Chapter 11

Comparing the Dynamic Vector Evaluated Particle Swarm Optimisation Algorithm against State-of-the-art Dynamic Multi-objective Optimisation Algorithms

> *"Learn to adjust yourself to the conditions you have to endure, but make a point of trying to alter or correct conditions so that they are most favorable to you."* – William Frederick Book

In order to determine how efficiently a DMOO algorithm solves DMOOPs, its performance should be compared against the performance of other DMOO algorithms. This chapter discusses experiments conducted to compare the performance of DVEPSO against four state-of-the art DMOO algorithms. The experimental setup is discussed in Section 11.1. Section 11.2 discusses the results that were obtained by the various DMOO algorithms. Finally, the chapter is summarised in Section 11.3.

11.1 Experimental Setup

This section describes the experimental setup of the experiments discussed in this chapter. The process followed to optimise the parameters of the DMOO algorithms are discussed in Section 11.1.1. Section 11.1.2 discusses the experiments that were conducted to compare the performance of the DMOO algorithms. Furthermore, the statistical analysis that were performed on the obtained results are described.

11.1.1 Parameter Optimisation of Dynamic Multi-objective Optimisation Algorithms

The four DMOO algorithms that DVEPSO were compared against in the experiments are:

- The DNSGA-II-A algorithm, an NSGA-II algorithm adapted for DMOO and proposed by Deb *et al.* [46]. If a change in the environment is detected, a percentage of individuals are randomly selected and replaced with newly created individuals.
- The DNSGA-II-B algorithm, an NSGA-II algorithm that selects a percentage of individuals randomly and replaces them with individuals that are mutated from existing individuals when a change is detected. DNSGA-II-B was proposed by Deb *et al.* [46].
- The dCOEA algorithm, a dynamic competitive-cooperative coevolutionary algorithm proposed by Goh and Tan [67].
- The DMOPSO algorithm, a MOPSO algorithm adapted for DMOO by Lechuga [102].

The DNSGA-II algorithms were selected since NSGA-II performed so well with MOO that it became a benchmark in the field. DMOPSO is one of only two PSO algorithms proposed for DMOO and was one of the first PSO algorithms proposed to solve MOOPs. The dCOEA algorithm was selected as a multi-population approach where the sub-populations co-operate with one another, as is the case with DVEPSO. These four algorithms are discussed in more detail in Chapter 8.

The source code of the dCOEA algorithm was obtained from the authors of [67]. The source code of the static NSGA-II algorithm was obtained from [109] and was adapted for DMOO according to [46]. The source code of MOPSO was obtained from the authors of [102], adapted for DMOO according to [102] and extended using sentry particles to detect a change in the environment.

For each of the three EAs listed above, the following approach was followed to optimise its parameters: the parameters and values for the parameters were identified from the literature. For each parameter value, the algorithm executed 30 independent runs and each run continued for 1000 iterations. Fifteen benchmark functions were used as discussed in Section 9.4.1. For all benchmark functions, the severity of change (n_t) was set to 1, 10 and 20 and the frequency of change (τ_t) was set to 10, 25 and 50. Three performance measures were used as discussed in Section 9.4.1. Wins and losses were calculated for each parameter value as discussed in Section 9.4.1. Based on the wins and losses, the best value for the parameter was selected. For dCOEA there was no statistical significant difference in the performance of the algorithm for different values of the SC_{ratio} and R_{size} parameters. Therefore, for these two parameters the default values of [67] were selected. For the PSO-based algorithms, the same c_1, c_2 and w values were used. The best configuration for DVEPSO was selected based on the results of the experiments discussed in Chapters 9 and 10.

According to Malan and Engelbrecht, using the same number of particles or individuals when comparing algorithms are not adequate to ensure a fair comparison [115]. Therefore, for these experiments the number of particles or individuals assigned to each algorithm depended upon the amount of new information that each individual contributed after an algorithm iteration. For the DNSGA-II algorithms, DVEPSO, and dynamic MOPSO (DMOPSO) each individual or particle provides a value for each decision variable, i.e. a complete solution. However, dCOEA's individuals only provide a value for one decision variable. Therefore, DNSGA-II, DVEPSO and DMOPSO were each assigned 100 individuals and dCOEA were assigned $100n_x$ individuals. The selected configuration for each of the DMOO algorithms is presented in Tables 11.1 and 11.2.

11.1.2 Experiments Comparing the Performance of Dynamic Multi-objective Optimisation Algorithms

Similar to the experiments discussed in Section 11.1.1, all experiments comparing the performance of the DMOAs consisted of 30 independent runs and each run continued

Table 11.1: Parameter values of the DMOEAs

DMOEA	n_i	p_c	p_m	**Other**
DNSGA-II-A	100	0.9	$\frac{4}{x_k}$	polynomial mutation, SBX crossover, $\zeta = 30$
DNSGA-II-B	100	0.8	$\frac{5}{x_k}$	polynomial mutation, SBX crossover, $\zeta = 30$
dCOEA	$100x_k$	0.5	$\frac{9}{L}$	$SC_{ratio} = 0.7$, $R_{size} = 5$

Table 11.2: Parameter values of the PSO-based DMOAs

DMOA	n_i	c_1	c_2	w	**Other**
DMOPSO	100	1.49	1.49	0.72	self-adapting σ_{share}
DVEPSO	100	1.49	1.49	0.72	guide update: p_s-g_r, boundary violation management: cl, knowledge sharing: ra-t, response applied to archive: ac, response applied to particles: ri-30-c

for 1000 iterations. For all benchmark functions, the severity of change (n_t) was set to 1, 10 and 20 and the frequency of change (τ_t) was set to 10, 25 and 50.

Eighteen benchmark functions were used to compare the performance of the five DMOO algorithms. The fifteen benchmark functions discussed in Section 9.4.1 were selected. In addition, three three-objective DMOOPs were used, namely FDA5, FDA5$_{iso}$ and FDA5$_{dec}$.

Three performance measures were used to quantify the performance of algorithms as discussed in Section 9.4.1, namely acc, $stab$ and NS.

Statistical analysis of the obtained data was performed as discussed in Section 9.4.1. The null hypotheses for these experiments was that there is no statistical significant difference between the performance of the five DMOO algorithms. The alternative hypothesis is that there is a difference in mean performance.

11.2 Results

This section presents the results obtained by the various DMOAs. The results are discussed considering the various n_t-τ_t combinations, with regards to three performance measures and with regards to DMOOP Types I to III. General observations are also highlighted. Tables 11.3 to 11.28 present the wins and losses. Only the tables highlighting interesting trends and that are therefore discussed, are presented in this section. The other wins and losses tables are presented in Appendix D. Only statistical significant values are included in the tables. The p-values obtained for the various Mann-Whitney U tests, as well as the average performance measure values, are presented in Appendix D.

Results with regards to Performance Measures

Table 11.3 presents the wins and losses for each performance measure calculated over all DMOOPs and all n_t-τ_t combinations.

Table 11.3: Overall Wins and Losses for Various Performance Measures

PM	**Results**	**DMOO Algorithm**				
		DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
acc	Wins	94	109	129	118	119
acc	Losses	120	104	164	86	95
acc	Diff	-26	5	-35	32	24
acc	Rank	4	3	5	**1**	2
stab	Wins	67	94	39	117	96
stab	Losses	89	66	200	39	50
stab	Diff	-22	28	-161	78	46
stab	Rank	4	3	5	**1**	2
NS	Wins	185	187	116	53	111
NS	Losses	83	78	202	195	129
NS	Diff	102	109	-86	-142	-18
NS	Rank	2	**1**	4	5	3

The following observations are made:

- DMOPSO obtained the best performance for *acc* and *stab* and ranked the worst for NS.
- DNSGA-II-B performed the best for NS.
- DNSGA-II-A and dCOEA were awarded more losses than wins for both *acc* and *stab*. In addition, dCOEA also obtained more losses than wins for NS.

- Both DNSGA-II algorithms obtained more wins than losses for NS. All other algorithms obtained more losses than wins for NS.
- The worst rank for *acc* and *stab* was obtained by dCOEA. Furthermore, DMOPSO obtained the worst rank for NS.
- DVEPSO obtained the second best rank for *acc* and *stab*, and the third best rank for NS.

Results with regards to Various Frequencies and Severities of Change

The wins and losses calculated over all performance measures and DMOOPs for the various n_t-τ_t combinations are presented in Table 11.4.

Table 11.4: Overall Wins and Losses for Various Frequencies and Severities of Change

n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
10	10	all	Wins	65	77	91	62	89
10	10	all	Losses	97	84	113	97	59
10	10	all	Diff	-32	-7	-22	-35	30
10	10	all	Rank	4	2	3	5	**1**
10	25	all	Wins	72	80	40	46	55
10	25	all	Losses	37	31	114	60	51
10	25	all	Diff	35	49	-74	-14	4
10	25	all	Rank	2	**1**	5	4	3
10	50	all	Wins	69	76	47	35	49
10	50	all	Losses	36	32	93	63	52
10	50	all	Diff	33	44	-46	-28	-3
10	50	all	Rank	2	**1**	5	4	3
1	10	all	Wins	66	72	49	87	73
1	10	all	Losses	72	62	115	42	56
1	10	all	Diff	-6	10	-66	45	17
1	10	all	Rank	4	3	5	**1**	2
20	10	all	Wins	74	85	57	58	60
20	10	all	Losses	50	39	131	58	56
20	10	all	Diff	24	46	-74	0	4
20	10	all	Rank	2	**1**	5	4	3

With regards to the various environments, the following are observed:

- Mixed results were obtained with regards to the various environments. DVEPSO performed the best for $n_t = 10$ and $\tau_t = 10$, DMOPSO performed the best for $n_t = 1$ and $\tau_t = 10$ and DNSGA-II-B performed the best for $n_t = 10$ and $\tau_t = 25$,

$n_t = 10$ and $\tau_t = 50$, and $n_t = 20$ and $\tau_t = 10$.

- DVEPSO was ranked in the top three for all environments.
- The worst performance for all environments, except $n_t = 10$ and $\tau_t = 10$, was obtained by dCOEA. For $n_t = 10$ and $\tau_t = 10$, DMOPSO performed the worst.
- DNSGA-II-B and DVEPSO performed well, obtaining more wins than losses for all environments, except one. More losses than wins were obtained by dCOEA for all environments. DMOPSO obtained more losses than wins for three environments, namely $n_t = 10$ and $\tau_t = 10$, $n_t = 10$ and $\tau_t = 25$, and $n_t = 10$ and $\tau_t = 50$. DNSGA-II-A was awarded more losses than wins for two environments, namely $n_t = 10$ and $\tau_t = 10$, and $n_t = 1$ and $\tau_t = 10$.

From Table 11.3 it is clear that the wins and losses obtained for NS may scew the results. Therefore, the wins and losses calculated over all DMOOPs and over *acc* and *stab* (excluding NS) for the different environment types are presented in Table 11.5.

Table 11.5: Overall Wins and Losses for Various Frequencies and Severities of Change measured over *acc* and *stab*

$\mathbf{n_t}$	$\boldsymbol{\tau_t}$	PM	Results	DMOO Algorithm				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
10	10	all	Wins	33	42	48	57	67
10	10	all	Losses	66	57	85	40	30
10	10	all	Diff	-33	-15	-37	17	37
10	10	all	Rank	4	3	5	2	**1**
10	25	all	Wins	34	41	16	38	32
10	25	all	Losses	24	19	76	18	24
10	25	all	Diff	10	22	-60	20	8
10	25	all	Rank	3	**1**	5	2	4
10	50	all	Wins	32	39	25	29	27
10	50	all	Losses	22	17	59	26	28
10	50	all	Diff	10	22	-34	3	-1
10	50	all	Rank	2	**1**	5	3	4
1	10	all	Wins	28	37	34	63	51
1	10	all	Losses	54	44	65	19	31
1	10	all	Diff	-26	-7	-31	44	20
1	10	all	Rank	4	3	5	**1**	2
20	10	all	Wins	34	44	45	48	38
20	10	all	Losses	43	33	79	22	32
20	10	all	Diff	-9	11	-34	26	6
20	10	all	Rank	4	2	5	**1**	3

The following observations are made:

- DVEPSO performed well in fast environments, obtaining the first, second and third rank in $n_t = 10$ and $\tau_t = 10$, $n_t = 1$ and $\tau_t = 10$, and $n_t = 20$ and $\tau_t = 10$, respectively. In slower changing environments it obtained the second lowest rank.
- DMOPSO performed well, being ranked in the top three in all environments and being ranked the best for $n_t = 1$ and $\tau_t = 10$, and $n_t = 20$ and $\tau_t = 10$.
- In slower changing environments ($n_t = 10$ and $\tau_t = 25$, and $n_t = 10$ and $\tau_t = 50$) DNSGA-II-B performed the best. In addition, DNSGA-II-B was ranked in the top three in all environments, similar to DMOPSO.
- DNSGA-II-A performed not so good, obtaining the second lowest rank in all environments with $\tau_t = 10$. In slower changing environments ($n_t = 10$ and $\tau_t = 25$, and $n_t = 10$ and $\tau_t = 50$) it obtained the second and third best rank.

Results for Various Dynamic Multi-objective Optimisation Problem Types

For the different DMOOP Types, the POS or POF or both changes over time. This section discusses the performance of the DMOAs with regards to the DMOOP Types I, II and III.

Type I DMOOPs

This section discusses the wins and losses of the guide update approaches for Type I DMOOPs, namely DIMP2 and dMOP3. The wins and losses for Type I DMOOPs obtained by the DMOAs over all n_t-τ_t combinations are presented in Table 11.6.

The following observations are made:

- Both DVEPSO and dCOEA obtained the best rank for *acc*. DNSGA-II-A performed the worst with regards to *acc* and was awarded 10 more losses than wins.
- With regards to *stab*, DNSGA-II-B and DMOPSO performed the best and dCOEA obtained the third best rank. The worst performance for *stab* was obtained by DVEPSO, obtaining thirteen more losses than wins.
- For NS, DNSGA-II-B performed the best and DVEPSO the worst. In addition, DNSGA-II-B was the only MOAs that obtained more wins than losses for NS.

Table 11.6: Overall Wins and Losses solving Type I DMOOPs for Various Performance Measures

$\mathbf{n_t}$	$\tau_\mathbf{t}$	**PM**	**Results**	**DMOO Algorithm**				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
all	all	*acc*	Wins	7	11	23	9	20
all	all	*acc*	Losses	17	13	15	13	12
all	all	*acc*	Diff	-10	-2	8	-4	8
all	all	*acc*	Rank	5	3	**1**	4	**1**
all	all	*stab*	Wins	6	15	6	13	1
all	all	*stab*	Losses	18	14	14	12	14
all	all	*stab*	Diff	-12	1	-8	1	-13
all	all	*stab*	Rank	4	**1**	3	**1**	5
all	all	*NS*	Wins	14	16	8	6	0
all	all	*NS*	Losses	15	13	17	19	15
all	all	*NS*	Diff	-1	3	-9	-13	-15
all	all	*NS*	Rank	2	**1**	3	4	5

Table 11.7 presents the wins and losses for Type I DMOOPs obtained by the DMOAs in various types of environments.

The following is observed for the various n_t-τ_t combinations:

- For $n_t = 10$ and $\tau_t = 10$, dCOEA performed the best and DNSGA-II-A performed the worst. All algorithms, except dCOEA, obtained more losses than wins.
- DNSGA-II-B obtained the best rank for slow changing environments, namely $n_t = 10$ and $\tau_t = 25$, and $n_t = 10$ and $\tau_t = 50$. For $n_t = 10$ and $\tau_t = 25$, dCOEA performed the worst and for $n_t = 10$ and $\tau_t = 50$, DMOPSO performed the worst.
- In fast and severely changing environments ($n_t = 1$ and $\tau_t = 10$), DMOPSO obtained the best rank and was the only algorithm being awarded more wins than losses. For $n_t = 1$ and $\tau_t = 10$, the worst performance was obtained by DVEPSO.
- In slow and gradually changing environments ($n_t = 20$ and $\tau_t = 10$), DNSGA-II-B performed the best and DNSGA-II-A performed the worst.

Table 11.7: Overall Wins and Losses solving Type I DMOOPs for Various Frequencies and Severities of Change

n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
10	10	all	Wins	4	7	12	6	4
10	10	all	Losses	14	10	5	9	9
10	10	all	Diff	-10	-3	7	-3	-5
10	10	all	Rank	5	2	**1**	2	4
10	25	all	Wins	5	8	6	5	4
10	25	all	Losses	8	7	11	8	7
10	25	all	Diff	-3	1	-5	-3	-3
10	25	all	Rank	2	**1**	5	2	2
10	50	all	Wins	9	12	7	2	4
10	50	all	Losses	8	6	11	13	9
10	50	all	Diff	1	6	-4	-11	-5
10	50	all	Rank	2	**1**	3	5	4
1	10	all	Wins	5	6	6	9	4
1	10	all	Losses	9	10	9	6	9
1	10	all	Diff	-4	-4	-3	3	-5
1	10	all	Rank	3	3	2	**1**	5
20	10	all	Wins	4	9	6	6	5
20	10	all	Losses	11	7	10	8	7
20	10	all	Diff	-7	2	-4	-2	-2
20	10	all	Rank	5	**1**	4	2	2

The wins and losses for all Type I DMOOPs over all performance measures and all n_t-τ_t combinations are presented in Table 11.8. DNSGA-II-B performed the best, with dCOEA obtaining the second best rank. DNSGA-II-B was the only algorithm that obtained more wins than losses, while all the other MOAs obtained more losses than wins.

Table 11.8: Overall Wins and Losses solving Type I DMOOPs

n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
all	all	all	Wins	27	42	37	28	21
all	all	all	Losses	50	40	46	44	41
all	all	all	Diff	-23	2	-9	-16	-20
all	all	all	Rank	5	**1**	2	3	4

Furthermore, it should be noted that only DVEPSO and dCOEA were able to solve DIMP2, and DVEPSO was the only algorithm that converged successfully to the true

POF for DIMP2. The DNSGA-II algorithms found only one solution per run and DMOPSO did not find any solutions.

The wins and losses obtained by DVEPSO and dCOEA for DIMP2 are presented in Table 11.9. The following are observed:

- DVEPSO completely outperformed dCOEA with regards to *acc*.
- For $n_t = 10$ and $\tau_t = 10$ there was no statistically significant difference in the *stab* values of the two algorithms. However, for all other n_t-τ_t combinations DVEPSO outperformed dCOEA.
- There was no statistically significant difference in the performance of DVEPSO and dCOEA with regards to NS.
- Measuring the performance of the algorithms for each n_t-τ_t combination over all performance measures, DVEPSO outperformed dCOEA for all n_t-τ_t combinations.
- DVEPSO completely outperformed dCOEA, obtaining 9 wins, with dCOEA obtaining zero wins.

Table 11.9: Wins and Losses of DIMP2 obtained by the DMOO algorithms

$\mathbf{n_t}$	τ_t	PM	Results	DMOO Algorithm	
				dCOEA	DVEPSO
10	10	*acc*	Wins	0	1
10	10	*acc*	Losses	1	0
10	10	*acc*	Diff	-1	1
10	10	*acc*	Rank	2	**1**
10	25	*acc*	Wins	0	1
10	25	*acc*	Losses	1	0
10	25	*acc*	Diff	-1	1
10	25	*acc*	Rank	2	**1**
10	50	*acc*	Wins	0	1
10	50	*acc*	Losses	1	0
10	50	*acc*	Diff	-1	1
10	50	*acc*	Rank	2	**1**
1	10	*acc*	Wins	0	1
1	10	*acc*	Losses	1	0
1	10	*acc*	Diff	-1	1
1	10	*acc*	Rank	2	**1**
20	10	*acc*	Wins	0	1
20	10	*acc*	Losses	1	0
20	10	*acc*	Diff	-1	1
20	10	*acc*	Rank	2	**1**
all	all	*acc*	Wins	0	5
all	all	*acc*	Losses	5	0
all	all	*acc*	Diff	-5	5
all	all	*acc*	Rank	2	**1**

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n_t	τ_t	PM	Results	DMOO Algorithm	
				dCOEA	DVEPSO
10	10	*stab*	Wins	0	0
10	10	*stab*	Losses	0	0
10	10	*stab*	Diff	0	0
10	10	*stab*	Rank	**1**	**1**
10	25	*stab*	Wins	0	1
10	25	*stab*	Losses	1	0
10	25	*stab*	Diff	-1	1
10	25	*stab*	Rank	2	**1**
10	50	*stab*	Wins	0	1
10	50	*stab*	Losses	1	0
10	50	*stab*	Diff	-1	1
10	50	*stab*	Rank	2	**1**
1	10	*stab*	Wins	0	1
1	10	*stab*	Losses	1	0
1	10	*stab*	Diff	-1	1
1	10	*stab*	Rank	2	**1**
20	10	*stab*	Wins	0	1
20	10	*stab*	Losses	1	0
20	10	*stab*	Diff	-1	1
20	10	*stab*	Rank	2	**1**
all	all	*stab*	Wins	0	4
all	all	*stab*	Losses	4	0
all	all	*stab*	Diff	-4	4
all	all	*stab*	Rank	2	**1**
10	10	NS	Wins	0	0
10	10	NS	Losses	0	0
10	10	NS	Diff	0	0
10	10	NS	Rank	**1**	**1**
10	25	NS	Wins	0	0
10	25	NS	Losses	0	0
10	25	NS	Diff	0	0
10	25	NS	Rank	**1**	**1**
10	50	NS	Wins	0	0
10	50	NS	Losses	0	0
10	50	NS	Diff	0	0
10	50	NS	Rank	**1**	**1**
1	10	NS	Wins	0	0
1	10	NS	Losses	0	0
1	10	NS	Diff	0	0
1	10	NS	Rank	**1**	**1**
20	10	NS	Wins	0	0
20	10	NS	Losses	0	0
20	10	NS	Diff	0	0
20	10	NS	Rank	**1**	**1**
all	all	NS	Wins	0	0
all	all	NS	Losses	0	0
all	all	NS	Diff	0	0
all	all	NS	Rank	**1**	**1**
10	10	all	Wins	0	1
10	10	all	Losses	1	0

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n_t	τ_t	PM	Results	DMOO Algorithm	
				dCOEA	DVEPSO
10	10	all	Diff	-1	1
10	10	all	Rank	2	**1**
10	25	all	Wins	0	2
10	25	all	Losses	2	0
10	25	all	Diff	-2	2
10	25	all	Rank	2	**1**
10	50	all	Wins	0	2
10	50	all	Losses	2	0
10	50	all	Diff	-2	2
10	50	all	Rank	2	**1**
1	10	all	Wins	0	2
1	10	all	Losses	2	0
1	10	all	Diff	-2	2
1	10	all	Rank	2	**1**
20	10	all	Wins	0	2
20	10	all	Losses	2	0
20	10	all	Diff	-2	2
20	10	all	Rank	2	**1**
all	all	all	Wins	0	9
all	all	all	Losses	9	0
all	all	all	Diff	-9	9
all	all	all	Rank	2	**1**

Type II DMOOPs

The wins and losses for Type II DMOOPs obtained by the DMOAs over all n_t-τ_t combinations are presented in Table 11.10. The Type II DMOOPs are FDA1$_{Zhou}$, FDA2, FDA3, FDA3$_{Camara}$, dMOP2, dMOP2$_{iso}$, dMOP2$_{dec}$, FDA5, FDA5$_{iso}$ and FDA5$_{dec}$.

Table 11.10: Overall Wins and Losses solving Type II DMOOPs for Various Performance Measures

n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
all	all	*acc*	Wins	55	55	36	56	63
all	all	*acc*	Losses	43	41	106	41	34
all	all	*acc*	Diff	12	14	-70	15	29
all	all	*acc*	Rank	4	3	5	2	**1**
all	all	*stab*	Wins	36	45	18	53	59
all	all	*stab*	Losses	43	30	104	20	14
all	all	*stab*	Diff	-7	15	-86	33	45
all	all	*stab*	Rank	4	3	5	2	**1**
all	all	*NS*	Wins	96	92	60	31	90
all	all	*NS*	Losses	48	50	116	105	50
all	all	*NS*	Diff	48	42	-56	-74	40
all	all	*NS*	Rank	**1**	2	4	5	3

The following observations are made:

- DVEPSO performed the best with regards to *acc* and *stab*. The best performance with regards to NS was obtained by DNSGA-II-A.
- The worst performance for both *acc* and *stab* was obtained by dCOEA. For NS, DMOPSO performed the worst.
- dCOEA performed poorly with regards to *acc*, being awarded more losses than wins.
- For *stab*, both DNSGA-II-A and dCOEA obtained more losses than wins.
- Only DMOPSO and dCOEA obtained more losses than wins for NS. All other algorithms performed well, obtaining more wins than losses for NS.

Table 11.11 presents the wins and losses for Type II DMOOPs obtained by the DMOAs in various types of environments.

Table 11.11: Overall Wins and Losses solving Type II DMOOPs for Various Frequencies and Severities of Change

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**DMOO Algorithm**				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
10	10	all	Wins	39	39	29	31	52
10	10	all	Losses	32	29	69	41	19
10	10	all	Diff	7	10	-40	-10	33
10	10	all	Rank	3	2	5	4	**1**
10	25	all	Wins	39	40	13	22	38
10	25	all	Losses	16	16	66	35	19
10	25	all	Diff	23	24	-53	-13	19
10	25	all	Rank	2	**1**	5	4	3
10	50	all	Wins	37	39	15	11	28
10	50	all	Losses	12	12	52	33	21
10	50	all	Diff	25	27	-37	-22	7
10	50	all	Rank	2	**1**	5	4	3
1	10	all	Wins	33	34	23	47	55
1	10	all	Losses	47	40	67	23	15
1	10	all	Diff	-14	-6	-44	24	40
1	10	all	Rank	4	3	5	2	**1**
20	10	all	Wins	39	40	34	29	39
20	10	all	Losses	27	24	72	34	24
20	10	all	Diff	12	16	-38	-5	15
20	10	all	Rank	3	**1**	5	4	2

The following are observed with regards to the various n_t-τ_t combinations:

- In fast changing environments ($n_t = 1$ and $\tau_t = 10$, and $n_t = 10$ and $\tau_t = 10$) DVEPSO obtained the best rank. Furthermore, DVEPSO was ranked in the top three for all environments.
- DNSGA-II-B obtained the best rank for $n_t = 10$ and $\tau_t = 25$, $n_t = 10$ and $\tau_t = 50$ and $n_t = 20$ and $\tau_t = 10$. Similar to DVEPSO, DNSGA-II-B was ranked in the top three for all environments.
- dCOEA performed the worst for all environments.
- DVEPSO performed really well, being the only algorithm that obtained more wins than losses for all environments. DNSGA-II-A and DNSGA-II-B also performed well, being awarded more losses than wins for only $n_t = 1$ and $\tau_t = 10$. More losses than wins were obtained by DMOPSO for all environments, except $n_t = 1$ and $\tau_t = 10$. dCOEA performed poorly, being awarded more losses than wins for all environments.

The results indicate that when solving Type II DMOOPs, DMOPSO obtains so much losses for NS, that it performs poorly even though its performance for *acc* and *stab* is good (refer to Table 11.10). Table 11.12 presents the wins and losses for Type II DMOOPs for various n_t-τ_t combinations, measured over *acc* and *stab* and therefore not taking NS into account.

The following are observed:

- DVEPSO performed the best for $n_t = 10$ and $\tau_t = 10$, and $n_t = 1$ and $\tau_t = 10$. For $n_t = 20$ and $\tau_t = 10$, DVEPSO obtained the second best rank and for the rest of the environments DVEPSO obtained the third best rank.
- DNSGA-II-B was awarded the best rank for $n_t = 10$ and $\tau_t = 25$, and $n_t = 10$ and $\tau_t = 50$. For all other environments, DNSGA-II-B obtained the third best rank.
- DMOPSO performed the best for $n_t = 20$ and $\tau_t = 10$. For $n_t = 10$ and $\tau_t = 10$, and $n_t = 1$ and $\tau_t = 10$, DMOPSO obtained the second best rank. However, for $n_t = 10$ and $\tau_t = 25$, and $n_t = 10$ and $\tau_t = 50$, DMOPSO was awarded the second lowest rank.
- dCOEA performed the worst for all environments.

Table 11.12: Overall Wins and Losses solving Type II DMOOPs for Various Frequencies and Severities of Change measured over *acc* and *stab*

n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
10	10	all	Wins	22	21	6	28	33
10	10	all	Losses	19	17	53	13	8
10	10	all	Diff	3	4	-47	15	25
10	10	all	Rank	4	3	5	2	**1**
10	25	all	Wins	19	21	1	18	19
10	25	all	Losses	8	7	44	10	9
10	25	all	Diff	11	14	-43	8	10
10	25	all	Rank	2	**1**	5	4	3
10	50	all	Wins	17	20	5	9	12
10	50	all	Losses	6	4	32	12	9
10	50	all	Diff	11	16	-27	-3	3
10	50	all	Rank	2	**1**	5	4	3
1	10	all	Wins	14	18	15	30	35
1	10	all	Losses	31	24	38	12	7
1	10	all	Diff	-17	-6	-23	18	28
1	10	all	Rank	4	3	5	2	**1**
20	10	all	Wins	19	20	27	24	23
20	10	all	Losses	22	19	43	14	15
20	10	all	Diff	-3	1	-16	10	8
20	10	all	Rank	4	3	5	**1**	2

The wins and losses for all Type II DMOOPs over all performance measures and all n_t-τ_t combinations are presented in Table 11.13. DVEPSO outperformed the other algorithms, obtaining the best rank and being awarded 114 more wins than losses. The worst rank was awarded to dCOEA. Both DMOPSO and dCOEA performed poorly, obtaining more losses than wins. In contrast, both DNSGA-II algorithms were awarded more wins than losses, with DNSGA-II-B and DNSGA-II-A obtaining the second and third best rank respectively.

Table 11.13: Overall Wins and Losses solving Type II DMOOPs

n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
all	all	all	Wins	187	192	114	140	212
all	all	all	Losses	134	121	326	166	98
all	all	all	Diff	53	71	-212	-26	114
all	all	all	Rank	3	2	5	4	**1**

The wins and losses measured over *acc* and *stab* (and not taking NS into account) are presented in Table 11.14. DVEPSO performed the best and DMOPSO obtained the second best rank. dCOEA performed the worst, obtaining 156 more losses than wins.

Table 11.14: Overall Wins and Losses solving Type II DMOOPs measured over *acc* and *stab*

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**DMOO Algorithm**				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
all	all	all	Wins	91	100	54	109	122
all	all	all	Losses	86	71	210	61	48
all	all	all	Diff	5	29	-156	48	74
all	all	all	Rank	4	3	5	2	**1**

Type III DMOOPs

Table 11.15 presents the wins and losses with regards to the various performance measures obtained by the DMOA for Type III DMOOPs. The Type III DMOOPs are HE1, HE2, HE6 to HE9 and FDA2$_{Camara}$.

Table 11.15: Overall Wins and Losses solving Type III DMOOPs for Various Performance Measures

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**DMOO Algorithm**				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
all	all	*acc*	Wins	32	43	70	53	36
all	all	*acc*	Losses	60	50	43	32	49
all	all	*acc*	Diff	-28	-7	27	21	-13
all	all	*acc*	Rank	5	3	**1**	2	4
all	all	*stab*	Wins	25	34	15	51	36
all	all	*stab*	Losses	28	22	82	7	22
all	all	*stab*	Diff	-3	12	-67	44	14
all	all	*stab*	Rank	4	3	5	**1**	2
all	all	NS	Wins	75	79	48	16	21
all	all	NS	Losses	20	15	69	71	64
all	all	NS	Diff	55	64	-21	-55	-43
all	all	NS	Rank	2	**1**	3	5	4

The following observations are made:

- dCOEA performed the best with regards to *acc*, with DNSGA-II-A performing the worst. Only dCOEA and DMOPSO obtained more wins than losses for *acc*.
- For *stab*, DMOPSO obtained the best performance and dCOEA performed the worst. Two DMOAs, dCOEA and DNSGA-II-A, were awarded more losses than

wins for *stab*.

- The best rank for *NS* was obtained by DNSGA-II-B and the worst rank was awarded to DMOPSO. More wins than losses were obtained by dCOEA and DMOPSO.
- DVEPSO obtained the second lowest rank for *acc* and *NS*, since it struggled to converge to discontinous POFs. For *stab*, DVEPSO obtained the second best rank.

The wins and losses with regards to the various environment types for Type III DMOOPs are presented in Table 11.16.

Table 11.16: Overall Wins and Losses solving Type III DMOOPs for Various Frequencies and Severities of Change

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**DMOO Algorithm**				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
10	10	all	Wins	22	27	38	25	17
10	10	all	Losses	27	21	27	23	31
10	10	all	Diff	-5	6	11	2	-14
10	10	all	Rank	4	2	**1**	3	5
10	25	all	Wins	28	33	24	19	17
10	25	all	Losses	19	14	40	23	25
10	25	all	Diff	9	19	-16	-4	-8
10	25	all	Rank	2	**1**	5	3	4
10	50	all	Wins	23	26	28	22	21
10	50	all	Losses	22	20	33	23	22
10	50	all	Diff	1	6	-5	-1	-1
10	50	all	Rank	2	**1**	5	3	3
1	10	all	Wins	28	33	23	31	18
1	10	all	Losses	22	18	42	19	32
1	10	all	Diff	6	15	-19	12	-14
1	10	all	Rank	3	**1**	5	2	4
20	10	all	Wins	31	37	20	23	20
20	10	all	Losses	18	14	52	22	25
20	10	all	Diff	13	23	-32	1	-5
20	10	all	Rank	2	**1**	5	3	4

For the various n_t-τ_t combinations, the following are observed:

- DNSGA-II-B obtained the best rank for all environments, except for $n_t = 10$ and $\tau_t = 10$, where it obtained the second best rank.
- dCOEA performed the worst in all environments, except $n_t = 10$ and $\tau_t = 10$. For $n_t = 10$ and $\tau_t = 10$, it performed the best.

- DMOPSO performed reasonably well, obtaining either the second or third best rank for all environments.
- DVEPSO obtained the third best rank for $n_t = 10$ and $\tau_t = 50$. However, it obtained the second lowest rank for $n_t = 10$ and $\tau_t = 25$, $n_t = 1$ and $\tau_t = 10$ and $n_t = 20$ and $\tau_t = 10$. Furthermore, DVEPSO obtained the worst rank for $n_t = 10$ and $\tau_t = 10$.
- DNSGA-II-B was the only algorithm that obtained more wins than losses for all n_t-τ_t combinations.

Table 11.17: Overall Wins and Losses solving Type III DMOOPs for Various Frequencies and Severities of Change

$\mathbf{n_t}$	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
10	10	all	Wins	22	27	38	25	17
10	10	all	Losses	27	21	27	23	31
10	10	all	Diff	-5	6	11	2	-14
10	10	all	Rank	4	2	**1**	3	5
10	25	all	Wins	28	33	24	19	17
10	25	all	Losses	19	14	40	23	25
10	25	all	Diff	9	19	-16	-4	-8
10	25	all	Rank	2	**1**	5	3	4
10	50	all	Wins	23	26	28	22	21
10	50	all	Losses	22	20	33	23	22
10	50	all	Diff	1	6	-5	-1	-1
10	50	all	Rank	2	**1**	5	3	3
1	10	all	Wins	28	33	23	31	18
1	10	all	Losses	22	18	42	19	32
1	10	all	Diff	6	15	-19	12	-14
1	10	all	Rank	3	**1**	5	2	4
20	10	all	Wins	31	37	20	23	20
20	10	all	Losses	18	14	52	22	25
20	10	all	Diff	13	23	-32	1	-5
20	10	all	Rank	2	**1**	5	3	4

Table 11.18 presents the wins and losses for Type III DMOOPs measured over all performance measures and all n_t-τ_t combinations. The best overall performance for Type III DMOOPs was obtained by DNSGA-II-B, with dCOEA performing the worst. DNSGA-II-A, DNSGA-II-B and DMOPSO were awarded more wins than losses. The DNSGA-II algorithms obtained the top two ranks and therefore outperformed the PSO-

based DMOAs.

Table 11.18: Overall Wins and Losses solving Type III DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**DMOO Algorithm**				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
all	all	all	Wins	132	156	133	120	93
all	all	all	Losses	108	87	194	110	135
all	all	all	Diff	24	69	-61	10	-42
all	all	all	Rank	2	**1**	5	3	4

Overall Performance

This section discusses the overall performance of the DMOAs. The overall wins and losses over all DMOOPs, n_t-τ_t combinations and performance measures are presented in Table 11.19.

Table 11.19: Overall Wins and Losses by the various DMOO algorithms

Results	**DMOO Algorithm**				
	DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
Wins	346	390	284	288	326
Losses	292	248	566	320	274
Diff	54	142	-282	-32	52
Rank	2	**1**	5	4	3

DNSGA-II-B performed the best obtaining 142 more wins than losses. The second best performance was obtained by DNSGA-II-A, being awarded 54 more wins than losses. DVEPSO was awarded the third best rank, obtaining 52 more wins than losses. The second worst rank was obtained by dCOEA and DMOPSO performed the worst. All DMOEAs obtained more wins than losses. On the other hand, all PSO-based DMOAs obtained more losses than wins. Therefore, the DMOEAs completely outperformed the PSO-based DMOAs.

However, many wins and losses obtained by the DMOEAs are for NS. Table 11.20 represents the overall wins and losses over all DMOOPs and n_t-τ_t combinations, without taking NS into account. The effect of the wins and losses with regards to NS can clearly be seen. DMOPSO now obtains the highest rank with 110 more wins than losses. The second best rank is awarded to DVEPSO, with DVEPSO being awarded 70 more wins than losses. DNSGA-II-B is ranked third, obtaining 33 more wins than losses.

DNSGA-II-A and dCOEA obtains the second lowest and lowest rank respectively, with both algorithms being awarded more losses than wins. Therefore, with regards to *acc* and *stab*, the PSO-based DMOAs completely outperforms the DMOEAs.

Table 11.20: Overall Wins and Losses for *acc* and *stab* by the various DMOO algorithms

Results	DMOO Algorithm				
	DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
Wins	161	203	168	235	215
Losses	209	170	364	125	145
Diff	-48	33	-196	110	70
Rank	4	3	5	**1**	2

General Observations

This section discusses general observations that were made about the performance of the DMOAs. Trends that vary from the overall performance of the DMOA (presented in Table 11.19) are highlighted below.

The difference in performance for dMOP2, dMOP2$_{iso}$ and dMOP2$_{dec}$ are presented in Tables 11.21 to 11.23. For dMOP2, DVEPSO obtained the best overall performance and DMOPSO obtained the second best rank. The lowest two ranks were awarded to DNSGA-II-A and dCOEA, with dCOEA obtaining more losses than wins. In contrast, for dMOP2$_{iso}$ the DNSGA-II algorithms obtained the top two ranks, with DVEPSO obtaining the third best rank. The DNSGA-II algorithms and DVEPSO obtained more wins than losses. Furthermore, dCOEA performed the worst. For dMOP2$_{dec}$, DVEPSO performed the best, with DNSGA-II-B and DNSGA-II-A obtaining the second and third best rank respectively. dCOEA performed the worst, and was the only algorithm that was awarded more losses than wins.

Table 11.21: Wins and Losses of dMOP2 obtained by the DMOO algorithms

$\mathbf{n_t}$	τ_t	**PM**	**Results**	DMOO Algorithm				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
10	10	*acc*	Wins	0	0	0	3	3
10	10	*acc*	Losses	2	2	2	0	0
10	10	*acc*	Diff	-2	-2	-2	3	3
10	10	*acc*	Rank	3	3	3	**1**	**1**
10	25	*acc*	Wins	1	1	0	1	1
10	25	*acc*	Losses	0	0	4	0	0

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n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
10	25	*acc*	Diff	1	1	-4	1	1
10	25	*acc*	Rank	**1**	**1**	5	**1**	**1**
10	50	*acc*	Wins	3	4	0	1	1
10	50	*acc*	Losses	1	0	4	2	2
10	50	*acc*	Diff	2	4	-4	-1	-1
10	50	*acc*	Rank	2	**1**	5	3	3
1	10	*acc*	Wins	0	1	1	3	3
1	10	*acc*	Losses	3	2	3	0	0
1	10	*acc*	Diff	-3	-1	-2	3	3
1	10	*acc*	Rank	5	3	4	**1**	**1**
20	10	*acc*	Wins	1	1	0	1	1
20	10	*acc*	Losses	0	0	4	0	0
20	10	*acc*	Diff	1	1	-4	1	1
20	10	*acc*	Rank	**1**	**1**	5	**1**	**1**
all	all	*acc*	Wins	5	7	1	9	9
all	all	*acc*	Losses	6	4	17	2	2
all	all	*acc*	Diff	-1	3	-16	7	7
all	all	*acc*	Rank	4	3	5	**1**	**1**
10	10	*stab*	Wins	1	1	0	1	1
10	10	*stab*	Losses	0	0	4	0	0
10	10	*stab*	Diff	1	1	-4	1	1
10	10	*stab*	Rank	**1**	**1**	5	**1**	**1**
10	25	*stab*	Wins	1	1	0	1	1
10	25	*stab*	Losses	0	0	4	0	0
10	25	*stab*	Diff	1	1	-4	1	1
10	25	*stab*	Rank	**1**	**1**	5	**1**	**1**
10	50	*stab*	Wins	1	1	0	1	1
10	50	*stab*	Losses	0	0	4	0	0
10	50	*stab*	Diff	1	1	-4	1	1
10	50	*stab*	Rank	**1**	**1**	5	**1**	**1**
1	10	*stab*	Wins	0	0	0	3	3
1	10	*stab*	Losses	2	2	2	0	0
1	10	*stab*	Diff	-2	-2	-2	3	3
1	10	*stab*	Rank	3	3	3	**1**	**1**
20	10	*stab*	Wins	1	1	0	3	3
20	10	*stab*	Losses	2	2	4	0	0
20	10	*stab*	Diff	-1	-1	-4	3	3
20	10	*stab*	Rank	3	3	5	**1**	**1**
all	all	*stab*	Wins	4	4	0	9	9
all	all	*stab*	Losses	4	4	18	0	0
all	all	*stab*	Diff	0	0	-18	9	9
all	all	*stab*	Rank	3	3	5	**1**	**1**
10	10	*NS*	Wins	1	1	3	0	3
10	10	*NS*	Losses	2	2	1	3	0
10	10	*NS*	Diff	-1	-1	2	-3	3
10	10	*NS*	Rank	3	3	2	5	**1**
10	25	*NS*	Wins	2	2	0	0	2
10	25	*NS*	Losses	1	1	2	2	0
10	25	*NS*	Diff	1	1	-2	-2	2
10	25	*NS*	Rank	2	2	4	4	**1**

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n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
10	50	NS	Wins	2	2	0	0	2
10	50	NS	Losses	1	1	2	2	0
10	50	NS	Diff	1	1	-2	-2	2
10	50	NS	Rank	2	2	4	4	**1**
1	10	NS	Wins	2	1	0	3	3
1	10	NS	Losses	2	3	4	0	0
1	10	NS	Diff	0	-2	-4	3	3
1	10	NS	Rank	3	4	5	**1**	**1**
20	10	NS	Wins	2	2	0	1	3
20	10	NS	Losses	1	1	4	2	0
20	10	NS	Diff	1	1	-4	-1	3
20	10	NS	Rank	2	2	5	4	**1**
all	all	NS	Wins	9	8	3	4	13
all	all	NS	Losses	7	8	13	9	0
all	all	NS	Diff	2	0	-10	-5	13
all	all	NS	Rank	2	3	5	4	**1**
10	10	all	Wins	2	2	3	4	7
10	10	all	Losses	4	4	7	3	0
10	10	all	Diff	-2	-2	-4	1	7
10	10	all	Rank	3	3	5	2	**1**
10	25	all	Wins	4	4	0	2	4
10	25	all	Losses	1	1	10	2	0
10	25	all	Diff	3	3	-10	0	4
10	25	all	Rank	2	2	5	4	**1**
10	50	all	Wins	6	7	0	2	4
10	50	all	Losses	2	1	10	4	2
10	50	all	Diff	4	6	-10	-2	2
10	50	all	Rank	2	**1**	5	4	3
1	10	all	Wins	2	2	1	9	9
1	10	all	Losses	7	7	9	0	0
1	10	all	Diff	-5	-5	-8	9	9
1	10	all	Rank	3	3	5	**1**	**1**
all	all	all	Wins	18	19	4	22	31
all	all	all	Losses	17	16	48	11	2
all	all	all	Diff	1	3	-44	11	29
all	all	all	Rank	4	3	5	2	**1**

Table 11.22: Wins and Losses of dMOP2$_{iso}$ obtained by the DMOO algorithms

n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
10	10	acc	Wins	3	3	2	0	0
10	10	acc	Losses	0	0	2	3	3
10	10	acc	Diff	3	3	0	-3	-3
10	10	acc	Rank	**1**	**1**	3	4	4
10	25	acc	Wins	3	3	0	0	0
10	25	acc	Losses	0	0	2	2	2
10	25	acc	Diff	3	3	-2	-2	-2
10	25	acc	Rank	**1**	**1**	3	3	3

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n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
10	50	*acc*	Wins	3	3	1	0	1
10	50	*acc*	Losses	0	0	3	3	2
10	50	*acc*	Diff	3	3	-2	-3	-1
10	50	*acc*	Rank	**1**	**1**	4	5	3
1	10	*acc*	Wins	3	3	0	1	1
1	10	*acc*	Losses	0	0	4	2	2
1	10	*acc*	Diff	3	3	-4	-1	-1
1	10	*acc*	Rank	**1**	**1**	5	3	3
20	10	*acc*	Wins	3	3	0	1	1
20	10	*acc*	Losses	0	0	4	2	2
20	10	*acc*	Diff	3	3	-4	-1	-1
20	10	*acc*	Rank	**1**	**1**	5	3	3
all	all	*acc*	Wins	15	15	3	2	3
all	all	*acc*	Losses	0	0	15	12	11
all	all	*acc*	Diff	15	15	-12	-10	-8
all	all	*acc*	Rank	**1**	**1**	5	4	3
10	10	*stab*	Wins	2	2	0	0	2
10	10	*stab*	Losses	1	1	2	2	0
10	10	*stab*	Diff	1	1	-2	-2	2
10	10	*stab*	Rank	2	2	4	4	**1**
10	25	*stab*	Wins	3	3	0	1	1
10	25	*stab*	Losses	0	0	4	2	2
10	25	*stab*	Diff	3	3	-4	-1	-1
10	25	*stab*	Rank	**1**	**1**	5	3	3
10	50	*stab*	Wins	1	1	0	0	0
10	50	*stab*	Losses	0	0	2	0	0
10	50	*stab*	Diff	1	1	-2	0	0
10	50	*stab*	Rank	**1**	**1**	5	3	3
1	10	*stab*	Wins	2	2	0	1	3
1	10	*stab*	Losses	1	1	4	2	0
1	10	*stab*	Diff	1	1	-4	-1	3
1	10	*stab*	Rank	2	2	5	4	**1**
20	10	*stab*	Wins	2	3	0	1	3
20	10	*stab*	Losses	2	1	4	2	0
20	10	*stab*	Diff	0	2	-4	-1	3
20	10	*stab*	Rank	3	2	5	4	**1**
all	all	*stab*	Wins	10	11	0	3	9
all	all	*stab*	Losses	4	3	16	8	2
all	all	*stab*	Diff	6	8	-16	-5	7
all	all	*stab*	Rank	3	**1**	5	4	2
10	10	*NS*	Wins	1	1	1	0	2
10	10	*NS*	Losses	1	1	0	3	0
10	10	*NS*	Diff	0	0	1	-3	2
10	10	*NS*	Rank	3	3	2	5	**1**
10	25	*NS*	Wins	1	1	1	0	3
10	25	*NS*	Losses	1	1	1	3	0
10	25	*NS*	Diff	0	0	0	-3	3
10	25	*NS*	Rank	2	2	2	5	**1**
10	50	*NS*	Wins	1	1	1	0	3
10	50	*NS*	Losses	1	1	1	3	0

Continued on next page

$\mathbf{n_t}$	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
10	50	*NS*	Diff	0	0	0	-3	3
10	50	*NS*	Rank	2	2	2	5	**1**
1	10	*NS*	Wins	2	2	0	1	3
1	10	*NS*	Losses	1	1	4	2	0
1	10	*NS*	Diff	1	1	-4	-1	3
1	10	*NS*	Rank	2	2	5	4	**1**
20	10	*NS*	Wins	2	2	0	1	3
20	10	*NS*	Losses	1	1	4	2	0
20	10	*NS*	Diff	1	1	-4	-1	3
20	10	*NS*	Rank	2	2	5	4	**1**
all	all	*NS*	Wins	7	7	3	2	14
all	all	*NS*	Losses	5	5	10	13	0
all	all	*NS*	Diff	2	2	-7	-11	14
all	all	*NS*	Rank	2	2	4	5	**1**
10	10	all	Wins	6	6	3	0	4
10	10	all	Losses	2	2	4	8	3
10	10	all	Diff	4	4	-1	-8	1
10	10	all	Rank	**1**	**1**	4	5	3
10	25	all	Wins	7	7	1	1	4
10	25	all	Losses	1	1	7	7	4
10	25	all	Diff	6	6	-6	-6	0
10	25	all	Rank	**1**	**1**	4	4	3
10	50	all	Wins	5	5	2	0	4
10	50	all	Losses	1	1	6	6	2
10	50	all	Diff	4	4	-4	-6	2
10	50	all	Rank	**1**	**1**	4	5	3
1	10	all	Wins	7	7	0	3	7
1	10	all	Losses	2	2	12	6	2
1	10	all	Diff	5	5	-12	-3	5
1	10	all	Rank	**1**	**1**	5	4	**1**
20	10	all	Wins	7	8	0	3	7
20	10	all	Losses	3	2	12	6	2
20	10	all	Diff	4	6	-12	-3	5
20	10	all	Rank	3	**1**	5	4	2
all	all	all	Wins	32	33	6	7	26
all	all	all	Losses	9	8	41	33	13
all	all	all	Diff	23	25	-35	-26	13
all	all	all	Rank	2	**1**	5	4	3

Table 11.23: Wins and Losses of dMOP2$_{dec}$ obtained by the DMOO algorithms

$\mathbf{n_t}$	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
10	10	*acc*	Wins	3	3	0	1	1
10	10	*acc*	Losses	0	0	4	2	2
10	10	*acc*	Diff	3	3	-4	-1	-1
10	10	*acc*	Rank	**1**	**1**	5	3	3
10	25	*acc*	Wins	3	3	0	1	1
10	25	*acc*	Losses	0	0	4	2	2

Continued on next page

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**DMOO Algorithm**				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
10	25	*acc*	Diff	3	3	-4	-1	-1
10	25	*acc*	Rank	**1**	**1**	5	3	3
10	50	*acc*	Wins	3	3	0	1	1
10	50	*acc*	Losses	0	0	4	2	2
10	50	*acc*	Diff	3	3	-4	-1	-1
10	50	*acc*	Rank	**1**	**1**	5	3	3
1	10	*acc*	Wins	3	3	0	1	1
1	10	*acc*	Losses	0	0	4	2	2
1	10	*acc*	Diff	3	3	-4	-1	-1
1	10	*acc*	Rank	**1**	**1**	5	3	3
20	10	*acc*	Wins	1	1	0	3	3
20	10	*acc*	Losses	2	2	4	0	0
20	10	*acc*	Diff	-1	-1	-4	3	3
20	10	*acc*	Rank	3	3	5	**1**	**1**
all	all	*acc*	Wins	13	13	0	7	7
all	all	*acc*	Losses	2	2	20	8	8
all	all	*acc*	Diff	11	11	-20	-1	-1
all	all	*acc*	Rank	**1**	**1**	5	3	3
10	10	*stab*	Wins	2	3	0	1	3
10	10	*stab*	Losses	2	1	4	2	0
10	10	*stab*	Diff	0	2	-4	-1	3
10	10	*stab*	Rank	3	2	5	4	**1**
10	25	*stab*	Wins	2	3	0	1	3
10	25	*stab*	Losses	2	1	4	2	0
10	25	*stab*	Diff	0	2	-4	-1	3
10	25	*stab*	Rank	3	2	5	4	**1**
10	50	*stab*	Wins	2	3	0	1	3
10	50	*stab*	Losses	2	1	4	2	0
10	50	*stab*	Diff	0	2	-4	-1	3
10	50	*stab*	Rank	3	2	5	4	**1**
1	10	*stab*	Wins	2	3	0	1	3
1	10	*stab*	Losses	2	1	4	2	0
1	10	*stab*	Diff	0	2	-4	-1	3
1	10	*stab*	Rank	3	2	5	4	**1**
20	10	*stab*	Wins	1	1	0	3	3
20	10	*stab*	Losses	2	2	4	0	0
20	10	*stab*	Diff	-1	-1	-4	3	3
20	10	*stab*	Rank	3	3	5	**1**	**1**
all	all	*stab*	Wins	9	13	0	7	15
all	all	*stab*	Losses	10	6	20	8	0
all	all	*stab*	Diff	-1	7	-20	-1	15
all	all	*stab*	Rank	3	2	5	3	**1**
10	10	*NS*	Wins	1	1	3	0	3
10	10	*NS*	Losses	2	2	1	3	0
10	10	*NS*	Diff	-1	-1	2	-3	3
10	10	*NS*	Rank	3	3	2	5	**1**
10	25	*NS*	Wins	1	1	1	0	3
10	25	*NS*	Losses	1	1	1	3	0
10	25	*NS*	Diff	0	0	0	-3	3
10	25	*NS*	Rank	2	2	2	5	**1**

Continued on next page

n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
10	50	NS	Wins	1	1	1	0	3
10	50	NS	Losses	1	1	1	3	0
10	50	NS	Diff	0	0	0	-3	3
10	50	NS	Rank	2	2	2	5	**1**
1	10	NS	Wins	1	1	0	3	3
1	10	NS	Losses	2	2	4	0	0
1	10	NS	Diff	-1	-1	-4	3	3
1	10	NS	Rank	3	3	5	**1**	**1**
20	10	NS	Wins	2	2	0	1	3
20	10	NS	Losses	1	1	4	2	0
20	10	NS	Diff	1	1	-4	-1	3
20	10	NS	Rank	2	2	5	4	**1**
all	all	NS	Wins	6	6	5	4	15
all	all	NS	Losses	7	7	11	11	0
all	all	NS	Diff	-1	-1	-6	-7	15
all	all	NS	Rank	2	2	4	5	**1**
10	10	all	Wins	6	7	3	2	7
10	10	all	Losses	4	3	9	7	2
10	10	all	Diff	2	4	-6	-5	5
10	10	all	Rank	3	2	5	4	**1**
10	25	all	Wins	6	7	1	2	7
10	25	all	Losses	3	2	9	7	2
10	25	all	Diff	3	5	-8	-5	5
10	25	all	Rank	3	**1**	5	4	**1**
10	50	all	Wins	6	7	1	2	7
10	50	all	Losses	3	2	9	7	2
10	50	all	Diff	3	5	-8	-5	5
10	50	all	Rank	3	**1**	5	4	**1**
1	10	all	Wins	6	7	0	5	7
1	10	all	Losses	4	3	12	4	2
1	10	all	Diff	2	4	-12	1	5
1	10	all	Rank	3	2	5	4	**1**
20	10	all	Wins	4	4	0	7	9
20	10	all	Losses	5	5	12	2	0
20	10	all	Diff	-1	-1	-12	5	9
20	10	all	Rank	3	3	5	2	**1**
all	all	all	Wins	28	32	5	18	37
all	all	all	Losses	19	15	51	27	8
all	all	all	Diff	9	17	-46	-9	29
all	all	all	Rank	3	2	5	4	**1**

Tables 11.24 to 11.26 present the wins and losses for FDA5, FDA5$_{iso}$ and FDA5$_{dec}$. For FDA5, dCOEA obtained the best overall performance and DNSGA-II-B obtained the second best rank. The worst performance was obtained by DVEPSO. Both PSO-based DMOAs were awarded more losses than wins. However, all other DMOAs obtained more wins than losses. The best overall performance for FDA5$_{iso}$ was obtained by DNSGA-II-A, with DNSGA-II-B obtaining the second best rank. dCOEA performed the worst,

obtaining the worst rank for all performance measures. For FDA5$_{dec}$, both DNSGA-II algorithms performed the best and both PSO-based DMOAs performed the worst. dCOEA performed the best with regards to *acc* and *stab*, but the worst with regards to *NS*.

Table 11.24: Wins and Losses of FDA5

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**DMOO Algorithm**				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
1	10	*acc*	Wins	2	0	0	0	0
1	10	*acc*	Losses	0	0	0	1	1
1	10	*acc*	Diff	2	0	0	-1	-1
1	10	*acc*	Rank	**1**	2	2	4	4
20	10	*acc*	Wins	0	0	4	0	0
20	10	*acc*	Losses	1	1	0	1	1
20	10	*acc*	Diff	-1	-1	4	-1	-1
20	10	*acc*	Rank	2	2	**1**	2	2
all	all	*acc*	Wins	2	0	4	0	0
all	all	*acc*	Losses	1	1	0	2	2
all	all	*acc*	Diff	1	-1	4	-2	-2
all	all	*acc*	Rank	2	3	**1**	4	4
1	10	*stab*	Wins	0	1	0	1	1
1	10	*stab*	Losses	3	0	0	0	0
1	10	*stab*	Diff	-3	1	0	1	1
1	10	*stab*	Rank	5	**1**	4	**1**	**1**
all	all	*stab*	Wins	0	1	4	1	1
all	all	*stab*	Losses	4	1	0	1	1
all	all	*stab*	Diff	-4	0	4	0	0
all	all	*stab*	Rank	5	2	**1**	2	2
10	10	*NS*	Wins	2	2	2	0	0
10	10	*NS*	Losses	0	0	0	3	3
10	10	*NS*	Diff	2	2	2	-3	-3
10	10	*NS*	Rank	**1**	**1**	**1**	4	4
10	25	*NS*	Wins	3	2	3	0	0
10	25	*NS*	Losses	0	1	1	3	3
10	25	*NS*	Diff	3	1	2	-3	-3
10	25	*NS*	Rank	**1**	3	2	4	4
10	50	*NS*	Wins	3	2	3	0	0
10	50	*NS*	Losses	0	1	1	3	3
10	50	*NS*	Diff	3	1	2	-3	-3
10	50	*NS*	Rank	**1**	3	2	4	4
1	10	*NS*	Wins	2	1	4	2	0
1	10	*NS*	Losses	2	3	0	1	3
1	10	*NS*	Diff	0	-2	4	1	-3
1	10	*NS*	Rank	3	4	**1**	2	5
20	10	*NS*	Wins	3	3	0	1	1
20	10	*NS*	Losses	0	0	4	2	2
20	10	*NS*	Diff	3	3	-4	-1	-1
20	10	*NS*	Rank	**1**	**1**	5	3	3
all	all	*NS*	Wins	13	10	12	3	1

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$\mathbf{n_t}$	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
all	all	NS	Losses	2	5	6	12	14
all	all	NS	Diff	11	5	6	-9	-13
all	all	NS	Rank	**1**	3	2	4	5
10	10	all	Wins	2	2	2	0	0
10	10	all	Losses	0	0	0	3	3
10	10	all	Diff	2	2	2	-3	-3
10	10	all	Rank	**1**	**1**	**1**	4	4
10	25	all	Wins	3	2	3	0	0
10	25	all	Losses	0	1	1	3	3
10	25	all	Diff	3	1	2	-3	-3
10	25	all	Rank	**1**	3	2	4	4
10	50	all	Wins	3	2	3	0	0
10	50	all	Losses	0	1	1	3	3
10	50	all	Diff	3	1	2	-3	-3
10	50	all	Rank	**1**	3	2	4	4
1	10	all	Wins	4	2	4	3	1
1	10	all	Losses	5	3	0	2	4
1	10	all	Diff	-1	-1	4	1	-3
1	10	all	Rank	3	3	**1**	2	5
20	10	all	Wins	3	3	8	1	1
20	10	all	Losses	2	2	4	4	4
20	10	all	Diff	1	1	4	-3	-3
20	10	all	Rank	2	2	**1**	4	4
all	all	all	Wins	15	11	20	4	2
all	all	all	Losses	7	7	6	15	17
all	all	all	Diff	8	4	14	-11	-15
all	all	all	Rank	2	3	**1**	4	5

Table 11.25: Wins and Losses of FDA5$_{iso}$

$\mathbf{n_t}$	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
10	10	acc	Wins	1	1	0	1	1
10	10	acc	Losses	0	0	4	0	0
10	10	acc	Diff	1	1	-4	1	1
10	10	acc	Rank	**1**	**1**	5	**1**	**1**
10	25	acc	Wins	1	1	0	1	1
10	25	acc	Losses	0	0	4	0	0
10	25	acc	Diff	1	1	-4	1	1
10	25	acc	Rank	**1**	**1**	5	**1**	**1**
20	10	acc	Wins	0	0	4	0	0
20	10	acc	Losses	1	1	0	1	1
20	10	acc	Diff	-1	-1	4	-1	-1
20	10	acc	Rank	2	2	**1**	2	2
all	all	acc	Wins	2	2	4	2	2
all	all	acc	Losses	1	1	8	1	1
all	all	acc	Diff	1	1	-4	1	1
all	all	acc	Rank	**1**	**1**	5	**1**	**1**
10	10	$stab$	Wins	1	0	0	1	1

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n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
10	10	*stab*	Losses	0	0	3	0	0
10	10	*stab*	Diff	1	0	-3	1	1
10	10	*stab*	Rank	**1**	4	5	**1**	**1**
1	10	*stab*	Wins	0	1	0	1	1
1	10	*stab*	Losses	0	0	3	0	0
1	10	*stab*	Diff	0	1	-3	1	1
1	10	*stab*	Rank	4	**1**	5	**1**	**1**
20	10	*stab*	Wins	0	0	4	0	0
20	10	*stab*	Losses	1	1	0	1	1
20	10	*stab*	Diff	-1	-1	4	-1	-1
20	10	*stab*	Rank	2	2	**1**	2	2
all	all	*stab*	Wins	1	1	4	2	2
all	all	*stab*	Losses	1	1	6	1	1
all	all	*stab*	Diff	0	0	-2	1	1
all	all	*stab*	Rank	3	3	5	**1**	**1**
10	10	*NS*	Wins	3	3	0	1	1
10	10	*NS*	Losses	0	0	4	2	2
10	10	*NS*	Diff	3	3	-4	-1	-1
10	10	*NS*	Rank	**1**	**1**	5	3	3
10	25	*NS*	Wins	3	3	0	1	1
10	25	*NS*	Losses	0	0	4	2	2
10	25	*NS*	Diff	3	3	-4	-1	-1
10	25	*NS*	Rank	**1**	**1**	5	3	3
10	50	*NS*	Wins	3	3	0	1	1
10	50	*NS*	Losses	0	0	4	2	2
10	50	*NS*	Diff	3	3	-4	-1	-1
10	50	*NS*	Rank	**1**	**1**	5	3	3
1	10	*NS*	Wins	4	1	0	1	1
1	10	*NS*	Losses	0	1	4	1	1
1	10	*NS*	Diff	4	0	-4	0	0
1	10	*NS*	Rank	**1**	2	5	2	2
20	10	*NS*	Wins	3	3	0	1	1
20	10	*NS*	Losses	0	0	4	2	2
20	10	*NS*	Diff	3	3	-4	-1	-1
20	10	*NS*	Rank	**1**	**1**	5	3	3
all	all	*NS*	Wins	16	13	0	5	5
all	all	*NS*	Losses	0	1	20	9	9
all	all	*NS*	Diff	16	12	-20	-4	-4
all	all	*NS*	Rank	**1**	2	5	3	3
10	10	all	Wins	5	4	0	3	3
10	10	all	Losses	0	0	11	2	2
10	10	all	Diff	5	4	-11	1	1
10	10	all	Rank	**1**	2	5	3	3
10	25	all	Wins	4	4	0	2	2
10	25	all	Losses	0	0	8	2	2
10	25	all	Diff	4	4	-8	0	0
10	25	all	Rank	**1**	**1**	5	3	3
10	50	all	Wins	3	3	0	1	1
10	50	all	Losses	0	0	4	2	2
10	50	all	Diff	3	3	-4	-1	-1

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n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
10	50	all	Rank	**1**	**1**	5	3	3
1	10	all	Wins	4	2	0	2	2
1	10	all	Losses	0	1	7	1	1
1	10	all	Diff	4	1	-7	1	1
1	10	all	Rank	**1**	2	5	2	2
20	10	all	Wins	3	3	8	1	1
20	10	all	Losses	2	2	4	4	4
20	10	all	Diff	1	1	4	-3	-3
20	10	all	Rank	2	2	**1**	4	4
all	all	all	Wins	19	16	8	9	9
all	all	all	Losses	2	3	34	11	11
all	all	all	Diff	17	13	-26	-2	-2
all	all	all	Rank	**1**	2	5	3	3

Table 11.26: Wins and Losses of FDA5$_{dec}$

n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
10	10	*acc*	Wins	0	0	4	0	0
10	10	*acc*	Losses	1	1	0	1	1
10	10	*acc*	Diff	-1	-1	4	-1	-1
10	10	*acc*	Rank	2	2	**1**	2	2
1	10	*acc*	Wins	0	0	4	0	0
1	10	*acc*	Losses	1	1	0	1	1
1	10	*acc*	Diff	-1	-1	4	-1	-1
1	10	*acc*	Rank	2	2	**1**	2	2
20	10	*acc*	Wins	0	0	4	0	0
20	10	*acc*	Losses	1	1	0	1	1
20	10	*acc*	Diff	-1	-1	4	-1	-1
20	10	*acc*	Rank	2	2	**1**	2	2
all	all	*acc*	Wins	0	0	12	0	0
all	all	*acc*	Losses	3	3	0	3	3
all	all	*acc*	Diff	-3	-3	12	-3	-3
all	all	*acc*	Rank	2	2	**1**	2	2
1	10	*stab*	Wins	0	0	4	0	0
1	10	*stab*	Losses	1	1	0	1	1
1	10	*stab*	Diff	-1	-1	4	-1	-1
1	10	*stab*	Rank	2	2	**1**	2	2
20	10	*stab*	Wins	0	0	4	0	0
20	10	*stab*	Losses	1	1	0	1	1
20	10	*stab*	Diff	-1	-1	4	-1	-1
20	10	*stab*	Rank	2	2	**1**	2	2
all	all	*stab*	Wins	0	0	8	0	0
all	all	*stab*	Losses	2	2	0	2	2
all	all	*stab*	Diff	-2	-2	8	-2	-2
all	all	*stab*	Rank	2	2	**1**	2	2
10	10	NS	Wins	3	3	0	1	1
10	10	NS	Losses	0	0	4	2	2
10	10	NS	Diff	3.0	3.0	-4.0	-1.0	-1.0

Continued on next page

n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
10	10	*NS*	Rank	**1**	**1**	5	3	3
10	25	*NS*	Wins	3	3	0	1	1
10	25	*NS*	Losses	0	0	4	2	2
10	25	*NS*	Diff	3.0	3.0	-4.0	-1.0	-1.0
10	25	*NS*	Rank	**1**	**1**	5	3	3
10	50	*NS*	Wins	3	3	0	1	1
10	50	*NS*	Losses	0	0	4	2	2
10	50	*NS*	Diff	3.0	3.0	-4.0	-1.0	-1.0
10	50	*NS*	Rank	**1**	**1**	5	3	3
1	10	*NS*	Wins	3	3	0	1	1
1	10	*NS*	Losses	0	0	4	2	2
1	10	*NS*	Diff	3.0	3.0	-4.0	-1.0	-1.0
1	10	*NS*	Rank	**1**	**1**	5	3	3
20	10	*NS*	Wins	3	3	0	1	1
20	10	*NS*	Losses	0	0	4	2	2
20	10	*NS*	Diff	3.0	3.0	-4.0	-1.0	-1.0
20	10	*NS*	Rank	**1**	**1**	5	3	3
all	all	*NS*	Wins	15	15	0	5	5
all	all	*NS*	Losses	0	0	20	10	10
all	all	*NS*	Diff	15	15	-20	-5	-5
all	all	*NS*	Rank	**1**	**1**	5	3	3
10	10	all	Wins	3	3	4	1	1
10	10	all	Losses	1	1	4	3	3
10	10	all	Diff	2	2	0	-2	-2
10	10	all	Rank	**1**	**1**	3	4	4
10	25	all	Wins	3	3	0	1	1
10	25	all	Losses	0	0	4	2	2
10	25	all	Diff	3	3	-4	-1	-1
10	25	all	Rank	**1**	**1**	5	3	3
10	50	all	Wins	3	3	0	1	1
10	50	all	Losses	0	0	4	2	2
10	50	all	Diff	3	3	-4	-1	-1
10	50	all	Rank	**1**	**1**	5	3	3
1	10	all	Wins	3	3	8	1	1
1	10	all	Losses	2	2	4	4	4
1	10	all	Diff	1	1	4	-3	-3
1	10	all	Rank	2	2	**1**	4	4
20	10	all	Wins	3	3	8	1	1
20	10	all	Losses	2	2	4	4	4
20	10	all	Diff	1	1	4	-3	-3
20	10	all	Rank	2	2	**1**	4	4
all	all	all	Wins	15	15	20	5	5
all	all	all	Losses	5	5	20	15	15
all	all	all	Diff	10	10	0	-10	-10
all	all	all	Rank	**1**	**1**	3	4	4

The results for the DMOOPs with a discontinous POF, HE1 and HE2, are presented in Tables 11.27 to 11.28. For HE1, DNSGA-II-B obtained the best performance and DVEPSO performed the worst. DMOPSO obtained the second best rank. Both

DVEPSO and dCOEA performed poorly, obtaining more losses than wins. For HE2, the best performance was obtained by DNSGA-II-A and DNSGA-II-B. DVEPSO and dCOEA performed poorly, obtaining more losses than wins, with DVEPSO obtaining the worst rank. From the results it can clearly be seen that DVEPSO struggles to converge towards discontinous POFs.

Table 11.27: Wins and Losses of HE1 obtained by the DMOO algorithms

n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
10	10	*acc*	Wins	1	2	3	3	0
10	10	*acc*	Losses	3	2	1	0	3
10	10	*acc*	Diff	-2	0	2	3	-3
10	10	*acc*	Rank	4	3	2	**1**	5
10	25	*acc*	Wins	2	3	1	3	0
10	25	*acc*	Losses	2	1	3	0	3
10	25	*acc*	Diff	0	2	-2	3	-3
10	25	*acc*	Rank	3	2	4	**1**	5
10	50	*acc*	Wins	1	2	1	3	0
10	50	*acc*	Losses	1	1	2	0	3
10	50	*acc*	Diff	0	1	-1	3	-3
10	50	*acc*	Rank	3	2	4	**1**	5
1	10	*acc*	Wins	2	3	1	3	0
1	10	*acc*	Losses	2	1	3	0	3
1	10	*acc*	Diff	0	2	-2	3	-3
1	10	*acc*	Rank	3	2	4	**1**	5
20	10	*acc*	Wins	2	3	1	3	0
20	10	*acc*	Losses	2	1	3	0	3
20	10	*acc*	Diff	0	2	-2	3	-3
20	10	*acc*	Rank	3	2	4	**1**	5
all	all	*acc*	Wins	8	13	7	15	0
all	all	*acc*	Losses	10	6	12	0	15
all	all	*acc*	Diff	-2	7	-5	15	-15
all	all	*acc*	Rank	3	2	4	**1**	5
10	10	*stab*	Wins	0	2	3	3	1
10	10	*stab*	Losses	4	2	1	0	2
10	10	*stab*	Diff	-4	0	2	3	-1
10	10	*stab*	Rank	5	3	2	**1**	4
10	25	*stab*	Wins	2	3	0	3	1
10	25	*stab*	Losses	2	1	4	0	2
10	25	*stab*	Diff	0	2	-4	3	-1
10	25	*stab*	Rank	3	2	5	**1**	4
10	50	*stab*	Wins	0	1	0	3	2
10	50	*stab*	Losses	2	1	2	0	1
10	50	*stab*	Diff	-2	0	-2	3	1
10	50	*stab*	Rank	4	3	4	**1**	2
1	10	*stab*	Wins	2	3	0	3	1
1	10	*stab*	Losses	2	1	4	0	2
1	10	*stab*	Diff	0	2	-4	3	-1

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n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
1	10	*stab*	Rank	3	2	5	**1**	4
20	10	*stab*	Wins	2	3	0	3	1
20	10	*stab*	Losses	2	1	4	0	2
20	10	*stab*	Diff	0	2	-4	3	-1
20	10	*stab*	Rank	3	2	5	**1**	4
all	all	*stab*	Wins	6	12	3	15	6
all	all	*stab*	Losses	12	6	15	0	9
all	all	*stab*	Diff	-6	6	-12	15	-3
all	all	*stab*	Rank	4	2	5	**1**	3
10	10	*NS*	Wins	3	3	1	1	0
10	10	*NS*	Losses	0	0	3	2	3
10	10	*NS*	Diff	3	3	-2	-1	-3
10	10	*NS*	Rank	**1**	**1**	4	3	5
10	25	*NS*	Wins	3	3	1	1	0
10	25	*NS*	Losses	0	0	3	2	3
10	25	*NS*	Diff	3	3	-2	-1	-3
10	25	*NS*	Rank	**1**	**1**	4	3	5
10	50	*NS*	Wins	3	3	1	1	0
10	50	*NS*	Losses	0	0	3	2	3
10	50	*NS*	Diff	3	3	-2	-1	-3
10	50	*NS*	Rank	**1**	**1**	4	3	5
1	10	*NS*	Wins	3	3	1	1	0
1	10	*NS*	Losses	0	0	3	2	3
1	10	*NS*	Diff	3	3	-2	-1	-3
1	10	*NS*	Rank	**1**	**1**	4	3	5
20	10	*NS*	Wins	3	3	1	1	0
20	10	*NS*	Losses	0	0	3	2	3
20	10	*NS*	Diff	3	3	-2	-1	-3
20	10	*NS*	Rank	**1**	**1**	4	3	5
all	all	*NS*	Wins	15	15	5	5	0
all	all	*NS*	Losses	0	0	15	10	15
all	all	*NS*	Diff	15	15	-10	-5	-15
all	all	*NS*	Rank	**1**	**1**	4	3	5
10	10	all	Wins	4	7	7	7	1
10	10	all	Losses	7	4	5	2	8
10	10	all	Diff	-3	3	2	5	-7
10	10	all	Rank	4	2	3	**1**	5
10	25	all	Wins	7	9	2	7	1
10	25	all	Losses	4	2	10	2	8
10	25	all	Diff	3	7	-8	5	-7
10	25	all	Rank	3	**1**	5	2	4
10	50	all	Wins	4	6	2	7	2
10	50	all	Losses	3	2	7	2	7
10	50	all	Diff	1	4	-5	5	-5
10	50	all	Rank	3	2	4	**1**	4
1	10	all	Wins	7	9	2	7	1
1	10	all	Losses	4	2	10	2	8
1	10	all	Diff	3	7	-8	5	-7
1	10	all	Rank	3	**1**	5	2	4
20	10	all	Wins	7	9	2	7	1

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n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
20	10	all	Losses	4	2	10	2	8
20	10	all	Diff	3	7	-8	5	-7
20	10	all	Rank	3	**1**	5	2	4
all	all	all	Wins	29	40	15	35	6
all	all	all	Losses	22	12	42	10	39
all	all	all	Diff	7	28	-27	25	-33
all	all	all	Rank	3	**1**	4	2	5

Table 11.28: Wins and Losses of HE2 obtained by the DMOO algorithms

n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	DNSGA-II-B	dCOEA	DMOPSO	DVEPSO
10	10	*acc*	Wins	2	2	1	3	0
10	10	*acc*	Losses	1	1	3	0	3
10	10	*acc*	Diff	1	1	-2	3	-3
10	10	*acc*	Rank	2	2	4	**1**	5
10	25	*acc*	Wins	1	1	1	1	0
10	25	*acc*	Losses	0	0	3	0	1
10	25	*acc*	Diff	1	1	-2	1	-1
10	25	*acc*	Rank	**1**	**1**	5	**1**	4
10	50	*acc*	Wins	2	2	1	3	0
10	50	*acc*	Losses	1	1	3	0	3
10	50	*acc*	Diff	1	1	-2	3	-3
10	50	*acc*	Rank	2	2	4	**1**	5
1	10	*acc*	Wins	2	2	1	3	0
1	10	*acc*	Losses	1	1	3	0	3
1	10	*acc*	Diff	1	1	-2	3	-3
1	10	*acc*	Rank	2	2	4	**1**	5
20	10	*acc*	Wins	1	1	1	1	0
20	10	*acc*	Losses	0	0	3	0	1
20	10	*acc*	Diff	1	1	-2	1	-1
20	10	*acc*	Rank	**1**	**1**	5	**1**	4
all	all	*acc*	Wins	8	8	5	11	0
all	all	*acc*	Losses	3	3	15	0	11
all	all	*acc*	Diff	5	5	-10	11	-11
all	all	*acc*	Rank	2	2	4	**1**	5
10	10	*stab*	Wins	1	1	1	3	0
10	10	*stab*	Losses	1	1	1	0	3
10	10	*stab*	Diff	0	0	0	3	-3
10	10	*stab*	Rank	2	2	2	**1**	5
10	25	*stab*	Wins	1	1	1	1	0
10	25	*stab*	Losses	0	0	3	0	1
10	25	*stab*	Diff	1	1	-2	1	-1
10	25	*stab*	Rank	**1**	**1**	5	**1**	4
10	50	*stab*	Wins	1	1	1	1	0
10	50	*stab*	Losses	0	0	3	0	1
10	50	*stab*	Diff	1	1	-2	1	-1
10	50	*stab*	Rank	**1**	**1**	5	**1**	4
1	10	*stab*	Wins	2	2	1	3	0

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n_t	τ_t	PM	Results	DMOO Algorithm				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
1	10	*stab*	Losses	1	1	3	0	3
1	10	*stab*	Diff	1	1	-2	3	-3
1	10	*stab*	Rank	2	2	4	**1**	5
20	10	*stab*	Wins	2	2	0	3	1
20	10	*stab*	Losses	1	1	4	0	2
20	10	*stab*	Diff	1	1	-4	3	-1
20	10	*stab*	Rank	2	2	5	**1**	4
all	all	*stab*	Wins	7	7	4	11	1
all	all	*stab*	Losses	3	3	14	0	10
all	all	*stab*	Diff	4	4	-10	11	-9
all	all	*stab*	Rank	2	2	5	**1**	4
10	10	*NS*	Wins	3	3	1	1	0
10	10	*NS*	Losses	0	0	3	2	3
10	10	*NS*	Diff	3	3	-2	-1	-3
10	10	*NS*	Rank	**1**	**1**	4	3	5
10	25	*NS*	Wins	3	3	1	1	0
10	25	*NS*	Losses	0	0	3	2	3
10	25	*NS*	Diff	3	3	-2	-1	-3
10	25	*NS*	Rank	**1**	**1**	4	3	5
10	50	*NS*	Wins	3	3	1	1	0
10	50	*NS*	Losses	0	0	3	2	3
10	50	*NS*	Diff	3	3	-2	-1	-3
10	50	*NS*	Rank	**1**	**1**	4	3	5
1	10	*NS*	Wins	3	3	1	1	0
1	10	*NS*	Losses	0	0	3	2	3
1	10	*NS*	Diff	3	3	-2	-1	-3
1	10	*NS*	Rank	**1**	**1**	4	3	5
20	10	*NS*	Wins	3	3	1	1	0
20	10	*NS*	Losses	0	0	3	2	3
20	10	*NS*	Diff	3	3	-2	-1	-3
20	10	*NS*	Rank	**1**	**1**	4	3	5
all	all	*NS*	Wins	15	15	5	5	0
all	all	*NS*	Losses	0	0	15	10	15
all	all	*NS*	Diff	15	15	-10	-5	-15
all	all	*NS*	Rank	**1**	**1**	4	3	5
10	10	all	Wins	6	6	3	7	0
10	10	all	Losses	2	2	7	2	9
10	10	all	Diff	4	4	-4	5	-9
10	10	all	Rank	2	2	4	**1**	5
10	25	all	Wins	5	5	3	3	0
10	25	all	Losses	0	0	9	2	5
10	25	all	Diff	5	5	-6	1	-5
10	25	all	Rank	**1**	**1**	5	3	4
10	50	all	Wins	6	6	3	5	0
10	50	all	Losses	1	1	9	2	7
10	50	all	Diff	5	5	-6	3	-7
10	50	all	Rank	**1**	**1**	4	3	5
1	10	all	Wins	7	7	3	7	0
1	10	all	Losses	2	2	9	2	9
1	10	all	Diff	5	5	-6	5	-9

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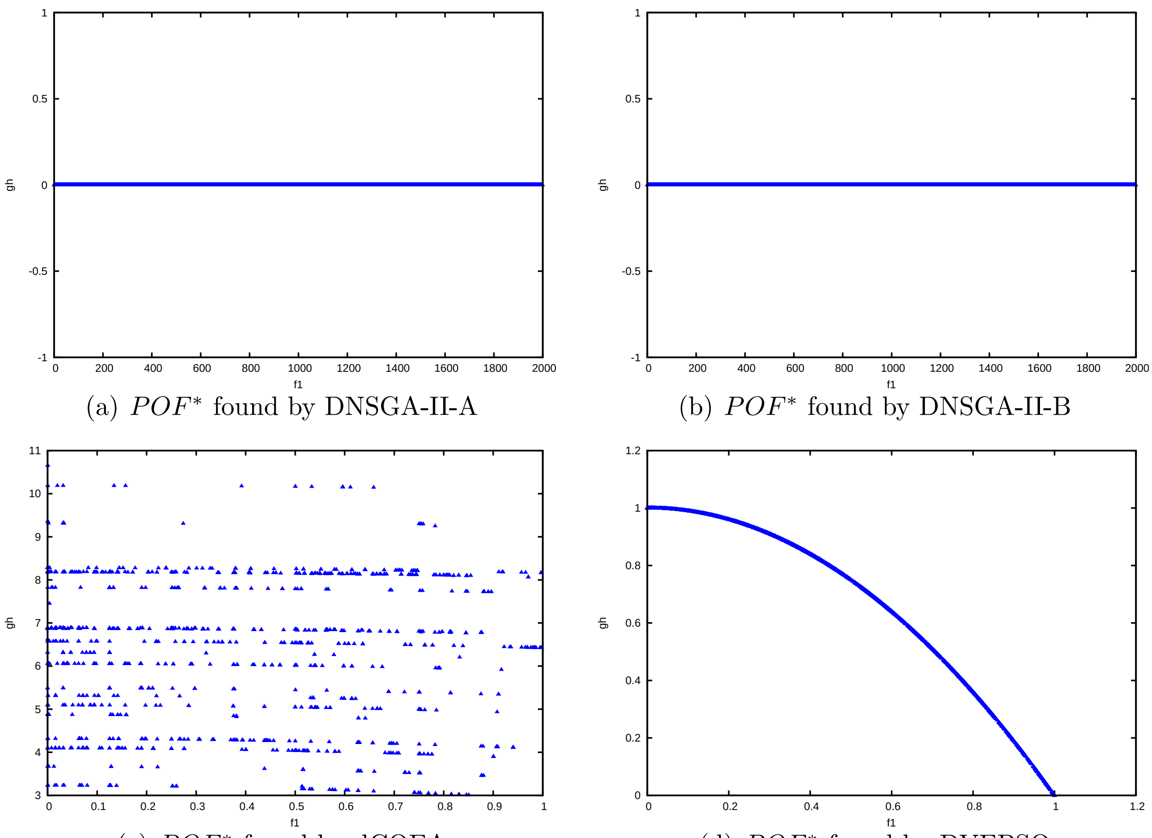
$\mathbf{n_t}$	τ_t	**PM**	**Results**	**DMOO Algorithm**				
				DNSGA-II-A	**DNSGA-II-B**	**dCOEA**	**DMOPSO**	**DVEPSO**
1	10	all	Rank	**1**	**1**	4	**1**	5
20	10	all	Wins	6	6	2	5	1
20	10	all	Losses	1	1	10	2	6
20	10	all	Diff	5	5	-8	3	-5
20	10	all	Rank	**1**	**1**	5	3	4
all	all	all	Wins	30	30	14	27	1
all	all	all	Losses	6	6	44	10	36
all	all	all	Diff	24	24	-30	17	-35
all	all	all	Rank	**1**	**1**	4	3	5

The POF^* found by the DMOAs for DIMP2, dMOP3, FDA5, FDA5$_{iso}$, FDA5$_{dec}$, and HE2 are illustrated in Figures 11.1 to 11.5.



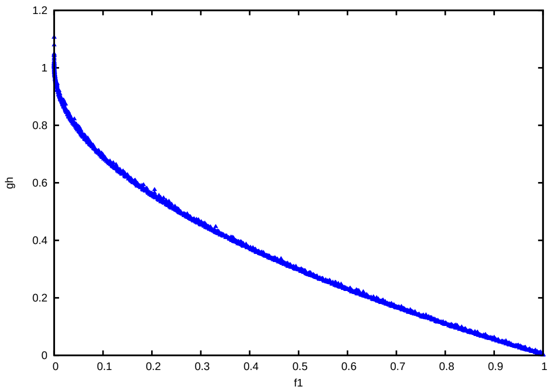
(a) POF^* found by DNSGA-II-A

(b) POF^* found by DNSGA-II-B

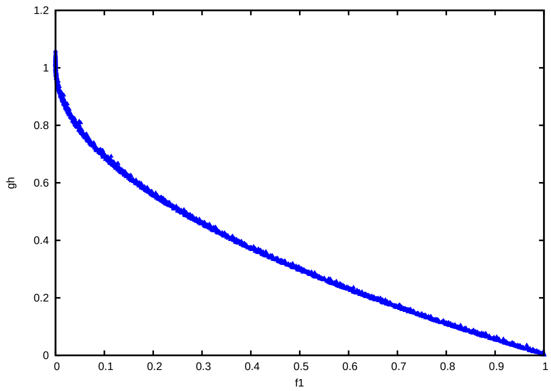
(c) POF^* found by dCOEA

(d) POF^* found by DVEPSO

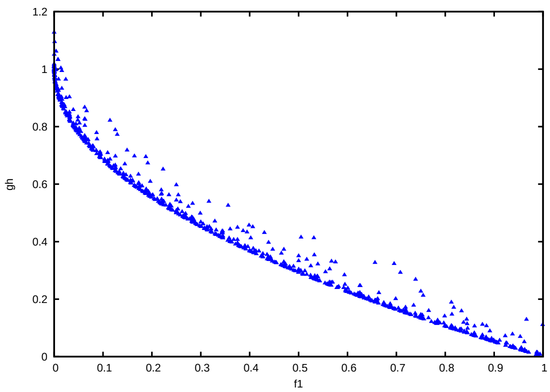
Figure 11.1: POF^* for DIMP2 for $n_t = 10$ and $\tau_t = 50$ found by the various DMOAs. DMOPSO did not find any solutions.



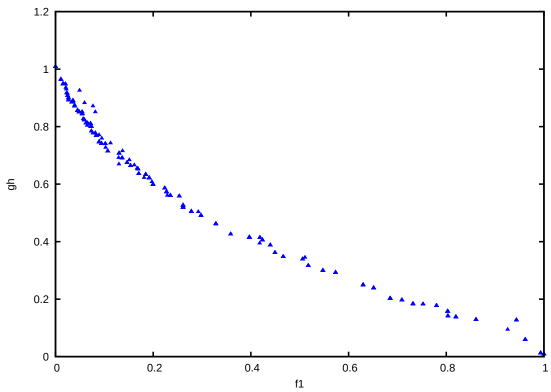
(a) POF^* found by DNSGA-II-A



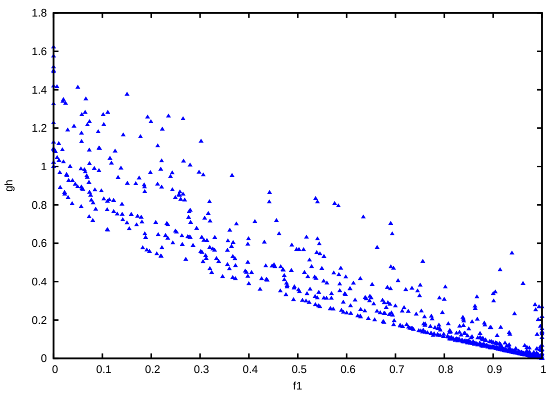
(b) POF^* found by DNSGA-II-B



(c) POF^* found by dCOEA

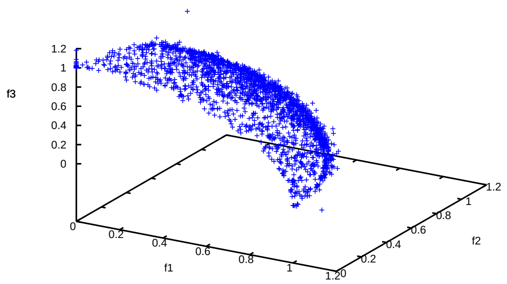


(d) POF^* found by DMOPSO

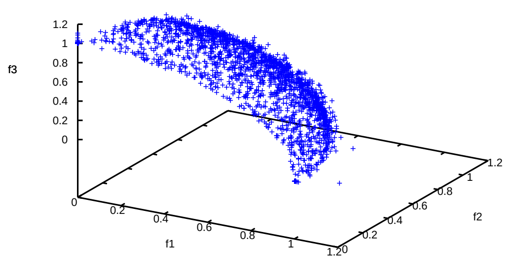


(e) POF^* found by DVEPSO

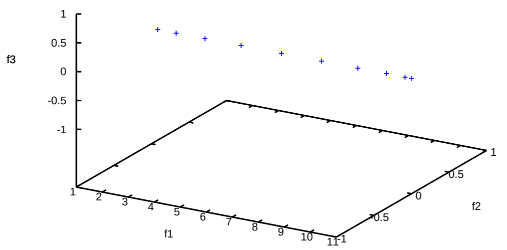
Figure 11.2: POF^* for dMOP3 for $n_t = 10$ and $\tau_t = 50$ found by the various DMOAs



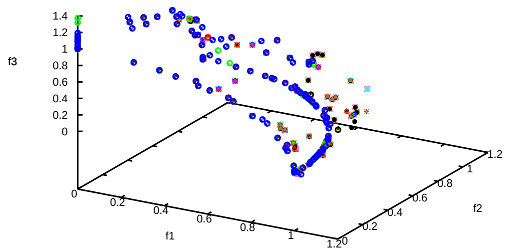
(a) POF^* found by DNSGA-II-A



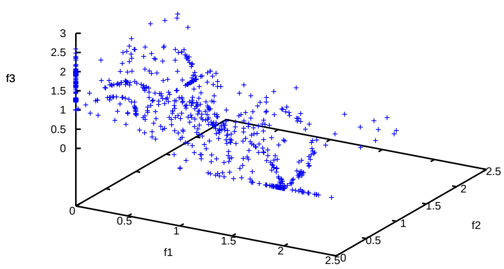
(b) POF^* found by DNSGA-II-B



(c) POF^* found by dCOEA

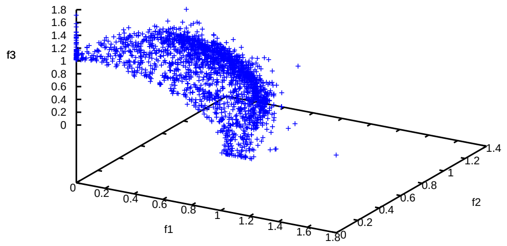


(d) POF^* found by DMOPSO

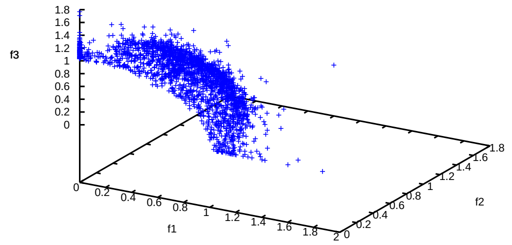


(e) POF^* found by DVEPSO

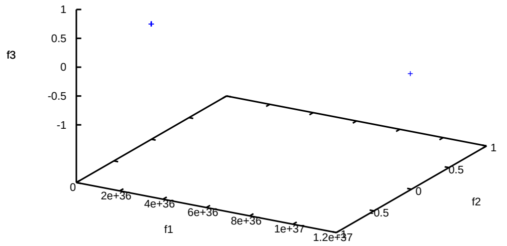
Figure 11.3: POF^* for FDA5$_{iso}$ for $n_t = 10$ and $\tau_t = 50$ found by the various DMOAs



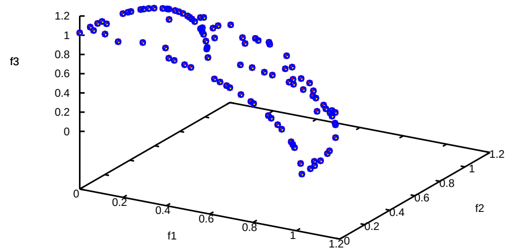
(a) POF^* found by DNSGA-II-A



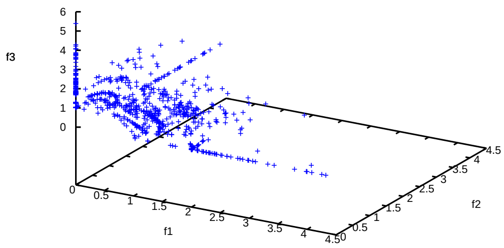
(b) POF^* found by DNSGA-II-B



(c) POF^* found by dCOEA

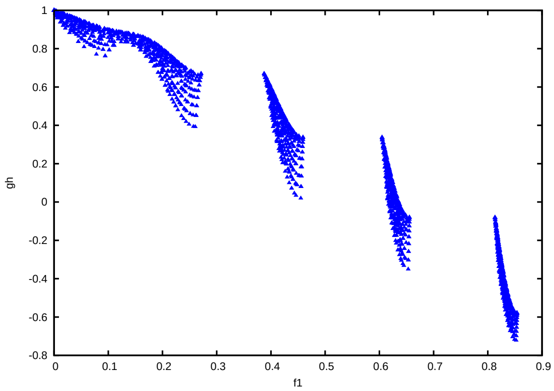


(d) POF^* found by DMOPSO

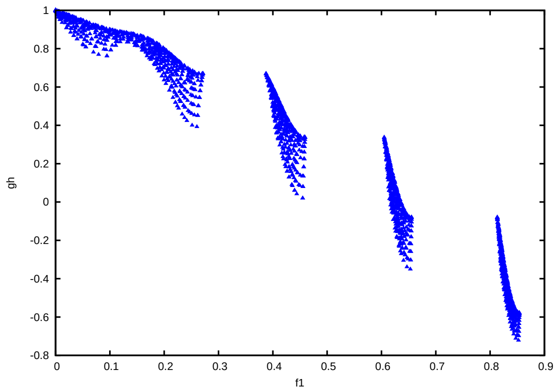


(e) POF^* found by DVEPSO

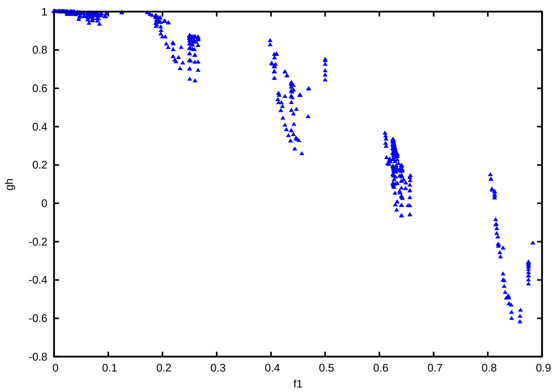
Figure 11.4: POF^* for FDA5$_{dec}$ for $n_t = 10$ and $\tau_t = 50$ found by the various DMOAs



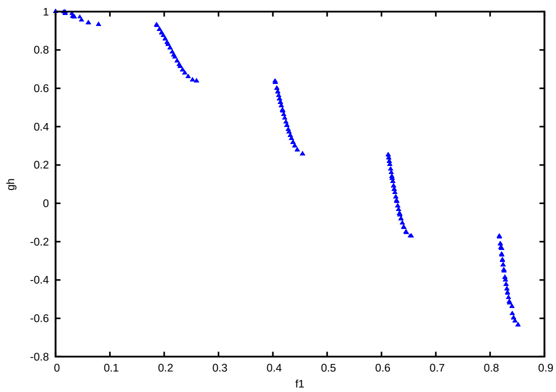
(a) POF^* found by DNSGA-II-A



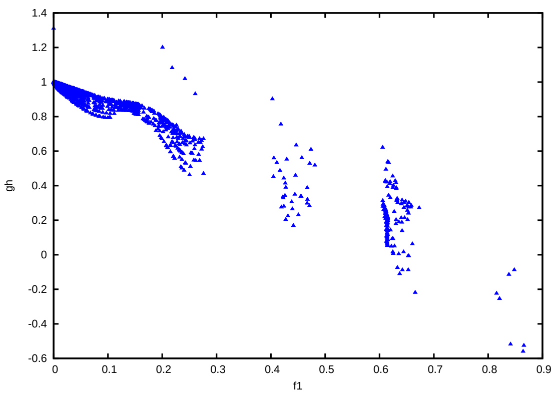
(b) POF^* found by DNSGA-II-B



(c) POF^* found by dCOEA



(d) POF^* found by DMOPSO



(e) POF^* found by DVEPSO

Figure 11.5: POF^* for HE2 for $n_t = 10$ and $\tau_t = 50$ found by the various DMOAs

For DIMP2, DVEPSO outperformed the other DMOAs and found a POF^* close to the true POF (POF). All other DMOAs struggled to converge towards POF. The DNSGA-II algorithms converged really well towards POF of dMOP3. DMOPSO also converged well, but found a worse spread of solutions than the DNSGA-II algorithms. dCOEA performed well, but also found a few solutions that were further away from POF. DVEPSO found a number of solutions further away from POF, but did manage to find solutions close to POF. For FDA5$_{iso}$, the DNSGA-II algorithms found a diverse set of solutions. DVEPSO found less solutions than the DNSGA-II algorithms, but a better spread of solutions than MOPSO. However, dCOEA struggled to converge towards POF of FDA5$_{iso}$. A similar trend than FDA5$_{iso}$ was observed for FDA5$_{dec}$. For HE2, all algorithms converged towards POF. However, DVEPSO found a few solutions further away from POF.

11.3 Summary

This chapter investigated the performance of five DMOAs solving 2-objective and 3-objective DMOOPs of Types I to III, with various frequencies and severities of change. Three performance measures were used to quantify the algorithms' performance, namely accuracy that measures the difference in HV values of the approximated and true POF (acc), stability that measures the effect of the environmental change on the accuracy of the algorithm ($stab$) and the number of non-dominated solutions found by the algorithm (NS).

The five DMOAs investigated were: an NSGA-II algorithm adapted for DMOO where if a change in the environment is detected, a percentage of individuals are randomly selected and replaced with newly created individuals (DNSGA-II-A); an NSGA-II algorithm that selects a percentage of individuals randomly and replaces them with individuals that are mutated from existing individuals when a change is detected (DNSGA-II-B); a dynamic competitive-cooperative coevolutionary algorithm (dCOEA); a MOPSO algorithm adapted for DMOO (DMOPSO); and the multi-swarm PSO-based algorithm proposed in this thesis (DVEPSO).

With regards to the various DMOOP types, DVEPSO obtained the second lowest rank for Type I DMOOPs, obtaining the best rank for *acc*, but the lowest rank for *stab* and NS. DVEPSO struggled to converge to the POF of dMOP3 where the variable that controls the spread of solutions randomly changes over time. However, DVEPSO was the only algorithm that successfully converged towards the POF of DIMP2, where the decision variables change at different rates over time. DVEPSO obtained the overall best rank for Type II DMOOPs, and obtained the best rank for both *acc* and *stab*, and the third best rank for NS. For Type III DMOOPs, DVEPSO obtained the fourth, second and fourth rank for *acc*, *stab* and NS respectively. DVEPSO struggled to converge towards discontinuous POFs and therefore did not perform as well with regards to the Type III DMOOPs.

Measuring the algorithms' performance over all performance measures, DNSGA-II-B obtained the best overall performance, with DNSGA-II-A obtaining the second best rank and DVEPSO the third best rank. dCOEA obtained the overall worst rank. However, all DMOEAs obtained a high number of wins for NS. Measuring the algorithms' performance over *acc* and *stab* and therefore not taking NS into account lead to DMOPSO obtaining the best performance, DVEPSO obtaining the second best rank and DNSGA-II-B being awarded the third best rank. Once again, dCOEA performed the worst. The DMOEAs obtained the most solutions in general. However, with regards to *acc* and *stab*, the PSO-based MOAs completely outperformed the MOEAs.

In addition, at least one DMOEA outperformed the PSO-based DMOAs on DMOOPs with either an isolated or deceptive POF. Furthermore, most of the DMOAs outperformed DVEPSO on DMOOPs with a discontinuous POF.

The next chapter concludes the research presented in this thesis and proposes possible future work.