

Trickle Flow

Hydrodynamic Multiplicity

Trickle Flow Hydrodynamic Multiplicity

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“Hydrodynamics has little significance for the engineer because of the great mathematical knowledge required for an understanding of it and the negligible possibility of applying its results.”

Prandtl (1904)

ABSTRACT

Title: Trickle Flow Hydrodynamic Multiplicity

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Trickle flow is encountered in a variety of process engineering applications where gas and liquid flow through a packed bed of stationary solid. Owing to the complexities of three interacting phases, a fundamentally exhaustive description of trickle flow hydrodynamics has not been achieved. A complicating factor in describing the hydrodynamics is the fact that the hydrodynamic state is dependent not only on the present operating conditions but also on their entire history, including fluid flow rate changes and pre-wetting procedures. This phenomenon is termed hydrodynamic multiplicity and is the subject of this work. Hydrodynamic multiplicity greatly complicates both the experimental investigation into the behaviour of a trickle flow column and the theoretical modelling of the observed behaviour.

Broadly speaking, this study addresses hydrodynamic multiplicity on three levels. First, a conceptual framework is proposed that can be used to study hydrodynamic multiplicity with limited resources. It is based on the absolute limiting values that the hydrodynamic parameters can adopt for a certain set of conditions, and encompasses both flow rate hysteresis loops and pre-wetting procedures. There are 5 such hydrodynamic modes. When the existing literature is critically evaluated in light of this framework, it is established that the reported experimental studies have not addressed all the issues. Previous modelling

attempts are also shown to be unable to qualitatively explain all the existing data. Moreover, authors have suggested different (and often contradictory) physical mechanisms responsible for hydrodynamic multiplicity.

Secondly, an experimental investigation intended to supplement the existing literature and illustrate the utility of the proposed framework is launched. This includes bed-scale measurements of liquid holdup, pressure drop and gas-liquid mass transfer for a variety of conditions including different flow rates, pressures, particle shapes, particle porosity and surface tension. The second part of the experimental effort uses radiography and tomography in new ways to visualise the temporal and spatial characteristics of the different hydrodynamic modes. The tomographic investigation incorporates advanced image processing techniques in order to culminate in a pore-level evaluation of the hydrodynamic modes that reveals additional features of hydrodynamic multiplicity.

Thirdly, the experimental insights are condensed into a set of characteristic trends that highlight the features of hydrodynamic multiplicity. A pore-level capillary mechanism is then introduced to qualitatively explain the observed behaviour. The mechanism shows how the differences in advancing and receding contact angles and the characteristics of the packed structure (or pore geometries) are ultimately responsible for the observed hydrodynamic multiplicity behaviour.

Lastly, the effect of hydrodynamic multiplicity on trickle bed reactor performance is discussed. It is established experimentally that depending on the reaction conditions, different modes yield optimal performance. The idea of optimizing the performance by manipulating the hydrodynamic state is introduced.

In totality, this work advances the understanding of trickle flow hydrodynamics in general and hydrodynamic multiplicity in particular.

Keywords: hydrodynamics, trickle flow, multiphase, hysteresis, pre-wetting, tomography, radiography, image processing, packed bed, multiplicity.

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Nomenclature

| <i>Symbol</i> | <i>Definition (units)</i> |
|----------------|---|
| A | Cross-sectional area (m ²) |
| A | Convolution target in Chapter 6 |
| A | Ergun constant in Chapters 2 and 4 |
| a | Hole area in Chapter 7 (m ²) |
| $a1, a2, a3$ | Size of A |
| $A_{\alpha S}$ | Interfacial area between phase α and the solid (m ²) |
| B | Convolution kernel in Chapter 6 |
| B | Second Ergun constant |
| b | Stoichiometric coefficient |
| $b1, b2, b3$ | Size of B |
| C | Concentration in Chapter 8 (kmol/m ³) |
| C | Constant in Chapter 7 |
| C | Convolution of A with B |
| $c1, c2, c3$ | Size of C |
| d | Capillary diameter in Chapter 7 |
| D | Column diameter (cm) |
| D | Difference operator (convolution kernel) |
| D | Diffusivity in Chapter 8 (m ² /s) |
| d | Particle diameter (m) |
| D_{AX} | Axial dispersion coefficient (m ² /s) |
| D_h | Hydraulic diameter (m) |
| d_t | Hydraulic diameter of tube in Chapter 7 (m) |
| E | Attenuation coefficient (1/m) |
| E | Edge image |
| ex, ey, ez | Voxels coordinates |
| F | Filtered image |

| | |
|--------------------------|---|
| F | Force (N) |
| f | Wetting efficiency |
| f_s, f_v | Shear and velocity slip factors |
| F_α | F-function parameter |
| G | Gas superficial flux (kg/m ² s) |
| g | Gravity acceleration (m/s ²) |
| Ga | Galileo number |
| h | Height in Chapter 7 (m) |
| h | Hough-transform value in Chapter 6 (normalized) |
| H | Packed height (cm) |
| I | Image |
| $k_{GL}a_{GL}$ | Volumetric gas-liquid mass transfer coefficient (1/s) |
| $k_{LS}a_{LS}$ | Volumetric liquid-solid mass transfer coefficient (1/s) |
| $k_\alpha, k_{\alpha i}$ | Relative permeabilities of phase α |
| l | Characteristic tube length (m) |
| L | Liquid superficial flux (kg/m ² s) |
| M | Maldistribution factor |
| m | Mass (kg) |
| \mathbf{n} | Normal vector |
| N | Number of observations (Chapter 2) |
| N | Number of particles in Chapter 6 |
| P | Pressure in Chapter 4 (Pa) |
| P | Productivity in Chapter 8 (kmol/s) |
| Q | Volumetric flow rate (m ³ /s) |
| r | Particle radius (m) |
| r_c | Radial distance from central axis (pixels or m) |
| Re | Reynolds number |
| S | Solids image |
| T | Ternary image |

| | |
|---------------------|---|
| t | Threshold in Chapter 6 |
| t | Time (s) |
| u | Direction normal to edge in Chapter 6 |
| U | Interstitial (or characteristic) velocity (m/s) |
| u | Velocity (m/s or mm/s) |
| \mathbf{u} | Velocity vector (m/s) |
| $u_{L,pulse}$ | Pulsing velocity (m/s) |
| v | Number of overlaps |
| V | Volume (m ³) |
| W_{Pd} | Mass palladium (kg) |
| X | Conversion |
| x | Generic hydrodynamic parameter in Chapter 2 |
| x | Thickness in Chapter 5 (m) |
| z | Length (m) |
| <i>Greek</i> | <i>Definition (units)</i> |
| ΔH | Heat of reaction (kJ/kmol) |
| $\Delta P/\Delta z$ | Pressure drop (kPa/m) |
| Δs | Image resolution (pixels or m) |
| Ω | Stresses |
| Ψ | Momentum vector |
| $\beta_{L/G}$ | Liquid/Gas saturation |
| δ | Reduced liquid saturation in Chapter 2 and 4 |
| ε | Porosity |
| ε_L | Liquid holdup |
| ϕ | Capillary correction factor for liquid in Chapter 7 |
| γ | Gas- or liquid limitation factor in Chapter 8 |
| λ, ψ | Parameters in Chapter 2 |

| | |
|-------------------|---|
| μ | Viscosity (Pa.s) |
| θ, ϕ | Spherical voxel coordinates in Chapter 6 |
| θ, θ' | Contact angle and bottom contact angle in Chapter 6 |
| ρ | Density (kg/m ³) |
| ρ_i | Intensity density (1/m ³) |
| σ | Surface tension (N/m) |
| σ_α | Tension vector (N/m) |
| ψ | Capillary correction factor for gas in Chapter 7 |

Subscripts

| | |
|--------------------|--|
| $0, o$ | Reference |
| A | Gas phase reagent in Chapter 8 |
| B | Liquid phase reagent in Chapter 8 |
| $crit, rec$ | Critical, receding (Chapter 7) |
| G, L, g, l | Gas, liquid |
| i, j, k, m, n, p | Indices in Chapter 6 |
| p | Particle |
| res | Residual |
| sub | Substrate |
| T | Total |
| v | Volumetric |
| α | Indicates phase α |
| β, γ | Indicates liquid and gas phases in Chapter 2 |