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APPENDIX A: FLEXURAL TEST RESULTS











* The LVDT displacement recording malfunctioned during this set of disk tests. The results plotted are the actuator displacement vs .load. The actuator LVDT has limited accuracy, resulting in the aberrant load-displacement curves.









* The LVDT displacement recording malfunctioned during the test on specimen Disk1-E-2. The result plotted for this specimen is the actuator displacement vs .load. The actuator LVDT has limited accuracy, resulting in the aberrant load-displacement curve.



APPENDIX B: CYCLIC TEST RESULTS









APPENDIX C: EXPONENTIAL SOFTENING



C-1







APPENDIX D: SIMULATION USING CRACK TIP SINGULARITY









APPENDIX E: SIMULATION OF FLEXURAL DISKS



* The LVDT displacement recording malfunctioned during this set of disk tests. The results plotted are the actuator displacement vs .load. The actuator LVDT has limited accuracy, resulting in the aberrant load-displacement curves.