PROBABILISTIC ANALYSIS OF REPAIRABLE REDUNDANT SYSTEMS

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PROBALISTIC ANALYSIS OF REPAIRABLE REDUNDANT SYSTEMS

by

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SUMMARY

Two well-known methods of improving the reliability of a system are

(i) provision of redundant units, and

(ii) repair maintenance.

In a redundant system more units are made available for performing the system function when fewer are actually required. There are two major types of redundancy – parallel and standby.

Some of the typical assumptions made in the study of standby redundant systems are:

(a) the repair facility can take up a failed unit for repair at any time, if no other unit is undergoing repair

(b) the state of the standby unit is either cold or warm throughout

(c) the random variables like failure times and repair times are independent

(d) the failures can be in one mode

(e) estimation of operating characteristics.

In this testis, an attempt is made to study a few complex and novel models of standby redundant repairable systems by relaxing one or more of these assumptions.

A number of interesting and important characteristics useful for reliability practioners and system designers are obtained for several models. Further, emphasis is also laid on the construction of comprehensive cost functions and their numerical optimization. We give below the conclusions and the possible extensions for future work. These conclusions are drawn from a limited but reasonably exhaustive numerical work carried out.

The thesis contains six chapters. Chapter 1 is introductory in nature and contains a brief description of various types of systems and the mathematical techniques used in the analysis of redundant systems.

In Chapter 2, a stochastic model of an urea decomposition system in the fertilizer industry is studied. A set of difference-differential equations for the state probabilities are formulated under suitable conditions. The state probabilities are obtained explicitly and the steady state availability of the system is obtained analytically as well as illustrated numerically. Confidence limits for the steady state availability are also obtained.

A two dissimilar unit system with different modes of failure is studied in Chapter 3. The system is a priority system in which one of the units is a priority unit and the one other unit is an ordinary unit. The concept of ‘dead time’ is introduced with the assumption that the ‘dead time’ is an arbitrarily distributed random variable. The operating characteristics like MTSF, Expected up time, Expected down time, and the busy period analysis, as well as the cost benefit analysis is studied. These characteristics have been demonstrated numerically.

Chapter 4 is a study of a two unit cold standby system with varying physical conditions of the repair facility. The system measures like MTSF, Availability, Busy period of the repairman, etc. are studied. Confidence limits, the steady state availability and the busy period of the repairman in the steady state are also obtained.

In most of the available literature on n-unit standby systems, many of the associated distributions are taken to be exponential, one of the main reasons for this assumption is the number of built-in difficulties otherwise faced while analysing such systems. In Chapter 5, this exponential nature of the distributions is relaxed and a general model of a three unit cold standby redundant system, where the failure and repair time distributions are arbitrary, is studied.

In Chapter 6, a stochastic model of a reliability system which is operated by a human operator is studied. The system fails due to the failure of the human operator. Once again, it is assumed that the human operator can be in any one of the three states; namely, normal stress, moderate stress or extreme stress. Different operating characteristics like availability, mean number of visits to a particular state and the expected profit are obtained. The results are illustrated numerically.