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APPENDIX 1

To compute the R matrix, we use the following set of non linear equations. This can be solved by using Gauss-Siedel iterative process. The equations are derived by exploiting the coefficient matrices appearing in chapter 2 (2).

For $i = 0$,

$$Z_{(i,i)}^{(0)}(I_{m1} \otimes D_1) + R_{(i,i)}^{(0)}[C_0 \oplus D_0 - \beta(I_{m1} \otimes I_{m2})] + R_{(i,i+1)}^{(0)}\gamma(I_{m1} \otimes I_{m2})$$

$$+R_{(i,i+1)}^{(1)}\mu(I_{m1} \otimes I_{m2}) + C_1 \otimes I_{m2} = 0,$$

For $i = 1,2, \dots, c-1$

$$Z_{(i,i)}^{(k)}(I_{m1} \otimes D_1) + R_{(i,i)}^{(k)}[C_0 \oplus D_0 - (i\gamma + \beta + \theta)(I_{m1} \otimes I_{m2})] + R_{(i,i+1)}^{(k)}(i + 1 - k)\gamma(I_{m1} \otimes I_{m2}) + R_{(i,i+1)}^{(k+1)}(k + 1)\mu(I_{m1} \otimes I_{m2}) = 0$$

$$k = 0$$

$$Z_{(i,i)}^{(k-1)}\theta(I_{m1} \otimes I_{m2})+Z_{(i,i)}^{(k)}(I_{m1} \otimes D_1)+R_{(i,i)}^{(k-1)}(C_1 \otimes I_{m2}) + R_{(i,i)}^{(k)}[C_0 \oplus D_0 - ((i-k)\gamma$$
$$+ k\mu + \beta + \theta)(I_{m1} \otimes I_{m2})] + R_{(i,i+1)}^{(k)}(i + 1 - k)\gamma(I_{m1} \otimes I_{m2})$$
$$+ R_{(i,i+1)}^{(k+1)}(k + 1)\mu(I_{m1} \otimes I_{m2}) = 0$$

$$k = 1,2, \dots, i-1$$

$$Z_{(i,i)}^{(k-1)}\theta(I_{m1} \otimes I_{m2})+Z_{(i,i)}^{(k)}(I_{m1} \otimes D_1)+R_{(i,i)}^{(k-1)}(C_1 \otimes I_{m2}) + R_{(i,i)}^{(k)}[C_0 \oplus D_0 - ((i-k)\gamma$$
$$+ k\mu + \beta)(I_{m1} \otimes I_{m2})] + R_{(i,i+1)}^{(k)}(i + 1 - k)\gamma(I_{m1} \otimes I_{m2})$$
$$+ R_{(i,i+1)}^{(k+1)}(k + 1)\mu(I_{m1} \otimes I_{m2}) + C_1 \otimes I_{m2} = 0$$

$$k = i$$

For $i = c, c+1, \dots, Q-1$

$$Z_{(i,i)}^{(k)}(I_{m1} \otimes D_1) + R_{(i,i)}^{(k)}[C_0 \oplus D_0 - (i\gamma + h(s-i)\beta + \theta)(I_{m1} \otimes I_{m2})] + R_{(i,i+1)}^{(k)}(i+1-k)\gamma(I_{m1} \otimes I_{m2}) + R_{(i,i+1)}^{(k+1)}(k+1)\mu(I_{m1} \otimes I_{m2}) = 0$$

$k = 0$

$$Z_{(i,i)}^{(k-1)}\theta(I_{m1} \otimes I_{m2}) + Z_{(i,i)}^{(k)}(I_{m1} \otimes D_1) + R_{(i,i)}^{(k-1)}(C_1 \otimes I_{m2}) + R_{(i,i)}^{(k)}[C_0 \oplus D_0 - ((i-k)\gamma + k\mu + h(s-i)\beta + \theta)(I_{m1} \otimes I_{m2})] + R_{(i,i+1)}^{(k)}(i+1-k)\gamma(I_{m1} \otimes I_{m2}) + R_{(i,i+1)}^{(k+1)}(k+1)\mu(I_{m1} \otimes I_{m2}) = 0$$

$k = 1,2, \dots, c-1$

$$Z_{(i,i)}^{(k-1)}\theta(I_{m1} \otimes I_{m2}) + Z_{(i,i)}^{(k)}(I_{m1} \otimes D_1) + R_{(i,i)}^{(k-1)}(C_1 \otimes I_{m2}) + R_{(i,i)}^{(k)}[C_0 \oplus D_0 - ((i-k)\gamma + k\mu + h(s-i)\beta)(I_{m1} \otimes I_{m2})] + R_{(i,i+1)}^{(k)}(i+1-k)\gamma(I_{m1} \otimes I_{m2}) + C_1 \otimes I_{m2} = 0$$

$k = c$

For $i = Q, Q+1, \dots, Q+c-1$

$$Z_{(i,i)}^{(k)}(I_{m1} \otimes D_1) + R_{(i,i)}^{(k)}[C_0 \oplus D_0 - (i\gamma + \theta)(I_{m1} \otimes I_{m2})] + R_{(i,i+1)}^{(k)}(i+1-k)\gamma(I_{m1} \otimes I_{m2}) + R_{(i,i+1)}^{(k+1)}(k+1)\mu(I_{m1} \otimes I_{m2}) + R_{(i,i-Q)}^{(k)}\beta(I_{m1} \otimes I_{m2}) = 0$$

$k = 0$

$$Z_{(i,i)}^{(k-1)}\theta(I_{m1} \otimes I_{m2}) Z_{(i,i)}^{(k)}(I_{m1} \otimes D_1) + R_{(i,i)}^{(k-1)}(C_1 \otimes I_{m2}) + R_{(i,i)}^{(k)}[C_0 \oplus D_0 - ((i-k)\gamma + k\mu + \theta)(I_{m1} \otimes I_{m2})] + R_{(i,i+1)}^{(k)}(i+1-k)\gamma(I_{m1} \otimes I_{m2}) + R_{(i,i+1)}^{(k+1)}(k+1)\mu(I_{m1} \otimes I_{m2}) + h(i-Q-k)R_{(i,i-Q)}^{(k)}\beta(I_{m1} \otimes I_{m2}) = 0$$

$k = 1,2, \dots, c-1$

$$Z_{(i,i)}^{(k-1)}\theta(I_{m1}\otimes I_{m2})+Z_{(i,i)}^{(k)}(I_{m1}\otimes D_1)+R_{(i,i)}^{(k-1)}(C_1\otimes I_{m2})+R_{(i,i)}^{(k)}[C_0\oplus D_0-((i-k)\gamma+k\mu)(I_{m1}\otimes I_{m2})]+R_{(i,i+1)}^{(k)}(i+1-k)\gamma(I_{m1}\otimes I_{m2})+C_1\otimes I_{m2}=0$$

$$k=c$$

For $i=Q+c,Q+C+1,\dots,S$

$$Z_{(i,i)}^{(k)}(I_{m1}\otimes D_1)+R_{(i,i)}^{(k)}[C_0\oplus D_0-(i\gamma+\theta)(I_{m1}\otimes I_{m2})]+\bar{\delta}_{(i,S)}R_{(i,i+1)}^{(k)}(i+1-k)\gamma(I_{m1}\otimes I_{m2})+\bar{\delta}_{(i,S)}R_{(i,i+1)}^{(k+1)}(k+1)\mu(I_{m1}\otimes I_{m2})+R_{(i,i-Q)}^{(k)}\beta(I_{m1}\otimes I_{m2})=0$$

$$k=0$$

$$Z_{(i,i)}^{(k-1)}\theta(I_{m1}\otimes I_{m2})+Z_{(i,i)}^{(k)}(I_{m1}\otimes D_1)+R_{(i,i)}^{(k-1)}(C_1\otimes I_{m2})+R_{(i,i)}^{(k)}[C_0\oplus D_0-\big((i-k)\gamma+k\mu+\theta\big)(I_{m1}\otimes I_{m2})]+\bar{\delta}_{(i,S)}R_{(i,i+1)}^{(k)}(i+1-k)\gamma(I_{m1}\otimes I_{m2})+\bar{\delta}_{(i,S)}R_{(i,i+1)}^{(k+1)}(k+1)\mu(I_{m1}\otimes I_{m2})+R_{(i,i-Q)}^{(k)}\beta(I_{m1}\otimes I_{m2})=0$$

$$k=1,2,\dots,c-1$$

$$Z_{(i,i)}^{(k-1)}\theta(I_{m1}\otimes I_{m2})+Z_{(i,i)}^{(k)}(I_{m1}\otimes D_1)+R_{(i,i)}^{(k-1)}(C_1\otimes I_{m2})+R_{(i,i)}^{(k)}[C_0\oplus D_0-((i-k)\gamma+k\mu)(I_{m1}\otimes I_{m2})]+\bar{\delta}_{(i,S)}R_{(i,i+1)}^{(k)}(i+1-k)\gamma(I_{m1}\otimes I_{m2})+R_{(i,i-Q)}^{(k)}\beta(I_{m1}\otimes I_{m2})+C_1\otimes I_{m2}=0$$

$$k=c$$

For $i=0,1,\dots,c-1,\ j=i+1,i+2,\dots,c\ \ or\ i=1,2,\dots,c, j=0,1,\dots,i-1$

$$Z_{(i,j)}^{k}(I_{m1}\otimes D_1)+R_{(i,j)}^{k}[C_0\oplus D_0-(j\gamma+\beta+\theta)(I_{m1}\otimes I_{m2})]+R_{(i,j+1)}^{k}(j+1-k)\gamma(I_{m1}\otimes I_{m2})+R_{(i,j+1)}^{k+1}\mu(I_{m1}\otimes I_{m2})=0$$

$$k=0$$

For $i = 0,1, \dots\ c-1, j = c+1, c+2, \dots, Q-1,\ or\ i = c, c+1, \dots, Q-2, j = i+1, i+2, \dots, Q-1\ \ or\ i = c+1, c+2, \dots, Q, j = c, c+1, \dots, i-1\ \ or\ i = Q+1, Q+2, \dots, S\ \ j = c, c+1, \dots, Q-1,$

$$Z_{(i,j)}^{(k)}(I_{m1} \otimes D_1) + R_{(i,j)}^{(k)}[C_0 \oplus D_0 - (j\gamma + h(s-j)\beta + \theta)(I_{m1} \otimes I_{m2})] + R_{(i,j+1)}^{(k)}(j+1-k)\gamma(I_{m1} \otimes I_{m2}) + R_{(i,j+1)}^{(k+1)}(k+1)\mu(I_{m1} \otimes I_{m2}) = 0$$

$$k = 0$$

$$Z_{(i,j)}^{(k-1)}\theta(I_{m1} \otimes I_{m2}) + Z_{(i,j)}^{(k)}(I_{m1} \otimes D_1) + R_{(i,j)}^{(k-1)}(C_1 \otimes I_{m2}) + R_{(i,j)}^{(k)}[C_0 \oplus D_0 - ((j-k)\gamma + k\mu + h(s-j)\beta + \theta)(I_{m1} \otimes I_{m2})] + R_{(i,j+1)}^{(k)}(j+1-k)\gamma(I_{m1} \otimes I_{m2}) + R_{(i,j+1)}^{(k+1)}(k+1)\mu(I_{m1} \otimes I_{m2}) = 0$$

$$k = 1,2, \dots, c-1$$

$$Z_{(i,j)}^{(k-1)}\theta(I_{m1} \otimes I_{m2}) + Z_{(i,j)}^{(k)}(I_{m1} \otimes D_1) + R_{(i,j)}^{(k-1)}(C_1 \otimes I_{m2}) + R_{(i,j)}^{(k)}[C_0 \oplus D_0 - ((j-k)\gamma + k\mu + h(s-j)\beta)(I_{m1} \otimes I_{m2})] + R_{(i,j+1)}^{(k)}(j+1-k)\gamma(I_{m1} \otimes I_{m2}) = 0$$

$$k = c$$

For $i = 0,1, \dots\ c-1, j = Q, Q+1, \dots, Q+c-1,\ or\ i = c, c+1, \dots, Q-1, j = Q, Q+1, \dots, Q+c\ \ or\ i = Q, Q+1, \dots, Q+c-1, j = i+1, i+2, \dots, Q+c\ \ or\ i = Q+1, Q+2, \dots, Q+c,\ j = Q, Q+1, \dots, i-1,\ or\ i = Q+c+1, Q+c+2, \dots, S, j = Q, Q+1, \dots, Q+c$

$$Z_{(i,j)}^{(k)}(I_{m1} \otimes D_1) + R_{(i,j)}^{(k)}[C_0 \oplus D_0 - (j\gamma + \theta)(I_{m1} \otimes I_{m2})] + R_{(i,j+1)}^{(k)}(j+1-k)\gamma(I_{m1} \otimes I_{m2}) + R_{(i,j+1)}^{(k+1)}(k+1)\mu(I_{m1} \otimes I_{m2}) + R_{(i,j-Q)}^{(k)}\beta I_{m1} \otimes I_{m2} = 0$$

$$k = 0$$

$$Z_{(i,j)}^{(k-1)}\theta(I_{m1}\otimes I_{m2})$$
$$+Z_{(i,j)}^{(k)}(I_{m1}\otimes D_1)+R_{(i,j)}^{(k-1)}(C_1\otimes I_{m2})+R_{(i,j)}^{(k)}[C_0\oplus D_0-((j-k)\gamma+k\mu$$
$$+\theta)(I_{m1}\otimes I_{m2})]+R_{(i,j+1)}^{(k)}(j+1-k)\gamma(I_{m1}\otimes I_{m2})$$
$$+R_{(i,j+1)}^{(k+1)}(k+1)\mu(I_{m1}\otimes I_{m2})+h(j-Q-k)R_{(i,j-Q)}^{(k)}\beta I_{m1}\otimes I_{m2}=0$$

$$k=1,2,\dots,c-1$$

$$Z_{(i,j)}^{(k-1)}\theta(I_{m1}\otimes I_{m2})+Z_{(i,j)}^{(k)}(I_{m1}\otimes D_1)+R_{(i,j)}^{(k-1)}(C_1\otimes I_{m2})+R_{(i,j)}^{(k)}[C_0\oplus D_0-((j-k)\gamma$$
$$+k\mu)(I_{m1}\otimes I_{m2})]+R_{(i,j+1)}^{(k)}(j+1-k)\gamma(I_{m1}\otimes I_{m2})=0$$

$$k=c$$

For $i=0,1,\dots c-1, j=Q+c,Q+c+1,\dots,S$ or $i=c,c+1,\dots,Q-1, j=Q+c+1,Q+c+2,\dots,S$ or $i=Q,Q+1,\dots,Q+c-1, j=Q+c+1,Q+c+2,\dots,S,$ or $i=Q+c,Q+c+1,\dots,S-1, j=i+1,i+2,\dots,S$

$$Z_{(i,j)}^{(k)}(I_{m1}\otimes D_1)+R_{(i,j)}^{(k)}[C_0\oplus D_0-(j\gamma+\theta)(I_{m1}\otimes I_{m2})]$$
$$+\bar{\delta}_{(j,S)}R_{(i,j+1)}^{(k)}(j+1-k)\gamma(I_{m1}\otimes I_{m2})+\bar{\delta}_{(j,S)}R_{(i,j+1)}^{(k+1)}(k+1)\mu(I_{m1}\otimes I_{m2})$$
$$+R_{(i,j-Q)}^{(k)}\beta I_{m1}\otimes I_{m2}=0$$

$$k=0$$

$$Z_{(i,j)}^{(k-1)}\theta(I_{m1}\otimes I_{m2})$$
$$+Z_{(i,j)}^{(k)}(I_{m1}\otimes D_1)+R_{(i,j)}^{(k-1)}(C_1\otimes I_{m2})+R_{(i,j)}^{(k)}[C_0\oplus D_0-((j-k)\gamma+k\mu$$
$$+\theta)(I_{m1}\otimes I_{m2})]+\bar{\delta}_{(j,S)}R_{(i,j+1)}^{(k)}(j+1-k)\gamma(I_{m1}\otimes I_{m2})$$
$$+\bar{\delta}_{(j,S)}R_{(i,j+1)}^{(k+1)}(k+1)\mu(I_{m1}\otimes I_{m2})+R_{(i,j-Q)}^{(k)}\beta I_{m1}\otimes I_{m2}=0$$

$$k=1,2,\dots,c-1$$

$$Z_{(i,j)}^{(k-1)}\theta(I_{m1} \otimes I_{m2})+Z_{(i,j)}^{(k)}(I_{m1} \otimes D_1)+R_{(i,j)}^{(k-1)}(C_1 \otimes I_{m2}) + R_{(i,j)}^{(k)}[C_0 \oplus D_0 - ((j-k)\gamma + k\mu)(I_{m1} \otimes I_{m2})] + \bar{\delta}_{(j,S)}R_{(i,j+1)}^{(k)}(j+1-k)\gamma(I_{m1} \otimes I_{m2}) + R_{(i,j-Q)}^{(k)}\beta I_{m1} \otimes I_{m2} = 0$$

$$k = c$$

APPENDIX 2

1. Renewal Processes

A renewal process is a sequence of independent non-negative random variables having identical distributions. Formally, if $\{N(i), i > 0\}$ is a counting process with $N(0) = 0$, and $N(i) = \sum_{j=1}^{i} x_j$ and $x_n = 1,2 \ldots$ the time between the $(n-1)$th and nth event of this process, $n \geq 1$. Let $\left\{g_i^{(n)} = P\{x_n = i\},\ i \geq 0\right\}$ be the distribution series of $x_n,\ n \geq 1$. If the sequence of $\{x_1, x_2 \ldots\}$ is independently and identically distributed from the second one, then the random sequence $v_n = max_{i\geq 0}\{i: N(i) \leq n\},\ n \geq 0$ is called the general discrete renewal process. This means $v_n$is the number of renewals until the instant n, inclusive.

The renewal process, v_n is said to be simple if $g_i^{(1)} = g_i,\ \ i \geq 0$. Also, v_n said to be stationary if the distribution series $\left\{g_i^{(1)},\ i \geq 0\right\}$ of the first instant of renewal $N(1) = x_1$ obeys the formula

$$g_0^1 = 0, \quad g_i^1 = \frac{1}{g}\sum_1^{\infty} g_j\,,\ \ i \geq 1\ \ and\ g = \boldsymbol{E}x_n = \sum_{i=1}^{\infty} i g_i\,,\ \ g < \infty\ .$$

The random variable v_n has moments of any order, and for any renewal process has moments of any order, and for any renewal process has moments of any order, and for any renewal process $\{v_n,\ n \geq 0\}$, and each $n \geq 0$, there exists a number $C = C(n)$, such that $Ev_n^k \leq C^k k!\ \ \forall\, k \geq 0$.

1.1. The renewal function

The renewal function, H_n, is the number of renewals up until the instant n inclusive and is given by $H_n = \boldsymbol{E}v_n$. The renewal series is the number of renewal at b, and is given by $h_n = H_n - H_{n-1},\ n \geq 1$. h_n can be considered to be the probability that a renewal occurs at the instant n.

The renewal series satisfies the renewal equation

$$h_n = g_n^{(1)} + \sum_{i=1}^{n} h_i g_{n-i}\,,\ n \geq 0;$$

Solving this equation with the generating function h_z defined over $z < 1$,

$$H_z = G_z^{(1)} + H_z G_z$$

From which

$$H_z = \frac{G_z^{(1)}}{1-G_z}$$

In the stationary case, this equation becomes

$H_z = \lambda \frac{z}{1-z}$, where $\lambda = {}^1/_g$.

From Blackwell and Smith theorems, as $n \to \infty$, if the skip is defined to mean the instant of the first after the nth renewal and the nth renewal, the distribution of the skip coincides with distribution of the instance of the first renewal and becomes

$\lambda \sum_{j=0}^{\infty} g_{j+i} = \frac{1}{g} \sum_{i}^{\infty} g_j$. This is the key renewal theorem for discrete case.

The above formulae easily generalise to the continuous case and becomes

$$h_t = g_t^{(1)} + \int_0^t g_{n-i} dh_i$$

And solving using the Laplace-Stieltjes transform

$\alpha(s) = \frac{\gamma^{(1)}(s)}{1-\gamma(s)}$, s being the Laplace variable

And with Blackwell and Smith theorems the stationary distribution of the skip becomes

$\int_0^t g(t-x)dH(x) \xrightarrow[t\to\infty]{} \lambda \int_0^{\infty} g(x)dx$. This is the key renewal theorem for continuous case.

2. Markov Processes

2.1. Markov Chain

A Markov chain is sequence of discrete random variables such that for any n, x_{n+1} is conditionally independent of $x_0, \ldots x_{n-1}$ given x_n. This means the future is independent of the past given the current state irrespective of how the current of how the current state has been reached.

Formally, this can be written as follows. Suppose a probability space (Ω, x, P) is defined such that $x_n: \Omega \rightarrow S,\ where\ S = 1, \ldots N\ or\ S = 1, \ldots$ i.e. S is finite or countably infinite.

$$\mathrm{P}\{\mathrm{x}_{\mathrm{n}+1} = \mathrm{j} \mid \mathrm{x}_0, \ldots, \mathrm{x}_\mathrm{n}\} = \mathrm{P}\{\mathrm{x}_{\mathrm{n}+1} = \mathrm{j} \mid \mathrm{x}_\mathrm{n}\} \quad \forall\, \mathrm{j} \in \mathrm{S} \text{ and } \mathrm{n} \in \mathrm{N}.$$

The Markov chain has a transition matrix, P, made up of classes of states that could be transient, recurrent null or recurrent non-null. This classification is important for solving problems using P.

The Markov property simplifies the manipulation of the Transition matrix such that For any $m, n \in N$,

$$P\{x_{n+m} = j \mid x_n = i\} = P^m(i,j).$$

The Chapman-Kolmogorov equation is important in manipulating the Markov chain. This provides that

$$P^{m+n} = P^m P^n$$

P can be used to find the potential matrix, R, of the variable x, and F, the time of first visit to a state, which are also useful in solving for the equilibrium distribution of its probabilities.

The matrices $R(i,j)$ = the potential matrix or expected number of visits to a state j from another state i and $F_k(i,j)$ = the time of first visit of state j from state i are important in classifying and also solving for the equilibrium distribution of the probabilities. They are defined as

$$F_k = \left\{\begin{matrix} P(i,j) & k = 1 \\ P(i,b)F_{k-1}(b,j) & k \geq 2 \end{matrix}\right\}$$

$$R(j,j) = E_j N_j = (I - P)^{-1}$$

where P is as defined earlier (the) one step transition matrix. Cinlar (1975) gives a detailed treatment of the foregoing.

2.2. Markov Process

A Markov chain is silent about the length of time spent in a given state, j. To address this, the time variable, t, is defined such that this variable, together with the Markov chain is used to define another random variable called the Markov process. The variable, t, would be taken such that

$$t_n\colon \Omega \rightarrow R_+,\ i.e. R = [0, \infty].$$

The process

$$\mathrm{P}\{\mathrm{x}_{\mathrm{t+m}} = \mathrm{j} \mid \mathrm{x}_{\mathrm{u}}; \mathrm{u} \leq \mathrm{t}\} = \mathrm{P}\{\mathrm{x}_{\mathrm{t+m}} = \mathrm{j} \mid \mathrm{x}_{\mathrm{t}}\} \quad \forall\, \mathrm{j} \in \mathrm{S} \text{ and } \mathrm{t}, \mathrm{s} \in \mathrm{R}_+.$$

The Markov process such that

$$P\{x_{n+m} = j \mid x_n = i\} = P_n(i,j)$$

or in the matrix form

$$P(m+n) = P(m)P(n)$$

holds is said to be time homogeneous, where P_n is the probability of being in state n. The Kolmogorov-Chapman equation still holds. The function $P_n(i,j)$ is called the transition function. The set of successive states visited by the process forms a Markov chain with the corresponding transition matrix, P, and the time of sojourn in each state has a probability distribution, which usually could be taken as exponential.

2.3. The Infinitesimal matrix

If it is assumed that the following holds everywhere

$$P_\Delta(i,j) \underset{\Delta\downarrow 0}{\longrightarrow} \delta(i,j) \quad i,j \in S,\ \delta(i,j)\ is\ the\ Kroneker\ symbol$$

Then there exists the limits

$$a(i,j) = \lim_{\Delta\downarrow 0} \frac{P_\Delta(i,j)}{\Delta}, \quad i,j \in S,\ i \neq j$$

$$-a(i,i) = \lim_{\Delta\downarrow 0} \frac{P_\Delta(i,i)-1}{\Delta}, \quad i \in S,$$

And

$0 \leq a(i,i) \leq \infty, \ o \leq a(i,j) \leq \infty, \ i,j \in S, \ i \neq j$

$\sum_{j \in S} a(i,j) \leq 0, \quad i \in S$

For a conservative (i.e. locally regular) matrix, the equation becomes

$\sum_{j \in S} a(i,j) = 0, \quad i \in S.$

The parameter $a(i,j)$ is the intensities of transition from state i to state j. Also, $a(i,i)$ is the intensity of exit from state i. The matrix $A = a(i,j)$ is the matrix of transition intensities or the infinitesimal matrix.

The transition matrix can be constructed from

$$q(i,j) = \begin{cases} \frac{a(i,j)}{a(i,i)}, & i \neq j \\ 0, & i = j \end{cases}$$

This is called the embedded Markov chain of the Markov process. The process is assumed to be conservative.

It is important that $a(i,i) > 0$, and also, to guarantee regularity of Markov chain, either

- $a(i,i)$ should be uniformly bounded, i.e. $a(i,i) < c < \infty, \ \forall \, i \in S$ or
- all the states of the Markov process should be recurrent.

3. Queuing Theory

Queuing is one of the areas in which stochastic processes in general and Markov processes in particular have had extensive applications. The essence of studying queuing is to understand how the properties of the system of interest will behave in the steady state and/or the transient state. Optimisation is not the actual goal of such analysis, but the results of such systems parameters as the expected queue length, expected waiting time, expected throughput time, facility utilisation etc (all usually expressed as a function of the traffic intensity) could find application in optimisation processes. Queuing techniques are particularly suitable in systems where there are flows, and where stocks are built up as a result of flows through such systems. This is actually characteristic of most production systems.

3.1. Properties and Classification of Queues

The idea of properties and classifications of queues are closely tied because queues are classified based on the values of these characteristics. The properties are: input pattern, service pattern, number of servers, location and sizes of buffer, the service discipline and the size of the calling source.

There have been many classifications based on all these properties, but the classification effort usually regarded as the first documented attempt was that of Kendall (1953). This makes use of the first three properties. Lee G was credited to have added the fourth property of service discipline. There are still other classifications depending on the problem being addressed.

3.2. Constructive Description of Model

Queuing models could be constructed by considering all the means through which entities enter (i.e. the birth process) and exit (i.e. the death process) the system of interest. This is summarised in the birth and death process of such queues and this immediately leads to the generation of the infinitesimal matrix of the process.

A more generalised and powerful approach for a conservative process is through the use of the global, local and partial (where necessary) balance of flows of probabilities between two states of the system. This approach is premised on the fact that at dynamic equilibrium, the ergodic properties of the system ensures that the flow of probabilities out of and into a stage cancels out.

If the states of a process are represented as nodes and all nodes that could be reached in one step of transition are connected by arcs, then the total flow into and out of a state of such system constitutes the global balance of flow. This is represented as

$$a_i p_i = \sum_{j \in S \setminus \{i\}} a_{ji} p_j$$

The local balance concerns flow between any two states. At equilibrium, the flow from a state m to another state n should be equal to the flow from state n back to state m. This is the same as looking at the ark between these two states and equating the flow across it. Formally,

$$\sum_{i \in S_1} \sum_{i \in S_2} a_{ij} p_i = \sum_{i \in S_1} \sum_{i \in S_2} a_{ji} p_j$$

The partial balance can be formally written as

$$a_{ij} p_i = a_{ji} p_j$$

The partial balance is not usually satisfied, but whenever it is satisfied, it gives some important consequences. In particular, it implies

$$p_i = \frac{a_{ji} p_j}{a_{ij}}$$

3.3. Solving the Flow Problems

The importance of the characteristic transformations in solving the problems encountered using the various distributions has been highlighted in the body of this thesis. But the two that are mostly applied appear to be the moment generating function when the random variable is defined on the integer space due to its simplicity, and the Laplace transform since it is defined on the Real field, and is simpler to handle than the characteristic function. The characteristic function is the only one applicable on the complex field. Other theorems and functions that are useful during the transformation process include the derivative function, the shifting theorems and the convolution theorem.

The transform of the derivatives, stated in general terms as

$$L[f^n](s) = SF(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

This is usually considered up until the first derivative only.

i.e. $L[f^{'}](s) = S^{n}F(s) - f(0)$

The first shifting theorems addresses a shift in the s variable of Laplace function and is written as

$$L(e^{at}f)(s) = F(s-a)$$

This means the Laplace transform of a function multiplied by an exponential function simply shifts the Laplace transform back by the coefficient of the exponent's variable. The inverse is also true.

The Heaviside, or unit step, function, in general, is

$$H(t-a) = 0 \quad if\ \ t < a, \qquad and \qquad H(t-a) = 1\ if \ \ t \geq a.$$

So, multiplying a function $f(x)$ by the Heaviside $H(t-a)$ turns off $f(x)$ if $x < a$ and on if $x \geq a$. Also, Combining two Heaviside functions $H(t-a) - H(t-b)$ produces the pulse function that turns off x before a, turns it on between a and b, and then off again after b.

The Heaviside function can be combined with the first shifting theorem to produce the Heaviside shifted function to produce another shifting theorem:

$$L[f(t-a)H(t-a)](s) = e^{-as}F(s).$$

These theorems are useful in manipulating the joint distribution of many Markovian random variables, seeing that the exponential distribution has the general form $1 - e^{\lambda t} \equiv 1 - e^{at}$.

3.4. Inputs Flows, Service Pattern and Nature of Queue

The Kendall classification, making use of the pattern of the input flow, service pattern and Queue size and location has been about the most important system of classification. A queue is said to be Markovian if the distribution of the input and output parameters conform to models that could be said to have Markov properties. This usually means the arrival pattern is Poisson (or compound Poisson) while the service pattern is exponential. But there are other input patterns said to follow the Markovian Arrival Pattern (MAP) that have become important. Also, the repertoire of Markov queuing models has been extended by the service pattern said to be Phase (PH) distribution. Another class extension of the queue type is the class of virtual queues called the retrial queues. These three extensions have further enriched the study of queuing systems and expanded the scope of applications of queuing principles to problems encountered daily.

3.5. System and Queue Structure

The system could be such that once a customer or job has been served in a facility, the customer or job exits the system. Such systems are referred to as single stage systems. Some other systems are such that when a customer has been served at one stage, the customer might move to another stage for another service. Such systems are referred to as multistage queuing systems, or in some instances, network systems.

Buffers refer to places where jobs or customers still (may be in process) are kept. There could be no buffers, real buffers or virtual buffers in a system. Systems without buffers are special cases of balking queues. If the buffers are real, it could have finite or infinite capacity. This is characteristic of most queues. In a virtual buffering system, the system does not have an actual place where customers waiting to be served could stay. Such customers would join a virtual buffer (sometimes called an orbit) where they could make subsequent attempts or leave the system altogether. Such type of buffering is characteristic of retrial queues.

3.6. Pattern of Input Flow

There are two main ways of describing the nature of the random input flow into a queuing system. The first is through the joint distribution of the times between the subsequent arrivals. If $\tau_1, \tau_{2\ldots,}\tau_{1\geq}0$, is a sequence of non-decreasing time of occurrence of certain event, and $\xi_i = \tau_i - \tau_{i-1}$ is the time between the $i - 1th$ and the ith arrival, then this is represented as

$$F_{\xi 1, \xi 2, \dots, \xi k}(x_1, x_2, \dots, x_n) = P(\xi_1 < x_1, \xi_2 < x_2, \dots, \xi_k < x_k,$$

where ξ_k is the time of arrival of the kth customer and x_k is a stopping time.

The second approach is based on the consideration of the likelihood of an event of interest occurring in some set of families of intervals $[0, t_i), [t_1, t_2), \dots [t_{k-1}, t_k),\ k \geq 1$ and defining the joint distribution function as

$$G(m_1, m_2, \dots, m_k; t_1, t_2, \dots, t_k) = P(\varsigma_1 = m_1, \varsigma_2 = m_2, \dots, \varsigma_k < m_k,$$

where m_k is the interval $[t_{k-1}, t_k)$ and ς_k is the arrival of the kth customer.

3.7. Poisson Input Flow

The Poisson input flow is the assumption of most Markov models, and the pattern of input flow is said to be Poisson of the probability, $p_i(t)$ of the ith customer arriving at time t is

$$p_i(t) = \frac{(\lambda t)^i}{i!} e^{-\lambda t}$$

The distribution parameter is λ and the time between arrivals is exponentially distributed exponentially with the same parameter λ. Since the Poisson flow is stationary and memoryless, with another assumption of ordinariness, the transition intensity matrix becomes

$$a_{ij} = \begin{cases} -\lambda, & j = i \\ \lambda, & j = i + 1 \\ 0 & otherwise \end{cases}$$

The compound (or superposed) Poisson process has the arrival rate

$\lambda = \sum_{i=1}^{n} \lambda_i, \ \sum \lambda_i = 1$

where λ_i is the weight of the component i of the superposed flows.

The convolution theorem comes in handy to solve the problem of the product of two functions. Unlike the addition function, the Laplace transform of the product of two functions is not equal to the product of the Laplace transform of the functions i.e. $L[f * g] = L[f] * L[g]$. But the convolution of two functions defined as

$$(f©g)(t) = \int_0^t f(t-\tau)g(\tau)d\tau$$

has the property that $L[f©g] = L[f]L[g]$. This makes it to handle the problems of the renewal equation which has that general form. O'neil (1995) treats this to further details.

3.8. Markov Input Flow

Some systems demand input flow that is more complex than the ordinary or compound Poisson, but still Markovian. An example of such flow is the Markov Arrival Process (MAP). They are a generalisation of the Poisson and compound Poisson flows.

If $v(t)$ is the number of customers that arrive in the time interval $[0, t)$, and $\tau_1, \tau_{2\dots,}\tau_{1\geq}0$ the instants of their arrival, and there exists a Markov process $\xi(t)$ defined on the finite state $S = \{1,2, \dots, l\}$. Also, define $\eta(t) = \{\xi(t), v(t)\}$. Then the process state set $\{\eta(t), t \geq 0\}$ is representable as

$\cup_{k=0}^{\infty} S_k$, where $S_k = \{(i,k), \ i = 1,2, \dots l, \ \ k \geq 0\}$.

$\eta(t)$ is said to be in state $(i,k), \ i = 1,2, \dots l, \ \ k \geq 0$ if k customers arrive at the instant t, and the process $\xi(t), t \geq 0$ is in the state i.

The flow $\{\tau_j, j \geq 1\}$ is said to be a Markov flow with respect to the process $\{\xi(t), t \geq 0\}$ if the random process $\{\eta(t), t \geq 0\}$ is a homogenous Markov process and its matrix, A, of transition intensities is of the block form

$$A = \begin{bmatrix} \Upsilon & N & 0 & 0 & 0 & 0 & \dots \dots \\ 0 & \Upsilon & N & 0 & 0 & 0 & \dots \dots \\ 0 & 0 & \Upsilon & N & 0 & 0 & \dots \; . \\ . & . & . & . & . & . & . \end{bmatrix}$$

where Υ and N are square matrices of order l,. $\Upsilon^* = \Upsilon + N$ is the the matrix of transition intensities of the Markov process $\{\xi(t), t \geq 0\}$.

Other Markov models can be seen as special cases of this matrix. For instance, if l is 1, then flow is the ordinary Poisson process. If N is a diagonal matrix, then the flow is a Markov Modulated Poisson process. With l = 2 for matrix N and only one non-zero and strictly positive diagonal matrix, the flow is the Interrupted Poisson process. If N is representable as $N = v\alpha^T$, where v and α are column vectors of dimension l and α is a probability vector, then the flow is called the phase type (PH) flow.

It should be noted that if $\{\xi(t), t \geq 0\}$ is a stationary Markov process, then the resulting flow from $\{\eta(t), t \geq 0\}$ is also stationary.

3.9. Distribution of service time

Basically, the service time in Markov models is assumed to be exponentially distributed. Formally, the distribution and the density function respectively are

$F(x) = 1 - e^{\mu x}$ and $f(x) = \mu e^{\mu x}$

where μ is the service rate.

But some other possible distributions include: Erlang, which is useful for cases where service time is made up of a series of some exponentially distributed stages; hyper-exponential, hyper-erlang and phase type distributions.

Formally, the Erlang density function is of the form

$$f(x) = \frac{\mu^m x^{m-1}}{(m-1)!} e^{-\mu x}, \;\; x > 0, \;\; m = 1,2, \dots , \;\; 0 < \mu < \infty$$

The hyper-exponential distribution is

$B(x) = \sum_{j=1}^{m} \beta_j(1 - e^{-\mu_j x})$ where $x > 0,\ \beta_j > 0,\ 0 < \mu_j < \infty,\ j = 1,2,\dots,m,$ $\Sigma\beta_j = 1$

And the hyper-Erlang distribution is

$B(x) = \sum_{j=1}^{m} \beta_j E_{mj}(x),$ where $\beta_j > 0,\ j = 1,2,\dots,m,\ \Sigma\beta_j = 1$ and E_{mj} is the Erlang distribution with the parameter m_j and μ_j.

In fact, Erlang, hyper-exponential and hyper-Erlang distributions are special cases of a more general class of distributions said to have fictitious phases, as coined by Erlang, or commonly called the phase type distributions.

3.10. PH distribution of service time

Some Markov models have flows that are more generalised than those discussed earlier. These can be got from the PH distribution. Generally, PH distributions admit the form

$$F(x) = 1 - \boldsymbol{f}^T e^{Gx}\mathbf{1},\ x > 0$$

where $\boldsymbol{f}$ is a probability vector, G is a probability matrix, $\sum_{j=1}^{m} f_j \le 1,\ f_j \ge 0,\ j = 1,2,\dots m,\ \sum_{j=1}^{m} G_{ij} \le 0,\ i \ne j,\ G_{ii} < 0,\ i,j = 1,2,\dots m,$ and $\sum_{j=1}^{m} G_{ij} < 0$ for at least one i. The pair $(\boldsymbol{f}, G)$ is called the PH-representation of order m of the distribution function $F(x)$.

The distribution function of the PH type of a non-negative random variable admits probabilistic interpretation based on the concept of phase. Let $v_i, i = 1 \dots m,\ v_i \ge -G_{ii}$ be some real numbers, the numbers $\theta_{ij}, i,j = 1,2,..m$ obey the formula

$$\theta_{ij} = \begin{cases} 1 + \frac{G_{ii}}{v_i}, & i = j \\ \frac{G_{ij}}{v_i} & i \neq j \end{cases}$$

This is synonymous to the embedded Markov chain of the Markov process. And the matrix of transitional intensities becomes

$$G_{ij} = \begin{cases} v_i(\theta_{ii} - 1), & i = j \\ v_i\theta_{ij}, & i \neq j \end{cases}$$

The matrix of transitional intensities satisfies the set of Kolmogorov differential equations

$$\frac{d}{dt}P(t) = P(t)G$$

With the initial condition $P(0) = I$ and the solution obeying the formula $P(t) = e^{Gt}$. And so,

$$P\{\tau < x\} = 1 - \boldsymbol{f}^T e^{Gx} 1.$$

$F(x)$ indicates the distribution of the customer sojourn in the queue network.

3.11. Solution Methods

Solving the problems of models with PH distribution requires special mathematical machinery which is found in the matrix theoretic functions. The Kronecker product of two matrices, A and B, is defined as

$$A \otimes B = \begin{bmatrix} a_{11}B & \dots & a_{1n}B \\ \vdots & . & \vdots \\ a_{m1}B & \dots & a_{mn}B \end{bmatrix}$$

The Kronecker sum of two matrices, A and B, is defined as

$A \oplus B = A \otimes I_n + I_m \otimes$

where I is the identity matrix, where m and n are the orders of the matrices A and B respectively.

The Kronecker product has many properties like scalar multiplication of the entries of the matrices, distributivity, associativity, identity matrices, zero matrices, transposition, inverse matrices, mixed product of matrices, vectorisation, eigen factors and vectors, determinants etc.

Some properties of the Kronecker sum and products that make them very useful, however, are that the products and sums are defined irrespective of the orders of the matrices A and B involved, and probably more importantly that while the expression

$$e^{A+B} = e^{A} * e^{B}$$

is true if and only if A and B commute, the expression

$$e^{A \oplus B} = e^{A \otimes I} * e^{I \otimes B}$$

is true irrespective of commutativity. This property makes the Kronecker product and sum very useful in the manipulation of PH distributed variables. Detailed treatment of Matrix theoretic functions are contained in Graham (1975) and Latouche and Ramaswami (1999).

APPENDIX 3

Table 2.2: Fraction of successful rate of retrials

$S = 25$, $s = 8$, $\lambda_{-1} = 2.5$, $\beta = 3$, $\mu = 10$, $\gamma = 0.1$, $\theta = 5$.

λ_1	c		Exp-	Erl-	HExp-	MNC-	MPC-
4	1	Exp+	0.420649	0.420426	0.421149	0.420994	0.421106
		Erl+	0.413698	0.411711	0.417077	0.415944	0.416344
		HExp+	0.428533	0.429046	0.427033	0.427875	0.427426
		MNC+	0.419197	0.419555	0.418566	0.418736	0.418499
		MPC+	0.419571	0.41978	0.419045	0.419251	0.419029
	2	Exp+	0.485599	0.482846	0.488151	0.489082	0.489177
		Erl+	0.40133	0.390861	0.416056	0.414357	0.413997
		HExp+	0.516639	0.515427	0.516175	0.518226	0.518234
		MNC+	0.508838	0.510211	0.505402	0.507208	0.506529
		MPC+	0.510315	0.5116	0.506864	0.508413	0.507263
	3	Exp+	0.401298	0.387547	0.421536	0.419646	0.419622
		Erl+	0.07422	0.059093	0.123982	0.099355	0.104827
		HExp+	0.477132	0.464921	0.492656	0.493288	0.494844
		MNC+	0.57606	0.576866	0.571744	0.575422	0.574373
		MPC+	0.596288	0.59917	0.588457	0.592109	0.589718
	4	Exp+	0.152172	0.129549	0.211352	0.185908	0.194186
		Erl+	0.027208	0.011397	0.092098	0.055607	0.065265
		HExp+	0.29284	0.26799	0.342831	0.32796	0.337393
		MNC+	0.562215	0.553123	0.575204	0.575434	0.576236
		MPC+	0.687067	0.688647	0.682241	0.685003	0.683503
4.5	1	Exp+	0.422597	0.422295	0.423307	0.423047	0.423229
		Erl+	0.416912	0.414786	0.420653	0.419324	0.419877
		HExp+	0.429211	0.429685	0.427853	0.428586	0.428131
		MNC+	0.419622	0.419857	0.419252	0.419323	0.419161
		MPC=	0.419908	0.420045	0.419582	0.419695	0.419544
	2	Exp+	0.489948	0.486681	0.493598	0.494093	0.494419
		Erl+	0.408121	0.395695	0.42552	0.423192	0.423394
		HExp+	0.522583	0.521683	0.521688	0.523836	0.523829
		MNC+	0.511748	0.512884	0.508933	0.510426	0.50983
		MPC+	0.51167	0.512708	0.508889	0.510125	0.509155
	3	Exp+	0.407207	0.391574	0.430702	0.427854	0.428886
		Erl+	0.097437	0.077082	0.15461	0.128575	0.13588
		HExp+	0.493367	0.481972	0.507929	0.508694	0.510701
		MNC+	0.577522	0.577711	0.574611	0.5777	0.576937
		MPC+	0.596487	0.598866	0.589923	0.59301	0.590933
	4	Exp+	0.177112	0.151344	0.240088	0.214447	0.224531
		Erl+	0.044207	0.022633	0.11876	0.079291	0.091906
		HExp+	0.331235	0.30736	0.377949	0.365196	0.374875
		MNC+	0.565337	0.555503	0.580159	0.579569	0.581233
		MPC+	0.687487	0.68858	0.683919	0.686072	0.684942
5	1	Exp+	0.424699	0.424358	0.425494	0.425195	0.42543
		Erl+	0.420374	0.41822	0.42421	0.42283	0.423529
		HExp+	0.429535	0.429936	0.428404	0.428989	0.428566
		MNC+	0.420284	0.420424	0.42009	0.420103	0.420003
		MPC+	0.42048	0.420564	0.420298	0.420349	0.420254
	2	Exp+	0.494902	0.491266	0.499425	0.499545	0.500113
		Erl+	0.416833	0.402967	0.436063	0.433359	0.434136
		HExp+	0.527899	0.527373	0.52646	0.528721	0.528638
		MNC+	0.514858	0.515783	0.512594	0.513806	0.513298
		MPC+	0.513523	0.514357	0.511304	0.512277	0.511466
	3	Exp+	0.416118	0.399252	0.441691	0.438292	0.440294
		Erl+	0.124192	0.099401	0.185896	0.160052	0.168928
		HExp+	0.509105	0.498813	0.522305	0.523227	0.525513
		MNC+	0.579764	0.579456	0.578037	0.580608	0.580129
		MPC+	0.59742	0.599373	0.591959	0.59455	0.592763
	4	Exp+	0.204386	0.176595	0.268793	0.243957	0.255451
		Erl+	0.065624	0.039159	0.146214	0.105902	0.121116
		HExp+	0.367325	0.344936	0.410367	0.3995	0.409157
		MNC+	0.570096	0.559872	0.586099	0.584909	0.587309
		MPC+	0.722301	0.720412	0.728029	0.72524	0.727214

Table 2.3: Fraction of successful rate of retrials

S = 25, s = 8, λ =5, β = 3, μ = 10, γ = 0.1, θ = 5.

λ−1	c		Exp-	Erl-	HExp-	MNC-	MPC-
2.5	1	Exp+	0.424699	0.424358	0.425494	0.425195	0.42543
		Erl+	0.420374	0.41822	0.42421	0.42283	0.423529
		HExp+	0.429535	0.429936	0.428404	0.428989	0.428566
		MNC+	0.420284	0.420424	0.42009	0.420103	0.420003
		MPC+	0.42048	0.420564	0.420298	0.420349	0.420254
	2	Exp+	0.494902	0.491266	0.499425	0.499545	0.500113
		Erl+	0.416833	0.402967	0.436063	0.433359	0.434136
		HExp+	0.527899	0.527373	0.52646	0.528721	0.528638
		MNC+	0.514858	0.515783	0.512594	0.513806	0.513298
		MPC+	0.513523	0.514357	0.511304	0.512277	0.511466
	3	Exp+	0.416118	0.399252	0.441691	0.438292	0.440294
		Erl+	0.124192	0.099401	0.185896	0.160052	0.168928
		HExp+	0.509105	0.498813	0.522305	0.523227	0.525513
		MNC+	0.579764	0.579456	0.578037	0.580608	0.580129
		MPC+	0.59742	0.599373	0.591959	0.59455	0.592763
	4	Exp+	0.204386	0.176595	0.268793	0.243957	0.255451
		Erl+	0.065624	0.039159	0.146214	0.105902	0.121116
		HExp+	0.367325	0.344936	0.410367	0.3995	0.409157
		MNC+	0.570096	0.559872	0.586099	0.584909	0.587309
		MPC+	0.688112	0.688782	0.685658	0.687277	0.686493
3	1	Exp+	0.424236	0.423805	0.425171	0.424833	0.425082
		Erl+	0.418926	0.416387	0.423444	0.421743	0.422452
		HExp+	0.430661	0.431176	0.429145	0.429986	0.429486
		MNC+	0.420678	0.420912	0.420312	0.420393	0.420244
		MPC+	0.420939	0.421077	0.420619	0.42073	0.420587
	2	Exp+	0.492639	0.488047	0.498434	0.498222	0.498808
		Erl+	0.411212	0.396107	0.433891	0.429285	0.429796
		HExp+	0.52706	0.525758	0.526211	0.528674	0.528728
		MNC+	0.51557	0.51646	0.512907	0.514551	0.514029
		MPC+	0.515374	0.516383	0.512507	0.513882	0.512935
	3	Exp+	0.407721	0.388812	0.438088	0.432295	0.43398
		Erl+	0.108785	0.085701	0.177118	0.144507	0.15286
		HExp+	0.501356	0.48825	0.518882	0.518511	0.521037
		MNC+	0.579717	0.578749	0.578074	0.581243	0.580724
		MPC+	0.601037	0.603265	0.594285	0.597771	0.59575
	4	Exp+	0.182832	0.154249	0.257382	0.224562	0.236167
		Erl+	0.044882	0.021858	0.133629	0.083737	0.098757
		HExp+	0.344754	0.318419	0.398409	0.381895	0.392501
		MNC+	0.562977	0.550732	0.582746	0.580216	0.582648
		MPC+	0.689763	0.690446	0.686752	0.688904	0.688021
3.5	1	Exp+	0.423811	0.42327	0.424889	0.424514	0.424776
		Erl+	0.417655	0.414815	0.422788	0.420778	0.421482
		HExp+	0.431583	0.432163	0.429747	0.430829	0.430284
		MNC+	0.421068	0.421391	0.420514	0.420684	0.420494
		MPC+	0.421407	0.421601	0.420931	0.421118	0.420928
	2	Exp+	0.490536	0.485064	0.49754	0.496979	0.497559
		Erl+	0.406691	0.391045	0.43197	0.425741	0.425913
		HExp+	0.525948	0.523757	0.525933	0.528437	0.528634
		MNC+	0.516073	0.516825	0.513138	0.515176	0.514664
		MPC+	0.517085	0.518242	0.5136	0.515383	0.514324
	3	Exp+	0.400912	0.380702	0.434977	0.427164	0.428388
		Erl+	0.097879	0.077658	0.169225	0.131923	0.139338
		HExp+	0.494083	0.478355	0.515908	0.51404	0.516711
		MNC+	0.579501	0.57778	0.578082	0.581772	0.581219
		MPC+	0.60435	0.606787	0.596402	0.60077	0.598559
	4	Exp+	0.165834	0.137955	0.247285	0.208087	0.219285
		Erl+	0.030232	0.011885	0.122218	0.065774	0.079954
		HExp+	0.324474	0.294937	0.387858	0.365761	0.376986
		MNC+	0.55675	0.542772	0.579843	0.576077	0.578415
		MPC+	0.69132	0.691983	0.687776	0.690469	0.689503

Table 2.4: Fraction of successful rate of retrials

$S = 25$, $s = 8$, $\lambda = 5$, $\lambda_{-1} = 2$, $\beta = 3$, $\mu = 10$, $\gamma = 0.3$, $\theta = 3$.

μ	c		Exp-	Erl-	HExp-	MNC-	MPC-
10	1	Exp+	0.487758	0.487521	0.488273	0.488107	0.488283
		Erl+	0.483721	0.482082	0.486377	0.485646	0.48627
		HExp+	0.489472	0.489586	0.489164	0.489315	0.489182
		MNC+	0.485473	0.485519	0.485408	0.485407	0.485356
		MPC+	0.485418	0.485443	0.485356	0.485375	0.485338
	2	Exp+	0.567485	0.562344	0.574365	0.574354	0.575461
		Erl+	0.4517	0.431961	0.477431	0.47573	0.478222
		HExp+	0.599593	0.598368	0.600738	0.601675	0.602219
		MNC+	0.605164	0.60551	0.604363	0.604893	0.604635
		MPC+	0.606362	0.606763	0.605424	0.605744	0.605313
	3	Exp+	0.430634	0.409823	0.462658	0.458819	0.464102
		Erl+	0.174204	0.142482	0.236018	0.216877	0.230031
		HExp+	0.555182	0.544349	0.571402	0.571214	0.575242
		MNC+	0.648973	0.646531	0.652211	0.653029	0.653674
		MPC+	0.684333	0.684978	0.682823	0.683404	0.682766
	4	Exp+	0.271087	0.240744	0.331204	0.313398	0.328415
		Erl+	0.162945	0.126146	0.238481	0.212774	0.230613
		HExp+	0.438024	0.418016	0.476583	0.467996	0.479591
		MNC+	0.60289	0.590112	0.624668	0.621917	0.627027
		MPC+	0.754603	0.753086	0.75832	0.757064	0.758286
11	1	Exp+	0.488842	0.488574	0.489429	0.489239	0.489427
		Erl+	0.483631	0.481732	0.486645	0.485853	0.486487
		HExp+	0.491195	0.491337	0.490818	0.491004	0.49085
		MNC+	0.486676	0.486735	0.486593	0.48659	0.486526
		MPC+	0.486625	0.486658	0.486561	0.48657	0.486526
	2	Exp+	0.562341	0.556418	0.57013	0.570204	0.571372
		Erl+	0.421074	0.398857	0.450726	0.448365	0.451455
		HExp+	0.598419	0.596728	0.600219	0.601174	0.60187
		MNC+	0.608125	0.60853	0.60721	0.607795	0.607501
		MPC+	0.609787	0.610256	0.608703	0.609063	0.608569
	3	Exp+	0.401582	0.379403	0.436687	0.43173	0.437846
		Erl+	0.161662	0.129979	0.224599	0.20439	0.218329
		HExp+	0.533881	0.521672	0.552462	0.551917	0.556498
		MNC+	0.644886	0.642009	0.648763	0.649588	0.650331
		MPC+	0.686251	0.68694	0.684675	0.685263	0.684594
	4	Exp+	0.260157	0.229583	0.321615	0.302832	0.318618
		Erl+	0.165573	0.128363	0.241526	0.215887	0.233903
		HExp+	0.41988	0.399606	0.460019	0.450326	0.462772
		MNC+	0.585109	0.571241	0.608933	0.605719	0.611345
		MPC+	0.751069	0.74929	0.755366	0.753945	0.755349
12	1	Exp+	0.489772	0.489474	0.490422	0.490214	0.490411
		Erl+	0.483292	0.481127	0.48663	0.485814	0.486451
		HExp+	0.492764	0.492931	0.492323	0.492542	0.492371
		MNC+	0.487774	0.487849	0.487674	0.487668	0.487592
		MPC+	0.487733	0.487775	0.487656	0.487664	0.487611
	2	Exp+	0.556093	0.54935	0.564863	0.565003	0.566245
		Erl+	0.389838	0.36549	0.423312	0.42003	0.423874
		HExp+	0.596045	0.59384	0.598558	0.599533	0.600382
		MNC+	0.610457	0.610906	0.609456	0.610086	0.609765
		MPC+	0.61268	0.613214	0.611463	0.611857	0.611305
	3	Exp+	0.375521	0.352295	0.413315	0.407203	0.414154
		Erl+	0.154371	0.122685	0.217984	0.197154	0.2116
		HExp+	0.513104	0.499735	0.533841	0.532852	0.537991
		MNC+	0.639922	0.636582	0.644489	0.645305	0.646159
		MPC+	0.687357	0.688072	0.685758	0.686342	0.685658
	4	Exp+	0.25293	0.222184	0.315356	0.295874	0.312239
		Erl+	0.168034	0.13047	0.244328	0.218765	0.23692
		HExp+	0.406244	0.38585	0.44755	0.436931	0.45008
		MNC+	0.568124	0.553288	0.593846	0.590142	0.596276
		MPC+	0.746607	0.744543	0.751526	0.749934	0.751535

Table 2.5: Fraction of successful rate of retrials

S = 25, s = 8, λ =5, λ−1 = 2, μ= 10, γ = 0.3, θ = 5.

β	c		Exp-	Erl-	HExp-	MNC-	MPC-
3	1	Exp+	0.487758	0.487521	0.488273	0.488107	0.488283
		Erl+	0.483721	0.482082	0.486377	0.485646	0.48627
		HExp+	0.489472	0.489586	0.489164	0.489315	0.489182
		MNC+	0.485473	0.485519	0.485408	0.485407	0.485356
		MPC+	0.485418	0.485443	0.485356	0.485375	0.485338
	2	Exp+	0.567485	0.562344	0.574365	0.574354	0.575461
		Erl+	0.4517	0.431961	0.477431	0.47573	0.478222
		HExp+	0.599593	0.598368	0.600738	0.601675	0.602219
		MNC+	0.605164	0.60551	0.604363	0.604893	0.604635
		MPC+	0.606362	0.606763	0.605424	0.605744	0.605313
	3	Exp+	0.430634	0.409823	0.462658	0.458819	0.464102
		Erl+	0.174204	0.142482	0.236018	0.216877	0.230031
		HExp+	0.555182	0.544349	0.571402	0.571214	0.575242
		MNC+	0.648973	0.646531	0.652211	0.653029	0.653674
		MPC+	0.684333	0.684978	0.682823	0.683404	0.682766
	4	Exp+	0.271087	0.240744	0.331204	0.313398	0.328415
		Erl+	0.162945	0.126146	0.238481	0.212774	0.230613
		HExp+	0.438024	0.418016	0.476583	0.467996	0.479591
		MNC+	0.60289	0.590112	0.624668	0.621917	0.627027
		MPC+	0.754603	0.753086	0.75832	0.757064	0.758286
3.5	1	Exp+	0.472677	0.472592	0.472913	0.472824	0.472901
		Erl+	0.469953	0.46887	0.471739	0.471245	0.471668
		HExp+	0.473971	0.47408	0.473689	0.473814	0.473681
		MNC+	0.470198	0.470261	0.470087	0.470103	0.470026
		MPC+	0.470077	0.470109	0.469992	0.470023	0.469974
	2	Exp+	0.556055	0.553307	0.559422	0.559843	0.560369
		Erl+	0.475621	0.461846	0.492227	0.492089	0.493259
		HExp+	0.578784	0.578526	0.578347	0.579461	0.57952
		MNC+	0.577624	0.578413	0.576053	0.576705	0.576208
		MPC+	0.576118	0.576624	0.574915	0.57534	0.574797
	3	Exp+	0.467484	0.451333	0.490012	0.489048	0.491945
		Erl+	0.180173	0.149466	0.236103	0.220782	0.230845
		HExp+	0.563506	0.555436	0.574336	0.575379	0.577863
		MNC+	0.636986	0.636751	0.636564	0.637907	0.63769
		MPC+	0.654967	0.656134	0.65224	0.653247	0.652154
	4	Exp+	0.278936	0.249145	0.333956	0.319793	0.331985
		Erl+	0.143099	0.106822	0.215467	0.191681	0.207221
		HExp+	0.446092	0.426312	0.480926	0.475126	0.484652
		MNC+	0.626588	0.617291	0.641306	0.64049	0.643596
		MPC+	0.742802	0.742502	0.743643	0.743422	0.743619
4	1	Exp+	0.460268	0.460282	0.460314	0.460282	0.460291
		Erl+	0.458307	0.457591	0.459493	0.459179	0.45947
		HExp+	0.461266	0.461363	0.461029	0.461125	0.461004
		MNC+	0.457975	0.458049	0.457832	0.457861	0.457767
		MPC+	0.457796	0.457832	0.457697	0.457736	0.45768
	2	Exp+	0.54035	0.539088	0.541647	0.542262	0.542474
		Erl+	0.485744	0.476583	0.496101	0.496655	0.497232
		HExp+	0.557496	0.557845	0.556148	0.557302	0.557081
		MNC+	0.552466	0.553492	0.550506	0.551197	0.550575
		MPC+	0.549518	0.55007	0.548209	0.54867	0.54808
	3	Exp+	0.494754	0.483391	0.509306	0.509909	0.51134
		Erl+	0.199658	0.170729	0.248655	0.237324	0.244472
		HExp+	0.565116	0.559674	0.571544	0.57321	0.574612
		MNC+	0.618968	0.620218	0.616247	0.617807	0.617078
		MPC+	0.625527	0.627002	0.622125	0.623342	0.62199
	4	Exp+	0.297645	0.269239	0.346632	0.336057	0.34553
		Erl+	0.128148	0.092751	0.197091	0.175174	0.188695
		HExp+	0.459031	0.440262	0.48952	0.48611	0.493663
		MNC+	0.640549	0.63447	0.649547	0.649849	0.651535
		MPC+	0.725455	0.726025	0.724348	0.724763	0.724269

Table 2.6: Fraction of successful rate of retrials

$S = 25, \quad s = 8, \ \lambda = 5, \quad \lambda_{-1} = 2, \quad \beta = 4, \quad \mu = 10, \quad \gamma = 0.3.$

θ	c		Exp-	Erl-	HExp-	MNC-	MPC-
3	1	Exp+	0.460268	0.460282	0.460314	0.460282	0.460291
		Erl+	0.458307	0.457591	0.459493	0.459179	0.45947
		HExp+	0.461266	0.461363	0.461029	0.461125	0.461004
		MNC+	0.457975	0.458049	0.457832	0.457861	0.457767
		MPC+	0.457796	0.457832	0.457697	0.457736	0.45768
	2	Exp+	0.54035	0.539088	0.541647	0.542262	0.542474
		Erl+	0.485744	0.476583	0.496101	0.496655	0.497232
		HExp+	0.557496	0.557845	0.556148	0.557302	0.557081
		MNC+	0.552466	0.553492	0.550506	0.551197	0.550575
		MPC+	0.549518	0.55007	0.548209	0.54867	0.54808
	3	Exp+	0.494754	0.483391	0.509306	0.509909	0.51134
		Erl+	0.199658	0.170729	0.248655	0.237324	0.244472
		HExp+	0.565116	0.559674	0.571544	0.57321	0.574612
		MNC+	0.618968	0.620218	0.616247	0.617807	0.617078
		MPC+	0.625527	0.627002	0.622125	0.623342	0.62199
	4	Exp+	0.297645	0.269239	0.346632	0.336057	0.34553
		Erl+	0.128148	0.092751	0.197091	0.175174	0.188695
		HExp+	0.459031	0.440262	0.48952	0.48611	0.493663
		MNC+	0.640549	0.63447	0.649547	0.649849	0.651535
		MPC+	0.725455	0.726025	0.724348	0.724763	0.724269
4	1	Exp+	0.461157	0.461096	0.461336	0.46127	0.461324
		Erl+	0.458284	0.457448	0.459673	0.45932	0.459644
		HExp+	0.462867	0.462965	0.462611	0.462728	0.462609
		MNC+	0.459821	0.459901	0.459666	0.459695	0.459598
		MPC+	0.459679	0.459721	0.459576	0.459609	0.459547
	2	Exp+	0.539029	0.537359	0.540858	0.541549	0.541837
		Erl+	0.477489	0.469291	0.486663	0.48779	0.488231
		HExp+	0.559354	0.559143	0.5591	0.559961	0.560026
		MNC+	0.557211	0.557861	0.555959	0.556458	0.556066
		MPC+	0.557246	0.557696	0.556212	0.556551	0.556076
	3	Exp+	0.480343	0.470249	0.492946	0.494478	0.495609
		Erl+	0.171678	0.149662	0.209045	0.201371	0.206869
		HExp+	0.557082	0.551373	0.563984	0.565882	0.567393
		MNC+	0.621728	0.622113	0.620532	0.621782	0.62138
		MPC+	0.636838	0.637962	0.634362	0.63517	0.634131
	4	Exp+	0.260879	0.238188	0.299249	0.292482	0.300122
		Erl+	0.092667	0.066625	0.143854	0.128228	0.138604
		HExp+	0.427444	0.410849	0.453384	0.452146	0.458931
		MNC+	0.627813	0.621971	0.636051	0.637086	0.638625
		MPC+	0.729295	0.729529	0.728875	0.729104	0.728841
5	1	Exp+	0.461845	0.461714	0.462146	0.462053	0.462147
		Erl+	0.458168	0.457238	0.459699	0.459338	0.459683
		HExp+	0.46427	0.464355	0.464032	0.464154	0.464052
		MNC+	0.461408	0.46148	0.461276	0.461295	0.461211
		MPC+	0.461381	0.461422	0.46128	0.461309	0.461249
	2	Exp+	0.537068	0.535154	0.539187	0.539985	0.540314
		Erl+	0.46992	0.462579	0.478067	0.479551	0.47987
		HExp+	0.559939	0.559327	0.560427	0.561142	0.561404
		MNC+	0.560427	0.560765	0.559726	0.56012	0.559909
		MPC+	0.563307	0.563653	0.562533	0.562773	0.56241
	3	Exp+	0.467087	0.458165	0.478002	0.480105	0.480964
		Erl+	0.151006	0.133994	0.180012	0.174624	0.17885
		HExp+	0.548487	0.542772	0.555405	0.557556	0.559081
		MNC+	0.622225	0.622012	0.622003	0.623156	0.622977
		MPC+	0.645099	0.645926	0.643338	0.643881	0.643102
	4	Exp+	0.232153	0.213881	0.262668	0.258248	0.264398
		Erl+	0.068331	0.048922	0.106915	0.09541	0.1034
		HExp+	0.399992	0.385327	0.422291	0.422415	0.428475
		MNC+	0.614959	0.609477	0.622411	0.623955	0.625316
		MPC+	0.730643	0.730622	0.730696	0.730842	0.730751

Table 2.7: Fraction of successful rate of retrials

$S = 25$, $s = 8$, $\lambda = 5$, $\lambda - 1 = 2$, $\beta = 4$, $\mu = 10$, $\theta = 5$.

γ	c		Exp-	Erl-	HExp-	MNC-	MPC-
0.2	1	Exp+	0.436775	0.436613	0.437144	0.437026	0.43714
		Erl+	0.432755	0.431715	0.434517	0.434054	0.43444
		HExp+	0.440201	0.440329	0.439838	0.440022	0.439865
		MNC+	0.436056	0.436134	0.435914	0.435929	0.435841
		MPC+	0.436155	0.436204	0.43603	0.43607	0.435998
	2	Exp+	0.50383	0.502276	0.505483	0.506271	0.506548
		Erl+	0.449861	0.444245	0.455989	0.457368	0.457559
		HExp+	0.528796	0.528434	0.528807	0.529628	0.529763
		MNC+	0.522528	0.522907	0.521755	0.522164	0.521958
		MPC+	0.525345	0.525725	0.524472	0.524758	0.524366
	3	Exp+	0.465914	0.459122	0.473806	0.476083	0.476425
		Erl+	0.163422	0.149332	0.187381	0.183416	0.186025
		HExp+	0.53225	0.52765	0.537453	0.539676	0.540736
		MNC+	0.58998	0.590247	0.588994	0.590204	0.589885
		MPC+	0.607385	0.6084	0.605165	0.605882	0.604947
	4	Exp+	0.238128	0.22233	0.264437	0.261036	0.265509
		Erl+	0.050832	0.035261	0.084827	0.073229	0.079946
		HExp+	0.394155	0.380758	0.413989	0.414712	0.41961
		MNC+	0.606515	0.602511	0.611597	0.613374	0.614088
		MPC+	0.702292	0.702796	0.701175	0.701698	0.701173
0.3	1	Exp+	0.461845	0.461714	0.462146	0.462053	0.462147
		Erl+	0.458168	0.457238	0.459699	0.459338	0.459683
		HExp+	0.46427	0.464355	0.464032	0.464154	0.464052
		MNC+	0.461408	0.46148	0.461276	0.461295	0.461211
		MPC+	0.461381	0.461422	0.46128	0.461309	0.461249
	2	Exp+	0.537068	0.535154	0.539187	0.539985	0.540314
		Erl+	0.46992	0.462579	0.478067	0.479551	0.47987
		HExp+	0.559939	0.559327	0.560427	0.561142	0.561404
		MNC+	0.560427	0.560765	0.559726	0.56012	0.559909
		MPC+	0.563307	0.563653	0.562533	0.562773	0.56241
	3	Exp+	0.467087	0.458165	0.478002	0.480105	0.480964
		Erl+	0.151006	0.133994	0.180012	0.174624	0.17885
		HExp+	0.548487	0.542772	0.555405	0.557556	0.559081
		MNC+	0.622225	0.622012	0.622003	0.623156	0.622977
		MPC+	0.645099	0.645926	0.643338	0.643881	0.643102
	4	Exp+	0.232153	0.213881	0.262668	0.258248	0.264398
		Erl+	0.068331	0.048922	0.106915	0.09541	0.1034
		HExp+	0.399992	0.385327	0.422291	0.422415	0.428475
		MNC+	0.614959	0.609477	0.622411	0.623955	0.625316
		MPC+	0.730643	0.730622	0.730696	0.730842	0.730751
0.4	1	Exp+	0.482632	0.482491	0.482953	0.482855	0.482957
		Erl+	0.479071	0.478156	0.480554	0.480229	0.480566
		HExp+	0.484591	0.484645	0.484435	0.484522	0.484461
		MNC+	0.482488	0.482543	0.482395	0.482403	0.482337
		MPC+	0.482433	0.482464	0.482367	0.482379	0.482335
	2	Exp+	0.563523	0.561085	0.56634	0.567141	0.567569
		Erl+	0.481098	0.471785	0.491574	0.493174	0.493709
		HExp+	0.586571	0.58564	0.587632	0.588248	0.588676
		MNC+	0.592583	0.592806	0.5921	0.592447	0.592283
		MPC+	0.596068	0.596351	0.595458	0.595628	0.595328
	3	Exp+	0.462938	0.451876	0.47695	0.478819	0.480354
		Erl+	0.147514	0.127804	0.180546	0.174434	0.18006
		HExp+	0.560939	0.554105	0.569605	0.571663	0.573707
		MNC+	0.647334	0.646497	0.648104	0.64918	0.649224
		MPC+	0.676958	0.677544	0.675758	0.676102	0.675534
	4	Exp+	0.233412	0.212984	0.26725	0.262298	0.269899
		Erl+	0.087464	0.064646	0.129614	0.118668	0.127738
		HExp+	0.407888	0.392172	0.432232	0.431885	0.438997
		MNC+	0.619003	0.612034	0.628852	0.630148	0.632241
		MPC+	0.753679	0.753098	0.754944	0.754722	0.755106

Table 2.8: Blocking Probability

S = 25, s = 8, λ_{-1} = 2, β = 4, μ = 10, γ = 0.3, θ = 5.

λ_1	c		Exp-	Erl-	HExp-	MNC-	MPC-
4.5	1	Exp+	0.494634	0.494672	0.494592	0.494559	0.494531
		Erl+	0.493193	0.493182	0.493252	0.493206	0.49321
		HExp+	0.501079	0.501252	0.500602	0.500774	0.500556
		MNC+	0.496617	0.496756	0.496376	0.496359	0.496213
		MPC+	0.496934	0.497025	0.496717	0.496764	0.496641
	2	Exp+	0.30636	0.306303	0.306431	0.306469	0.306486
		Erl+	0.303884	0.303871	0.303896	0.303912	0.303913
		HExp+	0.314138	0.314047	0.314265	0.314317	0.314371
		MNC+	0.309648	0.309568	0.309786	0.309801	0.309863
		MPC+	0.309572	0.309515	0.3097	0.309682	0.309754
	3	Exp+	0.232376	0.232366	0.232386	0.232401	0.232404
		Erl+	0.229905	0.229904	0.229904	0.22991	0.229911
		HExp+	0.239222	0.239184	0.239267	0.239302	0.239318
		MNC+	0.236747	0.236695	0.236816	0.23685	0.236879
		MPC+	0.236997	0.236937	0.237107	0.237114	0.237179
	4	Exp+	0.176361	0.176354	0.176368	0.176377	0.176381
		Erl+	0.177323	0.177319	0.177325	0.177331	0.177333
		HExp+	0.177598	0.177575	0.17763	0.177647	0.177662
		MNC+	0.175403	0.175356	0.175464	0.175495	0.17552
		MPC+	0.176122	0.176027	0.176291	0.176307	0.176399
5	1	Exp+	0.494296	0.494304	0.494308	0.494274	0.494267
		Erl+	0.493185	0.493136	0.493318	0.493264	0.49329
		HExp+	0.499859	0.500013	0.499431	0.499585	0.499371
		MNC+	0.495688	0.495788	0.495518	0.495503	0.495387
		MPC+	0.495919	0.495987	0.495759	0.495791	0.495695
	2	Exp+	0.307628	0.307546	0.30774	0.307787	0.307815
		Erl+	0.305195	0.305171	0.305222	0.305246	0.305249
		HExp+	0.315348	0.315239	0.315512	0.315565	0.315641
		MNC+	0.310565	0.310461	0.310754	0.310764	0.310851
		MPC+	0.310222	0.31015	0.310392	0.310361	0.310455
	3	Exp+	0.233717	0.2337	0.233734	0.233754	0.233759
		Erl+	0.231004	0.231003	0.231003	0.231012	0.231013
		HExp+	0.240939	0.240884	0.241009	0.241052	0.241078
		MNC+	0.238167	0.238098	0.238263	0.238302	0.238343
		MPC+	0.238133	0.23806	0.238275	0.238276	0.238358
	4	Exp+	0.179362	0.179352	0.179373	0.179384	0.17939
		Erl+	0.180171	0.180168	0.180174	0.180181	0.180184
		HExp+	0.180859	0.180825	0.180909	0.180929	0.180951
		MNC+	0.178608	0.178547	0.178689	0.178726	0.178759
		MPC+	0.179216	0.179105	0.17942	0.179431	0.179543
5.5	1	Exp+	0.494197	0.49418	0.494255	0.494219	0.494235
		Erl+	0.493416	0.493331	0.493614	0.493553	0.493608
		HExp+	0.498712	0.498836	0.498366	0.498488	0.4983
		MNC+	0.495003	0.495066	0.494898	0.494884	0.494805
		MPC+	0.495157	0.495202	0.495053	0.495072	0.495005
	2	Exp+	0.309193	0.309081	0.309355	0.309409	0.309454
		Erl+	0.306852	0.306811	0.306904	0.306936	0.306943
		HExp+	0.316728	0.316604	0.316926	0.316976	0.317076
		MNC+	0.311722	0.311595	0.311965	0.311966	0.312081
		MPC+	0.311128	0.311042	0.311339	0.311294	0.311411
	3	Exp+	0.235368	0.235342	0.235397	0.235422	0.235431
		Erl+	0.232429	0.232427	0.23243	0.232439	0.232441
		HExp+	0.242926	0.242851	0.243027	0.243077	0.243117
		MNC+	0.239837	0.23975	0.239966	0.240008	0.240064
		MPC+	0.23951	0.239423	0.239687	0.239679	0.239781
	4	Exp+	0.182529	0.182516	0.182547	0.182559	0.182568
		Erl+	0.183149	0.183145	0.183154	0.183162	0.183166
		HExp+	0.184335	0.184288	0.184408	0.184432	0.184463
		MNC+	0.181982	0.181907	0.182089	0.182131	0.182175
		MPC+	0.18245	0.182324	0.182693	0.182695	0.182829

Table 2.9: Blocking Probability

$S = 25$, $s = 8$, $\lambda_1 = 5$, $\beta = 4$, $\mu = 10$, $\gamma = 0.3$, $\theta = 5$.

λ_{-1}	c		Exp-	Erl-	HExp-	MNC-	MPC-
2	1	Exp+	0.494296	0.494304	0.494308	0.494274	0.494267
		Erl+	0.493185	0.493136	0.493318	0.493264	0.49329
		HExp+	0.499859	0.500013	0.499431	0.499585	0.499371
		MNC+	0.495688	0.495788	0.495518	0.495503	0.495387
		MPC+	0.495919	0.495987	0.495759	0.495791	0.495695
	2	Exp+	0.307628	0.307546	0.30774	0.307787	0.307815
		Erl+	0.305195	0.305171	0.305222	0.305246	0.305249
		HExp+	0.315348	0.315239	0.315512	0.315565	0.315641
		MNC+	0.310565	0.310461	0.310754	0.310764	0.310851
		MPC+	0.310222	0.31015	0.310392	0.310361	0.310455
	3	Exp+	0.233717	0.2337	0.233734	0.233754	0.233759
		Erl+	0.231004	0.231003	0.231003	0.231012	0.231013
		HExp+	0.240939	0.240884	0.241009	0.241052	0.241078
		MNC+	0.238167	0.238098	0.238263	0.238302	0.238343
		MPC+	0.238133	0.23806	0.238275	0.238276	0.238358
	4	Exp+	0.179362	0.179352	0.179373	0.179384	0.17939
		Erl+	0.180171	0.180168	0.180174	0.180181	0.180184
		HExp+	0.180859	0.180825	0.180909	0.180929	0.180951
		MNC+	0.178608	0.178547	0.178689	0.178726	0.178759
		MPC+	0.179216	0.179105	0.17942	0.179431	0.179543
2.5	1	Exp+	0.494332	0.494349	0.494335	0.494297	0.494286
		Erl+	0.493151	0.493112	0.493284	0.493219	0.49324
		HExp+	0.500498	0.50068	0.499976	0.500184	0.499952
		MNC+	0.496095	0.496231	0.495859	0.495853	0.495713
		MPC+	0.496349	0.496438	0.496136	0.496186	0.496069
	2	Exp+	0.307528	0.307436	0.307663	0.307703	0.307734
		Erl+	0.305168	0.30514	0.3052	0.305225	0.305228
		HExp+	0.315147	0.315018	0.315354	0.315396	0.315481
		MNC+	0.310359	0.310242	0.310587	0.310581	0.310676
		MPC+	0.309964	0.309882	0.310168	0.31012	0.310226
	3	Exp+	0.23369	0.233669	0.233712	0.233734	0.23374
		Erl+	0.230997	0.230994	0.230996	0.231007	0.231009
		HExp+	0.24085	0.240785	0.240938	0.240979	0.241009
		MNC+	0.23805	0.237971	0.238171	0.238203	0.23825
		MPC+	0.237914	0.23783	0.238091	0.238077	0.238171
	4	Exp+	0.179343	0.17933	0.179357	0.179369	0.179376
		Erl+	0.180162	0.180156	0.180166	0.180175	0.180178
		HExp+	0.180794	0.180754	0.180856	0.180874	0.180899
		MNC+	0.178508	0.178438	0.178611	0.178642	0.17868
		MPC+	0.178925	0.178798	0.179179	0.179167	0.179295
3	1	Exp+	0.494368	0.494394	0.494362	0.494322	0.494307
		Erl+	0.493133	0.493105	0.49326	0.493189	0.493205
		HExp+	0.500969	0.501163	0.500386	0.500636	0.500402
		MNC+	0.496438	0.496605	0.496143	0.496149	0.495992
		MPC+	0.496705	0.496812	0.496447	0.496514	0.496383
	2	Exp+	0.307443	0.307343	0.307598	0.307629	0.307662
		Erl+	0.305143	0.305111	0.305181	0.305205	0.305209
		HExp+	0.314974	0.314829	0.315218	0.315248	0.315339
		MNC+	0.310184	0.310058	0.310445	0.310423	0.310524
		MPC+	0.309742	0.309652	0.309975	0.309911	0.310025
	3	Exp+	0.233666	0.233641	0.233693	0.233715	0.233723
		Erl+	0.230989	0.230983	0.23099	0.231002	0.231005
		HExp+	0.240772	0.240699	0.240876	0.240914	0.240947
		MNC+	0.237948	0.23786	0.23809	0.238114	0.238166
		MPC+	0.237722	0.237628	0.237929	0.2379	0.238003
	4	Exp+	0.179325	0.17931	0.179343	0.179355	0.179362
		Erl+	0.180153	0.180144	0.18016	0.180168	0.180172
		HExp+	0.180736	0.180692	0.180809	0.180824	0.180852
		MNC+	0.178421	0.178343	0.178543	0.178566	0.178609
		MPC+	0.178671	0.178531	0.178969	0.178934	0.179074

Table 2.10: Blocking Probability

$S = 25,\quad s = 8,\quad \lambda_1 = 5,\quad \lambda_{-1} = 3,\quad \mu = 10,\quad \gamma = 0.3,\quad \theta = 5.$

β	c		Exp-	Erl-	HExp-	MNC-	MPC-
4	1	Exp+	0.494368	0.494394	0.494362	0.494322	0.494307
		Erl+	0.493133	0.493105	0.49326	0.493189	0.493205
		HExp+	0.500969	0.501163	0.500386	0.500636	0.500402
		MNC+	0.496438	0.496605	0.496143	0.496149	0.495992
		MPC+	0.496705	0.496812	0.496447	0.496514	0.496383
	2	Exp+	0.307443	0.307343	0.307598	0.307629	0.307662
		Erl+	0.305143	0.305111	0.305181	0.305205	0.305209
		HExp+	0.314974	0.314829	0.315218	0.315248	0.315339
		MNC+	0.310184	0.310058	0.310445	0.310423	0.310524
		MPC+	0.309742	0.309652	0.309975	0.309911	0.310025
	3	Exp+	0.233666	0.233641	0.233693	0.233715	0.233723
		Erl+	0.230989	0.230983	0.23099	0.231002	0.231005
		HExp+	0.240772	0.240699	0.240876	0.240914	0.240947
		MNC+	0.237948	0.23786	0.23809	0.238114	0.238166
		MPC+	0.237722	0.237628	0.237929	0.2379	0.238003
	4	Exp+	0.179325	0.17931	0.179343	0.179355	0.179362
		Erl+	0.180153	0.180144	0.18016	0.180168	0.180172
		HExp+	0.180736	0.180692	0.180809	0.180824	0.180852
		MNC+	0.178421	0.178343	0.178543	0.178566	0.178609
		MPC+	0.178671	0.178531	0.178969	0.178934	0.179074
5	1	Exp+	0.476097	0.476283	0.475753	0.475768	0.475634
		Erl+	0.47393	0.474039	0.473774	0.473739	0.473691
		HExp+	0.483688	0.484003	0.482801	0.483139	0.482759
		MNC+	0.479561	0.479882	0.478931	0.479003	0.47868
		MPC+	0.479802	0.480003	0.479284	0.47944	0.479182
	2	Exp+	0.286944	0.286894	0.287021	0.287036	0.287053
		Erl+	0.2843	0.284283	0.28432	0.284331	0.284333
		HExp+	0.295113	0.295046	0.295202	0.29524	0.295279
		MNC+	0.290553	0.290521	0.290622	0.290615	0.290644
		MPC+	0.290527	0.290511	0.29057	0.29056	0.290584
	3	Exp+	0.215053	0.215038	0.215068	0.215081	0.215084
		Erl+	0.212316	0.212313	0.212316	0.212322	0.212322
		HExp+	0.22224	0.222195	0.222299	0.222328	0.222346
		MNC+	0.219619	0.219571	0.219694	0.219711	0.219739
		MPC+	0.219651	0.219603	0.21975	0.219742	0.219796
	4	Exp+	0.164032	0.164024	0.164041	0.164048	0.164051
		Erl+	0.165064	0.165061	0.165067	0.16507	0.165072
		HExp+	0.164801	0.164772	0.164842	0.164857	0.164871
		MNC+	0.162822	0.162765	0.162907	0.162928	0.162956
		MPC+	0.162859	0.162751	0.163085	0.163061	0.163167
6	1	Exp+	0.46356	0.463856	0.462986	0.463037	0.462822
		Erl+	0.460758	0.460961	0.460408	0.460397	0.460305
		HExp+	0.471829	0.472226	0.470733	0.471131	0.47065
		MNC+	0.467981	0.468407	0.46712	0.467237	0.4668
		MPC+	0.468205	0.46847	0.467508	0.467726	0.467379
	2	Exp+	0.272511	0.272496	0.272536	0.27254	0.272545
		Erl+	0.269584	0.269578	0.269592	0.269595	0.269596
		HExp+	0.28119	0.281178	0.281173	0.281216	0.281221
		MNC+	0.276765	0.276801	0.276701	0.276703	0.276683
		MPC+	0.277042	0.277078	0.276951	0.276979	0.27694
	3	Exp+	0.201652	0.201644	0.201661	0.201669	0.201671
		Erl+	0.198761	0.19876	0.198761	0.198764	0.198764
		HExp+	0.209108	0.209079	0.209141	0.209163	0.209173
		MNC+	0.206528	0.206506	0.206559	0.206571	0.206584
		MPC+	0.206772	0.206755	0.206801	0.206807	0.206827
	4	Exp+	0.152823	0.152818	0.152829	0.152833	0.152834
		Erl+	0.15382	0.153818	0.153821	0.153822	0.153823
		HExp+	0.153492	0.153472	0.15352	0.153532	0.15354
		MNC+	0.151595	0.15155	0.151662	0.151679	0.1517
		MPC+	0.151514	0.151426	0.1517	0.15168	0.151767

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Table 2.11: Blocking Probability

$S = 25$, $s = 8$, $\lambda_1 = 5$, $\lambda_{-1} = 3$, $\beta = 4$, $\gamma = 0.3$, $\theta = 5$.

μ	c		Exp-	Erl-	HExp-	MNC-	MPC-
10	1	Exp+	0.494368	0.494394	0.494362	0.494322	0.494307
		Erl+	0.493133	0.493105	0.49326	0.493189	0.493205
		HExp+	0.500969	0.501163	0.500386	0.500636	0.500402
		MNC+	0.496438	0.496605	0.496143	0.496149	0.495992
		MPC+	0.496705	0.496812	0.496447	0.496514	0.496383
	2	Exp+	0.307443	0.307343	0.307598	0.307629	0.307662
		Erl+	0.305143	0.305111	0.305181	0.305205	0.305209
		HExp+	0.314974	0.314829	0.315218	0.315248	0.315339
		MNC+	0.310184	0.310058	0.310445	0.310423	0.310524
		MPC+	0.309742	0.309652	0.309975	0.309911	0.310025
	3	Exp+	0.233666	0.233641	0.233693	0.233715	0.233723
		Erl+	0.230989	0.230983	0.23099	0.231002	0.231005
		HExp+	0.240772	0.240699	0.240876	0.240914	0.240947
		MNC+	0.237948	0.23786	0.23809	0.238114	0.238166
		MPC+	0.237722	0.237628	0.237929	0.2379	0.238003
	4	Exp+	0.179325	0.17931	0.179343	0.179355	0.179362
		Erl+	0.180153	0.180144	0.18016	0.180168	0.180172
		HExp+	0.180736	0.180692	0.180809	0.180824	0.180852
		MNC+	0.178421	0.178343	0.178543	0.178566	0.178609
		MPC+	0.178671	0.178531	0.178969	0.178934	0.179074
11	1	Exp+	0.496514	0.496496	0.496598	0.496547	0.496564
		Erl+	0.495456	0.495399	0.495634	0.495566	0.495592
		HExp+	0.503266	0.503418	0.502785	0.503009	0.502831
		MNC+	0.498327	0.498455	0.498123	0.498107	0.497998
		MPC+	0.498646	0.498727	0.498458	0.498501	0.498406
	2	Exp+	0.309519	0.309427	0.309656	0.309692	0.30972
		Erl+	0.307358	0.307333	0.307384	0.307408	0.30741
		HExp+	0.31683	0.316681	0.317077	0.317111	0.317197
		MNC+	0.3121	0.31196	0.312376	0.312361	0.312466
		MPC+	0.311647	0.311546	0.311906	0.311839	0.311965
	3	Exp+	0.235584	0.235562	0.235604	0.235626	0.235632
		Erl+	0.233222	0.233217	0.233222	0.233235	0.233237
		HExp+	0.242184	0.24212	0.242272	0.242309	0.242336
		MNC+	0.239558	0.239472	0.239693	0.239719	0.239767
		MPC+	0.239372	0.239275	0.239582	0.239555	0.23966
	4	Exp+	0.179234	0.17922	0.17925	0.179261	0.179268
		Erl+	0.180191	0.180182	0.180197	0.180206	0.18021
		HExp+	0.18048	0.180442	0.180543	0.180556	0.180581
		MNC+	0.178173	0.178103	0.178281	0.178304	0.178342
		MPC+	0.178521	0.178387	0.178802	0.178774	0.178906
12	1	Exp+	0.498395	0.498339	0.498552	0.498495	0.498535
		Erl+	0.497488	0.497411	0.497696	0.497638	0.497667
		HExp+	0.505282	0.505392	0.5049	0.505097	0.504971
		MNC+	0.499973	0.500064	0.499853	0.499818	0.499751
		MPC+	0.500342	0.5004	0.500218	0.500239	0.500176
	2	Exp+	0.311299	0.311213	0.31142	0.311459	0.311483
		Erl+	0.309272	0.309252	0.30929	0.309312	0.309314
		HExp+	0.318412	0.318263	0.318654	0.318692	0.318772
		MNC+	0.313729	0.31358	0.314015	0.314007	0.314114
		MPC+	0.313274	0.313163	0.313552	0.313484	0.313619
	3	Exp+	0.237245	0.237226	0.237262	0.237282	0.237288
		Erl+	0.235169	0.235163	0.235169	0.235181	0.235183
		HExp+	0.243411	0.243355	0.243486	0.243521	0.243545
		MNC+	0.240925	0.240842	0.241053	0.241081	0.241126
		MPC+	0.240778	0.240678	0.240991	0.240967	0.241073
	4	Exp+	0.179158	0.179146	0.179173	0.179184	0.179191
		Erl+	0.180225	0.180217	0.180231	0.18024	0.180244
		HExp+	0.180265	0.180231	0.180321	0.180333	0.180356
		MNC+	0.177954	0.17789	0.178051	0.178073	0.178107
		MPC+	0.17839	0.17826	0.178656	0.178633	0.178758

Table 2.12: Blocking Probability

$S = 25$, $s = 8$, $\lambda_1 = 5$, $\lambda_{-1} = 2$, $\beta = 6$, $\mu = 10$, $\gamma = 0.3$.

θ	c		Exp-	Erl-	HExp-	MNC-	MPC-
5	1	Exp+	0.46356	0.463856	0.462986	0.463037	0.462822
		Erl+	0.460758	0.460961	0.460408	0.460397	0.460305
		HExp+	0.471829	0.472226	0.470733	0.471131	0.47065
		MNC+	0.467981	0.468407	0.46712	0.467237	0.4668
		MPC+	0.468205	0.46847	0.467508	0.467726	0.467379
	2	Exp+	0.272511	0.272496	0.272536	0.27254	0.272545
		Erl+	0.269584	0.269578	0.269592	0.269595	0.269596
		HExp+	0.28119	0.281178	0.281173	0.281216	0.281221
		MNC+	0.276765	0.276801	0.276701	0.276703	0.276683
		MPC+	0.277042	0.277078	0.276951	0.276979	0.27694
	3	Exp+	0.201652	0.201644	0.201661	0.201669	0.201671
		Erl+	0.198761	0.19876	0.198761	0.198764	0.198764
		HExp+	0.209108	0.209079	0.209141	0.209163	0.209173
		MNC+	0.206528	0.206506	0.206559	0.206571	0.206584
		MPC+	0.206772	0.206755	0.206801	0.206807	0.206827
	4	Exp+	0.152823	0.152818	0.152829	0.152833	0.152834
		Erl+	0.15382	0.153818	0.153821	0.153822	0.153823
		HExp+	0.153492	0.153472	0.15352	0.153532	0.15354
		MNC+	0.151595	0.15155	0.151662	0.151679	0.1517
		MPC+	0.151514	0.151426	0.1517	0.15168	0.151767
6	1	Exp+	0.463344	0.463638	0.462792	0.462817	0.462607
		Erl+	0.460607	0.460814	0.460268	0.460235	0.460146
		HExp+	0.471739	0.472124	0.470689	0.471052	0.47059
		MNC+	0.467632	0.468055	0.466793	0.466878	0.466449
		MPC+	0.467945	0.468211	0.467259	0.467459	0.467115
	2	Exp+	0.272539	0.272522	0.272567	0.272573	0.272579
		Erl+	0.26959	0.269584	0.269598	0.269602	0.269603
		HExp+	0.281297	0.281273	0.281311	0.281345	0.281358
		MNC+	0.276854	0.276876	0.276824	0.276816	0.276807
		MPC+	0.277245	0.277267	0.277194	0.277206	0.277184
	3	Exp+	0.201664	0.201655	0.201672	0.201681	0.201683
		Erl+	0.198763	0.198762	0.198764	0.198766	0.198766
		HExp+	0.209159	0.209129	0.209193	0.209218	0.209229
		MNC+	0.206596	0.206571	0.206632	0.206646	0.206661
		MPC+	0.206969	0.206944	0.207014	0.207018	0.207046
	4	Exp+	0.152828	0.152823	0.152833	0.152838	0.15284
		Erl+	0.153821	0.15382	0.153822	0.153824	0.153824
		HExp+	0.153516	0.153496	0.153541	0.153555	0.153563
		MNC+	0.151638	0.151594	0.151698	0.151719	0.151739
		MPC+	0.151684	0.151597	0.151852	0.151847	0.15193
7	1	Exp+	0.463172	0.46346	0.462647	0.462646	0.462443
		Erl+	0.460481	0.460688	0.460156	0.460104	0.460016
		HExp+	0.471689	0.472061	0.470691	0.47102	0.470578
		MNC+	0.467373	0.467788	0.466569	0.466622	0.466206
		MPC+	0.467773	0.468035	0.467107	0.467287	0.46695
	2	Exp+	0.272565	0.272546	0.272595	0.272603	0.272609
		Erl+	0.269596	0.26959	0.269603	0.269608	0.269609
		HExp+	0.281393	0.281361	0.281429	0.281459	0.281478
		MNC+	0.276942	0.276952	0.276936	0.276924	0.276923
		MPC+	0.277439	0.277449	0.27742	0.27742	0.277413
	3	Exp+	0.201674	0.201665	0.201682	0.201691	0.201693
		Erl+	0.198765	0.198763	0.198766	0.198768	0.198768
		HExp+	0.209203	0.209172	0.209238	0.209265	0.209277
		MNC+	0.206658	0.20663	0.206696	0.206713	0.206729
		MPC+	0.207149	0.207118	0.207205	0.207209	0.207243
	4	Exp+	0.152832	0.152827	0.152837	0.152842	0.152844
		Erl+	0.153822	0.153821	0.153823	0.153825	0.153825
		HExp+	0.153536	0.153516	0.153559	0.153574	0.153582
		MNC+	0.151673	0.151631	0.151727	0.151751	0.15177
		MPC+	0.151826	0.151742	0.151979	0.151985	0.152066

Table 2.13: Blocking Probability

$S = 25, \quad s = 8, \quad \lambda_1 = 5, \quad \lambda_{-1} = 3, \quad \beta = 6, \quad \mu = 10, \quad \theta = 5.$

γ	c		Exp-	Erl-	HExp-	MNC-	MPC-
0.2	1	Exp+	0.441381	0.441649	0.440882	0.440909	0.440717
		Erl+	0.4388	0.438979	0.438513	0.438488	0.438409
		HExp+	0.450281	0.450677	0.44918	0.449581	0.449105
		MNC+	0.445723	0.446143	0.444908	0.444992	0.444569
		MPC+	0.446044	0.446308	0.445371	0.445568	0.445229
	2	Exp+	0.253259	0.253227	0.253312	0.253319	0.25333
		Erl+	0.250641	0.250629	0.250656	0.250662	0.250664
		HExp+	0.261439	0.261401	0.261482	0.261512	0.261535
		MNC+	0.256996	0.256998	0.257006	0.256995	0.257001
		MPC+	0.257183	0.257192	0.257166	0.257166	0.257161
	3	Exp+	0.185234	0.185223	0.185247	0.185256	0.185258
		Erl+	0.1826	0.182598	0.182601	0.182603	0.182603
		HExp+	0.191957	0.19192	0.192005	0.192027	0.192041
		MNC+	0.189593	0.189556	0.189651	0.189662	0.189683
		MPC+	0.189556	0.189519	0.189635	0.189626	0.189668
	4	Exp+	0.139303	0.139297	0.139309	0.139312	0.139314
		Erl+	0.140447	0.140446	0.140448	0.140449	0.14045
		HExp+	0.139381	0.139359	0.13941	0.139423	0.139431
		MNC+	0.137767	0.137718	0.137839	0.137856	0.137878
		MPC+	0.137271	0.137173	0.137478	0.137453	0.137549
0.25	1	Exp+	0.453116	0.453404	0.452567	0.452609	0.4524
		Erl+	0.450377	0.450572	0.45005	0.450032	0.449945
		HExp+	0.461814	0.462217	0.460697	0.461105	0.460618
		MNC+	0.457564	0.457993	0.456712	0.456816	0.456379
		MPC+	0.457835	0.458104	0.457139	0.457351	0.457003
	2	Exp+	0.263277	0.263255	0.263314	0.26332	0.263327
		Erl+	0.260481	0.260473	0.260492	0.260497	0.260498
		HExp+	0.271848	0.271824	0.271857	0.271894	0.271907
		MNC+	0.267322	0.267344	0.26729	0.267286	0.267277
		MPC+	0.267578	0.267603	0.267518	0.267535	0.26751
	3	Exp+	0.193741	0.193731	0.193751	0.193759	0.193761
		Erl+	0.190968	0.190966	0.190969	0.19097	0.190971
		HExp+	0.200929	0.200897	0.200969	0.200991	0.201004
		MNC+	0.198385	0.198356	0.198428	0.19844	0.198457
		MPC+	0.198511	0.198485	0.198563	0.198562	0.198592
	4	Exp+	0.146304	0.146299	0.14631	0.146314	0.146316
		Erl+	0.147379	0.147378	0.14738	0.147381	0.147381
		HExp+	0.146712	0.14669	0.14674	0.146753	0.146761
		MNC+	0.144921	0.144874	0.144991	0.145008	0.145029
		MPC+	0.144647	0.144554	0.144842	0.14482	0.144911
0.3	1	Exp+	0.46356	0.463856	0.462986	0.463037	0.462822
		Erl+	0.460758	0.460961	0.460408	0.460397	0.460305
		HExp+	0.471829	0.472226	0.470733	0.471131	0.47065
		MNC+	0.467981	0.468407	0.46712	0.467237	0.4668
		MPC+	0.468205	0.46847	0.467508	0.467726	0.467379
	2	Exp+	0.272511	0.272496	0.272536	0.27254	0.272545
		Erl+	0.269584	0.269578	0.269592	0.269595	0.269596
		HExp+	0.28119	0.281178	0.281173	0.281216	0.281221
		MNC+	0.276765	0.276801	0.276701	0.276703	0.276683
		MPC+	0.277042	0.277078	0.276951	0.276979	0.27694
	3	Exp+	0.201652	0.201644	0.201661	0.201669	0.201671
		Erl+	0.198761	0.19876	0.198761	0.198764	0.198764
		HExp+	0.209108	0.209079	0.209141	0.209163	0.209173
		MNC+	0.206528	0.206506	0.206559	0.206571	0.206584
		MPC+	0.206772	0.206755	0.206801	0.206807	0.206827
	4	Exp+	0.152823	0.152818	0.152829	0.152833	0.152834
		Erl+	0.15382	0.153818	0.153821	0.153822	0.153823
		HExp+	0.153492	0.153472	0.15352	0.153532	0.15354
		MNC+	0.151595	0.15155	0.151662	0.151679	0.1517
		MPC+	0.151514	0.151426	0.1517	0.15168	0.151767

Table 2.14: Mean number of Idle Servers

$S = 25$, $s = 8$, $\lambda_{-1} = 2$, $\beta = 4$, $\mu = 10$, $\gamma = 0.3$, $\theta = 5$.

λ_1	c		Exp-	Erl-	HExp-	MNC-	MPC-
4.5	1	Exp+	0.506841	0.506744	0.507007	0.507023	0.507097
		Erl+	0.508277	0.508228	0.508337	0.508372	0.508395
		HExp+	0.50086	0.500659	0.5014	0.501222	0.501478
		MNC+	0.504654	0.504479	0.50498	0.50498	0.505171
		MPC+	0.50439	0.504277	0.50467	0.504602	0.504758
	2	Exp+	1.130471	1.130619	1.130283	1.130185	1.130139
		Erl+	1.132947	1.132985	1.132913	1.132865	1.132861
		HExp+	1.118357	1.118562	1.118055	1.117958	1.117836
		MNC+	1.127076	1.127328	1.126644	1.126599	1.126411
		MPC+	1.12813	1.128324	1.127695	1.127761	1.127522
	3	Exp+	1.75473	1.754793	1.754671	1.754593	1.75457
		Erl+	1.753625	1.753641	1.753622	1.753581	1.753572
		HExp+	1.749937	1.750093	1.749741	1.749618	1.749545
		MNC+	1.756176	1.756476	1.755758	1.755611	1.755446
		MPC+	1.762416	1.762811	1.761649	1.761669	1.761252
	4	Exp+	2.267027	2.267072	2.266996	2.266915	2.266885
		Erl+	2.243383	2.243411	2.243373	2.243313	2.243297
		HExp+	2.292411	2.292504	2.292294	2.292185	2.292112
		MNC+	2.306713	2.306956	2.306405	2.306246	2.30611
		MPC+	2.319806	2.320332	2.318866	2.318812	2.318306
5	1	Exp+	0.507566	0.507491	0.507695	0.507707	0.507774
		Erl+	0.508727	0.508698	0.508753	0.508785	0.508804
		HExp+	0.502437	0.502259	0.502922	0.502761	0.503013
		MNC+	0.505872	0.505737	0.506124	0.506122	0.506284
		MPC+	0.505685	0.505595	0.505906	0.505853	0.505982
	2	Exp+	1.126544	1.126741	1.126276	1.126163	1.126093
		Erl+	1.128428	1.128488	1.128363	1.128299	1.12829
		HExp+	1.115979	1.116219	1.115599	1.115511	1.115344
		MNC+	1.12417	1.124468	1.123631	1.123602	1.123359
		MPC+	1.125717	1.12594	1.125198	1.125293	1.125006
	3	Exp+	1.744545	1.744629	1.74446	1.744364	1.744333
		Erl+	1.742773	1.742788	1.74277	1.742718	1.742707
		HExp+	1.742887	1.743093	1.742604	1.742469	1.742361
		MNC+	1.747152	1.747511	1.74663	1.746474	1.746262
		MPC+	1.754146	1.754587	1.753257	1.753311	1.752824
	4	Exp+	2.248576	2.248626	2.248539	2.248442	2.248405
		Erl+	2.223203	2.223228	2.2232	2.223122	2.223103
		HExp+	2.280753	2.280878	2.280579	2.280461	2.280362
		MNC+	2.290425	2.290715	2.290038	2.289863	2.289691
		MPC+	2.304108	2.304692	2.303024	2.303001	2.302412
5.5	1	Exp+	0.50809	0.508037	0.508179	0.508191	0.508244
		Erl+	0.508987	0.508979	0.508981	0.509011	0.50902
		HExp+	0.503946	0.503803	0.50434	0.504211	0.504433
		MNC+	0.506873	0.506779	0.507047	0.507046	0.507169
		MPC+	0.506752	0.506688	0.506909	0.506871	0.506968
	2	Exp+	1.122283	1.122534	1.121922	1.1218	1.121697
		Erl+	1.123541	1.123633	1.123432	1.123349	1.123334
		HExp+	1.113437	1.113707	1.112978	1.112908	1.112692
		MNC+	1.120918	1.121258	1.120274	1.120269	1.119967
		MPC+	1.122888	1.123135	1.12229	1.122416	1.122084
	3	Exp+	1.734204	1.734313	1.734083	1.73397	1.733927
		Erl+	1.73178	1.731796	1.731775	1.731714	1.7317
		HExp+	1.735776	1.73604	1.735388	1.735248	1.735096
		MNC+	1.737918	1.738336	1.737283	1.737127	1.73686
		MPC+	1.745552	1.746035	1.744543	1.744635	1.74408
	4	Exp+	2.230683	2.230737	2.230636	2.230527	2.230482
		Erl+	2.203849	2.203868	2.203853	2.203761	2.203738
		HExp+	2.269533	2.269696	2.269284	2.269164	2.269032
		MNC+	2.274301	2.274641	2.273828	2.273642	2.27343
		MPC+	2.288393	2.28903	2.287169	2.287182	2.286512

Table 2.15: Mean number of Idle Servers

$S = 25$, $s = 8$, $\lambda_1 = 5$, $\beta = 4$, $\mu = 10$, $\gamma = 0.3$, $\theta = 5$.

λ_{-1}	c		Exp-	Erl-	HExp-	MNC-	MPC-
2	1	Exp+	0.507566	0.507491	0.507695	0.507707	0.507774
		Erl+	0.508727	0.508698	0.508753	0.508785	0.508804
		HExp+	0.502437	0.502259	0.502922	0.502761	0.503013
		MNC+	0.505872	0.505737	0.506124	0.506122	0.506284
		MPC+	0.505685	0.505595	0.505906	0.505853	0.505982
	2	Exp+	1.126544	1.126741	1.126276	1.126163	1.126093
		Erl+	1.128428	1.128488	1.128363	1.128299	1.12829
		HExp+	1.115979	1.116219	1.115599	1.115511	1.115344
		MNC+	1.12417	1.124468	1.123631	1.123602	1.123359
		MPC+	1.125717	1.12594	1.125198	1.125293	1.125006
	3	Exp+	1.744545	1.744629	1.74446	1.744364	1.744333
		Erl+	1.742773	1.742788	1.74277	1.742718	1.742707
		HExp+	1.742887	1.743093	1.742604	1.742469	1.742361
		MNC+	1.747152	1.747511	1.74663	1.746474	1.746262
		MPC+	1.754146	1.754587	1.753257	1.753311	1.752824
	4	Exp+	2.248576	2.248626	2.248539	2.248442	2.248405
		Erl+	2.223203	2.223228	2.2232	2.223122	2.223103
		HExp+	2.280753	2.280878	2.280579	2.280461	2.280362
		MNC+	2.290425	2.290715	2.290038	2.289863	2.289691
		MPC+	2.304108	2.304692	2.303024	2.303001	2.302412
2.5	1	Exp+	0.507404	0.507314	0.507561	0.507569	0.507641
		Erl+	0.508672	0.508632	0.508711	0.508745	0.508766
		HExp+	0.501666	0.501452	0.502262	0.502044	0.50232
		MNC+	0.505317	0.505136	0.505655	0.505637	0.505831
		MPC+	0.505111	0.504995	0.505401	0.505323	0.505479
	2	Exp+	1.126787	1.127009	1.126462	1.126367	1.126289
		Erl+	1.128502	1.128573	1.128422	1.128354	1.128344
		HExp+	1.116404	1.116683	1.115934	1.115876	1.115691
		MNC+	1.124744	1.125081	1.124092	1.124114	1.123847
		MPC+	1.12649	1.126744	1.125861	1.126013	1.125692
	3	Exp+	1.744681	1.744786	1.74457	1.744469	1.74443
		Erl+	1.742825	1.742855	1.742816	1.742753	1.742738
		HExp+	1.743229	1.743467	1.742881	1.742757	1.742633
		MNC+	1.747725	1.748133	1.747076	1.746972	1.746731
		MPC+	1.755378	1.75588	1.754281	1.754443	1.753895
	4	Exp+	2.248711	2.248786	2.248655	2.248543	2.248497
		Erl+	2.223285	2.223334	2.223269	2.223176	2.223151
		HExp+	2.281067	2.281219	2.280851	2.280724	2.280605
		MNC+	2.290924	2.291262	2.290435	2.290289	2.290089
		MPC+	2.305595	2.306263	2.304247	2.304356	2.303688
3	1	Exp+	0.507271	0.507168	0.507452	0.507454	0.507528
		Erl+	0.508623	0.508573	0.508674	0.508709	0.508731
		HExp+	0.501084	0.50085	0.501757	0.501493	0.501777
		MNC+	0.504854	0.504637	0.505269	0.505232	0.505447
		MPC+	0.50464	0.504503	0.504987	0.504886	0.50506
	2	Exp+	1.126994	1.127234	1.12662	1.126546	1.126463
		Erl+	1.128567	1.128651	1.128474	1.128405	1.128393
		HExp+	1.116767	1.117074	1.11622	1.116194	1.115997
		MNC+	1.125229	1.125593	1.124482	1.124556	1.124273
		MPC+	1.127152	1.12743	1.126431	1.126635	1.12629
	3	Exp+	1.744805	1.744929	1.744668	1.744566	1.744521
		Erl+	1.742877	1.742925	1.742857	1.742787	1.742769
		HExp+	1.74353	1.743794	1.743124	1.743016	1.742878
		MNC+	1.74822	1.748661	1.74746	1.747413	1.74715
		MPC+	1.756445	1.756992	1.75517	1.755438	1.754843
	4	Exp+	2.248841	2.248942	2.248761	2.24864	2.248584
		Erl+	2.223366	2.22344	2.22333	2.22323	2.223199
		HExp+	2.281353	2.281533	2.281097	2.280964	2.280828
		MNC+	2.291364	2.29174	2.290782	2.290675	2.29045
		MPC+	2.306892	2.30762	2.305312	2.305555	2.304826

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Table 2.16: Mean number of Idle Servers

$S = 25$, $s = 8$, $\lambda_1 = 5$, $\lambda_{-1} = 3$, $\mu = 10$, $\gamma = 0.3$, $\theta = 5$.

β	c		Exp-	Erl-	HExp-	MNC-	MPC-
4	1	Exp+	0.507271	0.507168	0.507452	0.507454	0.507528
		Erl+	0.508623	0.508573	0.508674	0.508709	0.508731
		HExp+	0.501084	0.50085	0.501757	0.501493	0.501777
		MNC+	0.504854	0.504637	0.505269	0.505232	0.505447
		MPC+	0.50464	0.504503	0.504987	0.504886	0.50506
	2	Exp+	1.126994	1.127234	1.12662	1.126546	1.126463
		Erl+	1.128567	1.128651	1.128474	1.128405	1.128393
		HExp+	1.116767	1.117074	1.11622	1.116194	1.115997
		MNC+	1.125229	1.125593	1.124482	1.124556	1.124273
		MPC+	1.127152	1.12743	1.126431	1.126635	1.12629
	3	Exp+	1.744805	1.744929	1.744668	1.744566	1.744521
		Erl+	1.742877	1.742925	1.742857	1.742787	1.742769
		HExp+	1.74353	1.743794	1.743124	1.743016	1.742878
		MNC+	1.74822	1.748661	1.74746	1.747413	1.74715
		MPC+	1.756445	1.756992	1.75517	1.755438	1.754843
	4	Exp+	2.248841	2.248942	2.248761	2.24864	2.248584
		Erl+	2.223366	2.22344	2.22333	2.22323	2.223199
		HExp+	2.281353	2.281533	2.281097	2.280964	2.280828
		MNC+	2.291364	2.29174	2.290782	2.290675	2.29045
		MPC+	2.306892	2.30762	2.305312	2.305555	2.304826
5	1	Exp+	0.524477	0.524261	0.52489	0.524859	0.525016
		Erl+	0.52666	0.52652	0.526888	0.526907	0.52697
		HExp+	0.517097	0.516766	0.518018	0.517676	0.518076
		MNC+	0.520894	0.520555	0.52157	0.521486	0.521831
		MPC+	0.520677	0.520465	0.521229	0.52106	0.521334
	2	Exp+	1.183111	1.183298	1.182817	1.182766	1.182704
		Erl+	1.186663	1.186727	1.186583	1.186542	1.186535
		HExp+	1.167882	1.168112	1.167509	1.167453	1.167312
		MNC+	1.178451	1.178687	1.177967	1.178014	1.177828
		MPC+	1.179282	1.17945	1.178849	1.17897	1.178757
	3	Exp+	1.844566	1.844669	1.844443	1.844375	1.844348
		Erl+	1.846282	1.846305	1.846267	1.84624	1.846233
		HExp+	1.833504	1.833753	1.833138	1.833034	1.832927
		MNC+	1.842686	1.84309	1.842006	1.841948	1.841722
		MPC+	1.848828	1.849313	1.847714	1.847933	1.84741
	4	Exp+	2.378727	2.37879	2.378668	2.378607	2.37858
		Erl+	2.357812	2.357844	2.357791	2.357755	2.357744
		HExp+	2.398879	2.399032	2.398673	2.398572	2.398488
		MNC+	2.414239	2.41459	2.413701	2.413602	2.413421
		MPC+	2.427574	2.428284	2.426048	2.426271	2.425575
6	1	Exp+	0.536663	0.536354	0.537266	0.537207	0.537433
		Erl+	0.53946	0.539244	0.539841	0.539845	0.539943
		HExp+	0.5285	0.528096	0.52961	0.529211	0.529699
		MNC+	0.532198	0.531764	0.533077	0.532955	0.533401
		MPC+	0.531986	0.531715	0.532695	0.532473	0.532826
	2	Exp+	1.224637	1.224762	1.224437	1.224405	1.224364
		Erl+	1.229577	1.229621	1.229519	1.229496	1.229492
		HExp+	1.206238	1.20638	1.206049	1.205973	1.20589
		MNC+	1.217753	1.217855	1.217536	1.217561	1.217473
		MPC+	1.217686	1.217745	1.217535	1.217574	1.217493
	3	Exp+	1.921137	1.921223	1.921033	1.920982	1.920963
		Erl+	1.925554	1.925566	1.925544	1.925531	1.925529
		HExp+	1.903387	1.903607	1.903072	1.902977	1.902891
		MNC+	1.915155	1.915498	1.914587	1.914528	1.914341
		MPC+	1.919385	1.919785	1.918482	1.918645	1.918215
	4	Exp+	2.482824	2.482866	2.482781	2.482746	2.482733
		Erl+	2.465847	2.46586	2.465837	2.465823	2.465819
		HExp+	2.493508	2.493637	2.493339	2.493258	2.493202
		MNC+	2.512287	2.512609	2.511801	2.511706	2.511556
		MPC+	2.523356	2.52402	2.521941	2.522135	2.521492

Table 2.17: Mean number of Idle Servers

$S = 25, \quad s = 8, \quad \lambda_1 = 5, \quad \lambda_{-1} = 3, \quad \beta = 4, \quad \gamma = 0.3, \quad \theta = 5.$

μ	c		Exp-	Erl-	HExp-	MNC-	MPC-
10	1	Exp+	0.507271	0.507168	0.507452	0.507454	0.507528
		Erl+	0.508623	0.508573	0.508674	0.508709	0.508731
		HExp+	0.501084	0.50085	0.501757	0.501493	0.501777
		MNC+	0.504854	0.504637	0.505269	0.505232	0.505447
		MPC+	0.50464	0.504503	0.504987	0.504886	0.50506
	2	Exp+	1.126994	1.127234	1.12662	1.126546	1.126463
		Erl+	1.128567	1.128651	1.128474	1.128405	1.128393
		HExp	1.116767	1.117074	1.11622	1.116194	1.115997
		MNC+	1.125229	1.125593	1.124482	1.124556	1.124273
		MPC+	1.127152	1.12743	1.126431	1.126635	1.12629
	3	Exp+	1.744805	1.744929	1.744668	1.744566	1.744521
		Erl+	1.742877	1.742925	1.742857	1.742787	1.742769
		HExp+	1.74353	1.743794	1.743124	1.743016	1.742878
		MNC+	1.74822	1.748661	1.74746	1.747413	1.74715
		MPC+	1.756445	1.756992	1.75517	1.755438	1.754843
	4	Exp+	2.248841	2.248942	2.248761	2.24864	2.248584
		Erl+	2.223366	2.22344	2.22333	2.22323	2.223199
		HExp+	2.281353	2.281533	2.281097	2.280964	2.280828
		MNC+	2.291364	2.29174	2.290782	2.290675	2.29045
		MPC+	2.306892	2.30762	2.305312	2.305555	2.304826
11	1	Exp+	0.50528	0.505214	0.505382	0.505397	0.50544
		Erl+	0.50644	0.506418	0.506439	0.506474	0.506483
		HExp+	0.499047	0.498846	0.499634	0.499395	0.49963
		MNC+	0.503097	0.502912	0.503435	0.503418	0.503591
		MPC+	0.502843	0.502728	0.50313	0.50305	0.503193
	2	Exp+	1.123003	1.123218	1.122684	1.1226	1.122532
		Erl+	1.124186	1.124252	1.124123	1.124058	1.124049
		HExp+	1.113578	1.113875	1.113062	1.113023	1.112847
		MNC+	1.121778	1.122152	1.121036	1.121089	1.120813
		MPC+	1.123841	1.124134	1.123098	1.123298	1.122943
	3	Exp+	1.739418	1.739525	1.739308	1.739211	1.739171
		Erl+	1.736326	1.736371	1.736308	1.73624	1.736222
		HExp+	1.740169	1.740392	1.739839	1.73973	1.739614
		MNC+	1.744405	1.744815	1.743717	1.743658	1.743423
		MPC+	1.753161	1.7537	1.751927	1.752171	1.751599
	4	Exp+	2.249043	2.249139	2.248971	2.248852	2.248798
		Erl+	2.222677	2.222751	2.222641	2.222543	2.222512
		HExp+	2.282566	2.282725	2.282347	2.282216	2.282089
		MNC+	2.293367	2.293706	2.292853	2.292742	2.292538
		MPC+	2.310065	2.310763	2.30858	2.308786	2.308103
12	1	Exp+	0.503542	0.503509	0.50358	0.503602	0.503622
		Erl+	0.504531	0.504531	0.504493	0.504526	0.504526
		HExp+	0.49728	0.497113	0.497784	0.49757	0.497761
		MNC+	0.501575	0.501421	0.501843	0.501843	0.501979
		MPC+	0.501284	0.501189	0.501518	0.501456	0.501572
	2	Exp+	1.119604	1.119798	1.11933	1.119242	1.119185
		Erl+	1.120416	1.120468	1.120372	1.120312	1.120304
		HExp+	1.110901	1.111183	1.110423	1.110372	1.110217
		MNC+	1.118892	1.119271	1.118161	1.118196	1.117929
		MPC+	1.121089	1.121393	1.120331	1.120526	1.120164
	3	Exp+	1.734785	1.734879	1.734695	1.734601	1.734564
		Erl+	1.730633	1.730677	1.730617	1.730549	1.730531
		HExp+	1.737288	1.737479	1.737015	1.736905	1.736804
		MNC+	1.74126	1.741642	1.74063	1.740564	1.74035
		MPC+	1.750551	1.751082	1.749335	1.749578	1.749026
	4	Exp+	2.249332	2.249424	2.249265	2.249148	2.249095
		Erl+	2.222195	2.222269	2.22216	2.222062	2.222032
		HExp+	2.283725	2.283868	2.283533	2.283403	2.283282
		MNC+	2.295266	2.295576	2.294805	2.294693	2.294506
		MPC+	2.313106	2.313776	2.311705	2.311879	2.311236

Table 2.18: Mean number of Idle Servers

$S = 25, \quad s = 8, \quad \lambda_1 = 5, \quad \lambda_{-1} = 2, \quad \beta = 6, \quad \mu = 10, \quad \gamma = 0.3.$

θ	c		Exp-	Erl-	HExp-	MNC-	MPC-
5	1	Exp+	0.536663	0.536354	0.537266	0.537207	0.537433
		Erl+	0.53946	0.539244	0.539841	0.539845	0.539943
		HExp+	0.5285	0.528096	0.52961	0.529211	0.529699
		MNC+	0.532198	0.531764	0.533077	0.532955	0.533401
		MPC+	0.531986	0.531715	0.532695	0.532473	0.532826
	2	Exp+	1.224637	1.224762	1.224437	1.224405	1.224364
		Erl+	1.229577	1.229621	1.229519	1.229496	1.229492
		HExp+	1.206238	1.20638	1.206049	1.205973	1.20589
		MNC+	1.217753	1.217855	1.217536	1.217561	1.217473
		MPC+	1.217686	1.217745	1.217535	1.217574	1.217493
	3	Exp+	1.921137	1.921223	1.921033	1.920982	1.920963
		Erl+	1.925554	1.925566	1.925544	1.925531	1.925529
		HExp+	1.903387	1.903607	1.903072	1.902977	1.902891
		MNC+	1.915155	1.915498	1.914587	1.914528	1.914341
		MPC+	1.919385	1.919785	1.918482	1.918645	1.918215
	4	Exp+	2.482824	2.482866	2.482781	2.482746	2.482733
		Erl+	2.465847	2.46586	2.465837	2.465823	2.465819
		HExp+	2.493508	2.493637	2.493339	2.493258	2.493202
		MNC+	2.512287	2.512609	2.511801	2.511706	2.511556
		MPC+	2.523356	2.52402	2.521941	2.522135	2.521492
6	1	Exp+	0.536892	0.536584	0.537477	0.537445	0.537666
		Erl+	0.539619	0.539398	0.53999	0.540016	0.540112
		HExp+	0.528617	0.528223	0.529684	0.529319	0.529791
		MNC+	0.53257	0.532138	0.533431	0.53334	0.533779
		MPC+	0.532271	0.532	0.532974	0.532767	0.533119
	2	Exp+	1.224534	1.22466	1.224343	1.224298	1.224258
		Erl+	1.229549	1.229591	1.229497	1.229469	1.229465
		HExp+	1.205957	1.206118	1.205723	1.205651	1.205555
		MNC+	1.217452	1.217577	1.217194	1.217216	1.217113
		MPC+	1.217105	1.217191	1.216888	1.216944	1.216831
	3	Exp+	1.921083	1.921164	1.92099	1.920933	1.920914
		Erl+	1.925543	1.925555	1.925533	1.925519	1.925517
		HExp+	1.903191	1.903405	1.902903	1.902786	1.902703
		MNC+	1.914868	1.915202	1.914352	1.914254	1.914075
		MPC+	1.918645	1.919046	1.917789	1.917896	1.91747
	4	Exp+	2.482785	2.482827	2.482745	2.482706	2.482692
		Erl+	2.465834	2.465849	2.465823	2.465808	2.465804
		HExp+	2.493361	2.493484	2.493213	2.493118	2.493063
		MNC+	2.51212	2.512422	2.511698	2.511571	2.51143
		MPC+	2.522815	2.523444	2.521571	2.521654	2.521053
7	1	Exp+	0.537077	0.536773	0.537638	0.537631	0.537845
		Erl+	0.539752	0.539529	0.540111	0.540156	0.540251
		HExp+	0.528691	0.52831	0.52971	0.529378	0.529832
		MNC+	0.53285	0.532424	0.53368	0.53362	0.534047
		MPC+	0.532469	0.5322	0.533154	0.532966	0.533312
	2	Exp+	1.224445	1.224571	1.224265	1.22421	1.22417
		Erl+	1.229525	1.229566	1.229478	1.229447	1.229443
		HExp+	1.205715	1.205889	1.205454	1.205379	1.205275
		MNC+	1.217187	1.21733	1.216905	1.216919	1.216806
		MPC+	1.2166	1.216706	1.216338	1.2164	1.216264
	3	Exp+	1.921039	1.921117	1.920955	1.920894	1.920876
		Erl+	1.925534	1.925547	1.925524	1.92551	1.925507
		HExp+	1.90303	1.903237	1.902767	1.902634	1.902553
		MNC+	1.91464	1.914961	1.914169	1.914042	1.913872
		MPC+	1.918059	1.918456	1.917255	1.917314	1.916896
	4	Exp+	2.482754	2.482795	2.482715	2.482675	2.48266
		Erl+	2.465824	2.465839	2.465812	2.465797	2.465793
		HExp+	2.49324	2.493358	2.493109	2.493004	2.49295
		MNC+	2.511999	2.512281	2.511629	2.511482	2.511349
		MPC+	2.522457	2.523047	2.521357	2.521358	2.520797

Table 2.19: Mean number of Idle Servers

S = 25, s = 8, λ_1 = 5, λ_{-1} = 3, β = 6, μ = 10, θ = 5.

γ	c		Exp-	Erl-	HExp-	MNC-	MPC-
0.2	1	Exp+	0.558737	0.558461	0.559255	0.559223	0.559421
		Erl+	0.56131	0.561123	0.561617	0.561636	0.561719
		HExp+	0.549917	0.549516	0.551027	0.550625	0.551107
		MNC+	0.554372	0.553947	0.555198	0.555111	0.555539
		MPC+	0.554059	0.553793	0.55474	0.55454	0.554882
	2	Exp+	1.280978	1.281161	1.280685	1.280642	1.280582
		Erl+	1.285306	1.285369	1.285223	1.28519	1.285184
		HExp+	1.263517	1.263745	1.263138	1.263095	1.262952
		MNC+	1.275332	1.275541	1.274883	1.274943	1.274772
		MPC+	1.275562	1.275703	1.275178	1.2753	1.275114
	3	Exp+	2.014689	2.014793	2.014559	2.014502	2.01448
		Erl+	2.018676	2.018688	2.018664	2.018654	2.018652
		HExp+	1.998994	1.999268	1.99859	1.998487	1.998379
		MNC+	2.010126	2.01056	2.009397	2.009332	2.009097
		MPC+	2.015747	2.016262	2.014554	2.014795	2.014238
	4	Exp+	2.616494	2.616538	2.616446	2.616414	2.616401
		Erl+	2.599141	2.599151	2.599132	2.599122	2.599119
		HExp+	2.630792	2.630945	2.630587	2.630502	2.630437
		MNC+	2.647079	2.647449	2.646514	2.646412	2.646241
		MPC+	2.66141	2.662193	2.659721	2.659975	2.659216
0.25	1	Exp+	0.547049	0.546751	0.547622	0.547574	0.54779
		Erl+	0.549781	0.549575	0.550133	0.550145	0.550237
		HExp+	0.538444	0.538035	0.539573	0.539164	0.539657
		MNC+	0.542568	0.542133	0.543435	0.543327	0.543771
		MPC+	0.542307	0.542035	0.543013	0.542798	0.54315
	2	Exp+	1.251672	1.251823	1.251431	1.251394	1.251345
		Erl+	1.256368	1.25642	1.256299	1.256272	1.256266
		HExp+	1.233369	1.233549	1.233097	1.233034	1.232924
		MNC+	1.245277	1.245426	1.244958	1.245	1.244876
		MPC+	1.245294	1.245388	1.245042	1.245118	1.244991
	3	Exp+	1.966311	1.966405	1.966194	1.96614	1.96612
		Erl+	1.970544	1.970556	1.970533	1.970521	1.970519
		HExp+	1.94918	1.949426	1.948824	1.948724	1.948628
		MNC+	1.96094	1.961326	1.960297	1.960236	1.960026
		MPC+	1.965771	1.966224	1.964733	1.964932	1.964443
	4	Exp+	2.547198	2.547241	2.547153	2.547119	2.547106
		Erl+	2.529999	2.530011	2.529989	2.529978	2.529974
		HExp+	2.559356	2.559496	2.559169	2.559087	2.559026
		MNC+	2.577246	2.577592	2.576723	2.576625	2.576464
		MPC+	2.589827	2.590548	2.588282	2.588504	2.587806
0.3	1	Exp+	0.536663	0.536354	0.537266	0.537207	0.537433
		Erl+	0.53946	0.539244	0.539841	0.539845	0.539943
		HExp+	0.5285	0.528096	0.52961	0.529211	0.529699
		MNC+	0.532198	0.531764	0.533077	0.532955	0.533401
		MPC+	0.531986	0.531715	0.532695	0.532473	0.532826
	2	Exp+	1.224637	1.224762	1.224437	1.224405	1.224364
		Erl+	1.229577	1.229621	1.229519	1.229496	1.229492
		HExp+	1.206238	1.20638	1.206049	1.205973	1.20589
		MNC+	1.217753	1.217855	1.217536	1.217561	1.217473
		MPC+	1.217686	1.217745	1.217535	1.217574	1.217493
	3	Exp+	1.921137	1.921223	1.921033	1.920982	1.920963
		Erl+	1.925554	1.925566	1.925544	1.925531	1.925529
		HExp+	1.903387	1.903607	1.903072	1.902977	1.902891
		MNC+	1.915155	1.915498	1.914587	1.914528	1.914341
		MPC+	1.919385	1.919785	1.918482	1.918645	1.918215
	4	Exp+	2.482824	2.482866	2.482781	2.482746	2.482733
		Erl+	2.465847	2.46586	2.465837	2.465823	2.465819
		HExp+	2.493508	2.493637	2.493339	2.493258	2.493202
		MNC+	2.512287	2.512609	2.511801	2.511706	2.511556
		MPC+	2.523356	2.52402	2.521941	2.522135	2.521492