CHAPTER 3

A SINGLE PRODUCT PERISHING INVENTORY MODEL WITH DEMAND INTERACTION

A modified version of chapter has been published in ORiON, Vol 20, No 2, 2004.

3.1 INTRODUCTION

In inventory models of perishing products the lifetime of the products in the inventory model is described in alternative ways. One assumption is that the product has a fixed lifetime and if no demand occurs for the product within its lifetime, it is considered as perished and removed from the inventory. Nahmias (1982) has given an exhaustive survey of the fixed-life perishable inventory literature. Another description of the lifetime is that the product deteriorates continuously in quality over time and eventually perishes. Raafat (1991) has presented a review of the literature on deteriorating (decaying) inventory models. Apart from the lifetime consideration, the deteriorating items have one important kind of interaction on the demand process in the sense that, in addition to the usual demand, there may also be a separate demand for items slightly deteriorated in quality if the cost is comparatively lesser than a new one. For example, vegetables, food, meat and fish loose their lustre as time elapse. A day old vegetable is slightly inferior in quality compared to a new one. Such items may be accepted by some customers in the event of non-availability of new ones. There may also be a significant number of demands for slightly deteriorated items due to the fact that they are less expensive. Some of continuous review inventory models have been studied recently by Beyer and Girlich (1994), Yadavalli et al (2001), Yadavalli & Joubert (2003) and Yadavalli et al (2004).

In this chapter, an attempt is made to incorporate the above kind of interaction in the study of deteriorating product inventory systems. Specifically, a continuous review of perishing inventory models is considered with the assumption that if there is no demand for product in inventory, it passes through two phases and then perishes. An item in Phase I is fresh and in Phase II slightly deteriorated. On leaving Phase II, it is considered as being perished and removed from inventory or scrapped. Independent demand takes place at constant rates for items in both phases. Demand for an item during Phase I stock-out may be satisfied by an item in Phase II based on a probability measure. Demand for product in phase II during stock-out is lost. Using the regeneration point technique, various measures of the inventory model are obtained.

The organization of this chapter is as follows: Section 3.2 lists various assumptions and notations in the description of the inventory model and also provides the auxiliary functions which are needed to describe the behaviour of the process between two successive regeneration points of the underlying stochastic process describing the inventory model. Various performance measures of the inventory model are obtained in Section 3.3. A cost analysis is provided in Section 3.4 and some numerical results are presented in Section 3.5.

3.2 ASSUMPTIONS AND AUXILIARY FUNCTION

The following assumptions are considered in the continuous inventory model with:

1. The item under consideration is perishable.

2. The lifetime distribution of an item is a generalized Erlang distribution with two phases. For convenience the items in Phase I are designated as Product 1 and that in Phase II as Product 2.

3. The demand for product i occurs at a constant rate λ_i, $i = 1, 2$.

4. Maximum storage capacity or total capacity of the inventory level is S and re-order takes place if the total inventory level is s.

5. At the epoch of replenishment, all items of Product 2 are scrapped (deleted) and the inventory level is raised to S.

6. The lead-time is arbitrary with pdf $f(.)$, and survivor function $\overline{F}(t) = 1 - F(t)$, where *F(t)* is the cdf. The arbitrary distribution is selected as an approximation of complex problems.

7. A demand for Product 1 occurring during the stock-out period can be substituted by an item of Product 2 with probability p if available, $0 \le p \le 1$.

8. A demand for Product 2 occurring during the stock-out period is lost, that is no backlogging is possible.

The following notation are used in this chapter:

a_j : Event that a re-order takes place when the inventory level of Product 2 is j, $0 \le j \le s$.

a : Any a_j-event, $0 \le j \le s$.

r_{ij} : Event that a stock replenishment occurs. $S - i$ units of Product 1 are added to the inventory and j units of Product 2 scrapped from the inventory.

r : Any r_{ij}-event, $0 \le i,\ j,\ \ i + j \le s.$

l_j : Event that a demand for product j is lost, j = 1, 2

g : Event that a demand for Product 1 is substituted by Product 2.

d_i : Event that a demand for product i is satisfied with product i, $i = 1, 2$.

k_1 : Event of Product 1 transitting as Product 2.

k_2 : Event of Product 2 perishing and being removed from the inventory.

$L_i(t)$: Inventory level of product i at time t; $i = 1, 2$.

$Z(t)$: $\left(L_1(t),\, L_2(t)\right)$.

λ_i : The demand rate of product i, $i = 1, 2$.

μ_i : The perishing rate of product i, $i = 1, 2$.

N(η,t): Number of η events in (0,t].

$E[N(a_j, \infty)]$: The mean stationary rate of re-order.

$E[N(k_1, \infty)]$: The mean stationary rate of transit of Product 1 as Product 2.

$E[N(k_2, \infty)]$: The mean stationary rate of perishing and removed from the inventory.

CR : Re-ordering cost.

CL_i : Cost of lost demand for product i, $i = 1, 2$.

CP : Salvage cost per scrapped (deleted) unit.

CB : Purchase price of one unit.

$C(S, s)$: Total expected cost per unit time.

© : Convolution symbol.

In order to study the stochastic process $(L_1(t), L_2(t))$, note that the r-events constitute a renewal process (see Figure 3.1 below). Consequently, it is sufficient to describe the behaviour of the inventory process between two successive renewals.

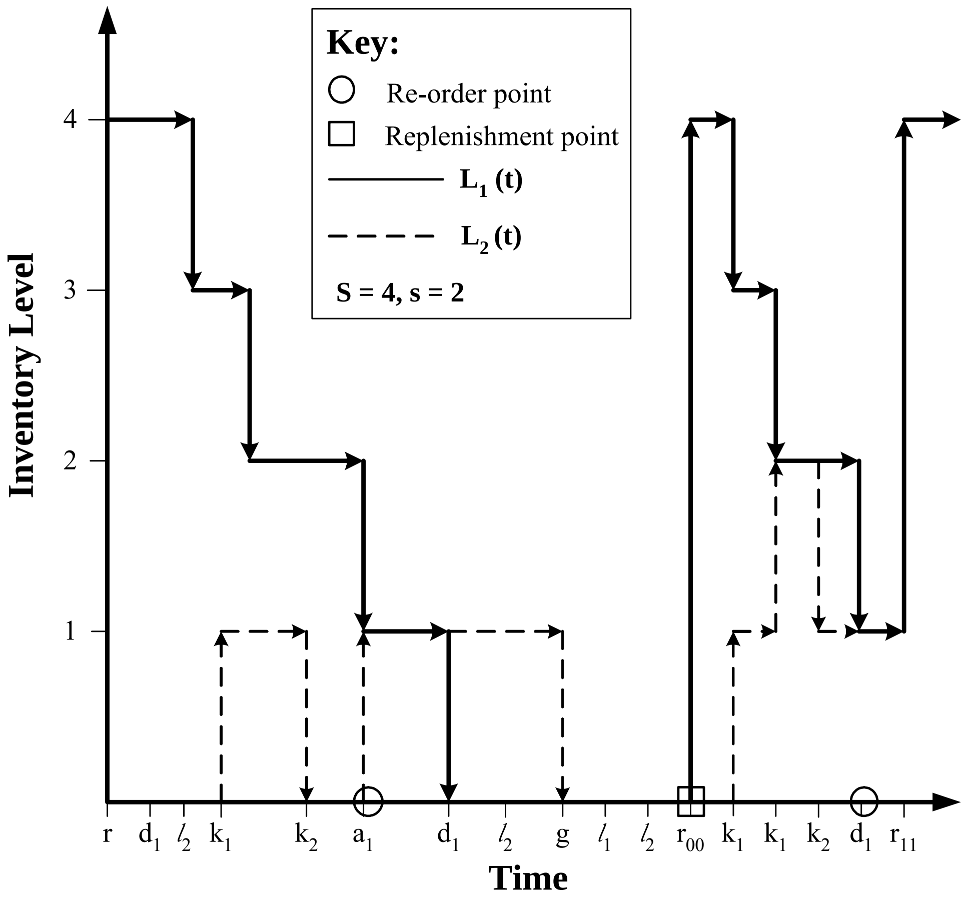

Figure 3.1: Realization of Events

The necessary auxiliary functions are introduced:

3.2.1 FUNCTION $P(k,l,t|i,j)$

We define

$$P(k,l,t|i,j) = P[Z(t)=(k,l), N(\eta,t)=0 | Z(0)=(i,j)] \quad , \quad \eta = a, r \, .$$

$P(k,l,t|\ i,j)$ represents the probability distribution of the inventory level in an interval in which neither reorder nor replenishment can occur. To derive an expression for this function, we note that a change in the inventory level may occur due to any one of the following possibilities:

1. A demand for Product i occurs and is satisfied by product i, $(i = 1,2)$
2. A unit of Product 1 perishes and transits as Product 2.
3. A unit of Product 2 perishes.
4. A demand for a unit of Product 1 occurs during the stock-out period and is substituted by Product 2 with probability p if it is available.

Accordingly, we have for

$$0 \le k + l \le i + j \le s \text{ or } s + 1 \le k + l \le i + j \le S ,$$

Case 1: $k > i$.

$$P(k,l,t|\ i,j) = 0 . \tag{3.1}$$

Case 2: $i > 0,\ \ j > 0,\ \ 0 < k < i,\ \ k + l < i + j$.

$$P(k,l,t|\ i,j) = \lambda_1\ e^{-(\lambda_1+\lambda_2+i\mu_1+j\mu_2)t} \copyright\ P(k,l,t|\ i-1,j) + i\mu_1 e^{-(\lambda_1+\lambda_2+i\mu_1+j\mu_2)t}$$
$$\copyright\ P(k,l,t|\ i-1,j+1) + (\lambda_2 + j\mu_2)\ e^{-(\lambda_1+\lambda_2+i\mu_1+j\mu_2)t} \copyright\ P(k,l,t|\ i,j-1) . \tag{3.2}$$

Case 3: $i > 0,\ j = 0, 0 \le k < i, l \ge 0, k + l < i$.

$$P[k,l,t|\ i,0] = \lambda_1\ e^{-(\lambda_1+\lambda_2+i\mu_1+j\mu_2)t} \copyright\ P[k,l,t|i-1,0] + i\mu_1 e^{-(\lambda_1+\lambda_2+i\mu_1+j\mu_2)t}$$
$$\copyright\ P(k,l,t|i+1). \qquad (3.3)$$

Case 4: $i > 0,\ j > 0,\ k = i,\ l = j$.

$$P[i,j,t|\ i,j] = e^{-(\lambda_1+\lambda_2+i\mu_1+j\mu_2)t}. \qquad (3.4)$$

Case 5: $i > 0,\ j > 0,\ k = i,\ 0 \le l < j$.

$$P[i,l,t|\ i,j] = (\lambda_2 + j\mu_2)e^{-(\lambda_1+\lambda_2+i\mu_1+j\mu_2)t} \copyright\ P[i,l,t|i,j-1]. \qquad (3.5)$$

Case 6: $i > 0,\ j = 0,\ k = i,\ l = 0$.

$$P[i,0,t|\ i,0] = e^{-(\lambda_1+i\mu_1)t}. \qquad (3.6)$$

Case 7: $i = 0,\ j > 0,\ k = 0,\ l = j$.

$$P[0,j,t|\ 0,j] = e^{-(\lambda_1 p+\lambda_2+j\mu_2)t}. \qquad (3.7)$$

Case 8: $i = 0,\ l \ge 0,\ k = 0,\ l < j$

$$P[0,l,t|\ 0,j] = (\lambda_1 p + \lambda_2 + j\mu_2)e^{-(\lambda_1 p+\lambda_2+j\mu_2)t} \copyright\ P[0,l,t|\ 0,j-1]. \qquad (3.8)$$

Case 9: $i = j = k = l = 0$.

$$P[0,0,t|0,0] = 1. \qquad (3.9)$$

3.2.2 FUNCTION $\phi_j(t)$

We define

$$\phi_j(t) = \lim_{\Delta \to 0} \frac{P[a_j - event\ in\ (t, t+\Delta),\ N(r,t) = 0 \mid r - event\ at\ t = 0]}{\Delta}.$$

The function $\phi_j(t)dt$ represents the probability that an a_j-event occurs in $(t, t+\Delta)$ and there is no replenishment in $(0,t]$, given that an r-event has occurred at $t = 0$. Hence, we have

$$\phi_j(t) = P[k+1, j, t \mid S, 0]\lambda_1 \overline{F}(t) + P[k, j+1, t \mid S, 0][\lambda_2 + (j+1)\mu_2 + \delta_{k0}\lambda_1 p]\overline{F}(t) \quad (3.10)$$

where $k + j = s, 0 \le k, j \le s$, and δ_{k0} is a Kronecker's delta function.

3.2.3 FUNCTION $W(i, j, t)$

We define

$$W(i, j, t) = P[Z(t) = (i, j),\ N(r,t) = 0 \mid Z(0) = (S, 0)].$$

Then the function $W(i, j, t)$ represents the probability that the inventory level is (i, j) at the time t, where t is the time elapsed since the last renewal. To obtain $W(i, j, t)$, we consider

Case 1: $0 \le i + j \le s$

In this case, exactly one re-order is made in $(0, t)$ and it does not materialize up to time t. Precisely, we have

(i) The system is in state $(S,0)$ at $t=0$.

(ii) The system enters the state (k,l) in $(u,u+du)$ where $k+l=s$ and $0<u<t$.

(iii) A re-order is placed in $(u,u+du)$.

(iv) The re-order does not materialize up to time t.

(v) The system enters the state (i,j) at time t.

Using probabilistic arguments,

$$W(i,j,t)=\sum_{l=0}^{s}\phi_l(t)\copyright\left\{\overline{F}(t)P(i,j,t|k,l)\right\}, \text{ where } 0\le k,l\le s \text{ and } k+l=s. \quad (3.11)$$

Case 2: $s+1\le i+j\le S$

In this case no re-order takes place in $(0,t)$. Hence,

$$W(i,j,t)=P[i,j,t|S,0] \quad (3.12)$$

The steady-state probabilities of the system are given by

$$\begin{aligned} W(i,j) &= \lim_{t\to\infty}W(i,j,t) \quad (3.13)\\ &= \lim_{\Delta\to 0}W^*(i,j,s) \end{aligned}$$

Where $W^*(i,j,s)$ is the Laplace transform of $W(i,j,t)$ (See Girlich, 2003)

$$W^*(i,j,s)=\int_0^\infty e^{-st}W(i,j,t)\,dt$$

3.3 MEASURES OF SYSTEM PERFORMANCE

To obtain explicit expressions for various performance measures of the presented model, we proceed to define the first-order product density

$$h_\eta(t) = \lim_{\Delta \to 0} \frac{P[\eta - event\ in\ (t, t+\Delta) | Z(0) = (S, 0)]}{\Delta}.$$

where $\eta = r, r_{ij}, a, a_j, d_1, d_2, l_1, l_2, g, k_1, k_2$.

3.3.1 MEAN NUMBER OF RE-ORDERS

Since a re-order is defined as an a_j-event, the expressions for $h_{ij}(t)$ are derived to obtain the mean number of re-orders. Note that a re-order takes place when the total inventory level enters s. Hence,

$$h_{a_j}(t) = \sum_{i+j=s}^{\infty} [W(i+1, j, t)\lambda_1 + W(i, j+1, t)\{\delta_{i0}\lambda_1 p + \lambda_2 + (j+1)\mu_2\}]. \tag{3.14}$$

The mean number of re-orders in $(0, t]$ is given by

$$E[N(a_j, t)] = \int_0^t h_{a_j}(u)du. \tag{3.15}$$

Consequently, the mean stationary rate of re-orders is given by

$$E[N(a_j, \infty)] \quad = \lim_{t \to \infty} \frac{1}{t} E[N(a_j, t)] = \lim_{t \to \infty} h_{a_j}(t)$$

$$= \sum_{i+j=s}^{\infty} [W(i+1, j)\lambda_1 + W(i, j+1)\{\delta_{i0}\lambda_1 p + \lambda_2 + (j+1)\mu_2\}]. \quad (3.16)$$

3.3.2 MEAN NUMBER OF DEMANDS FOR A PARTICULAR PRODUCT WHICH IS SATISFIED BY THE SAME PRODUCT

A demand for Product 1 being satisfied by Product 1 is represented by the d_1-event. Hence an expression for $h_{d_1}(t)$ is derived. Observe that a d_1-event occurs whenever a demand for Product 1 occurs when the inventory levels is (i, j) where, $1 \le i \le S$, $0 \le j \le S$ and $0 < i + j \le S$. Hence,

$$h_{d_1}(t) = \sum_{\substack{0 \le i+j \le S \\ i \ge 1, j \ge 0}}\sum W(i, j)\lambda_1 \ , \quad (3.17)$$

so that

$$E[N(d_1, t)] = \int_0^t h_{d_1}(u)du \, .$$

Therefore,

$$E[N(d_1, \infty)] = \sum_{\substack{0 \le i+j \le S \\ i \ge 1, j \ge 0}}\sum W(i, j)\lambda_1 . \quad (3.18)$$

In the same way,

$$h_{d_2}(t) = \sum_{\substack{0 \le i+j \le S \\ i \ge 1, j \ge 0}}\sum W(i, j, t)\lambda_2 , \quad (3.19)$$

so that

$$E[N(d_2,t)] = \int_0^t h_{d_2}(u)du$$

$$E[N(d_2,\infty)] = \sum\sum_{\substack{0\le i+j\le S \\ i\ge 1,\, j\ge 0}} W(i,j)\lambda_2 \,. \tag{3.20}$$

3.3.3 MEAN NUMBER OF LOST DEMAND

A demand for Product 1 is lost when the total inventory level is zero or when the inventory level of Product 1 is zero and that of Product 2 is positive, but when the demand is not substituted with Product 2. Therefore,

$$\begin{aligned} h_{l_1}(t) &= W(0,0,t)\lambda_1 + \sum_{j=1}^{S} W(0,j,t)\lambda_1(1-p) \\ &= \sum_{j=0}^{S} W(0,j,t)\{1-p+p\delta_{j0}\}\lambda_1 . \end{aligned} \tag{3.21}$$

The mean number of lost demands for Product 1 is given by

$$E[N(l_1,t)] = \int_0^t h_{l_1}(u)du \,,$$

so that the mean stationary rate of lost demand for Product 1 is given by

$$E[N(l_1,\infty)] = \sum_{j=0}^{S} W(0,j)[1-p+p\delta_{j0}]\lambda_1 \,. \tag{3.22}$$

In the same way, for the events l_2,

$$h_{l_2}(t) = \sum_{i=0}^{S} W(i,0,t)\lambda_2 \ , \tag{3.23}$$

$$E[N(l_2,t)] = \int_0^t h_{l_2}(u)du$$

and

$$E[N(l_2,\infty)] = \sum_{i=0}^{S} W(i,0)\lambda_2 \ . \tag{3.24}$$

3.3.4 MEAN NUMBER OF DEMANDS OF PRODUCT 1 BEING SUBSTITUTED BY PRODUCT 2

A demand for Product 1 being substituted by Product 2 is denoted by the g-event. Note that a g-event occurs in $(t, t+\Delta)$ if the inventory level of the system at time t equals $(0, j), 1 \le j \le S$ and if a demand for Product 1 occurs in $(t, t+\Delta)$ being substituted by Product 2. Hence,

$$h_g(t) = \sum_{j=1}^{S} W(0,j,t)\lambda_1 p \tag{3.25}$$

and

$$E[N(g,t)] = \int_0^t h_g(u)du \ .$$

Therefore,

$$E[N(g,\infty)] = \sum_{j=1}^{S} W(0,j)\lambda_1 p \ . \tag{3.26}$$

3.3.5 MEAN NUMBER OF UNITS DETERIORATED FROM PRODUCT 1 AND TRANSITTED AS PRODUCT 2

Since a k_1-event pertains to the event of a unit of Product 1 deteriorates and transits as Product 2 and a k_1-event occurs in $(t, t+\Delta)$ if the system is in state (i, j) at time t, $1 \le i \le S$, $0 \le j \le S$ and $1 \le i + j \le S$ and a unit in Product 1 transits as Product 2 in $(t, t+\Delta)$, we have

$$h_{k_1}(t) = \sum_{\substack{0 \le i+j \le S \\ i \ge 1,\, j \ge 0}}\sum W(i, j, t) i \mu_1 \tag{3.27}$$

The mean number of units of Product 1 that have transitted as Product 2 in $(0, t]$ is given by

$$E[N(k_1, t)] = \int_0^t h_{k_1}(u) du$$

and the mean stationary rate of units of Product 1 transiting as Product 2 is given by

$$E[N(k_1, \infty)] = \sum_{\substack{0 \le i+j \le S \\ i \ge 1,\, j \ge 0}}\sum W(i, j) i \mu_1 . \tag{3.28}$$

3.3.6 MEAN NUMBER OF PRODUCT 2 PERISHED AND REMOVED FROM THE INVENTORY

The first order product density of k_2 is given by

$$h_{k_2}(t) = \sum_{\substack{0 \le i+j \le S \\ i \ge 1,\, j \ge 0}}\sum W(i, j, t) j \mu_2 \ . \tag{3.29}$$

Hence the mean number of units of Product 2 that have perished and removed from the inventory in $(0,t]$ is given by

$$E[N(k_2,t)] = \int_0^t h_{k_2}(u)du .$$

Consequently, the mean stationary rate of perishing of Product 2 is given by

$$E[N(k_2,\infty)] = \sum_{\substack{0\le i+j\le S \\ i\ge 1,\, j\ge 0}}\sum W(i,j)\, j\mu_2 . \tag{3.30}$$

3.3.7 MEAN NUMBER OF REPLENISHMENTS

Consider the renewal process of r-events and derive its first-order product density $h_r(t)$. Firstly, an expression for the pdf $g(t)$ of the interval between two successive occurrences of the r-events is derived. By definition,

$$g(t) = \lim_{\Delta\to 0}\frac{P[r - event\ in\ (t,t+\Delta),\, N(r,t) = 0 \,|\, Z(0) = (S,0)]}{\Delta} .$$

In order to derive $g(t)$, its survival function $\overline{G}(t)$ is determined. Since $\overline{G}(t)$ denotes the probability that a replenishment has not occurred up to t, we have two mutually exclusive cases for $\overline{G}(t)$:

(i) A re-order does not occur up to time t.

(ii) A re-order is placed in $(u,u+\Delta)$, $0 < u < t$, but it has not been realized up to time t.

Hence,

$$\overline{G}(t) = \sum_{\substack{s+1 \le k+l \le S \\ k \ge 0, l \ge 0}} \sum P(k,l,t|\ S,0) + \sum_{l=0}^{s} \phi_l(t) © \left\{ \overline{F}(t) + \sum_{k_1=0}^{s-l} \sum_{l_1=0}^{l} P(k_1,l_1,t|\ s-l,l \right\}. \quad (3.31)$$

However,

$$h_r(t) = \sum_{n=1}^{\infty} g^{(n)}(t),$$

and

$$E[N(r,t)] = \int_0^t h_r(u)du.$$

Hence, by renewal theory, the mean stationary rate of replenishment is given by

$$E[N(r,\infty)] = \lim_{t\to\infty} \frac{1}{t} \int_0^t h_r(u)du = \frac{1}{\int_0^\infty \overline{G}(t)dt}. \quad (3.32)$$

3.3.8 MEAN NUMBER OF REPLENISHMENTS

First, the product density is defined

$$h_{r_{ij}}(t) = \lim_{\Delta\to 0} \frac{P[r_{ij} - event\ in\ (t,t+\Delta)|\ Z(0) = (S,0)]}{\Delta}$$

Next a relation between $h_{r_{ij}}(t)$ and $h_r(t)$ is obtained.

Therefore, the following function is define

$$f_{ij}(t) = \lim_{\Delta \to 0} \frac{P[r_{ij} - event\ in\ (t, t+\Delta), N(r,t) = 0 \mid Z(0) = (S,0)]}{\Delta}$$

Observe that

$$f_{ij}(t) = \sum_{\substack{0 \le k+l \le S \\ k \ge 0, l \ge 0}} \sum [P(k+1, l, t \mid S, 0)\lambda_1 + P(k, l+1, t \mid S, 0)\{\lambda_2 + (l+1)\mu_2 + \delta_{k0}\lambda_1 p\}] ©$$

$$f(t) P(i, j, t \mid k, l) . \qquad (3.33)$$

Consequently,

$$h_{r_{ij}}(t) = f_{ij}(t) + h_r(t) © f_{ij}(t) \qquad (3.34)$$

and

$$E[N(r_{ij}, t)] = \int_0^t h_{r_{ij}}(u) du .$$

Hence,

$$E[N(r_{ij}, \infty)] = \lim_{t \to \infty} \frac{1}{t} \int_0^t h_{r_{ij}}(u) du$$

$$= E[N(r, \infty)] \lim_{\theta \to 0} f_{ij}*(\theta) . \qquad (3.35)$$

Since at the occurrence of each r_{ij}-event, $S - i$ units of Product 1 are added to the inventory, the mean number of Product 1 items added to the inventory per unit time is given by

$$\sum\sum_{\substack{0\le i+j\le S\\ i\ge 0,\, j\ge 0}} E[N(r_{ij},\infty)](S-i) \;=\; E[N(r,\infty)]\sum\sum_{\substack{0\le i+j\le S\\ i\ge 0,\, j\ge 0}} \lim_{\theta\to 0} f_{ij}*(\theta)\,. \tag{3.36}$$

3.3.9 MEAN NUMBER OF UNITS SCRAPPED FROM THE INVENTORY

Since, at the occurrence of an r_{ij}-event, j units of Product 2 are scrapped from the inventory per unit time, we have

$$\sum\sum_{\substack{0\le i+j\le S\\ i\ge 0,\, j\ge 0}} E[N(r_{ij},\infty)]j \;=\; E[N(r,\infty)]\sum\sum_{\substack{0\le i+j\le S\\ i\ge 0,\, j\ge 0}} \lim_{\theta\to 0} f_{ij}*(\theta)\,. \tag{3.37}$$

3.4 COST ANALYSIS

Since $E[N(l_1,\infty)]$ and $E[N(l_2,\infty)]$ are respectively the mean stationary rates of the two types of lost demands. The cost due to lost demand is given by

$$E[N(l_1,\infty)]CL_1 + E[N(l_2,\infty)]CL_2 \tag{3.38}$$

The cost corresponding to items of Product 2 perished and removed from the inventory is $E[N(k_2,\infty)]CP$. The number of items of Product 2 that are scrapped from the inventory per unit time is

$$\sum_{\substack{0\le i+j\le S \\ i\ge 0,\, j\ge 0}}\sum E[N(r_{ij},\infty)]\,j\,. \tag{3.39}$$

The cost due to this is

$$\sum_{\substack{0\le i+j\le S \\ i\ge 0,\, j\ge 0}}\sum E[N(r_{ij},\infty)]\,jCP\,. \tag{3.40}$$

Hence the total expected cost per unit time is:

$$\begin{aligned} C(S,s) = {} & E[N(a,\infty)]CR + E[N(l_1,\infty)]CL_1 + E[N(l_2,\infty)]CL_2 \\ & + [E[N(k_2,\infty)] + \sum_{\substack{0\le i+j\le S \\ i\ge 0,\, j\ge 0}}\sum E[N(r_{ij},\infty)]\,j]CP + \sum_{\substack{0\le i+j\le S \\ i\ge 0,\, j\ge 0}}\sum E[N(r_{ij},\infty)](s-i)CB\,. \end{aligned} \tag{3.41}$$

3.5 NUMERICAL EXAMPLE

For illustration purposes, consider the following numerical example. Let

$f(t) = \theta e^{-\theta t}, t > 0, \theta > 0$

$\lambda_1 = 4.0$,

$\lambda_2 = 6.0$,

$\mu_1 = 2.5$,

$\mu_2 = 2.5$,

$\theta = 2.0$,

CR = 10.0,

$CL_1 = 6.0$,

$CL_2 = 5.0$,

CP = 4.0, and

CB = 10.0

By varying the probability p from 0.1 to 0.9 and varying S from 2 to 4, with corresponding possible values for s, the values of the mean stationary rates of the following variables are obtained:

(i) Demand satisfied (ED_1, ED_2)
(ii) Demands substituted (EG)
(iii) Lost demands (EL_1, EL_2)
(iv) Items perished (EK_2)
(v) Re-orders (ES)
(vi) Replenishments (RRATE)
(vii) Units replenished (EUR)
(viii) Units scrapped or deleted (EUS)
(ix) Total expected cost (COST)

The numerical results of the relationship between p and the above variables are summarised in Table 3.1 below:

	S=2, s=1	S=3, s=1	S=3, s=2	S=4, s=1	S=4, s=2	S=4, s=3
ED1	increases	increases	increases	increases	increases	increases
ED2	decreases	decreases	decreases	decreases	decreases	decreases
EG	increases	increases	increases	increases	increases	increases
EL1	decreases	decreases	decreases	decreases	decreases	decreases
EL2	increases	increases	increases	increases	increases	increases
EK2	decreases	decreases	decreases	decreases	decreases	decreases
EA	increases	increases	increases	increases	increases	increases
RRATE	increases	increases	increases	increases	increases	increases
EUR	increases	increases	increases	increases	increases	increases
EUS	decreases	decreases	decreases	decreases	decreases	decreases
COST	decreases	decreases	decreases	decreases	decreases	decreases

Table 3.1: Relationship between p and selected variables for varying S and s

Per illustration, the relationships of Total Expected Cost (COST) and Lost Demand (EL2) versus increasing values of p are shown graphically in Figure 3.2.

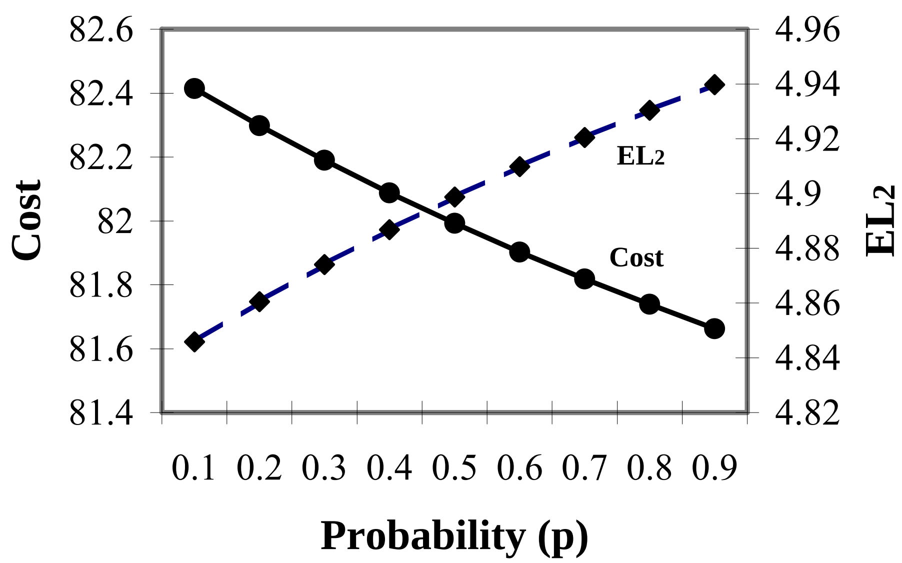

Figure 3.2: Relationship of COST and EL2 versus p for S =3, s =1

The detail results of the numerical example are given in Table 3.2 to Table 3.7 for varying values of S and s.

p	ED1	ED2	EG	EL1	EL2	EK2
0.1	1.447408	0.802104	0.030899	2.521693	5.197896	0.365150
0.2	1.448016	0.786138	0.059651	2.492333	5.213861	0.357462
0.3	1.448584	0.771246	0.086471	2.464945	5.228754	0.350288
0.4	1.449116	0.757323	0.111549	2.439334	5.242677	0.343578
0.5	1.449616	0.744276	0.135049	2.415335	5.255723	0.337289
0.6	1.450087	0.732027	0.157114	2.392799	5.267973	0.331382
0.7	1.450530	0.720503	0.177874	2.371595	5.279497	0.325824
0.8	1.450949	0.709642	0.197440	2.351611	5.290359	0.320585
0.9	1.451345	0.699389	0.215912	2.332742	5.300611	0.315637

p	EA	RRATE	EUR	EUS	COST
0.1	1.523762	1.523762	2.775306	0.129744	86.0899
0.2	1.524402	1.524402	2.776470	0.125240	85.9627
0.3	1.525000	1.525000	2.777560	0.120971	85.8441
0.4	1.525560	1.525560	2.778581	0.117015	85.7332
0.5	1.526087	1.526087	2.779540	0.113309	85.6293
0.6	1.526582	1.526582	2.780442	0.109832	85.5318
0.7	1.527049	1.527049	2.781292	0.106561	85.4400
0.8	1.527490	1.527490	2.782095	0.103480	85.3536
0.9	1.527907	1.527907	2.782854	0.100571	85.2720

Table 3.2: Numerical Results for S = 2, s =1

p	ED1	ED2	EG	EL1	EL2	EK2
0.1	2.015121	1.154118	0.029509	1.955370	4.845882	0.591228
0.2	2.016546	1.139571	0.057012	1.926442	4.860429	0.583235
0.3	2.017882	1.125985	0.082706	1.899412	4.874015	0.575761
0.4	2.019136	1.113267	0.106766	1.874098	4.886733	0.568758
0.5	2.020316	1.101338	0.129342	1.850342	4.898662	0.562181
0.6	2.021429	1.090125	0.150568	1.828003	4.909874	0.555993
0.7	2.022480	1.079566	0.170562	1.806958	4.920434	0.550161
0.8	2.023474	1.069605	0.189429	1.787098	4.930395	0.544654
0.9	2.024415	1.060193	0.207260	1.768325	4.939806	0.539447

p	EA	RRATE	EUR	EUS	COST
0.1	1.139148	1.139148	3.227835	0.104645	82.4150
0.2	1.139954	1.139954	3.230117	0.101013	82.2985
0.3	1.140709	1.140709	3.232256	0.097624	82.1897
0.4	1.141418	1.141418	3.234266	0.094455	82.0879
0.5	1.142085	1.142085	3.236156	0.091486	81.9924
0.6	1.142714	1.142714	3.237938	0.088698	81.9027
0.7	1.143308	1.143308	3.239621	0.086075	81.8182
0.8	1.143870	1.143870	3.241213	0.083603	81.7384
0.9	1.144402	1.144402	3.242721	0.081269	81.6631

Table 3.3: Numerical Results for S = 3, s = 1

p	ED1	ED2	EG	EL1	EL2	EK2
0.1	2.100194	1.209167	0.028260	1.871547	4.790833	0.619952
0.2	2.100425	1.195623	0.054702	1.844873	4.804378	0.612563
0.3	2.100644	1.182931	0.079498	1.819857	4.817068	0.605627
0.4	2.100852	1.171014	0.102799	1.796349	4.828986	0.599103
0.5	2.101049	1.159802	0.124737	1.774214	4.840198	0.592954
0.6	2.101236	1.149234	0.145429	1753335	4.850765	0.587149
0.7	2.101414	1.139256	0.164979	1.733607	4.860744	0.581660
0.8	2.101583	1.129818	0.183481	1.714936	4.870183	0.576461
0.9	2.101745	1.120878	0.201016	1.697240	4.879122	0.571530

p	EA	RRATE	EUR	EUS	COST
0.1	2.058707	1.436794	3.566840	0.231180	94.8435
0.2	2.058934	1.436952	3.567234	0.225902	94.7067
0.3	2.059149	1.437102	3.567606	0.220951	94.5783
0.4	2.059352	1.437244	3.567958	0.216298	94.4577
0.5	2.059545	1.437379	3.568292	0.211917	94.3441
0.6	2.059728	1.437507	3.568610	0.207783	94.2370
0.7	2.059903	1.437628	3.568912	0.203878	94.1357
0.8	2.060069	1.437744	3.569200	0.200181	94.0398
0.9	2.060227	1.437855	3.569474	0.196677	93.9489

Table 3.4: Numerical Results for S = 3, s = 2

p	ED1	ED2	EG	EL1	EL2	EK2
0.1	2.461397	1.550978	0.022888	1.515716	4.449023	0.824795
0.2	2.462760	1.540001	0.044223	1.493017	4.460000	0.818603
0.3	2.464039	1.529750	0.064159	1.471803	4.470250	0.812810
0.4	2.465240	1.520156	0.082829	1.451930	4.479844	0.807379
0.5	2.466372	1.511157	0.100352	1.433276	4.488843	0.802276
0.6	2.467440	1.502699	0.116830	1.415730	4.497301	0.797472
0.7	2.468449	1.494735	0.132354	1.399197	4.505266	0.792943
0.8	2.469404	1.487222	0.147005	1.383592	4.512778	0.788664
0.9	2.470309	1.480124	0.160855	1.368836	4.519876	0.784615

p	EA	RRATE	EUR	EUS	COST
0.1	0.884151	0.884151	3.387431	0.080117	77.6749
0.2	0.884640	0.884640	3.389307	0.077324	77.5813
0.3	0.885100	0.885100	3.391067	0.074719	77.4939
0.4	0.885531	0.885531	3.392720	0.072284	77.4120
0.5	0.885938	0.885938	3.394278	0.070030	77.3352
0.6	0.886321	0.886321	3.395747	0.067862	77.2629
0.7	0.886684	0.886684	3.397136	0.065848	77.1949
0.8	0.887027	0.887027	3.398450	0.063950	77.1037
0.9	0.887352	0.887352	3.399695	0.062158	77.0700

Table 3.5: Numerical Results for S = 4, s = 1

p	**ED1**	**ED2**	**EG**	**EL1**	**EL2**	**EK2**
0.1	2.571096	1.635669	0.021343	1.407561	4.364332	0.872010
0.2	2.571408	1.625571	0.041319	1.387274	4.374429	0.866307
0.3	2.571703	1.616109	0.060056	1.368241	4.383891	0.860949
0.4	2.571984	1.607225	0.077667	1.350350	4.392776	0.855907
0.5	2.572251	1.598865	0.094251	1.333499	4.401135	0.851153
0.6	2.572504	1.590985	0.109898	1.317598	4.409014	0.846663
0.7	2.572746	1.583545	0.124684	1.302570	4.416455	0.842414
0.8	2.572976	1.576508	0.138680	1.288344	4.423493	0.838389
0.9	2.573195	1.569841	0.151948	1.274857	4.430159	0.834568

p	**EA**	**RRATE**	**EUR**	**EUS**	**COST**
0.1	1.557317	1.082949	3.764555	0.170350	87.6552
0.2	1.557506	1.083081	3.765011	0.166502	87.5522
0.3	1.557685	1.083205	3.765444	0.162895	87.4556
0.4	1.557855	1.083323	3.765855	0.159505	87.3647
0.5	1.558017	1.083436	3.766245	0.156314	87.2792
0.6	1.558170	1.083542	3.766616	0.155504	87.1984
0.7	1.558316	1.083644	3.766970	0.150461	87.1221
0.8	1.558456	1.083741	3.767308	0.147770	87.0498
0.9	1.558589	1.083834	3.767629	0.145219	86.9813

Table 3.6: Numerical Results for S = 4, s = 2

p	ED1	ED2	EG	EL1	EL2	EK2
0.1	2.639271	1.675820	0.020484	1.340245	4.324180	0.902930
0.2	2.639348	1.666208	0.039680	1.320971	4.333792	0.897530
0.3	2.639421	1.657192	0.057710	1.302868	4.342807	0.892449
0.4	2.639492	1.648719	0.074677	1.285831	4.351282	0.887662
0.5	2.639560	1.640739	0.090675	1.269766	4.359261	0.883141
0.6	2.639623	1.633211	0.105786	1.254591	4.366789	0.878865
0.7	2.639685	1.626096	0.120081	1.240234	4.373904	0.874815
0.8	2.639744	1.619362	0.133628	1.226628	4.380638	0.870973
0.9	2.639800	1.612977	0.146483	1.213717	4.387023	0.867323

p	EA	RRATE	EUR	EUS	COST
0.1	2.405319	1.401584	4.151391	0.363537	100.2953
0.2	2.405389	1.401625	4.151512	0.359440	100.1917
0.3	2.405456	1.401664	4.151628	0.355589	100.0942
0.4	2.405520	1.401702	4.151738	0.351962	100.0025
0.5	2.405581	1.401737	4.151844	0.348539	99.9159
0.6	2.405640	1.401771	4.151945	0.345305	99.8340
0.7	2.405696	1.401804	4.152041	0.342243	99.7565
0.8	2.405749	1.401835	4.152133	0.339339	99.6830
0.9	2.405801	1.401865	4.152223	0.336583	99.6133

Table 3.7: Numerical Results for S = 4, s = 3

3.6 CONCLUSION

This chapter described a single perishing product inventory model where items deteriorate in two phases and then perish. Independent demand takes place at constant rates for items in both phases. Demand for an item in Phase I not satisfied may be satisfied by an item in Phase II based on a probability measure. Demand for items in Phase II during stock-out is lost. The re-ordering policy is an adjustable (S, s) policy with the lead-time following an arbitrary distribution. Identifying the underlying stochastic process as a renewal process, the probability distribution of the inventory level at any arbitrary point in time is obtained. The expressions for the mean stationary rates of lost demand, substituted demand, perished units and scrapped units are also derived. A numerical example is considered to highlight the obtained results.