

# **ROBUST NONLINEAR MODEL PREDICTIVE CONTROL OF A CLOSED RUN-OF-MINE ORE MILLING CIRCUIT**

by

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## SUMMARY

Title: Robust Nonlinear Model Predictive Control of a Closed Run-Of-Mine Ore Milling Circuit  
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This thesis presents a robust nonlinear model predictive controller (RNMPC), nominal nonlinear model predictive controller (NMPC) and single-loop proportional-integral-derivative (PID) controllers that are applied to a nonlinear model of a run-of-mine (ROM) ore milling circuit. The model consists of nonlinear modules for the individual process units of the milling circuit (such as the mill, sump and cyclone), which allow arbitrary milling circuit configurations to be modelled easily.

This study aims to cast a complex problem of a ROM ore milling circuit into an RNMPC framework without losing the flexibility of the modularised nonlinear model and implement the RNMPC using open-source software modules.

The three controllers are compared in a simulations study to determine the performance of the controllers subject to severe disturbances and model parameter variations. The disturbances include changes to the feed ore hardness, changes in the feed ore size distributions and spillage water being added to the sump.

The simulations show that the RNMPC and NMPC perform better than the PID controllers with regard to the economic objectives, assuming full-state feedback is available, especially when actuator constraints become active. The execution time of the RNMPC, however, is much too long for real-time implementation and would require further research to improve the efficiency of the implementation.

**Keywords:** Run-of-mine ore milling circuit, Robust Nonlinear Model Predictive Control, ROM, RNMPC.



## OPSOMMING

Titel: Robuuste nie-lineêre-model voorspellende beheer van 'n geslote maalkring wat onbehandelde erts maal  
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Die tesis beskryf 'n robuuste nie-lineêre-model voorspellende beheerder (RNMVB), 'n nominale nie-lineêre-model voorspellende beheerder (NMVB) en proporsioneel-integraal-afgeleide (PIA)-beheerders wat toegepas word op 'n nie-nielineêre-model van 'n geslote maalkring wat onbehandelde erts maal. Die model bestaan uit nie-lineêre modules vir die individuele proses-eenhede (soos die meule, opvangbak en sikloon) wat dit moontlik maak om arbitrêre proseskonfigurasies te modelleer.

Die studie beoog om 'n komplekse probleem van 'n geslote maalkring wat onbehandelde erts maal in 'n RNMVB raamwerk te plaas sonder om die voordeel van 'n modulêre nie-lineêre-model te verloor. Die RNMVB-implimentering gebruik oop-bronkode sagtewaremodules.

Die drie beheerders word met mekaar vergelyk deur van 'n simulasiestudie gebruik te maak om die werkverrigting van die beheerders te bepaal as beduidende sturings en model-parametervariasies teenwoordig is. Die sturings sluit in veranderinge in die hardheid en groottesamestelling van die erts wat na die meule gevoer word, asook water wat in die opvangbak gestort word.

Die simulاسies wys dat die RNMVB en NMVB beter werkverrigting lewer as die PIA-beheerders as hulle vergelyk word wat betref die ekonomiese doelwitte, veral as die ak-tueerders hulle limiete bereik. Die simulاسies neem aan dat alle proses toestandsveran-derlikes terugvoerbaar is. Die berekeninge van die RNMVB neem te lank om dit te kan implementeer op 'n werklike aanleg en vereis verdere navorsing om die implimentering te bespoedig.

**Sleutelwoorde:** Maalkring wat onbehandelde erts maal, Robuuste Nie-lineêre-model Voor-spellende Beheer.



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# LIST OF ABBREVIATIONS

ADMC	Adaptive Dynamic Matrix Control
AG	Autogenous
ARX	Auto-Regressive with eXogenous input
BC	The rate of steel Ball Consumption [ $\text{m}^3/\text{hour}$ ]
CFD	Computational Fluid Dynamics
CFF	The volumetric flow-rate of slurry from the sump to the cyclone. [ $\text{m}^3/\text{hour}$ ]
CPPAD	Software to perform automatic differentiation of software code through operator overloading.
DCS	Distributed Control System
DEM	Discrete Element Method
DMC	Dynamic Matrix Control
FBS	Frequency Based Specifications
FP	The rate of Fines Production [ $\text{m}^3/\text{hour}$ ]
GPC	Generalized Predictive Control
IDCOM	IDentification and COMmand
INA	Inverse Nyquist Array
IPOPT	Software to solve large scale nonlinear programming problems.
ISpS	Input-to-State practical Stability
ISS	Input-To-State Stability
JKMRC	Julius Kruttschnitt Mineral Research Centre



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LMI	Linear Matrix Inequality
LOAD	The total charge of the mill. [%]
LQ	Linear Quadratic
MFB	The feed-rate of steel balls to the circuit. [tons/hour]
MFS	The feed-rate of ore to the circuit (consists of rocks, coarse and fine ore). [tons/hour]
MHE	Moving Horizon Estimators
MHSO	Moving Horizon State Observers
MIMO	Multi-Input-Multi-Output
MIW	The volumetric flow-rate of water to the circuit. [m <sup>3</sup> /hour]
MPC	Model Predictive Control
NMPC	Nonlinear Model Predictive Controller
ODEs	Ordinary Differential Equations
ORC	Override Control
PI	Proportional-Integral
PID	Proportional-Integral-Derivative
PSE	Product particle-size. [% < 75 $\mu$ m]
QDMC	Quadratic Dynamic Matrix Control
QP	Quadratic Programming
RBF	Radial Basis Function
RBF-ARX	Radial Basis Function – Auto-Regressive with eXogenous input.
RC	The rate of Rock Consumption [m <sup>3</sup> /hour]
RHC	Receding Horizon Control
RMHSO	Robust Moving Horizon State Observer
RNMPC	Robust Nonlinear Model Predictive Control
ROM	Run-of-Mine
RPM	Revolutions Per Minute



Runge-Kutta	Method to solve ordinary differential equations numerically.
SAG	Semi-Autogenous
SANE	Symbiotic Adaptive Neuro-Evolution
SDP	Semidefinite Programming
SFW	The volumetric flow-rate of extra water to the sump. [ $\text{m}^3/\text{hour}$ ]
SID	System Identification
SIMC	Skogestad Internal Model Control
SISO	Single-Input-Single-Output
SLEV	The level of the sump. [ $\text{m}^3$ ]
SMOC	Shell Multi-variable Optimizing Control
SPH	Smoothed Particle Hydrodynamics
THROUGHPUT	Product throughput consisting of coarse and fine solids. [ $\text{tons}/\text{hour}$ ]
TITO	Two-Input-Two-Output