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Process Integration of Complex Cooling Water Systems

by

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Synopsis

Cooling water systems are generally designed with a set of heat exchangers arranged in parallel. This arrangement results in higher cooling water flowrate and low cooling water return temperature thus reducing cooling tower efficiency. Previous research on cooling water systems has focused mainly on heat exchanger network thus excluding the interaction between heat exchanger network and the cooling towers. The studies completed on cooling water system in which the interaction between the cooling tower and the heat exchanger network was taken into consideration were limited to systems with single cooling tower.

The main aim of this study was to develop a design methodology for synthesis and optimization of cooling water systems with multiple cooling towers. The design intends to debottleneck the cooling towers by reducing the circulating water flowrate. The study focuses mainly on cooling systems consisting of multiple cooling towers that supply a common set of heat exchangers.

In this work the mathematical optimization technique was developed for optimization and synthesis of cooling water system. The heat exchanger network was synthesized using the mathematical optimization technique. This technique is based on superstructure in which all opportunities for cooling water reuse are explored. The cooling tower model was used to predict the thermal performance of the cooling towers while taking the thermal conditions of the associated heat exchanger network into account.

The propose technique debotlleneck the cooling towers by decreasing the circulating water flowrate. This implies that a given set of cooling towers can manage an increased heat load. From the case studies, 22% decrease in circulating water flowrate was realized. The blowdown and makeup were also decreased by 7%. Furthermore, the cooling tower effectiveness was also improved by 4%. A decrease in the overall circulation water has an added benefit of decreasing the overall power consumption of the circulating pumps. There is also a potential for the reduction of makeup and blowdown water flowrate.

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1 INTRODUCTION

1.1 Background

Cooling water systems are used to remove waste energy from the process to the atmosphere. The systems consist of cooling towers, re-circulating system and cooling water network. Water is used as a medium to remove energy from the process through the cooling towers into the atmosphere. The cooling towers use the mechanism of evaporative cooling to remove heat from the cooling water. Evaporation of water from the cooling towers results in an increase in the concentration of the dissolved solids in the circulating cooling water. Thus blowdown mechanism is employed to maintain the concentration of the dissolved solids at an allowable level. Makeup water is then added to replace evaporation and blowdown losses. The flowrates for the blowdown and makeup depend on the rate of evaporation loss and the cycles of concentration.

The cooling water network consists of a set of cooling water using operations which are generally arranged in parallel, i.e. they use only freshwater from the cooling source. After process cooling, water which gained heat from the process is recycled back to the cooling source.

Research done on cooling water systems has focused mainly on individual components of the system. The thermal performance of a cooling tower was studied by Bernier (1994), Gharaghei *et al.* (2007), and Lemouari *et al.* (2007). Lefevre (1984) presented ways of reducing water losses in a wet cooling tower. Kim and Smith (2001) presented a methodology for grassroot design of a cooling water system with one cooling source taking into account the cooling tower performance. Majozi and Moodley (2008) developed a technique for debottlenecking a cooling water system consisting of multiple cooling towers through the cooling water network synthesis.

Few authors (Kim & Smith, 2001; Feng *et al., 2005*; Panjeshahi *et al.,* 2009; Ponce-Ortega *et al.,* 2010) conducted studies on cooling water system design by taking into account the interaction between the cooling tower and the heat exchanger network. The notable results from the above mentioned studies were the decrease in the circulating water flowrate with an increase in the cooling water return temperature. Although the authors managed to debottleneck the cooling tower, these studies were focused on cooling water systems consisting of a single cooling tower, while in practice there are cooling water systems with multiple cooling towers. Majozi & Moodley (2008) and Majozi & Nyathi (2007) addressed problems with multiple cooling towers, however their work did not take into consideration the performance of the cooling towers.

Process integration techniques have been used to design cooling water systems. Majozi & Nyathi (2007), Kim & Smith (2001) and Panjeshahi *et al.* (2009) applied the principles of pinch analysis to design the cooling water system. Although the principles of pinch analysis have been successfully applied, this graphical technique is not flexible enough to accommodate practical constraints, e.g. cost functions, pressure drop models etc. These limitations led few authors (Majozi & Moodley, 2008; Kim & Smith, 2003; Ponce-Ortega *et al.,* 2010) to use mathematical modelling techniques to optimize the cooling water systems. This technique involves an optimization of superstructure in which all possible network features are explored. The superstructure is optimized subject to material and energy balances condition at each node and across each unit. The strength of this technique lies on its ability to handle many practical constraints. For instance; forced or forbidden matches, capital cost functions, control and safety constraints.

Although the mathematical modelling techniques are more robust to solve optimization problems, the mathematical models are more often nonconvex nonlinear programs (NLP) or mixed integer nonlinear programs (MINLP) due to presence of bilinear terms. Optimization of nonconvex nonlinear problems generally yields a local optimum solution or results in infeasibilities. Thus it is important to best initialize the problem. The starting point can be found by first linearizing/relaxing the model where the solution for the linearized/relaxed model is used as a starting point for the exact NLP or MINLP model. Sherali and Alameddine (1992) developed a reformulation linearization technique to deal with bilinear programming problems. This convexification technique was imbedded into a branch-and-bound algorithm to find the global optimum solution. This procedure is computationally expensive particularly when dealing with large problems. Savelski and Bagajewicz (2000) linearized the bilinear terms by identifying necessary conditions of optimality in water allocation problems. This is an exact linearization technique which yields globally optimal solution. However, this technique is limited to cases where conditions of optimality can be readily specified.

1.2 Basis and objectives of this study

Cooling water systems are generally designed with a set of heat exchangers arranged in parallel. This design means that all heat exchangers receive cooling water at the same supply temperature and the outlet streams are mixed before being returned to the cooling tower. In cases where some of the heat exchangers do not necessarily require cooling water at the cooling water supply temperature, this arrangement results in higher cooling water flowrate and low cooling water return temperature thus reducing the cooling tower efficiency (Bernier, 2004).

The main aim of this study was to develop a design methodology for synthesis and optimization of cooling water systems with multiple cooling towers. The design intends to debottleneck the cooling towers by reducing the circulating water flowrate.

Problem statement

The problem addressed in this study can be stated as follows:

Given,

(a) a set of cooling towers with their dedicated set of cooling water using operations

(b) the cooling water using operations with their limiting temperatures and heat duties,

(c) the limiting temperature for each cooling tower fill

(d) the dimensions for each cooling tower

(e) the coefficient of performance correlation for each cooling tower

determine the minimum amount of cooling water required for the overall cooling water network.

1.3 Thesis scope

The scope of this research was to develop a synthesis and optimization methodology for cooling water networks with multiple cooling water sources. This required an appropriate mathematical model to predict the thermal performance for each cooling tower as well as cooling water network model to synthesize and optimize the cooling water network. One cooling tower model was used for all cooling towers in the cooling water system. The only distinction in the cooling towers was the correlations for the cooling tower coefficient of performance $\left(\frac{KaV}{m_w}\right)$. The model predicted the outlet water temperature, effectiveness, evaporation, makeup and blowdown for each cooling tower.

The model for cooling water network was developed from the superstructure in which all opportunities for cooling water reuse were explored. The following two practical cases were considered:

Case I. Any cooling tower can supply any cooling water using operation whilst the cooling water using operation can return to any cooling tower.

Case II. This is similar to Case I except that the geographic constrains are taken into account. A particular cooling tower can only supply a particular set of cooling water using operations and these cooling water using operations can only return water to the same supplier.

The models for the cooling towers and the cooling water network were solved iteratively to yield the optimum cooling water system.

1.4 Thesis structure

Chapter 1 introduces the thesis by giving a brief background to cooling water systems. The motivation and basis for the study are stated followed by the objectives. The Literature study which entails a brief review of the latest developments on cooling water system design is given in Chapter 2. Chapter 3 details the derivation of all mathematical models developed and used in this study. In Chapter 4 the developed models are applied to a case study and the results are discussed. Conclusions and recommendations are given in Chapter 5.

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2 LITERATURE REVIEW

2.1 Introduction

This section details a review on process integration techniques for energy and utility savings. The section starts by introducing pinch analysis technique followed by various systematic methods which where adapted from this technique. The adapted methods include mass exchange pinch and water pinch. This is followed by a discussion on mathematical optimization techniques. The major part of this section is on cooling water systems. The mathematical modelling of a cooling tower will be presented followed by latest development on synthesis and optimization of cooling water systems. This section will be concluded by summarizing the drawbacks identified from previous studies conducted on synthesis and optimization of cooling water systems.

2.2 Process Integration: Heat pinch

Since the oil crisis in the 70's, several heat exchanger network synthesis methods have been developed to minimize utility requirement for the process. In all these methods, a unique set of rules were laid to find matches between heat exchangers and finally generating the network. Although these methods have been applied successfully, the results were not always optimum (Linnhoff & Flower, 1978). The most successfully applied method was pinch analysis developed by Linnhoff and coworkers (Linnhoff & Flower, 1978; Linnhoff *et al.*, 1979; Flower & Linnhoff, 1980; Linnhoff & Hindmarsh, 1983). This is a graphical energy saving technique that involves integration of heat exchanger network by adhering to thermodynamic rules. The technique starts by targeting the minimum utility requirement for the process. By adhering to thermodynamic rules, the heat exchanger network is then synthesized to meet the target. Application of pinch analysis technique could be found in both grassroots and retrofit design with the benefits ranging from 6% to 60% for energy savings and 30% for capital savings (Linnhoff, *et al.*, 1982). The following sections give a brief overview of pinch analysis.

2.2.1 Targeting using Composite Curve

The process starts by constructing composite curves for both hot and cold streams as shown in Figure 2-1. The hot streams are streams that need to be cooled while cold streams are the streams that need to be heated. By moving the curves close to each other in a horizontal direction, one can determine the minimum hot and cold utility for a given minimum approach temperature(ΔT_{min}) between the cold and hot streams.

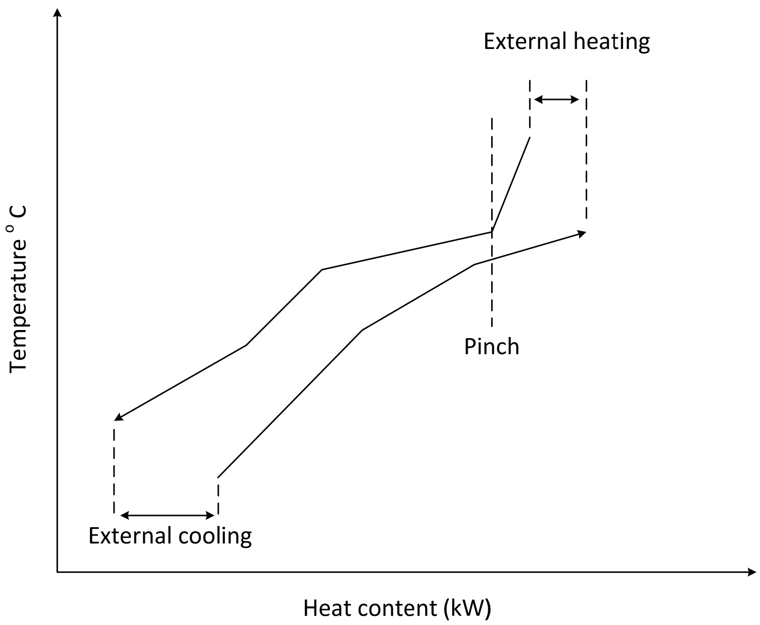

Figure 2-1 Prediction of energy targets using composite curves

The overshoot of the hot composite curve represents the minimum target for the external cooling while the overshoot of cold composite curve represents the minimum target for the external heating. The point where the two curves are close to each other is called pinch point, which is a point of minimum driving force. It is clear from Figure 2-1 that shifting the curves further apart increases ΔT_{min} and thus the utility requirements also increase.

2.2.2 Heat Exchanger Network Synthesis Using Grid Diagram

The heat exchanger network is synthesized by first dividing the problem into two sections; above and below pinch as shown in Figure 2-2. In each section, streams were matched to satisfy their heat duties starting the process at pinch and moving away. The following sets of rules were obeyed when matching streams:

- No heat transfer across pinch
- No cold utility above pinch
- No hot utility below pinch
- Above pinch $CP_{HOT} \leq CP_{COLD}$
- Below pinch $CP_{HOT} \geq CP_{COLD}$
- The ΔT_{min} rule must always be adhered to

where *CP* is heat capacity flowrate.

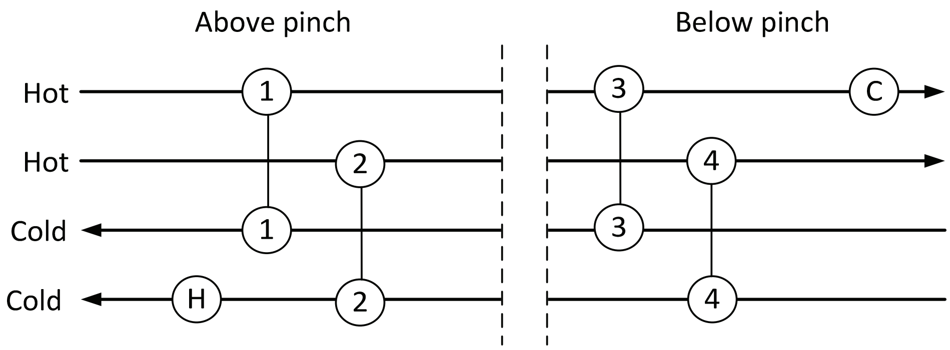

Figure 2-2 Heat exchanger network synthesis

A stream can also be split to satisfy the minimum utility requirement. Splitting is done simple by dividing the utility flowrate, thus changing the *CP* value. The algorithm for stream splitting is shown in Figure 2-3.

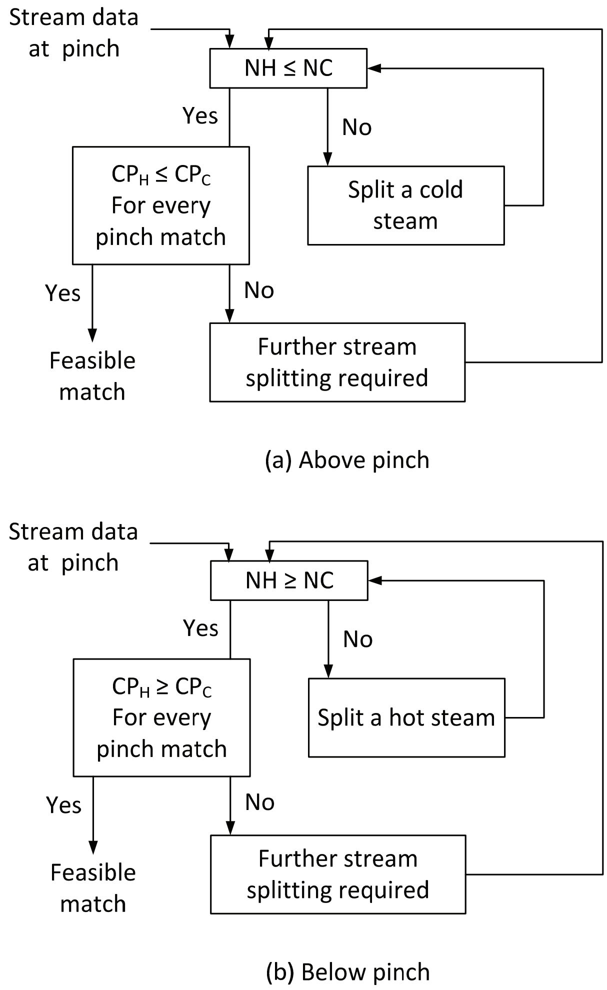

Figure 2-3 Hot end pinch design procedure. (b) Cold end pinch design procedure (Linnhoff & Hindmarsh, 1983).

2.2.3 Targeting Multiple Utilities using Grand Composite Curve

The synthesis as stipulated above only shows the minimum utilities requirement for the process. One or more utilities can be used to satisfy utilities requirement for the process depending on availability and cost. The grand composite curve shown in Figure 2-4 depicts how various levels of utilities can be used. The curve could be used to maximize the most economical utility. The procedure for constructing a grand composite curve was outlined by Linnhoff *et al.* (1982).

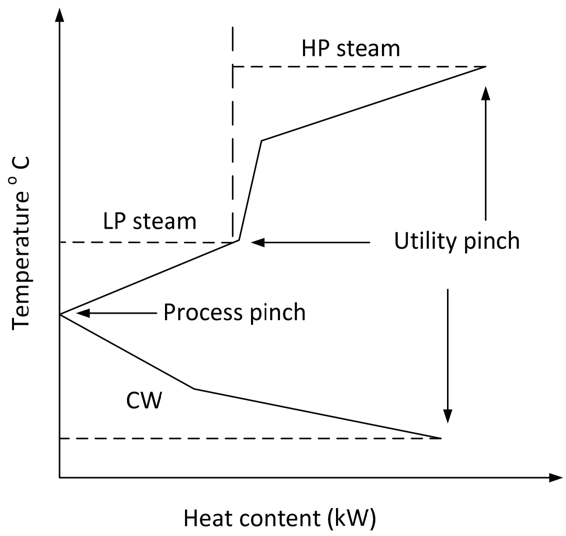

Figure 2-4 Grand composite curve

Using the same principle as described above, pinch analysis was further applied in mass integration and wastewater minimization problems. This part of pinch analysis was called mass pinch and is described in the following sections.

2.3 Process Integration: Mass Pinch

Techniques derived for mass integration or heat integration problems are very similar and can be adapted to both problems. Minor adjustments are normally required to apply heat integration technique to mass integration problems and visa versa. In mass integration the plots are given as mass load vs. concentration while in heat integration the plots are given as enthalpy vs. temperature. The governing equations used in both problems are also similar. The mass balance equation for wastewater minimization is shown in Equation 2-1 and the energy balance equation is shown in Equation 2-2. Another commonality between the two techniques is the thermodynamic bottleneck. In both techniques the thermodynamic bottleneck plays a major role in determining the minimum utility requirement.

$$ML = F(C_{out} - C_{in}) \tag{2-1}$$

$$\Delta H = Fc_p\left(T_{out} - T_{in}\right) \tag{2-2}$$

El-Halwagi and Manousiouthakis (1989) adapted the technique from heat pinch to address the problem of mass exchange networks. The targeting involves plotting cumulative exchanged mass vs. composition for rich and lean streams. This method only applied to single contaminant problems and was later automated by El-Halwagi and Manousiouthakis (1990a). The author used thermodynamic constraints to formulate a linear programming (LP) problem and objective function minimized the cost of mass separating agents and the pinch point. Then, mixed integer linear program (MILP) transshipment model was solved to minimize number of mass exchange units. El-Halwagi and Manousiouthakis (1990b) further used mathematical technique to solve the problem of mass exchange network with regeneration.

Wang and Smith (1994a) developed a technique of wastewater minimization in industry. Their method which is known as water pinch was used for targeting the minimum freshwater. This was done by drawing the limiting composite curve on a mass load vs. concentration plot. The procedure for drawing limiting composite curve was adapted from heat pinch. The water line which was assumed to have a zero contaminant concentration, was matched against the limiting composite curve. The slope of the water line was increased until it touches the limiting composite curve thus forming pinch as illustrated in Figure 2-5. The flowrate of freshwater was then calculated from the inverse of the slope of the water line. The network synthesis was completed using one of the following methods:

Method A. Maximize the use of the available concentration driving forces in the individual process

Method B. Allow the minimum number of water sources to be used for individual processes via bypass and mixing

The authors further included regeneration reuse/recycle into their design. These techniques were used for both single and multiple contaminants problems. Although these were graphical approaches, the author attempted to incorporate mass transfer driving forces such as corrosion and fouling limitation.

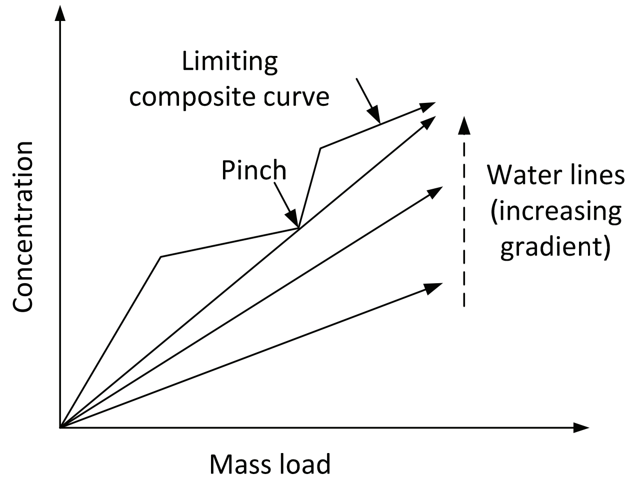

Figure 2-5: Limiting composite curve and water supply line

Kuo and Smith (1998a) simplified the network design presented by Wang and Smith (1994a). The new method was called water mains method. The limiting composite curve was divided into pockets as shown in Figure 2-6. A pocket represents a section on limiting composite curve enveloped by minimum flowrate water line. At the beginning and end of each pocket there are imaginary water mains.

Starting with fresh water, water extracted from each main was allocated to water using operation to satisfy the mass balance subject to limiting concentration constraint. The mains were then removed to complete the network.

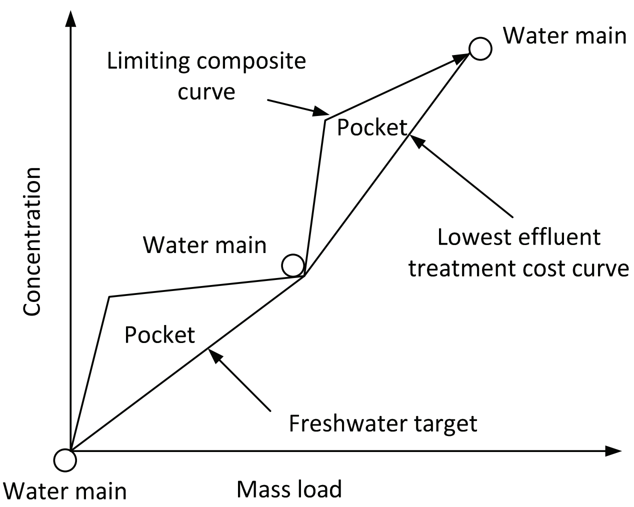

Figure 2-6 Targeting freshwater and effluent.

Kuo and Smith (1998a,b) improved the targeting method that allow for regeneration presented by Wang and Smith (1994a). In this method water using operations were divided into two groups. Group I was supplied by freshwater and Group II was supplied by regenerated water. For further freshwater reduction, operations could be moved from one group to the other.

Wang and Smith (1994b) applied the principles of pinch analysis in a slightly different environment. They presented a methodology for distributed effluent systems design for both single and multiple contaminants systems. The design procedure starts by plotting the composite curve for all effluent streams. The treatment line was then drawn against the composite curve to target minimum treatment flowrate. The distributed effluent system was then designed using the grouping rule which could be summarized as follows:

- All effluent streams with the concentration above pinch must pass through the treatment process
- All effluent streams with the concentration located at pinch partially bypass the treatment process
- All effluent streams with the concentration below pinch must bypass the treatment process

Kuo and Smith (1997) extended the work of Wang and Smith (1994b) by addressing problems with multiple treatment processes.

Although pinch analysis has been applied successfully, this graphical technique is not adaptable enough to accommodate all practical constraints. Performance model (e.g. pressure drop model) cannot readily be incorporated with this graphical technique.

2.4 Mathematical Optimization Techniques

Takama *et al.* (1980) were first to use mathematical programming for targeting and designing water using networks in the refinery. This technique involved superstructure in which all possible network features were explored. The possible features included recycles and reuse as shown in Figure 2-7. The superstructure was optimized subject to material balance at each node and across each unit. The strength of this technique lies in its ability to handle many practical constraints, e.g. forced or forbidden matches, capital cost functions, control and safety constraints.

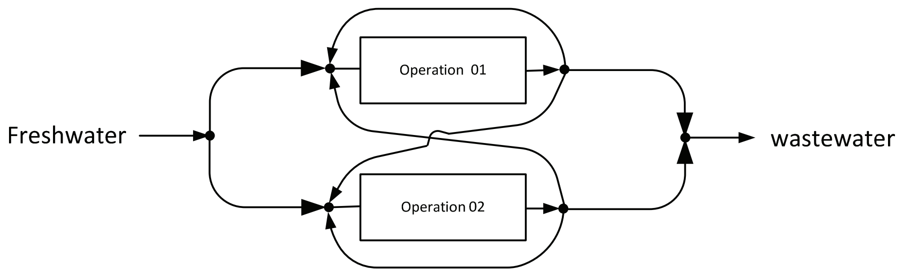

Figure 2-7 The superstructure for water using network

Takama *et al.* (1980) formulated their problem as nonlinear program (NLP) which was solved by minimizing the total cost. The authors used sequential procedure to find the optimum solution. The problem was transformed into series of problems without inequality constraints by using penalty function. The optimization was carried out using complex method.

Doyle and Smith (1997) presented a methodology for targeting maximum water reuse for multicontaminant problems. Their formulation was also NLP which was solved by using a combined LP-NLP approach. The model was first linearized by using fix outlet concentration and the linear model was solved to obtain an initial starting point for the exact nonlinear problem. This approach was also used by Alva-Argáez *et al.* (1998) for wastewater minimization of industrial systems. However, Alva-Argáez *et al.* (1998) formulation was MINLP and the water treatment operations were also included. Teles *et al.* (2008) also applied the linearization strategy proposed by Doyle and Smith (1997). However, in their method the chances of finding global solution were improved by introducing a multiple starting point solution procedure.

Gauratnam *et al.* (2005) solved the problem of total water system by adopting the solution procedure that generates the initial starting point. Their problem was formulated as MINLP problem. Two-stage optimization strategy was used to initialize the problem. The formulated problem was decomposed into MILP and LP subproblems and the solution was obtained by solving the subproblems iteratively to yield a starting point for the exact MINLP. The solvers used were OSL for LP and MILP problems, and DICOPT for MINLP problem. The solvers used for DICOPT subproblem were not mentioned.

Dong *et al.* (2008) and Chen *et al.* (2010) used mathematical technique to simultaneously address water allocation and heat exchanger network problems. These types of problems arise when there is a need to cool/heat streams in a water using networks. Streams that require cooling are matched with streams that require heating subject to thermodynamic constraints. The water using network is then optimized to minimize the freshwater consumption while the heat exchanger network is optimized to minimize utility cost.

Dong *et al.* (2008) developed a mathematical technique for simultaneous synthesis of the multicontaminant water allocation and heat exchanger networks (WAHEN). Their formulation was based on a state-space superstructure shown in Figure 2-8. The superstructure was divided into two interconnected blocks, distribution network (DN) for splitters and mixers and process operator (OP) for WAN and HEN.

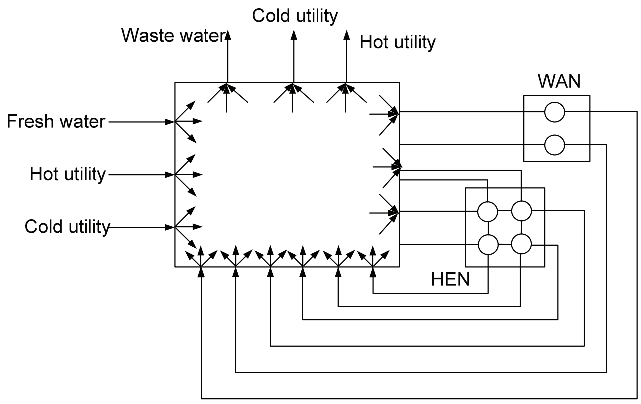

Figure 2-8 State-space superstructure for WAHEN.

Opportunities for wastewater treatment, water reuse and indirect heat exchange were explored and the mathematical model was formulated as MINLP. The authors adopted a 3-step approach to solve the model. In the first step a feasible solution was found by randomly generating initial guesses. The second step then improves the solution from the first step using perturbation techniques. The third step improves the network structures. The DICOPT solver was used in GAMS platform to solve the MINLP models. The solvers used for NLP and MIP subproblems were not mentioned. The major setback in this method was the long solution procedure which starts with random feasible starting point. Random starting point can be anywhere in the search space and thus global optimality cannot be guaranteed.

Chen *et al.* (2010) also developed mathematical model for synthesis of heat-integrated WUN. The mathematical model was developed based on superstructural approach. The formulation was MINLP and the authors did not attempt to relax the model or applied any technique to address the problem of suboptimality associated with MINLP problems.

Two-step sequential approach was adopted in which WUN was synthesized by minimizing freshwater consumption. With this target, the throughput for water using units and regenerator could be further minimized. From the optimized WUN, the HEN was synthesized by minimizing the utility cost or total annualized cost. CPLEX solver was used for MILP problem and BARON solver was used for MINLP problems. The setback with this study was that the objective function was not based on simultaneous optimization of WUN and HEN models thus the solution cannot be regarded as globally optimum. Kim *et al.* (2009) already addressed the issues related to two-step approach by using one objective function which represent the total cost for both WUN and HEN problems.

As discussed above, the main challenge of using mathematical optimization technique is the solution procedure for NLP or MINLP models. The challenge is finding a feasible good starting point. The starting point can be found by first linearizing/relaxing the model where the solution for the linearized/relaxed model is used as a starting point for the exact NLP or MINLP models. Various techniques have been developed to relax or linearize the bilinear terms.

Sherali and Alameddine (1992) developed a reformulation linearization technique, which was based on McCormick (1976) convex and concave envelop to relax bilinear terms in the NLP formulations. This convexification technique was imbedded into a branch-and-bound algorithm to find the global optimum solution. Quesada and Grossmann (1995) improved the work of Sherali and Alameddine (1992) by improving the tightness for the bounds. Wicaksono and Karimi (2008) used different relaxation technique which was called piecewise MILP relaxation. This was also based on a convex envelope for bilinear terms, however the search space was divided into smaller partitions within the feasible region. Consequently, the bounds were tighter. These procedures are inexact linearization technique and thus global optimality cannot be guaranteed.

Savelski and Bagajewicz (2000) used a different approach to linearize the NLP problems. The author presented necessary optimal conditions for single contaminant water allocation problem. The conditions for optimality were then used to linearize the bilinear terms. This was an exact linearization technique which yielded globally optimal solution. However, this technique was limited to cases where conditions of optimality can be readily specified.

2.5 Cooling Water Systems

This section gives a preview of studies conducted on cooling water system. The studies include cooling tower modelling, optimization of cooling water system where the cooling water network is not synthesized, optimization of cooling water system where the cooling water network is synthesized and the synthesis of cooling water system for effluent thermal treatment.

2.5.1 Cooling Tower

Cooling tower is a direct contact heat exchangers that cool warm water stream by evaporating some of the water into the air stream. They normally contain parking (fill) inside which serve to increase the contact surface area between water and air (Kern, 1950). Two main types of cooling towers are used in the industry namely; mechanical draft and atmospheric towers. The mechanical draft tower, which is the most common, uses a fan to draw air into the tower. The air flow can either be counter or crosscurrent relative to the water depending on the design. Atmospheric towers do not use a fan but rather rely on the buoyancy effect of the heated air that naturally moves from the bottom to the top of the tower (Qureshi & Zubair, 2006). Applications of cooling towers are commonly found in power plants, petrochemical plants, explosive industries, refrigeration and air conditioning processes (Naphton, 2005).

2.5.1.1 Diffusion and Evaporative Cooling

When unsaturated air is brought into contact with water at the same temperature, water vapour will escape from the water interface into the unsaturated air interface. This process takes place because of the concentration gradient that exists between water and the air interface (Kern, 1950). As more water evaporates into the air interface, the humidity of air increases until the saturation concentration is reached. At this stage diffusion becomes minimal. The water vapor entering the air interface carries with it the latent heat of vaporization. This energy comes from the water itself thus the water temperature decreases. About 90% of energy transfer between air and water is through evaporation of water into the air interface (Bernier, 1994).

The mass transfer from the liquid interface to the gas is given by:

$K_a adz(w_s - w) = m_a dw$ (Coulson & Richardson, 1996) (2-3)

The difference in temperature between the air and the liquid results in convective heat transfer at the interface as shown in Equation 2-4.

$h_w adz(T_l - T_i) = m_w c_p dT$ (Coulson & Richardson, 1996) (2-4)

The overall mass and energy balance equations across the tower are given by:

$$dm_w = m_a dw \tag{2-5}$$

$$m_a dH_a = m_w dH_w \tag{2-6}$$

2.5.1.2 Mass Transfer Coefficient

The coefficient of mass and heat transfer are not known in most cases. They are normally determined practically by carrying out tests on a laboratory scale or pilot scale (Kern, 1950). Coulson and Richardson (1996), Kern (1950) and Bernier (1994) have presented correlations which directly show the dependence of mass and heat transfer coefficient on water and air flowrates. In all three cases, the general format for the correlation is as shown below.

$$K_a a = x\left(m_a{}^y m_w{}^z\right) \tag{2-7}$$

$$h_w a = x\left(m_a{}^y m_w{}^z\right) \tag{2-8}$$

where *x, y* and *z* are parameters.

2.5.1.3 Cooling Tower Performance

Several authors (Crozier, 1980; Gharaghei *et al.,*2007; Lemouari *et al.,* 2007) have published work relating to the effect of operating parameters on the performance of a cooling tower. The main variables were the water to air flowrate ratio, inlet air wet bulb temperature and the inlet water temperature. The performance of a cooling tower can be described using Figure 2-9 presented by Perry *et al.* (1997). The *y* and *x* axis represent the enthalpy and temperature respectively. Line CD represents the air operating line and line AB is the water operating line.

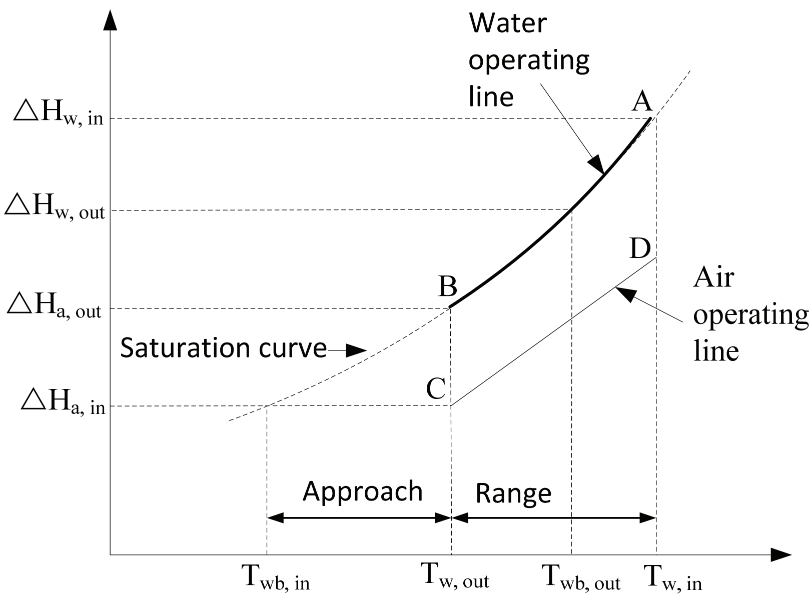

Figure 2-9 Cooling tower process heat balance

Factors affecting cooling tower performance

- **Water to air flowrate ratio**

The thermal performance of a cooling tower was studied by Gharaghei *et al.* (2007) and Lemouari *et al.* (2007). The tower characteristics given in Equation 2-9 were used to evaluate the cooling tower performance.

$$\frac{KaV}{m_w} = c_{pw} \int_{T_{w,in}}^{T_{w,out}} \frac{dT_w}{H_w - H_a} \tag{2-9}$$

The result for the above mentioned studies showed that increasing water to air ratio in the cooling tower decreases the tower performance characteristics. This would further decrease the efficiency of the cooling. Lemouari *et al.* (2007) further showed that cooling tower range ($T_{w,\ in}$-$T_{w,\ out}$) increases with a decrease in the water to air ratio. Thus, the best cooling is obtained at lower water flowrate.

- **Inlet air wet bulb temperature**

The inlet air wet bulb temperature governs the lowest possible temperature to which water can be cooled in a cooling tower. This is because when water attains the wet bulb temperature, its vapour pressure is the same as the vapour pressure of air thus resulting in zero diffusion potential (Kern, 1950). Therefore, a cooling tower operating in an environment with lower wet bulb temperature has a greater chance of producing cooler water.

Although wet bulb temperature is an indication of how low the cooling tower water outlet temperature can get, it does not mean that all cooling towers are able to attain the wet bulb temperature. Hence the performance of a cooling tower is normally described in terms of the approach, which is the difference between the outlet and the wet bulb temperature. A lower approach means a good performing cooling tower.

- **Inlet water temperature and flowrate**

  Crozier (1980) suggested that by minimizing the return water flowrate thus increasing the return temperature, the thermal driving force between warm water and cold air is increased. This will improve the cooling tower thermal performance. Kim and Smith (2001) further elaborated this concept by calculating the effectiveness of the cooling tower when the inlet water temperature is increased. The results are depicted in Figure 10. The added benefits related to a high return temperature were: (i) A reduction in the cooling tower capital and operational cost. (ii) Reduced power consumption for the circulating pump (Crozier, 1980). However, higher inlet temperature also results in higher evaporation rate in the cooling tower. Consequently, makeup water and blowdown flowrates will increase. This is evident from the Equation 2-10 to 2-12 given by Perry *et al.* (1997).

$$Evaporation = 0.0008 F_w \left(T_{w,in} - T_{w,out}\right) \tag{2-10}$$

$$F_M = F_E \frac{CC}{CC - 1} \quad \text{where CC, cycles of concentration} \tag{2-11}$$

$$F_M = F_B + F_E \tag{2-12}$$

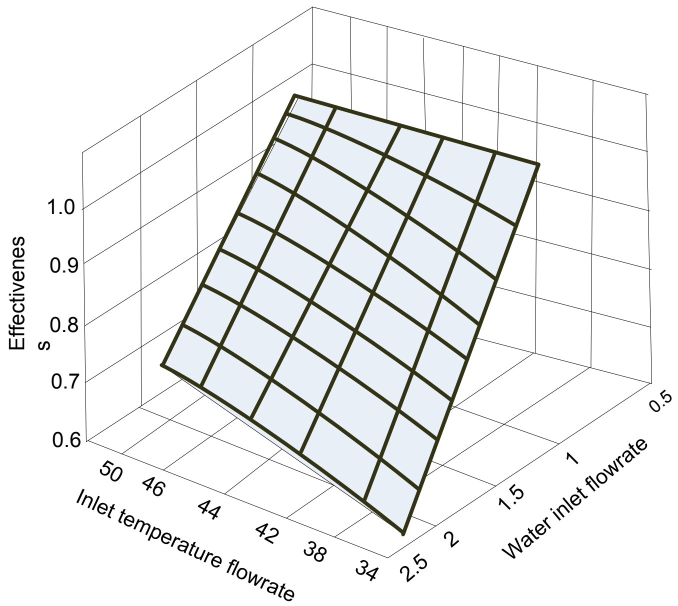

Figure 2-10 Cooling tower performance

2.5.1.4 Water loss Reduction in a Cooling Tower

Lefevre (1984) presented ways of reducing water losses in a wet cooling tower. The author identified the major losses as evaporation and blowdown. Blowdown depends on evaporation thus reducing evaporation will inevitable reduce blowdown as shown in Equation 2-11 and 2-12.

Nonetheless, evaporation depends mainly on ambient conditions such as pressure, temperature and relative humidity. All these factors cannot be controlled by the operator thus other factors need to be considered. Lefevre (1984) suggested higher air to water ratio in a relatively low humidity environment as a way of decreasing evaporation. The author also suggested range as a way of reducing water consumption. The author observed that the cooling tower operating at a higher range evaporates less water. This is contrary to Equation 2-10 which suggests that evaporation is directly proportional to cooling range.

2.5.1.5 Cooling Tower Model

The prediction of cooling tower thermal performance dates back to 1925 by Merkel. Bernier (1994) and Kröger (2004) applied Merkel's theory to develop the cooling tower model. Bernier (1994) further evaluated the cooling tower thermal performance by deriving a one dimensional model based on the thermal behavior of the water droplet in a spray type cooling tower. The model was able to predict the cooling tower outlet temperature and change in air humidity. The major assumptions for this model were as follows:

- Lewis factor is unity
- No parking inside the tower

The author further used Merkel's theory to predict the coefficient of performance $\left(\frac{KaV}{m_w}\right)$ as shown in Equation 2-13. This equation was derived from energy balance between the surrounding air and water droplet. The following assumptions were used:

- Water evaporation inside the tower is negligible
- The resistance surrounding water droplets is negligible
- Constant heat capacity of water
- Transfer coefficients are independent on temperature

This implies that the effect of inlet wet bulb temperature and the inlet water temperature on the coefficient of performance is negligible.

$$\frac{KaV}{m_w} = x\left[\frac{m_w}{m_a}\right]^y \tag{2-13}$$

The values of *x* and *y* parameters could be determined experimentally for a given cooling tower packing. The experimental work completed by the author showed a good approximation for $\frac{KaV}{m_w}$. The coefficient of correlation for the regression was in magnitude of 0.99.

Richardson and Coulson (1996) suggested similar correlation as shown below.

$$Ka \quad \alpha \quad m_w^{1-n} m_a^{n} \tag{2-14}$$

where n varies from 0.4 to 0.8.

Fisenko *et al.* (2004) also derived a one dimensional mathematical model for a counter flow mechanical draft cooling tower by solving heat transfer, mass transfer and dynamic equations of a falling water droplet. Equation 2-15 was used to evaluate the cooling tower efficiency.

$$\eta = \frac{T_{w,out} - T_{w,in}}{T_{w,out} - T_{wb}} \tag{2-15}$$

Qureshi and Zubair (2006) developed a cooling tower model which accounts for heat transfer in the spray zone, packing and rain zone. They further developed a fouling model to predict fouling on packing. The mass transfer coefficient was calculated from the same correlation used by Bernier (1994). Equation 2-16 was used to evaluate the cooling tower effectiveness. The effectiveness was defined thermodynamically as the ratio of actual heat transferred over the maximum theoretical amount of heat that can be transferred. This should not be confused with the cooling tower efficiency by Fisenko *et al.* (2004).

$$\varepsilon = \frac{H_{a,out} - H_{a,in}}{H_{s,w} - H_{a,in}} \tag{2-16}$$

The detailed analysis of the cooling tower effectiveness was already presented by Jaber and Webb (1961). The authors adopted the effectiveness-NTU equations for counter flow heat exchanger to derive the equations for the cooling tower effectiveness. Two possible cases were considered.

Case A. Water capacity rate (m_{cap}) is less than the air capacity rate (m_a). In this case the effectiveness was given as follows:

$$\varepsilon = \frac{m_w c_{pw}(T_{wi} - T_{wo})}{m_{cap}(H_{masi} - H_{mai})} \tag{2-17}$$

Kröger (2004) further showed that the effectiveness can be expressed in terms of enthalpies as shown in Equation 2-18.

$$\varepsilon = \frac{H_{masi} - H_{maso}}{H_{masi} - H_{mai}} \tag{2-18}$$

If the exit water temperature is equal to the inlet air wet bulb temperature, then the effectiveness can be expressed as:

$$\varepsilon = \frac{T_{wi} - T_{wo}}{T_{wi} - T_{wb}} \tag{2-19}$$

Case B. Water capacity rate (m_{cap}) is greater than the air capacity rate (m_a).In this case the effectiveness was given as follows:

$$\varepsilon = \frac{m_w c_{pw}(T_{wi} - T_{wo})}{m_a(H_{masi} - \delta - H_{mai})} \tag{2-20}$$

where δ is the correction factor for nonlinearity of $H_{mas} vs \;\; T_w$ given in Equation 2-21

$$\delta = \frac{H_{maso} + H_{masi} - 2H_{mas}}{4} \tag{2-21}$$

where i_{mas} is the enthalpy of saturated air at mean water temperature.

Kim and Smith (2001) also developed the cooling tower model which predicts the thermal performance a cooling tower. The author derived a cooling tower model with the following major assumptions:

- Adiabatic operation in the cooling tower
- Water and dry air flowrate are constant
- Drift and leakage losses neglected

The performance of the cooling tower was assessed by changing the inlet conditions of water and air. The cooling tower performance was then measured by calculating the effectiveness.

Khan and Zubair (2001) also developed a model which incorporates the evaporation and drift losses. Khan et al (2004) extended the work done by Khan and Zubair (2001) to investigate fouling on the thermal performance of a cooling tower

Kröger (2004) developed a model for a cooling tower by considering a control volume as shown Figure 2-11.

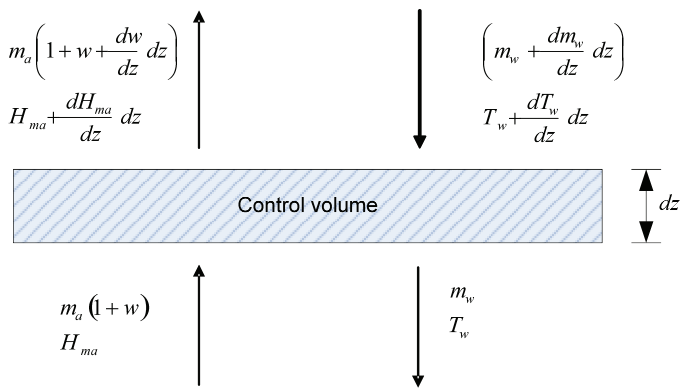

Figure 2-11 Control volume

The following assumptions were made:

- Interface water temperature is the same as the bulk temperature
- Air and water properties are the same at any horizontal cross section
- Heat and mass transfer area is identical

The governing equations that predict the thermal performance of a cooling tower are given by Equations 2-22, 2-23 and 2-24. Equations 2-22 and 2-23 define the mass and energy balance for the control volume, respectively. Equation 2-24 defines the air enthalpy change for the control volume.

$$\frac{dm_w}{dz} = m_a \frac{dw}{dz} \tag{2-22}$$

$$\frac{dT_w}{dz} = \frac{m_a}{c_{pw} m_w}\left(\frac{1}{c_{pw}}\frac{dH_a}{dz} - T_w \frac{dw}{dz}\right) \tag{2-23}$$

$$\frac{dH_a}{dz} = \frac{Ka_{fi}A_{fi}}{m_a}\left(Le_f(H_{as} - H_a) + (1 - Le_f)H_v(w_s - w)\right) \tag{2-24}$$

In Equation 2-24 a_{fi} is the wetted area divided by the corresponding volume of the fill and A_{fr} is a frontal area. The Lewis factor, Le_f, appearing in Equation 2-24 is the relationship between the heat transfer coefficient and the mass transfer coefficient, i.e. $\frac{h}{Kc_{pma}} = Le_f$. Lewis factor appears in many governing heat and mass transfer equations. A number of authors like Bernier (2004) assumed the Lewis factor to be unity. Klopper and Kröger (2005) used expression given in Equation 2-25 to predict the value of Lewis factor. The authors studied the influence of Lewis factor on the performance prediction of a wet cooling tower. Their findings were that the influence of Lewis factor diminishes when the inlet ambient air is relatively hot and humid.

$$Le_f = 0.866^{0.667}\left(\frac{w_s + 0.622}{w + 0.622} - 1\right) / ln\left(\frac{w_s + 0.622}{w + 0.622}\right) \tag{2-25}$$

They further elaborated that increasing Lewis factor increases heat rejection, decreases water outlet temperature and decreases water evaporation rate.

2.5.1.6 Cooling Water Systems Optimization with no Cooling Water Reuse.

Castro *et al.* (2000) developed a cooling water system model which consists of a cooling tower, pumping system and the heat exchanger network. The model took into account the pressure drop through the lines and the heat exchangers. This work was based on modelling the cooling water system with parallel heat exchanger network configuration and optimizing the system for any change in operating variables.

The authors did not develop a detailed cooling tower model. Instead, they used correlation functions that predict operating variables in the cooling tower. The correlations predicted the cooling tower outlet temperature, evaporation, makeup, blowdown and forced hot blowdown. The heat exchanger network model was based on energy balance and pressure drop balance at each node and across each unit.

The objective was to minimize the total operating cost, i.e. cooling water and electricity costs. The optimization variables were the cooling tower fan speed, forced hot blowdown flowrate and the equivalent length of the valves in each branch. The model was applied in a case study to evaluate the effect of cooling tower outlet temperature and the monthly climatic conditions on cost. The results showed a decrease in cooling tower return temperature that yielded a decrease in cooling range. This implies less removal of heat by the cooling tower. The flowrate of the forced hot blowdown was increased to reject heat that cannot be removed by the cooling tower. Consequently, the makeup water flowrate was increased.

The result for the effect of weather showed the highest operating cost was high during July month which is the month with highest humidity and lower temperatures in the region where this work was done. This observation confirms that the humidity affects the cooling tower performance more significantly than the air temperature.

The setback with the proposed work was the use of correlation equations for the cooling tower model and the accuracy of these equations was not provided. The authors analyzed the results by focusing on air and circulating water flowrate without taking into account the cooling tower performance.

Cortinovis *et al.* (2009a) also developed a mathematical model for cooling water system which takes into account the cooling tower performance, as well as hydraulic and thermal performance of the heat exchanger network. This work was based on modeling the cooling water system and validating the model with the experimental data.

The cross flow cooling tower model was developed based on the following major assumptions:

- Constant water flowrate along the tower
- Constant water and air specific heats
- Lewis factor is equal to 1
- Wet bulb temperature is equal to the saturation temperature
- Air-water interface temperature is equal to that of the bulk liquid

The correlation for the fill mass transfer coefficient was developed using the power law equation and the experimental data as shown in Equation 2-26.

$$Ka = 0.63G^{0.82}L^{0.48} \tag{2-26}$$

The hydraulic model, which includes heat exchanger network mass balance and mechanical energy balance, and the thermal model, which caters for enthalpy balance across each heat exchanger, were developed and validated against the experimental data. The three models were combined into one cooling water system model which was also validated against the experimental data.

Although the authors tried to incorporate the cooling tower performance in the cooling water system model, the model could not predict the evaporating, makeup and blowdown flowrate.

The minimum temperature approach (ΔT_{min}) between the process and the cooling water was also not taken into consideration.

This work was extended by Cortinovis *et al*. (2009b) for optimization of cooling water system that incorporates the economical aspects of the operation. The cooling tower model was improved to cater for evaporation and entrainment losses, blowdown, makeup and forced hot blowdown. The entrainment was assumed to be 0.1% of the circulating water flowrate and the evaporation was calculated from Equation 2-10.

$$Evaporation = 0.0008F_w\left(T_{w,in} - T_{w,out}\right) \qquad (2\text{-}10)$$

where F_w is the circulating cooling water flowrate, $T_{CT,in}$ and $T_{CT,out}$ are inlet and outlet cooling tower temperatures respectively.

The model was optimized to minimize the total operating cost, i.e. cooling water and electricity cost. The optimization variables were the cooling tower fan speed and forced hot blowdown flowrate. The model was applied in a case study to evaluate the effect of change in heat load, makeup water temperature and the air temperature on cost. The results showed that an increase in heat load of the process requires optimum performance of the cooling tower. The authors suggested that this can be achieved by higher air flowrate or higher forced hot blowdown flowrate depending on which option is more cost effective. Furthermore, an increase in air and makeup water temperature resulted in an increase in forced hot water blowdown flowrate.

The authors analyzed the results by focusing on air and circulating water flowrate without taking into account the cooling tower performance. Kim & Smith (2001) showed that by exploring the opportunities for cooling water reuse, the circulating water flowrate can be reduced, thus debottlenecking the cooling tower. However, this option was not considered by the authors.

2.5.1.7 Cooling Water Systems optimization with Cooling Water Reuse

Heat exchanger networks are generally designed with a set of heat exchangers arranged in parallel. This design means that all heat exchangers receive cooling water at the same supply temperature and the outlet streams are mixed before being returned to the cooling tower. This arrangement results in higher cooling water flowrate and low cooling water return temperature thus reducing the cooling tower efficiency (Bernier, 1994).

In most cases not all heat exchanger inlet temperatures should be at the cooling source supply temperature. Higher inlet temperatures can be tolerated as long as the ΔT_{min} approach is satisfied. If the outlet temperature of the cooling water from one heat exchanger is at least ΔT_{min} lower than the process temperature in any heat exchanger, it could be reused to supply those heat exchangers thereby reducing the freshwater consumption. Mixing two or more cooling water outlet streams from various heat exchangers can result in a stream with temperature at least ΔT_{min} lower than the process temperature in other heat exchangers thus giving an opportunity for cooling water reuse.

The recent study on cooling water system design was conducted by Kim and Smith (2001). They developed a methodology for grassroot design of a cooling water system with one cooling source taking into account the cooling tower performance. The cooling tower and cooling water network were examined discretely. The cooling tower model was developed to predict the outlet conditions of water from the cooling tower.

The authors further used the principles of pinch analysis for targeting and design of cooling water network. The water mains method by Kuo and Smith (1998a) was readily used for cases were maximum reuse was allowed, i.e. the heat exchanger outlet temperature was allowed to go as high as possible. In most practical situations the return water temperature is constrained. The cooling water supply line for these situations does not make a pinch with the composite curve, which implies that the water mains method cannot be readily applied. The concept of pinch migration and temperature shift was introduced to handle problems were process pinch does not exist. From the illustrative example, a decrease of 15.4% in recirculating cooling water flowrate was realized.

The author further looked into other debottlenicking procedure for cooling water systems. The cooling tower was bottlenecked by increasing the process heat load and the following options were considered:

- Targeting by using feasible cooling water line and the isothemal line of the cooling system outlet temperature
- Introduce hot blowdown
- Installing air cooler to reduce the heat load

Although the authors designed the cooling water network by considering cooling tower performance, their work was only limited to one cooling source which is not the case in most practical situations.

Kim and Smith (2003) extended their work (Kim & Smith, 2001) by incorporating pressure drop into their design. Targeting was done by applying the method of Kim and Smith (2001). The mathematical optimization technique was used to synthesize the heat exchanger network with the least pressure drop. Their formulation was MINLP model which was solved by first linearizing the problem through setting the outlet temperature of each heat exchanger to maximum value and using linear estimation of pressure drop correlations. The solution of the linearized problem was then used as a starting point for MINLP model. The model was solved in a GAMS platform using OSL solver for MILP problem and DICOPT for MINLP.

Kim and Smith (2004) further presented a systematic method for the design of cooling water system to reduce makeup water. The makeup water reduction was achieved by reuse of water from wastewater generating processes. The authors assumed that wastewater was available at allowable contaminant concentration and with unlimited quantities. To reduce freshwater makeup usage, Kim and Smith (2004) added wastewater before or after the cooling tower depending on its temperature. If the wastewater temperature was higher than the inlet temperature of the cooling water network, the wastewater was added at the inlet of the cooling tower. This ensured that the cooling water did not gain heat before entering the cooling water network. Thus, wastewater with lower temperature was added at the outlet of the cooling tower. The cooling water network was synthesized using the procedure developed by Kim and Smith (2001).

Feng *et al.* (2005) developed a model for synthesis of recirculation cooling water networks using a superstructural approach. The superstructure was divided into two sections consisting of three mains, namely cooling water supply main, intermediate cooling water main and cooling water return main. Cooling water using operations can get water from cooling water supply main and/or intermediate cooling water main as shown in Figure 2-12. Cooling water that cannot be reused goes to the return main which feed the cooling tower. The use of water main was justified by the fact that when cooling water operations are connected in series, instabilities (heat load or temperature fluctuations) in the first operation will affect the second operation. The authors also suggested that the cooling water circuit was simpler.

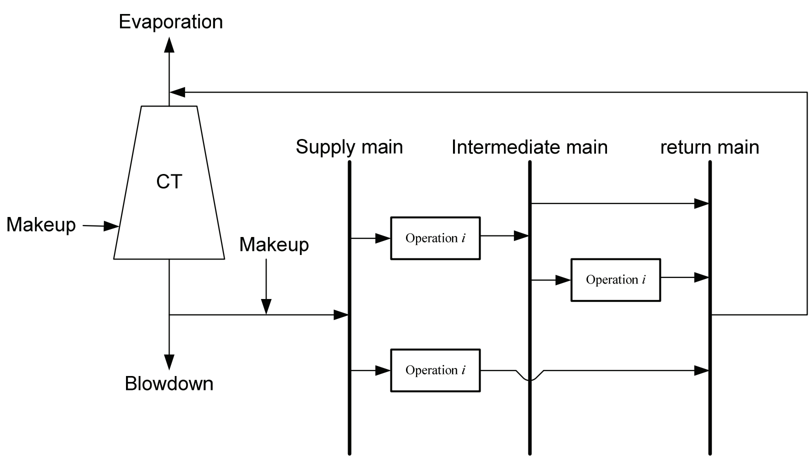

Figure 2-12 Superstructure for cooling water network with water mains.

The developed mathematical model was based on energy and mass balance constraints. The most important feature to note in the model was energy balance across the intermediate main. The temperature of this main was an optimization variable and could not be more than the maximum of all the cooling water using operations maximum outlet temperature. The authors classified their formulation as MINLP although no constraint with integer variables was shown. The NLP or MINLP problems exhibit multiple local optimum solutions and it is often a challenge to find globally optimal solution using commercial solvers. In this work the authors did not attempt to address this problem. The objective function minimized the circulating water flowrate.

The results obtained using this method showed a higher circulating cooling water flowrate when compared with the cooling water networks where direct connections between cooling water using operations were allowed. The main setback in this method was the intermediate main which restrict flexibility in terms of water reuse. Mixing of streams at different temperatures in this main also resulted in heating of colder streams which might have been used in other operations. Thus, water reuse opportunities were not maximized.

Ponce-Ortega *et al.* (2007) presented a methodology for synthesis of cooling water networks that was based on a stage wise superstructural approach. In this approach the superstructure was divided into a number of stages which were equivalent to the number of hot streams to be cooled. This allowed for parallel and series combination or both. Bypass of fresh utility was also catered for. The superstructure is shown in Figure 2-13.

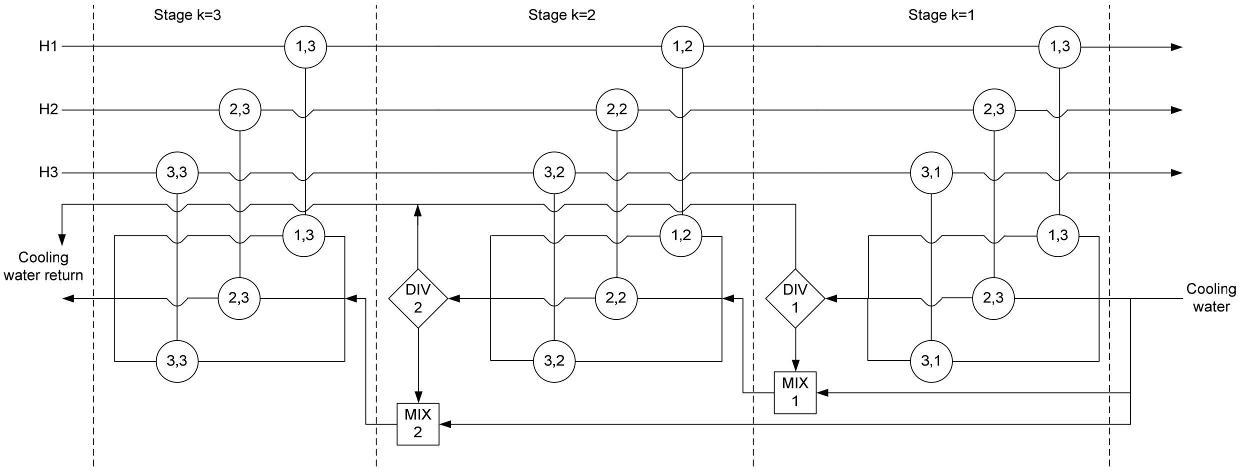

Figure 2-13 Superstructure for cooling water network with three hot process streams.

The mathematical model was developed based on energy and mass balance constraints. The cooling tower model was not included in the formulation thus the supply water temperature was assumed to be constant. A logical constraint which has an integer variable was used to determine if the heat exchanger is in use. This resulted in a formulation being a mixed integer nonlinear program (MINLP). The objective function minimized the annual cost which was given by annualized capital cost for cooling water using operations and the utility cost.

The model was solved using DICOPT in a GAMS platform. The solvers used for the MIP and NLP subproblems were not specified. A case study was taken from Kim and Smith (2001) to elaborate the effectiveness of this method. The results showed an improvement in annual cost, however the cooling water flowrate increased by more than two folds. Increasing the circulating water flowrate results in a decrease in cooling water return temperature for constant cooling duty, consequently the performance of the cooling tower is compromised. Higher flowrate can also overload the cooling tower. Another setback with this method was nonconvex nonlinear terms in the mathematical model. MINLP models have multiple local optimum solutions, thus global optimality cannot be guaranteed if the model is solved directly using a commercial solver.

This formulation was later improved by Ponce-Ortega *et al.* (2010) by including the cooling tower model which was based on Merkel's theory. The authors developed a mathematical model which considers detailed design for the cooling towers and coolers. For a cooling tower, three types of fills and two types of fans were available and the optimization process selects the one that gives an optimum cooling water system. The cooling water network model was improved by including the pressure drop for the coolers in the hot streams side. The pressure drop from each cooler at each stage was also catered for. The model also chose whether a cooler should be single pass or multiple pass heat exchangers. The formulation was still mixed integer nonlinear program (MINLP).

The objective function minimizes the annual cost given by annualized capital cost for cooling water using operations and the utility cost. This cost includes the cooling tower, heat exchangers, pumps for process streams, cooling water and electricity. To address the problem of suboptimality related to MINLP models, the authors developed a two-step solution procedure. The cooling tower was first optimized assuming constant heat transfer coefficients and the original superstructure by Ponce-Ortega *et al.* (2007) was solved. The results were then used as the initial starting points for the exact MINLP model. The model was solved using DICOPT in a GAMS platform. CPLEX solver was used for MIP subproblem and CONOPT solver was used for NLP subproblem.

The proposed methodology was also applied in a case study from Kim and Smith (2001) and the results showed an improvement of 9.7% on annual cost. There was no significant change in the circulating water flowrate although the return temperature was increased with a decrease in cooling tower outlet temperature. Although the authors tried to address the problem of nonlinearity, global optimality could not be guaranteed.

Majozi and Nyathi (2007) derived a methodology for synthesis of cooling water system consisting of multiple cooling sources by combining graphical approach and mathematical programming. The graphical approach was used for targeting minimum cooling water flowrate while the mathematical optimization technique was used for synthesis of cooling water network. The technique involves a superstructure in which all possible water reuse and recycle opportunities are exploited. The cooling water system was debottlenecked by simultaneously determining the minimum amount of the overall circulating cooling water and the corresponding cooling water networks. The following two cases were considered:

Case A. Unspecified cooling water return temperature to the cooling tower without a dedicated source or sink for any cooling water using operation. The formulation entailed bilinear terms which were nonconvex thereby rendering the model NLP

Case B. Unspecified cooling water return temperature to the cooling tower with a dedicated source or sink for any cooling water using operation. The formulation also consisted of bilinear terms and binary variables thus rendering the model MINLP

Cases A and B were linearized using the technique by Savelski and Bagajewicz (2000). The technique was derived for water utilization systems but equally applies in cooling water systems design. Savelski and Bagajewicz (2000) stated that the condition for optimality exists when the outlet concentration is at its maximum allowable level. If this condition is satisfied, the water flowrate will be at its minimum. In the case of heat exchanger networks, Majozi and Nyathi (2007) demonstrated that the optimal solution exists if the outlet temperature is at its maximum allowable level. The proposed methodology was applied in an industrial case study and results showed that the cooling tower could be debottlenecked by decreasing the circulating water flowrate by 23%.

Majozi and Moodley (2008) also developed a mathematical optimization technique for synthesis of cooling water systems comprising at least two cooling towers. The authors used the similar mathematical formulation given Majozi and Nyathi (2007), however the following two additional cases were considered.

Case C. Specified maximum cooling water return temperature to the cooling tower without a dedicated source or sink for any cooling water using operation. The formulation entails bilinear terms which are nonconvex thereby rendering the model NLP

Case D. Specified maximum cooling water return temperature to the cooling tower with a dedicated source or sink for any cooling water using operation. The formulation also consists of bilinear terms and binary variables thus rendering the model MINLP.

The reformulation linearization technique by Quesada and Grossmann (1995) was used to linearize Cases C and D. A relaxed model was solved to obtain the global optimum solution for LP problem. This solution was used as an initial starting point for the exact NLP (Case C) and MINLP (Case D) models. GAMS/DICOPT solver was used for MINLP models where CPLEX solver was used for MILP subproblems and CONOPT solver was used for NLP subproblems. Case studies were presented and the results showed that an improvement of about 30%-40% in targeting could be realized.

The drawback of the work by Majozi and Nyathi (2007) and Majozi and Moodley (2008) was the exclusion of cooling tower model. Changing the cooling water return conditions (temperature and flowrate) will affect the thermal performance of a cooling tower thus optimization of cooling system without the cooling tower model will not yield true optimality.

Panjeshahi and Ataei (2008) extended the work of Kim and Smith (2001) on cooling water system design by incorporating a comprehensive cooling tower model and the ozone treatment for recirculation cooling water. The mathematical model derived by Kröger (2004) was used to predict the thermal performance of the cooling tower and the effectiveness was calculated from expression given by Khan *et al.* (2004).

Mathematical optimization technique was used to target the operating conditions of the cooling tower using the procedure outlined by Kim *et al.* (2001). The procedure starts by drawing a composite curve for all operations and identifying feasible operating cooling water line as shown in Figure 2-14.

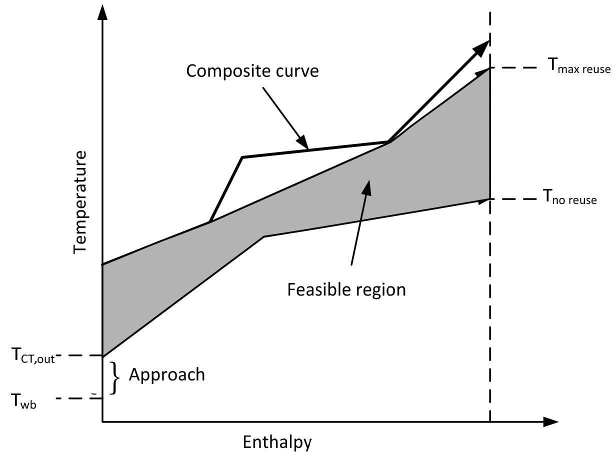

Figure 2-14 Identification of optimization constraints.

Constraints for the optimization model were derived by considering the feasible region given in Figure 2-14. The objective function minimized the total annual cost for the cooling tower and the ozone treatment facilities. After the target was set, the heat exchanger network was synthesized using the water mains method proposed by Kuo and Smith (1997). An illustrative example was presented to demonstrate the proposed technique. By including ozone treatment, the cycles of concentration was increased to 15, which resulted in a significant reduction in blowdown. The authors compared their proposed method with Kim and Smith (2001) method and results showed 46% and 93% reduction in makeup and blowdown respectively. There was also 33.4% savings in total cost.

The main difference between the two methods was targeting and the maximum allowable cycles of concentration. Kim and Smith (2001) targeting was based on fixed cooling tower outlet temperature while in Panjeshahi and Ataei (2008) method, the cooling tower outlet temperature was not fixed. This resulted in opportunities for lower circulating water flowrate and higher cooling tower return temperature. The cycles of concentration in Kim and Smith (2001) design was 3, which resulted in higher blowdown as compared cycles of concentration of 15 proposed by Panjeshahi and Ataei (2008).

Panjeshahi *et al.* (2009) and Ataei *et al.* (2010) used similar design methods proposed by Panjeshahi and Ataei (2008), however Ataei *et al.* (2010) used the cooling tower model which accounted for rain zone and spray zone while Panjeshahi *et al.* (2009) applied pinch migration technique to solve problems where the cooling water line does not form pinch with the composite curve.

A different approach was taken by Ataei *et al.* (2009a). They developed a technique for grassroot design of cooling water systems by incorporating regeneration. In this context, regeneration refers to an air cooler which is used to reduce the cooling tower load. The cooling water target was set using the procedure outlined by Kuo and Smith (1998) for mass exchanger networks. This procedure was adopted for cooling water using networks and could be summarized as follows:

- Divide the cooling water using operations into two groups. Group I is supplied by freshwater and Group II is supplied by regenerated water as shown in Figure 2-15
- For further freshwater reduction, operations can be moved from one group to other

The heat exchanger network was synthesized using the water main method.

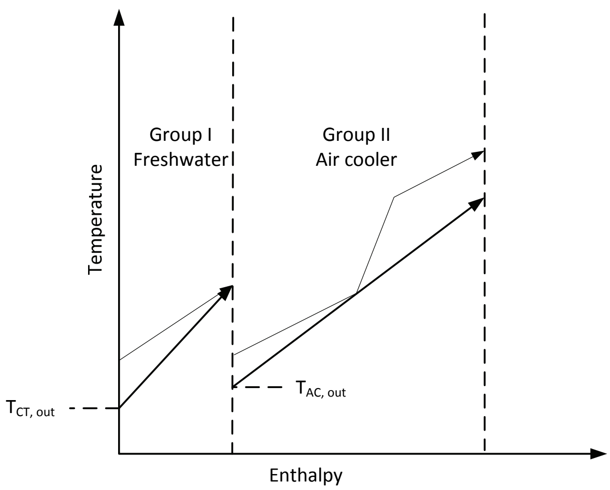

Figure 2-15 Targeting with regeneration.

The work of Panjeshahi and Ataei (2008), Panjeshahi *et al.* (2009), Ataei *et al.* (2010) and Ataei *et al.* (2009a) was limited to the cooling water systems consisting of a single cooling tower while in practice there are many cases with multiple cooling towers.

2.5.1.8 Cooling Water Systems Optimization for Effluent Thermal Treatment

In most practice effluent streams from various processes are mixed and treated in central cooling facility. However mixing of streams at different temperatures reduces the opportunities to recover heat and the driving forces for cooling system are also degraded. Kim *et al.* (2001) introduced a method for the design of effluent cooling water systems. This work was based on Wang and Smith (1994b) and Kuo and Smith (1997) design methodologies for distributed wastewater treatment systems and was adopted for cooling water systems. The design procedure, which was based on the principles of pinch analysis, starts by plotting the composite curve for all effluent streams. The cooling water line was then drawn against the composite curve. The distributed cooling system was designed using the grouping rule proposed by Kuo and Smith (1997) for distributed wastewater treatment systems. For cooling water systems, the grouping rules were summarized as follows:

- All effluent streams with the temperature above pinch must go to the cooling source
- All effluent streams with the temperature located at pinch are partially cooled and partially bypassed
- All effluent streams with the temperature below pinch must bypass the cooling

The authors further used mathematical optimization techniques to obtain the optimum cooling water line. This was done by identifying the cooling water line feasible region using the principles of pinch analysis as shown in Figure 2-16.

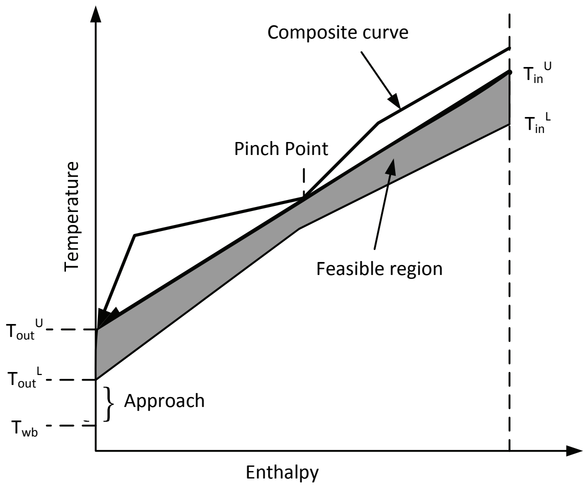

Figure 2-16 Identification of optimization constraints.

The objective function minimized the total cost for the cooling tower, which included both operating and capital cost. From the case study, it was shown that 41.8% saving could be achieved when applying the proposed methodology.

In this work the cooling tower performance was not taken into consideration. The authors assumed fixed cooling tower exit temperature. Ataei *et al.* (2009b) extended this work by including a detailed cooling tower model. The evaporative effect on the cooling tower supply flowrate was also considered, which resulted in a cooling line without pinch. However, the grouping rule was based on problems with pinch, thus the concept of pinch migration developed by Kim & Smith (2001) was used to modify the composite curve to form a new pinch. The grouping rule was then used to design the distributed cooling system.

Panjeshahi *et al.* (2010) used a slightly different targeting procedure. The authors targeted the cooling line by using the cooling tower model and the feasible water supply line. The target was found from the intersection between the feasible water supply line and the cooling tower line. This target also resulted in a cooling line without pinch. The authors followed the same procedure as described by Ataei *et al.* (2009b) to design the distributed cooling system.

2.6 Conclusions

A review on process integration techniques has been presented with a special focus on pinch analysis and its evolution into mass exchange pinch and water pinch. Mathematical optimization techniques and the challenges related to finding the global optimum solutions were also presented. Several authors attempted to address these challenges and the latest developments in this regards were discussed.

It was evident from the presented techniques that graphical techniques have more limitation in terms of its practical application. This technique cannot readily handle many practical constraints e.g. cost functions, forced/forbidden matches, mass transfer function, pressure drop function etc. On contrary, mathematical techniques offer more flexibility to handle practical constraints.

Various studies conducted on cooling water system were also presented. It is apparent from the presented work that most researches were conducted on individual component of the cooling water system. The cooling tower and cooling water network were analyzed separately. Few authors (Kim and Smith, 2001; Panjeshahi & Ataei, 2008; Ataei *et al.,* 2009a; Ponce-Ortega *et al.,* 2010) applied a holistic approach in synthesizing the cooling water system. The work done by those who took into consideration the interaction between the cooling source and cooling water networks was only limited to one cooling source. In practice, operations have two or more cooling towers thus there is a need to conduct a detailed study on cooling water systems with multiple cooling towers.

The important observations which form basis for cooling water system synthesis could be summarized as follows:

- Reusing water in the cooling water network reduces the circulating water flowrate thus debottlenecking the cooling tower

- Maximizing cooling water return temperature with reduced flowrate improve the thermal performance of a cooling tower

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3 MODEL DEVELOPMENT

3.1 Introduction

The cooling water system consists of cooling towers and heat exchanger networks thus, the mathematical model for synthesis and optimization of the cooling water system entails the heat exchanger network model and the cooling tower model. The heat exchanger model is based on a superstructure in which all possible cooling water reuse are explored. The interaction between the heat exchanger network and the cooling towers is investigated using the cooling tower model derived by Kröger (2004). The following sections detail the development of the cooling tower model and the heat exchanger network model.

3.2 Cooling Tower Model

The model was derived by considering a control volume as shown Figure 3-1.

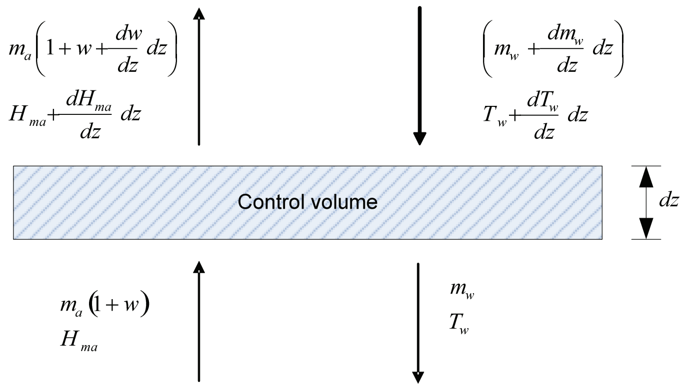

Figure 3-1 Control volume

The following assumptions were made:

- Interface water temperature is the same as the bulk temperature
- Air and water properties are the same at any horizontal cross section
- Heat and mass transfer area is identical

The governing equations that predict the thermal performance of a cooling tower are given by Equation 3-2, 3-4 and 3-14.

The mass and energy balance equations for the control volume are given in Equation 3-1 and 3-3 respectively.

$$m_a(1+w)+\left(m_w+\frac{dm_w}{dz}dz\right)=m_a\left[1+\left(w+\frac{dw}{dz}dz\right)\right]+m_w \qquad (3\text{-}1)$$

The above equation can be further simplified

$$\frac{dm_w}{dz}=m_a\frac{dw}{dz} \qquad (3\text{-}2)$$

$$m_aH_a+\left(m_w+\frac{dm_w}{dz}dz\right)c_{pw}\left(T_w+\frac{dT_w}{dz}dz\right)=m_a\left(H_a+\frac{dH_a}{dz}dz\right)+m_wc_{pw}T_w \qquad (3\text{-}3)$$

Ignoring the second order terms, the equation can further be simplified

$$\frac{dT_w}{dz}=\frac{m_a}{cp_wm_w}\left(\frac{1}{cp_w}\frac{dH_a}{dz}-T_w\frac{dw}{dz}\right) \qquad (3\text{-}4)$$

The enthalpy transfer between the air and water interface is given in equation (3-5).

$$dQ=dQ_m+dQ_c \qquad (3\text{-}5)$$

where,

$dQ_m=H_VK(w_s-w)dA$ (Mass transfer enthalpy) (3-6)

$dQ_c=h(T_w-T_a)dA$ (Convective heat transfer) (3-7)

therefore, $dQ=H_VK(w_s-w)dA+h(T_w-T_a)dA$ (3-8)

The enthalpy of unsaturated air and saturated air are given as

$$H_a = c_{pa}T_a + w(H_{fgwo} + c_{pv}T_a) \tag{3-9}$$

$$H_{as} = c_{pa}T_w + w_s(H_{fgwo} + c_{pv}T_w) = c_{pa}T_w + w_sH_V \tag{3-10}$$

The difference of the above equations

$$H_{as} - H_a = (c_{pa} + wc_{pv})(T_w - T_a) + (w_s - w)H_v \tag{3-11}$$

Or

$$T_w - T_a = \left[(H_{as} - H_a) - (w_s - w)H_v\right] \big/ c_{pma} \tag{3-12}$$

where $c_{pma} = c_{pa} + wc_{pv}$

The enthalpy transfer between the air and water interface

Substituting the expression of $T_w - T_a$ into the enthalpy transfer between the air and water interface equation.

$$dQ = h\left[\frac{h}{Kc_{pma}}(H_{as} - H_a) + \left(1 + \frac{h}{Kc_{pma}}\right)H_v(w_s - w)\right]dA \tag{3-13}$$

The enthalpy change must be equal to the enthalpy change of air stream

$$\frac{dH_a}{dz} = \frac{1}{m_a}\frac{dQ}{dz} = \frac{Ka_{fi}A_{fi}}{m_a}\left(Le_f(H_{as} - H_a) + (1 - Le_f)H_v(w_s - w)\right) \tag{3-14}$$

where $dA = a_{fi}A_{fr}dz$ and $\frac{h}{Kc_{pma}} = Le$

a_{fi} is the wetted area divided by the corresponding volume of the fill and A_{fr} is a frontal area.

$\frac{h}{Kc_{pma}} = Le$ is a Lewis factor.

The Lewis factor was calculated from expression given in Equation 2-25 which was used by Klopper and Kröger (2005) to study the influence of the Lewis factor on the performance prediction of wet cooling tower.

$$Le_f = 0.866^{0.667}\left(\frac{w_s + 0.622}{w + 0.622} - 1\right) / ln\left(\frac{w_s + 0.622}{w + 0.622}\right) \qquad (2\text{-}25)$$

Coefficient of performance

Kröger (2004) suggested a correlation for counter flow fills as shown Equation (3-15)

$$\frac{Ka_{fi}}{m_w} = \frac{a_d}{A_{fr}}\left(\frac{m_w}{m_a}\right)^{d_{da}} ATD^{b_{db}} \qquad (3\text{-}15)$$

where a_d, d_{da}, ATD and b_{db} are system parameters.

Makeup and blow down

Equation 3-16 and 3-17 were used to calculate the makeup and blowdown flowrates.

$$F_M = F_E \frac{CC}{CC - 1} \quad \text{where CC, cycle of concentration} \qquad (3\text{-}16)$$

$$F_M = F_B + F_E \qquad (3\text{-}17)$$

3.3 Heat Exchanger Network Model

The heat exchanger network model was based on the following two possible practical cases.

Case I. Specified maximum cooling water return temperature to the cooling tower without a dedicated source or sink for any cooling water using operation. This situation arises when packing material inside the cooling tower is sensitive to temperature and any cooling tower can supply any water using operation whilst the water using operation can return to any cooling tower.

Case II. Specified maximum cooling water return temperature to the cooling tower with a dedicated source or sink for any cooling water using operation. This is similar to Case I except that the geographic constraints are taken into account. A particular cooling tower can only supply a particular set of heat exchangers and these heat exchangers can only return water to the same supplier.

Mathematical formulation

The mathematical formulation was the extension of the work of Majozi and Moodley (2008). Additional constraints were included in the model to cater for cooling tower, makeup, blowdown, evaporation and variable outlet temperature. The formulation entailed the following sets, parameters, continuous variables and constraints:

Sets:

i = { *i* | *i* is a cooling water using operation}

n = { *n* | *n* is a cooling tower}

Parameters:

$Q(i)$	Duty of cooling water using operation $i(kW)$
$T_{ctout}(n)$	Cooling water supply temperature from cooling tower $n(^oC)$
$OS^n(n)$	Maximum design capacity of cooling tower $n(kg/s)$
$T^U_{out}(i)$	Limiting outlet temperature of cooling water using operation $i(^oC)$
$T^U_{in}(i)$	Limiting inlet temperature of cooling water using operation $i(^oC)$
$F^U_{in}(i)$	Maximum inlet flowrate of cooling water using operation $i(kg/s)$
$T^U_{ret}(n)$	Limiting inlet temperature of cooling water using operation $n(^oC)$
$B(n)$	Blowdown flowrate for cooling tower $n(kg/s)$

$M(n)$ Makeup flowrate for cooling tower $n(kg/s)$

$E(n)$ Blowdown flowrate for cooling tower $n(kg/s)$

c_p Specific heat capacity of water 4.2 (kJ/kg^oC).

T_{amb} Ambient temperature (^oC)

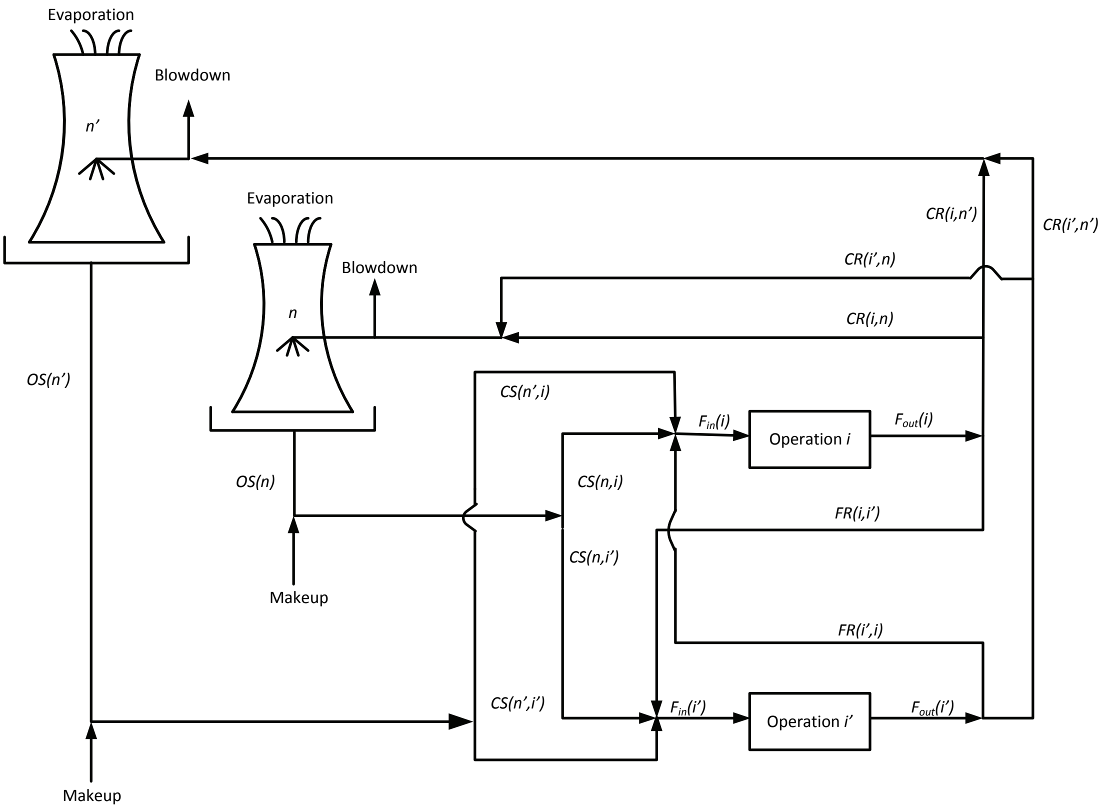

Figure 3-2 Superstructure for a cooling system

Continuous variables:

$OS(n)$ Operating capacity of cooling tower $n(kg/s)$

CW Overall cooling water supply from all cooling tower (kg/s)

$CS(n,i)$ Cooling water supply from cooling tower *n* to cooling water using operating $i(kg/s)$

$CR(i,n)$	Return cooling water to cooling tower *n* from cooling water using operating $i(kg/s)$
$FR(i',i)$	Reuse cooling water to cooling water using operating *i'* from cooling water using operation $i(kg/s)$
$F_{in}(i)$	Total cooling water into cooling water using operation $i(kg/s)$
$F_{out}(i)$	Total cooling water from cooling water using operation $i(kg/s)$
$T_{in}(i)$	Inlet temperature of cooling water to cooling water using operation $i(^oC)$
$T_{out}(i)$	Outlet temperature of cooling water to cooling water using operation $i(^oC)$
$T_{st}(n)$	Cooling water supply temperature form cooling tower *n* after adding makeup (^oC)
$crt(i,n)$	Linearization variable for relaxation technique
$frt(i',i)$	Linearization variable for relaxation technique
$fnt(i)$	Linearization variable for relaxation technique
$tcs(n,i)$	Linearization variable for relaxation technique

The mathematical optimization formulation was developed from the superstructure given in Figure 3-2 by considering energy and mass balance equations across each cooling water using operation and at each node. Two cases that were considered are given in the following sections.

3.3.1.1 Case I

In this case there was no dedicated source or sink for any cooling water using operation. The water using operation can be supplied by one or more cooling towers. The maximum cooling water return temperatures to the cooling towers were also specified. This situation arises when packing material inside the cooling tower is sensitive to temperature and any cooling tower can supply any water using operation and the water using operation can return to any cooling tower.

The model was developed by considering mass and energy balance constraints across each equipment and at each node. The mass and energy balance constraints are given by constraints 3-18 to 3-23 and constraints 3-24 to 3-28 respectively. The design constraints are given by constraints 3-29 to 3-31.

Mass balance constraints

Constraint 3-18 stipulates that the total cooling water is made up of circulating water from all cooling towers:

$$CW = \sum_{n \in N} OS(n) \tag{3-18}$$

Constraints 3-19 and 3-20 ensure that the inlet and outlet cooling water flowrates for any cooling tower are equal. These are the mass balance constraints across any cooling tower.

$$OS(n) = \sum_{n \in N} CS(n,i) - M(n) \qquad \forall n \in N \tag{3-19}$$

$$OS(n) = \sum_{i \in I} CR(i,n) - B(n) - E(n) \qquad \forall n \in N \tag{3-20}$$

The water using operation inlet flowrate is defined by constraint 3-21. It stipulates that the total water flowrate to water using operation i is made up of cooling water from cooling towers and reuse cooling water from other operations.

$$F_{in}(i) = \sum_{n \in N} CS(n,i) + \sum_{i' \in I} FR(i',i) \qquad \forall i \in I \tag{3-21}$$

The outlet flowrate for any water using operation is given by constraint 3-22. The constraint states that the total water flowrate from water using operation *i* is made up of reuse cooling water to other operations and cooling water recycling back to the cooling towers.

$$F_{out}(i) = \sum_{i \in I} CR(i,n) + \sum_{i' \in I} FR(i,i') \qquad \forall i \in I \qquad (3\text{-}22)$$

Constraint 3-23 ensures that water is conserved through each cooling water using operation.

$$F_{in}(i) = F_{out}(i) \qquad \forall i \in I \qquad (3\text{-}23)$$

Energy balance constraints

All energy balance constraints were derived considering the energy balance at a node where streams are mixing. This was done to calculate the resultant temperature for mixing streams. The energy balance for all streams supplying any operation *i* yielded constraint 3-24, which is the definition of inlet temperature into operation *i.*

$$T_{in}(i) = \frac{\sum_{i' \in I} FR(i,i')T_{out}(i') + \sum_{n \in N} CS(n,i)T(n)}{F_{in}(i)} \qquad \forall i \in I \qquad (3\text{-}24)$$

The addition of makeup water results in a change in cooling water temperature. This change is catered for by the energy balance constraint 3-25.

$$T_s(n) = \frac{M(n)T_{amb} + OS(n)T_{ctout}(n)}{\left(CS(n)\right)} \qquad \forall n \in N \qquad (3\text{-}25)$$

The return temperature to any cooling tower is calculated from the energy balance constraint 3-26. This constraint was derived by considering the energy balance for all streams supplying a cooling tower

$$T_{ret}(n) = \frac{\sum_{i \in I} CR(i,n) T_{out}(i)}{\sum_{i \in I} CR(i,n)} \qquad \forall n \in N \qquad (3\text{-}26)$$

Energy balance across water using operation *i* is given by constraint 3-27.

$$\left(T_{out}(i) - T_{in}(i)\right) F_{in}(i) c_p = Q(i) \qquad \forall i \in I \qquad (3\text{-}27)$$

By substituting constraint 3-25 into constraint 3-27, the bilinear term $F_{in}(i)T_{in}(i)$ can be eliminated and constraint 3-25 and constraint 3-27 will be replaced by constraint 3-28.

$$Q(i) + c_p \sum_{n \in N} CS(n,i) T(n) + c_p \sum_{i' \in I} FR(i',i) T_{out}(i') = F_{in}(i) c_p T_{out}(i) \qquad \forall i \in I \qquad (3\text{-}28)$$

Design constraints

The equipments within the cooling water system have the maximum allowable flowrates and temperatures. The design constraints ensure that all the equipments are operated within their specified design limits.

Constraints 3-29 and 3-30 ensure that the cooling towers are operated below their maximum throughputs and the maximum allowable temperatures respectively.

$$OS(n) \le OS^u(n) \qquad \forall n \in N \qquad (3\text{-}29)$$

$$T_{ret}(n) \leq T_{ret}^{u}(n) \qquad \forall n \in N \qquad \text{(3-30)}$$

Constraint 3-31 ensures that the flowrate through water using operations does not exceed its maximum design flowrate.

$$F_{in}(i) \leq F_{in}^{u}(i) \qquad \forall i \in I \qquad \text{(3-31)}$$

The formulation for Case I entails constraints 3-18 to 3-23, 3-25, 3-26 and 3-28 to 3-31. The objective function of this model is to minimize the total cooling water as given in constraint 3-18. Constraints 3-25, 3-26 and 3-28 consist of bilinear terms which are nonconvex thus rendering the model NLP. This model is difficult to initialize because the starting point might be infeasible or the solution might be locally optimum. To overcome these difficulties the technique proposed by Quesada and Grossmann (1995) was used to linearize the bilinear terms. This technique uses the upper and the lower bounds to create a convex space for the bilinear terms as shown in the next section.

Reformulation linearization technique

The technique starts by assigning a variable to all bilinear terms as shown in below.

Let,

$$crt(i,n) = CR(i,n)T_{out}(i) \quad \forall n \in N, i \in I$$

$$frt(i',i) = FR(i',i)T_{out}(i) \quad i \in I, i' \in I$$

$$fnt(i) = F_{in}(i)T_{out}(i) \quad \forall i \in I$$

$tcs(n,i) = CS(n,i)T_s(n) \quad \forall n \in N, i \in I$

The upper and the lower bound for variables in each bilinear term were defined as follows: The lower bound for the flowrates is zero and the upper bound was given a value. The lower bound for the temperatures is the wet bulb temperature and the upper bound was also assigned a value. The bilinear term $CR(i,n)T_{out}(i)$ can now be defined by constraints 3-32 to 3-35.

$$crt(i,n) \geq OS^u(n)T_{out}(i) + CR(i,n)T_{out}^u(i) - OS^u(n)T_{out}^u(i) \quad \forall n \in N, \forall i \in I \tag{3-32}$$

$$crt(i,n) \geq CR(i,n)T_{out}^L(i) \quad \forall n \in N, \forall i \in I \tag{3-33}$$

$$crt(i,n) \leq OS^u(n)T_{out}(i) + CR(i,n)T_{out}^L(i) - OS^u(n)T_{out}^u(i) \quad \forall n \in N, \forall i \in I \tag{3-34}$$

$$crt(i,n) \leq CR(i,n)T_{out}^u(i) \quad \forall n \in N, \forall i \in I \tag{3-35}$$

Similarly, the bilinear term $FR(i',i)T_{out}(i)$ is defined by constraints 3-36 to 3-39, the bilinear term $F_{in}(i)T_{out}(i)$ by constraints 3-40 to 3-43 and the bilinear term $CS(n,i)T_s(n)$ by constraints 3-44 to 3-47.

$$frt(i',i) \geq F_{in}^u(i)T_{out}(i') + FR(i',i)T_{out}^u(i) - F_{in}^u(i)T_{out}^u(i) \quad i \in I, i' \in I \tag{3-36}$$

$$frt(i',i) \geq FR(i',i)T_{out}^L(i) \quad i \in I, i' \in I \tag{3-37}$$

$$frt(i',i) \leq F_{in}^{u}(i)T_{out}(i') + FR(i',i)T_{out}^{L}(i) - F_{in}^{u}(i)T_{out}^{L}(i) \quad i \in I, i' \in I \tag{3-38}$$

$$frt(i',i) \leq FR(i',i)T_{out}^{u}(i) \quad i \in I, i' \in I \tag{3-39}$$

$$fnt(i) \geq F_{in}^{u}(i)T_{out}(i) + F_{in}(i)T_{out}^{u}(i) - F_{in}^{u}(i)T_{out}^{u}(i) \quad \forall i \in I \tag{3-40}$$

$$fnt(i) \geq F_{in}(i)T_{out}^{L}(i) \quad \forall i \in I \tag{3-41}$$

$$fnt(i) \leq F_{in}^{u}(i)T_{out}(i) + F_{in}(i)T_{out}^{L}(i) - F_{in}^{u}(i)T_{out}^{L}(i) \quad \forall i \in I \tag{3-42}$$

$$fnt(i) \leq F_{in}(i)T_{out}^{u}(i) \quad \forall i \in I \tag{3-43}$$

$$tcs(n,i) \geq OS^{u}(n)T_{s}(n) + CS(n,i)T_{s}^{u}(n) - OS^{u}(n)T_{s}^{u}(n) \quad \forall n \in N, i \in I \tag{3-44}$$

$$tcs(n,i) \geq CS(n,i)T_{s}^{L}(n) \quad \forall n \in N, i \in I \tag{3-45}$$

$$tcs(n,i) \geq OS^{u}(n)T_{s}(n) + CS(n,i)T_{s}^{L}(n) - OS^{u}(n)T_{s}^{L}(n) \quad \forall n \in N, i \in I \tag{3-46}$$

$$tcs(n,i) \geq CS(n,i)T_{s}^{u}(n) \quad \forall n \in N, i \in I \tag{3-47}$$

Introduction of linearization variables require constraint 3-26, constraint 3-28 and constraint 3-25 to be modified as shown in constraint 3-48, constraint 3-49 and constraint 3-50, respectively.

$$T_{ret}(n) = \frac{\sum_{i \in I} crt(i,n)}{\sum_{i \in I} CR(i,n)} \qquad \forall n \in N \tag{3-48}$$

$$Q(i) + c_p \sum_{n \in N} tcs(n,i) + c_p \sum_{i' \in I} frt(i',i) = fnt(i) c_p \quad \forall i \in I \tag{3-49}$$

$$\sum_{i \in I} tcs(n,i) = M(n) T_{amb} + OS(n) T_{ctout}(n) \qquad \forall n \in N \tag{3-50}$$

The relaxed LP model for Case I consists of Constraints 3-18 to 3-23, 3-29 to 3-31 and 3-32 to 3-50. To get the solution for Case 1, the relaxed model is first solved by minimizing the total cooling water. The solution of the relaxed model is then used as a starting point for solving the exact model.

3.3.1.2 Case II

In this case there are dedicated source and sink for any cooling water using operation. This implies that no pre-mixing or post-splitting of cooling water return is allowed. A set of heat exchanger can only be supplied by one cooling tower. Furthermore, the return cooling water from cooling water using operation must supply the source cooling tower. However, reuse of water within the network is still allowed. All the constraints in Case I are still applicable. Few constraints needed to be added to control the source and the sink.

Constraints 3-51 and 3-52 prevent pre-mixing. Constraint 3-51 ensures that the supply flowrate from any cooling tower to operation *i* cannot exceed the maximum flowrate. Constraint 3-52 ensures that cooling water using operation *i* can only be supplied by a maximum of one cooling tower.

$$CS(n,i) \le F_{in}^{u}(i)ys(n,i) \qquad \forall i \in I \qquad \forall n \in N \qquad (3\text{-}51)$$

$$\sum_{n \in N} ys(n,i) \le 1 \qquad \forall n \in N \qquad (3\text{-}52)$$

Post-splitting can also be prevented by constraints 3-53 and 3-54. Constraint 3-53 ensures that the return flowrate from operation *i* to any cooling tower cannot exceed the maximum flowrate into that operation. Constraint 3-54 ensures that cooling water using operation *i* can supply a maximum of one cooling tower.

$$CR(i,n) \le F_{in}^{u}(i)yr(i,n) \qquad \forall i \in I \qquad \forall n \in N \qquad (3\text{-}53)$$

$$\sum_{n \in N} yr(i,n) \le 1 \qquad \forall n \in N \qquad (3\text{-}54)$$

Constraints 3-55 and 3-56 ensure that the source and the sink cooling water supply is the same for a particular cooling water using operation.

$$yr(i,n) \le ys(n,i) + \left(2 - \sum_{n \in N} ys(n,i) - \sum_{n \in N} yr(i,n)\right) \forall i \in I \ \forall n \in N \quad (3\text{-}55)$$

$$yr(i,n) \geq ys(n,i) - \left(2 - \sum_{n \in N} ys(n,i) - \sum_{n \in N} yr(i,n) \right) \forall i \in I \ \forall n \in N \quad \text{(3-56)}$$

The formulation for Case II entails constraints 3-18 to 3-23, 3-25, 3-26, 3-28 to 3-31, 3-51 to 3-56. Constraints 3-51 to 3-56 consist of binary variables while constraints 3-25, 3-26 and 2-31 consist of bilinear terms which are nonconvex. This renders the model MINLP. Similar to Case I, the model was linearized using the linearization relaxation procedure by Quesada and Grossmann (1995) as shown in constraints 3-32 to 3-50. The relaxed model entails constraints 3-18 to 3-23, 3-29 to 3-30 and 3-32 to 3-56.

The solution procedure starts by first solving the relaxed model by minimizing the total cooling water. The solution of the relaxed model is then used as a starting point for solving the exact model.

To synthesize the overall cooling water system, both the cooling tower model and the heat exchanger network model should be solved simultaneously. The algorithm for synthesizing the overall cooling water system is given in the following section.

3.4 The overall effectiveness for multiple cooling towers

Bernier (2004) evaluated the heat rejection performance of a cooling tower using term called coefficient of performance $\left({KaV}/{m_w} \right)$. The author showed that the coefficient of performance for the cooling tower increases with a decrease in water flowrate. Based on this observation, Smith and Kim (2001) further showed that the cooling tower effectiveness increases as the return temperature increases and/or when the water flowrate decreases. In this section the overall cooling tower effectiveness for cooling water system consisting of multiple cooling towers is presented. Using the thermodynamic definition of effectiveness for one cooling tower, the overall cooling towers effectiveness is calculated from Equation 3-57.

$$\varepsilon_{averall} = \frac{\sum_{n=1}^{N} Q_{act}^{n}}{\sum_{n=1}^{N} Q_{\max}^{n}} \tag{3-57}$$

where superscript n represents cooling tower n in circuit consisting of N number of cooling towers. The term $\sum_{n=1}^{N} Q_{act}^{n}$ represents the total amount of actual heat transferred and $\sum_{n=1}^{N} Q_{max}^{n}$ represents the total maximum theoretical heat that can be transferred.

The expression for Q_{act} and Q_{max} used for each cooling tower was provided by Jaber and Webb (1987) as shown in Chapter 2.

3.5 Solution algorithm

The solution procedure can be applied for both cases considered. The first step is to optimize the heat exchanger network model without the cooling towers. The results from the first iteration, which are cooling water return (CWR) temperatures and flowrates, become the input to the cooling tower models. Each cooling tower model then predicts the outlet water temperatures and flowrates. This is done by first assuming the outlet water temperature of a cooling tower. The assumption is done by subtracting 0.5 ^{o}C from the given cooling tower inlet temperature. The three governing mass and heat transfer equations, i.e. Equation 3-2, 3-4 and 2-14 are then solved numerically using forth order Runge_Kutta method starting from the bottom of the cooling tower moving upwards at stepsize Δz. When the maximum height is reached, the temperature at this point will be compared with the CWR temperature. If the two agree within a specified tolerance, the cooling tower model will stop and the outlet temperature will be given as the assumed temperature, else the inlet temperature will be adjusted until the CWR temperature agrees with the calculated temperature.

The predicted outlet cooling tower temperatures and flowrates then become the input to the heat exchanger network model. If the outlet temperature of the cooling tower model agrees with the previous inlet temperature to the heat exchanger network model, the algorithm stops which implies that final results have been obtained. Otherwise the iteration continues. The solution algorithm flowchart is shown in Figure 3-3.

The cooling tower model was solved using MATLAB while the cooling water network was solved using GAMS platform. The procedure used to link MATLAB and GAMS was presented by Ferris (2005).

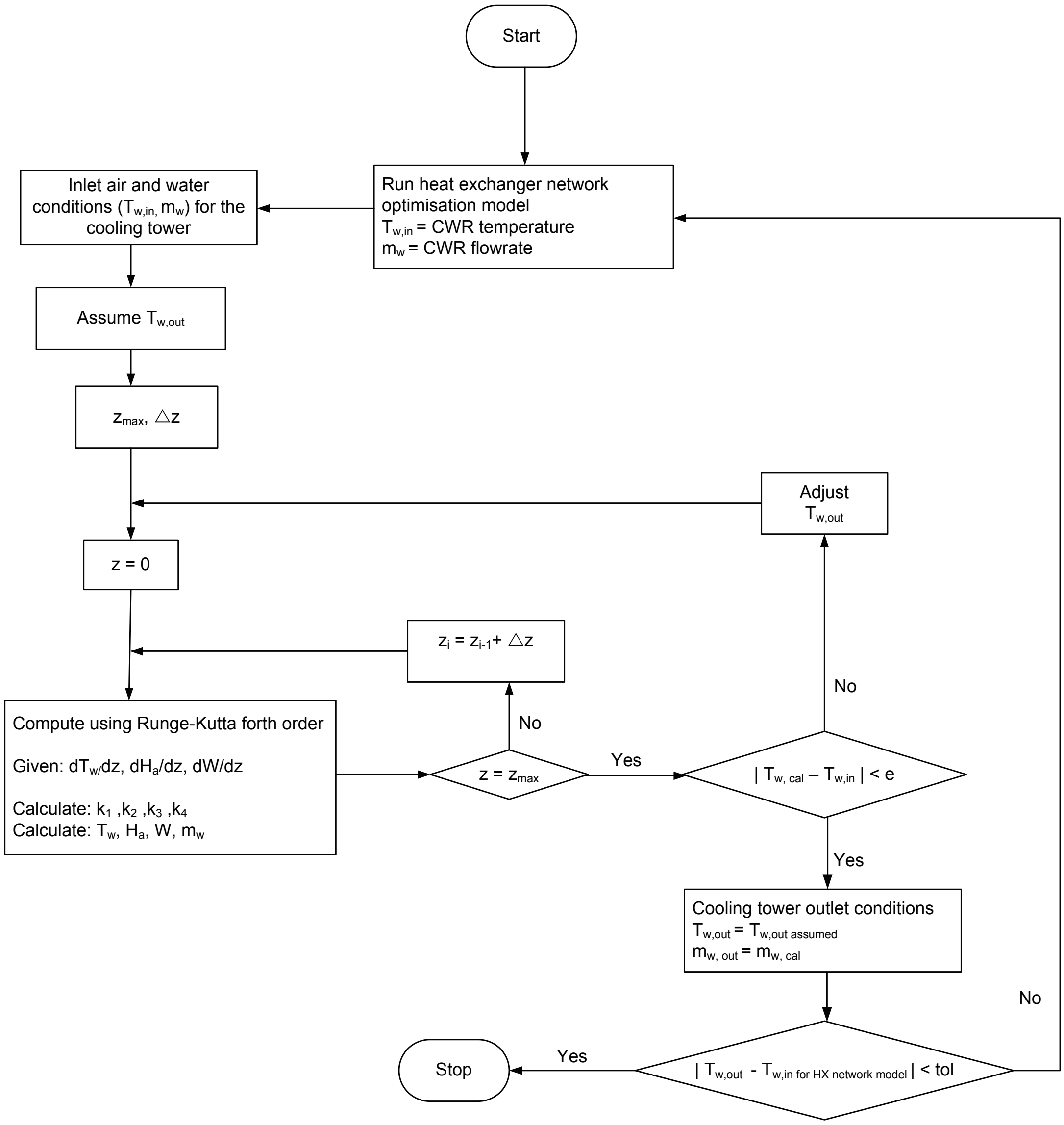

Figure 3-3 Flowchart for cooling water system model

3.6 Conclusion

The mathematical formulation for synthesis and optimization cooling water system with multiple cooling towers has been presented. The formulation consists of the heat exchanger network and the cooling tower model. Two practical cases were considered when developing the heat exchanger network model. The first case resulted in nonlinear programming (NLP) formulation and the second case yield mixed integer nonlinear programming (MINLP).

References

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2. Ferris, M.C. (2005). MATLAB and GAMS: Interfacing optimization and visualization software. *Computer Sciences Department, University of Wisconsin.*
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4. Kim, J.K., & Smith, R. (2001). Cooling water system design. *Chemical Engineering Science*, 56, 3641-3658.
5. Klopper, J.C., & Kröger, D.G. (2005). The Lewis factor and its influence on the performance prediction of wet-cooling towers. *International Journal of Thermal Science*, 44, 879-884.
6. Kröger, D.G. (2004). Air-cooled heat exchangers and cooling towers: mass transfer and evaporative cooling. Penn Well Corporation, USA.

4 CASE STUDIES

4.1 Introduction

In this section the application of the proposed technique is demonstrated by considering a case study taken from the paper by Majozi and Moodley (2008). Two practical cases which were illustrated in Chapter 3 are considered.

4.2 Base case

The cooling water system in Figure 4-1 shows a set of heat exchanger networks which are supplied by a set of cooling towers. Each cooling water using operation is supplied by fresh water from the cooling tower and return back to the cooling tower. The implication of these arrangements results in higher return cooling water flow rate and low return cooling water temperature thus reducing cooling tower efficiency (Bernier, 1994).

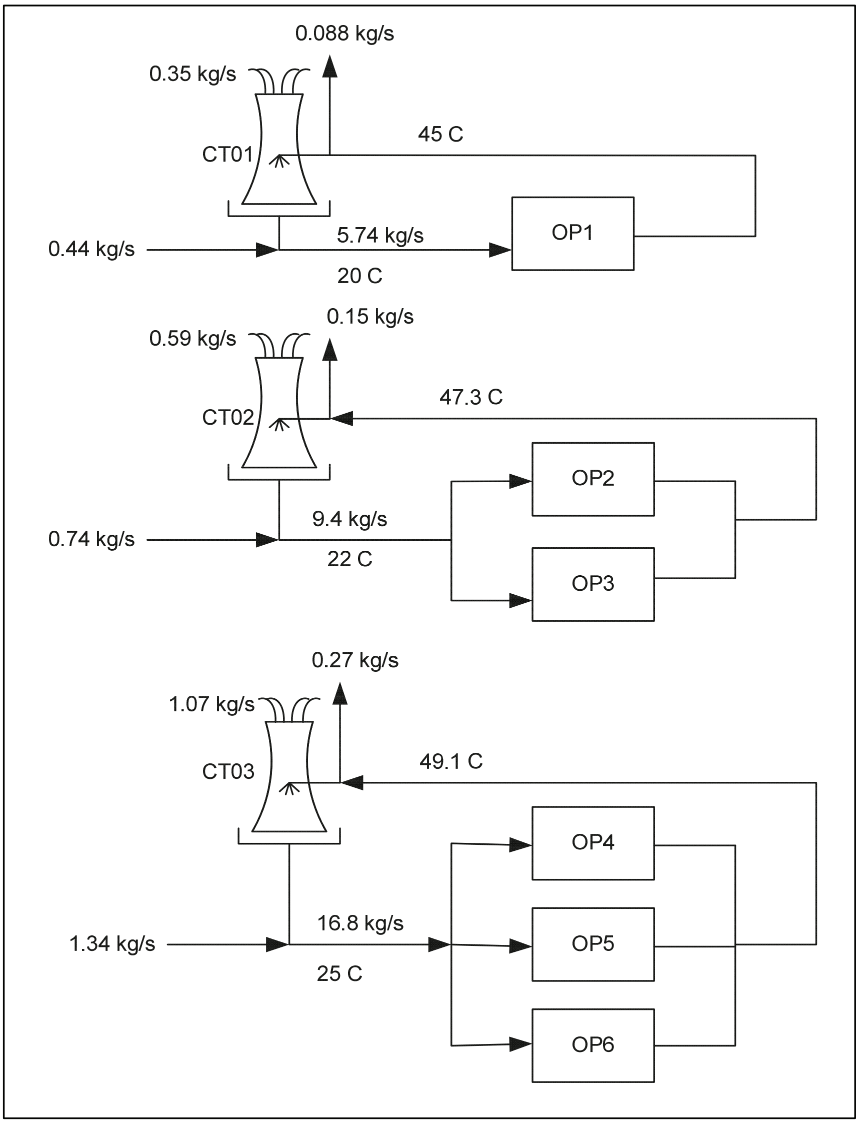

Figure 4-1 Base case

The heat duties, temperature limits and design information are shown in Table 4-1 and Table 4-2. T_{ret}^{u} is the maximum allowable temperature for packing inside the cooling towers while OS^{u} is the maximum flowrate of the cooling tower before flooding. T_{in}^{u} and T_{out}^{u} are the thermodynamic temperature limits for the inlet and outlet temperature of the cooling water using operation respectively.

Table 4-1 Cooling towers design information

Cooling towers	T_{ret}^{u} **(C)**	OS^{u}
CT01	50	9.6
CT02	50	16
CT03	55	20

Table 4-2 Limiting cooling water data

Operations	T_{in}^{u} **(C)**	T_{out}^{u} **(C)**	F_{in} **(kg/s)**	**Q(i)(kW)**
OP01	30	45	9.52	600
OP02	40	60	3.57	300
OP03	25	50	7.62	800
OP04	45	60	7.14	600
OP05	40	55	4.76	300
OP06	30	45	11.1	700

4.3 Case I

In this case each cooling tower can supply any cooling water using operation. The return streams from any cooling water using operation can go to any cooling tower. The return temperature to any cooling tower is however specified. The heat exchanger network model consists of bilinear terms which are nonconvex thus rendering the formulation NLP problem. The objective function of this model was to minimize the total cooling water. The model was solved using MINOS in a GAMS platform.

Figure 4-2 shows the heat exchanger network after applying the methodology described above.

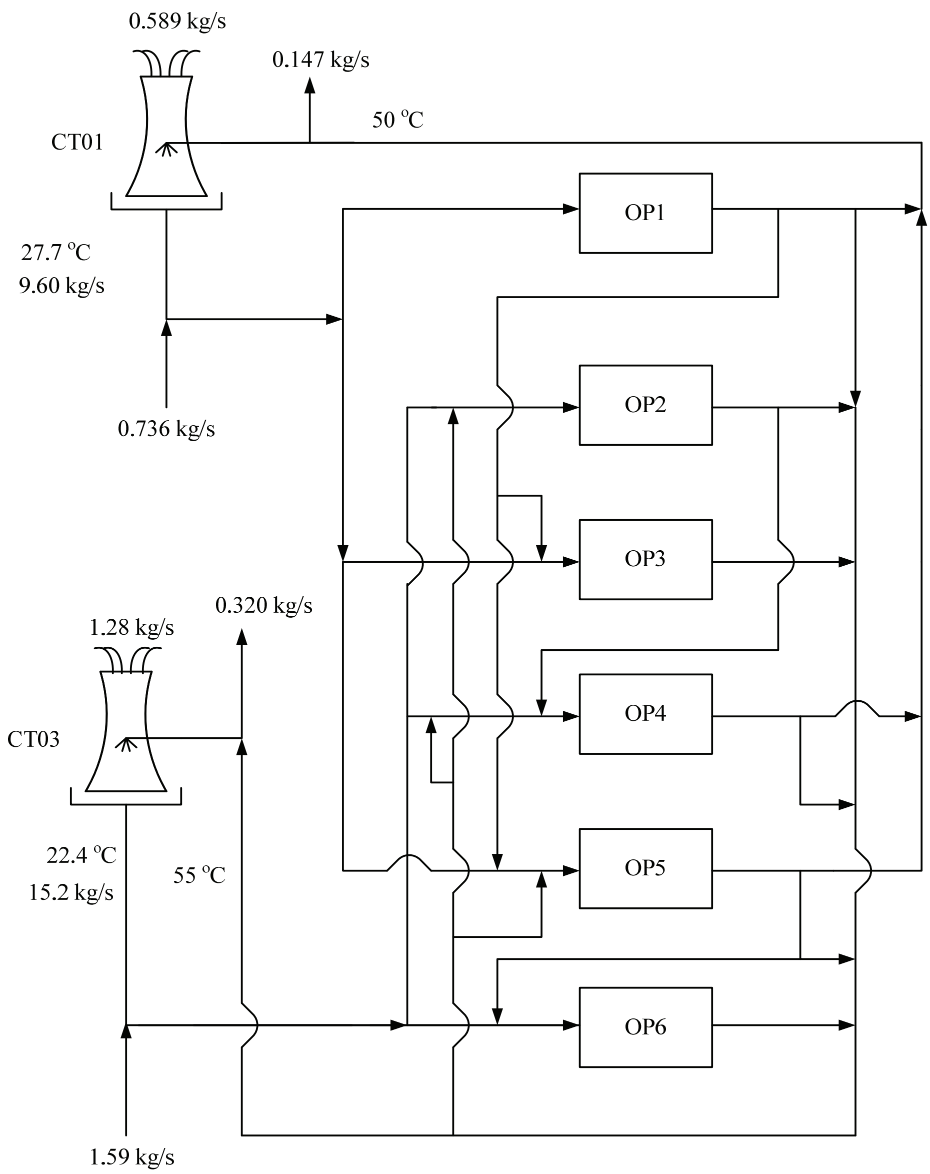

Figure 4-2 Final design of the cooling water system

By exploiting the opportunity for cooling water reuse, the overall circulating water decreased by 22 % and one cooling tower was eliminated. The cooling tower inlet temperatures are at their maximum values.

These results show the opportunity to increase the heat duties, through expansions, without investing on a new cooling tower. The only additional investment required is on piping for reuse streams. For this case study the makeup and the blowdown was also decreased by 7%. However the decrease in makeup and blowdown cannot be guaranteed for all practical case studies.

Table 4-3 Results summary

Stream	base case(kg/s)	results(kg/s)
Make up	2.52	2.33
Blowdown	0.50	0.47
Circulating water	31.94	24.80

4.4 Case II

In this case a cooling tower can only supply a dedicated set of heat exchangers. This implies that each operation can only be supplied by one cooling tower. The return streams from any cooling water using operation can only go to its supplier cooling tower. The return temperatures to the cooling towers are also specified. The heat exchanger network model also consists of binary and bilinear terms which rendered MINLP. The model was solved using SBB solver in a GAMS platform. CONOPT solver was used for NLP subproblems. Figure 4-3 shows the heat exchanger network after applying the methodology described above.

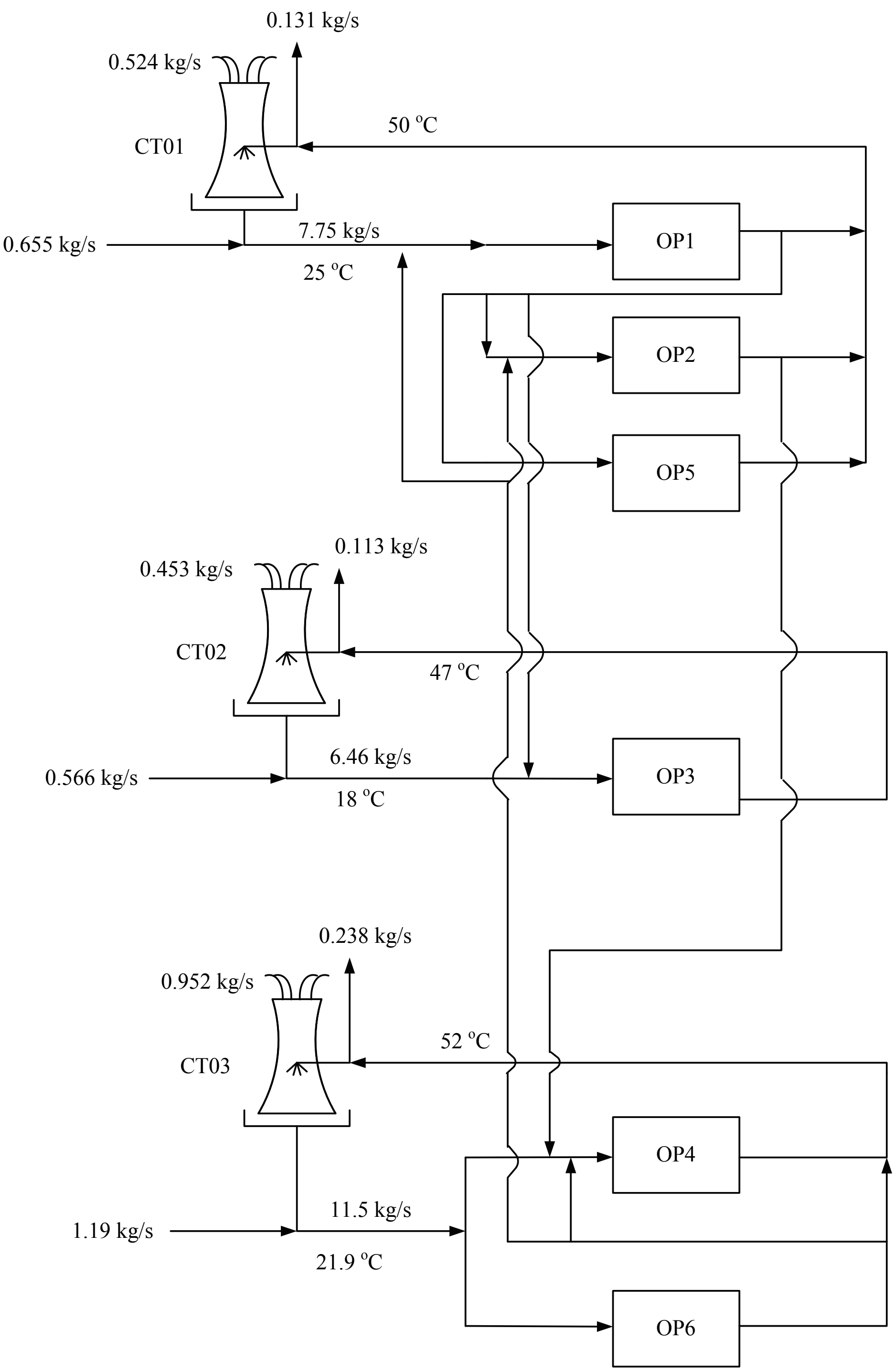

Figure 4-3 Final design of the cooling water system

By allowing for the cooling water reuse, the overall circulating water decreased by 20 %. This will decrease the pumping power requirement for the circulating pump thus reducing the pumping cost. The cooling towers spare capacity is also increased giving opportunities for increased heat load without investing in a new cooling tower. To satisfy the required heat duties with the reduced flowrate, the return temperature to the cooling towers is increased to the maximum value. The makeup and the blowdown are also decreased by 4%. As abovementioned, the decrease in makeup and blowdown cannot be guaranteed for all practical case studies.

Table 4-4 Results summary

Stream	base case(kg/s)	results(kg/s)
Make up	2.52	2.41
Blowdown	0.50	0.48
Circulating water	31.94	25.69

4.5 The overall effectiveness for multiple cooling towers

The overall effectiveness was calculated based on the procedure outlined in Chapter 3. Table 4-5 and 4-6 shows an increase in the overall effectiveness of the cooling tower when applying the proposed design methodology. The increase in the overall cooling towers effectiveness is attributed to a decrease in the overall circulating water and an increase in return water temperature.

The results in Table 4-5 are based on optimization where the total cooling water flowrate is minimized. In this case the individual cooling tower flowrate is allowed to increase or decrease as long as the total flowrate for the circuit is minimized. This implies that the individual cooling tower effectiveness can increase or decrease depending on the flowrate and return temperature. In this case the decrease in the total circuit flowrate does not necessarily mean an increase in the overall effectiveness. This is due to the fact that some cooling towers might be operating at their maximum capacities.

Table 4-5 Effectiveness for base Case, Case I and Case II

	Effectiveness	Circulating water (kg/s)
Base case	0.90	31.94
Case I	0.91	24.80
Case II	0.94	25.69

Table 4-6 shows the results where optimization was based on increasing the overall effectiveness. This was done by setting the flowrate for each cooling tower in Figure 4.1 as the maximum allowable flowrate for a given cooling tower. The objective function minimized the total circulating water flowrate as before. The results showed a decrease in total circulating water flowrate with an increase in the overall effectiveness. In this case an increase in overall effectiveness can be guaranteed because the flowrate for each cooling tower will decrease or remains de same while the return temperature is maximized.

Table 4-6 Effectiveness for base Case, Case I and Case II

	Effectiveness	Circulating water (kg/s)
Base case	0.90	31.94
Case I	0.93	24.98
Case II	0.94	24.69

4.6 Conclusion

Case studies to demonstrate the proposed technique were presented. The results obtained using this technique shows that by exploring the opportunities for cooling water reuse the cooling tower can be debottlenecked. This implies that a given set of cooling towers can manage an increased heat load. The proposed technique can also improve the overall effectiveness for the cooling towers. There is also a potential for the reduction of makeup and blowdown water flowrate.

References

1. Bernier, M.A. (1994). Cooling tower performance: Theory and experiments. *ASHRAE Transactions Research*, 100, 114-121.
2. Majozi, T., & Moodley, A.(2008), Simultaneous targeting and design for cooling water systems with multiple cooling water supplies. *Computer and Chemical Engineering*, 32, 540-551.

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5 CONCLUSIONS AND RECOMMENDATIONS

The mathematical technique for optimization and synthesis cooling water system with multiple cooling towers has been presented. This technique is more holistic because it caters for the effect of cooling tower performance on heat exchanger network. The cooling tower thermal performance was predicted using the mathematical model. The results obtained using this technique are more practical, since all components of the cooling water system are included in the analysis.

5.1 Debottlenecking the cooling towers

The propose technique debotllenecked the cooling towers by decreasing the circulating water flowrate. This implies that a given set of cooling towers can manage an increased heat load. From the case studies, 22% decrease in circulating water flowrate was realized. A decrease in the overall circulation water has an added benefit of decreasing the overall power consumption of the circulating pumps. The blowdown and makeup were also decreased by 7%, however this cannot be guaranteed for all practical case studies.

5.2 Overall effectiveness for multiple cooling towers

An improvement of up to 4% was realized by applying the proposed technique in a case study. It is important to note that in a cooling water systems consisting of multiple cooling towers, a decrease in circulating water flowrate does not necessarily imply an improvement in the overall effectiveness. However, improving the effectiveness will yield a decreased in circulating water flowrate. Thus, the results which show a better effectiveness and the reduced flowrate can be achieved by setting the objective function as maximizing effectiveness as oppose to minimizing the circulating water flowrate.

5.3 Recommendations

Although the proposed methodology explore the interactions between the cooling towers and the cooling water networks, there are still more features that needs to be included in the model.

Pressure drop

The proposed technique results in a more complex network due to reuse streams that are introduced, thus it is important for future work to study pressure drop. This will also explore the need to invest on a new pump.

Costs

The proposed network will also require extra piping for reuse streams, thus it is imperative to study the economical impacts of the proposed methodology.

Environment

Although the case studies showed a decrease in makeup and blowdown, this cannot be guaranteed for all practical cases. More work still need to be done to qualify and quantify these observations.

Global optimality

In the proposed methodology the cooling tower model and the cooling water network model are solved using different platforms thus global optimality cannot be guaranteed for the whole system. The issue of global optimality can be better addressed by formulating the problem using a single platform for the entire cooling water system.

NOMENCLATURE

a, a_{fi}	surface area per unit volume (m^2/m^3)
A	area (m^2)
A_{fr}	frontal area (m^2)
C	concentration (g/l)
CC	cycles of concentration
CP	specific heat capacity times flowrate (kJ/ (s.°C))
c_p	specific heat capacity (J/ (kg.°C))
CW	cooling water
CWR	cooling water return
F	Flowrateb (kg/s)
H	Enthalpy (J/kg)
Q	rate of heat transfer (W)
h	heat transfer coefficient (W/ (m^2.s))
HP	high pressure steam
HX	heat exchanger
K, K_a	mass transfer coefficient (kg/ (m^2.s))
Le_f	Lewis factor
LP	low pressure steam
m	flowrate (kg/s)
ML	mass load (kg)
NC	number of cold streams
NH	number of hot streams
T	temperature (K)
V	volume (m^3)
w	humidity (kg/kg)
z	cooling tower height (m)
ρ	density (kg/m^3)
η	efficiency
ε	effectiveness
δ	enthalpy correction factor (kJ/kg)

a_d, d_{da}, ATD, b_{db}, x,y,z,n Cooling towers fill parameters/constants

Subscripts

a	air
B	blowdown
c	cold
CT	cooling tower
E	evaporation
h	hot
in	inlet
M	makeup
ma	moist air
min	minimum
max	maximum
out	oulet
s	saturation
v	vapor
w	water
wb	wet bulb

APPENDICES

APPENDIX A: Case I

Initial model

```
Sets
i cooling water using operation /hx1*hx6/
n cooling towers            /ct1*ct3/
alias(i,j);
Parameters
Q(i) heat loads /hx1 600
            hx2 300
            hx3 800
            hx4 600
            hx5 300
            hx6 700/
Toutmax(i) limiting outlet temperature /hx1 45
                            hx2 60
                            hx3 50
                            hx4 60
                            hx5 55
                            hx6 45/
Tinmax(i)  limiting inlet temperature /hx1 30
                           hx2 40
                           hx3 25
                           hx4 45
                           hx5 40
                           hx6 30/
Finmax(i) maximum flowrate through cooling water using operation /hx1 9.52
                                               hx2 3.57
                                               hx3 7.62
                                               hx4 7.14
                                               hx5 4.76
                                               hx6 11.1/
OSmax(n) maximum cooling tower capacity /ct1 9.6
                             ct2 16
                             ct3 20/
```

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Tctout(n) cooling tower supply temperature /ct1 20

ct2 22

ct3 25/

Tretmax(n) maximum return temperature /ct1 50

ct2 50

ct3 55/

E(n) evaporation /ct1 0.22

ct2 0.36

ct3 0.62/

Tsmax(n) maximum circuit supply temperature /ct1 50

ct2 50

ct3 55/

Tmin minimum temperature is equal to wet bulb temperature /15/

cp /4.2/

Tamb /25/

CC /5/

B(n)/ct1 0.05

ct2 0.09

ct3 0.15/

M(n)/ct1 0.27

ct2 0.45

ct3 0.77/;

Variables

CW overall cooling water supply

OS(n) operating cooling tower flowrate

OSin(n)

CS(n,i) fresh cooling water supply from n cooling tower to i operation

CR(i,n) return cooling water from i operation to n cooling tower

CTin cooling tower inlet flowrate

CTout cooling tower outlet flowrate

FR(j,i) cooling water reuse from j to i

APPENDICES

Fin(i) inlet cooling water using operation flowrate

Fout(i) outlet cooling water using operation flowrate

Tin(i) inlet cooling water using operation temperature

Tout(i) outlet cooling water using operation temperature

Tret(n) return temperature to the cooling tower n

Ts(n) temperature after adding makeup

Mt total make up

Bt total blow down

Et total evaporation

crt(i,n) linearization variable CR*Tout

frt(j,i) linearization variable FR*Tout

fnt(i)   linearization variable F*Tout

tos(n) linearization variable Ts*OS

tcs(n,i) linearization variable Ts*CS;

Positive variable

OS

OSin

CS

CR

FR

Fin

Fout

Tin

Tout

Tret

Ts

Bt

Mt

Et

crt

frt

fnt

tos

tcs;

APPENDICES

Temperature Limits(lower bounds)

Tin.lo(i) = 15;

Tout.lo(i) = 15;

Tret.lo(n) = 15;

Ts.lo(n) = 15;

Equations

Linear Model

overall_cooling_water total circulating water

cooling_towerMB1(n) cooling water mass balance from the top

cooling_towerMB2(n) cooling water mass balance from the bottom

circuit_supply_temperature(n) supply temperature after adding makeup water

operation_inlet_flowrate(i) inlet temperature to water using operation

operation_recycle(i) operation outlet flowrate

total_blowdown total blowdown

total_makeup total makeup

total_evaporation total evaporation

operationMB(i) operation mass balance

cooling_tower_design(n) cooling tower flowrate limit

operation_design(i) operation flowrate limit

return_temperature_coolingtower(n) return temperature to the cooling tower

return_temp_limit_operation(i) limit of the return temperature to the cooling tower

operationEB(i) operation energy balance

operation_inlet_temp(i) operation inlet temperature calculation

linearization1(i,n) linerization of CR*Tout

linearization2(i,n) linerization of CR*Tout

linearization3(i,n) linerization of CR*Tout

linearization4(i,n) linerization of CR*Tout

linearization5(j,i) linerization of FR*Tout

linearization6(j,i) linerization of FR*Tout

linearization7(j,i) linerization of FR*Tout

linearization8(j,i) linerization of FR*Tout

linearization9(i) linerization of Fin*Tout

linearization10(i)	linerization of Fin*Tout
linearization11(i)	linerization of Fin*Tout
linearization12(i)	linerization of Fin*Tout
linearization13(n)	linerization of OS*Ts
linearization14(n)	linerization of OS*Ts
linearization15(n)	linerization of OS*Ts
linearization16(n)	linerization of OS*Ts
linearization17(n,i)	linerization of CS*Ts
linearization18(n,i)	linerization of CS*Ts
linearization19(n,i)	linerization of CS*Ts
linearization20(n,i)	linerization of CS*Ts

```
*Nonlinear Model*
loverall_cooling_water
lcooling_towerMB1(n)
lcooling_towerMB2(n)
lcircuit_supply_temperature(n)
loperation_inlet_flowrate(i)
loperation_recycle(i)
ltotal_makeup
ltotal_evaporation
loperationMB(i)
lcooling_tower_design(n)
loperation_design(i)
lreturn_temp_coolingtower(n)
lreturn_temp_limit_operation(i)
loperationEB1(i)
lCWR_temp(n)
lCWR_flow(n);

overall_cooling_water.. CW =e= sum(n,OS(n));
cooling_towerMB1(n).. OS(n)+M(n) =e= sum(i,CS(n,i));
cooling_towerMB2(n).. OS(n) =e= sum(i,CR(i,n))-B(n)-E(n);
circuit_supply_temperature(n).. sum(i,tcs(n,i)) =e=  Tamb*M(n) +  Tctout(n)*OS(n);
operation_inlet_flowrate(i).. Fin(i) =e= sum(n,CS(n,i))+ sum(j,FR(j,i)$(ord(i) ne ord(j)));
```

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```
operation_recycle(i).. Fout(i) =e= sum(j,FR(i,j)$(ord(i) ne ord(j))) + sum(n,CR(i,n));


total_blowdown.. Bt =e= sum(n,B(n));
total_makeup..Mt =e= sum(n,M(n));
total_evaporation.. Et =e= sum(n,E(n));
operationMB(i)..Fin(i) =e= Fout(i);
cooling_tower_design(n).. OS(n) =l= OSmax(n);
operation_design(i).. Fin(i)=l= Finmax(i);
return_temperature_coolingtower(n).. Tretmax(n)*sum(i,CR(i,n))=g= sum(i,crt(i,n));
return_temp_limit_operation(i).. Tout(i) =l= Toutmax(i);
operationEB(i).. cp*fnt(i)  =e= Q(i) + cp*sum(j,frt(j,i)$(ord(i) ne ord(j)))+ cp*sum(n,tcs(n,i));


linearization1(i,n).. crt(i,n) =g= OSmax(n)*Tout(i) + CR(i,n)*Toutmax(i)-OSmax(n)*Toutmax(i);
linearization2(i,n).. crt(i,n) =g= CR(i,n)*Tmin;
linearization3(i,n).. crt(i,n) =l= OSmax(n)*Tout(i)+CR(i,n)*Tmin-OSmax(n)*Tmin;
linearization4(i,n).. crt(i,n) =l= CR(i,n)*Toutmax(i);
linearization5(j,i)$(ord(i) ne ord(j)).. frt(j,i) =g= Finmax(i)*Tout(j) + FR(j,i)*Toutmax(i)-Finmax(i)*Toutmax(j);
linearization6(j,i)$(ord(i) ne ord(j)).. frt(j,i) =g= FR(j,i)*Tmin;
linearization7(j,i)$(ord(i) ne ord(j)).. frt(j,i) =l= Finmax(i)*Tout(j)+FR(j,i)*Tmin-Finmax(i)*Tmin;
linearization8(j,i)$(ord(i) ne ord(j)).. frt(j,i) =l= FR(j,i)*Toutmax(j);
linearization9(i).. fnt(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutmax(i)-Finmax(i)*Toutmax(i);
linearization10(i).. fnt(i) =g= Fin(i)*Tmin;
linearization11(i).. fnt(i) =l= Finmax(i)*Tout(i)+Fin(i)*Tmin-Finmax(i)*Tmin;
linearization12(i).. fnt(i) =l= Fin(i)*Toutmax(i);
linearization13(n).. tos(n) =g= OSmax(n)*Ts(n) + OS(n)*Tsmax(n)-OSmax(n)*Tsmax(n);
linearization14(n).. tos(n) =g= OS(n)*Tmin;
linearization15(n).. tos(n) =l= OSmax(n)*Ts(n)+OS(n)*Tmin+OSmax(n)*Tmin;
linearization16(n).. tos(n) =l= OS(n)*Tsmax(n);
linearization17(n,i).. tcs(n,i) =g= OSmax(n)*Ts(n) + CS(n,i)*Tsmax(n)-OSmax(n)*Tsmax(n);
linearization18(n,i).. tcs(n,i) =g= CS(n,i)*Tmin;
linearization19(n,i).. tcs(n,i) =l= OSmax(n)*Ts(n)+CS(n,i)*Tmin-OSmax(n)*Tmin;
linearization20(n,i).. tcs(n,i) =l= CS(n,i)*Tsmax(n);
```

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```
Model
linear
/overall_cooling_water,cooling_towerMB1,cooling_towerMB2,circuit_supply_temperature,operation_inlet_flowrate,operation_recycle,total_blowdown,total_makeup,total_evaporation,operationMB,cooling_tower_design,operation_design,return_temperature_coolingtower,return_temp_limit_operation,operationEB,linearization1,linearization2,linearization3,linearization4,linearization5,linearization6,linearization7,linearization8,linearization9,linearization10,linearization11,linearization12,linearization17,linearization18,linearization19,linearization20/;

loverall_cooling_water.. CW =e= sum(n,OS(n));
lcooling_towerMB1(n).. OS(n) =e= sum(i,CS(n,i))-M(n);
lcooling_towerMB2(n).. OS(n) =e= sum(i,CR(i,n))-B(n)-E(n);
lcircuit_supply_temperature(n).. Ts(n)*(OS(n)+M(n)) =e=  Tamb*M(n) +  Tctout(n)*OS(n);
loperation_inlet_flowrate(i).. Fin(i) =e= sum(n,CS(n,i))+ sum(j,FR(j,i)$(ord(i) ne ord(j)));
loperation_recycle(i).. Fout(i) =e= sum(j,FR(i,j)$(ord(i) ne ord(j))) + sum(n,CR(i,n));
ltotal_makeup..Mt =e= sum(n,M(n));
ltotal_evaporation.. Et =e=sum(n,E(n));

loperationMB(i)..Fin(i) =e= Fout(i);
lcooling_tower_design(n).. OS(n) =l= OSmax(n);
loperation_design(i).. Fin(i)=l= Finmax(i);
lreturn_temp_coolingtower(n).. Tretmax(n)*(sum(i,CR(i,n)))=g= sum(i,CR(i,n)*Tout(i));
lreturn_temp_limit_operation(i).. Tout(i) =l= Toutmax(i);
loperationEB1(i).. Q(i) + cp*sum(j,FR(j,i)$(ord(i) ne ord(j))*Tout(j))+cp*sum(n,CS(n,i)*Ts(n)) =e= Fin(i)*cp*Tout(i);
lCWR_temp(n).. Tret(n)*(sum(i,CR(i,n))) =e= sum(i,CR(i,n)*Tout(i));
lCWR_flow(n).. OSin(n) =e= OS(n);

Model
exact/overall_cooling_water,lcooling_towerMB1,lcooling_towerMB2,lcircuit_supply_temperature,loperation_inlet_flowrate,loperation_recycle,ltotal_makeup,ltotal_evaporation,loperationMB,lcooling_tower_design,loperation_design,lreturn_temp_coolingtower,lreturn_temp_limit_operation,lCWR_temp,lCWR_flow,loperationEB1/;

option nlp = minos;
Solve linear using LP minimizing CW;
Solve exact using NLP minimizing CW;
*Exporting data to Matlab*
```

```
set stat /modelstat,solvestat,d/;
parameter returnStat(stat);
returnStat('modelstat') = exact.modelstat;
returnStat('solvestat') = exact.solvestat;
returnStat('d') = 0;
$libinclude matout Tret.l n
$libinclude matout OSin.l n
$libinclude matout returnStat stat
```

Main model

```
Sets
i cooling water using operation /hx1*hx6/
n cooling towers            /1*3/
alias(i,j);
Parameters
Q(i) heat loads /hx1 600
         hx2 300
         hx3 800
         hx4 500
         hx5 300
         hx6 700/
Toutmax(i) limiting outlet temperature /hx1 45
                    hx2 60
                    hx3 50
                    hx4 60
                    hx5 55
                    hx6 45/
Tinmax(i)  limiting inlet temperature /hx1 30
                    hx2 40
                    hx3 25
                    hx4 45
                    hx5 40
                    hx6 30/
Finmax(i) maximum flowrate through cooling water using operation /hx1 9.52
                                   hx2 3.57
                                   hx3 7.62
```

```
                                      hx4 7.14
                                      hx5 4.76
                                      hx6 11.1/
OSmax(n) maximum cooling tower capacity /1 9.6
                        2 16
                        3 20/
Tretmax(n) maximum return temperature   /1 50
                       2 50
                       3 55/
Tsmax(n) maximum circuit supply temperature /1 50
                        2 50
                        3 55/
Tmin minimum temperature is equal to wet bulb temperature /15/

cp /4.2/
Tamb /25/
CC /5/ ;
parameter E(n);
parameter B(n);
parameter M(n);
parameter Tctout(n);
parameter OS(n);
*Importing data from Matlab*
$if exist matdata.gms $include matdata.gms
Variables
CW overall cooling water supply
CS(n,i) fresh cooling water supply from n cooling tower to i operation
CR(i,n) return cooling water from i operation to n cooling tower
CTin cooling tower inlet flowrate
CTout cooling tower outlet flowrate

FR(j,i) cooling water reuse from j to i
Fin(i) inlet cooling water using operation flowrate
Fout(i) outlet cooling water using operation flowrate
Tin(i) inlet cooling water using operation temperature
```

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Tout(i) outlet cooling water using operation temperature

Tret(n) return temperature to the cooling tower n

Ts(n) temperature after adding makeup

Mt total make up

Bt total blow down

Et total evaporation

crt(i,n) linearization variable CR*Tout

frt(j,i) linearization variable FR*Tout

fnt(i)   linearization variable F*Tout

tos(n) linearization variable Ts*OS

tcs(n,i) linearization variable Ts*CS

OSin(n);

Positive variable

CS

CR

FR

Fin

Fout

Tin

Tout

Tret

Ts

Bt

Mt

Et

crt

frt

fnt

tos

tcs

OSin;

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*Temperature limits(lower bounds)

Tin.lo(i) = 15;

Tout.lo(i) = 15;

Tret.lo(n) = 15;

Ts.lo(n) = 15;

Equations

Linear Model

overall_cooling_water	total circulating water
cooling_towerMB1(n)	cooling water mass balance from the top
cooling_towerMB2(n)	cooling water mass balance from the bottom
circuit_supply_temperature(n)	supply temperature after adding makeup water
operation_inlet_flowrate(i)	inlet temperature to water using operation
operation_recycle(i)	operation oulet flowrate
total_blowdown	total blowdown
total_makeup	total makeup
total_evaporation	total evaporation
operationMB(i)	operation mass balance
cooling_tower_design(n)	cooling tower flowrate limit
operation_design(i)	operation flowrate limit
return_temperature_coolingtower(n)	return temperature to the cooling tower
return_temp_limit_operation(i)	limit of the return temperature to the cooling tower
operationEB(i)	operation energy balance
operation_inlet_temp(i)	operation inlet temperature calculation

linearization1(i,n)	linerization of CR*Tout
linearization2(i,n)	linerization of CR*Tout
linearization3(i,n)	linerization of CR*Tout
linearization4(i,n)	linerization of CR*Tout
linearization5(j,i)	linerization of FR*Tout
linearization6(j,i)	linerization of FR*Tout
linearization7(j,i)	linerization of FR*Tout
linearization8(j,i)	linerization of FR*Tout
linearization9(i)	linerization of Fin*Tout
linearization10(i)	linerization of Fin*Tout

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```
linearization11(i)            linerization of Fin*Tout
linearization12(i)            linerization of Fin*Tout
linearization13(n)            linerization of OS*Ts
linearization14(n)            linerization of OS*Ts
linearization15(n)            linerization of OS*Ts
linearization16(n)            linerization of OS*Ts
linearization17(n,i)          linerization of CS*Ts
linearization18(n,i)          linerization of CS*Ts
linearization19(n,i)          linerization of CS*Ts
linearization20(n,i)          linerization of CS*Ts

*Nonlinear Model*
loverall_cooling_water
lcooling_towerMB1(n)
lcooling_towerMB2(n)
lcircuit_supply_temperature(n)
loperation_inlet_flowrate(i)
loperation_recycle(i)
ltotal_makeup
ltotal_evaporation
loperationMB(i)
lcooling_tower_design(n)
loperation_design(i)
lreturn_temp_coolingtower(n)
lreturn_temp_limit_operation(i)
loperationEB1(i)
lCWR_temp(n)
lCWR_flow(n);

overall_cooling_water.. CW =e= sum(n,OS(n));
cooling_towerMB1(n).. OS(n)+M(n) =e= sum(i,CS(n,i));
cooling_towerMB2(n).. OS(n) =e= sum(i,CR(i,n))-B(n)-E(n);
circuit_supply_temperature(n).. sum(i,tcs(n,i)) =e=  Tamb*M(n) +  Tctout(n)*OS(n);
operation_inlet_flowrate(i).. Fin(i) =e= sum(n,CS(n,i))+ sum(j,FR(j,i)$(ord(i) ne ord(j)));
operation_recycle(i).. Fout(i) =e= sum(j,FR(i,j)$(ord(i) ne ord(j))) + sum(n,CR(i,n));
```

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```
total_blowdown.. Bt =e= sum(n,B(n));
total_makeup..Mt =e= sum(n,M(n));
total_evaporation.. Et =e= sum(n,E(n));
operationMB(i)..Fin(i) =e= Fout(i);
cooling_tower_design(n).. OS(n) =l= OSmax(n);
operation_design(i).. Fin(i)=l= Finmax(i);
return_temperature_coolingtower(n).. Tretmax(n)*sum(i,CR(i,n))=g= sum(i,crt(i,n));
return_temp_limit_operation(i).. Tout(i) =l= Toutmax(i);
operationEB(i).. cp*fnt(i)  =e= Q(i) + cp*sum(j,frt(j,i)$(ord(i) ne ord(j)))+ cp*sum(n,tcs(n,i));

linearization1(i,n).. crt(i,n) =g= OSmax(n)*Tout(i) + CR(i,n)*Toutmax(i)-OSmax(n)*Toutmax(i);
linearization2(i,n).. crt(i,n) =g= CR(i,n)*Tmin;
linearization3(i,n).. crt(i,n) =l= OSmax(n)*Tout(i)+CR(i,n)*Tmin-OSmax(n)*Tmin;
linearization4(i,n).. crt(i,n) =l= CR(i,n)*Toutmax(i);
linearization5(j,i)$(ord(i) ne ord(j)).. frt(j,i) =g= Finmax(i)*Tout(j) + FR(j,i)*Toutmax(i)-Finmax(i)*Toutmax(j);
linearization6(j,i)$(ord(i) ne ord(j)).. frt(j,i) =g= FR(j,i)*Tmin;
linearization7(j,i)$(ord(i) ne ord(j)).. frt(j,i) =l= Finmax(i)*Tout(j)+FR(j,i)*Tmin-Finmax(i)*Tmin;
linearization8(j,i)$(ord(i) ne ord(j)).. frt(j,i) =l= FR(j,i)*Toutmax(j);
linearization9(i).. fnt(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutmax(i)-Finmax(i)*Toutmax(i);
linearization10(i).. fnt(i) =g= Fin(i)*Tmin;
linearization11(i).. fnt(i) =l= Finmax(i)*Tout(i)+Fin(i)*Tmin-Finmax(i)*Tmin;
linearization12(i).. fnt(i) =l= Fin(i)*Toutmax(i);
linearization13(n).. tos(n) =g= OSmax(n)*Ts(n) + OS(n)*Tsmax(n)-OSmax(n)*Tsmax(n);
linearization14(n).. tos(n) =g= OS(n)*Tmin;
linearization15(n).. tos(n) =l= OSmax(n)*Ts(n)+OS(n)*Tmin+OSmax(n)*Tmin;
linearization16(n).. tos(n) =l= OS(n)*Tsmax(n);
linearization17(n,i).. tcs(n,i) =g= OSmax(n)*Ts(n) + CS(n,i)*Tsmax(n)-OSmax(n)*Tsmax(n);
linearization18(n,i).. tcs(n,i) =g= CS(n,i)*Tmin;
linearization19(n,i).. tcs(n,i) =l= OSmax(n)*Ts(n)+CS(n,i)*Tmin-OSmax(n)*Tmin;
linearization20(n,i).. tcs(n,i) =l= CS(n,i)*Tsmax(n);

Model
linear
/overall_cooling_water,cooling_towerMB1,cooling_towerMB2,circuit_supply_temperature,operation_inlet_flowra
```

```
te,operation_recycle,total_blowdown,total_makeup,total_evaporation,operationMB,cooling_tower_design,operat
ion_design,return_temperature_coolingtower,return_temp_limit_operation,operationEB,linearization1,linearizati
on2,linearization3,linearization4,linearization5,linearization6,linearization7,linearization8,linearization9,linearizati
on10,linearization11,linearization12,linearization17,linearization18,linearization19,linearization20/;

loverall_cooling_water.. CW =e= sum(n,OS(n));
lcooling_towerMB1(n).. OS(n) =e= sum(i,CS(n,i))-M(n);
lcooling_towerMB2(n).. OS(n) =e= sum(i,CR(i,n))-B(n)-E(n);
lcircuit_supply_temperature(n).. Ts(n)*(OS(n)+M(n)) =e=  Tamb*M(n) +  Tctout(n)*OS(n);
loperation_inlet_flowrate(i).. Fin(i) =e= sum(n,CS(n,i))+ sum(j,FR(j,i)$(ord(i) ne ord(j)));
loperation_recycle(i).. Fout(i) =e= sum(j,FR(i,j)$(ord(i) ne ord(j))) + sum(n,CR(i,n));
ltotal_makeup..Mt =e= sum(n,M(n));
ltotal_evaporation.. Et =e=sum(n,E(n));
loperationMB(i)..Fin(i) =e= Fout(i);
lcooling_tower_design(n).. OS(n) =l= OSmax(n);
loperation_design(i).. Fin(i)=l= Finmax(i);
lreturn_temp_coolingtower(n).. Tretmax(n)*(sum(i,CR(i,n)))=g= sum(i,CR(i,n)*Tout(i));
lreturn_temp_limit_operation(i).. Tout(i) =l= Toutmax(i);
loperationEB1(i).. Q(i) + cp*sum(j,FR(j,i)$(ord(i) ne ord(j))*Tout(j))+cp*sum(n,CS(n,i)*Ts(n)) =e= Fin(i)*cp*Tout(i);
lCWR_temp(n).. Tret(n)*(sum(i,CR(i,n))) =e= sum(i,CR(i,n)*Tout(i));
lCWR_flow(n).. OSin(n) =e= OS(n);

Model
exact/loverall_cooling_water,cooling_towerMB1,lcooling_towerMB2,lcircuit_supply_temperature,loperation_inlet
_flowrate,loperation_recycle,ltotal_makeup,ltotal_evaporation,loperationMB,lcooling_tower_design,loperation_d
esign,lreturn_temp_coolingtower,lreturn_temp_limit_operation,lCWR_temp,lCWR_flow,loperationEB1/;

option nlp = minos;
*Exporting data to Matlab*
$if exist matdata.gms $include matdata.gms
Solve linear using LP minimizing CW;
$if exist matdata.gms $include matdata.gms
Solve exact using NLP minimizing CW;
*Exporting data to Matlab*
set stat /modelstat,solvestat,d/;
```

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parameter returnStat(stat);

returnStat('modelstat') = exact.modelstat;

returnStat('solvestat') = exact.solvestat;

returnStat('d') = 0;

$libinclude matout Tret.l n

$libinclude matout OSin.l n

$libinclude matout returnStat stat

Cooling tower model

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
                    %%%The cooling water systems model%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
error = 2;
gg=0;
maxstepp = 10;
while error > 0.1
    gg = gg + 1;
%Recalling Gams models
if gg == 1
gams_output = 'std';
[CT,CM,s] = gams('start');

else

gams_output = 'std';
[CT,CM,s] = gams('optimum','E','B','M','Tctout','OS');

end

%NN number of cooling towers
NN = 3;

%model status
model_status = s(1);
solve_status = s(2);

%Flowrates from the Gams model
mwin(1) = CM(1,1);
mwin(2) = CM(2,1);
mwin(3) = CM(3,1);

twctin(1) = CT(1,1);
twctin(2) = CT(2,1);
twctin(3) = CT(3,1);
eff = zeros(3,1);
E = zeros(3,1);
```

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```
B= zeros(3,1);
M = zeros(3,1);
Tctout=zeros(3,1);
OS= zeros(3,1);

for m = 1:NN

err = 2;
j = 1;

maxstep = 10000;
while err > 1

mw = mwin(m);% water flowrate
mamax(1)=9.6;
mamax(2)=16;
mamax(3)=20;

ma = mamax(m);%air flowrate
twin = twctin(m)+273;

if mw <= 1
       twout = 295;
       Evap_loss = 0;
       mwin(m)  = 0;
    break
end

twexp = twin;

ta = 290.15;

twb = 288.95;
z =0;% cooling tower height
a = z;
b = 2.438;
Af = 5.943;
V = 14.49;
N = 1000;
h = (b-a)/N;

p = 101325;%total pressure
ifwo = 2.5016e+006;%vapor formation at 0C)
tw = twin-j/100;

%initializing variables
x = zeros(1,N);
y = zeros(1,N);
y1 = zeros(1,N);
y2 = zeros(1,N);
y3 = zeros(1,N);
y4 = zeros(1,N);
y5 = zeros(1,N);
y6 = zeros(1,N);
```

```
cpw = 0;
cpa = 0;
cpv = 0;
hfv = 0;
pvwb = 0;
w= 0;
cpma= 0;
pvs= 0;
ws= 0;
Hv = 0;
Hma = 0;
Hmas= 0;
Ka= 0;
Le = 0;
k11=0;
k12=0;
k13=0;
k14=0;
k21=0;
k22=0;
k23=0;
k24=0;
k31=0;
k32=0;
k33=0;
k34=0;
k41=0;
k42=0;
k43=0;
k44=0;

cpa = (1.045356e3)-(3.161783e-1)*(ta/2)+(7.083814e-4)*...
        ((ta/2)^2)-(2.705209e-7)*((ta/2)^3);

cpv = (1.3605e3)+(2.31334)*(ta/2)-(2.46784e-10)*((ta/2)^5)...
        -(5.91332e-13)*((ta/2)^6);

hfv =  (3.4831814e6)-(5.8627703e3)*ta+(12.139568)*(ta^2)...
    -(1.40290431e-2)*(ta^3);


pvwb = 10^(10.79586*(1-273.16/twb)+5.02808*log10(273.16/twb)+...
      (1.50474*10^-4)*(1-10^(-8.29692*(twb/273.16-1)))+...
      (4.2873*10^-4)*(10^(4.76955*(1-273.16/twb))-1)+2.7861);

w = ((2501.6-2.3263*(twb-273.16))/(2501.6 + 1.8577*(ta-273.16)-...
        4.184*(twb-273.16)))*(0.62509*pvwb/(p-1.005*pvwb))-...
        ((1.00416*(ta-twb))/(2501.6 + 1.8577*...
        (ta-273.16)-4.184*(twb-273.16)));
cpma = cpa + w*cpv;
pvs = 10^(10.79586*(1-273.16/tw)+5.02808*log10(273.16/tw)+...
      (1.50474*10^-4)*(1-10^(-8.29692*(tw/273.16-1)))+...
      (4.2873*10^-4)*(10^(4.76955*(1-273.16/tw))-1)+2.7861);
```

APPENDICES

```
ws = 0.622*pvs/(p-pvs);
Hv =ifwo+ ((1.3605e3)+(2.31334)*(ta/2)-(2.46784e-10)*...
    ((ta/2)^5)-(5.91332e-13)*((ta/2)^6))*ta;
Hma = cpa*ta+w*(ifwo+cpv*ta);
Hmas = ((1.045356e3)-(3.161783e-1)*(ta/2)+(7.083814e-4)*...
    ((ta/2)^2)-(2.705209e-7)*((ta/2)^3))*ta+ws*Hv;
 twout =tw;
 win = w;
 Hmain = Hma;
 Hmasin =Hmas;
 wsin=ws;

%Expressions for cooling tower performance characteristics(Ka)
ss(1) = (mw/Af)*2.69*((mw/ma)^-0.67)*(0.609^-0.062);
ss(2) = (mw/Af)*1.664*((mw/ma)^-0.62)*(0.914^-0.27);
ss(3) = (mw/Af)*4.69*((mw/ma)^-0.67)*(0.914^-0.062);

Ka = ss(m);

if tw < twb
       twout = 295;
       Evap_loss = 0.35;
    break
end

for i = 1:N

     ifwo = 2.5016e+006;%vapor formation at 0C)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    cpw = (8.15599e3)-(2.80627*10)*tw/2+(5.11283e-2)*((tw/2)^2)...
        -(2.17582e-13)*((tw/2)^6);

    pvs = 10^(10.79586*(1-273.16/tw)+5.02808*log10(273.16/tw)+...
      (1.50474*10^-4)*(1-10^(-8.29692*(tw/273.16-1)))+...
      (4.2873*10^-4)*(10^(4.76955*(1-273.16/tw))-1)+2.7861);

    ws = 0.622*pvs/(p-pvs);
    if ws < 0
       break
    end

    cpa = (1.045356e3)-(3.161783e-1)*(ta/2)+(7.083814e-4)*...
        ((ta/2)^2)-(2.705209e-7)*((ta/2)^3);

    cpv = (1.3605e3)+(2.31334)*(ta/2)-(2.46784e-10)*((ta/2)^5)...
        -(5.91332e-13)*((ta/2)^6);

    cpma = cpa + w*cpv;

    Hv =ifwo+ ((1.3605e3)+(2.31334)*(tw/2)-(2.46784e-10)*...
    ((tw/2)^5)-(5.91332e-13)*((tw/2)^6))*tw;

    Hmas = ((1.045356e3)-(3.161783e-1)*(tw/2)+(7.083814e-4)*...
    ((tw/2)^2)-(2.705209e-7)*((tw/2)^3))*tw+ws*Hv;
```

```
    Le = (0.866^0.667)*((ws+0.622)/(w+0.622)-1)/log((ws+0.622)/(w+0.622));


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


    k11 = h*((Ka*Af/(cpw*mw))*(-(ws-w)*cpw*tw+(Hmas-Hma)+(Le-1)*(Hmas-Hma-
(ws-w)*Hv)));
    k12 = h * (Ka*Af*(ws-w));
    k13 = h * (Ka*Af*(ws-w)/ma);
    k14 = h * ((Ka*Af/ma)*(Le*(Hmas-Hma)+(1-Le)*Hv*(ws-w)));
%calculating k2 needs w + k1/2,
    tw = tw + k11/2;
    mw = mw + k12/2;
    w  = w + k13/2;
    Hma = Hma + k14/2;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    cpw = (8.15599e3)-(2.80627*10)*tw/2+(5.11283e-2)*((tw/2)^2)...
        -(2.17582e-13)*((tw/2)^6);

    pvs = 10^(10.79586*(1-273.16/tw)+5.02808*log10(273.16/tw)+...
       (1.50474*10^-4)*(1-10^(-8.29692*(tw/273.16-1)))+...
       (4.2873*10^-4)*(10^(4.76955*(1-273.16/tw))-1)+2.7861);

    ws = 0.622*pvs/(p-pvs);

    cpa = (1.045356e3)-(3.161783e-1)*(ta/2)+(7.083814e-4)*...
        ((ta/2)^2)-(2.705209e-7)*((ta/2)^3);

    cpv = (1.3605e3)+(2.31334)*(ta/2)-(2.46784e-10)*((ta/2)^5)...
        -(5.91332e-13)*((ta/2)^6);

    cpma = cpa + w*cpv;

    Hv =ifwo+ ((1.3605e3)+(2.31334)*(tw/2)-(2.46784e-10)*...
    ((tw/2)^5)-(5.91332e-13)*((tw/2)^6))*tw;
    Hmas = ((1.045356e3)-(3.161783e-1)*(tw/2)+(7.083814e-4)*...
    ((tw/2)^2)-(2.705209e-7)*((tw/2)^3))*tw+ws*Hv;

    Le = (0.866^0.667)*((ws+0.622)/(w+0.622)-1)/log((ws+0.622)/(w+0.622));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    k21 = h*((Ka*Af/(cpw*mw))*(-(ws-w)*cpw*tw+(Hmas-Hma)+(Le-1)*(Hmas-Hma-
(ws-w)*Hv)));
    k22 = h * (Ka*Af*(ws-w));
    k23 = h * (Ka*Af*(ws-w)/ma);
    k24 = h * ((Ka*Af/ma)*(Le*(Hmas-Hma)+(1-Le)*Hv*(ws-w)));

    tw = tw - k11/2;
    mw = mw - k12/2;
    w  = w  - k13/2;
    Hma = Hma - k14/2;
%calculating k3 needs w + k2/2,
```

```
    tw = tw + k21/2;
    mw = mw + k22/2;
    w  = w  + k23/2;
    Hma = Hma + k24/2;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    cpw = (8.15599e3)-(2.80627*10)*tw/2+(5.11283e-2)*((tw/2)^2)...
        -(2.17582e-13)*((tw/2)^6);

    pvs = 10^(10.79586*(1-273.16/tw)+5.02808*log10(273.16/tw)+...
      (1.50474*10^-4)*(1-10^(-8.29692*(tw/273.16-1)))+...
      (4.2873*10^-4)*(10^(4.76955*(1-273.16/tw))-1)+2.7861);

    ws = 0.622*pvs/(p-pvs);

    cpa = (1.045356e3)-(3.161783e-1)*(ta/2)+(7.083814e-4)*...
        ((ta/2)^2)-(2.705209e-7)*((ta/2)^3);

    cpv = (1.3605e3)+(2.31334)*(ta/2)-(2.46784e-10)*((ta/2)^5)...
        -(5.91332e-13)*((ta/2)^6);

    cpma = cpa + w*cpv;

    Hv =ifwo+ ((1.3605e3)+(2.31334)*(tw/2)-(2.46784e-10)*...
    ((tw/2)^5)-(5.91332e-13)*((tw/2)^6))*tw;
    Hmas = ((1.045356e3)-(3.161783e-1)*(tw/2)+(7.083814e-4)*...
    ((tw/2)^2)-(2.705209e-7)*((tw/2)^3))*tw+ws*Hv;

    Le = (0.866^0.667)*((ws+0.622)/(w+0.622)-1)/log((ws+0.622)/(w+0.622));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    k31 = h*((Ka*Af/(cpw*mw))*(-(ws-w)*cpw*tw+(Hmas-Hma)+(Le-1)*(Hmas-Hma-
(ws-w)*Hv)));
    k32 = h * (Ka*Af*(ws-w));
    k33 = h * (Ka*Af*(ws-w)/ma);
    k34 = h * ((Ka*Af/ma)*(Le*(Hmas-Hma)+(1-Le)*Hv*(ws-w)));

    tw = tw - k21/2;
    mw = mw - k22/2;
    w  = w  - k23/2;
    Hma = Hma - k24/2;
 %calculating k4 needs k2 + w,
    tw = tw + k31;
    mw = mw + k32;
    w  = w  + k33;
    Hma = Hma + k34;

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   cpw = (8.15599e3)-(2.80627*10)*tw/2+(5.11283e-2)*((tw/2)^2)...
        -(2.17582e-13)*((tw/2)^6);

    pvs = 10^(10.79586*(1-273.16/tw)+5.02808*log10(273.16/tw)+...
      (1.50474*10^-4)*(1-10^(-8.29692*(tw/273.16-1)))+...
      (4.2873*10^-4)*(10^(4.76955*(1-273.16/tw))-1)+2.7861);
```

```
    ws = 0.622*pvs/(p-pvs);

    cpa = (1.045356e3)-(3.161783e-1)*(ta/2)+(7.083814e-4)*...
        ((ta/2)^2)-(2.705209e-7)*((ta/2)^3);

    cpv = (1.3605e3)+(2.31334)*(ta/2)-(2.46784e-10)*((ta/2)^5)...
        -(5.91332e-13)*((ta/2)^6);

    cpma = cpa + w*cpv;

    Hv =ifwo+ ((1.3605e3)+(2.31334)*(tw/2)-(2.46784e-10)*...
    ((tw/2)^5)-(5.91332e-13)*((tw/2)^6))*tw;
    Hmas = ((1.045356e3)-(3.161783e-1)*(tw/2)+(7.083814e-4)*...
    ((tw/2)^2)-(2.705209e-7)*((tw/2)^3))*tw+ws*Hv;

    Le = (0.866^0.667)*((ws+0.622)/(w+0.622)-1)/log((ws+0.622)/(w+0.622));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    k41 = h*((Ka*Af/(cpw*mw))*(-(ws-w)*cpw*tw+(Hmas-Hma)+(Le-1)*(Hmas-Hma-
(ws-w)*Hv)));
    k42 = h * (Ka*Af*(ws-w));
    k43 = h * (Ka*Af*(ws-w)/ma);
    k44 = h * ((Ka*Af/ma)*(Le*(Hmas-Hma)+(1-Le)*Hv*(ws-w)));

    tw = tw - k31;
    mw = mw - k32;
    w  = w  - k33;
    Hma = Hma - k34;

    tw = tw + (k11 + 2*k21 + 2*k31 + k41)/6;
    mw = mw + (k12 + 2*k22 + 2*k32 + k42)/6;
    w = w   + (k13 + 2*k23 + 2*k33 + k43)/6;
    Hma = Hma + (k14 + 2*k24 + 2*k34+ k44)/6;

    ta = tw - ((Hmas-Hma)-(ws-w)*Hv)/cpma;

    z = a + i*h;


    %storage for ploting data
    x(i) = z; %distance
    y(i)=tw;  %water temperature
    y1(i)=ta; %air temperature

    %storage for effectiveness calculations
    y2(i) = Hmas;
    y3(i) = Hma;
    y4(i) = ws;
    y5(i) = w;
    y6(i) = mw;

end

if imag(tw)== 0 && i == N
```

```
err = abs(tw -twexp);

Evap_loss = ma*(w -win);

end

  j = j + 1;

  if j >= maxstep
      err = 0.01;
      fprintf(1,'maximum limit reached for cooling tower %d \n',m);
      display(twin)
  end
end
CC=5;%Cycles of concentration

if mw > 0
Cemin = (mw*cpw/((Hmas-Hmasin)/(twin-twout)));

eff(m) = mw*cpw*(twin-twout)/(Cemin*(Hmas-Hmain));

E(m) = real(Evap_loss);
B(m) = E(m)/(CC - 1);
M(m) = E(m)*CC/(CC-1);
Tctout(m) = twout - 273;
OS(m) = mwin(m);

%averall effectiveness
Hact(m)= mw*cpw*(twin-twout);
Hmax(m) = (Cemin*(Hmas-Hmain));
else
eff(m) = 0;
E(m) = 0;
B(m) = E(m)/(CC - 1);
M(m) = E(m)*CC/(CC-1);
Tctout(m) = 0;
OS(m) = mwin(m);

%averall effectiveness
Hact(m)= 0;
Hmax(m) = 0;
end
end

if gg >= 2
    error = abs(XX1-Tctout(1))+abs(XX2-Tctout(2))+abs(XX3-Tctout(3))

end

XX1 = Tctout(1);
XX2 = Tctout(2);
XX3 = Tctout(3);
```

```
  if gg >= maxstepp
      error = 0.01;
      fprintf(1,'maximum iteration limit reached \n');

  end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


gg
averall_eff = sum(Hact)/sum(Hmax);

display('Gams model status')
display(model_status)

display('inlet temperatures')
display(twctin)

display('outlet temperatures')
display(Tctout)

display('outlet flowrates')
display(OS)

display('Evaporation')
display(E)

display('Blowdown')
display(B)

display('Make up')
display(M)

display('effectiveness')
display(eff)

display('averall effectiveness')
display(averall_eff)
end
```

APPENDICES

APPENDIX B: Case II

Initial model

```
option iterlim = 100000;
Sets
i cooling water using operation /hx1*hx6/
n cooling towers            /ct1*ct3/
alias(i,j);
Parameters
Q(i) heat loads /hx1 600
            hx2 300
            hx3 800
            hx4 600
            hx5 300
            hx6 700/
Toutmax(i) limiting outlet temperature /hx1 45
                            hx2 60
                            hx3 50
                            hx4 60
                            hx5 55
                            hx6 45/
Tinmax(i)  limiting inlet temperature /hx1 30
                            hx2 40
                            hx3 25
                            hx4 45
                            hx5 40
                            hx6 30/
Finmax(i) maximum flowrate through cooling water using operation /hx1 9.52
                                                  hx2 3.57
                                                  hx3 7.62
                                                  hx4 7.14
                                                  hx5 4.76
                                                  hx6 11.1/
OSmax(n) maximum cooling tower capacity /ct1 9.6
                              ct2 16
```

ct3 20/

Tctout(n) cooling tower supply temperature /ct1 20

ct2 22

ct3 25/

Tretmax(n) maximum return temperature /ct1 50

ct2 50

ct3 55/

E(n) evaporation /ct1 0.31

ct2 0.49

ct3 0.82/

Tsmax(n) maximum circuit supply temperature /ct1 50

ct2 50

ct3 55/

Tmin minimum temperature is equal to wet bulb temperature /15/

cp /4.2/

Tamb /25/

CC /5/

B(n)/ct1 0.08

ct2 0.12

ct3 0.21/

M(n)/ct1 0.39

ct2 0.61

ct3 1.03/;

Variables

CW overall cooling water supply

OS(n) operating cooling tower flowrate

OSin(n)

CS(n,i) fresh cooling water supply from n cooling tower to i operation

CR(i,n) return cooling water from i operation to n cooling tower

CTin cooling tower inlet flowrate

CTout cooling tower outlet flowrate

FR(j,i) cooling water reuse from j to i

Fin(i) inlet cooling water using operation flowrate

Fout(i) outlet cooling water using operation flowrate

APPENDICES

Tin(i) inlet cooling water using operation temperature

Tout(i) outlet cooling water using operation temperature

Tret(n) return temperature to the cooling tower n

Ts(n) temperature after adding makeup

Mt total make up

Bt total blow down

Et total evaporation

crt(i,n) linearization variable CR*Tout

frt(j,i) linearization variable FR*Tout

fnt(i)   linearization variable F*Tout

tos(n) linearization variable Ts*OS

tcs(n,i) linearization variable Ts*CS;

Positive variable

OS

OSin

CS

CR

FR

Fin

Fout

Tin

Tout

Tret

Ts

Bt

Mt

Et

crt

frt

fnt

tcs;

variables

APPENDICES

xx(i) sum of binary variables to from cooling tower n to opeartion i

yy(i) sum of binary variables to from opeartion i to cooling tower n;

binary variables

ys(n,i) node from cooling tower n to operation i

yr(i,n) node from operation i to cooling tower n;

Temperature Limits(lower bounds)

Tin.lo(i) = 15;

Tout.lo(i) = 15;

Tret.lo(n) = 15;

Ts.lo(n) = 15;

Equations

Linear Model

overall_cooling_water total circulating water

cooling_towerMB1(n) cooling water mass balance from the top

cooling_towerMB2(n) cooling water mass balance from the bottom

circuit_supply_temperature(n) supply temperature after adding makeup water

operation_inlet_flowrate(i) inlet temperature to water using operation

operation_recycle(i) operation outlet flowrate

total_blowdown total blowdown

total_makeup total makeup

total_evaporation total evaporation

operationMB(i) operation mass balance

cooling_tower_design(n) cooling tower flowrate limit

operation_design(i) operation flowrate limit

return_temperature_coolingtower(n) return temperature to the cooling tower

return_temp_limit_operation(i) limit of the return temperature to the cooling tower

operationEB(i) operation energy balance

operation_inlet_temp(i) operation inlet temperature calculation

nopremix1(n,i) preventing premixing from different cooling towers n to operation i

nopremix2(i) preventing premixing from different in operation i

nopostmix1(i,n) preventing postmixing from different cooling towers n to operation i

nopostmix2(i)	preventing postmixing from different in operation i
samesource1(i,n)	returning water to the same cooling source
samesource2(i,n)	returning water to the same cooling source
same1(i)	sum of binary variables to from cooling tower n to opeartion i
same2(i)	sum of binary variables to from opeartion i to cooling tower n
linearization1(i,n)	linerization of CR*Tout
linearization2(i,n)	linerization of CR*Tout
linearization3(i,n)	linerization of CR*Tout
linearization4(i,n)	linerization of CR*Tout
linearization5(j,i)	linerization of FR*Tout
linearization6(j,i)	linerization of FR*Tout
linearization7(j,i)	linerization of FR*Tout
linearization8(j,i)	linerization of FR*Tout
linearization9(i)	linerization of Fin*Tout
linearization10(i)	linerization of Fin*Tout
linearization11(i)	linerization of Fin*Tout
linearization12(i)	linerization of Fin*Tout
linearization13(n)	linerization of OS*Ts
linearization14(n)	linerization of OS*Ts
linearization15(n)	linerization of OS*Ts
linearization16(n)	linerization of OS*Ts
linearization17(n,i)	linerization of CS*Ts
linearization18(n,i)	linerization of CS*Ts
linearization19(n,i)	linerization of CS*Ts
linearization20(n,i)	linerization of CS*Ts

Nonlinear Model

loverall_cooling_water

lcooling_towerMB1(n)

lcooling_towerMB2(n)

lcircuit_supply_temperature(n)

loperation_inlet_flowrate(i)

loperation_recycle(i)

```
ltotal_makeup
ltotal_evaporation
loperationMB(i)
lcooling_tower_design(n)
loperation_design(i)
lreturn_temp_coolingtower(n)
lreturn_temp_limit_operation(i)
loperationEB1(i)
lCWR_temp(n)
lCWR_flow(n)
lsame1(i)
lsame2(i)

lnopremix1(n,i)
lnopremix2(i)
lnopostmix1(i,n)
lnopostmix2(i)
lsamesource1(i,n)
lsamesource2(i,n);

overall_cooling_water.. CW =e= sum(n,OS(n));
cooling_towerMB1(n).. OS(n)+M(n) =e= sum(i,CS(n,i));
cooling_towerMB2(n).. OS(n) =e= sum(i,CR(i,n))-B(n)-E(n);
circuit_supply_temperature(n).. sum(i,tcs(n,i)) =e=  Tamb*M(n) +  Tctout(n)*OS(n);
operation_inlet_flowrate(i).. Fin(i) =e= sum(n,CS(n,i))+ sum(j,FR(j,i)$(ord(i) ne ord(j)));
operation_recycle(i).. Fout(i) =e= sum(j,FR(i,j)$(ord(i) ne ord(j))) + sum(n,CR(i,n));
total_blowdown.. Bt =e= sum(n,B(n));
total_makeup..Mt =e= sum(n,M(n));
total_evaporation.. Et =e= sum(n,E(n));
operationMB(i)..Fin(i) =e= Fout(i);
cooling_tower_design(n).. OS(n) =l= OSmax(n);
operation_design(i).. Fin(i)=l= Finmax(i);
return_temperature_coolingtower(n).. Tretmax(n)*sum(i,CR(i,n))=g= sum(i,crt(i,n));
return_temp_limit_operation(i).. Tout(i) =l= Toutmax(i);
operationEB(i).. cp*fnt(i)  =e= Q(i) + cp*sum(j,frt(j,i)$(ord(i) ne ord(j)))+ cp*sum(n,tcs(n,i));
```

```
nopremix1(n,i).. CS(n,i) =l= Finmax(i)*ys(n,i);
nopremix2(i).. sum(n,ys(n,i)) =l= 1;
nopostmix1(i,n).. CR(i,n) =l= Finmax(i)*yr(i,n);
nopostmix2(i).. sum(n,yr(i,n)) =l= 1;
same1(i).. xx(i) =e= sum(n,ys(n,i));
same2(i).. yy(i) =e= sum(n,yr(i,n));
samesource1(i,n).. yr(i,n) =l= ys(n,i) + (2 - xx(i)- yy(i));
samesource2(i,n).. yr(i,n) =g= ys(n,i) - (2 - xx(i)- yy(i));

linearization1(i,n).. crt(i,n) =g= OSmax(n)*Tout(i) + CR(i,n)*Toutmax(i)-OSmax(n)*Toutmax(i);
linearization2(i,n).. crt(i,n) =g= CR(i,n)*Tmin;
linearization3(i,n).. crt(i,n) =l= OSmax(n)*Tout(i)+CR(i,n)*Tmin-OSmax(n)*Tmin;
linearization4(i,n).. crt(i,n) =l= CR(i,n)*Toutmax(i);
linearization5(j,i)$(ord(i) ne ord(j)).. frt(j,i) =g= Finmax(i)*Tout(j) + FR(j,i)*Toutmax(i)-Finmax(i)*Toutmax(j);
linearization6(j,i)$(ord(i) ne ord(j)).. frt(j,i) =g= FR(j,i)*Tmin;
linearization7(j,i)$(ord(i) ne ord(j)).. frt(j,i) =l= Finmax(i)*Tout(j)+FR(j,i)*Tmin-Finmax(i)*Tmin;
linearization8(j,i)$(ord(i) ne ord(j)).. frt(j,i) =l= FR(j,i)*Toutmax(j);
linearization9(i).. fnt(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutmax(i)-Finmax(i)*Toutmax(i);
linearization10(i).. fnt(i) =g= Fin(i)*Tmin;
linearization11(i).. fnt(i) =l= Finmax(i)*Tout(i)+Fin(i)*Tmin-Finmax(i)*Tmin;
linearization12(i).. fnt(i) =l= Fin(i)*Toutmax(i);
linearization13(n).. tos(n) =g= OSmax(n)*Ts(n) + OS(n)*Tsmax(n)-OSmax(n)*Tsmax(n);
linearization14(n).. tos(n) =g= OS(n)*Tmin;
linearization15(n).. tos(n) =l= OSmax(n)*Ts(n)+OS(n)*Tmin+OSmax(n)*Tmin;
linearization16(n).. tos(n) =l= OS(n)*Tsmax(n);
linearization17(n,i).. tcs(n,i) =g= OSmax(n)*Ts(n) + CS(n,i)*Tsmax(n)-OSmax(n)*Tsmax(n);
linearization18(n,i).. tcs(n,i) =g= CS(n,i)*Tmin;
linearization19(n,i).. tcs(n,i) =l= OSmax(n)*Ts(n)+CS(n,i)*Tmin-OSmax(n)*Tmin;
linearization20(n,i).. tcs(n,i) =l= CS(n,i)*Tsmax(n);
Model
linear
/overall_cooling_water,cooling_towerMB1,cooling_towerMB2,circuit_supply_temperature,operation_inlet_flowra
te,operation_recycle,total_blowdown,total_makeup,total_evaporation,operationMB,cooling_tower_design,operat
ion_design,return_temperature_coolingtower,return_temp_limit_operation,operationEB,nopremix1,nopremix2,n
```

APPENDICES

```
opostmix1,nopostmix2,samesource1,samesource2,same1,same2,linearization1,linearization2,linearization3,linearization4,linearization5,linearization6,linearization7,linearization8,linearization9,linearization10,linearization11,linearization12,linearization17,linearization18,linearization19,linearization20/;

loverall_cooling_water.. CW =e= sum(n,OS(n));
lcooling_towerMB1(n).. OS(n) =e= sum(i,CS(n,i))-M(n);
lcooling_towerMB2(n).. OS(n) =e= sum(i,CR(i,n))-B(n)-E(n);
lcircuit_supply_temperature(n).. Ts(n)*(OS(n)+M(n)) =e=  Tamb*M(n) +  Tctout(n)*OS(n);
loperation_inlet_flowrate(i).. Fin(i) =e= sum(n,CS(n,i))+ sum(j,FR(j,i)$(ord(i) ne ord(j)));
loperation_recycle(i).. Fout(i) =e= sum(j,FR(i,j)$(ord(i) ne ord(j))) + sum(n,CR(i,n));
ltotal_makeup..Mt =e= sum(n,M(n));
ltotal_evaporation.. Et =e=sum(n,E(n));
loperationMB(i)..Fin(i) =e= Fout(i);
lcooling_tower_design(n).. OS(n) =l= OSmax(n);
loperation_design(i).. Fin(i)=l= Finmax(i);
lreturn_temp_coolingtower(n).. Tretmax(n)*(sum(i,CR(i,n)))=g= sum(i,CR(i,n)*Tout(i));
lreturn_temp_limit_operation(i).. Tout(i) =l= Toutmax(i);
loperationEB1(i).. Q(i) + cp*sum(j,FR(j,i)$(ord(i) ne ord(j))*Tout(j))+cp*sum(n,CS(n,i)*Ts(n)) =e= Fin(i)*cp*Tout(i);
lCWR_temp(n).. Tret(n)*(sum(i,CR(i,n))) =e= sum(i,CR(i,n)*Tout(i));
lCWR_flow(n).. OSin(n) =e= OS(n);

lnopremix1(n,i).. CS(n,i) =l= Finmax(i)*ys(n,i);
lnopremix2(i).. sum(n,ys(n,i)) =l= 1;
lnopostmix1(i,n).. CR(i,n) =l= Finmax(i)*yr(i,n);
lnopostmix2(i).. sum(n,yr(i,n)) =l= 1;
lsame1(i).. xx(i) =e= sum(n,ys(n,i));
lsame2(i).. yy(i) =e= sum(n,yr(i,n));
lsamesource1(i,n).. yr(i,n) =l= ys(n,i) + (2 - xx(i)- yy(i));
lsamesource2(i,n).. yr(i,n) =g= ys(n,i) - (2 - xx(i)- yy(i));

Model
exact/loverall_cooling_water,lcooling_towerMB1,lcooling_towerMB2,lcircuit_supply_temperature,loperation_inlet_flowrate,loperation_recycle,ltotal_makeup,ltotal_evaporation,loperationMB,lcooling_tower_design,loperation_design,lreturn_temp_coolingtower,lreturn_temp_limit_operation,lCWR_temp,lCWR_flow,loperationEB1,lnopremix1,lnopremix2,lnopostmix1,lnopostmix2,lsamesource1,lsamesource2,lsame1,lsame2/;
```

```
option minlp = sbb;
Solve linear using MIP minimizing CW;
Solve exact using MINLP minimizing CW;
display Tret.l
display OSin.l
*Exporting data to Matlab*
set stat /modelstat,solvestat,d/;
parameter returnStat(stat);
returnStat('modelstat') = exact.modelstat;
returnStat('solvestat') = exact.solvestat;
returnStat('d') = 0;

$libinclude matout Tret.l n
$libinclude matout OSin.l n
$libinclude matout returnStat stat
```

Main model

```
option iterlim = 100000;
Sets
i cooling water using operation /hx1*hx6/
n cooling towers            /1*3/
alias(i,j);
Parameters
Q(i) heat loads /hx1 600
          hx2 300
          hx3 800
          hx4 600
          hx5 300
          hx6 700/
Toutmax(i) limiting outlet temperature /hx1 45
                    hx2 60
                    hx3 50
                    hx4 60
                    hx5 55
```

```
                        hx6 45/
Tinmax(i)  limiting inlet temperature /hx1 30
                       hx2 40
                       hx3 25
                       hx4 45
                       hx5 40
                       hx6 30/
Finmax(i) maximum flowrate through cooling water using operation /hx1 9.52
                                        hx2 3.57
                                        hx3 7.62
                                        hx4 7.14
                                        hx5 4.76
                                        hx6 11.1/
OSmax(n) maximum cooling tower capacity /1 9.6
                         2 16
                         3 20/
Tretmax(n) maximum return temperature   /1 50
                        2 50
                        3 55/
Tsmax(n) maximum circuit supply temperature /1 50
                         2 50
                         3 55/
Tmin minimum temperature is equal to wet bulb temperature /15/
cp /4.2/
Tamb /25/
CC /5/ ;
parameter E(n);
parameter B(n);
parameter M(n);
parameter Tctout(n);
parameter OS(n);
*Importing data from Matlab*
$if exist matdata.gms $include matdata.gms
```

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Variables

CW overall cooling water supply

CS(n,i) fresh cooling water supply from n cooling tower to i operation

CR(i,n) return cooling water from i operation to n cooling tower

CTin cooling tower inlet flowrate

CTout cooling tower outlet flowrate

FR(j,i) cooling water reuse from j to i

Fin(i) inlet cooling water using operation flowrate

Fout(i) outlet cooling water using operation flowrate

Tin(i) inlet cooling water using operation temperature

Tout(i) outlet cooling water using operation temperature

Tret(n) return temperature to the cooling tower n

Ts(n) temperature after adding makeup

Mt total make up

Bt total blow down

Et total evaporation

crt(i,n) linearization variable CR*Tout

frt(j,i) linearization variable FR*Tout

fnt(i) linearization variable F*Tout

tos(n) linearization variable Ts*OS

tcs(n,i) linearization variable Ts*CS

OSin(n);

Positive variable

CS

CR

FR

Fin

Fout

Tin

Tout

Tret

Ts

Bt

Mt

Et

crt

frt

fnt

tcs

OSin;

variables

xx(i) sum of binary variables to from cooling tower n to opeartion i

yy(i) sum of binary variables to from opeartion i to cooling tower n;

binary variables

ys(n,i) node from cooling tower n to operation i

yr(i,n) node from operation i to cooling tower n;

Temperature Limits(lower bounds)

Tin.lo(i) = 15;

Tout.lo(i) = 15;

Tret.lo(n) = 15;

Ts.lo(n) = 15;

Equations

Linear Model

overall_cooling_water	total circulating water
cooling_towerMB1(n)	cooling water mass balance from the top
cooling_towerMB2(n)	cooling water mass balance from the bottom
circuit_supply_temperature(n)	supply temperature after adding makeup water
operation_inlet_flowrate(i)	inlet temperature to water using operation
operation_recycle(i)	operation outlet flowrate
total_blowdown	total blowdown
total_makeup	total makeup
total_evaporation	total evaporation
operationMB(i)	operation mass balance
cooling_tower_design(n)	cooling tower flowrate limit

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operation_design(i)	operation flowrate limit
return_temperature_coolingtower(n)	return temperature to the cooling tower
return_temp_limit_operation(i)	limit of the return temperature to the cooling tower
operationEB(i)	operation energy balance
operation_inlet_temp(i)	operation inlet temperature calculation
nopremix1(n,i)	preventing premixing from different cooling towers n to operation i
nopremix2(i)	preventing premixing from different in operation i
nopostmix1(i,n)	preventing postmixing from different cooling towers n to operation i
nopostmix2(i)	preventing postmixing from different in operation i
samesource1(i,n)	returning water to the same cooling source
samesource2(i,n)	returning water to the same cooling source
same1(i)	sum of binary variables to from cooling tower n to opeartion i
same2(i)	sum of binary variables to from opeartion i to cooling tower n
linearization1(i,n)	linerization of CR*Tout
linearization2(i,n)	linerization of CR*Tout
linearization3(i,n)	linerization of CR*Tout
linearization4(i,n)	linerization of CR*Tout
linearization5(j,i)	linerization of FR*Tout
linearization6(j,i)	linerization of FR*Tout
linearization7(j,i)	linerization of FR*Tout
linearization8(j,i)	linerization of FR*Tout
linearization9(i)	linerization of Fin*Tout
linearization10(i)	linerization of Fin*Tout
linearization11(i)	linerization of Fin*Tout
linearization12(i)	linerization of Fin*Tout
linearization13(n)	linerization of OS*Ts
linearization14(n)	linerization of OS*Ts
linearization15(n)	linerization of OS*Ts
linearization16(n)	linerization of OS*Ts
linearization17(n,i)	linerization of CS*Ts
linearization18(n,i)	linerization of CS*Ts
linearization19(n,i)	linerization of CS*Ts
linearization20(n,i)	linerization of CS*Ts

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Nonlinear Model

```
loverall_cooling_water
lcooling_towerMB1(n)
lcooling_towerMB2(n)
lcircuit_supply_temperature(n)
loperation_inlet_flowrate(i)
loperation_recycle(i)
ltotal_makeup
ltotal_evaporation
loperationMB(i)
lcooling_tower_design(n)
loperation_design(i)
lreturn_temp_coolingtower(n)
lreturn_temp_limit_operation(i)
loperationEB1(i)
lCWR_temp(n)
lCWR_flow(n)

lsame1(i)
lsame2(i)

lnopremix1(n,i)
lnopremix2(i)
lnopostmix1(i,n)
lnopostmix2(i)
lsamesource1(i,n)
lsamesource2(i,n);

overall_cooling_water.. CW =e= sum(n,OS(n));
cooling_towerMB1(n).. OS(n)+M(n) =e= sum(i,CS(n,i));
cooling_towerMB2(n).. OS(n) =e= sum(i,CR(i,n))-B(n)-E(n);
circuit_supply_temperature(n).. sum(i,tcs(n,i)) =e=  Tamb*M(n) +  Tctout(n)*OS(n);
operation_inlet_flowrate(i).. Fin(i) =e= sum(n,CS(n,i))+ sum(j,FR(j,i)$(ord(i) ne ord(j)));
operation_recycle(i).. Fout(i) =e= sum(j,FR(i,j)$(ord(i) ne ord(j))) + sum(n,CR(i,n));
```

```
total_blowdown.. Bt =e= sum(n,B(n));
total_makeup..Mt =e= sum(n,M(n));
total_evaporation.. Et =e= sum(n,E(n));
operationMB(i)..Fin(i) =e= Fout(i);
cooling_tower_design(n).. OS(n) =l= OSmax(n);
operation_design(i).. Fin(i)=l= Finmax(i);
return_temperature_coolingtower(n).. Tretmax(n)*sum(i,CR(i,n))=g= sum(i,crt(i,n));
return_temp_limit_operation(i).. Tout(i) =l= Toutmax(i);

operationEB(i).. cp*fnt(i)  =e= Q(i) + cp*sum(j,frt(j,i)$(ord(i) ne ord(j)))+ cp*sum(n,tcs(n,i));

*operation_inlet_temp(i).. fnti(i) =e=  sum(j,frt(j,i)$(ord(i) ne ord(j)))+ sum(n,tcs(n,i));
nopremix1(n,i).. CS(n,i) =l= Finmax(i)*ys(n,i);
nopremix2(i).. sum(n,ys(n,i)) =l= 1;
nopostmix1(i,n).. CR(i,n) =l= Finmax(i)*yr(i,n);
nopostmix2(i).. sum(n,yr(i,n)) =l= 1;
same1(i).. xx(i) =e= sum(n,ys(n,i));
same2(i).. yy(i) =e= sum(n,yr(i,n));
samesource1(i,n).. yr(i,n) =l= ys(n,i) + (2 - xx(i)- yy(i));
samesource2(i,n).. yr(i,n) =g= ys(n,i) - (2 - xx(i)- yy(i));

linearization1(i,n).. crt(i,n) =g= OSmax(n)*Tout(i) + CR(i,n)*Toutmax(i)-OSmax(n)*Toutmax(i);
linearization2(i,n).. crt(i,n) =g= CR(i,n)*Tmin;
linearization3(i,n).. crt(i,n) =l= OSmax(n)*Tout(i)+CR(i,n)*Tmin-OSmax(n)*Tmin;
linearization4(i,n).. crt(i,n) =l= CR(i,n)*Toutmax(i);
linearization5(j,i)$(ord(i) ne ord(j)).. frt(j,i) =g= Finmax(i)*Tout(j) + FR(j,i)*Toutmax(i)-Finmax(i)*Toutmax(j);
linearization6(j,i)$(ord(i) ne ord(j)).. frt(j,i) =g= FR(j,i)*Tmin;
linearization7(j,i)$(ord(i) ne ord(j)).. frt(j,i) =l= Finmax(i)*Tout(j)+FR(j,i)*Tmin-Finmax(i)*Tmin;
linearization8(j,i)$(ord(i) ne ord(j)).. frt(j,i) =l= FR(j,i)*Toutmax(j);
linearization9(i).. fnt(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutmax(i)-Finmax(i)*Toutmax(i);
linearization10(i).. fnt(i) =g= Fin(i)*Tmin;
linearization11(i).. fnt(i) =l= Finmax(i)*Tout(i)+Fin(i)*Tmin-Finmax(i)*Tmin;
linearization12(i).. fnt(i) =l= Fin(i)*Toutmax(i);
linearization13(n).. tos(n) =g= OSmax(n)*Ts(n) + OS(n)*Tsmax(n)-OSmax(n)*Tsmax(n);
```

```
linearization14(n).. tos(n) =g= OS(n)*Tmin;
linearization15(n).. tos(n) =l= OSmax(n)*Ts(n)+OS(n)*Tmin+OSmax(n)*Tmin;
linearization16(n).. tos(n) =l= OS(n)*Tsmax(n);
linearization17(n,i).. tcs(n,i) =g= OSmax(n)*Ts(n) + CS(n,i)*Tsmax(n)-OSmax(n)*Tsmax(n);
linearization18(n,i).. tcs(n,i) =g= CS(n,i)*Tmin;
linearization19(n,i).. tcs(n,i) =l= OSmax(n)*Ts(n)+CS(n,i)*Tmin-OSmax(n)*Tmin;
linearization20(n,i).. tcs(n,i) =l= CS(n,i)*Tsmax(n);

Model
linear
/overall_cooling_water,cooling_towerMB1,cooling_towerMB2,circuit_supply_temperature,operation_inlet_flowra
te,operation_recycle,total_blowdown,total_makeup,total_evaporation,operationMB,cooling_tower_design,operat
ion_design,return_temperature_coolingtower,return_temp_limit_operation,operationEB,nopremix1,nopremix2,n
opostmix1,nopostmix2,samesource1,samesource2,same1,same2,linearization1,linearization2,linearization3,lineari
zation4,linearization5,linearization6,linearization7,linearization8,linearization9,linearization10,linearization11,linea
rization12,linearization17,linearization18,linearization19,linearization20/;

loverall_cooling_water.. CW =e= sum(n,OS(n));
lcooling_towerMB1(n).. OS(n) =e= sum(i,CS(n,i))-M(n);
lcooling_towerMB2(n).. OS(n) =e= sum(i,CR(i,n))-B(n)-E(n);
lcircuit_supply_temperature(n).. Ts(n)*(OS(n)+M(n)) =e=  Tamb*M(n) +  Tctout(n)*OS(n);
loperation_inlet_flowrate(i).. Fin(i) =e= sum(n,CS(n,i))+ sum(j,FR(j,i)$(ord(i) ne ord(j)));
loperation_recycle(i).. Fout(i) =e= sum(j,FR(i,j)$(ord(i) ne ord(j))) + sum(n,CR(i,n));
ltotal_makeup..Mt =e= sum(n,M(n));
ltotal_evaporation.. Et =e=sum(n,E(n));
loperationMB(i)..Fin(i) =e= Fout(i);
lcooling_tower_design(n).. OS(n) =l= OSmax(n);
loperation_design(i).. Fin(i)=l= Finmax(i);
lreturn_temp_coolingtower(n).. Tretmax(n)*(sum(i,CR(i,n)))=g= sum(i,CR(i,n)*Tout(i));
lreturn_temp_limit_operation(i).. Tout(i) =l= Toutmax(i);
loperationEB1(i).. Q(i) + cp*sum(j,FR(j,i)$(ord(i) ne ord(j))*Tout(j))+cp*sum(n,CS(n,i)*Ts(n)) =e= Fin(i)*cp*Tout(i);
lCWR_temp(n).. Tret(n)*(sum(i,CR(i,n))) =e= sum(i,CR(i,n)*Tout(i));
lCWR_flow(n).. OSin(n) =e= OS(n);

lnopremix1(n,i).. CS(n,i) =l= Finmax(i)*ys(n,i);
```

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```
lnopremix2(i).. sum(n,ys(n,i)) =l= 1;
lnopostmix1(i,n).. CR(i,n) =l= Finmax(i)*yr(i,n);
lnopostmix2(i).. sum(n,yr(i,n)) =l= 1;
lsame1(i).. xx(i) =e= sum(n,ys(n,i));
lsame2(i).. yy(i) =e= sum(n,yr(i,n));
lsamesource1(i,n).. yr(i,n) =l= ys(n,i) + (2 - xx(i)- yy(i));
lsamesource2(i,n).. yr(i,n) =g= ys(n,i) - (2 - xx(i)- yy(i));

Model
exact/
loverall_cooling_water,lcooling_towerMB1,lcooling_towerMB2,lcircuit_supply_temperature,loperation_inlet_flow
rate,loperation_recycle,ltotal_makeup,ltotal_evaporation,loperationMB,lcooling_tower_design,loperation_design,
lreturn_temp_coolingtower,lreturn_temp_limit_operation,lCWR_temp,lCWR_flow,loperationEB1,lnopremix1,lnop
remix2,lnopostmix1,lnopostmix2,lsamesource1,lsamesource2,lsame1,lsame2/;

option minlp = sbb;
$if exist matdata.gms $include matdata.gms
Solve linear using MIP minimizing CW;
$if exist matdata.gms $include matdata.gms
Solve exact using MINLP minimizing CW;
*Exporting data to Matlab*
set stat /modelstat,solvestat,d/;
parameter returnStat(stat);
returnStat('modelstat') = exact.modelstat;
returnStat('solvestat') = exact.solvestat;
returnStat('d') = 0;

$libinclude matout Tret.l n
$libinclude matout OSin.l n
$libinclude matout returnStat stat
```

Cooling tower model

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
                    %%%The cooling water systems model%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
error = 2;
gg=0;
maxstepp = 10;
while error > 0.1
    gg = gg + 1;
%Recalling Gams models
if gg == 1
gams_output = 'std';
[CT,CM,s] = gams('start');

else

gams_output = 'std';
[CT,CM,s] = gams('optimum','E','B','M','Tctout','OS');

end

%NN number of cooling towers
NN = 3;

%model status
model_status = s(1);
solve_status = s(2);

%Flowrates from the Gams model
mwin(1) = CM(1,1);
mwin(2) = CM(2,1);
mwin(3) = CM(3,1);

twctin(1) = CT(1,1);
twctin(2) = CT(2,1);
twctin(3) = CT(3,1);
eff = zeros(3,1);
E = zeros(3,1);
B= zeros(3,1);
M = zeros(3,1);
Tctout=zeros(3,1);
OS= zeros(3,1);

for m = 1:NN

err = 2;
j = 1;

maxstep = 10000;
while err > 1
```

```
mw = mwin(m);% water flowrate
mamax(1)=9.6;
mamax(2)=16;
mamax(3)=20;

ma = mamax(m);%air flowrate
twin = twctin(m)+273;

if mw <= 1
       twout = 295;
       Evap_loss = 0;
       mwin(m)  = 0;
    break
end

twexp = twin;

ta = 290.15;

twb = 288.95;
z =0;% cooling tower height
a = z;
b = 2.438;
Af = 5.943;
V = 14.49;
N = 1000;
h = (b-a)/N;

p = 101325;%total pressure
ifwo = 2.5016e+006;%vapor formation at 0C)
tw = twin-j/100;

%initializing variables
x = zeros(1,N);
y = zeros(1,N);
y1 = zeros(1,N);
y2 = zeros(1,N);
y3 = zeros(1,N);
y4 = zeros(1,N);
y5 = zeros(1,N);
y6 = zeros(1,N);

cpw = 0;
cpa = 0;
cpv = 0;
hfv = 0;
pvwb = 0;
w= 0;
cpma= 0;
pvs= 0;
ws= 0;
Hv = 0;
Hma = 0;
Hmas= 0;
```

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```
Ka= 0;
Le = 0;
k11=0;
k12=0;
k13=0;
k14=0;
k21=0;
k22=0;
k23=0;
k24=0;
k31=0;
k32=0;
k33=0;
k34=0;
k41=0;
k42=0;
k43=0;
k44=0;

cpa = (1.045356e3)-(3.161783e-1)*(ta/2)+(7.083814e-4)*...
        ((ta/2)^2)-(2.705209e-7)*((ta/2)^3);

cpv = (1.3605e3)+(2.31334)*(ta/2)-(2.46784e-10)*((ta/2)^5)...
        -(5.91332e-13)*((ta/2)^6);

hfv =  (3.4831814e6)-(5.8627703e3)*ta+(12.139568)*(ta^2)...
    -(1.40290431e-2)*(ta^3);


pvwb = 10^(10.79586*(1-273.16/twb)+5.02808*log10(273.16/twb)+...
      (1.50474*10^-4)*(1-10^(-8.29692*(twb/273.16-1)))+...
      (4.2873*10^-4)*(10^(4.76955*(1-273.16/twb))-1)+2.7861);

w = ((2501.6-2.3263*(twb-273.16))/(2501.6 + 1.8577*(ta-273.16)-...
        4.184*(twb-273.16)))*(0.62509*pvwb/(p-1.005*pvwb))-...
        ((1.00416*(ta-twb))/(2501.6 + 1.8577*...
        (ta-273.16)-4.184*(twb-273.16)));
cpma = cpa + w*cpv;
pvs = 10^(10.79586*(1-273.16/tw)+5.02808*log10(273.16/tw)+...
      (1.50474*10^-4)*(1-10^(-8.29692*(tw/273.16-1)))+...
      (4.2873*10^-4)*(10^(4.76955*(1-273.16/tw))-1)+2.7861);

ws = 0.622*pvs/(p-pvs);
Hv =ifwo+ ((1.3605e3)+(2.31334)*(ta/2)-(2.46784e-10)*...
    ((ta/2)^5)-(5.91332e-13)*((ta/2)^6))*ta;
Hma = cpa*ta+w*(ifwo+cpv*ta);
Hmas = ((1.045356e3)-(3.161783e-1)*(ta/2)+(7.083814e-4)*...
    ((ta/2)^2)-(2.705209e-7)*((ta/2)^3))*ta+ws*Hv;
 twout =tw;
 win = w;
 Hmain = Hma;
 Hmasin =Hmas;
 wsin=ws;

%Expressions for cooling tower performance characteristics(Ka)
```

APPENDICES

```
ss(1)  =  (mw/Af)*2.69*((mw/ma)^-0.67)*(0.609^-0.062);
ss(2)  =  (mw/Af)*1.664*((mw/ma)^-0.62)*(0.914^-0.27);
ss(3)  =  (mw/Af)*4.69*((mw/ma)^-0.67)*(0.914^-0.062);

Ka = ss(m);

if tw < twb
       twout = 295;
       Evap_loss = 0.35;
    break
end

for i = 1:N

     ifwo = 2.5016e+006;%vapor formation at 0C)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    cpw = (8.15599e3)-(2.80627*10)*tw/2+(5.11283e-2)*((tw/2)^2)...
        -(2.17582e-13)*((tw/2)^6);

    pvs = 10^(10.79586*(1-273.16/tw)+5.02808*log10(273.16/tw)+...
      (1.50474*10^-4)*(1-10^(-8.29692*(tw/273.16-1)))+...
      (4.2873*10^-4)*(10^(4.76955*(1-273.16/tw))-1)+2.7861);

    ws = 0.622*pvs/(p-pvs);
    if ws < 0
       break
    end

    cpa = (1.045356e3)-(3.161783e-1)*(ta/2)+(7.083814e-4)*...
        ((ta/2)^2)-(2.705209e-7)*((ta/2)^3);

    cpv = (1.3605e3)+(2.31334)*(ta/2)-(2.46784e-10)*((ta/2)^5)...
        -(5.91332e-13)*((ta/2)^6);

    cpma = cpa + w*cpv;

    Hv =ifwo+ ((1.3605e3)+(2.31334)*(tw/2)-(2.46784e-10)*...
    ((tw/2)^5)-(5.91332e-13)*((tw/2)^6))*tw;

    Hmas = ((1.045356e3)-(3.161783e-1)*(tw/2)+(7.083814e-4)*...
    ((tw/2)^2)-(2.705209e-7)*((tw/2)^3))*tw+ws*Hv;

    Le = (0.866^0.667)*((ws+0.622)/(w+0.622)-1)/log((ws+0.622)/(w+0.622));


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


    k11 = h*((Ka*Af/(cpw*mw))*(-(ws-w)*cpw*tw+(Hmas-Hma)+(Le-1)*(Hmas-Hma-
(ws-w)*Hv)));
    k12 = h * (Ka*Af*(ws-w));
    k13 = h * (Ka*Af*(ws-w)/ma);
    k14 = h * ((Ka*Af/ma)*(Le*(Hmas-Hma)+(1-Le)*Hv*(ws-w)));
%calculating k2 needs w + k1/2,
```

```
    tw = tw + k11/2;
    mw = mw + k12/2;
    w  = w + k13/2;
    Hma = Hma + k14/2;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    cpw = (8.15599e3)-(2.80627*10)*tw/2+(5.11283e-2)*((tw/2)^2)...
        -(2.17582e-13)*((tw/2)^6);

    pvs = 10^(10.79586*(1-273.16/tw)+5.02808*log10(273.16/tw)+...
      (1.50474*10^-4)*(1-10^(-8.29692*(tw/273.16-1)))+...
      (4.2873*10^-4)*(10^(4.76955*(1-273.16/tw))-1)+2.7861);

    ws = 0.622*pvs/(p-pvs);

    cpa = (1.045356e3)-(3.161783e-1)*(ta/2)+(7.083814e-4)*...
        ((ta/2)^2)-(2.705209e-7)*((ta/2)^3);

    cpv = (1.3605e3)+(2.31334)*(ta/2)-(2.46784e-10)*((ta/2)^5)...
        -(5.91332e-13)*((ta/2)^6);

    cpma = cpa + w*cpv;

    Hv =ifwo+ ((1.3605e3)+(2.31334)*(tw/2)-(2.46784e-10)*...
    ((tw/2)^5)-(5.91332e-13)*((tw/2)^6))*tw;
    Hmas = ((1.045356e3)-(3.161783e-1)*(tw/2)+(7.083814e-4)*...
    ((tw/2)^2)-(2.705209e-7)*((tw/2)^3))*tw+ws*Hv;

    Le = (0.866^0.667)*((ws+0.622)/(w+0.622)-1)/log((ws+0.622)/(w+0.622));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    k21 = h*((Ka*Af/(cpw*mw))*(-(ws-w)*cpw*tw+(Hmas-Hma)+(Le-1)*(Hmas-Hma-
(ws-w)*Hv)));
    k22 = h * (Ka*Af*(ws-w));
    k23 = h * (Ka*Af*(ws-w)/ma);
    k24 = h * ((Ka*Af/ma)*(Le*(Hmas-Hma)+(1-Le)*Hv*(ws-w)));

    tw = tw - k11/2;
    mw = mw - k12/2;
    w  = w  - k13/2;
    Hma = Hma - k14/2;
%calculating k3 needs w + k2/2,
    tw = tw + k21/2;
    mw = mw + k22/2;
    w  = w  + k23/2;
    Hma = Hma + k24/2;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    cpw = (8.15599e3)-(2.80627*10)*tw/2+(5.11283e-2)*((tw/2)^2)...
        -(2.17582e-13)*((tw/2)^6);

    pvs = 10^(10.79586*(1-273.16/tw)+5.02808*log10(273.16/tw)+...
      (1.50474*10^-4)*(1-10^(-8.29692*(tw/273.16-1)))+...
      (4.2873*10^-4)*(10^(4.76955*(1-273.16/tw))-1)+2.7861);

    ws = 0.622*pvs/(p-pvs);
```

```
    cpa = (1.045356e3)-(3.161783e-1)*(ta/2)+(7.083814e-4)*...
        ((ta/2)^2)-(2.705209e-7)*((ta/2)^3);

    cpv = (1.3605e3)+(2.31334)*(ta/2)-(2.46784e-10)*((ta/2)^5)...
        -(5.91332e-13)*((ta/2)^6);

    cpma = cpa + w*cpv;

    Hv =ifwo+ ((1.3605e3)+(2.31334)*(tw/2)-(2.46784e-10)*...
    ((tw/2)^5)-(5.91332e-13)*((tw/2)^6))*tw;
    Hmas = ((1.045356e3)-(3.161783e-1)*(tw/2)+(7.083814e-4)*...
    ((tw/2)^2)-(2.705209e-7)*((tw/2)^3))*tw+ws*Hv;

    Le = (0.866^0.667)*((ws+0.622)/(w+0.622)-1)/log((ws+0.622)/(w+0.622));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    k31 = h*((Ka*Af/(cpw*mw))*(-(ws-w)*cpw*tw+(Hmas-Hma)+(Le-1)*(Hmas-Hma-
(ws-w)*Hv)));
    k32 = h * (Ka*Af*(ws-w));
    k33 = h * (Ka*Af*(ws-w)/ma);
    k34 = h * ((Ka*Af/ma)*(Le*(Hmas-Hma)+(1-Le)*Hv*(ws-w)));

    tw = tw - k21/2;
    mw = mw - k22/2;
    w  = w  - k23/2;
    Hma = Hma - k24/2;
 %calculating k4 needs k2 + w,
    tw = tw + k31;
    mw = mw + k32;
    w  = w  + k33;
    Hma = Hma + k34;

 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   cpw = (8.15599e3)-(2.80627*10)*tw/2+(5.11283e-2)*((tw/2)^2)...
        -(2.17582e-13)*((tw/2)^6);

    pvs = 10^(10.79586*(1-273.16/tw)+5.02808*log10(273.16/tw)+...
      (1.50474*10^-4)*(1-10^(-8.29692*(tw/273.16-1)))+...
      (4.2873*10^-4)*(10^(4.76955*(1-273.16/tw))-1)+2.7861);

    ws = 0.622*pvs/(p-pvs);

    cpa = (1.045356e3)-(3.161783e-1)*(ta/2)+(7.083814e-4)*...
        ((ta/2)^2)-(2.705209e-7)*((ta/2)^3);

    cpv = (1.3605e3)+(2.31334)*(ta/2)-(2.46784e-10)*((ta/2)^5)...
        -(5.91332e-13)*((ta/2)^6);

    cpma = cpa + w*cpv;

    Hv =ifwo+ ((1.3605e3)+(2.31334)*(tw/2)-(2.46784e-10)*...
    ((tw/2)^5)-(5.91332e-13)*((tw/2)^6))*tw;
    Hmas = ((1.045356e3)-(3.161783e-1)*(tw/2)+(7.083814e-4)*...
```

```
    ((tw/2)^2)-(2.705209e-7)*((tw/2)^3))*tw+ws*Hv;

    Le = (0.866^0.667)*((ws+0.622)/(w+0.622)-1)/log((ws+0.622)/(w+0.622));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    k41 = h*((Ka*Af/(cpw*mw))*(-(ws-w)*cpw*tw+(Hmas-Hma)+(Le-1)*(Hmas-Hma-
(ws-w)*Hv)));
    k42 = h * (Ka*Af*(ws-w));
    k43 = h * (Ka*Af*(ws-w)/ma);
    k44 = h * ((Ka*Af/ma)*(Le*(Hmas-Hma)+(1-Le)*Hv*(ws-w)));

    tw = tw - k31;
    mw = mw - k32;
    w  = w  - k33;
    Hma = Hma - k34;

    tw = tw + (k11 + 2*k21 + 2*k31 + k41)/6;
    mw = mw + (k12 + 2*k22 + 2*k32 + k42)/6;
    w = w   + (k13 + 2*k23 + 2*k33 + k43)/6;
    Hma = Hma + (k14 + 2*k24 + 2*k34+ k44)/6;

    ta = tw - ((Hmas-Hma)-(ws-w)*Hv)/cpma;

    z = a + i*h;


    %storage for ploting data
    x(i) = z; %distance
    y(i)=tw;  %water temperature
    y1(i)=ta; %air temperature

    %storage for effectiveness calculations
    y2(i)  = Hmas;
    y3(i)  = Hma;
    y4(i)  = ws;
    y5(i)  = w;
    y6(i)  = mw;

end

if imag(tw)== 0 && i == N

err = abs(tw -twexp);

Evap_loss = ma*(w -win);

end

  j = j + 1;

  if j >= maxstep
      err = 0.01;
      fprintf(1,'maximum limit reached for cooling tower %d \n',m);
      display(twin)
```

```
  end
end
CC=5;%Cycles of concentration

if mw > 0
Cemin = (mw*cpw/((Hmas-Hmasin)/(twin-twout)));

eff(m) = mw*cpw*(twin-twout)/(Cemin*(Hmas-Hmain));

E(m) = real(Evap_loss);
B(m) = E(m)/(CC - 1);
M(m) = E(m)*CC/(CC-1);
Tctout(m) = twout - 273;
OS(m) = mwin(m);

%averall effectiveness
Hact(m)= mw*cpw*(twin-twout);
Hmax(m) = (Cemin*(Hmas-Hmain));
else
eff(m) = 0;
E(m) = 0;
B(m) = E(m)/(CC - 1);
M(m) = E(m)*CC/(CC-1);
Tctout(m) = 0;
OS(m) = mwin(m);

%averall effectiveness
Hact(m)= 0;
Hmax(m) = 0;
end
end

if gg >= 2
    error = abs(XX1-Tctout(1))+abs(XX2-Tctout(2))+abs(XX3-Tctout(3))

end

XX1 = Tctout(1);
XX2 = Tctout(2);
XX3 = Tctout(3);



  if gg >= maxstepp
      error = 0.01;
      fprintf(1,'maximum iteration limit reached \n');

  end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


gg
averall_eff = sum(Hact)/sum(Hmax);

display('Gams model status')
display(model_status)

display('inlet temperatures')
display(twctin)

display('outlet temperatures')
display(Tctout)

display('outlet flowrates')
display(OS)

display('Evaporation')
display(E)

display('Blowdown')
display(B)

display('Make up')
display(M)

display('effectiveness')
display(eff)

display('averall effectiveness')
display(averall_eff)
end
```