were teaching at the time of the observations. Alice's MCK was not always coherent, and she made several mistakes in the written examples on the board as well as during her verbal explanations. This finding, where one of the four teachers in my study had insufficient MCK of the ML topics taught, is inconsistent with Hechter's (2011a) finding that both the teachers in her study had insufficient MCK of the ML topics taught (*their knowledge was not coherent and some errors were made with respect to the mathematical content dealt with in the classrooms,* p. 149). My finding is also inconsistent with Bansilal's (2008) finding that most of the ML teachers in her study had insufficient knowledge of the ML topics taught.

4.6.6.2 ML teachers' knowledge and beliefs regarding their learners

Denise and Elaine (as experienced former Mathematics teachers) demonstrated specific knowledge of the ML learners' prior knowledge and what content should be emphasised and how the content would be understood by the learners. They were able to predict learners' common errors and misconceptions and could act appropriately to facilitate learning. Compared to Denise and Elaine, Monty and Alice (as novice teachers) demonstrated superficial knowledge. As far as I could establish, no study has reported this finding before.

[69] See Table 4.3 under Section 4.3.2.2: Theme 2: ML teachers' PCK and beliefs.

Denise and Elaine once again had similar beliefs on how learners come to understand mathematical content as did Monty and Alice. Denise believed that learners learn by explaining the work to each other while Elaine believed they learn when the teacher builds on their existing knowledge, and they then talk about their thinking. Conversely, Monty believed that learners gain understanding by studying several examples and through a lot practice while Alice believed that learners learn by practising the work in the presence of someone who gives them confidence. My finding that two of the four teachers in my study believed that learners reach understanding through active involvement in the lessons is strongly consistent with the finding of Sidiropolous (2008) that one of the two teachers in her study believed that learners reach understanding through critical and creative engagement in the lessons. However, apart from this observation, my literature control did not yield any other reportable findings, that is, findings that I could realistically compare with my own.

4.6.6.3 ML teachers' knowledge and beliefs regarding the teaching of ML

Knowledge regarding the teaching of ML refers to teachers' ability to know what prior knowledge should be present for learners to understand new work; to use various representations and resources to facilitate learner understanding; to transform their own knowledge into forms that are pedagogically powerful; to sequence content; and to choose appropriate instructional strategies and materials (Artzt, et al., 2008; Ball, 1990; Borko & Putnam, 1996; Hill et al., 2008; Shulman 1986; Shulman, 1987).

In the interviews prior to the observed lessons, only Denise and Elaine could identify the prior knowledge that should have been present for the learners to understand the work. Monty and Alice said *everything* was prior knowledge as they believed all the content had already been done in Grade 10. Regarding the various representations such as tables, figures and graphs that teachers can use to facilitate learners' understanding, both Alice and Elaine conformed to this requirement. Only Alice did not have the ability to transform her own knowledge into forms that were pedagogically powerful. During the interview prior to the lesson on data-handling, it seemed she had sufficient knowledge of the topic, but her lesson presentation was incoherent, and the learners were confused and frustrated. As far as I could establish, these findings have not been reported before.

It is difficult to comment on the teachers' ability to use various resources as three of the four teachers in my study taught content only when only a textbook and calculators were used. Elaine was busy with her revision programme at the time of the study and used various textbooks, previous examination papers and calculators as resources. This finding, namely that three of the four teachers in my study used a textbook and calculators only, is consistent with the finding of Sidiropolous (2008) where both teachers in her study used only a textbook and calculators. The teachers' ability to sequence the content[70] and choose appropriate instructional material[71] as well as their ability to choose appropriate instructional strategies[72] has already been discussed.

4.6.6.4 ML teachers' knowledge and beliefs regarding the ML curriculum

- **Content-context issue**

Denise and Elaine believed that ML teaching differs from teaching Mathematics. This finding is strongly consistent with Sidiropolous' (2008) finding that one of the two teachers in her study believed that ML teaching is different from teaching Mathematics. Elaine demonstrated that her belief and instructional practice conformed to the requirement of the DoE (2003a) of *engaging with contexts rather than applying mathematics already learned to contexts* (p. 43). The other three teachers believed that teachers should initially explain the content and then apply the content to contexts using discussions if time permitted. My finding that only one of the four teachers in my study used relevant contexts in order for the learners to explore the content is inconsistent with the finding of Venkat (2010) where the teacher in her study used contexts but consistent with the finding of Sidiropolous (2008) as one of the teachers in her study did not use contexts. My finding that all four teachers in my study however believed contexts should be used is strongly consistent with both Sidiropolous' (2008) and Hechter's (2011a) finding where both teachers in each study believed that contexts should be used to facilitate learning.

- **Integration of ML with other subjects**

Apart from the fact that ML requires a different teaching approach to that of Mathematics, the ML curriculum also requires ML to be taught in a de-compartmentalised manner and to be integrated with other school subjects (DoE, 2003a; De Villiers, 2007; North, 2005; Venkat & Graven, 2007). All four teachers in my study believed ML should be integrated with other school subjects. My finding is strongly consistent with both Sidiropolous' (2008) and Fransman's (2010) findings where all the teachers in their studies believed ML should be integrated with other disciplines. Only Elaine could identify other school subjects that ML should be integrated with, and only she had knowledge of their curricula. She believed the ML curriculum should address the mathematical needs of those subjects. As far as I could establish, this finding has not been reported before.

- **Other ML curriculum issues**

Only Elaine had sufficient knowledge of other ML curriculum issues such as knowledge of the appropriate use of various instructional materials, the strengths and weaknesses of textbooks, the use of departmental documents as guidelines as well as knowledge of topics that were taught in preceding and would be taught in subsequent years, i.e. Grades 10 and 12. Sidiropolous (2008) reported that both the

[70] See Discussion of Theme 1: Tasks.
[71] See Discussion of Theme 1: Tasks.
[72] See Discussion of Theme 1: Learning environment.

teachers in her study had insufficient knowledge of other curriculum issues (as explained above). My finding that three of the four teachers in my study had insufficient knowledge of other ML curriculum issues is consistent with Sidiropolous' finding.

- **Teachers' beliefs about the nature of mathematics as a discipline**

All four teachers in my study had a constructivist perspective on teaching and learning mathematics. Monty and Alice believed mathematics is a logical discipline while Denise and Elaine considered mathematics as being flexible and creative. As far as I could establish, no study up to now has reported on ML teachers' beliefs about the nature of mathematics as a discipline.

- **Teachers' beliefs about the nature of ML as a subject**

Only Alice considered ML similar to Mathematics but at a lower level − the other three teachers viewed ML as a unique subject. This finding that only one of the four teachers in my study considered ML to be similar to Mathematics but at a lower level provides some (albeit limited) support for Fransman's (2010) finding (only 2 out of the 58 teachers in her study viewed ML as similar to Mathematics but at a lower level) and is consistent with Hechter's (2011a) finding where half of the teachers in her study viewed ML as similar to Mathematics but at a lower level, but is inconsistent with Sidiropolous' (2008) finding that all the teachers in her study viewed ML as similar to Mathematics but at a lower level.

- **Teachers' beliefs about the value of ML as a subject**

All four teachers in my study believed ML has great value as learners obtain knowledge they can use in their everyday lives and work situations in the future. My finding that all four teachers in my study valued ML is strongly inconsistent with Sidiropolous' (2008) finding that neither of the two teachers in her study believed ML has great value in helping learners obtain knowledge they can use in their everyday lives and work situations in the future.

4.6.6.5 Summary of discussion on Theme 2

To summarise: All four teachers in my study believed that MCK is a prerequisite for teaching ML, and three of them appeared to have sufficient MCK of the topics they were teaching at the time of the observations. Compared to Monty and Alice who demonstrated superficial knowledge of the learners' prior knowledge, Denise and Elaine demonstrated specific knowledge of the learners' prior knowledge, which content to emphasise and how it would be understood by learners to the extent that they could accurately predict the learners' problems with the content. Monty and Alice believed learners gain understanding by studying several examples while Denise and Elaine believed a teacher should build on learners' prior knowledge and involve learners in discussions. Only Elaine based her teaching on life-related problems while both she and Denise used appropriate instructional strategies to facilitate learning. Monty and Alice believed the teaching of ML is no different to teaching Mathematics.

Only Elaine knew which subjects are integrated with ML although all four teachers believed that ML should be integrated with other disciplines. Apart from Elaine, the teachers in my study had a superficial understanding of the ML curriculum. All four teachers had a constructivist perspective on teaching and learning mathematics. Only Alice viewed ML as a lower level of Mathematics while the other teachers viewed ML as a unique subject. All four teachers regarded ML as valuable because, through it, learners could obtain knowledge they could use in their daily lives.

4.7 Findings, trends and explanations

An analysis of the discussions on Theme 1 and Theme 2 was done and resulted in the following summary in which the findings, trends and explanations are delineated.

- **Experiences and beliefs shared by all four teachers in my study**

I found that all four teachers were positive about teaching ML and all shared the following beliefs: Mathematics is a subject that is best mastered by implementing a constructivist approach to teaching and learning; ML is a valuable subject; ML should be integrated with other school subjects; and MCK is a prerequisite for teaching ML. Many teachers in recent studies share this positive attitude towards teaching ML and attach value to the subject. Sidiropolous (2008), however, found that both the teachers in her study had negative attitudes towards ML. This discrepancy in findings can perhaps be attributed to teachers becoming aware of the uniqueness of the subject and starting to realise its value during the past four years − Sidiropolous conducted her study only a year after ML had been introduced at a time when teachers tended to be negative about this unfamiliar subject. Research indicates that almost all ML teachers believe that MCK is a prerequisite for teaching ML and that all ML teachers should have some form of tertiary mathematics training. With Alice having some form of tertiary mathematics training but still making several errors and not being able to *transform [her] own knowledge into forms that are pedagogical powerful* (Shulman, 1987, p. 15), it seems that not only general training in the content of mathematics but also mathematics teacher training (i.e. training in the teaching and learning of mathematics) is required to teach ML proficiently. A concern is what happens in ML classes where teachers from other disciplines with no formal mathematics teacher training teach ML.

- **Two differing cases**

In my study, two highly differing cases were Alice and Elaine. Alice, as a novice teacher with no mathematics teacher training, was the only teacher who communicated judgmentally with the learners; did not work at a slower pace as stipulated in the ML curriculum (DoE, 2003a); did not have the ability to transform her own knowledge into forms that were pedagogically powerful; and viewed ML as similar but inferior to mathematics. Elaine, a former mathematics teacher with years of experience, was the only teacher who used contextual tasks effectively; pointed out the value of mathematics to the learners; selected tasks from all four levels of the ML Assessment Taxonomy; required the learners to explain their answers; posed a variety of oral questions on different levels; and had sufficient curriculum knowledge. From this comparison, it seems experience and mathematics teacher training play a crucial role in the instructional practice of the ML teachers.

- **Role of teaching experience in ML teachers' instructional practice**

In comparing the instructional practices of experienced and inexperienced teachers from my own and other research studies, I found that not all experienced teachers taught ML in a satisfactory manner. For example, the experienced Denise in my study did not comply with all the requirements regarding the instructional approach to teaching ML as set out by the DoE (2003a). Another example is the study of Sidiropolous (2008) where the instructional practices of the two experienced, but negative, teachers were not aligned with the curriculum or with their claimed beliefs. Conversely, there were inexperienced teachers in the studies (one from each study) of Fransman (2010) and Hechter (2011a) whose practices were aligned with the curriculum and with their claimed beliefs − teachers who had developed a *new status identity* (Fransman, 2010, p. 184) of being a ML teacher. It seems that apart from having sufficient teaching experience, the success of a ML teacher's practice can be improved by being positive about and taking ownership of teaching ML as well as developing a new status identity of being a ML teacher.

- **Value of teacher and ML training**

Apart from Monty and Alice both being novice teachers, a major difference in their instructional practices is that Alice had no teacher training but had completed a mathematics course as part of her Management degree whereas Monty had completed a BEd degree with Mathematics as a major. In contrast to Alice, Monty communicated with the learners in a non-judgmental manner; assisted them individually; worked at a slower pace; demonstrated sufficient MCK; and viewed ML as a unique subject. These findings suggest that teacher training plays a role in teacher-learner interactions and in establishing a manageable pace of work. Only Monty attended short courses for in-service ML teachers. A comparison of the teachers in my study with the teachers who were part of the ACE (ML) programme of Fransman (2010) and Hechter (2011a) revealed that ML teacher training had a positive influence on the instructional practices and beliefs of the teachers who were enrolled for the ACE (ML) course. Examples of this include their ability to provide scaffolding and to teach the mathematical content using contextualised tasks.

- **On a continuum from teacher-centred to learner-centred**

If I could place the teachers in my study on a continuum from teacher-centred on the left to learner-centred on the right, this would be the order from left to right: Alice, Monty, Denise and Elaine. Monty and Alice as novice teachers adopted a teacher-centred approach. Denise as a former Mathematics teacher, with many years' experience of teaching Mathematics, leant more towards a learner-centred approach. Elaine, also a former Mathematics teacher with many years of experience in teaching Mathematics, also adopted a learner-centred approach. It appears that teacher training and experience influenced the teachers' ability to facilitate learning effectively in the ML classroom in my study. Although both teachers in Sidiropolous' (2008) study had mathematics training and teaching experience, they nevertheless still used a teacher-centred approach. This can perhaps be attributed to factors such as their having had no ML teacher training or their beliefs regarding the teaching of ML. Venkat and Graven (2008) and Venkat (2010) reported that the teachers in their studies were trained and experienced, but it was after making an actual decision, as Elaine did, to change their pedagogic practices to a learner-centred and activity-based approach, that the teachers as well as the learners could experience the value of ML.

- **Contradictions between teachers' beliefs and their instructional practices**

The following beliefs held by the teachers in my study were contradicted in their practices.

Table 4.16: Contradictions between teachers' beliefs and their instructional practices

Teacher	Belief	Practice
Monty, Alice,	Mathematics is a constructivist discipline	Direct instruction (lecturing) was used.
Monty, Alice	ML should be integrated with other subjects	No integration was evident in their lessons. Could not explain how ML could be integrated with other subjects.
Monty, Alice, Denise	Real-life application problems should be used	With the exception of one real-life example in one of Alice's classes, only content was taught.
Monty, Denise	ML is a unique subject	Lessons were typical Mathematics lessons
Denise	Learners learn by explaining the work to each other in small groups	She used their solutions to discuss the work with them.
Denise	ML teaching differs from Mathematics teaching	Content was lectured and if there was enough time, discussions took place.

Such contradictions were also evident in Sidiropolous' (2008) study where both teachers believed *ML is a maths only better than nothing* and even *the maths of oranges and bananas* (p. 225) while, in later interviews, they complained about the difficulty level of the subject. They also believed that a teacher could work at a much slower pace, but, during the interviews, they complained about not having enough time to teach the way they knew they were supposed to teach ML. A possible reason for this contradiction is that the theory they espoused was difficult to carry out in practice.

To summarise: The following are some issues that need attention by ML teachers in their instructional practices.

1. Knowledge of the pedagogic approach in which teaching content is integrated with contexts.
2. Knowledge of varied, applicable instructional strategies and material.
3. Selection of tasks according to the ML Assessment Taxonomy.
4. Different types and level of oral questioning in the classroom.
5. Increasing learner participation by giving them the opportunity to verbalise their thinking.
6. How to facilitate learner-learner interactions.
7. Establishing and maintaining a positive learning atmosphere.
8. Having a positive attitude towards the subject and the learners.
9. Choice and efficient use of appropriate instructional strategies.

To ensure proficient ML teaching, it seems that teachers require the following: ML teacher training; a certain level of MCK (it was not the purpose of the study to determine the required level of MCK); experience; and a positive attitude towards and the desire to change their instructional practices.

4.8 Conclusion

In this chapter, I discussed the data collection process that took place in Pretoria during the second quarter of 2011. I initially had five participants, but, during the data analysis process, I realised that the one case did not add value to my study, and so I decided to continue only with the other four cases. Data were collected by means of three lesson observations per teacher with interviews conducted prior to the second and third observations and a third and last in-depth interview conducted after the observations based on the observed lessons as well as their knowledge and beliefs. I adopted a deductive approach to coding the data as I had identified two themes: ML teachers' instructional practices and ML teachers' knowledge and beliefs prior to the data collection stage. Different categories for each theme were chosen according to the work of Artzt et al. (2009), Ball (1990), Borko and Putnam (1996), Hill et al. (2008), Shulman (1986) and Shulman (1987) apropos of which the raw data were analysed. In this chapter I also presented the data of the four participants and discussed my findings on the basis of a literature control. I finally identified trends and possible explanations for the trends.

In the next chapter, the research questions are answered, and I reflect on my research study and draw conclusions from the case study. I also discuss the limitations and significance of the study and make recommendations for further research.