CHAPTER 6

A STOCHASTIC MODEL OF A RELIABILITY SYSTEM
WITH A HUMAN OPERATOR
6.1 INTRODUCTION

With the advancement of methods and developments in artificial intelligence, computer systems and electronic systems, we find a high degree of automation all around. Nevertheless, it cannot be overlooked that human beings are inseparable parts of systems. Radars, motor vehicles, aircrafts and ships are examples of systems which are continuously monitored by human operators. Errors that are caused by human operators are referred to as ‘human-errors’ (Dhillon, 1980, 1984; Yadavalli & Bekker, 2005).

Dhillon has analysed several models incorporating the concepts of human reliability (1981, 1984). Yadavalli & Bekker (2005) studied a stochastic model of a two unit system with human error and common-cause failures. Their main focus was on the estimation of the steady-state availability (both classical and Bayesian) with the assumption of human error and common-cause failures. One common feature of all the models is that they are Markovian in nature. Kumar et al. (1986) analysed systems operating in fluctuating weather conditions and subject to critical human error, the models are being Markovian. Dhillon and Rayapati (1985) studied five models for a system which needs a human operator. In their models, a human operator is assumed to be working in one of the three states: normal, moderate stress and extreme stress. In all three states, the system is assumed to be subject to two failure modes, i.e. from each state, the system may fail because of self-corrected-human error and non-self-corrected human error. The system can recover from a self-corrected-human failure state, whereas it remains in a failed state when this occurs because of non-self-corrected human error. All the underlying failure distributions are assumed to be exponential. The organization of this chapter is as follows: Section 6.2 presents the
system description and relevant notation; in Section 6.3, we represent transition probability functions and sojourn times, which are used in the subsequent analyses; Section 6.4 is a study of reliability analysis and the mean time to system failure; the availability analysis is presented in Section 6.5; the study of expected number of visits to a state and the profit analysis are presented in Section 6.6.

6.2 SYSTEM DESCRIPTION AND NOTATION

1. The system is operated by a human operator. The system may fail because of its built-in nature or because of the human operator.

2. The human operator working on the system can be in one of three states: normal, moderate stress or extreme stress. The human operator is more prone to commit errors while in extreme stress state than in the other two states.

3. The system is subject to two failure modes irrespective of the state of the human operator: (a) failure because of self-corrected human-error, (b) failure because of non-self-corrected human error.

4. The rates of change of human operator condition from normal to moderate stress, to extreme stress and vice versa are all exponential. Further, human errors also occur at an exponential rate.

5. Repair time distributions for the system failed from the three human operator conditions are arbitrary and different from one another.

6. Failures are statistically independent.

7. The repaired system is as good as new.
NOTATION

$\lambda_{ij}$  Constant rate of occurrence of human error, where $i$ denotes the state of human operator and $j$ denotes the system states

$\alpha_{lm}$ Constant rate of change of the state of human operator from $l^{th}$ state to $m^{th}$ state; $l = 1, 2, 3$ and $m = 1, 2, 3 \ (l \neq m)$

$G_{ij}(t)$ Repair time distribution for the system, where $i$ denotes the state of human operator from which the system failed and $j$ denotes the system state; $i = 1, 2, 3$ and $j = 1, 2$.

$P_{ij}$ One-step transition probability from state $S_i$ to $S_j$

$P$ Transition probability matrix

$\mu_i$ Mean unconditional sojourn time of the system in $S_i$

$\mu^{(1)}$ The diag ($\mu_0$, $\mu_3$, $\mu_6$)

$\xi$ $(1, 1, 1)^\prime$, column vector

$N_{ij}$ Total time spent in a transit state $S_j$ before the system failure, given that the system starts in $S_i$

$N^{(1)} = \begin{bmatrix} N_{ij} \end{bmatrix}$, matrix

$t'_i$ Total time spent in up-states given that the system starts to $S_i$

$v_i$ $E(t'_i)$

$V^{(1)} = (v_0, v_3, v_6)^\prime$, column vector

$b_{ij}$ $P[\text{System is absorbed in } S_j \mid \text{system started in } S_i]$

$B = \begin{bmatrix} b_{ij} \end{bmatrix}$
E \{0, 1, 2, ... , 8\}
E_1 \{0, 3, 6\}, up-states
E_2 \{1, 2, 4, 5, 7, 8\}, down-states
\Pi_i \text{ Limiting probability that the Markov-Chain in } S_i, (i \in E)
d_i \text{ Determinant of the minor of } D
\psi_i \text{ Limiting probability of the system being in } S_i, (i \in E)
Y_i \text{ Earning rate of the system per unit time in } S_j
g \text{ Expected profit per unit time in steady state}
r_{ij} \text{ Fixed transition reward for a transition from } S_i \text{ to } S_j

### 6.3 TRANSITION PROBABILITIES AND SOJOURN TIMES

At any instant, the system can be in one of the following states: (See figure 6.1)

(i) \text{ Up-states:}
S_0 (1, 0), \ S_3 (2, 0), \ S_6 (3, 0),

(ii) \text{ Down-states:}
S_1 (1, 1), \ S_2 (1, 2), \ S_4 (2, 1),
S_5 (2, 2), S_7 (3, 1), S_8 (3, 2).

The first symbol in parenthesis denotes the human operator state and the second symbol denotes the state of the system.
System failure due to non-self-corrected-human error in normal state (1)  
$S_2$

Human operator performing his task correctly in normal state (1)  
$S_0$

System failure due to self-corrected-human error in normal state (1)  
$S_1$

System failure due to non-self-corrected-human error in moderate stress (2)  
$S_5$

Human operator performing his task correctly at moderate stress (2)  
$S_3$

System failure due to self-corrected-human error at moderate stress (1)  
$S_4$

System failure due to non-self-corrected-human error in extreme stress (2)  
$S_8$

Human operator performing his task correctly at extreme stress (3)  
$S_6$

System failure due to self-corrected-human error at extreme stress (1)  
$S_7$

$\alpha_{21}$  
$\alpha_{22}$  
$\alpha_{31}$  
$\alpha_{32}$  
$\alpha_{33}$

Figure 6.1: Transition Diagram
As indicated earlier, the states of human operator are (1) normal (2) moderate stress and (3) extreme stress. Similarly, the system states are 0: operative, 1: failed because of self-corrected-human error and 2: failed because of non-self-corrected-human error.

The system behavior can be described by a stochastic process \( \{Z(t), t \geq 0\} \) with state space \( E = \{0,1,...,8\} \), where \( Z(t) \) denotes the state of the system at time \( t \). It may be noted that the process \( Z(t) \), is a semi-Markovian and as such the well-known properties of Semi-Markovian Process (SMP) (see Cinlar (1975)) are applied to study the system behavior in detail.

The transition probabilities are given by:

\[
P_{01} = \int_0^\infty \lambda_1 e^{-(\lambda_1 + \lambda_{12} + \alpha_{12} + \alpha_{13})t} dt = \frac{\lambda_1}{\lambda_1 + \lambda_{12} + \alpha_{12} + \alpha_{13}}
\]

(6.1)

\[
P_{02} = \int_0^\infty \lambda_2 e^{-(\lambda_{12} + \lambda_{11} + \alpha_{12} + \alpha_{13})t} dt = \frac{\lambda_2}{\lambda_{11} + \lambda_{12} + \alpha_{12} + \alpha_{13}}
\]

(6.2)

\[
P_{03} = \int_0^\infty \alpha_{12} e^{-(\alpha_{12} + \lambda_{11} + \lambda_{12} + \alpha_{13})t} dt = \frac{\alpha_{12}}{\lambda_{11} + \lambda_{12} + \alpha_{12} + \alpha_{13}}
\]

(6.3)

\[
P_{06} = \int_0^\infty \alpha_{13} e^{-(\alpha_{13} + \alpha_{12} + \lambda_{11} + \lambda_{12})t} dt
\]
\[ P_{10} = P_{20} = 1 \]  
\[ P_{30} = \int_0^\infty e^{-(\alpha_{21} + \lambda_{21} + \lambda_{22} + \lambda_{23})t} dt \]
\[ = \frac{\alpha_{21}}{\lambda_{21} + \lambda_{22} + \alpha_{21} + \alpha_{23}} \]  
\[ P_{34} = \int_0^\infty e^{-(\lambda_{21} + \lambda_{22} + \alpha_{21} + \alpha_{23})t} dt \]
\[ = \frac{\lambda_{21}}{\lambda_{21} + \lambda_{22} + \alpha_{21} + \alpha_{23}} \]  
\[ P_{35} = \int_0^\infty e^{-(\lambda_{22} + \alpha_{21} + \alpha_{23})t} dt \]
\[ = \frac{\lambda_{22}}{\lambda_{21} + \lambda_{22} + \alpha_{21} + \alpha_{23}} \]  
\[ P_{36} = \int_0^\infty e^{-(\alpha_{23} + \lambda_{21} + \lambda_{21} + \alpha_{21})t} dt \]
\[ = \frac{\alpha_{23}}{\lambda_{21} + \lambda_{22} + \alpha_{21} + \alpha_{23}} \]  
\[ P_{43} = P_{53} = 1 \]  
\[ P_{67} = \int_0^\infty e^{-(\lambda_{31} + \lambda_{32} + \alpha_{31})t} dt \]
\[
P_{68} = \int_{0}^{\infty} \lambda_{32} e^{-(\lambda_{32} + \lambda_{31} + \alpha_{31} + \alpha_{32})t} dt
= \frac{\lambda_{32}}{\lambda_{31} + \lambda_{32} + \alpha_{31} + \alpha_{32}}
\] (6.12)

\[
P_{60} = \int_{0}^{\infty} \alpha_{31} e^{-(\alpha_{31} + \lambda_{31} + \lambda_{32} + \alpha_{32})t} dt
= \frac{\alpha_{31}}{\alpha_{31} + \lambda_{31} + \lambda_{32} + \alpha_{32}}
\] (6.13)

\[
P_{63} = \int_{0}^{\infty} \alpha_{32} e^{-(\alpha_{32} + \alpha_{31} + \lambda_{31} + \lambda_{32})t} dt
= \frac{\alpha_{32}}{\lambda_{31} + \lambda_{32} + \alpha_{31} + \alpha_{32}}
\] (6.14)

and \( P_{76} = P_{86} = 1. \) (6.15)

The unconditional mean sojourn time of the system in state \( S_i \) are given below:

\[
\mu_0 = \int_{0}^{\infty} e^{-(\lambda_{11} + \lambda_{12} + \alpha_{12} + \alpha_{13})t} dt
= \frac{1}{\lambda_{11} + \lambda_{12} + \alpha_{12} + \alpha_{13}}
\] (6.16)

\[
\mu_1 = \int_{0}^{\infty} \lambda_{11}(t) dt = m_1 \text{ (say)}
\] (6.17)
\[ \mu_2 = \int_0^{\infty} t \lambda_2(t) dt = m_2 \text{ (say)} \]  
\[ \mu_3 = \int_0^{\infty} e^{(\lambda_2 + \lambda_3 + \alpha_2 + \alpha_3) t} dt \]  
\[ = \frac{1}{\lambda_2 + \lambda_3 + \alpha_2 + \alpha_3} \]  
\[ \mu_4 = \int_0^{\infty} t \lambda_2(t) dt = m_4 \text{ (say)} \]  
\[ \mu_5 = \int_0^{\infty} t \lambda_2(t) dt = m_5 \text{ (say)} \]  
\[ \mu_6 = \int_0^{\infty} e^{(\lambda_3 + \lambda_2 + \alpha_3 + \alpha_2) t} dt \]  
\[ = \frac{1}{\lambda_3 + \lambda_2 + \alpha_3 + \alpha_2} \]  
\[ \mu_7 = \int_0^{\infty} t \lambda_2(t) dt = m_7 \text{ (say)} \]  
\[ \mu_8 = \int_0^{\infty} t \lambda_2(t) dt = m_8 \text{ (say)} \]  

6.4 RELIABILITY ANALYSIS

We need to find the MTSF using reliability analysis. To obtain MTSF, we convert the down-states of the system into absorbing states, so that the transition probability matrix
(TPM) is steadily available in a canonical form and to obtain MTSF by the SMP approach (Agarwal et al. (1986)).

Then we know that (See Agarwal et al. (1986))

\[ N^{(1)} = (I - Q)^{-1} M^{(1)} \]

\[ V^{(1)} = N^{(1)} \]

The elements of \( V^{(1)} \) produces the MTSF, given that the system starts in a transient state.
Specifically,

\[
\mathbf{N}^{(1)} = \frac{1}{D} \begin{bmatrix}
-P_{36}P_{63} & P_{03} + P_{06}P_{63} & P_{06} + P_{03}P_{36} \\
0 + P_{60}P_{36} & (1 - P_{06}P_{60}) & (P_{36} + P_{06}P_{30}) \\
0 + P_{30}P_{63} & (P_{63} + P_{03}P_{60}) & (1 - P_{03}P_{30})
\end{bmatrix} \mu_0 = V^{(1)}
\]

(6.25)

\[
V^{(1)} = \frac{1}{D} \begin{bmatrix}
-P_{36}P_{63} & P_{03} + P_{06}P_{63} & P_{06} + P_{03}P_{36} \\
0 + P_{60}P_{36} & (1 - P_{06}P_{60}) & (P_{36} + P_{06}P_{30}) \\
0 + P_{30}P_{63} & (P_{63} + P_{03}P_{60}) & (1 - P_{03}P_{30})
\end{bmatrix} \mu_0 + (P_{03} + P_{06}P_{63}) \mu_3 + (P_{06} + P_{03}P_{36}) \mu_6
\]

(6.26)

where

\[
D = \left[(1 - P_{36}P_{63}) - P_{36}(P_{03} + P_{06}P_{63}) + P_{60}(P_{06} + P_{03}P_{36})\right].
\]

After substituting the required values into (6.25) and (6.26), we obtain

\[
V_0 = \frac{B_0}{B}
\]

(6.27)

\[
V_3 = \frac{B_3}{B}
\]

(6.28)

\[
V_6 = \frac{B_6}{B}
\]

(6.29)

where

\[
B_0 = M_1 + (\alpha_{21} + \alpha_{12} + \alpha_{23})M_1M_2 + (\alpha_{32} + \alpha_{31} + \alpha_{13})M_1M_3 + PM_1M_2M_3
\]

\[
B_3 = M_2 + (\alpha_{31} + \alpha_{32} + \alpha_{23})M_1M_2 + (\alpha_{21} + \alpha_{12} + \alpha_{13})M_2M_3 + PM_1M_2M_3
\]

\[
B_6 = M_3 + (\alpha_{31} + \alpha_{12} + \alpha_{13})M_1M_3 + (\alpha_{32} + \alpha_{23} + \alpha_{21})M_2M_3 + PM_1M_2M_3
\]

\[
B = 1 + (\alpha_{12} + \alpha_{13})M_1 + (\alpha_{23} + \alpha_{31})M_2
\]

\[
+ (\alpha_{31} + \alpha_{32})M_3 + K_1M_2M_3 + K_2M_1M_3 + K_3M_1M_2
\]
\[ K_1 = \alpha_{31}(\alpha_{23} + \alpha_{21}) + \alpha_{21}\alpha_{32} \]
\[ K_2 = \alpha_{12}(\alpha_{32} + \alpha_{41}) + \alpha_{32}\alpha_{13} \]
\[ K_3 = \alpha_{23}(\alpha_{42} + \alpha_{13}) + \alpha_{21}\alpha_{32} \]  \hspace{1cm} (6.30)

\[ P = (\alpha_{12}\alpha_{23} + \alpha_{22}\alpha_{31} + \alpha_{31}\alpha_{12}) + (\alpha_{23}\alpha_{31} + \alpha_{32}\alpha_{12} + \alpha_{12}\alpha_{31}) \\
+ (\alpha_{12}\alpha_{32} + \alpha_{31}\alpha_{21} + \alpha_{21}\alpha_{31}) \]  \hspace{1cm} (6.31)

\[ M_1 = \frac{1}{\lambda_{11} + \lambda_{12}} ; \quad M_2 = \frac{1}{\lambda_{21} + \lambda_{22}} ; \quad M_3 = \frac{1}{\lambda_{31} + \lambda_{32}} . \]

In fact, \( M_1, M_2 \) and \( M_3 \) are MTSF’s of respective decomposed subsystems, corresponding to normal, moderate and extreme stress states. It can be noted that (6.27) is in agreement with Dhillon and Rayapati (1985).

According to Agarwal et al. (1986), the absorption probabilities, i.e. the probabilities that the process starting from \( S_i \) (\( i = 0, 3, 6 \)) enters the absorbing states \( S_j \) (\( j = 1, 2, 4, 5, 7, 8 \)) are given by the matrix \( B = [b_{ij}] \), namely

\[ B = [I - Q]^{-1} R \]

\[
\begin{bmatrix}
0 & P_{01}C_{01} & P_{02}C_{01} & P_{04}C_{02} & P_{05}C_{02} & P_{06}C_{03} & P_{07}C_{03} & P_{08}C_{03} \\
1 & C_{01} & P_{02}C_{01} & P_{04}C_{02} & P_{05}C_{02} & P_{06}C_{03} & P_{07}C_{03} & P_{08}C_{03} \\
2 & C_{01} & P_{02}C_{01} & P_{04}C_{02} & P_{05}C_{02} & P_{06}C_{03} & P_{07}C_{03} & P_{08}C_{03} \\
3 & C_{01} & P_{02}C_{01} & P_{04}C_{02} & P_{05}C_{02} & P_{06}C_{03} & P_{07}C_{03} & P_{08}C_{03} \\
4 & C_{01} & P_{02}C_{01} & P_{04}C_{02} & P_{05}C_{02} & P_{06}C_{03} & P_{07}C_{03} & P_{08}C_{03} \\
5 & C_{01} & P_{02}C_{01} & P_{04}C_{02} & P_{05}C_{02} & P_{06}C_{03} & P_{07}C_{03} & P_{08}C_{03} \\
6 & C_{01} & P_{02}C_{01} & P_{04}C_{02} & P_{05}C_{02} & P_{06}C_{03} & P_{07}C_{03} & P_{08}C_{03} \\
7 & C_{01} & P_{02}C_{01} & P_{04}C_{02} & P_{05}C_{02} & P_{06}C_{03} & P_{07}C_{03} & P_{08}C_{03} \\
8 & C_{01} & P_{02}C_{01} & P_{04}C_{02} & P_{05}C_{02} & P_{06}C_{03} & P_{07}C_{03} & P_{08}C_{03} \\
\end{bmatrix}
\]

where

\[ C_{01} = (1 - P_{36}P_{63}) ; \quad C_{02} = (P_{03} + P_{06}P_{63}) ; \]
\[ C_{03} = (P_{06} + P_{03}P_{63}) ; \quad C_{04} = (P_{03} + P_{06}P_{63}) ; \]
\[ C_{32} = (1 - P_{06}P_{60}) ; \quad C_{33} = (P_{36} + P_{06}P_{60}) ; \]
\[ C_{61} = (P_{60} + P_{30}P_{63}) ; \quad C_{62} = (P_{63} + P_{03}P_{60}) ; \]
\[ C_{63} = (1 - P_{03}P_{60}) . \]  \hspace{1cm} (6.32)
6.5 AVAILABILITY ANALYSIS

To obtain the steady-state availability $A_{\infty}$, the limiting probabilities $\alpha_i$’s are required

$$
\psi_i = \frac{\mu_i \pi_i}{\sum_{j \in E} \mu_j \pi_j}
$$

$$
= \frac{\mu_i d_i}{\sum_{j \in E} \mu_j d_j} ; \ i \in E_1 \quad (6.33)
$$

$$
A_{\infty} = \psi_0 + \psi_3 + \psi_6 . \quad (6.34)
$$

Since $d_i$’s are the determinants of the minors of $D$,

$$
d_0 = (1 - P_{34} - P_{35})(1 - P_{67} - P_{68}) - P_{63} P_{36} \quad (6.35)
$$

$$
d_1 = (1 - P_{02})[(1 - P_{34} - P_{35})(1 - P_{67} - P_{68}) - P_{63} P_{36}]
- P_{30} [P_{30} (1 - P_{67} - P_{68}) + P_{63} P_{36}]
- P_{60} [P_{30} P_{36} + P_{60} (1 - P_{34} - P_{35})] \quad (6.36)
$$

$$
d_2 = (1 - P_{01})[(1 - P_{34} - P_{35})(1 - P_{67} - P_{68}) - P_{63} P_{36}]
- P_{30} [P_{30} (1 - P_{67} - P_{68}) + P_{63} P_{36}]
- P_{60} [P_{30} P_{36} + P_{60} (1 - P_{34} - P_{35})] \quad (6.37)
$$

$$
d_3 = (1 - P_{01} - P_{02})(1 - P_{67} - P_{68}) - P_{60} P_{06} \quad (6.38)
$$

$$
d_4 = (1 - P_{01} - P_{02})[(1 - P_{35})(1 - P_{67} - P_{68}) - P_{63} P_{36}]
- P_{30} [P_{30} (1 - P_{67} - P_{68}) + P_{63} P_{06}]
- P_{60} P_{06} (1 - P_{34} - P_{35}) - P_{60} P_{30} P_{36} \quad (6.39)
$$

$$
d_5 = (1 - P_{01} - P_{02})[(1 - P_{34})(1 - P_{67} - P_{68}) - P_{63} P_{36}]
- P_{30} [P_{30} (1 - P_{67} - P_{68}) + P_{63} P_{06}]$$
\[ -P_{60} P_{60} (1 - P_{34}) - P_{60} P_{63} P_{36} \]  

\[ d_6 = (1 - P_{01} - P_{02})(1 - P_{34} - P_{35}) - P_{30} P_{04} \]  

\[ d_7 = (1 - P_{01} - P_{02})[(1 - P_{34} - P_{35})(1 - P_{36} - P_{37} - P_{36})] \] 

\[ -P_{30} [P_{03} (1 - P_{36}) + P_{06} P_{63}] \] 

\[ -P_{60} [P_{03} P_{36} + P_{06} (1 - P_{34} - P_{36})] \]  

\[ d_8 = (1 - P_{01} - P_{02})[(1 - P_{34} - P_{35})(1 - P_{36} - P_{37} - P_{36})] \] 

\[ -P_{30} [P_{03} (1 - P_{36}) + P_{06} P_{63}] \] 

\[ -P_{60} [P_{03} P_{36} + P_{06} (1 - P_{34} - P_{36})] \].  

Using the \( d_i \)'s, the steady-state availability \( A_\infty \), can be obtained as

\[ A_\infty = \frac{K_1 + K_2 + K_3}{K_1 + K_2 + K_3} \]  

\[ \frac{A_1}{A_2 + A_3} \]  

where

\[ A_1 = \frac{1}{1 + a_{11} + a_{12}} \] 

\[ A_2 = \frac{1}{1 + a_{21} + a_{22}} \] 

\[ A_3 = \frac{1}{1 + a_{31} + a_{32}} \] 

\[ a_{11} = \lambda_{11} \cdot M_1 \ ; \quad a_{12} = \lambda_{12} \cdot M_2 \ ; \] 

\[ a_{21} = \lambda_{21} \cdot M_4 \ ; \quad a_{22} = \lambda_{22} \cdot M_5 \ ; \] 

\[ a_{31} = \lambda_{31} \cdot M_7 \ ; \quad a_{32} = \lambda_{32} \cdot M_8 \].

In fact, \( A_1, A_2 \) and \( A_3 \) are the steady-state availabilities of the decomposed sub-systems corresponding to normal, moderate and extreme stress states respectively. It may be noted
that $A_\infty$ is the harmonic mean of $A_1$, $A_2$ and $A_3$ with the weights $K_1$, $K_2$ and $K_3$ which are functions of $\alpha_i$’s. Also

(i) \[ \min_{1 \leq i \leq 3} A_i \leq A_\infty \leq \max_{1 \leq i \leq 3} A_i \]

(ii) The above range of $A_\infty$ is independent of $\alpha_i$’s.

### 6.6 EXPECTED NUMBER OF VISITS TO A STATE AND EXPECTED PROFIT

$$\delta_i = \frac{\mu_i}{\sum_{i \in E} \mu_i d_i}$$

Hence, from equations 6.16 – 6.24 and 6.35 – 6.43,

$$\delta_0 = \frac{K}{(\lambda_{11} + \lambda_{12} + \alpha_{12} + \alpha_{13})T}$$

$$\delta_1 = \frac{M_1 K}{T}$$

$$\delta_2 = \frac{M_2 K}{T}$$

$$\delta_3 = \frac{K}{(\lambda_{21} + \lambda_{22} + \alpha_{21} + \alpha_{23})T}$$

$$\delta_4 = \frac{m_2 K}{T}$$

$$\delta_5 = \frac{m_1 K}{T}$$

$$\delta_6 = \frac{K}{(\lambda_{31} + \lambda_{32} + \alpha_{31} + \alpha_{32})T}$$

$$\delta_7 = \frac{M_1 K}{T}$$
and \[ \delta_8 = \frac{M_8K}{T} \]

where

\[ K = (\lambda_{11} + \lambda_{12} + \alpha_{12} + \alpha_{13})(\lambda_{21} + \lambda_{22} + \alpha_{21} + \alpha_{23})(\lambda_{31} + \lambda_{32} + \alpha_{31} + \alpha_{32}) \]

\[ T = \frac{K_1}{A_1} + \frac{K_2}{A_2} + \frac{K_3}{A_3}. \]

We follow the same approach as in Agarwal (1988), to find the expected profit

\[ g = \frac{\sum_i \pi_i \mu_i V_i}{\sum_i \pi_i V_i} \]

\[ \mu_i V_i = \sum_j P_{ij} r_{ij} + y_i \mu_i. \]

Hence \( g \) can be calculated as

\[ g = \frac{Z}{T} \]

where

\[ g = \frac{K_1}{(\lambda_{11} + \lambda_{12} + \alpha_{12} + \alpha_{13})} \left[ \lambda_{11} r_{01} + \lambda_{12} r_{02} + \alpha_{12} r_{03} + \alpha_{13} r_{06} + y_0 \right] \]

\[ + \frac{\lambda_{11} K_1}{M_1} \left[ M_1 r_{10} + y_1 \right] + \frac{\lambda_{12} K_1}{M_2} \left[ M_2 r_{20} + y_2 \right] \]

\[ + \frac{K_2}{(\lambda_{21} + \lambda_{22} + \alpha_{23} + \alpha_{21})} \left[ \lambda_{21} r_{34} + \lambda_{22} r_{35} + \alpha_{23} r_{36} + \alpha_{21} r_{30} + y_3 \right] \]

\[ + \frac{\lambda_{21} K_2}{M_4} \left[ M_4 r_{43} + y_4 \right] + \frac{\lambda_{22} K_2}{M_5} \left[ M_5 r_{53} + y_5 \right] \]

\[ + \frac{K_3}{(\lambda_{31} + \lambda_{32} + \alpha_{32} + \alpha_{31})} \left[ \lambda_{31} r_{67} + \lambda_{32} r_{68} + \alpha_{32} r_{63} + \alpha_{32} r_{60} + y_6 \right] \]
6.7 NUMERICAL ILLUSTRATION

As a numerical illustration, the behaviour of $q_0$, the mean time to system failure, has been studied with respect to changes in $M_1$, $M_2$ and $M_3$, the mean times to failure of decomposed sub-systems. It may be noted that MTSF increases as $M_1$ increases but there is always an upper bound on the MTSF when other factors $M_2$ and $M_3$ are kept constant. This is convincing, as, after a certain change, any improvement in $M_1$ is not likely to improve the MTSF value. In fact, for fixed values of $M_2$ and $M_3$, $q_0$ takes the form

$$q_0 = \frac{M_1}{a + bM_2} \leq \frac{1}{b}$$

where $a$ and $b$ are functions of $M_2$ and $M_3$ and thus may be treated as constants as long as $M_2$ and $M_3$ are fixed. The above inequality gives the least upper bound for $q_0$.

For example, for $M_2 = 100$, least upper bounds for $q_0$ are given as follows for varying $M_3$:

<table>
<thead>
<tr>
<th>$M_3$</th>
<th>25</th>
<th>40</th>
<th>60</th>
<th>100</th>
<th>...</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>97.03</td>
<td>126.48</td>
<td>155.65</td>
<td>194.09</td>
<td>...</td>
<td>324.80</td>
</tr>
</tbody>
</table>
As the Mean time to failure of decomposed subsystem 1 ($M_1$) increases Mean Time to System Failure (MTSF) is an increasing function of $M_1$ (for different values of $M_3$ and fixed values of $\alpha_{12}, \alpha_{23}, \alpha_{31}, \alpha_{21}, \alpha_{32}$ and $\alpha_{13}$).

**Figure 6.1**

\[\alpha_{12} = 0.04, \alpha_{23} = 0.1, \alpha_{31} = 0.05, \alpha_{21} = 0.06, \alpha_{32} = 0.15, \alpha_{13} = 0.03\]
As the constant rate of change from first to second state, $\alpha_{12}$, increases Mean Time to System Failure (MTSF) is a decreasing function of $\alpha_{12}$ (for different values of $\alpha_{31}$ and fixed values of $M_1$, $M_2$, $M_3$, $\alpha_{23}$, $\alpha_{13}$, $\alpha_{21}$, and $\alpha_{32}$).

Figure 6.2
6.8 CONCLUSION

A repairable system with a human operator is considered. The operator could be in one of the three states – normal, moderate stress and extreme stress. The system can fail due to self-corrected and non-self-corrected errors. With the help of a semi-Markovian process and a regeneration point technique various characteristics, like availability and MTSF are obtained (results in Figures 6.1 – 6.2). A cost-benefit analysis is also obtained for such a system.