Chapter 2

Literature review and conceptual framework

2.1 Introduction

This literature study is a critical and integrative synthesis of various researchers' findings, justifying this research endeavour. It is imperative to remember that South Africa is the only country offering ML as a compulsory alternative to Mathematics in Grades 10 to 12. As the study concerns the ML teachers and the relationship between their knowledge and beliefs and their instructional practices, the literature review begins with a comparison of the international and national perspectives of mathematical literacy. Comparisons are made between the different conceptions of mathematical literacy; the contexts in which mathematical literacy can be applied; international studies measuring learners' mathematical knowledge and literacy skills; meanings and definitions of mathematical literacy; and the role mathematical literacy plays in some school curricula. Following the review on mathematical literacy is a discussion of the meaning of teachers' instructional practices and the value of various approaches to teaching. Moving to the core of the problem, literature regarding teachers' knowledge and beliefs about the subject they teach are discussed. Attention is given to the different domains of teachers' knowledge, teachers' belief systems and the relationship between their knowledge and beliefs and their instructional practices. The literature review concludes with the conceptual framework which is based on concepts and theories from relevant work in the literature[6].

2.2 Mathematical literacy

Mathematical literacy is not a clearly defined term and internationally there exists a range of different conceptions of mathematical literacy that are discussed in this section. As mathematical literacy (ML) is a school subject in South Africa, it is important to understand the motivation and purpose of ML in the South African curriculum and to compare it with the role mathematical literacy plays internationally.

[6] Several direct quotations are used in the literature review to avoid nuance chances of meaning to the matter under discussion.

2.2.1 International perspectives on mathematical literacy

In this section I mention the different terminology being used for mathematical literacy, compare different conceptions of mathematical literacy, discuss different contexts in which mathematics could be applied and refer to some international comparative studies that measure learners' mathematical literacy skills in order to derive a general meaning or definition of mathematical literacy.

There is an expanding body of literature that uses the terms "mathematical literacy" and "numeracy" as synonyms (Jablonka, 2003). The National Council on Education and the Disciplines however uses the term "quantitative literacy" to stress *the importance of enquiring into the meaning of numeracy in a society that keeps increasing the use of numbers and quantitative information* (Jablonka, 2003, p. 77). Jablonka prefers to use the term "mathematical literacy" *to focus attention on its connection to mathematics and to being literate*, in other words *to a mathematically educated and well-informed individual* (p. 77).

In a comprehensive study by Jablonka (2003) in which different international perspectives on mathematical literacy were investigated, she found that the perspectives basically differ according to the stakeholders' underlying principles and values. In her opinion there is a direct connection between a conception of mathematical literacy and a particular social practice. She acknowledges the difficulty of pointing out the distinct meaning of mathematical literacy as it varies according to the *culture and context of the stakeholders who promote it* (p. 76). The different conceptions of mathematical literacy relate to a number of relationships and factors. One of the relationships is between *mathematics, the surrounding culture, and the curriculum* (p. 80) while another is between *school mathematics and out-of-school mathematics* as mathematical literacy is *about the individual's ability to use the mathematics they are supposed to learn at school* (p. 97). Varying with respect to the culture and the context four possible perspectives of mathematical literacy are:

- *The ability to use basic computational and geometrical skills in everyday contexts.*
- *The knowledge and understanding of fundamental mathematical notions.*
- *The ability to develop sophisticated mathematical models.*
- *The capacity for understanding and evaluating another's use of numbers and mathematical models* (p. 76).

With the above-mentioned perspectives as background the different conceptions of mathematical literacy as found in the literature will subsequently be categorised.

2.2.1.1 Different conceptions of mathematical literacy

The literature revealed different conceptions of mathematical literacy but the resemblances between mathematical literacy and RME, mathematisation, mathematical modelling, as well as mathematics in action are most evident. As there is opacity as to what each of these conceptions entail and how they differ a clarification of the concepts and notions will be provided.

Realistic Mathematics Education

Hope (2007) expressed the resemblance of mathematical literacy with the theory of RME. RME *uses a theoretical framework that relies on real-world applications and modelling, a didactical belief propagated by Hans Freudenthal* (Gates & Vistro-Yu, 2003, p. 67). According to Van den Heuvel-Panhuizen (1998), Freudenthal and his colleagues laid the foundations of RME in the early seventies to address the world-wide need to reform the teaching and learning of mathematics and to move away from mechanistic mathematics education. Freudenthal's theory of RME rests upon the following five components:

- *Using a real-world context as a starting point for learning.*
- *Bridging the gap between abstract and applied mathematics by using visual models.*
- *Having students develop their own problem-solving strategies rather than memorise rules and procedures.*
- *Making mathematical communication, perhaps in the form of journaling or oral presentations, an integral part of the lesson.*
- *Making connections to other disciplines using meaningful real-world problems* (Hope, 2007, p. 30).

Hope (2007) further believes mathematical literacy is a matter of the appropriate pedagogy that should be used in teaching mathematics. According to these fundamental pedagogical aspects of teaching mathematics, it is comprehensible that the traditional school mathematics instruction is too formal, less intuitive, more abstract, less contextual, more symbolic, and less concrete than the type of instruction that would expand student thinking and develop mathematical literacy (p. 30).

Mathematisation

Freudenthal believed that the focus should not be on *mathematics as a closed system, but on the activity, on the process of mathematisation* (Van den Heuvel-Panhuizen, 1998), and that mathematics should be seen as a human activity that is connected to reality and relevant to society. Treffers (1978) formulated the idea of two types of mathematisation, namely horizontal and vertical mathematisation. He stated that *in horizontal mathematisation the students come up with mathematical tools which can help to organise and solve a problem located in a real-life situation* whereas *vertical mathematisation is the process of reorganization within the mathematical system itself, like, for instance, finding shortcuts and discovering connections between concepts and strategies and then applying these discoveries* (Van den Heuvel-Panhuizen, 1998). Freudenthal (1991) explained horizontal mathematisation as *going from the world of life into the world of symbols, while vertical mathematisation means moving within the world of symbols* (p. 24). It would seem that vertical mathematisation refers to the more formal mathematics while horizontal mathematisation refers to the informal mathematical literacy part.

According to Hope (2007) mathematising is a term used by The Organisation of Economic Co-operation and Development (OECD) which involves five elements:

- *Starting with a problem whose roots are situated in reality.*
- *Organising the information and data according to mathematical concepts.*
- *Transforming a real-world, concrete application to an abstract problem whose roots are situated in mathematics.*
- *Solving the mathematical problem.*
- *Reflecting back from the mathematical solution to the real-world situation to determine whether the answer makes sense* (p. 29).

Mathematical modelling

ML bears a strong resemblance to mathematical modelling in that both require an application of Polya's four basic steps in problem-solving namely: a) understanding the problem; b) designing a plan; c) carrying out the plan; and d) looking back on the problem. Mathematical modelling can further be described as *a matter of constructing an idealised, abstract model which may then be compared for its degree of similarity with a real system*. (Giere, 1999, p. 50).

Gellert et al. (2001) use mathematical literacy as a metaphor referring to well-educated and well-informed individuals. According to them different conceptions of mathematical literacy are based on the relationship between mathematics, reality and the society. Their concept of mathematical literacy involves *gaining a level of mathematical understanding that goes beyond the minimal abilities of calculating, estimating, and gaining some number sense, and basic geometrical understanding ... by seeing the power of mathematics in its potential of abstracting from concrete realities by generating concepts and structures for universal application* (p. 59). They further believe these abilities can be developed *by experiencing mathematical modes of thinking, such as searching for patterns, classifying, formalising and symbolising, seeking implications of premises, testing conjectures, arguing, and thinking propositionally* (p. 59) which form the basis of mathematical modelling. Mathematical literacy *requires the mathematical competence to understand the mathematical methods involved and the analytical competence to demystify the justifications for specific mathematical applications as well as to assess their consequences* (p. 66).

Mathematics in action

Although some of the above-mentioned conceptions are formal, involving higher-order mathematical skills, there are other researchers who regard mathematical literacy as a fundamental requirement for all people, recognising its essential value to learners in contexts forming part of their everyday living (McCrone & Dossey, 2007; Powell & Anderson, 2007; Skovsmose, 2007). McCrone and Dossey (2007) believe mathematical literacy is not about *studying higher levels of formal mathematics, but about making mathematics relevant and empowering for everyone* (p. 32). They further call for mathematics to play an even greater part in non-mathematics classes where teachers promote the mathematics embedded in their subjects.

Skovsmose (2007) refers to mathematical literacy as mathematics in action and considers the role of mathematical literacy in both mathematicians' and non-mathematicians' lives. He based his study on two types of literacy being either **functional** or **critical**, terms introduced by Apple in 1992. Functional literacy is defined by competencies a person possesses to *fulfil a particular job function* (p. 4) whereas critical literacy addresses themes such as working conditions and political issues. Skovsmose prefers to talk about **reflective** knowledge with respect to mathematics instead of **critical** literacy. Reflective knowledge refers to *a competence in evaluating how mathematics is used or could be used* (p. 4). Critical literacy is associated with the skill to create or design models using mathematics whereas functional literacy is the skill to use and apply those models. To make unambiguous distinctions between these two types is not that simple and it *could have very different interpretations depending on the context of the learner* (p. 4).

Clarification of basic concepts and notions

From the discussion above it is clear that the lines between the different concepts such as mathematisation and mathematical modelling are blurred. Blum and Niss (2010) provided a clarification of the different concepts and notions when they described the process of applied problem solving. The process of applied problem solving commences with a **real problem situation** and through a process of simplification, idealisation and structuring of the situation, the process ends with a **real model of the original situation**. Through the process of mathematisation, the real model is translated into mathematics. Mathematisation is therefore the process of converting *the data, concepts, relations, conditions and assumptions* (p. 208) of the real model into a **mathematical model of the original situation**. Mathematical modelling is the entire process leading from the real problem situation to the mathematical model. Then the mathematical model must be processed to obtain certain **mathematical results**. This includes mathematical activities such as *drawing conclusions, calculating and checking concrete examples, applying known mathematical methods and results as well as developing new ones etc.* (p. 208). The next process is to retranslate the results into the real world, *i.e. to be interpreted in relation to the* ***original situation*** (p. 208). The model is then validated and if discrepancies of any kind occur, they may lead to the modification of the model or replacement of the model by going through the process cycle more than once.

Summary

In the light of the above discussion of the different conceptions from the literature, mathematical literacy cannot adequately be described in terms of skills only, as it involves mathematical problems in contexts that require attributes such as conceptual understanding of formal mathematical knowledge and problem-solving skills (Gellert et al., 2001). Gellert et al. also believe that the differences between various conceptions of mathematical literacy *consist of the problems to which mathematics is applied* (p. 61). Hope (2007) on the other hand believes mathematical literacy is a matter of the appropriate pedagogy that should be used in teaching mathematics. All mathematics learners should therefore be provided with the opportunity to apply their knowledge and logic to real-world situations that form part of their daily lives. Mathematical literacy implies bridging the gap between abstract and applied mathematics where the contexts and degree of complexity differ. Jablonka's (2003) question of: mathematical literacy for what?, further calls attention to the need for discussing the different contexts in which mathematics could be applied to further explicate the purpose of mathematical literacy.

2.2.1.2 Some contexts in which mathematical literacy can be applied

Prescribing the different contexts in which mathematical literacy can be applied is as complicated as conceptualising mathematical literacy. In this section the different categories of contexts are discussed as well as the role technology plays in determining these contexts.

Context categories according to stakeholders' demands

Contexts in which mathematical literacy can be applied depend on the stakeholders' philosophy, view or principles and could be guided by, among other things, some socio-economic demands (Jablonka, 2003). Jablonka categorised the different, and in some cases, conflicting contexts in which mathematics can be applied as mathematical literacy for:

- Developing Human Capital – looking at the world through mathematical eyes where higher-order mathematical skills are applicable, where mathematics is not regarded as culture-bound and value-driven.
- Cultural Identity – incorporating ethno-mathematical practices to avoid privileging of Western academic mathematical knowledge.
- Social Change – to uncover and communicate aspects of social or political nature (such as unemployment, life expectancy, national income) in an attempt to overcome the dominance of academic mathematics in the curriculum.
- Environmental Awareness – the mathematical content comprises arguments underpinned by mathematical visualisations, qualitative mathematics that is characterised as not aiming at an analytical solution but serving as thought experiments and computational mathematics, which include the use of simulation packages, graphing calculators and spreadsheets.

- Evaluating Mathematics - includes reasoning with condensed measures and indexes, formalising transactions, reasoning with platonic models, constructing surface-models and numerology.

An example of conflict between some of these conceptions Jablonka referred to is where the application of mathematics in the Cultural Identity is restricted to contexts situated in a specific culture, where in the Environmental Awareness the focus is on applying mathematics to contexts of global nature. Then there is the complex problem of Cultural Identity, where Ethno-mathematics is suggested as a necessity for addressing cultural conflicts in the classroom. Knoblauch's (1990) categories are similar to Jablonka's, speaking of literacy for *professional competence in a technological world, for civic responsibility and the preservation of heritage, for personal growth and self-fulfilment, and for social and political change* (p. 76). Different contexts in which mathematical literacy can be applied can also be categorised according to some processes involved in applying mathematics in real-world contexts.

Context categories according to processes

By categorising the content in four processes, Skovsmose (2007) illustrated *how mathematics in action can operate in powerful ways, and power can be exercised through mathematics in action* (p. 8). The following categories show the different conceptions of mathematical literacy categorised as critical and functional literacy as discussed in par. 2.2.1.1.

- Construction – *includes systems of knowledge and techniques, by means of which technology, in the broadest interpretation of the term, is maintained and further developed* (p. 8) for example in the construction of the computer.
- Operating – *bringing technology into operation in work practices and job functions. The operator may not be aware of the mathematical content of the procedures he or she performs* (p. 11) for example, ticket reservations in the travel industry and procedures for buying and selling houses.
- Consuming – as citizens we are the consumers who need to listen to statements from experts that are expressed everyday on television and in the newspapers, for example *numbers and figures concerning elections, the economy, exchange rates and investments are mixed with advertising of any number of special offers* (p. 13).
- Marginalising – *a steady growth of favela-like neighbourhoods gloomily testifies that free-growing globalised capitalism is not an inclusive economy. Instead it marginalises in great measures people as being disposables.* Examples include drugdealing, *selling of sunglasses, lighters and other items possible to carry around along the streets where cars come to a stop* (p. 14).

Skovsmose (2007) concluded that mathematical literacy *could be either functional or critical* but that the *distinction is difficult to maintain, is vague, maybe illusive* (p. 17). In many of these processes technology plays a significant role in the process of context selection.

The role of technology

Gellert et al. (2001) pointed out that technology has taken over many processes in society where highly skilled mathematicians develop mathematical models and processes which people generally do not need to understand or even be aware of. There is little need for the majority of people *to learn more mathematics in a more successful way as it is based more on common sense than on rational reason or on justifiable evidence* (p. 58). The result is then *an increasing mathematisation of our society [which] is complemented by an increasing demathematisation of its individual members* (p. 58). In comparing their view with Skovsmose's (2007) categories stated in the preceding paragraph, it is a minority of people who use their advanced mathematical literacy skills to construct or bring technology into operation while the majority of people use their basic mathematical skills to operate and consume.

2.2.1.3 Studies measuring learners' mathematical literacy skills

There are various studies measuring learners' mathematical knowledge and skills, but in this study only two international comparative studies will be considered, namely the Organisation for Economic Co-operation and Development's (OECD) Programme for International Student Assessment's (PISA) as well as TIMSS.

The foci of PISA and TIMSS

Every three years PISA assesses 15-year-olds' reading, mathematical and scientific literacy in different countries. Its purpose is to measure *how well students can apply their knowledge and skills to problems within real-life contexts. PISA is designed to represent a 'yield' of learning at age 15, rather than a direct measure of attained curriculum knowledge* (National Centre for Education Statistics, 2008b, p. 3). In 2003, when 45 countries participated, the focus was on mathematics. Every four years TIMSS assesses fourth- and eighth-graders' mathematics and science performance in different countries (58 in 2007) to compare U.S. learners' performance with that of their peers in other countries. This study's purpose is to *measure the mathematics and science knowledge and skills broadly aligned with curricula of the participating countries* (National Centre for Education Statistics, 2008a, p. 5). The two studies differ in focus as *TIMSS seeks to find out how well students have mastered curriculum-based scientific and mathematical knowledge and skills* whereby the purpose of PISA is *to assess students' scientific and mathematical literacy, that is, their ability to apply scientific and mathematical concepts and thinking skills to everyday, non-school situations* (Nohara, 2001, p. 11). In the next section attention is given to PISA's definition of mathematical literacy and the criteria used in assessing learners' mathematical literacy skills.

PISA

According to the OECD (2004, p. 37), the content of school mathematics and science in the last decade was chosen *to provide the foundations for the professional training of a small number of mathematicians, scientists and engineers*. With the increased emphasis on the application value of science, mathematics and technology in modern life to all adults, the objectives of these three subjects changed to *personal fulfillment, employment and full participation in society*. Mathematical literacy is *concerned with the capacity of students to analyse, reason and communicate effectively as they pose, solve and interpret mathematical problems in a variety of situations involving quantitative, spatial, probabilistic or other mathematical concepts*. PISA's (OECD, 2003) definition of mathematical literacy is:

> *the capacity to identify, to understand and to engage in mathematics and make well-founded judgement about the role that mathematics plays, as needed for an individual's current and future life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen* (p. 20).

PISA was originally designed to measure the extent to which learners can apply their mathematical knowledge in realistic, everyday life situations. It involves the ability to analyse situations in *content areas involving quantity, shape and space, change and relationships and uncertainty* (McCrone et al., 2008, p. 35). In order to be able to assess mathematical literacy, PISA (OECD, 2003) identified three broad criteria to be used, namely:

- The content of mathematics – in terms of clusters of relevant, connected mathematical concepts that appear in real situations and contexts. These include quantity, space and shape, change and relationships, and uncertainty.
- The process of mathematics – different skills needed for mathematics such as **reproduction** – simple computations; **connections** – using of ideas and procedures to solve straightforward and familiar problems; and **reflection** – using of mathematical thinking, generalisations and insight to engage in analysis, identify mathematical elements in a situation, formulate questions and search for solutions.
- The contexts in which mathematics is used – the kinds of problems encountered in real life vary in terms of distance from individual, from effecting one directly regarding private life, school life, work and sports, local community and society and scientific, to scientific problems of more general interest.

From PISA's definition and assessment criteria it is clear that the focus is not just on applying routine procedures but to become cognitively involved in mathematical thought, using and applying formal mathematics to solve real-life problems. The contexts should involve realistic day-to-day situations involving people's personal, occupational and social lives enabling them to become reflective citizens.

Bearing in mind the conceptions, contexts and meanings of the concept mathematical literacy discussed above, I will subsequently discuss the role mathematical literacy plays in the FET school curricula in Australia and briefly mention the situations in the United Kingdom (UK) and United States (US).

2.2.1.4 Defining mathematical literacy

When referring to mathematical literacy or numeracy, many people in and outside the academic field tend to believe that only basic mathematical skills are involved or that numeracy refers only to primary school learners' mathematics. To conceptualise or define mathematical literacy is unfortunately not that simple and is far from being well-defined (Gellert et al., 2001; Jablonka, 2003; Skovsmose, 2007).

Different conceptions or definitions are held by researchers ranging from informal mathematics requiring basic mathematical skills (McCrone & Dossey, 2007; McCrone et al., 2008; Powell & Anderson, 2007; Skovsmose, 2007) to formal mathematics involving higher-order thinking skills (Gellert et al., 2001; Hope, 2007; Jablonka, 2003; Skovsmose, 2007). Gellert et al. (2001) pointed to especially primary school teachers who regard mathematical literacy as informal, defining mathematical literacy as *survival mathematics for all* (p. 68) with the exact purpose to propagate that mathematics is not just formal and complicated, but can be useful and beautiful to all people. Skovsmose (2007) believes that mathematical literacy can be related to notions such as autonomy, empowerment and globalisation, whereas Hope (2007) presumes it implies *that a person is able to reason, analyse, formulate, and solve problems in a real-world setting* (p. 29).

Gellert et al. (2001) and Jablonka (2003) perceive mathematical literacy in terms of higher-order mathematical skills. Jablonka is of the opinion that any attempt to define mathematical literacy *faces the problem that it cannot be conceptualised exclusively in terms of mathematical knowledge, because it is about an individual's capacity to* ***use*** *and* ***apply*** *this knowledge* (p. 78). She defined mathematical literacy as *a bundle of knowledge, skills and values that transcend the difficulties arising from cultural differences and economic inequalities because mathematics and mathematics education themselves are not seen as culture-bound and value-driven* (p. 81). She conceptualises mathematical literacy in terms of *higher-order mathematical skills* (p. 97) that are applicable to all kinds of contexts.

Although a definition of mathematical literacy is elusive, a golden thread running through all attempts to define mathematical literacy is that mathematical literacy is a valuable competence or skill a person possesses to put mathematics to work in solving real-life contextual problems. With the emphasis on globalisation and the information explosion in mind, mathematical literacy should imply the empowerment of learners to meet the demands of living in a 21st century (Gellert et al., 2001;

Queensland Government, 2007b; Skovsmose, 2007). It is informative to investigate the current situation regarding mathematical literacy in some international school curricula.

2.2.1.5 The role of mathematical literacy in some international school curricula

Instead of using the term "mathematical literacy" when referring to the competency of applying mathematical knowledge to life-related problems, Australia and the UK generally refer to the terms "numeracy", while the US refers to "quantitative literacy". A discussion regarding the role mathematical literacy plays in Australia's school curricula subsequently follows. I then briefly mention the situation in the UK and US.

AUSTRALIA

In 2008 all Australian regional governments agreed that instead of the eight different arrangements, only one national curriculum is to be implemented in 2013, which should play a key role in delivering quality education (Australian Curriculum, Assessment and Reporting Authority, n.d). Queensland is the second largest region and a study of the Education Department of Queensland provided insight into the role numeracy plays in the education system of Australia.

According to the PISA 2003 results when the focus of the study was on mathematics, Australia came 12th out of 41 countries (OECD, 2004). In the TIMSS 2007 they came 14th out of the 58 participating countries for both Grade 4 and Grade 8 learners (National Center for Education Statistics, 2008). For the past few years the raising of the numeracy levels of Australian learners received serious attention. There are numerous documents and guidelines available to teachers on how to develop learners' numeracy skills in the Mathematics classroom. There are also fact sheets available to parents with information on numeracy, providing some guiding principles on how to support their children in their numeracy development.

In a document called **Numeracy: Lifelong Confidence with Mathematics - Framework for Action 2007 – 2010**, which serves as an action plan to improve numeracy education, the Minister for Education and Training declared that the Queensland Government (QG) recognises numeracy as *a key pillar of learning and an essential component* (QG, 2007b, p. 1) of their curriculum. He also said that teachers have an important role to play in helping learners to become confident appliers of mathematics in their everyday lives. A Queensland Certificate in Education (QCE) is awarded at the end of Year 12 to a person who, in addition to achieving 20 credits in the required pattern of learning has met the requirements for literacy and numeracy. Learners can meet QCE numeracy requirements by satisfying a number of possible options including *a sound achievement* in one of their three Mathematics subjects in school or passing *a short course in numeracy developed by the Queensland Studies Authority* (QSA, 2009b, p. 1). Numeracy is clearly an important component in the Queensland school curriculum, but there is no indication or description of a connection between Mathematics and numeracy in this curriculum.

Mathematics and numeracy

The QSA (2009a) provided a clear explanation of Mathematics and numeracy and said the focus of Mathematics is on the development of learners' *knowledge and ways of working in a range of situations from real life to the purely mathematical* where *numeracy refers to the confident use of mathematical knowledge and problem-solving skills not only in the Mathematics classroom, but across the school curriculum and in everyday life, work or further learning* (p. 9). In the Queensland Government's (QG, 2007b) definition of numeracy it is stated that *to be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in the community and civic life* (p. 2). Mathematics and numeracy are interrelated and it is *the responsibility of the Mathematics curriculum to introduce and develop the mathematics which underpins the numeracy* (QSA, 2009a, p. 9). As numeracy refers to the ability to use mathematics in solving life-related problems, it is essential to determine the contexts in which mathematics could be applied and what the role of the teacher is in developing learners' skills in this regard.

The context and teaching of numeracy

In Year 10 to 12 the numeracy work learners *relate[s] to a specific context across a broad range of work and study options* (QG, 2007a) and involve:

- *Applying mathematical skills in new contexts such as: 1) analysing data to inform decision making; 2) deciding to estimate or calculate an answer depending on the purpose; 3) calculating dimensions and quantities of materials in vocational tasks such as construction or hospitality.*
- *Selecting, sequencing and evaluating information to understand texts and to communicate with other people.*
- *Using particular communication skills needed to effectively participate in the workplace such as industry terms and customer services* (QG, p. 1).

Teachers are the key role players in selecting contexts relevant to the learner. They need to *recognise numeracy demands and opportunities within the curriculum* (QG, 2007b, p. 10) enabling learners to develop their numerical knowledge, skills and confidence. Teachers should intentionally create opportunities in which learners can, among other things, explore mathematical ideas with concrete or visual representations and hands-on activities; experience practical and contextualised learning; communicate about mathematical issues; develop calculator and computer skills and use multiple solution strategies (QSA, 2006). According to the Queensland Government (QG, 2007b) teachers' understanding of mathematics content needs to be developed with respect to *the nature of mathematics as a discipline; the* *mathematics topics they teach; the relationship of those topics to further learning and everyday life; the impact of information and communication technologies on the teaching and learning of mathematics* (p. 4).

UNITED KINGDOM (UK)

England did not participate in the 2003 or 2006 PISA study, but performed very well in TIMSS 2007, taking the 7th position for both the 4th and 8th graders out of the 58 participating countries (National Center for Education Statistics, 2008a). By law all children between ages 5 and 16 must receive a full-time education. The UK introduced a National Curriculum in 1992 to which state schools need to adhere until learners reach the age of 16. National Curriculum core subjects are: English, Mathematics and Science which are offered at different levels.

The UK national curriculum

Within the framework of the National Curriculum, schools are free to plan and organise teaching and learning in the way that best meets the demands of their pupils. The Qualifications and Curriculum Development Agency (QCDA) provides guidelines and assistance in this regard. The National Curriculum is organised in four key stages: Key Stage 1 (5-7 years) and Key Stage 2 (7-11 years) form part of the Primary curriculum while Key Stage 3 (11-14 years) and Key Stage 4 (14-16 years) form part of the Secondary curriculum (Government of United Kingdom, 2010a). The aim of the Government is to address the literacy and numeracy levels of children in the first two Key Stages (5-11 years) in order to develop pupils' mathematical thinking and number skills, with a focus on understanding and application. A document addressed to learners, schools and families, called the **The Primary Framework for literacy and mathematics** makes recommendations on how literacy should be incorporated in daily mathematics lessons (Government of United Kingdom, 2010b). The secondary curriculum focuses on developing the skills and qualities that learners need not only to succeed in school, but also in the broader community.

Functional mathematical skills

Numeracy appears in the Early Year Foundation Stage (birth to 5 years) as part of the learning area: Problem solving, Reasoning and Numeracy. In the Primary (5-11 years) and Secondary (11-16 years) curricula Mathematics, and no longer numeracy, appears as one of the ten compulsory school subjects. **Functional mathematics** frequently appears in the Secondary curriculum referring to functional mathematical skills the learners should acquire. Learners need these skills and abilities to play an active and responsible role in their communities, in their everyday life, workplace and in the educational settings (QCDA, 2010a). Functional mathematical skills are a subset of the key processes set out in the programme of study. These key processes are representing, analysing, interpreting, evaluating, communicating and reflecting. All teaching needs to contribute to the development of these key processes. It requires pupils to be introduced to a range of real-life uses of mathematics, including its role in the modern workplace (QCDA, 2010b). These functional skills need to be developed in the five strands of Mathematics, namely Mathematical processes and applications; Number; Algebra; Geometry and measures; and Statistics. Individuals with functional mathematical skills understand a range of mathematical concepts and know how and when to use these concepts. They have the *confidence and capability to use mathematics to solve increasing complex problems; are able to use a range of tools, including integrated computer technologies as appropriate; possess the analytical and reasoning skills needed to draw conclusions, justify how these conclusions are reached and identify errors or inconsistencies; are able to validate and interpret results, judging the limits of the validity and using the results effectively and efficiently* (QCDA, 2010c).

UNITED STATES (US)

In the United States learners take part in both the PISA study and TIMSS. In the PISA 2003 when the focus of the study was on mathematics, they came 31st out of 41 countries (OECD, 2004). In the TIMSS 2007 results they took the 11th position for the 4th graders and the 9th position for the 8th graders out of the 58 countries participating (National Centre for Education Statistics, 2008a).

Although the term "quantitative literacy" is common in the discourse of US mathematics educators, it does not appear often in their curricula (J. Kilpatrick, personal communication, May 24, 2010). Kilpatrick is a mathematics expert, advisor, consultant and professor in Mathematics Education, University Georgia who serves on various mathematical boards and councils. The US does not have a single national curriculum in Mathematics. In search of the term "quantitative literacy" in National Curricula in the Departments of Education of Ohio and North Carolina, as suggested by Kilpatrick, a reference was eventually found on the webpage of the National Council of Teachers of Mathematics (2010) where it was stated that *consumer mathematics should develop a broader quantitative literacy and should consist primarily of work in informal statistics, such as organizing and interpreting quantitative information*.

Comparing the national curriculum documents of Australia, the UK as well as Ohio and North Carolina in the US, it is evident that Australia accentuates the importance of mathematical literacy in an education system. Through their national documents for learners, schools and parents they drive an intensive awareness campaign regarding the raising of the numeracy levels of their learners. Regardless of the terminology used, numeracy, functional mathematical skills and quantitative literacy are embedded in Mathematics and involve the competency or skill to use and apply mathematics to solve contextualised problems.

2.2.1.6 Summary

To define, value, position or conceptualise mathematical literacy is a daunting task. Different views exist but the most common descriptions of mathematical literacy are mathematics in action (Skovsmose, 2007); mathematics in context (McCrone & Dossey, 2007; Powell & Anderson, 2007); realistic mathematics education (Hope, 2007); and mathematising (Gellert et al., 2001; Hope, 2007). The different perspectives of mathematical literacy undoubtedly illustrate how the different conceptions vary in degree of complexity regarding the required mathematical knowledge and skills where in some notions advanced and expert mathematical knowledge and higher order cognitive skills are required. It is however PISA's definition and criteria for assessment that best describe the requirements of this study.

Although some researchers accentuate the formal application of mathematics by mathematicians to real-world contexts demanding a high level of mathematical knowledge and the competence to use and apply it (Gellert et al., 2001; Hope, 2007; Jablonka, 2003; Skovsmose, 2007), other researchers remain convinced that all people need some basic level of literacy to empower them to make well informed decisions in their daily lives, whether personally, to care for their families or to contribute in their workplace or society (McCrone & Dossey, 2007; McCrone et al., 2008; Powell & Anderson, 2007; Skovsmose, 2007). The value of being mathematically literate is evident but it remains uncertain to what extent mathematical literacy could address educational practices and contribute to an individual's quality of life or even the development of the country (Gellert et al., 2001; Jablonka, 2003; Skovsmose, 2007).

Nowhere in the literature has mathematical literacy been referred to as a specialised subject. It is rather regarded as specialised knowledge or a competency or skill embedded in the subject Mathematics. According to Hope (2007) mathematical literacy is a matter of the appropriate pedagogy that should be used in teaching mathematics. As mathematical literacy with its focus on the skill of using and applying mathematical knowledge forms part of Mathematics, the focus of Mathematics teaching should be on knowledge and the development of skills enabling learners to solve real-life application problems. The aforementioned perspectives and conceptions are wide and theoretical and to provide only one international definition of mathematical literacy is not viable as it depends primarily on a particular social practice and the context involved. With these international perspectives in mind, the South African perspective on mathematical literacy is discussed below.

2.2.2 An overview of ML

In this overview of ML in South Africa the history and principles of the subject are discussed. An overview of the two subjects ML and Mathematics is given, some general concerns about ML are discussed and lastly a comparison is made between the national and international perspectives on mathematical literacy.

2.2.2.1 The history of ML

Background information to ML

One of the reasons behind the implementation of ML as an alternative subject to Mathematics in the FET band was the low level of learners' mathematical knowledge and mathematical literacy skills as shown in the results of international studies (DoE, 2003a). The last time South Africa took part in TIMSS, an international study, was in 2003 when Grade 8 learners participated and came last out of 46 countries (National Centre for Education Statistics, 2008b). Recently The World Economic Forum ranked South Africa 120th for Mathematics and Science education, well behind our troubled neighbour Zimbabwe which was ranked 71st (Maths Excellence, 2009). At present the country's GET learners are not participating in any such studies as a four-year Foundations for Learning campaign was introduced in 2008 in the Foundation and Intermediate phases to improve the reading, writing and numeracy abilities of all South African children (DoE, 2008a). Another reason for implementing ML was to address the concern that *Mathematics is too abstract, catering primarily to prepare students to proceed to further mathematically or scientifically oriented studies* (Graven & Venkat, 2007, p. 340). They believe ML now offers an alternative to learners who do not need it for this purpose.

The ML curriculum reform

Curriculum 2005 was introduced in 1998, coinciding with the birth of a new democracy in South Africa's post-apartheid era, and was based on the principles of outcomes-based education (OBE) (DoE, 2009). This curriculum was revised in 2000 and in 2002 the NCS for the FET phase was developed. In 2009 a task team reviewed the curriculum and apart from problems related to learning materials and teacher training, the curriculum documents were deemed to be in need of streamlining (DoE, 2009). The task team found that some of these documents contradicted each other while at other times there were repetitions. The review supports the DoE's current move away from OBE and learning outcomes, which are now replaced with clear content, concept and skill standards as well as clear and concise assessment requirements (DoE, 2009). The various subject specific documents will be replaced with a single document called the Curriculum and Assessment Policy Statement (CAPS) (DoE, 2009). The date of implementation should be in 2012.

2.2.2.2 ML principles

This section examines the principles of ML such as the purpose, aims, definition, key elements, composition of the subject as well as the assessment taxonomy on which ML is based.

The purpose of ML

The purpose of ML is to ensure that all learners develop an understanding of mathematics and how it relates to the world in order to use mathematical information to make valuable decisions affecting their life, work and society. It is important that learners are able to interpret and critically analyse everyday situations and solve problems. With this purpose in mind, ML aims to ensure a broadening of the education of learners, preparing them to meet the demands of a modern world (DoE, 2003a). According to the DoE (2008b, 2011a), the purpose of the subject includes the ability of a learner to become:

- **A self-managing person** where the focus is on problems that relate to financial issues such as mortgage bonds, hire-purchase and investments, other personal issues such as the ability to estimate and calculate length, areas and volumes, to read a map and follow timetables and to understand house plans, sewing patterns and converting recipes.
- **A contributing worker** at the workplace requires the use of fundamental numerical and spatial skills with understanding in order to deal with work-related formulae, statistical charts and schedules and to understand instructions involving numerical components.
- **A participating citizen** where learners need to acquire a critical stance to mathematical arguments presented to them in the media or other platforms.

These three abilities as part of ML's purpose correspond with the international purpose for learner competence, stating that for learners to be competent means having more than just knowledge, they must know how to use and apply their mathematical knowledge (Gellert et al., 2001; Hope, 2007; Jablonka, 2003; Skovsmose, 2007).

The aims of ML

The main aim of ML is to equip learners to be skilled citizens, meeting the demands they will encounter in their future lives. The process to achieve this aim involves the mastering of mathematical content through solving contextualized problems. ML aims to develop the following learner abilities (DoE, 2008b):

- *The ability to use basic mathematics to solve problems encountered in everyday life and in work situations.*
- *The ability to understand information represented in mathematical ways.*
- *The ability to engage critically with mathematically based arguments encountered in daily life.*
- *The ability to communicate mathematically* (p. 8).

The definition of ML

The DoE's (2003a) national definition of ML reads as follows:

> *Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems* (p. 9).

There are, according to this definition, three key elements of ML namely 1) the mathematical content, 2) the contexts that should involve everyday life-related problems and 3) the abilities and behaviours that a mathematically literate person needs to possess which include problem solving through interpreting and analysing the problem with confidence (Bowie & Frith, 2006). In the new CAPS document (DoE, 2011a) these three key elements of ML have been extended to five key elements[7].

Comparing national and international purposes and definitions of ML, the national purpose and definition closely relate to that of PISA (National Centre for Education Statistics, 2008b; OECD, 2003). The purpose of PISA is *to measure the extent to which students can make use of their mathematical knowledge in realistic and day-to-day situations* (McCrone et al., 2008, p. 35). An international definition of mathematical literacy according to PISA (OECD, 2003) is:

> *the capacity to identify, to understand and to engage in mathematics and make well-founded judgements about the role that mathematics plays, as needed for an individual's current and future life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen* (p. 20).

The interface between the international and national views is the emphasis being placed on the role mathematics plays in the world and the value of applying mathematics in people's personal lives, at the workplace and as participating citizens. Further shared objectives are guiding learners to become engaged in mathematics and to understand and appreciate how it is embedded in everyday life situations. A point of difference however is that nationally mathematical literacy refers to both a subject and competence while internationally it refers to a competence (Christiansen, 2007).

The five key elements of ML

From the purpose, aims and definition of ML the DoE (2011a) lists five key elements involved in ML namely:

- **The use of elementary mathematical content:** The general idea is that the focus is not on formal abstract mathematical concepts and mathematical content should not be taught in the absence of context.

[7] The five elements of ML are discussed under the next subheading: *The five key elements of ML.*

- **Real-life contexts:** These contexts should be authentic and relevant, and should relate to learners' daily lives, their future workplace and the wider social, political and global environments.
- **Solving familiar and unfamiliar problems:** Learners should have the ability and skills to interpret both familiar and unfamiliar real-life contextual problems they encounter in the world. They should have the ability to apply both mathematical and non-mathematical techniques and considerations in order to explore and make sense of the context. The interplay between content, context and solving problems is illustrated in the following figure:

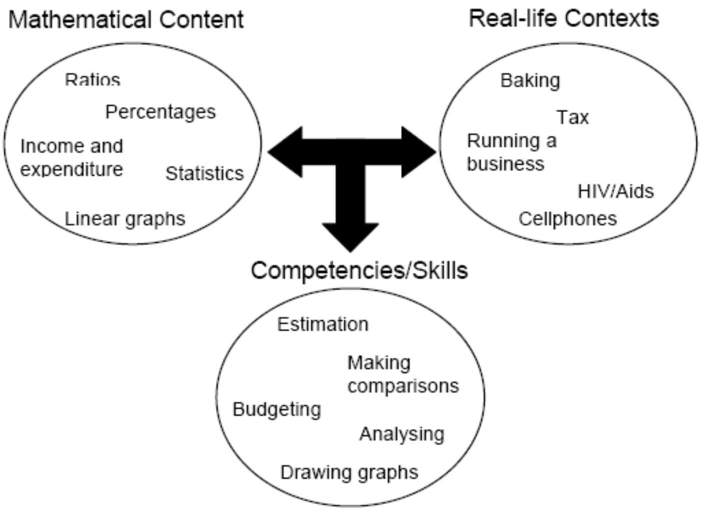

Figure 2.1: Interplay between content, context and problem-solving skills in ML (DoE, 2011a, p. 10)

- **Decision-making and communication:** A mathematically literate person should be able to compare solutions, make decisions regarding the most appropriate choice for a given set of conditions and communicate their decisions through the use of appropriate terminology.
- **The use of integrated content and/or skills in solving problems:** Since most real-life problems consist of a range of mathematical topics, learners need to use mathematical content and/or skills drawn from a range of topics and need to identify and use a range of techniques and skills integrated from a range of content topics.

The composition of ML

The topics in the new CAPS (DoE, 2011a) replace the learning outcomes from the current NCS for ML. The content, contexts and problem solving skills appropriate to ML are offered in topics and divided into two sets of topics, namely (DoE, 2011a):

- **Basic skills topics:** Much of the content in these topics includes the mathematical content and skills that learners have already been exposed to in Grade 9. Teachers therefore have the opportunity to revise important mathematical concepts and to provide learners now with the opportunity to explore and use these concepts in various contexts.
- **Application topics:** These topics contain contexts that can be related to situations from everyday life, the workplace and business environments as well as wider social, national and global issues that learners are expected to make sense of. A profound understanding of the content and skills from the **Basic skills topics** are required to make sense of the contexts and content from the **Application topics**. Figure 2.2 below shows an overview and weighting of the topics according to which the ML curriculum has been organized for Grades 10, 11 and 12.

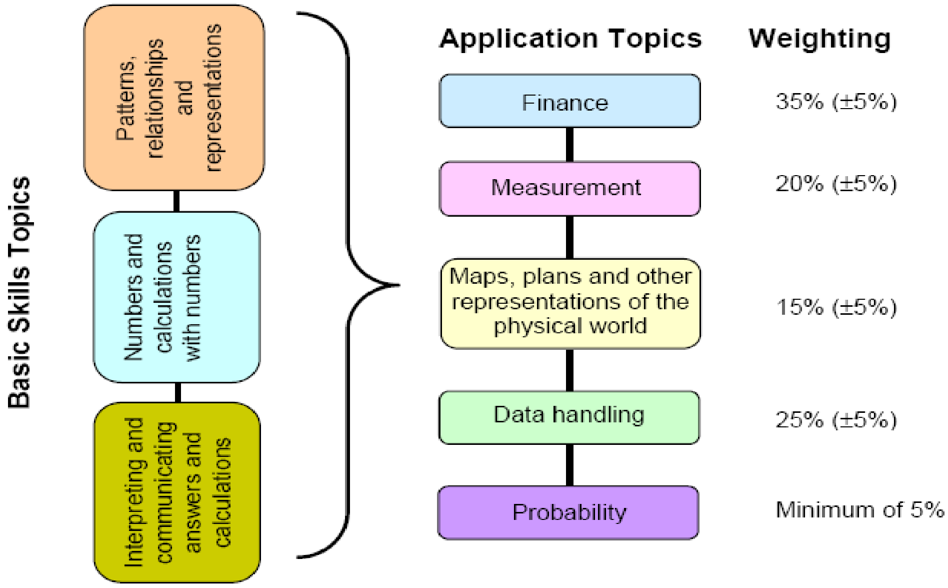

Figure 2.2: Overview and weighting of topics in Grades 10, 11 and 12 (DoE, 2011a, p. 14)

The ML assessment taxonomy

Assessment should be done at different levels of cognitive demand, from *simple reproduction of facts* to *detailed analysis and the use of varied and complex methods and approaches* (DoE, 2011a, p. 91). The following assessment taxonomy framework is used:

- Level 1: Knowing
- Level 2: Applying routine procedures in familiar contexts
- Level 3: Applying multi-step procedures in a variety of contexts
- Level 4: Reasoning and reflecting (p. 84).

According to Venkat, Graven, Lampen and Nalube (2009) the emphasis in Levels 1 and 2 is on routine calculations whereas the key aims of ML are located primarily in Levels 3 and 4. Level 3 refers to the ability of learners to think numerically and spatially whereas Level 4 refers to critically analysing everyday life situations.

The DoE (2011a) explicitly states that since ML *involves the use of both mathematical and non-mathematical techniques and considerations in exploring and making sense of authentic real-life scenarios* (p. 92), the taxonomy should be regarded as follows:

> *This taxonomy should not be seen as being associated exclusively with different levels of mathematical calculations and/or complexity. In determining the level of complexity and cognitive demand of a task, consideration should also be given to the extent to which the task requires the use of integrated content and skills drawn from different topics, the complexity of the context in which the problem is posed, the influence of non-mathematical considerations on the problem, and the extent to which the learner is required to make sense of the problem without guidance or assistance* (DoE, 2011a, p. 92).

2.2.2.3 Pedagogical approaches for teaching ML

The DoE (2011a) suggests that the focus of ML teaching is the integration of content and skills in real-life contexts. Teachers should provide learners with opportunities to *analyse problems and devise ways to work mathematically in solving them* (p. 9) and *develop and practice decision-making and communication skills* (p. 10). According to Brown and Schäfer (2006) the emphasis in the ML curriculum is on contextualised mathematics. These contexts should be realistic and demand real-life authenticity to provide learners with opportunities to apply and use mathematics in order to make sense of the world, instead of letting learners do more mathematical content (Bansilal, Mkhwanazl & Mahlaboratoryela, 2010). These problems should relate to a learner's daily life, the workplace and the wider social, political and global environment (DoE, 2011a, p. 12). Brown and Schäfer (2006) found *many similarities to that of mathematical modelling* but the *differences appeared to be that mathematical modelling is generally described using more advanced mathematics, in more technical contexts* (p. 46) but that the basic principles of modelling can be applied on elementary mathematics too. ML further focuses on *de-compartmentalisation, where mathematical topics are no longer taught in isolation of each other* (North, 2005, p. 35). All ML textbooks are written accordingly with the initial four learning outcomes being integrated to enable ML teachers to teach ML in a de-compartmentalised way.

In a longitudinal study performed by the Marang Centre at the University of the Witwatersrand, Graven and Venkat (2009) report that the learners who are part of the study, are for the most part positive about ML, which the researchers attribute to the teachers who substantially changed their pedagogic practices. This differs from traditional Mathematics teaching in that *the nature of tasks in ML (engagement* *with a scenario rather than application of maths in 'word problems') and the nature of interaction in ML (much slower pace, more discussion and group work)* (p. 2). Venkat (2007) argues that if learners are engaged in problems situated in real-life situations, they will develop valuable skills such as mathematical reasoning, sense-making, applying different procedures and decision-making.

In Table 2.1 given below, Graven and Venkat (2007) identify a spectrum of pedagogic agendas that *traverses across the question of the nature and degree of integration of context with mathematics within pedagogic situations* (p. 74). This spectrum of agendas is a tool for the ML teachers to think about the nature of their lessons and may assist the teachers to navigate their teaching along a whole spectrum of pedagogic agendas.

Table 2.1: A spectrum of pedagogic agendas (Graven & Venkat, 2007, p. 74-75)

1. Context driven (by learners' needs)	**2. Content and context driven**	**3. Mainly content driven**	**4. Content driven**
<u>Driving agenda</u>: To explore contexts that learners need to interact and engage with in their lives (current, future, citizenship) and to use maths to achieve this.	<u>Driving agenda</u>: To explore a context so as to deepen maths understanding and to learn maths (new or GET) and to deepen understanding of that context.	<u>Driving agenda</u>: To learn maths and then apply it to various contexts.	<u>Driving agenda</u>: To give learners a second chance to learn the basics of maths from the GET band.
<u>Pedagogic demands</u>: • Involves identifying contexts/scenarios needed for the above agenda. • Teaching needs increased discussion of contexts and critical engagement with them and the mathematics embedded in them. • Teaching might require revisiting or learning new maths but largely insofar as it will service critical engagement with and understanding of the context.	<u>Pedagogic demands</u>: • Involves selecting real contexts (possibly edited or adapted) that enable the above agenda. • Teaching needs discussion about contexts but this must be balanced with revising maths and learning new maths in new ways. Contextual and mathematical learning need to be balanced and connected in a dialectical relationship that enables the agenda.	<u>Pedagogic demands</u>: • Involves selecting contexts that GET maths can be applied to (contrived or more real) and editing these to enable application appropriate to the level of learning. • Teaching focuses on mathematical learning and its use in applications and does not necessarily require much discussion of context.	<u>Pedagogic demands</u>: • Involves revision of GET maths without the need for pedagogic change except in relation to slower pacing. • Contexts do not feature much except in relation to their use in teaching GET basics (e.g. in the case of fractions – using cakes for understanding fractions).

Graven and Venkat (2007) analysed ML's definition and purpose on context as stated by the DoE (2003a) and propose Agenda 2 to be the core business of ML. They call these four agendas a **spectrum** and not a **continuum** which *might imply that teachers move along it in one direction* (Graven & Venkat, 2007, p. 77). The idea is that teachers may use different agendas at different times as required. Although Agenda 2 is the *primary driving agenda,* a teacher can *adopt other agendas at different points in order to support this agenda and also to assist in meeting curricula demands* (p. 77).

2.2.2.4 The ML learner profile

Although there are positive and enthusiastic ML learners, the majority of learners are less interested and enthusiastic about mathematics and mathematical activities and many negative feelings result in fear of anything mathematical (Vermeulen, 2007). According to Vermeulen learners could avoid these negative experiences and feelings of anxiety in the past by not choosing mathematics, but now they need to confront them. He argues that it is the parents' and society's incorrect beliefs, teachers' teaching methods based on their beliefs and attitudes, teachers' attitudes towards the learners, and teachers' classroom culture that contribute to learners' negative feelings. According to Mbekwa (2007) it is a challenge to teach ML as learners lack understanding and motivation because ML is seen as the *dumping ground for mathematics underperformers* (p. 227).

2.2.2.5 Some general concerns about ML

The ML teachers

As ML is a relatively new subject and different to Mathematics, clear guidelines from the DoE regarding issues such as pedagogical approaches to teach ML should have been a given, but instead *the absence of precedents of what pedagogy and assessment should be like* (Graven & Venkat, 2007, p. 67) caused multifarious interpretations of the curriculum aims. Bowie and Frith (2006) were concerned about a perception that ML could be interpreted as *a slightly toned-downed standard grade Mathematics with word sums* (Bowie & Frith, 2006, p. 32). Experience and research have indicated that Mathematics learners and in many cases teachers too, find word or application problems requiring conceptual understanding more difficult than routine problems which require factual recall or the use of routine procedures (Abedi & Lord, 2001; Grobler, Grobler & Esterhuyse, 2001; Johari, 2003; Schoenfeld, 1988; White & Mitchelmore, 2002). My own experience like that of De Villiers (2007) confirms that Mathematics learners cope well with the theory of linear functions, but when it is put in real-life contexts they cannot solve such problems. Even in ML both teachers and learners find the process of mathematising contexts complex as a good understanding of both the context and the mathematical content is required (Bowie & Frith, 2006). A further concern is the number of ML teachers with other specialisations who also teach the subject (Mbekwa, 2007). It is known that in the past, before ML was introduced, there existed a shortage of appropriately qualified Mathematics teachers (Sidiropoulos, 2008) and the question arises as to the provenance of all the teachers who now teach both ML and Mathematics. Sidiropolous (2008) further suggested that *a change is required not only in pedagogical content knowledge, but also in understanding the nature and value of Mathematical Literacy* (p. 205).

The choice of context

Apart from the complexity of solving contextual problems, the contexts to which mathematics should be applied in ML are not clear to teachers. A further concern is how mathematical progression is made through the years regarding the complexity of contexts. A good understanding of the context is required by both teachers and learners in order to mathematise a context (Bowie & Frith, 2006). For example when working on personal finances, topics such as budgeting, compound interest, mortgage payments, and retirement options are not part of all teachers' and learners' life experiences. They have inadequate experiences of banks, interests, risks and return on investments. To teach one mathematical content topic requires several periods to first explain the context involved. What further complicates the situation is that in reality banks normally use their own formulae programmes and do not calculate interest as learners are taught to do (Christiansen, 2007). In choosing contexts, teachers may use the principle from PISA (OECD, 2003) that categorises contexts according to their distance from the learner. ML teachers can therefore include contexts from the learner's private life, school life, work and sport, and local community and society. It is crucial that contexts be authentic and applicable to the learners' environment.

The language issue

In South Africa the majority of learners are taught in English, which is often not their mother tongue. According to Graven and Venkat (2009) integration with the above-mentioned contexts could be problematic due to the increased English language demands. Many researchers reported on difficulties learners experience regarding the contextualised problems and the role language plays in conceptual understanding (Mbekwa, 2007; Setati, 2005). Maree (2000) posits that insufficient language skills and language usage play an important role in under-achievement of learners in Mathematics. In his classification of learners' mistakes in mathematics, language problems were the most significant problem identified. He expressed his concern about learners having to unravel problems in mathematics that require more sophisticated language skills while they actually lack the minimum language skills to even understand what is being asked.

Debate regarding ML as only alternative to Mathematics exists and many teachers expressed their concern about the existence of two extreme levels of mathematics, especially considering South Africa's diverse population (Maths Excellence, 2009). This group of teachers advises a three-level system consisting of two formal Mathematics subjects and ML as the third option to accommodate the diverse skills and needs of our learners. Their idea is that ML should then be taken by learners who do not wish to take either of the formal Mathematics courses. The ML learners can then be equipped with basic numeracy skills. According to them the current lack of curriculum flexibility could result in the downgrading of mathematical skills. On the other hand a number of people in the school education system are against a two-level system as was applicable in the South African schools up to 2007. According to Kitto (personal communication, February 23, 2011) a reason is that *very few township and rural schools offered higher grade under the old system, so an overwhelming percentage of the higher grade candidates were white. A huge number of competent black students were denied the chance to demonstrate their ability and get into engineering and other faculties*. She reasons that the policy makers are trying to make sure that everyone who has the ability to continue with careers in science and engineering has access to the mathematics that is needed.

2.2.2.6 Comparison between the national and international perspectives on mathematical literacy

In South Africa the term "mathematical literacy" refers *both to a school subject and to the competency of individuals,* where *internationally it is mainly the latter* (Christiansen, 2007, p. 91). The original NCS Grades 10-12 General (DoE, 2003a) is based on OBE, social transformation and integration, and applied competence. These principles encourage a learner-centred and activity-based approach.

With the increased international emphasis on the application value of mathematics, science and technology, the objectives of subjects such as ML changed to *personal fulfilment, employment and full participation in society* (OECD, 2004, p. 37). Internationally mathematical literacy as application skills is embedded in the subject Mathematics. For this purpose real-life contexts are used to re-contextualise mathematical concepts. From the literature it is evident that mathematical literacy varies in width and depth and that one needs to interpret it according to the purpose and context being used (Gellert et al., 2001; Hope, 2007; Jablonka, 2003; McCrone & Dossey, 2007; Powell & Anderson, 2007; Skovsmose, 2007). Jablonka (2003) states that the context in which mathematical literacy is applied, sometimes demands higher-order mathematical skills, whereas McCrone and Dossey (2007) believe mathematical literacy should be promoted even in non-mathematics classes to make mathematics relevant and to empower all learners. Nationally the subject ML focuses on *making sense of real-life contexts and scenarios*

(DoE, 2011a, p. 9) and *requires an understanding of only basic mathematical concepts and calculations, and does not require an understanding of complex and/or abstract mathematical principles* (DoE, 2011a, p. 11).

2.2.2.7 An overview of ML and Mathematics

Since ML is a compulsory subject for Grade 10 to 12 learners who do not choose Mathematics as subject, parents and learners should know what each subject entails and what the implications are for further studies. For example the DoE (2003a) states that learners who wish to proceed to tertiary studies of a mathematical nature such as engineering, architecture, natural sciences at tertiary institutions should not take ML. Issues dealt with in this overview are the subjects' premises, learning outcomes and topics to be covered as well as the pedagogical approach for teaching ML and the ML learner profile.

The premises of ML and Mathematics

In Table 2.2 below the premises of Mathematics and ML are discussed according to the subjects' purposes, aims, definitions and their educational and career links as set out by the DoE (2003a, 2003b, 2008b, 2011a, 2011b).

Table 2.2: The premises of ML and Mathematics

	ML	Mathematics
Purpose	Provide learner with an awareness and understanding of the role mathematics plays in the modern world, enabling learners to become self-managing people, contributing workers and participating citizens (DoE, 2003a, 2011a).	To create an appreciation of the discipline itself and a deeper understanding and successful application of knowledge and skills. This competence contributes not only to personal and social, but also to learners' scientific and economic development (DoE, 2003b).
Aim	To equip learners to understand information represented in mathematical ways and to solve problems encountered in everyday life and work situations (DoE, 2008b). ML learners should have the ability or skills to think mathematically, interpret, analyse and solve problems (DoE, 2003a).	To allow learners to develop into citizens who are able to deal with the mathematics that forms part of the society they live in and on their daily lives. It is more important for learners to acquire skills such as investigating, generalising and proving instead of only acquiring content knowledge for its own sake (DoE, 2003b).

Definition	*Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems* (DoE, 2003a).	*Mathematics enables creative and logical reasoning about problems in the psychical and social world and in the context of mathematics itself … is based on observing patterns, with rigorous logical thinking, this leads to theories of abstract relations … enables us to understand the world and make use of that understanding in our daily lives* (DoE, 2003b). According to CAPS mathematics is *a language that makes use of symbols and notations for describing numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves* (DoE, 2011b, p. 10).
Career links	ML should not be taken by learners who intend to study mathematically based disciplines such as natural sciences and engineering. ML learners proceeding to Higher Education institutions will have developed the skills needed to deal effectively with mathematically related requirements in disciplines such as the social and life sciences (DoE, 2003a).	The subject provides a platform for linkages to Mathematics in Higher Education institutions. Mathematics is essential for learners who intend to pursue a career in the psychical, mathematical, computer, life, earth, space and environmental sciences or in technology. Mathematics also plays an important role in the social, management and economic sciences (DoE, 2003b).

Studying these premises, it is clear that the two subjects are different in kind and should not be compared. Sidiropoulos (2008) is also of the opinion that the *distinction between ML and Mathematics is principally not a distinction in level, but a distinction in kind* (p. 208). Mathematics on the one hand is regarded as a purely academic subject with a reputation of being an abstract science, involving mathematical rigour and a high level of cognitive thinking and reasoning based on sound conceptual understanding of the content. ML on the other hand does not focus on abstract mathematical concepts but, instead, primarily on developing practical skills to use elementary mathematical content to find concrete solutions to numeric, spatial and statistical problems associated with everyday life experiences (DoE, 2011a; Maffessanti, 2009). A shared aim however is the development of competent learners who are able to use their mathematical knowledge to solve personal and social real-life problems.

The learning outcomes of ML and Mathematics

With the introduction of ML in 2008, the similarities between the learning outcomes for ML and Mathematics, as seen in the table below, were a major concern as some people thought of ML as a lower grade Mathematics subject. The learning outcomes as they were applied from 2008 to 2011 for the Senior Phase in the GET band (Grades 8-9), ML and Mathematics, both in the FET band, are listed in Table 2.3 below (DoE, 2003a, 2003b; 2010):

Table 2.3: Learning outcomes for ML and Mathematics

	Mathematics (GET: Senior Phase)	**ML (FET)**	**Mathematics (FET)**
Learning outcome 1	Numbers, Operations and Relationships	Number and Operations in Context	Number and Number Relationships
Learning outcome 2	Patterns, Functions and Algebra	Functional Relationships	Functions and Algebra
Learning outcome 3	Measurement	Space, Shape and Measurement	Space, Shape and Measurement
Learning outcome 4	Data Handling	Data Handling	Data Handling and Probability

The first two columns show how the learning outcomes for ML build on the learning outcomes for Mathematics in the GET band (DoE, 2005). Some researchers are of the opinion that ML, being a new subject with a different focus, should not have used the same content-based learning outcomes as Mathematics as this scenario ended up being stumbling blocks to the teachers (Bowie & Frith, 2006; Christiansen, 2007; North, 2005). This concern about similar learning outcomes has been addressed in the new CAPS (DoE, 2011a) and is discussed in the paragraph below.

Topics covered in ML and Mathematics

Different concerns regarding the content-context issue have been expressed by academics prior to the new CAPS. There were questions about what content knowledge should be taught by the ML teachers, which contexts should they use (Geldenhuys, Kruger & Moss, 2009; Julie, 2006; Vithal & Bishop, 2006), and whether the content should determine the context or vice versa (Bowie & Frith, 2006; Graven & Venkat, 2007). Although these issues were not elucidated clearly in the original NCS for ML (DoE, 2003a), the new CAPS (DoE, 2011a) addresses these issues.

In ML the topics are divided into two groups, namely the **Basic Skills Topics** which *comprise elementary mathematical content and skills that learners have already been exposed to in Grade 9* and the **Application Topics** which *contain the contexts related to scenarios involving daily life, workplace and business environments, and wider social, national and global issues* (DoE, 2011a, p. 13). For this purpose it is necessary to list the content areas and topics covered in the Mathematics Senior Phase[8] as well as the ML and Mathematics in the FET Phase. Different terminology for content areas and topics is used across the different bands (DoE, 2011a; DoE, 2011b; DoE, 2010) as indicated in Table 2.4 below:

[8] The Senior Phase band includes Grade 7 through to Grade 9.

Table 2.4: Comparison of the composition of ML and Mathematics across the different bands

COMPOSITION OF MATHEMATICS AND ML		
Senior Phase Mathematics	**FET ML**	**FET Mathematics**
Content areas: 1. Number, Operations and Relations 2. Patterns, Functions and Algebra 3. Space and Shape (Geometry) 4. Measurement 5. Data Handling (Statistics)	**Basic Skills Topics:** 1. Interpreting and communicating answers and calculations 2. Numbers and calculations with numbers 3. Patterns, relationships and representations **Application Topics:** 1. Finance 2. Measurement 3. Maps, plans and other representations of the physical world 4. Data handling 5. Probability	**Main content topics:** 1. Functions 2. Number patterns, sequences, series 3. Finance, growth and decay 4. Algebra 5. Differential calculus 6. Probability 7. Euclidean Geometry and measurement 8. Analytical geometry 9. Trigonometry 10. Statistics
Content topics: Example: Exponents, Integers, Fractions etc. under number 1 above are called the content topics.	**Content topics:** A range of content topics based on Senior Phase content topics only.	**Curriculum statement:** Instead of using *Content topics*, *Descriptions* is used to explain the content under each main topic. Example: *Practical problems involving optimisation and rates of change* (DoE, 2011b, p. 11) under number 5 above.

2.2.2.8 Summary

South Africa was the first country in the world to introduce ML as a school subject in 2006 in the FET band (Grades 10 to 12) (Christiansen, 2007). A major reason behind the implementation of a compulsory mathematics subject in the FET band is to improve the low level of learners' mathematical knowledge and mathematical literacy skills. One of ML's purposes is to provide the opportunity for each learner to become mathematically literate in order to effectively deal with *mathematically related requirements in disciplines such as the social and life sciences* (DoE, 2003, p. 11). In comparing the national and international perspectives the latter refers to various levels of specialised knowledge, skills and understanding that are required to apply formal mathematics to solve application problems in various contexts.

The approach to the teaching and learning of ML should provide opportunities to engage with mathematics in diverse contexts at a level that learners can access logically (DoE, 2003c). However, the teaching of ML in a contextualised and de-compartmentalised manner where the content topics are integrated, complicates the teaching of the subject as teachers lack the knowledge and skills to do so.

The *distinction between ML and Mathematics is principally not a distinction in level, but a distinction in kind* (Sidiropoulos, 2008, p. 208). Mathematics is regarded as a purely academic subject, an abstract science involving a high level of cognition. ML on the other hand does not focus on abstract mathematical concepts but primarily on developing practical skills to deal with everyday life experiences (DoE, 2011a; Maffessanti, 2009).

2.3 Teachers' instructional practices

The process of teaching and learning is extensive and involves many pedagogical concerns and influences. Teaching in general involves more than the activities in the classroom and includes activities such as working with parents, colleagues and engaging in professional development (Franke et al., 2007). However the instructional practice of the teacher occurs in the classroom where teachers' goals, knowledge and beliefs serve as driving forces behind their instructional efforts to guide and mentor learners in their search of knowledge (Artzt et al., 2008). Different terminology is used in the literature when referring to teachers' performances or the act of teaching in the classroom. Terminology such as teachers' behaviour, instructional behaviour, instructional practices, classroom practices, classroom processes, and classroom instruction are frequently used. Table 2.5 below provides a short definition of four of the frequently used terms when referring to teachers' practices:

Table 2.5: Different terminology used for teachers' practices

Classroom practice	Focus is on three features, namely discourse, norms and building relationships (Franke et al., 2007).
Classroom instruction	*Involves interactions among teachers and students around mathematical subject matter* (Kilpatrick, 2001, p. 107).
Classroom processes	Interaction taking place between the teacher and learner and all the factors influencing this interaction (Koehler & Grouws, 1992).
Instructional practice	Refers to the qualitative dimensions of teacher behaviour regarding their teaching (Englert et al., 1992).

The term "instructional practice" best portrays the focus of this study being the ML teachers' classroom behaviour. Englert et al. (1992) refer to teachers' instructional practices as teachers' qualitative dimensions in the teaching and learning process. Qualitative dimensions involve teachers' abilities to apply appropriate cognitive strategies in meaningful and purposive activities, promote classroom dialogues and adjust instruction as required, and establish classroom environments in which students cooperatively and collaboratively participate in enquiry-related activities. To examine teachers' instructional practices, Artzt et al. (2008) use a phase dimension framework that is built on three observable aspects of mathematical lessons, namely tasks, discourse and the learning environment[9] (Figure 2.3).

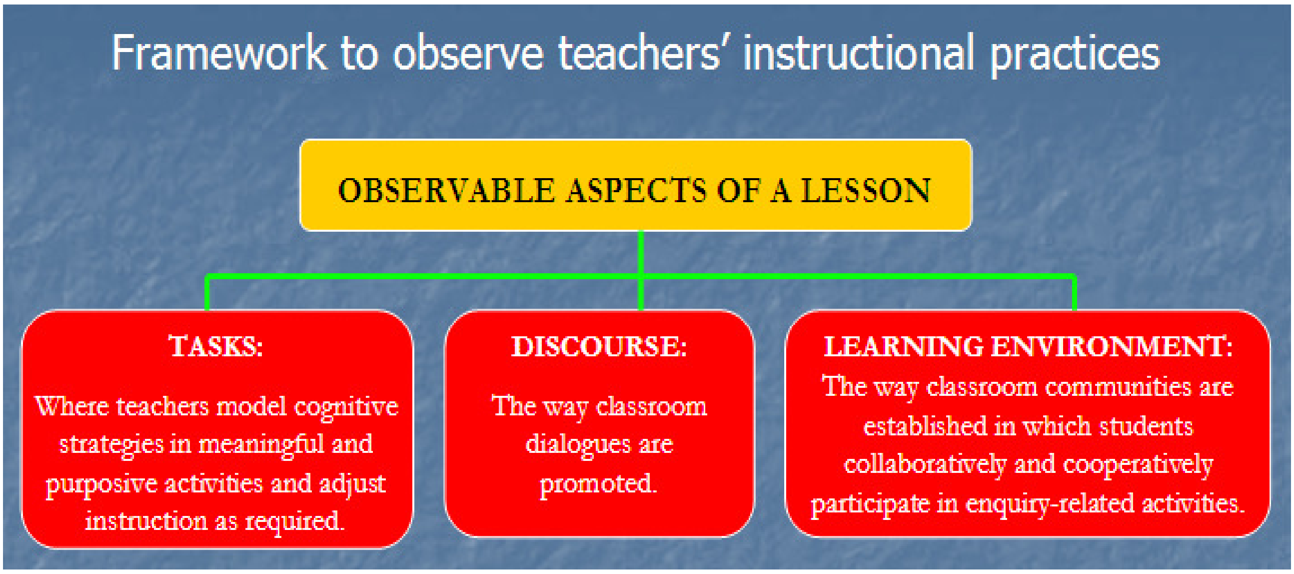

Figure 2.3: Framework to observe teachers' instructional practices (Adapted from Artzt et al., 2008; Englert et al., 1992)

In the light of my research paradigm of social constructivism which suggests that all knowledge is constructed and based upon not only prior knowledge, but also the cultural and social context (Ollerton, 2009), the participants' instructional practices are subsequently discussed. Franke et al. (2007) recognise a productive instructional practice as a practice creating ongoing opportunities for learning. There are different perceptions regarding the components of a teacher's instructional practice. Artzt et al.'s (2008) dimensions of instructional practices are tasks, discourse and learning environment whereas Franke et al. (2007) speak of discourse, norms and building relationships as the three features of classroom practices. From these two views the teachers' practices could be described as a social environment where all people in the classroom are in a relationship with one another, have the opportunity to construct and enhance their knowledge through communicating while solving and pursuing their conjectures of challenging tasks. For the purpose of my study the dimensions discussed by Artzt are most appropriate, since they address the practical issues of classroom practice which is fundamental in ML teaching.

[9] The characteristics of the observable aspects of a lesson are further discussed in par. 2.5.5.

I will now briefly discuss researchers' views on tasks, discourse and the learning environment and report on findings in the literature regarding the three aspects of ML teachers' lessons.

2.3.1 Tasks

Since knowledge is constructed and based upon, among other things, prior knowledge, the purpose of tasks is to *provide opportunities for learners to connect their knowledge to new information and to build on their knowledge and interest through active engagement in meaningful problem solving* (Artzt et al., 2008, p. 10).

Modes of representation

Franke et al. (2007) believe teaching *involves orchestrating the content*, that teachers' planning of their actions is crucial to enable learners to progress in their cumulative understanding of a particular content area (p. 228). According to Artzt et al. (2008) modes of representation are the forms for representing mathematical concepts *through the use of oral or written language, diagrams, manipulatives, computers, or calculators* (p. 12). Geldenhuys et al. (2009) recommended that teachers should increase the use of resources such as computers. Bransford, Brown and Cocking (2000) mentioned some people believe technology is money and time wasted whereas others regard the mere presence of computer technology in schools as enhancing the learning in the school. When computer technology is used correctly, Bransford et al. believe it has great potential to enhance student achievement. Since ML is related to real-life situations such as interest rates of home loans or personal income tax, computer technology could enhance the learners' understanding and interest in the subject and its application value as they could find the specific day's interest rates or even general information regarding income taxes.

Motivational strategies

The tasks teachers use in their lessons should *possess attributes that attract and sustain [the learners'] attention and emotional investment over time* (Artzt et al., 2008, p. 13). Dewey (as cited in Bransford et al., 2000) noted the following:

> *From the standpoint of the child, the great waste in school comes from his inability to utilize the experience he gets outside … while on the other hand, he is unable to apply in daily life what he is learning in school. That is the isolation of the school – its isolation from life* (p. 147).

In my view Dewey's concern is addressed by the DoE (2003a) when ML was implemented with its purpose of providing opportunities for learners to experience how mathematics relates to the world, enabling the learners to use mathematical information to make valuable decisions affecting their life, work and society (DoE, 2003a). The idea of connecting the school and home environments is consistent with Moll and Gonzalez (2004) who argued that teachers need to know and understand their learners' home environments which could be used to understand the learners' participation in the classroom. Especially in ML where the emphasis is on content being taught in context and making the subject applicable to real-life situations (DoE, 2003a), teachers need to take into consideration the knowledge their learners bring to their classrooms.

Sequencing and difficulty levels

The difficulty levels and sequencing of tasks *must allow students to use their past knowledge and experience to help them understand the requirements of the task* (Artzt et al., 2008, p. 13). Bransford et al. (2000) mentioned that tasks must be at the appropriate level of difficulty in order for learners to remain motivated. They stated too easy tasks cause learners to become bored while too difficult tasks cause frustration. Hechter (2011b) reported that the cognitive levels of the assessment tasks set by both teachers in her study were on a relatively low level. Bansilal (2008), whose study consisted of an analysis of the answers given by 38 ML teachers to various questions taken from a test and the final examination in a module of their Advanced Certificate in Education (ACE) (ML) programme, revealed that teachers found questions which had multi-steps, difficult.

2.3.2 Discourse

To contribute to learner understanding, the discourse in class should provide opportunities for learners to express themselves, to listen to, to question, to respond and to reflect on their thinking (Artzt, et al., 2008). Franke et al. (2007) believed classrooms involve *people who work in social, cultural and political contexts that shape how they do their work and how that work gets interpreted* (p. 227).

Teacher-learner interaction

The teacher plays a critical role in orchestrating discourse in class and should know how to use verbal and non-verbal strategies to communicate effectively (Artzt, et al., 2008). According to Franke et al. (2007) teaching is multifaceted and teaching should be seen as *deliberate work*, where the teacher should orchestrate the content, the representations of the content, as well as all people in the classroom in relation to one another. They mentioned that teachers need to have the ability to elicit and interpret what learners do and know, to act appropriately on that and be able to make decisions emerging from complex interactions. They do not regard learning as receiving information but rather as engaging in sense-making as the teacher and learners participate together. Although my study was not concerned with the influence of teacher-learner interaction on learners' performance, it is worth noting that Bansilal et al. (2010) found in their study that the continuous support and feedback the tutors provided to the practising ML teachers (students) in the ACE (ML) programme improved the students' performances over the semester. Bansilal et al. (2010) regarded this increasing interaction in communities of practice as providing positive learning opportunities to the students.

Learner-learner interaction

Contributing to learners' development of conceptual understanding are the opportunities learners have to interact with each other in such ways that they can support, strengthen and challenge each others' ideas (Artzt, et al., 2008). Lampert (2004) mentioned that the practice of teaching is not only about the *actions* of the teacher but the *evolution of relationships* between the teacher and learners and among learners themselves around mathematics *and engaging together in constructing mathematical meaning* (p. 2). Franke et al. (2007) expressed their concern that many mathematics classrooms do not provide sufficient opportunities for learners to develop mathematical understanding. They believe learners must have the opportunity to become encouraged and curious and *talk about and with mathematical expertise* (p. 229). National researchers emphasise the importance of learner-centred approaches where learners are involved in the lesson, taking part in discussions and group work (Brown & Schäfer, 2006; Venkat, 2007; Venkat & Graven, 2008).

Questioning

The value of proficient oral questioning is that *the teacher encourages students to make public their knowledge, skills, and attitudes in relation to the problem under consideration* (Artzt, et al., p. 16). Knowledge of learners' mathematical thinking will support the teachers to provide opportunities for asking questions which are linked to the learners' thinking, will elicit discussion and will draw on connections learners need to make to comprehend the work (Franke et al., 2007).

2.3.3 Learning environment

In my study I based my rationale for a learning environment on the work of Artzt et al. (2008) who state that a learning environment comprises a particular social and intellectual climate, the use of effective modes of instruction and pacing of the content and attending to certain administrative routines. Bransford et al. (2000) on the other hand, regard a learning environment as involving the rethinking of what should be taught, how it should be taught and how it should be assessed. When these two views are compared, a common aspect is **how** the content should be taught and **what** should be taught which forms part of the tasks, and how learners should be **assessed** does not form part of Artzt et al.'s learning environment.

Social and intellectual climate

The social and intellectual climate defines the tone, style, and manner of the interpersonal interactions in the classroom and contributes to learners' *social and cognitive growth and development* (Artzt et al., 2008, p. 14). Franke et al. (2007) stated that productive practices occur where learners *see themselves as comfortable, confident, and knowledgeable in their abilities to engage in mathematics* (p. 227). Silver, Smith and Nelson (1995) found that creating an atmosphere of trust and mutual respect was critical for the development of valuable discourse between the teacher and learners and among learners themselves.

Modes of strategies and pacing

Modes of strategies and pacing are the strategies teachers use in the classroom to help learners attain the objectives of the lesson and teachers should properly pace the activities so that learners have enough time to participate and construct new knowledge (Artzt et al., 2008). The use of cognitively guided instruction is suggested by researchers to support the development of learners' mathematical understanding (Carpenter et al., 2000; Bransford et al., 2000; Franke et al., 2007). This approach to teaching assists learners to overcome their misunderstandings and effectively change conceptual misconceptions. Another effective strategy mentioned is interactive lecture demonstrations (Franke et al., 2007). Nationally some researchers proposed effective strategies for teaching ML, namely mathematical modelling (Brown & Schäfer, 2006); discussions and group work (Venkat & Graven, 2008); co-operative learning (Frith & Prince, 2006) and project work (Vithal, 2006).

Administrative routines

According to Artzt et al. (2008) administrative routines are procedures or activities in classroom organisation and management. Kounin and Gump (1974) regard these routines as providing an ongoing sign of organisational and interpersonal behaviour in class.

2.4 Mathematics teachers' knowledge and beliefs about mathematics and the teaching thereof

In this section I mention the relationship between knowledge and beliefs, give an overview of different domains of teachers' knowledge, and discuss what is meant by teachers' belief systems. Lastly, I point out what the influence of teachers' knowledge and beliefs is on their instructional practices.

2.4.1 Relationship between knowledge and beliefs

There is no agreement on the definitions of knowledge and beliefs, their relationship or even their influence on teaching (Gess-Newsome, Lederman & Gess-Newsome, 2002). She points out some differences and relationships between knowledge and beliefs (Table 2.6) and emphasises that in practice the lines between knowledge and beliefs can easily become blurred.

Table 2.6: Relationship between knowledge and beliefs

	KNOWLEDGE	BELIEFS
Described as:	Evident, dynamic, emotionally neutral, internally structured.	Both evidential and non-evidential, static, emotionally bound, organised into systems.
Develops with:	Age and experience	Episodically
Functions:	• Conceptual knowledge (knowledge that is rich in relationships) is used in problem solving situations. • The amount, accessibility and organisation thereof distinguish experts from novices.	• Have both affective and evaluative functions; • Act as information filters; • Have an impact on how knowledge is used, organised and retrieved; • Are powerful predictors of behaviour which can either be consistent or inconsistent with beliefs.

Artzt et al. (2008) define teacher knowledge *as an integrated system of internalised information acquired over time about pupils, content and pedagogy* and beliefs are defined *as an integrated system of internalised assumptions about the subject, the students, the learning, and teaching* (p. 20). They further believe that *beliefs function as an interpretative filter for teachers' goals and knowledge and strongly affect classroom practice* (p. 20). Their views on knowledge and beliefs correspond with Gess-Newsome et al.'s (2002) except that Artzt et al. (2008) also describe knowledge as organised into systems.

Liljedahl (2008) strongly believes that any discussion on a teacher's knowledge cannot be restricted to knowledge of mathematics and knowledge of teaching mathematics but needs to include a discussion on teacher's beliefs. He believes teachers' actions in the classroom are strongly guided by what they believe about mathematics and the teaching thereof. He further states that it is a false dichotomy to distinguish between knowledge and beliefs, as a belief becomes knowledge once the *truth criterion is satisfied* (p. 2). Leatham (2006, p. 92) explains this argument as follows:

> *Of all the things we believe, there are some things that we 'just believe' and other things we 'more than believe – we know'. Those things we 'more than believe' we refer to as knowledge and those things we 'just believe' we refer to as beliefs. Thus beliefs and knowledge can profitably be viewed as complementary subsets of the things we believe.*

Borko and Putnam (1996) focus on two interrelated aspects of knowledge and beliefs. They argue that prospective and experienced teachers' knowledge and beliefs serve as filters through which their learning takes place and on the other hand knowledge and beliefs themselves are critical targets of change.

In student teacher training it is important that both students' mathematical knowledge and beliefs need to be developed and restructured. In my experience once a student's mathematical knowledge base is enhanced, the new or enriched knowledge influences the student's beliefs about mathematics, reorganising and broadening the student's existing belief system[10]. On the other hand when a student's beliefs about mathematics are restructured, they sometimes become more receptive to new mathematical knowledge.

2.4.2 Overview of the different domains of teachers' knowledge

The most fundamental aspect in effective and proficient teaching of mathematics is a high level of knowledge (Kilpatrick, 2001; Taylor, 2008). A teacher needs proper subject matter knowledge and a high level of PCK to assure effective teaching (Shulman, 1986; Ma, 1999). In Taylor's (2008) study short tests in literacy and mathematics amongs others were conducted in primary and secondary schools throughout South Africa and his finding was that teachers clearly do not have the knowledge that the curricula require to proficiently teach the learners. To address this problem of teachers' inadequacy, the school system has to re-establish the emphasis on expert knowledge (Taylor, 2008). Mathematics teaching is a specialised profession, requiring content knowledge, knowledge of the curriculum, knowledge about how to teach mathematics and knowledge about how learners learn mathematics. The question is how these different categories of mathematical knowledge are organised. Some of the leading mathematics researchers' categories or domains of mathematical knowledge are given in Table 2.7 below with a brief summary of each.

[10] Beliefs systems are discussed in Section 2.4.3.2.

Table 2.7: Overview of different domains of mathematical knowledge

OVERVIEW OF DIFFERENT DOMAINS OF MATHEMATICAL KNOWLEDGE				
SHULMAN Categories of knowledge 1986	**GROSSMAN Components of PCK** 1990	**BORKO AND PUTNAM Domains of knowledge** 1996	**BALL, THAMES AND PHELPS Domains of knowledge of teaching** 2005	**HILL, BALL AND SCHILLING Domain map for mathematical knowledge for teaching** 2008
1. Subject matter content knowledge	1. Purposes for teaching mathematics	1. General pedagogical knowledge and beliefs	1. Subject matter knowledge • Common knowledge of mathematics content • Specialised knowledge of mathematics content	1. Subject matter knowledge • Common content knowledge • Specialised content knowledge • Knowledge at the mathematical horizon
2. PCK	2. Learners' understanding, conceptions and misunderstandings	2. Subject matter knowledge and beliefs		
3. Curricular knowledge	3. Curriculum and curricular materials	3. PCK and beliefs	2. PCK • Knowledge of content and students • Knowledge of content and teaching	2. PCK • Knowledge of content and students • Knowledge of content and teaching • Knowledge of curriculum
	4. Instructional strategies and representations for teaching topics			

2.4.2.1 Shulman's (1986) categories of content knowledge

Shulman (1986) initiated the debate on different categories of knowledge a mathematics teacher needs. Figure 2.4 indicates his three categories of content knowledge, namely 1) subject matter content knowledge; 2) PCK; and 3) curricular knowledge.

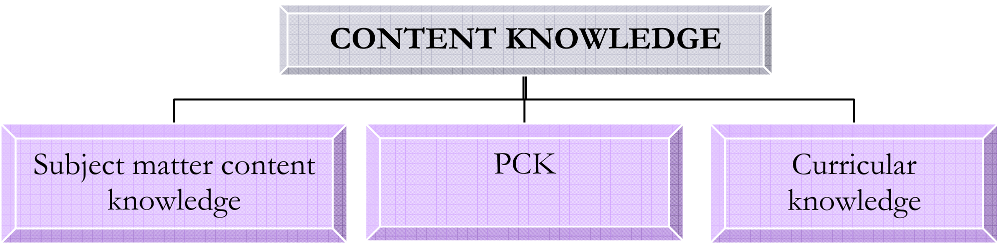

Figure 2.4: Shulman's (1986) three categories of content knowledge

Subject matter content knowledge as one of the categories of content knowledge goes beyond knowledge of the facts or concepts of a domain to understand the structures of the subject matter. The second category **PCK** refers to pedagogical knowledge that goes beyond subject matter knowledge to subject matter knowledge for teaching, also called teachers' professional knowledge. This knowledge includes *the most useful forms of representation of ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that makes it comprehensible to others* (Shulman, 1986, p. 9). The ability to use different representations may be derived from research or from years of experience in practice. This knowledge further includes a teacher's understanding of why certain topics are comprehensible and others not, and what preconceptions learners have that may be misconceptions that could actually be rectified and reorganised by the teacher through the use of different strategies. Shulman (1987) further describes PCK as *the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students* (p. 15), in other words, the knowledge of how to make the subject comprehensible to others. The third category of knowledge, **curricular knowledge**, refers to the knowledge about the full range of programmes designed for the teaching of different topics at given levels in a subject area. It further includes knowledge regarding the variety of instructional materials available to teach particular curriculum components. It is imperative for teachers to be familiar with the topics and their levels being taught in the same subject during the preceding and subsequent years in school. Teachers also need to be familiar with the curriculum materials studied by learners in other subjects at the same time (Shulman, 1986). Whereas Shulman's work is foundational in this area, other mathematics researchers' categorisations of mathematical knowledge needed for teaching are discussed below.

2.4.2.2 Grossman's (1990) components of PCK

Grossman (as cited in Sowder, 2007) (Figure 2.5) distinguishes between four components of PCK, namely 1) purposes for teaching mathematics; 2) learners' understandings, conceptions and potential misunderstandings; 3) curriculum and curricular materials; and 4) instructional strategies and representations for teaching particular topics.

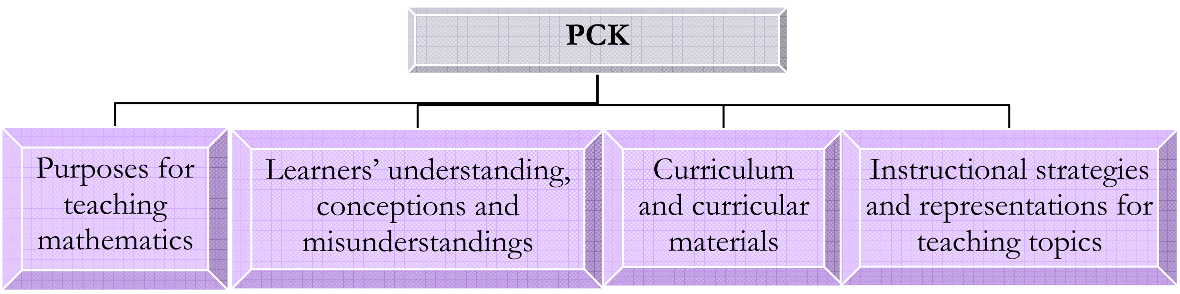

Figure 2.5: Grossman's (1990) four components of PCK

Borko and Putnam (1996) believe the **first component** serves as a conceptual map for the teacher's instructional decision-making, and as a basis for making decisions regarding classroom objectives, instructional strategies, student assignments, textbooks, curricular materials and the evaluation of student learning. This is a salient component of the professional knowledge base of teachers as it concerns teachers' knowledge about the nature of the subject and what is important for students to learn. The **second component** is knowledge a proficient teacher has to predict what mathematics learners will understand, how they will understand it, and what their potential misunderstandings will be. This knowledge enables a teacher to (Sowder, 2007):

> … *plan more effectively because they can anticipate learners' difficulties. They know what prior knowledge must be present to understand something new. They know how to scaffold knowledge to assist students in developing understanding. They know how to listen to students. Much of this knowledge comes from practice, but teachers with poor understanding of mathematics are unlikely to develop this type of knowledge, particularly when the mathematics in the curriculum becomes more sophisticated* (p. 165).

Teachers need to have an understanding of learners' preconceptions, misconceptions, and alternative conceptions of specific topics (Borko & Putnam, 1996). The **third component** includes the ability of teachers to recognise the particular strengths and weaknesses of textbooks and materials they use. Competent teachers normally have a collection of materials they use when teaching mathematics. This component also includes knowledge of how the topics are organised and structured both horizontally and vertically, i.e. within a grade level and across grades. The **fourth component** is characterised as a wide selection of significant representations and the ability to adapt these representations in various ways in order to meet specific goals for specific learners (Borko & Putnam, 1996).

2.4.2.3 Borko and Putnam's (1996) domains of knowledge

The framework (Figure 2.6) used by Borko and Putnam (1996) in their study "Learning to teach" was loosely based on Shulman's categories of knowledge. The proposed domains are 1) general pedagogical knowledge and beliefs; 2) subject matter knowledge and beliefs; and 3) PCK and beliefs. These three domains encompass teachers' knowledge of teaching, subject matter and learners, the three major determinants of what teachers do in their classrooms.

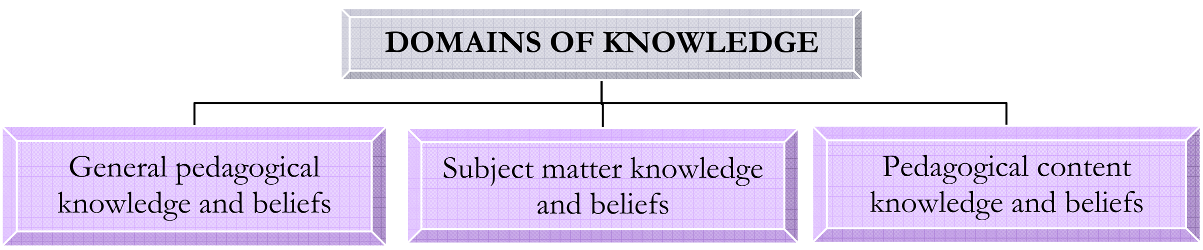

Figure 2.6: Borko and Putnam's (1996) three domains of knowledge

The **first domain** does not form part of this study's focus but refers to a teacher's knowledge and beliefs about teaching and learning in general, which include knowledge of strategies for effective classroom management, various instructional strategies for specific lesson topics, how to create a positive learning environment and most fundamental a thorough knowledge of learners, of how they learn and how learning can be fostered by teaching. Borko and Putnam (1996) do not prescribe a specific model or set of categories to be used regarding the **second domain** of knowledge, as long as the need to know more than just facts, terms and concepts of a discipline is recognised. Key aspects in this domain include knowledge of how to organise ideas, how to make connections among ideas and knowing different ways of thinking and argumentation. They briefly refer to the work of Shulman in 1986 as well as Ball in 1990 and 1991. Regarding their **third domain**, they again discuss the work of Shulman in 1986 and that of Grossman in 1990 emphasising the importance of PCK for teachers who want to teach for understanding.

2.4.2.4 Ball, Thames and Phelps' (2005) domains of knowledge for teaching

According to Silverman and Thompson (2008), Shulman invented the term "PCK" referring to specific content knowledge as applied to teaching. Since then many researchers within the field of mathematics teacher education have been developing this notion with special reference to the work of Ball in 1990 as well as Ball and Bass in 2000. Ball et al. (2005) use the term "knowledge of mathematics for teaching" when referring to the special knowledge needed to teach mathematics for understanding. *Their pioneering work has succeeded in identifying various examples of special ways in which one must know mathematical procedures and representations to interact productively with students in the context of teaching* (Thompson, 1992, p. 500). Ball et al. (2005) divide knowledge of mathematics for teaching in two domains, namely 1) subject matter knowledge and 2) PCK where each domain is divided in two sub-domains as indicated in Figure 2.7 below.

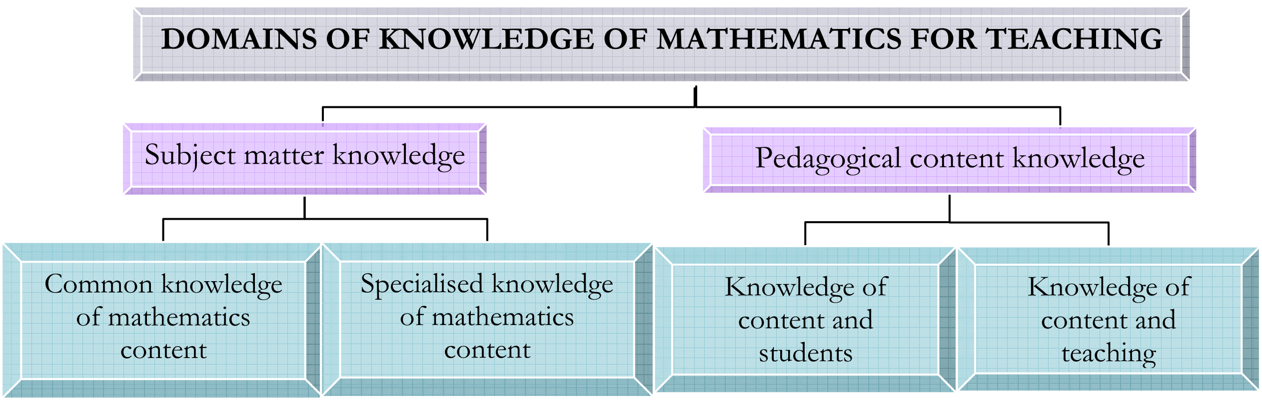

Figure 2.7: Ball, Thames and Phelps' (2005) domains of knowledge of mathematics for teaching

2.4.2.5 Hill, Ball and Schilling's (2008) domain map for mathematical knowledge for teaching

This overview concludes with the domain map for mathematical knowledge for teaching of Hill et al. (2008) as indicated in Figure 2.8. Similar to Ball et al. (2005), they also divide knowledge into two domains, namely 1) subject matter knowledge and 2) PCK, but included an additional subdomain under each domain. **Subject matter knowledge** now consists of 1) common content knowledge; 2) specialised content knowledge and 3) knowledge at the mathematical horizon. Common content knowledge involves knowing central facts, concepts and principles within a relationship while specialised content knowledge goes beyond common content knowledge. Teachers need to have specialised knowledge to know more than just explaining the content, but must be able to explain why it is so, why it is worth knowing and how to relate it to other learning outcomes and other disciplines, both in theory and practice. Knowledge at the mathematical horizon refers to having knowledge of the subject beyond the years for which a teacher is responsible for. **PCK** is now divided into 1) knowledge of content and students; 2) knowledge of content and teaching and 3) knowledge of the curriculum.

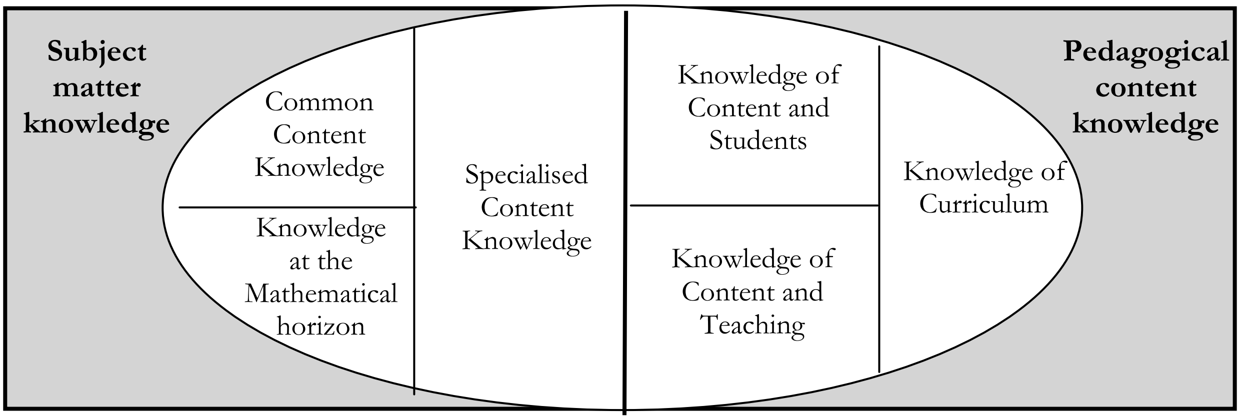

Figure 2.8: Hill, Ball and Schilling's (2008) domain map for mathematical knowledge for teaching

My study is based on the PCK domain and is discussed under the heading "Conceptual framework"[11]. Although the focus of this study is not on ML teachers' subject matter knowledge, the value of teachers having a deep knowledge base is still recognised as part of their complete cognitive knowledge base.

2.4.2.6 Summary

Borko and Putnam (1996) argue that the increased attention to teachers' knowledge in recent years has led to multiple schemes for categorising teachers' mathematical knowledge. One must bear in mind that any categorisation of teachers' knowledge is somewhat arbitrary, that there is no single definite system of categorisation of knowledge and that the boundaries between these categorisations are very vague

[11] The conceptual framework is discussed in Section 2.5.

(Borko & Putnam, 1996; Hill et al., 2008; Shulman, 1986; Sowder, 2007). Although knowledge is categorised in different domains, these domains are interwoven in teachers' instructional practices and teachers continually draw on all aspects of their knowledge (Koellner et al., 2007).

2.4.3 An overview of mathematics teachers' beliefs about mathematics and the teaching thereof

Thompson (1992) emphasises the complexity involved in distinguishing between beliefs and knowledge and found that in many cases teachers treat their beliefs as knowledge. There is no agreement on how beliefs are to be evaluated, as beliefs cannot be directly observed or measured, but must be inferred from what people say, intend and do (Pajares, 1992; Thompson, 1992). In this section I discuss the nature of beliefs and what is meant by teachers' belief systems.

2.4.3.1 The nature of beliefs

According to Pajares (1992, p. 316) beliefs are formed *through a process of enculturation*[12] *and social construction* and influence a person's perceptions, behaviour and the processing of new information. He suggests that beliefs created by individuals years ago are fixed and difficult to change whereas newly formed beliefs are most vulnerable. Listed below are some of the inferences and generalisations Pajares made regarding teachers' educational beliefs:

- *Beliefs are formed early and tend to self-perpetuate, persevering even against contradictions caused by reason, time, schooling, or experience.*
- *The belief system has an adaptive function in helping individuals define and understand the world and themselves.*
- *Epistemological beliefs play a key role in knowledge interpretation and cognitive monitoring.*
- *Beliefs are prioritised according to their connections or relationship to other beliefs or other cognitive and affective structures. Apparent inconsistencies may be explained by exploring the functional connections and centrality of the beliefs.*
- *By their very nature and origin, some beliefs are more incontrovertible than others.*
- *Belief change during adulthood is a relatively rare phenomenon, the most common cause being a conversion from one authority to another or a gestalt shift. Individuals tend to hold on to beliefs based on incorrect or incomplete knowledge, even after scientifically correct explanations are presented to them.*
- *Beliefs must be inferred, and this inference must take into account the congruence among individuals' belief statements, the intentionality to behave in a predisposed manner, and the behaviour related to the belief in question.*
- *Beliefs about teaching are well established by the time a student gets to college* (p. 324-326).

In Table 2.8 below, Schoenfeld (1988, p. 151) mentions four general beliefs held by learners and their effects in practice.

[12] Enculturation involves the incidental learning process individuals undergo through their lives and includes their assimilation through individual observation, participation and imitation (Pajares, 1992).

Table 2.8: Some beliefs held by learners and their effects in practice

BELIEF	EFFECTS IN PRACTICE
The processes of formal mathematics (e.g. 'proof') have little or nothing to do with discovery or invention.	*Students fail to use information from formal mathematics when they are in 'problem solving mode'.*
Students who understand the subject matter can solve assigned mathematics problems in five minutes or less.	*Students stop working on a problem after just a few minutes since, if they haven't solved it, they didn't understand the material (and therefore will not solve it).*
Only geniuses are capable of discovering, creating, or really understanding mathematics.	*Mathematics is studied passively, with students accepting what is passed down 'from above' without the expectation that they can make sense of it for themselves.*
One succeeds in school by performing the tasks, to the letter, as described by the teacher.	*Learning is an incidental by-product to 'getting the work done'.*

Learners cannot be blamed for holding such beliefs as many teachers and parents also hold these or similar beliefs. Beliefs play an important role in how people view ML. This is an influential factor in the success of ML as some teachers and learners have their view of ML influenced by the comments from people outside the mathematics field, and thus see ML as a worthless and insignificant subject.

2.4.3.2 Teachers' belief systems

Leatham (2006) argues that the way an individual's various beliefs are related to each other is just as important as what the individual believes. A belief system according to Thompson (1992) consists of conscious and subconscious beliefs, preferences concerning mathematics as discipline, concepts, meaning, rules, and mental images. In Thompson's (1992) study on teachers' conceptions consisting of beliefs, views and preferences, she points out that some people's actions and behaviours are influenced by the nature of their beliefs. She further describes a belief system as *a metaphor for examining and describing how an individual's beliefs are organised* (p. 130). She also typifies such systems as being dynamic in nature because they undergo change and restructuring as individuals evaluate their beliefs against their experience. According to Ball (1988) and Thompson (1992) preservice teachers' beliefs are formed through the development of a network of interrelated ideas about mathematics, the teaching and learning thereof and also through their experiences at schools.

2.4.4 The influence of teachers' knowledge and beliefs on their instructional practices

Pajares (1992) acknowledges the complexity of a psychological construct such as beliefs, but through his extensive study of numerous researchers' findings, he found *a strong relationship between teachers' educational beliefs and their planning, instructional decisions, and classroom practices* (p. 326), although the link to learner outcomes has not been explored extensively. Artzt et al. (2008) refer to teachers' goals, knowledge and beliefs as teachers' cognitions and describe them as *the driving forces* (p. 17) behind teachers' instructional practices. In this section I mention the influence of mathematics teachers' knowledge and beliefs on their learners as well as their teaching of the subject. I also report some findings from South African studies regarding the influence of ML teachers' knowledge and beliefs on their instructional practices.

2.4.4.1 The influence of teachers' knowledge and beliefs on the learners

Learners' beliefs were for the most part consistent with the beliefs and views held by their teachers (Thompson, 1992; Ford, 1994). Ford refers to a study he conducted in which teachers regard good problem solvers as the smarter learners. He found that this belief was then adopted by learners who claim that you need to be smart to be able to solve problems. This finding is supported by Mason's (2003) study in which learners with low achievement comment that in mathematics, intelligence counts 90% and effort 10% and the intelligence a person is born with, can be exploited but not improved, so a person either can or cannot do mathematics.

Teachers need to accept and acknowledge their responsibility towards learners and need to provide learners with opportunities for positive learning experiences. The teacher's attitude towards the subject is also significant. The teacher has the responsibility to ensure that mathematics comes alive, that learners find it constructive and develop a passion for the subject. Ollerton (2009) argues that teachers cannot force learners to have a positive relationship with their subject but they need to realise that they have a *massive impact* (p. 2) on their learners. The teacher has the knowledge and skills to create a positive learning atmosphere where sufficient opportunities are provided to build this relationship. In order to do this, teachers need a positive attitude towards the subject and its learners.

2.4.4.2 The influence of teachers' knowledge and beliefs on their teaching

In practice teachers spontaneously convey their ideas on mathematics to their learners (Ball, 1991). Teachers' beliefs about mathematics and the teaching thereof often serve as a foundation on which their instructional practices are built (Liljedahl, 2008; Pajares, 1992). Liljedahl mentions four researchers' notions of teachers' beliefs which in principle are very similar, each notion consisting of three different perspectives. Dionne's (1984) notion is divided into the traditional, formalist and constructivist perspective. Ernest's (1988) notion describes three philosophies of mathematics, namely instrumentalist, Platonist and problem solving while Törner and Grigutsch (1994) name their three perspectives the toolbox aspect, system aspect and process aspect, which are described as follows:

> *In the toolbox aspect mathematics is seen as a set of rules, formulae, skills and procedures while mathematical activity means calculating as well as using rules, procedures and formulae. The system aspect refers to teachers who*

believe mathematics is characterised by logic, rigorous proofs, exact definitions and a precise mathematical language and doing mathematics consists of accurate proofs as well as of the use of a precise and rigorous language. The process aspect refers to teachers who believe mathematics is a constructive process where relations between different notions and sentences play an important role. Here the mathematical activity involves creative steps, such as generating rules and formulae, thereby inventing or re-inventing the mathematics. (Liljedahl, 2008, p. 2-3)

Beliefs regarding the nature of mathematics influence a teacher's choice of teaching approach. Teachers holding a traditional belief most probably believe that mathematics is an abstract phenomenon that is far distant from reality. These teachers will then struggle to relate mathematics to real-life situations and tend to believe mathematics consists of a set of rules and procedures that must be learned mechanically with little or no connection to each other and hardly any relevance to their everyday lives. They also tend to separate mathematics from the discipline of discovery and creativity (White & Mitchelmore, 2002; Mason, 2003; Schoenfeld, 1988). Thom (as cited in Golafshani, 2002), also claims that *all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics* (p. 204). He comments on disparities that do occur where teachers' conceptions are not reflected in their instructional practices due to constraints such as fixed curricula, time pressure and other external factors.

Findings regarding the influence of ML teachers' knowledge and beliefs on their teaching of ML

Sidiropoulos (2008) found that ML teachers' instructional practices were neither aligned to the ML curriculum nor to their alleged beliefs and understanding. She states that external strategies and interventions that promote the required depth of ML teachers' understanding are required to change their instructional practices. She further found that the negative and low expectations those teachers have of their learners negatively affected their implementation of the curriculum in class. One of the two teachers believed that everyone could do ML if taught properly, but when that teacher was asked about his learners' poor performance, the blame was put on learners' past history with mathematics. Mhlolo (2008) believes that ML teachers were not equipped with conceptual skills required for the implementation of the subject and that they need to re-conceptualise their knowledge and beliefs about the subject. He further states that there is a problematic relationship between the idealised teacher in policy documents and teachers' personal identities. He calls it a mismatch, dislocation or disjuncture between espoused policy images and the personal identities of teachers. Although there are many ML teachers who do not meet the requirements as set out by the DoE, there are research studies such as those of Venkat and Graven (2007), telling stories of successful ML teachers.

Venkat and Graven (2007) report on their longitudinal study performed at an inner city school in Johannesburg on the difference positive and knowledgeable ML teachers make to learners' experience of the subject. They suggest learner negativity is associated with a lack of substantive change in teachers' pedagogic practice, that is where teachers still *incorporate the kinds of tasks and pedagogic practice that have predominated within learners' earlier experiences with Mathematics* (p. 81).

2.4.5 Summary

Liljedahl (2008) strongly believes that any discussion on a teacher's knowledge cannot be restricted to knowledge of mathematics and knowledge of teaching mathematics, but needs to include a discussion on teacher's beliefs. Different categories or domains of mathematical knowledge exist but any categorisation of teachers' knowledge is somewhat arbitrary as there is no single true system of categories and the boundaries between these categorisations are usually very vague (Borko & Putnam, 1996; Hill et al., 2008; Shulman, 1986; Sowder, 2007). The different categories of a teacher's knowledge are also interwoven in their instructional practices and teachers continually draw on all aspects of their knowledge (Koellner et al., 2007).

Beliefs consist of conscious and subconscious beliefs as well as preferences concerning mathematics as a discipline. Beliefs can further be defined as *convictions or opinions that are formed either by experience or by the intervention of ideas through the learning process* (Ford, 1994, p. 315). Teachers' beliefs about mathematics can be located on a perspective continuum from a traditional to a formalist, to a constructivist perspective (Dionne 1984).

Knowledge and beliefs are closely related and there is a constant interplay between the two, both influencing teachers' instructional practices. Borko and Putnam (1996) argue that on the one hand prospective and experienced teachers' knowledge and beliefs serve as filters through which their learning takes place and on the other hand knowledge and beliefs themselves are critical targets of change.

2.5 Conceptual framework

The focus of my study is to determine the relationship between ML teachers' knowledge and beliefs and their instructional practices. My conceptual framework (Figure 2.9) is based on an amalgamation of Artzt et al.'s (2008) phase dimension framework[13], Franke et al.'s (2007) view of a productive practice and Hill et al.'s (2008) domain map for mathematical knowledge for teaching[14].

13 See Section 2.3: Teachers' instructional practices.
14 See Section 2.4.2.5: Hill, Ball and Schilling's (2008) domain map for mathematical knowledge for teaching.

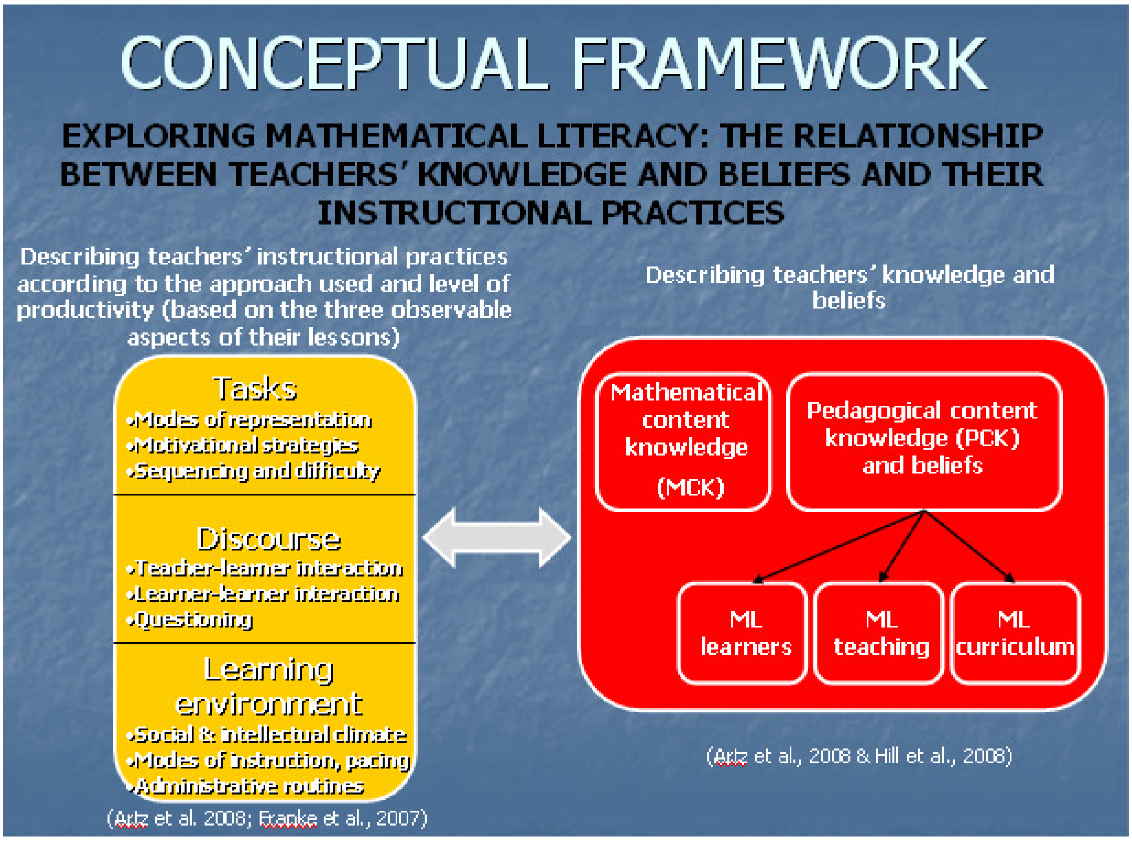

Figure 2.9: Conceptual framework: Instructional practice, knowledge and beliefs framework of analysis (adapted from Artzt et al., 2008; Franke et al., 2007; Hill et al., 2008)

Explanation of my conceptual framework

I believe teachers need to apply appropriate instructional strategies to provide learners with opportunities to develop their critical thinking and problem solving skills. Figure 2.9 illustrates the components of, and logic behind my framework. To enable me to determine the relationship between ML teachers' instructional practices and their knowledge and beliefs, their practices can be observed in terms of **tasks**, **discourse** and the **learning environment**[15]. From these observations the ML teachers' instructional practices can be described according to the instructional approach used and level of productivity of their practices. Teacher's instructional approaches will be described as either teacher-centred, learner-centred or a combination of teacher- and learner-centred (Artzt et al., 2008). The level of productivity of the teachers' instructional practices will be described based on Franke et al.'s (2007) view of a productive practice: A practice where the teacher listens to learners' mathematical thinking and aims to use it to encourage conversation that revolves around the mathematical ideas in the sequenced problems. Subsequently I will deal with some of the driving forces behind their lessons, namely teachers' knowledge and beliefs concerning content and learners[16], content and teaching[17] and

[15] See Section 2.5.5: Teachers' instructional practices.
[16] See Section 2.5.2: PCK and beliefs regarding content and learners.

the curriculum[18]. The three segments of PCK and beliefs, namely knowledge and beliefs of the ML learners, ML teaching and the ML curriculum, are strongly influenced by teachers' idiosyncratic beliefs about the nature of mathematics as a discipline and ML as subject. These idiosyncratic beliefs can typically be located on a perspective continuum from traditional to formalist to constructivist[19].

Included in my framework is Hill et al.'s (2008) PCK domain (learners, teaching and curriculum in Figure 2.9) and teachers' MCK which is similar to Hill et al.'s (2008) common content knowledge as part of their subject matter knowledge domain. The rationale for this decision is that ML focuses on solving contextualised problems using only basic mathematics. Notwithstanding the fact that ML teachers need to have MCK, the focus of my study is not on the assessment of their subject matter knowledge per se. PCK is defined by Hill et al. (2008) as teachers' *content knowledge intertwined with* (p. 375) knowledge of students; knowledge of teaching; and knowledge of the curriculum. Teachers' beliefs are integrated in my framework as I believe they are inseparable from teachers' knowledge. Incidentally, I excluded teachers' goals as part of teachers' cognitions in order to keep the study focused (Artzt et al., 2008).

Even though some of the headings in this section may come across as repetitive, the previous two sections focussed on some general views and background from the literature regarding mathematical knowledge, beliefs and instructional practices. In this section I relate the literature to my study concerning 1) a general view on mathematics teachers' PCK and beliefs; 2) PCK and beliefs regarding the learners, teaching and the curriculum; and 3) instructional practices.

2.5.1 General view on mathematics teachers' knowledge and beliefs

2.5.1.1 Mathematics teachers' MCK

Hill et al.'s (2008), domain map for mathematical knowledge for teaching (Figure 2.8) is used in an attempt to *conceptualise and develop measures of teachers' combined knowledge of content and students* (p. 372). Mathematical knowledge for teaching is divided into two domains, namely subject matter knowledge and PCK. The subject matter knowledge category consists of three strands, namely common content knowledge, specialised content knowledge, and knowledge at the horizon. For the purpose of this study I base ML teachers' MCK on common content knowledge that can be defined as a basic understanding

17 See Section 2.5.3: PCK and beliefs regarding content and teaching.
18 See Section 2.5.4: PCK and beliefs regarding curriculum.
19 See Section 2.5.1.2: Mathematics teachers' beliefs.

of mathematical skills, procedures, and concepts acquired by any well-educated adult enabling a teacher to solve mathematical problems in the prescribed curriculum (Ball et al., 2005).

2.5.1.2 Mathematics teachers' PCK

The conceptual knowledge demanded of teachers to teach school mathematics is different from the mathematical knowledge mathematicians[20] might have of advanced topics (Ball, 1990; Leinhardt et al., 1991). Dewey (1902) also addressed this issue when he wrote:

> *Every study or subject thus has two aspects: one for the scientist as a scientist; the other for the teacher as a teacher. … For the scientist, the subject matter represents simply a given body of truth to be employed in locating new problems, instituting new researches, and carrying them through to a verified outcome… The problem of the teacher is a different one… What concerns him as teacher is the ways in which that subject may become part of experience, what there is in the child's present that is usable with reference to it; how such elements are to be used; how his own knowledge of the subject-matter may assist in interpreting the child's needs and doings, and determine the medium in which the child should be placed in order that his growth may be properly directed. He is concerned, not with the subject-matter as such, but with the subject-matter as a related factor in a total and growing experience* (p. 162-163).

PCK is regarded as knowledge that is unique to teachers; knowledge that can only be developed over time through experience in the classroom or practice and can therefore not be taught (Ball, 1988; Ball et al., 2005; Koellner et al., 2007; Ma, 1999; Shulman, 1986; Sowder, 2007). Having profound understanding and knowledge of mathematical subject matter is a prerequisite to develop PCK (Ball, 1990; Van Driel, Verloop & De Vos, 1998). Sowder (2007) is of the opinion that it is only as mathematics increases in sophistication, that a deep content knowledge base becomes a prerequisite in developing PCK. Although ML teachers need to have mathematical content knowledge, the DoE (2003a) stated that the content in ML must not be an end in itself, but must serve the learning outcome of applying content to certain contexts. Since the emphasis in ML is on solving real-life contextualised problems using **basic** mathematics, it is debatable whether a high level of subject matter knowledge is required by the ML teacher. My belief is nevertheless that ML teachers do need to have conceptual knowledge of the subject matter involved in the curriculum to enable them to use their knowledge efficiently in preparing their learners for their future lives.

2.5.1.3 Mathematics teachers' beliefs

Teachers' beliefs about mathematics are powerful as they influence their representations of mathematics (Ball, 1990). She mentions a few beliefs of mathematics teachers that need to be examined such as their:

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[20] Mathematicians refer to people using higher levels of formal mathematics in their professions such as engineers and scientists.

> *understandings about the nature of mathematical knowledge and of mathematics as a field and the substance of mathematics. What counts as an answer in mathematics? What establishes the validity of an answer? What is involved in doing mathematics? What do mathematicians do? … What is the origin of some of the mathematics we use today and how does mathematics change? (p. 458) What do they think an explanation is? How do they sort out convention from logic with respect to particular principles or ideas? What do they think it means to 'know' or to 'do' mathematics?* (p. 459)

The saying 'we teach what we believe' emphasises the importance and far-reaching effects of teachers' beliefs on their instructional practice (Leatham, 2006). Ollerton (2009) believes that once teachers have articulated what their pedagogy is and obtain clarity on the beliefs and values they hold and which drive them, it will help them to strengthen effective practice. Mathematics teachers' beliefs about mathematics are located on a perspective continuum from traditional to formalist to constructivist (Dionne 1984). In my study I regard ML teachers' knowledge and beliefs as inseparable driving forces behind their instructional practices. In many cases teachers' beliefs are established by their knowledge and changing their knowledge base will change their belief system.

2.5.2 The three domains of PCK and beliefs

In this section I discuss my study's view on mathematics teachers' PCK and beliefs regarding the three domains, namely 1) content and learners; 2) content and teaching; and 3) the curriculum. In my discussion of each domain, I firstly mention some views from the literature regarding mathematics teachers in general and then discuss ML teachers' PCK and beliefs concerning that specific domain.

2.5.2.1 PCK and beliefs regarding content and learners

Knowledge of content and learners includes a teacher's ability to predict what mathematics learners will understand and how they will understand it, how learners will probably approach a task, understanding why certain topics are comprehensible and others not, what alternative conceptions and preconceptions learners have that could be misconceptions and that should be rectified and reorganised by the teacher through the use of different strategies (Ball, 1990; Borko & Putnam, 1996; Hill et al., 2008; Shulman, 1986; Sowder 2007). According to Sowder (2007), having this knowledge enables a teacher to:

> *plan more effectively because they can anticipate learners' difficulties. They know what prior knowledge must be present to understand something new. They know how to listen to students. Much of this knowledge comes from practice, but teachers who have poor understanding of mathematics themselves are unlikely to develop this type of knowledge, particularly when the mathematics in the curriculum becomes more sophisticated* … (p. 165)

This knowledge of content and learners should be taken into account when planning lessons (Koellner et al., 2007). Teachers must be able to **see** what learners do, **hear** what they think and then be able to **act** appropriately as mentors to facilitate the learning process (Hill et al., 2008).

ML teachers' knowledge and beliefs regarding their learners

Capturing learners' attention is particularly significant in the ML classrooms as many ML learners lack motivation, have negative attitudes and experience anxiety, causing teachers to be discouraged in teaching this subject (Mbekwa, 2007; Venkat, 2007; Venkat & Graven, 2007; Vermeulen, 2007). A shortcoming in the implementation process of ML is that teachers were not empowered to *deal with and assist learners with a past history of low attainment in mathematics* (Sidiropoulos, 2008, p. 250). To meet these challenges teachers need a firm knowledge base of the purpose and goal of the subject, its content, the teaching thereof, its learners and classroom management skills. Some teachers, for instance, only listen for correct answers and do not use incorrect answers to engage learners in mathematical thinking. Hill et al. (2007) emphasise the necessity for teachers to rephrase learners' questions to help them unravel the problem themselves.

2.5.2.2 Knowledge and beliefs regarding content and teaching

Knowledge regarding content and teaching includes *the most useful forms of representation of ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others* (Shulman, 1986, p. 9). He further describes this knowledge as *the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students* (Shulman, 1987, p. 15).

Teachers should have the ability to recognise the instructional advantages and constraints of using and adapting various representations depending on the content and needs of the learners and also have the ability to sequence content to facilitate student learning (Ball, 1990; Borko & Putnam, 1996; Koellner et al., 2007). Teachers furthermore need to be able to present subject matter in multiple ways like using story problems, pictures, situations and concrete materials. This knowledge is required to choose the appropriate pedagogical strategy and instructional material for a lesson, to consider which tasks to set and which assessment techniques to use. Knowledge of content and teaching further assists teachers to reflect on their own practice for the purpose of improvement (Koellner et al., 2007). Sowder (2007) feels teachers need to know how to scaffold knowledge to assist learners in developing understanding. Hill et al. (2008) concur with this view and assert that teachers need to know different ways of how to build on student mathematical thinking or how to remedy student errors.

Approaches to teaching ML

Approaches to the teaching and learning of ML should provide *extended opportunities to engage with ML in diverse contexts at a level that learners can access logically* (DoE, 2003c, p. 5). The teaching of mathematics in a contextualised and de-compartmentalised way however complicates the teaching of ML as some teachers lack the knowledge and skills to do so. Sidiropoulos (2008) found that teachers use ML textbooks where content is embedded in context, but *predominantly deliver the algorithmic content to the learners*, and afterwards *dress it up* with an artificial level of context, maybe using a picture (p. 227). Principles that guided Frith and Prince's (2006) curriculum in preparing in-service teachers to teach ML are the following:

- *That material should be context-based and make use of real relevant intrinsically motivating contexts, wherever possible.*
- *That curriculum tasks should require the exercise of several related competencies, such as writing and using computers, not just mathematical skills.*
- *That the production of a (mainly verbal) product as an outcome of mathematically literate practice is important (as well as the understanding and interpretation of existing information).*
- *That students' confidence should be promoted.*
- *That co-operative learning should be emphasised* (p. 55).

ML teachers' knowledge of different teaching approaches

Sidiropoulos (2008) believes ML teachers' PCK regarding different teaching strategies or approaches is inadequate. The success of ML depends largely on the skills of the teachers to apply appropriate teaching approaches such as discussions and problem solving (Brown & Schäfer, 2006; Venkat, 2007). Venkat (2007) reports that learners became positive about ML, enjoying the subject and finding it practical, useful and challenging when teachers changed the nature of tasks and interactions they used in the ML classroom. *Both these shifts provided openings for learners to communicate and participate in classroom activities, in addition to gaining understandings and make sense of the mathematics being used* (Venkat, 2007, p. 30). They enjoyed being active and focused and coming up with solutions to everyday problems, even sharing them with their parents at home. Vithal (2006) proposes project work as an approach since the purpose of project work is to improve effective participation and to provide learners with the opportunity to 'read the world' using mathematics, to develop mathematical power and to change their orientation towards mathematics. Project work is based on six conceptual principles, namely problem orientation, participant-directed, inter-disciplinarily, exemplarity, assessment, and practical organisation (Venkat, 2007). The notion of ML being inter-disciplinary implies that teachers from different disciplines should work together, drawing on their different disciplines to solve various problems.

ML teachers' beliefs regarding the teaching of ML

Learners' positive experiences normally stem from situations in which the teacher has a positive attitude, believes in the subject and uses approaches applicable to the requirements of the subject. Mathematics teachers hold a strong belief that teaching ML is a *major threat to their Mathematics teacher* *status-identity* (Sidiropoulos, 2008, p. 251). Labels teachers put on ML are *lesser maths; it is not real maths; it is the beginning of maths; it is a maths only better than nothing; it is the maths of oranges and bananas; it is a subject for the doffies [dim ones]* (p. 225). Even the learners and broader community *held a similar impoverished view* (p. 222) of teachers who teach ML as learners directly asked ML teachers if they are not as bright as the other teachers or if they are being punished for something they did wrong. Unless teachers undergo appropriate development programmes to seek a change in their behaviour, PCK and beliefs about the nature and value of ML, they will continue to fall back on *knowledge and beliefs already entrenched in their instructional practice* (p. 205-206). *The reality is that deep change is even more difficult to attain on an emotional level* (p. 225), is complex and also personal as *new teacher identities will require time to develop and unfold even under optimal conditions of reform* (p. 205). She further found that the ML teachers in her study do not want to change and they do not want to lower their status in society as Mathematics teachers. She is of the opinion that the best way to solve this problem is to recruit new ML teachers who do not need to undergo change in status-identity instead of trying to change the *qualified and experienced mathematics educators* (p. 226).

2.5.2.3 Knowledge and beliefs regarding the curriculum

Curricular knowledge refers to the knowledge of the full range of programmes designed for the teaching of different topics at given levels in a subject area. Teachers need to be familiar with the topics and level thereof being taught in the same subject during the preceding and later years in school, in other words how topics are organised horizontally and vertically. Curricular knowledge further includes knowledge regarding the variety of instructional materials available to teach particular curriculum components. Teachers need to recognise the particular strengths and weaknesses of textbooks and materials they are using. Competent teachers normally have a collection of materials they use when teaching mathematics. They also need to be familiar with the curriculum materials studied by learners in other subjects at the same time (Borko & Putnam, 1996; Shulman, 1986).

ML teachers and the curriculum

ML teachers need to be informed not only about the ML subject curriculum but all relevant departmental documents in order to understand what is expected of them to teach this relatively new subject. For example the DoE (2006) provided a list of resources needed to teach ML such as advertisements from the media containing contextual problems on percentage and interest rate, graphs and tables, etcetera. The new CAPS (DoE, 2011a) for ML will hopefully assist teachers concerning the issue of how to progress from one year to the next.

In Sidiropoulos' (2008) study on the implementation of the ML curriculum, she found that the purpose of the ML curriculum had not been well understood by the teachers and consequently they did not value the curriculum and the possibilities it provided for. Negative labels teachers put on ML stem from the fact that the ML curriculum, which is *distinctly different from curricula of the past was diktat on educators without due consideration on how substantial the required change would be in terms of understanding the purpose and possibilities of this new curriculum* (p. 249). She believes that if the broader purpose and value of the ML curriculum is well understood by teachers and all stakeholders, *this threat to identity may not have been as prominent as it was* (p. 225). She further found that the teachers' disjointed understandings of the ML curriculum put emphasis on the *complexity of bridging the gap between curriculum as intended and curriculum as implemented in the context of actual classrooms* (p. 225). Other problems are the fact that the curriculum assumes that all learners can be taught to become mathematically literate and that all *educators understood the concept of mathematical literacy that by its very own nature is distinctly dissimilar from that of mathematics or numeracy* (p. 250), the only known mathematical subjects taught by teachers in South Africa.

2.5.3 Teachers' instructional practices

In defining instructional practice, Englert et al. (1992) refer to the qualitative dimensions of teachers' behaviour in their practices. These dimensions involve teachers' abilities to model cognitive strategies in meaningful and purposive activities, adjust instruction as required, promote classroom dialogues, and establish classroom communities in which learners collaboratively and cooperatively participate in enquiry-related activities. A framework used to observe and describe teachers' instructional practices is built on three observable aspects of mathematics lessons, namely tasks, discourse and the learning environment (Artzt et al., 2008). The characteristics of tasks, discourse and the learning environment are provided in Table 2.9 below (Artzt et al., 2008, p. 10-12).

Table 2.9: The observable aspects of a lesson

TASKS Provide opportunities for learners to connect new knowledge to existing knowledge through active engagement in problem solving activities. Tasks should be motivational, at an appropriate level of difficulty and sequenced in a meaningful way to help learners clarifying their ideas.	
Modes of representation	Uses different representations such as symbols, diagrams, manipulatives, and computer representations to facilitate content clarity, enabling learners to connect new knowledge to prior knowledge and skills.
Motivational strategies	Uses tasks that capture learners' curiosity, inspiring them to reflect on their conjectures. The diversity of learners' interest and experiences should be taken into account.

Sequencing and difficulty levels	Sequences tasks in assisting learners to make connections between ideas and develop conceptual understanding. Uses tasks suitable to what learners already know and can do and what they need to learn.
DISCOURSE Describes the verbal exchange among members of the community in the classroom, both teachers and learners.	
Teacher-learner interaction	Communicates with learners in an accepting, non-judgmental manner, encouraging learner participation. Requires learners to explain and demonstrate their thinking while carefully listening to provide clarification.
Learner-learner interaction	Encourages learners to listen to, respond to and question one another in order to assess each other's ideas or solutions, and if necessary to rectify or adjust.
Questioning	Poses a variety of types and levels of questions and allowing enough time to elicit thinking and to follow their reasoning through.
LEARNING ENVIRONMENT Describe the conditions under which the teaching and learning process unfolds in the classroom and refer to the circumstances that affect the flow of action in the classroom. This should promote the development of learners' conceptual understanding.	
Social and intellectual climate	Establishes and maintains a positive culture with and among learners by valuing their ideas and showing respect. Enforces classroom rules to ensure positive learner behaviour.
Modes of instruction and pacing	Uses instructional strategies that encourage and support student involvement and purposefulness. Attends to time management to ensure learners have the opportunity to explore mathematical ideas and to express them.
Administrative routines	Uses effective procedures in organising and managing class activities to maximise learners' active involvement in the discourse and tasks.

Flowing from observing the teachers, their instructional practices will be described according to their instructional approaches and general level of productivity. Table 2.10 below indicates the patterns being identified in teachers' instructional practices (Artzt et al., 2008).

Table 2.10: Teacher-centred versus learner-centred instructional practices

	Teacher-centred	**Learner-centred**
Tasks	Impede learners' efforts to build on prior knowledge; unrelated to learners' interest; often too easy or too difficult; illogically sequenced.	Multiple accurate representations to facilitate content clarity; connect to learners' prior knowledge; relevant and interesting tasks; challenging and sequenced.
Discourse	Teacher judges learners' responses and resolves questions without learner input; learners give short responses, lacking explanation and justification; no interaction among learners; low-level,	Teacher has accepting attitude toward learners' ideas and encourage learners to think and reason; learners explain and justify their responses; learners listen to and respond to one another's

	leading questions are asked.	ideas; variety of levels and types of questions.
Learning environment	Tense and awkward atmosphere; superficial requests for and use of learners' input; use of strategies that discourage learner participation; pace too fast or too slow; learners uninvolved; disorder in class.	Relaxed yet businesslike atmosphere; focus on learner input; strategies focus on learner involvement; effective organising and managing of class; learners actively involved.

To describe the productivity of the teachers' instructional practices is complex as *it is consistently controversial and will remain controversial* to what constitutes good teaching (Franke et al., 2007, p. 226).

2.5.4 Summary

The conceptual framework for my study is based on the domain map for mathematical knowledge for teaching (Hill et al., 2008) and the categories of an instructional practice, namely tasks, discourse and learning environment (Artzt et al., 2008). MCK is based on the category common content knowledge as part of Hill et al's (2008) subject matter domain. PCK consists of knowledge of content and learners; knowledge of content and teaching; and knowledge of the curriculum. Knowledge of content and learners includes teachers' ability to understand and predict what learners will understand, how they will understand it, what their preconceptions are, what prior knowledge they need, what possible misconceptions and alternative conceptions they could have and why some topics are more comprehensible than others. Knowledge of content and teaching includes different pedagogical approaches, strategies and representations, use of meaningful sequencing of content and appropriate instructional material, all depending on the content and learners to make the subject comprehensible. Curriculum knowledge includes knowledge of the purpose, aim, learning outcomes and assessment criteria of the subject; the topics to be covered during the preceding, current and later years as well as the level thereof; the teaching strategies applicable to the subject; and the strengths and weaknesses of instructional materials.

2.6 Conclusion

Chapter 2 explored the international and national perspectives of mathematical literacy. Although the emphasis and terminology differ between different countries, researchers unanimously believe in the importance and value of learners' mathematical literacy skills that should be developed and enhanced (Gellert et al., 2001; Jablonka, 2003; Knoblauch, 1990; McCrone & Dossey, 2007; Queensland Government, 2007b; Skovsmose, 2007). In South Africa ML refers to both a subject and a competency whereas in other countries it is mainly the latter (Christiansen, 2007). ML may have become stigmatised as a subject having virtually no meaning when it comes to career opportunities, but a closer investigation showed that this subject has its own demands and requires specialised PCK and a positive belief system towards ML as a specialised subject. Teachers' instructional practices are strongly influenced not only by their MCK, but by their beliefs about the nature of ML as well as their PCK and beliefs about the learners, the teaching of the subject as well as the curriculum. Beliefs are powerful and many times the beliefs learners have are for the most part consistent with those of the teachers. The beliefs teachers hold regarding mathematics as discipline normally varies from a traditional perspective to a formalist perspective through to a constructivist perspective (Liljedahl, 2007). The next chapter provides a lay-out of the study's methodology.