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Appendix A

List of Symbols

Δ	spacing metric of Deb
d_i	Euclidean distance in the objective space between solution i of approximated POF and the nearest member of the sampled points of the true POF
d_k^e	distance between the extreme solutions of the approximated POF and the sampled solutions of the true POF
\prec	domination relational operator
\preceq	weak domination relational operator
\prec_ϵ	ϵ -domination relational operator
F	feasible space, $F \subseteq S$
f_k	objective function
$\mathbf{f}(\mathbf{x}, \mathbf{w}(t))$	dynamic objective function vector
$f(\mathbf{x}, \mathbf{w}(t))$	dynamic objective function
\mathbf{f}_{ref}	reference vector for hypervolume calculation
\mathbf{f}	objective function vector

g	inequality constraint
γ	path between two solutions in objective space
\mathbf{g}	inequality constraint vector
h	equality constraint
\mathbf{h}	equality constraint vector
F	neighbourhood of points, $N \subseteq F$
ND	number of non-dominated solutions
n_g	number of inequality constraints
n_h	number of equality constraints
n_k	number of objective functions
n_t	severity of change
n_x	number of decision variables
O_{space}	objective space
O_C	complete outperformance
O_S	strong outperformance
O_W	weak outperformance
O	outperforms
$PF^*(t)$	Pareto-optimal front at time t
PF^*	Pareto-optimal front
PF_ϵ^*	ϵ -approximate Pareto-optimal front
P^*	Pareto-optimal set

PF_g^*	global POF
PF_l^*	local POF
POF^*	approximated POF
n_{POF^*}	number of solutions in approximated POF
POF	true POF
$n_{POF'}$	number of sampled solutions in true POF
POF'	sampled solutions of true POF
S	unrestricted search space
τ	current iteration
τ_t	frequency of change
U	Set of utility functions
\mathbf{x}	solution vector
\mathbf{x}_i	decision variable
\mathbf{x}_{min}	lower bound of the feasible values for decision variable \mathbf{x}
\mathbf{x}_N	local optima
\mathbf{x}^*	global optimum solution
$\mathbf{x}^*(t)$	global optimum solution at time step t
\mathbf{x}_{max}	upper bound of the feasible values for decision variable \mathbf{x}

Appendix B

List of Acronyms

AIS

artificial immune system. 172, 173, 191

CCEA

cooperative-coevolution evolutionary algorithm. 6, 143, 151, 152, 154, 155, 177, 178

CI

computational intelligence. 28, 124, 126, 144, 156, 169, 191, 192

COEA

competitive-cooperative evolutionary algorithm. 177

D-QMOO

dynamic queuing multi-objective optimizer. 187, 188

dCCEA

dynamic CCEA. 176, 178, 179

dCOEA

dynamic COEA. 177–179, 327–329, 335–337, 339, 340, 346, 353, 362, 363, 366

DE

differential evolution. 7, 141, 156, 165, 172, 184

dMO-EGS

dynamic MO-EGS. 175, 176, 191

dMO-EGS-PG

dynamic MO-EGS with prediction gradient. 191

DMOA

dynamic MOA. 84, 327, 329, 332, 334–337, 339, 340, 346, 353, 357, 362, 363, 365–367, 399, 401

DMOEA

dynamic MOEA. 183, 189, 336, 337, 339, 363, 366

dMOEA

dynamic MOEA. 178, 179

DMOO

dynamic multi-objective optimisation. 2–7, 10, 11, 24, 27–31, 41, 42, 48, 54, 72, 74, 82, 84, 87, 88, 100, 101, 103, 107, 109, 110, 113, 114, 122, 124, 142, 143, 168–170, 172, 173, 175–177, 179, 180, 183, 191, 192, 201, 202, 206, 248, 249, 311, 322–326, 364–366, 368

DMOOP

dynamic multi-objective optimisation problem. 1–7, 20, 25–31, 41–45, 47, 48, 50–54, 56–75, 77–87, 92, 101, 102, 105, 111–113, 117, 119, 121, 123, 168–177, 179–187, 189–192, 194, 195, 197–199, 201–204, 206, 209, 211, 213, 215–218, 220, 222, 223, 225, 230, 234, 237, 239, 251, 253–261, 271, 273–280, 282, 289, 291, 293, 295–298, 302, 309, 311–313, 315, 316, 323, 326–329, 334–339, 352, 362–367, 397–399, 401

DMOPSO

dynamic MOPSO. 325, 327, 328, 334–340, 346, 352, 353, 362, 366

DNSGA-II

dynamic NSGA-II. 170, 171, 190, 327–329, 334–340, 346, 352, 353, 362, 366

DSOO

dynamic single-objective optimisation. 2, 10, 11, 21, 27, 107, 110

DSOOP

dynamic single-objective optimisation problem. 2, 11, 21, 22, 25–27, 63, 168, 169

DVEPSO

dynamic VEPSO. 194

DVEPSO

dynamic Vector Evaluated Particle Swarm Optimisation. 2–7, 121, 126, 143, 162, 192, 194–196, 198–202, 209, 222, 223, 225, 228, 248–251, 260, 261, 270, 288, 298, 322–325, 328, 329, 334–336, 338–340, 346, 352, 353, 362, 363, 365–367, 399

EA

evolutionary algorithm. 2, 28, 143–145, 166, 169, 173, 176, 177, 179, 181, 182, 185, 191, 192

GA

genetic algorithm. 6, 121, 126, 136–138, 141–145, 156, 170, 176, 184

GD

generational distance. 91, 92, 102, 173, 179, 180, 182, 186

gIDG

generalised immigrants-based diversity generator. 185

HV

hypervolume. 98–100, 106, 110, 113, 188, 365

HVD

hypervolume distance. 110

HVR

hypervolume ratio. 99, 100, 189

IDMOEA

individual diversity multi-objective optimization evolutionary algorithm. 174, 175

IGD

inverse generational distance. 92

MA

memetic algorithm. 181, 182

MO-EGS

dynamic multi-objective gradient search. 175, 191

MOA

multi-objective algorithm. 2, 3, 364, 365

MOEA

multi-objective evolutionary algorithm. 144, 145, 155, 156, 171–174, 176, 178, 179, 182, 183, 187, 189, 190, 334

MOGA

multi-objective genetic algorithm. 146, 155, 170

MOO

multi-objective optimisation. 2, 7, 10, 11, 15, 16, 20, 27–30, 43, 72, 74, 83, 84, 87, 88, 91, 92, 100, 124, 143–145, 149, 151, 152, 155, 156, 163, 166, 170, 171

MOOP

multi-objective optimisation problem. 2, 6, 7, 11, 14–20, 24, 27, 29, 30, 32, 37, 39, 56, 59, 64, 71, 72, 74, 75, 84, 88, 89, 99, 119, 128, 144, 151–153, 155, 156, 160, 163, 165–168, 182, 183, 187, 188, 195, 324

MOPSO

multi-objective Particle Swarm Optimisation. 6, 143, 149–151, 155, 179, 362

MSOPS

multiple single objective Pareto sampling. 171, 172

NSGA

non-dominated sorting genetic algorithm. 146, 149, 155

NSGA-II

non-dominated sorting genetic algorithm II. 6, 143, 146, 147, 149, 151, 155, 170–172, 183, 185, 187

PAES

pareto archived evolution strategy. 151, 176, 183

POF

Pareto-optimal front. 2–5, 7, 17–20, 25–27, 29–35, 37–52, 54–56, 58–64, 66–75, 77–83, 85, 86, 88, 89, 91, 96, 98–102, 104, 105, 110–115, 117, 118, 123, 124, 143–145, 151, 154, 155, 157, 163, 164, 168, 169, 171–174, 177–179, 181–183,

186–192, 196, 200–202, 209, 220, 222, 223, 225, 228, 230, 237, 240, 250, 251, 260, 261, 280, 316, 329, 352, 362–368, 390, 397

POS

Pareto-optimal set. 16, 18, 20, 26, 27, 30–36, 38, 39, 41–52, 54–56, 58–64, 66, 68–71, 73–75, 77–83, 85, 101, 123, 124, 169, 172, 176, 184, 189, 191, 192, 201, 209, 239, 250, 263, 316, 329, 364, 365, 367

PSO

particle swarm optimisation. 2–6, 28, 120, 126–129, 134, 135, 137, 141–143, 149, 150, 155, 156, 160, 167, 179, 180, 191, 192, 200, 324, 336, 339, 362–367

QMOO

queuing multi-objective optimizer. 188

SFGA

single front genetic algorithm. 187

SMOO

static MOO. 41, 122, 124, 169, 176, 191, 195, 248

SMOOP

static MOOP. 169, 181, 191

SOO

single-objective optimisation. 6, 11, 20, 22, 27

SOOP

single-objective optimisation problem. 6, 10, 11, 13–16, 20, 63, 124, 128, 156, 181

SPEA

strength Pareto evolutionary algorithm. 145, 172, 183

SPEA2

SPEA2. 171, 172, 187

SQP

sequential quadratic programming. 181

VD

variational distance. 101, 102

VEDE

vector evaluated differential evolution. 7, 156, 165–167

VEDEPSO

vector evaluated differential evolution particle swarm optimisation. 166, 167

VEGA

vector evaluated genetic algorithm. 7, 144, 156, 158, 164–167

VEPSO

vector evaluated particle swarm optimisation. 5, 7, 156, 158–167, 192, 194–196, 198, 199, 206, 208, 228, 248, 365, 366

Appendix C

Calculating the True POS and POF

This appendix discusses how *POS* and *POF* are determined for DMOOPs. Two examples are provided, namely FDA5 and FDA2 modified by Cámara *et al.* [17] [16] [138] referred to in this section as FDA2_{Cámara}.

C.1 Example 1: FDA2_{Cámara}

The FDA2_{Cámara} DMOOP has two objective functions (refer to Section 3.2.1) and is defined as

$$\left\{ \begin{array}{l} f_1(\mathbf{x}_I) = x_1 \\ g(\mathbf{x}_{II}) = 1 + \sum_{x_i \in \mathbf{x}_{II}} x_i^2 \\ h(\mathbf{x}_{III}, f_1, g, t) = 1 - \left(\frac{f_1}{g}\right)^{H_2(t)} \\ \text{where :} \\ H(t) = z^{-\cos(\pi t/4)}, \quad t = \frac{1}{n_t} \left\lfloor \frac{\tau}{\tau_t} \right\rfloor \\ H_2(t) = H(t) + \sum_{x_i \in \mathbf{x}_{III}} (x_i - H(t)/2)^2 \\ \mathbf{x}_I \in [0, 1]; \quad \mathbf{x}_{II}, \mathbf{x}_{III} \in [-1, 1] \end{array} \right.$$

The goal when solving FDA2_{Cámara} is to minimise the two objective functions, namely f_1 and $f_2 = gh$. Since f_1 only depends on x_1 the true POF depends on f_2 . In order to minimise gh , both g and h have to be minimised. h will be minimised if the term $\frac{f_1}{g}^{H_2(t)}$ is maximised (since this term is subtracted from 1). The term $\frac{f_1}{g}^{H_2(t)}$ is maximised if g is minimised (since f_1 is divided by g). g is minimised if the term $\sum_{x_i \in \mathbf{x}_{II}} x_i^2$ is minimised, i.e. if $\sum_{x_i \in \mathbf{x}_{II}} x_i^2$ is zero. Therefore, the optimal values for $x_i \in \mathbf{x}_{II}$ is

$x_i = 0$. If $\sum_{x_i \in \mathbf{x}_{\text{II}}} x_i^2 = 0$, $g = 1$. Replacing $g = 1$ into $f_2 = gh$, results in $f_2^* = 1 - f_1^{H_2(t)}$. In order to minimise f_2^* , $H_2(t)$ has to be minimised. $H_2(t)$ is minimised if the term $\sum_{x_i \in \mathbf{x}_{\text{III}}} (x_i - H(t)/2)^2$ is minimised, which results in $H_2^*(t) = H(t)$. Therefore, the optimal values of $x_i \in \mathbf{x}_{\text{III}}$ is $x_i = \frac{H(t)}{2}$. Replacing H_2 in f_2^* with H_2^* , results in $f_2 = 1 - f_1^{H(t)}$.

Therefore, *POF* is $1 - f_1^{H(t)}$. The decision variable values that lead to *POF* is *POS*, namely $x_i = 0$, $\forall x_i \in \mathbf{x}_{\text{II}}$ and $x_i = \frac{H(t)}{2}$, $\forall x_i \in \mathbf{x}_{\text{III}}$.

C.2 Example 2: FDA5

FDA5 is a three-objective DMOOP [58] (refer to Section 3.2.1), defined as

$$\text{FDA5} = \begin{cases} \text{Minimize : } \mathbf{f}(\mathbf{x}, t) = (f_1(\mathbf{x}, g(\mathbf{x}_{\text{II}}, t)), \dots, \\ \quad \quad \quad f_k(\mathbf{x}, g(\mathbf{x}_{\text{II}}, t))) \\ f_1(\mathbf{x}, g, t) = (1 + g(\mathbf{x}_{\text{II}}, t)) \prod_{i=1}^{M-1} \cos\left(\frac{y_i \pi}{2}\right) \\ f_k(\mathbf{x}, g, t) = (1 + g(\mathbf{x}_{\text{II}}, t)) \left(\prod_{i=1}^{M-1} \cos\left(\frac{y_i \pi}{2}\right) \right) \\ \quad \sin\left(\frac{y_{M-k+1} \pi}{2}\right), \forall k = 1, \dots, M-1 \\ \vdots \\ f_m(\mathbf{x}, g, t) = (1 + g(\mathbf{x}_{\text{II}}, t)) \prod_{i=1}^{M-1} \sin\left(\frac{y_i \pi}{2}\right) \\ \text{where :} \\ g(\mathbf{x}_{\text{II}}, t) = G(t) + \sum_{x_i \in \mathbf{x}_{\text{II}}} (x_i - G(t))^2 \\ G(t) = |\sin(0.5\pi t)|, \quad t = \frac{1}{n_t} \left\lfloor \frac{\tau}{\tau_t} \right\rfloor \\ y_i = x_i^{F(t)}, \quad \forall i = 1, \dots, (M-1) \\ F(t) = 1 + 100 \sin^4(0.5\pi t) \\ \mathbf{x}_{\text{II}} = (x_M, \dots, x_n) \\ x_i \in [0, 1], \quad \forall i = 1, \dots, n \end{cases} \quad (\text{C.1})$$

In order to minimise FDA5, each objective function has to be minimised. Therefore, g has to be minimised, i.e. the term $\sum_{x_i \in \mathbf{x}_{\text{II}}} (x_i - G(t))^2$ has to be minimised. This results in $g = G(t)$ and *POF* of $\sum_{i=1}^m f_i^2 = (1 + G(t))^2$ (refer to [49]). The decision variable values that minimises g is $x_i = G(t)$, $\forall x_i \in \mathbf{x}_{\text{II}}$, resulting in *POS* of $x_i = G(t)$, $\forall x_i \in \mathbf{x}_{\text{II}}$.

Appendix D

Additional Data and Figures for Conducted Experiments

This appendix provides additional data and figures of experiments discussed in Chapters 9, 10 and 11. Section D.1 presents performance measure values, p-values and figures with regards to the data of the experiments that were conducted to investigate the influence of various guide update approaches on the performance of DVEPSO (refer to Chapter 9). The performance measure values and p-values of the sensitivity analysis discussed in Chapter 10 are presented in Section D.2. Finally, Section D.3 presents the performance measure values and p-values of the DMOAs discussed in Chapter 11.

Due to space limitations, the tables and figures can be found on the included CD. Below a list is provided of the data that is available on the CD, as well as the file that contains the specific data.

D.1 Guide Update Approaches

The following additional data is provided on the CD:

- Tables containing performance measure values that were obtained by the various guide update approaches (guide-update-approaches-performance-measure-values.pdf)
- Figures of the wins and losses values obtained by the guide update approaches for each performance measure and DMOOP (guide-update-approaches-wins-and-

losses.pdf)

- Tables presenting the p-values of the Mann-Whitney U tests that were performed on the data obtained by the guide update approaches (guide-update-approaches-pvalues.pdf)

D.2 Sensitivity Analysis

The following additional data is provided on the CD with regards to various approaches used to manage boundary constraint violations:

- Tables containing performance measure values that were obtained by the boundary constraint management approaches (boundary-performance-measure-values.pdf)
- Tables presenting the p-values of the Mann-Whitney U tests that were performed on the data obtained by the boundary constraint violation management approaches (boundary-pvalues.pdf)

Additional data with regards to knowledge sharing approaches that can be found on the CD are:

- Tables containing performance measure values that were obtained by the knowledge sharing strategies (knowledge-performance-measure-values.pdf)
- Tables presenting the p-values of the Mann-Whitney U tests that were performed on the data obtained by the knowledge sharing approaches (knowledge-pvalues.pdf)

The following additional data is provided on the CD with regards to responses after a change in the environment occurs:

- Tables containing performance measure values that were obtained by the responses applied to particles of the sub-swarms (response-particles-performance-measure-values.pdf) and responses applied to the archive (response-archive-performance-measure-values.pdf)
- Tables presenting the p-values of the Mann-Whitney U tests that were performed on the data obtained by the responses applied to the particles (response-particles-pvalues.pdf) and responses applied to the archive (response-archive-pvalues.pdf)

D.3 Comparison of Dynamic Multi-objective Optimisation Algorithms

Additional data with regards to the comparison of DMOAs solving DMOOPs that can be found on the CD are:

- Tables containing performance measure values that were obtained by the DMOAs (comparison-performance-measure-values.pdf)
- Tables presenting the p-values of the Mann-Whitney U tests that were performed on the data obtained by the DMOAs (comparison-pvalues.pdf)

Appendix E

List of Publications

This appendix lists the publications that were derived from the research conducted for this thesis.

Journal Articles

M. Helbig and A.P. Engelbrecht. Dynamic multi-objective optimisation problems. Submitted for review.

M. Helbig and A.P. Engelbrecht. Performance measures for dynamic multi-objective optimisation algorithms. Submitted for review.

Conference Papers

M. Greeff and A.P. Engelbrecht. Solving dynamic multi-objective problems with vector evaluated particle swarm optimisation. In *Proc. of IEEE World Congress on Computational Intelligence: IEEE Congress on Evolutionary Computation*, pages 2917–2924, Hong Kong, June 2008.

M. Helbig and A.P. Engelbrecht. Archive management for dynamic multi-objective optimisation problems using vector evaluated particle swarm optimisation. In *Proc. of the*

IEEE Congress on Evolutionary Computation, pages 2047–2054, New Orleans, U.S.A., June 2011.

M. Helbig and A.P. Engelbrecht. Analyses of Guide Update Approaches for Vector Evaluated Particle Swarm Optimisation on Dynamic Multi-Objective Optimisation Problems. In *Proc. of IEEE World Congress on Computational Intelligence: IEEE Congress on Evolutionary Computation*, pages 2621–2628, Brisbane, June, 2012.

Book Chapters

M. Greeff and A.P. Engelbrecht. Dynamic multi-objective optimisation using PSO. In Nadia Nedjah, Leandro dos Santos Coelho, and Luiza de Macedo Mourelle, editors, *Multi-Objective Swarm Intelligent Systems*, volume 261 of *Studies in Computational Intelligence*, pages 105–123, Springer Berlin/Heidelberg, 2010.

M. Greeff and A.P. Engelbrecht. Dynamic multi-objective optimisation using PSO. In *Metaheuristics for Dynamic Optimization*, Springer, To be published.