

**EXPLORING MATHEMATICAL LITERACY: THE RELATIONSHIP
BETWEEN TEACHERS' KNOWLEDGE AND BELIEFS AND THEIR
INSTRUCTIONAL PRACTICES**

by

Johanna Jacoba Botha

A thesis submitted in fulfilment of the requirements for the degree of

Philosophiae Doctor (PhD)

Department of Science, Mathematics and Technology Education

Faculty of Education

University of Pretoria

Supervisor: Prof JG Maree

Co-supervisor: Dr GH Stols

October 2011

Acknowledgement

My deepest appreciation to the following:

The Lord, my Provider who promised me that if He brought me to it, He would bring me through it.
“The will of God will never take you where the grace of God cannot protect you”.

My supervisors: Professor Kobus Maree and Dr Gerrit Stols for all their guidance, advice and motivation.

My family: Jan for all his motivation and strong belief in my abilities, my parents, sisters and brother as well as all my friends.

My participants: ‘Monty’, ‘Alice’, ‘Denise’ and ‘Elaine’ who allowed me into their classrooms and hearts.

My colleagues: Sonja, Barbara and Ina.

My editors: Mr AK Welman, Mr Tim Steward and Mrs M Labuschagne.

Summary

South Africa is the first country in the world to offer Mathematical Literacy as a school subject. This subject was introduced in 2006 as an alternative to Mathematics in the Further Education and Training band. The purpose of this subject is to provide learners with an awareness and understanding of the role that mathematics plays in the modern world, but also with opportunities to engage in real-life problems in different contexts. A problem is the beliefs some people in and outside the classroom have regarding this subject such as teachers believing ML is the dumping ground for mathematics underperformers (Mbekwa, 2007). Another problem is the belief of some principals that any non-mathematics teacher can teach ML. In practice there is Mathematics teachers who teach ML in the same way that they teach Mathematics; non-Mathematics teachers who in many cases lack the necessary mathematical content knowledge and skills to teach ML competently; and Mathematics teachers who adapted their practices to teach ML using different approaches than those required for teaching Mathematics. Limited in-depth research has been done on the ML teachers, what they believe and what knowledge is required to teach this subject effectively and proficiently.

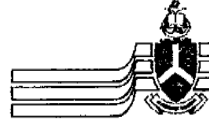
The purpose of this study is to investigate the way in which ML is taught in a limited number of classrooms with the view to exploring the relationship between ML teachers' knowledge and beliefs and their instructional practices. According to Artzt, Armour-Thomas and Curcio (2008) the instructional practice of the teacher plays out in the classroom where teachers' goals, knowledge and beliefs serve as the driving force behind their instructional efforts to guide and mentor learners in their search for knowledge. To accomplish this aim, an in-depth case study was conducted to explore the nature of teachers' knowledge and beliefs about ML as manifested in their instructional practices. A qualitative research approach was used in which observations and interviews served as data collection techniques enabling me to interpret the reality as I became part of the lives of the teachers.

My study revealed that there is a dynamic but complex relationship between ML teachers' knowledge and beliefs and their instructional practices. The teachers' knowledge, but not their stated beliefs were reflected in their instructional practices. Conversely, in one case, the teacher's instructional practice also had a positive influence on her knowledge and beliefs. It was further revealed that mathematics teacher training and teaching experience played a significant role in the productivity of the teachers' practices. The findings suggest that although mathematical content knowledge is required to develop PCK, it is teaching experience that plays a crucial role in the development of teachers' PCK.

Although the study's results cannot be generalised due to the small sample, I believe that the findings concerning the value of teachers' knowledge and the contradictions between their stated beliefs and

practices could possibly contribute to teacher training. Curriculum decision-makers should realise that the teaching of ML requires specially trained, competent, dedicated teachers who value the subject. This exploratory study concludes with recommendations for further research.

Key words: Mathematical literacy; Teachers; Learners; Curriculum; Instructional practice; Tasks; Discourse; Learning environment; Mathematical content knowledge; Pedagogical content knowledge; Beliefs.



UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
RESEARCH ETHICS COMMITTEE

CLEARANCE CERTIFICATE

CLEARANCE NUMBER :

SM 10/02/01

DEGREE AND PROJECT

PhD

INVESTIGATOR(S)

Johanna Jacoba Botha

DEPARTMENT

Science, Mathematics and Technology Education

DATE CONSIDERED

17 October 2011

DECISION OF THE COMMITTEE

APPROVED

Please note:

For Masters applications, ethical clearance is valid for 2 years

For PhD applications, ethical clearance is valid for 3 years.

**CHAIRPERSON OF ETHICS
COMMITTEE**

Prof L Ebersohn

DATE

17 October 2011

CC

Jeannie Beukes
Prof. J.G. Maree
Dr G. Stols

This ethical clearance certificate is issued subject to the following conditions:

1. A signed personal declaration of responsibility
2. If the research question changes significantly so as to alter the nature of the study, a new application for ethical clearance must be submitted
3. It remains the students' responsibility to ensure that all the necessary forms for informed consent are kept for future queries.

Please quote the clearance number in all enquiries.



UNIVERSITY OF PRETORIA

DECLARATION OF ORIGINALITY

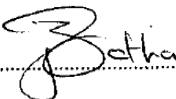
This document must be signed and submitted with every
essay, report, project, assignment, dissertation and/or thesis.


Full names of student: Johanna Jacoba Botha

Student number: 29474052

Declaration

1. I understand what plagiarism is and am aware of the University's policy in this regard.
2. I declare that this thesis (eg essay, report, project, assignment, dissertation, thesis, etc) is my own original work. Where other people's work has been used (either from a printed source, internet or any other source), this has been properly acknowledged and referenced in accordance with departmental requirements.
3. I have not used work previously produced by another student or any other person to hand in as my own.
4. I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as his or her own work.

SIGNATURE OF STUDENT: 

SIGNATURE OF SUPERVISOR: 



2.4.2.3	Borko and Putnam’s (1996) domains of knowledge.....	50
2.4.2.4	Ball, Thames and Phelps’ (2005) domains of knowledge for teaching	51
2.4.2.5	Hill, Ball and Schilling’s (2008) domain map for mathematical knowledge for teaching	52
2.4.2.6	Summary	52
2.4.3	An overview of mathematics teachers’ beliefs about mathematics and the teaching thereof	53
2.4.3.1	The nature of beliefs	53
2.4.3.2	Teachers’ belief systems	54
2.4.4	The influence of teachers’ knowledge and beliefs on their instructional practices	54
2.4.4.1	The influence of teachers’ knowledge and beliefs on the learners	55
2.4.4.2	The influence of teachers’ knowledge and beliefs on their teaching	55
2.4.5	Summary.....	57
2.5	Conceptual framework	57
2.5.1	General view on mathematics teachers’ knowledge and beliefs	59
2.5.1.1	Mathematics teachers’ MCK	59
2.5.1.2	Mathematics teachers’ PCK.....	60
2.5.1.3	Mathematics teachers’ beliefs.....	60
2.5.2	The three domains of PCK and beliefs	61
2.5.2.1	PCK and beliefs regarding content and learners	61
2.5.2.2	Knowledge and beliefs regarding content and teaching	62
2.5.2.3	Knowledge and beliefs regarding the curriculum	64
2.5.3	Teachers’ instructional practices	65
2.5.4	Summary.....	67
2.6	Conclusion.....	67
Chapter 3	69
Methodology	69
3.1	Introduction	69
3.2	Research paradigm and assumptions	69
3.2.1	Research paradigm.....	69
3.2.2	Paradigmatic assumptions	70
3.3	Research approach and design	71
3.3.1	Research approach	72
3.3.2	Research design	72
3.4	Research site and sampling	73
3.5	Data collection techniques	73
3.5.1	Observations	74
3.5.2	Interviews	76
3.6	Data analysis strategies	77
3.7	Quality assurance criteria	80
3.7.1	Trustworthiness of the study.....	80
3.7.2	Validity and reliability of the study	81
3.7.2.1	The Hawthorne effect.....	81
3.7.2.2	The Halo effect.....	82
3.8	Ethical considerations	82
3.9	Conclusion.....	83
Chapter 4	85
Presentation and discussion of the findings	85
4.1	Introduction	85
4.2	The data collection process	86

4.3	Data analysis strategies	87
4.3.1	Transcribing the data	87
4.3.2	Coding of the data.....	88
4.3.2.1	Theme 1: ML teachers’ instructional practices.....	88
4.3.2.2	Theme 2: ML teachers’ knowledge and beliefs	94
4.3.2.3	Inclusion criteria for coding the data	100
4.3.2.4	Exclusion criteria for coding the data	100
4.4	Information regarding the four participants	101
4.4.1	Monty.....	101
4.4.2	Alice.....	102
4.4.3	Denise	102
4.4.4	Elaine	103
4.5	Theme 1: The ML teachers’ instructional practices.....	104
4.5.1	Monty’s instructional practice	104
4.5.1.1	Tasks	104
4.5.1.2	Discourse.....	107
4.5.1.3	Learning environment	109
4.5.2	Alice’s instructional practice	112
4.5.2.1	Tasks	112
4.5.2.2	Discourse.....	118
4.5.2.3	Learning environment	120
4.5.3	Denise’s instructional practice.....	123
4.5.3.1	Tasks	123
4.5.3.2	Discourse.....	126
4.5.3.3	Learning environment	129
4.5.4	Elaine’s instructional practice.....	131
4.5.4.1	Tasks	131
4.5.4.2	Discourse.....	136
4.5.4.3	Learning environment	138
4.5.5	Summary of participants’ instructional practices	141
4.5.6	Discussion of Theme 1: ML teachers’ instructional practices	146
4.5.6.1	Tasks	146
4.5.6.2	Discourse.....	149
4.5.6.3	Learning environment	151
4.5.6.4	Summary of discussion on Theme 1	153
4.6	Theme 2: ML teachers’ knowledge and beliefs	154
4.6.1	Monty’s knowledge and beliefs.....	154
4.6.1.1	Mathematical content knowledge (MCK).....	154
4.6.1.2	Knowledge and beliefs regarding ML learners.....	155
4.6.1.3	Knowledge and beliefs regarding ML teaching.....	156
4.6.1.4	Knowledge and beliefs regarding ML curriculum	158
4.6.2	Alice’s knowledge and beliefs.....	160
4.6.2.1	Mathematical content knowledge (MCK).....	160
4.6.2.2	Knowledge and beliefs regarding ML learners.....	163
4.6.2.3	Knowledge and beliefs regarding ML teaching.....	165
4.6.2.4	Knowledge and beliefs regarding ML curriculum	167
4.6.3	Denise’s knowledge and beliefs	169
4.6.3.1	Mathematical content knowledge (MCK).....	169
4.6.3.2	Knowledge and beliefs regarding ML learners.....	169
4.6.3.3	Knowledge and beliefs regarding ML teaching.....	170



4.6.3.4	Knowledge and beliefs regarding ML curriculum.....	172
4.6.4	Elaine’s knowledge and beliefs	174
4.6.4.1	Mathematical content knowledge (MCK).....	174
4.6.4.2	Knowledge and beliefs regarding ML learners.....	175
4.6.4.3	Knowledge and beliefs regarding ML teaching.....	176
4.6.4.4	Knowledge and beliefs regarding ML curriculum.....	178
4.6.5	Summary of the participants’ knowledge and beliefs.....	181
4.6.6.	Discussion of Theme 2: ML teachers’ knowledge and beliefs.....	185
4.6.6.1	ML teachers’ mathematical content knowledge (MCK).....	185
4.6.6.2	ML teachers’ knowledge and beliefs regarding their learners.....	185
4.6.6.3	ML teachers’ knowledge and beliefs regarding the teaching of ML	186
4.6.6.4	ML teachers’ knowledge and beliefs regarding the ML curriculum	187
4.6.6.5	Summary of discussion on Theme 2.....	188
4.7	Findings, trends and explanations	189
4.8	Conclusion.....	192
Chapter 5	193
Conclusions and implications	193
5.1	Introduction	193
5.2	Chapter summary	193
5.3	Verification of research questions.....	194
5.3.1	Question 1: How can ML teachers’ instructional practices be described?.....	197
5.3.2	Question 2: What is the nature of ML teachers’ knowledge and beliefs?.....	199
5.3.3	Question 3: How do ML teachers’ knowledge and beliefs relate to their instructional practices?	201
5.3.4	Question 4: What are the possible implications of the findings from Questions 1, 2 and 3 for teacher training?	202
5.3.5	Question 5: What is the value of the study’s findings for theory building in teaching and learning ML?	204
5.3.6	Summary of verification of research questions	205
5.4	What would I have done differently?.....	206
5.5	Providing for errors in my conclusion	206
5.6	Conclusions	207
5.7	Recommendations for further research	208
5.8	Limitations of the study	209
5.9	Last reflections	209
References	211
Appendices	226

List of abbreviations

ACE	Advanced Certificate in Education
BEEd	Baccalaureus Educationis
BTech	Baccalaureus Technologiae
CAPS	Curriculum and Assessment Policy Statement
DoE	Department of Education (South Africa)
FET	Further Education and Training
GET	General Education and Training
HED	Higher Education Diploma
MCK	Mathematical content knowledge
ML	Mathematical Literacy (the subject)
NCS	National curriculum statement
OBE	Outcomes-based education
OECD	Organisation for Economic Co-operation and Development
PCK	Pedagogical content knowledge
PISA	Programme for International Student Assessment
QCDA	Qualifications and Curriculum Development Agency
QCE	Queensland Certificate in Education
QG	Queensland Government
QSA	Queensland Studies Authority
RME	Realistic Mathematics Education
TIMSS	Trends in International Mathematics and Science Study
UK	United Kingdom
UP	University of Pretoria
US	United States

List of figures

Figure 2.1	Interplay between content, context and problem-solving skills in ML	29
Figure 2.2	Overview and weighting of topics in Grades 10, 11 and 12	30
Figure 2.3	Framework to observe teachers' instructional practices	41
Figure 2.4	Shulman's (1986) three categories of content knowledge	48
Figure 2.5	Grossman's (1990) four components of PCK	49
Figure 2.6	Borko and Putnam's (1996) three domains of knowledge	50
Figure 2.7	Ball, Thames and Phelps' (2005) domains of knowledge of mathematics for teaching	51
Figure 2.8	Hill, Ball and Schilling's (2008) domain map for mathematical knowledge for teaching	52
Figure 2.9	Conceptual framework: Instructional practice, knowledge and beliefs framework of analysis	58
Figure 3.1	The data collection process	74
Figure 3.2	Elucidation of the character of the lesson observations	75
Figure 4.1	Conceptual framework: Instructional practice, knowledge and beliefs framework of analysis	85
Figure 4.2	ML instructional practices: Tasks	92
Figure 4.3	ML teachers' instructional practices: Discourse	93
Figure 4.4	ML teachers' instructional practices: Learning environment	94
Figure 4.5	ML teachers' PCK and beliefs: ML learners	98
Figure 4.6	ML teachers' PCK and beliefs: ML teaching	99
Figure 4.7	ML teachers' PCK and beliefs: ML curriculum	99

List of tables

Table 2.1	A spectrum of pedagogic agendas	32
Table 2.2	The premises of ML and Mathematics	36
Table 2.3	Learning Outcomes for ML and Mathematics	38
Table 2.4	Comparison of the composition of ML and Mathematics across the different bands	39
Table 2.5	Different terminology used for teachers' practices	40
Table 2.6	Relationship between knowledge and beliefs	46
Table 2.7	Overview of different domains of mathematical knowledge	48
Table 2.8	Some beliefs held by learners and their effects in practice	54
Table 2.9	The observable aspects of a lesson	65
Table 2.10	Teacher-centred versus learner-centred instructional practices	66
Table 3.1	Synopsis of methodology	71
Table 3.2	Criteria justifying inclusion and exclusion in the sample	73
Table 3.3	Elucidation of the character of the interviews	76
Table 3.4	Collection, analysis and reporting data	78
Table 4.1	Timeline of the data collection process	87
Table 4.2	Lesson dimensions and dimension indicators as inclusion criteria for coding the data	89
Table 4.3	PCK and beliefs dimensions and its indicators as inclusion criteria for coding the data	95
Table 4.4	Exclusion criteria for coding of the data	100
Table 4.5	Biographical information of the four participants	103
Table 4.6	Summary of Monty's instructional practice	111
Table 4.7	Summary of Alice's instructional practice	122
Table 4.8	Summary of Denise's instructional practice	130
Table 4.9	Summary of Elaine's instructional practice	140
Table 4.10	Snapshot of the four participants and their instructional practices	142
Table 4.11	Summary of Monty's PCK and beliefs	160
Table 4.12	Summary of Alice's PCK and beliefs	168
Table 4.13	Summary of Denise's PCK and beliefs	174
Table 4.14	Summary of Elaine's PCK and beliefs	180
Table 4.15	Snapshot of the four participants' PCK and beliefs	182
Table 4.16	Contradictions between teachers' beliefs and their instructional practices	191
Table 5.1	Summary of participants' information	196
Table 5.2	Comparison of the participants' instructional practices	198
Table 5.3	Summary of verification of research questions	205

List of appendices

Appendix A	Letter of consent to the ML learners
Appendix B	Letter of consent to the principals
Appendix C	Letter of permission to the department
Appendix D	Ethical clearance certificate
Appendix E	Observation sheet for observing ML teachers' lessons
Appendix F	Interview schedule 1 (Prior to lessons 2 and 3)
Appendix G	Interview schedule 2 (Final interview)
Appendix H	List of research studies for Literature Control
Appendix I	Analysis of discussions on Theme 1 and Theme 2
Appendix J	Additional information verifying Question 1
Appendix K	Additional information verifying Question 1
Appendix L	Declaration: External coder