BUILDING BLOCKS FOR SUPPLY CHAIN MANAGEMENT – A STUDY OF INVENTORY MODELLING

by

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ABSTRACT

Title: Building Blocks for Supply Chain Management – A Study of Inventory Modelling

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This thesis presents a study of stochastic models of continuous review of inventory systems of perishable and non-perishable products, as well as inventory systems operating in random environment. It contains five chapters. The first chapter is introductory in nature, containing the motivation for the study and the techniques required for the analysis of respective models described in the remaining chapters.

Chapter 2 provides a model of perishable product inventory system operating in a random environment. For the sake of simplicity, the stochastic environment is considered to alternate randomly over time between two states 0 and 1 according to an alternating renewal process. When the environment is in state k, the items in inventory have a perishable rate μ_k, the demand rate is λ_k and the replenishment cost is CR_k. The performance of various measures of the system evolution are obtained, assuming instantaneous replenishment at the epoch of the first demand after the stock-out and associating a Markov renewal process with the inventory level.

In Chapter 3, a continuous review single product perishable inventory model is considered. Items deteriorate in two phases and then perish. Independent demands occur at constant rates for items in phase I and in phase II. Demand that occurs for an item in phase I during its stock-out period is satisfied by an item in phase II with some

probability. However a demand for an item in phase II occurring during its stock-out period is lost. The reordering policy is an adjustable (S,s) policy with the lead-time following an arbitrary distribution. Identifying the stochastic process as a renewal process, the probability distribution of the inventory level at any arbitrary instant of time is obtained. The expressions for the mean stationary rates of demands lost, demands substituted, perished units and scrapped units are also derived. A numerical example is considered to highlight the results obtained.

Chapter 4 is a study of a two-commodity inventory system under continuous review. The maximum storage capacity for the *i*-th item is S_i (*i*=1, 2). The demand points for each commodity are assumed to form an independent Poisson process, with unit demand for one item and bulk demand for the other. The order level is fixed as s_i for the *i*-th commodity (*i*=1, 2) and the ordering policy is to place an order for Q_i ($= S_i - s_i$, i = 1,2) items for the *i*-the commodity when both the inventory levels are less than or equal to their respective reorder levels. The lead-time is assumed to be exponential. The joint probability distribution for both commodities is obtained in both transient and steady state cases. Various measures of systems performance and the total expected cost rate in the steady state are derived. The results are illustrated with numerical examples.

Chapter 5 provides an analysis of a continuous review of two-product system with two types of demands and with individual (S,s) ordering policy. The lead-time distribution of product 1 is arbitrary and that of product 2 exponential. Two types of demands occur at constant rates either for both products or for product 2 alone. Expressions for the stationary distribution of the inventory level are obtained by identifying the underlying stochastic processes as a semi-regenerative process. The mean stationary rates of the lost demands, the demands that are satisfied and the number of reorders are obtained and these measures are used to provide an expression for the cost rate.

The main objective of this thesis is to improve the state of art of continuous review inventory systems. The salient features of the thesis are summarized below:

(a) Consideration of
  (i) The impact of the stochastic environment on inventory systems;
  (ii) The interactions existing among the products in multi-product systems;
  (iii) Individual and joint-ordering policies;

(b) Discussion of inventory systems with perishable products;
(c) Effective use of the regeneration point technique to derive expressions for various system measures;
(d) Illustration of the various results by extensive numerical work;
(e) Relevant optimization problems

Key Words:

Inventory optimization
Inventory modelling
Inventory systems in a stochastic environment
Perishable product inventory systems
Continuous review inventory systems
Two-commodity inventory system
Individual and joint-ordering policies
(S, s) order policy
Regeneration point technique
Renewal process

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