Inventory Management in Supply Chain with Stochastic Inputs

by

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ABSTRACT

Title: Inventory Management in Supply Chain with Stochastic Inputs

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This thesis studies and proposes some new ways to manage inventory in supply chains with stochastic demand and lead time. In particular, it uses queuing principles to determine the parameters of supply chain stations with delayed differentiation (typical assemble-to-order systems) and went on to apply some previously known results of steady state of some queuing systems to the management of flow and work in process inventory in supply chain stations. Consideration was also given to the problem of joint replenishment in partially dependent demand conditions. The first chapter introduces the important concepts of supply chain, the role of inventory in a supply chain, and developing stochastic models for such system. It then went on to review the pertinent literature that has been contributed to the inventory management, especially using stochastic models.

Chapter two presents a perishable inventory model with a multi-server system, where some services, having an exponentially distributed lead time, have to be done on the product before it is delivered to the customer. Customers whose demands are not met immediately are put in an orbit from where they send in random retrial requests for selection. The input stream follows a Markov Arrival Process, $\boldsymbol{MAP}$, and another flow of negative customers (typical of a competitive environment with customer poaching), also following an $\boldsymbol{MAP}$, takes customers away from the orbit. An $(\boldsymbol{s},\boldsymbol{S})$ replenishment policy was used. The joint probability distribution of the number of busy servers, the inventory level and the number of customers in the orbit is obtained in the steady state. Various measures of

stationary system performance are computed and the total expected cost per unit time is calculated. Numerical illustrations were made.

Chapter three is also a continuous review retrial inventory system with a finite source of customers and identical multiple servers in parallel. The customers are assumed to arrive following a quasi-random distribution. Items demanded are also made available after some service, exponentially distributed, has been done on the demanded item. Customers with unsatisfied orders join an orbit from where they can make retrials only if selected following a special rule. Replenishment follows an $(\boldsymbol{s}, \boldsymbol{S})$ policy and also has an exponentially distributed lead time. The intervals separating two successive repeated attempts are exponentially distributed with rate $\theta + iv$, when the orbit has i customers $i \geq 1$. The joint probability distribution of the number of customers in the orbit, the number of busy servers and the inventory level is obtained in the steady state case. Various measures of stationary system performance are computed and the total expected cost per unit time is calculated.

Chapter four is a two-commodity continuous review inventory system, with three customer input flows, following the $\boldsymbol{MAP}$; one for individual demand for product 1; another for bulk demand for product 2; and the third for a joint individual demand for product 1 and bulk demand for product 2. The ordering policy is to place orders for both commodities when the inventory levels are below prefixed levels for both commodities, using $(\boldsymbol{s_1 s_2}, \boldsymbol{S_1 S_2})$ replenishment. The replenishment lead time is assumed to have phase type distribution and the demands that occur during stock out period are assumed to be lost. The joint probability distribution for both commodities is obtained in the steady state case. Various measures of system performance and the total expected cost rate in the steady state are derived. Numerical illustrations were then done.

Chapter five is a model that shows how the steady state parameters of a typical queuing system can be used in the dynamic management of flow and buffer in a Theory of Constraints ($\boldsymbol{TOC}$) environment. This chapter is in two parts, and the typical $M/M/1/\infty$ production environment with $0 < \rho < 1$ was assumed. The optimal feed rate for maximum profit was obtained. In the first part, the model was considered without consideration for shortage cost. This model was then extended in the second part to a case where a fixed cost is charged for every unit shortage from the desired production level. Part A result was shown to be a special case of part B result; the unit shortage cost has been implicitly taken to be zero in part A.

Chapter six is the concluding chapter, where the various possible applications of the models developed and opportunities for possible future expansions of models and areas of research were highlighted.

The main contributions of this work are in the Supply Chain area of delayed differentiation of products and service lead time. Others include management of joint replenishment and optimisation of flow in a TOC environment. The key contributions to knowledge made in this thesis include:

- A model of a multi-server retrial queue with MAP arrival and negative arrival, and deteriorating inventory system in which inventory items are made available only after some work has been done on the inventory item before it is delivered to the customer. No previous model is known to have considered any queuing system with such multi-server system ahead of this chapter.
- A model of a retrial queuing system with multi-server rule based in which the arrival pattern is quasi-random, the calling population is finite, and an exponentially distributed system service is done on the inventory item before being delivered to the customer. It has not been found in literatures that such models have been developed elsewhere.
- A stochastic model of joint replenishment of stocks in which two products are being ordered together; one of such is ordered in bulk and the other in single units, but both could be ordered together and unfilled order during the replenishment lead-time is lost. No published work is known to have also addressed such systems.
- The management of flow in a theory of constraint environment, which explicitly utilises the holding cost, shortage cost, product margin, the level of utilisation of the resource and the effect of such on the stocks (WIP inventory) build up in the system. Such flows are then explicitly considered in the process of buffering the system. Most works have been known to focus on buffer and not the flow of the products in order to optimise the system profit goal.

Some of the insights derived include

- An understanding of how the system cost rate is affected by the choice of the replenishment policy in systems with $\boldsymbol{MAP}$ arrival pattern so that controlling policies (reorder point and capacity) could be chosen to optimise system profit
- The effect of correlated arrival in $\boldsymbol{MAP}$ input system on the cost rate of the system
- How the nature of input pattern and their level of correlation affect the fraction of the retrials in a retrial queue in a competitive environment that are successful and how many of such customers are likely to be poached away by the randomly arriving competitors. This has direct effect on the future market size.
- The nature of utilisation, blocking and idleness of servers in typical retrial queues, such that there could be yet-to-be-served customers in the orbit while there are still idle serves in such systems
- Management of utilisation of resources in stochastic input and processing environment with respect to the throughput rate of such systems. It was shown that it may not be profitable to strive to always seek to fully utilise the full capacity of a Capacity Constrained Resource, even in the face of unmet demands. Increase in utilisation should always be considered in the light of the effect of such on the throughput time of the products and the consequence on the system’s profit goal. This decision is also important in determining the necessity and level of buffers allowable in the production system.

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