Chapter 10

Sensitivity Analysis of Dynamic Vector Evaluated Particle Swarm Optimisation Algorithm

"It is not the strongest of the species that survives, nor the most intelligent. It is the one that is the most adaptable to change." – Charles Darwin

A self-adapting DMOO algorithm that does not require the optimisation of parameters is the ideal. However, before a self-adapting DVEPSO algorithm can be developed, the influence of the various parameters on the performance of DVEPSO has to be understood. Therefore, this chapter investigates how knowledge sharing swarm topologies, approaches to manage boundary constraint violations, and approaches to respond to changes in the environment effect the performance of DVEPSO.

Section 10.1 describes the experimental setup for this study. The results of the experiments are presented in Section 10.2. Finally, Section 10.3 provides a summary of this chapter.

10.1 Experimental Setup

This section describes the experimental setup of the experiments discussed in this chapter. The experiments investigate the approaches discussed in Section 7.2.2 to manage boundary constraints to determine the influence of the approaches on the performance of DVEPSO.

The search process of DVEPSO is driven through guides. Since each swarm only optimises one objective, knowledge about other objectives are obtained through guides from other swarms. The knowledge sharing stategies discussed in Section 7.2.2 were investigated to determine the approaches' effect on the performance of DVEPSO.

After an environment change, DVEPSO has to respond in an appropriate manner to enable tracking of the changing POF or POS. Two response categories are investigated, namely managing the particles and managing the archive. The approaches to manage the particles after a change occurred were discussed in Section 9.2. Section 7.2.2 discussed the archive management approaches that were investigated in this study.

All simulations were run on an Intel Core i7 X990 8-core machine with a 3.47 GHz processor.

Fifteen benchmark functions were used as discussed in Section 9.4.1. Three performance measures were used to quantify the performance of algorithms, as discussed in Section 9.4.1.

The same default configuration of DVEPSO was used for these experiments as discussed in Section 9.4.1. However, p_s-g_r was used for the guide updates.

Statistical analysis of the obtained data was performed as discussed in Section 9.4.1. The three null hypotheses for these experiments were:

1. There are no statistical significant difference between the performance of the various knowledge sharing approaches.
2. There are no statistical significant difference between the performance of the various responses applied to the particles after a change in the environment occurs.
3. There are no statistical significant difference between the performance of the various responses applied to the archive after an environmental change.

The alternative hypothesis for all three cases above is that there is a difference in mean performance.

The next section discusses the results of the experiments.

10.2 Results

This section discusses the results obtained from the experiments. Section 10.2.1 discusses the results of the various strategies that were used to manage particles that moved outside of the search space and therefore violated the boundary constraints of the DMOOP. The approaches used to share knowledge between the sub-swarms of DVEPSO are discussed in Section 10.2.2. Section 10.2.3 discusses responses to a change in the environment applied to either the particles or the archive. Only the tables highlighting interesting trends and that are therefore discussed, are presented in this section. The other wins and losses tables are presented in Appendix D. Only statistical significant values are included in the tables. The p-values obtained for the various Mann-Whitney U tests, as well as the average performance measure values, are presented in Appendix D.

10.2.1 Management of Boundary Constraint Violations

This section discusses the results obtained by various approaches used to manage particles that moved outside the search space. The results are discussed with regards to each performance measure and each n_t-τ_t combination. Results obtained for each of the three DMOOP Types (Type I, II and III) are also presented. Each approach's overall performance is also discussed. Furthermore, general observations with regards to DVEPSO's performance are also highlighted.

The wins and losses of the various boundary constraint management approaches are presented in Tables 10.1 to 10.12. In Tables 10.1 to 10.12, cl, pe, ra and re refer to the clamping, per element re-initialisation, random, and re-initialisation approaches discussed in Section 7.2.2 respectively. The other approaches proposed to manage boundary constraint violations discussed in Section 7.2.2 are not included in the results, since these approaches found solutions so far away from the true POF, that huge reference vectors (larger than 10^{260}) and therefore huge HV values were obtained.

Results with regards to Performance Measures

This section discusses the results obtained by the various approaches used to manage boundary constraint violations for the various performance measures. The wins and losses obtained by these approaches over all n_t-τ_t combinations for the various performance measures are presented in Table 10.1.

The following observations are made:

- *cl* performed the best with regards to all performance measures. The worst performance for all performance measures was obtained by *pe*.
- With regards to *acc*, both *pe* and *re* obtained more losses than wins.
- For *stab*, all approaches, except *cl*, obtained more losses than wins. Therefore, the other approaches were completely outperformed by *cl*.
- Similar to *stab*, for *NS* only *cl* was awarded more wins than losses.

Table 10.1: Overall wins and losses for various performance measures obtained by various boundary management strategies

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
all	all	*acc*	Wins	117	30	78	64
all	all	*acc*	Losses	55	104	61	69
all	all	*acc*	Diff	62	-74	17	-5
all	all	*acc*	Rank	**1**	4	2	3
all	all	*stab*	Wins	125	7	37	17
all	all	*stab*	Losses	5	73	55	53
all	all	*stab*	Diff	120	-66	-18	-36
all	all	*stab*	Rank	**1**	4	2	3
all	all	*NS*	Wins	87	24	40	39
all	all	*NS*	Losses	54	53	46	37
all	all	*NS*	Diff	33	-29	-6	2
all	all	*NS*	Rank	**1**	4	3	2

Results with regards to Various Frequencies and Severities of Change

This section discusses the results obtained for the various n_t-τ_t combinations. The wins and losses obtained by the approaches for management of boundary constraint violations are presented in Table 10.2.

Table 10.2: Overall wins and losses for various frequencies and severities of change obtained by various boundary management strategies

n_t	τ_t	PM	Results	Boundary management strategies			
				cl	**pe**	**ra**	**re**
10	10	all	Wins	72	13	38	31
10	10	all	Losses	22	55	38	39
10	10	all	Diff	50	-42	0	-8
10	10	all	Rank	**1**	4	2	3
10	25	all	Wins	64	12	29	18
10	25	all	Losses	18	43	33	29
10	25	all	Diff	46	-31	-4	-11
10	25	all	Rank	**1**	4	2	3
10	50	all	Wins	59	12	25	15
10	50	all	Losses	28	32	24	27
10	50	all	Diff	31	-20	1	-12
10	50	all	Rank	**1**	4	2	3
1	10	all	Wins	71	13	35	29
1	10	all	Losses	25	54	35	34
1	10	all	Diff	46	-41	0	-5
1	10	all	Rank	**1**	4	2	3
20	10	all	Wins	63	11	28	27
20	10	all	Losses	21	46	32	30
20	10	all	Diff	42	-35	-4	-3
20	10	all	Rank	**1**	4	3	2

From the obtained results, the following observations are made:

- For all n_t-τ_t combinations cl obtained the best performance.
- A bad performance was obtained by pe and re, with more losses than wins for all n_t-τ_t combinations.
- Even though ra obtained more losses than wins for only $n_t = 10$ and $\tau_t = 25$, and $n_t = 20$ and $\tau_t = 10$, ra achieved only marginally more wins than losses for the other n_t-τ_t combinations.

Results for Various Dynamic Multi-objective Optimisation Problem Types

This section discusses the obtained results with regards to three DMOOP types, namely Type I to III.

Type I DMOOPs

The wins and losses for Type I DMOOPs are presented in Table 10.3.

Table 10.3: Overall wins and losses for various performance measures obtained by various boundary management strategies solving Type I DMOOPs

$\mathbf{n_t}$	$\tau_\mathbf{t}$	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
all	all	*acc*	Wins	15	5	15	7
all	all	*acc*	Losses	15	11	6	10
all	all	*acc*	Diff	0	-6	9	-3
all	all	*acc*	Rank	2	4	**1**	3
all	all	*stab*	Wins	15	1	10	1
all	all	*stab*	Losses	3	9	5	10
all	all	*stab*	Diff	12	-8	5	-9
all	all	*stab*	Rank	**1**	3	2	4
all	all	*NS*	Wins	0	5	7	9
all	all	*NS*	Losses	15	4	2	0
all	all	*NS*	Diff	-15	1	5	9
all	all	*NS*	Rank	4	3	2	**1**

The following are observed:

- The best performance for *acc* was obtained by *ra* and *pe* performed the worst. Both *pe* and *re* obtained more losses than wins. Furthermore, an equal number of wins and losses were achieved by *cl*.
- The best rank for *stab* was achieved by *cl* and the worst by *re*. More losses than wins were obtained by *pe* and *re*.
- For *NS*, the best performance was obtained by *pe*. *cl* performed the worst and obtained more losses than wins.

Table 10.4 presents the wins and losses for Type I DMOOPs with regards to the various n_t-τ_t combinations.

With regards to the various types of environments, the following observations are made:

- *ra* performed the best for all environments, except $n_t = 20$ and $\tau_t = 10$. For $n_t = 20$ and $\tau_t = 10$, *ra* obtained the second best rank. For $n_t = 10$ and $\tau_t = 10$, *cl* performed the worst and both *cl* and *pe* were awarded more losses than wins. For $n_t = 10$ and $\tau_t = 25$, and $n_t = 1$ and $\tau_t = 10$, *pe* obtained the worst rank. For $n_t = 10$ and $\tau_t = 50$, *re* performed the worst.
- For gradually changing environments ($n_t = 20$ and $\tau_t = 10$), *cl* performed the best obtaining only wins and no losses. In contrast, *pe* and *re* were awarded only losses and no wins. Both *pe* and *re* obtained the lowest rank.

Table 10.4: Overall wins and losses for various frequencies and severities of change obtained by various boundary management strategies solving Type I DMOOPs

$\mathbf{n_t}$	$\boldsymbol{\tau_t}$	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
10	10	all	Wins	6	5	10	9
10	10	all	Losses	15	7	4	4
10	10	all	Diff	-9	-2	6	5
10	10	all	Rank	4	3	**1**	2
10	25	all	Wins	6	2	7	2
10	25	all	Losses	6	5	2	4
10	25	all	Diff	0	-3	5	-2
10	25	all	Rank	2	4	**1**	3
10	50	all	Wins	6	2	5	2
10	50	all	Losses	6	3	2	4
10	50	all	Diff	0	-1	3	-2
10	50	all	Rank	2	3	**1**	4
1	10	all	Wins	6	2	6	4
1	10	all	Losses	6	5	3	4
1	10	all	Diff	0	-3	3	0
1	10	all	Rank	2	4	**1**	2
20	10	all	Wins	6	0	4	0
20	10	all	Losses	0	4	2	4
20	10	all	Diff	6	-4	2	-4
20	10	all	Rank	**1**	3	2	3

The wins and losses for all Type I DMOOPs over all performance measures and all n_t-τ_t combinations are presented in Table 10.5. The best overall rank for Type I DMOOPs was obtained by ra, with pe obtaining the worst rank. Only ra scored more wins than losses, while the other three approaches were awarded more losses than wins.

Table 10.5: Overall wins and losses obtained by various boundary management strategies solving Type I DMOOPs

$\mathbf{n_t}$	$\boldsymbol{\tau_t}$	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
all	all	all	Wins	30	11	32	17
all	all	all	Losses	33	24	13	20
all	all	all	Diff	-3	-13	19	-3
all	all	all	Rank	2	4	**1**	2

Type II DMOOPs

Table 10.6 presents the wins and losses with regards to the various performance measures for the boundary constraint management approaches solving Type II DMOOPs.

Table 10.6: Overall wins and losses for various performance measures obtained by various boundary management strategies solving Type II DMOOPs

$\mathbf{n_t}$	$\tau_\mathbf{t}$	PM	Results	Boundary management strategies			
				cl	**pe**	**ra**	**re**
all	all	*acc*	Wins	72	12	55	28
all	all	*acc*	Losses	16	73	29	49
all	all	*acc*	Diff	56	-61	26	-21
all	all	*acc*	Rank	**1**	4	2	3
all	all	*stab*	Wins	69	4	27	15
all	all	*stab*	Losses	1	51	34	29
all	all	*stab*	Diff	68	-47	-7	-14
all	all	*stab*	Rank	**1**	4	2	3
all	all	*NS*	Wins	24	8	20	2
all	all	*NS*	Losses	15	16	10	13
all	all	*NS*	Diff	9	-8	10	-11
all	all	*NS*	Rank	2	3	**1**	4

The following observations are made:

- The best performance for *acc* was achieved by *cl* and the worst by *pe*. More losses than wins were obtained by *pe* and *re*.
- With regards to *stab*, *cl* performed the best and *pe* the worst. Only one loss was obtained by *cl*, where as the other three approaches were all awarded more losses than wins.
- For *NS* the highest rank was obtained by *ra* and the worst rank by *re*. Both *pe* and *re* performed poorly, obtaining more losses than wins.

The wins and losses with regards to the various n_t-τ_t combinations are presented in Table 10.7.

With regards to the various environment types, the following are observed:

- For all environment types, *cl* performed the best and *pe* the worst.
- More losses than wins were awarded to *pe* and *re* for all n_t-τ_t combinations.

Table 10.7: Overall wins and losses for various frequencies and severities of change obtained by various boundary management strategies solving Type II DMOOPs

$\mathbf{n_t}$	$\tau_\mathbf{t}$	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
10	10	all	Wins	40	4	26	13
10	10	all	Losses	4	36	18	25
10	10	all	Diff	36	-32	8	-12
10	10	all	Rank	**1**	4	2	3
10	25	all	Wins	32	5	20	8
10	25	all	Losses	5	27	17	16
10	25	all	Diff	27	-22	3	-8
10	25	all	Rank	**1**	4	2	3
10	50	all	Wins	28	7	17	4
10	50	all	Losses	13	19	11	13
10	50	all	Diff	15	-12	6	-9
10	50	all	Rank	**1**	4	2	3
1	10	all	Wins	35	6	23	11
1	10	all	Losses	7	33	15	20
1	10	all	Diff	28	-27	8	-9
1	10	all	Rank	**1**	4	2	3
20	10	all	Wins	30	2	16	9
20	10	all	Losses	3	25	12	17
20	10	all	Diff	27	-23	4	-8
20	10	all	Rank	**1**	4	2	3

Table 10.8 presents the wins and losses for all Type II DMOOPs over all performance measures and all n_t-τ_t combinations. For Type II DMOOPs, *cl* obtained the best overall rank and *pe* the worst rank. Similar to the results for Type I DMOOPs, *pe* and *re* performed poorly, being awarded more losses than wins. All other approaches were completely outperformed by *cl*, with *cl* obtaining 133 more wins than losses and *ra* that ranked second obtained only 29 more wins than losses.

Table 10.8: Overall wins and losses obtained by various boundary management strategies solving Type II DMOOPs

$\mathbf{n_t}$	$\tau_\mathbf{t}$	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
all	all	all	Wins	165	24	102	45
all	all	all	Losses	32	140	73	91
all	all	all	Diff	133	-116	29	-46
all	all	all	Rank	**1**	4	2	3

Type III DMOOPs

Table 10.9 presents the wins and losses with regards to the various performance measures for the boundary management approaches solving Type III DMOOPs.

The following observations are made:

- With regards to *acc*, *re* performed the best and *ra* performed the worst. Furthermore, both *pe* and *ra* obtained more losses than wins.
- For *stab*, the highest rank was obtained by *cl* and the lowest rank by *ra*. All approaches, except *cl*, were awarded more losses than wins.
- The best performance for *NS* was achieved by *cl* and *pe* performed the worst. Both *pe* and *ra* scored more losses than wins.

Table 10.9: Overall wins and losses for various performance measures obtained by various boundary management strategies solving Type III DMOOP

$\mathbf{n_t}$	τ_t	PM	Results	Boundary management strategies			
				cl	**pe**	**ra**	**re**
all	all	*acc*	Wins	30	13	8	29
all	all	*acc*	Losses	24	20	26	10
all	all	*acc*	Diff	6	-7	-18	19
all	all	*acc*	Rank	2	3	4	**1**
all	all	*stab*	Wins	41	2	0	1
all	all	*stab*	Losses	1	13	16	14
all	all	*stab*	Diff	40	-11	-16	-13
all	all	*stab*	Rank	**1**	2	4	3
all	all	*NS*	Wins	63	11	13	28
all	all	*NS*	Losses	24	33	34	24
all	all	*NS*	Diff	39	-22	-21	4
all	all	*NS*	Rank	**1**	4	3	2

The wins and losses with regards to the various environment types for Type III DMOOPs are presented in Table 10.10.

Observations made with regards to the various n_t-τ_t combinations are:

- *cl* performed the best in all environments, and *ra* the worst. In addition, for $n_t = 1$ and $\tau_t = 10$, *pe* ranked the lowest together with *ra*.
- *cl* was the only approach that obtained more wins than losses in all environments. *pe* and *ra* performed poorly, obtaining more losses than wins for all environments.
- *re* obtained the second best rank in all environments, and obtained more losses

than wins for only the slowly changing environments ($n_t = 10$ and $\tau_t = 25$, and $n_t = 10$ and $\tau_t = 50$).

Table 10.10: Overall wins and losses for various frequencies and severities of change obtained by various boundary management strategies solving Type III DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
10	10	all	Wins	26	7	5	14
10	10	all	Losses	12	13	17	10
10	10	all	Diff	14	-6	-12	4
10	10	all	Rank	**1**	3	4	2
10	25	all	Wins	26	5	2	8
10	25	all	Losses	7	11	14	9
10	25	all	Diff	19	-6	-12	-1
10	25	all	Rank	**1**	3	4	2
10	50	all	Wins	25	3	3	9
10	50	all	Losses	9	10	11	10
10	50	all	Diff	16	-7	-8	-1
10	50	all	Rank	**1**	3	4	2
1	10	all	Wins	30	5	6	14
1	10	all	Losses	12	16	17	10
1	10	all	Diff	18	-11	-11	4
1	10	all	Rank	**1**	3	3	2
20	10	all	Wins	27	6	5	13
20	10	all	Losses	9	16	17	9
20	10	all	Diff	18	-10	-12	4
20	10	all	Rank	**1**	3	4	2

Table 10.11 presents the wins and losses for Type III DMOOPs measured over all performance measures and all n_t-τ_t combinations. The best overall performance for Type III DMOOPs was obtained by *cl*, with *ra* performing the worst.

Table 10.11: Overall wins and losses obtained by various boundary management strategies solving Type III DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
all	all	all	Wins	134	26	21	58
all	all	all	Losses	49	66	76	48
all	all	all	Diff	85	-40	-55	10
all	all	all	Rank	**1**	3	4	2

Overall Performance

This section discusses the overall performance of the various approaches used to manage boundary constraint violations. The overall wins and losses over all DMOOPs, n_t-τ_t combinations and performance measures are presented in Table 11.19.

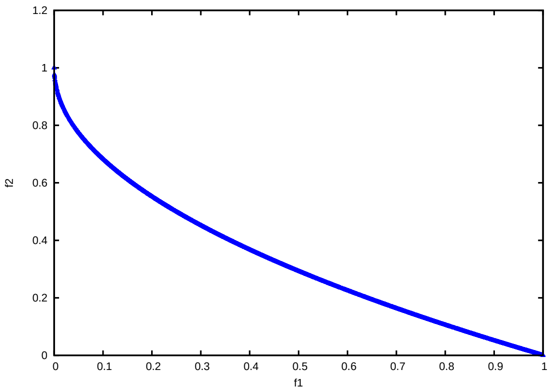
Table 10.12: Overall wins and losses by various boundary management strategies

$\mathbf{n_t}$	$\tau_\mathbf{t}$	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
all	all	all	Wins	329	61	155	120
all	all	all	Losses	114	230	162	159
all	all	all	Diff	215	-169	-7	-39
all	all	all	Rank	**1**	4	2	3

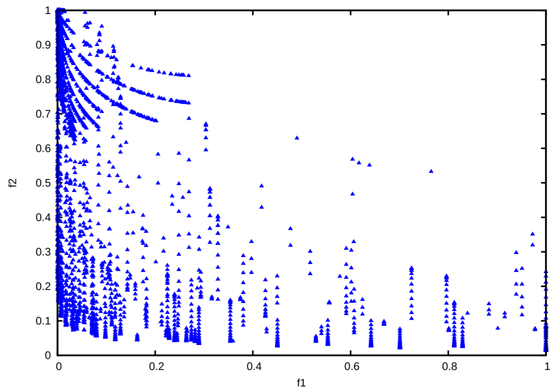
The following are observed:

- The best performance was achieved by *cl*, completely outperforming the other approaches with 215 more wins than losses.
- All approaches, except *cl*, were awarded more losses than wins.
- The lowest rank was obtained by *pe*, with 169 more losses than wins.

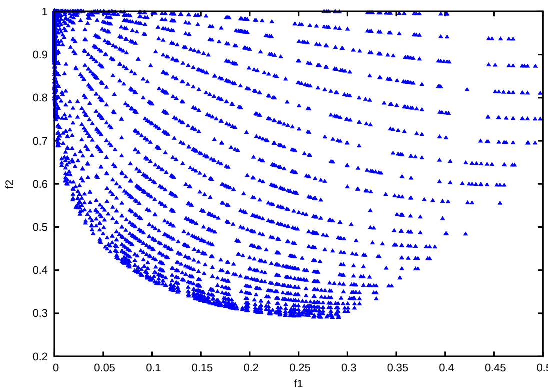
The POF^*s found by *cl* for $n_t = 10$ and $\tau_t = 10$ are illustrated in Figures 10.1 to 10.3. The Figures indicate that DVEPSO successfully tracked the changing POF for FDA1$_{Zhou}$, FDA2$_{Camara}$, FDA3, FDA3$_{Camara}$, DIMP2, dMOP2, dMOP2$_{iso}$ and dMOP2$_{dec}$. Even though DVEPSO struggled with FDA2, it did manage to find some of the POFs, but with a bad spread of solutions. For dMOP3, DVEPSO found solutions close to the true POF, but also quite a few solutions that are a bit further away. With the discontinous function HE1, DVEPSO struggled to find the different continuous sections of the discontinous POF. However, for the discontinous function HE2, DVEPSO managed to find solutions for all the continuous parts of the POF, but the found solutions were not very close to the true POF. For HE6 and HE7 DVEPSO did manage to find some of the POFs, but did not always track the POF successfully with a good spread of solutions. However, for HE9 DVEPSO failed to track the changing POF.



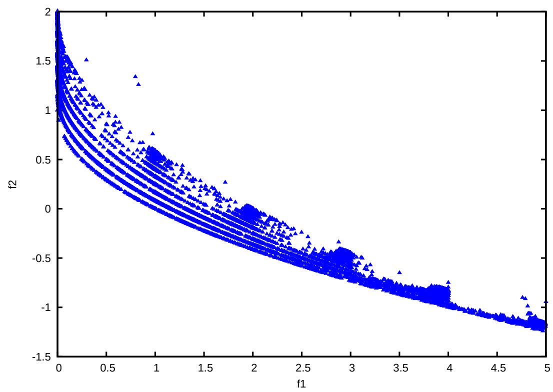
(a) POF^* for FDA1$_{Zhou}$



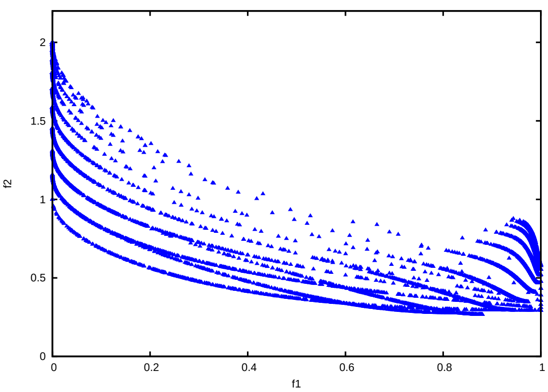
(b) POF^* for FDA2



(c) POF^* for FDA2$_{Camara}$

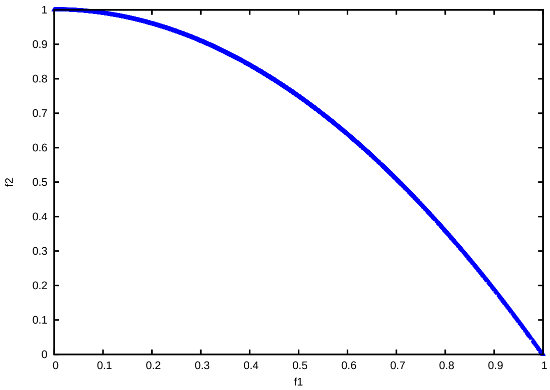


(d) POF^* for FDA3

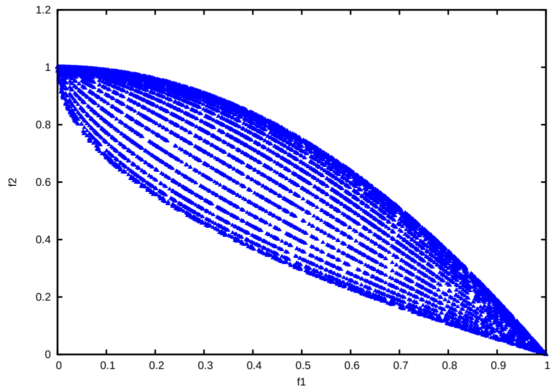


(e) POF^* for FDA3$_{Camara}$

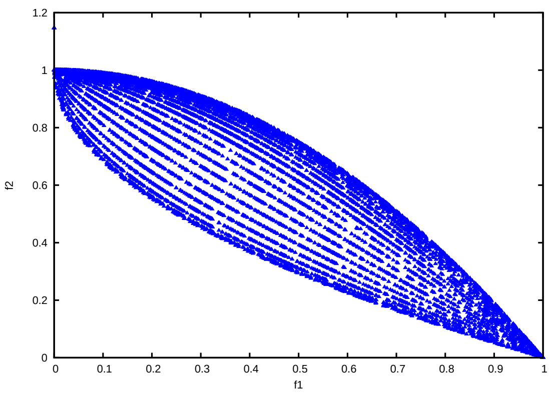
Figure 10.1: POF^* for the FDA functions of DVEPSO using cl for $n_t = 10$ and $\tau_t = 10$



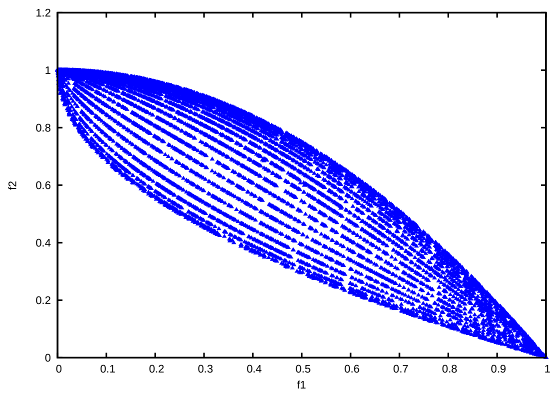
(a) POF^* for DIMP2



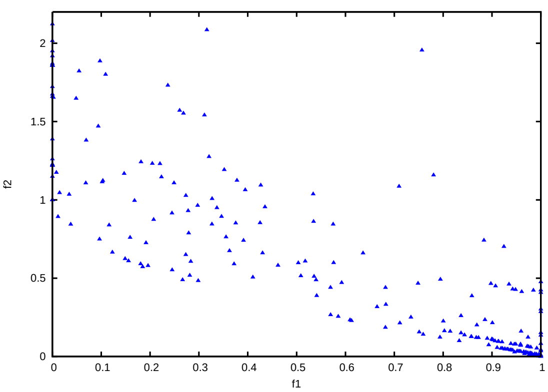
(b) POF^* for dMOP2



(c) POF^* for dMOP2$_{iso}$

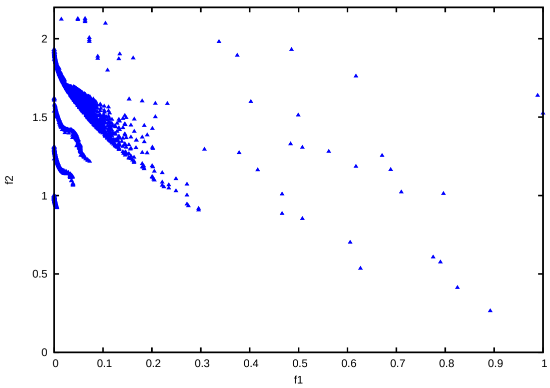


(d) POF^* for dMOP2$_{dec}$

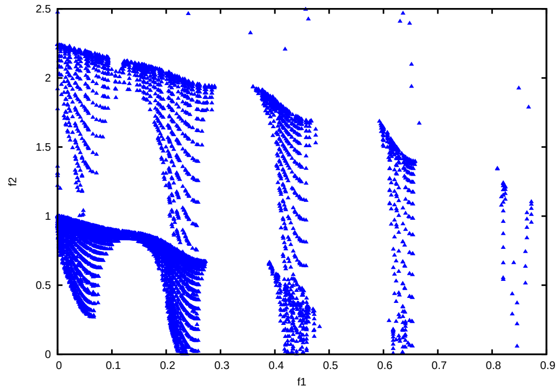


(e) POF^* for dMOP3

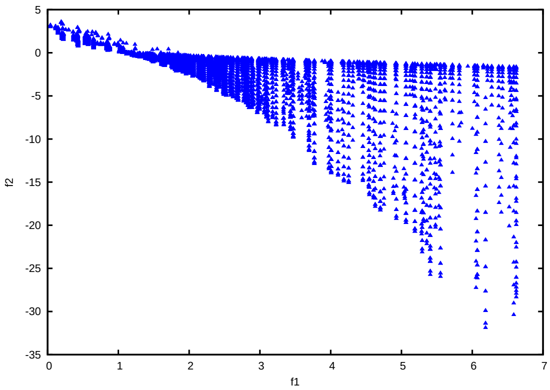
Figure 10.2: POF^* for DIMP2 and the dMOP functions of DVEPSO using cl for $n_t = 10$ and $\tau_t = 10$



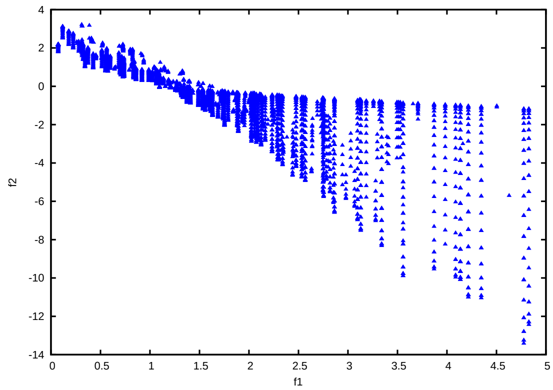
(a) POF^{*} for HE1



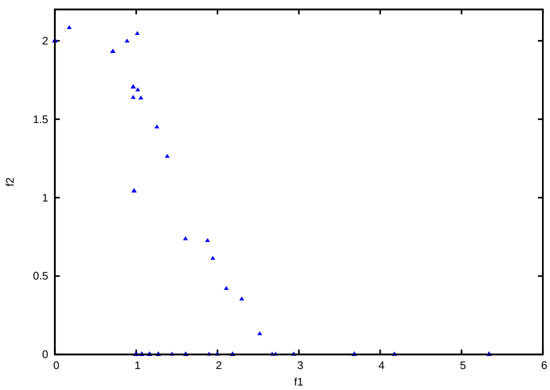
(b) POF^{*} for HE2



(c) POF^{*} for HE6



(d) POF^{*} for HE7



(e) POF^{*} for HE9

Figure 10.3: POF^{*} for the HE functions of DVEPSO using cl for $n_t = 10$ and $\tau_t = 10$

General Observations

This section discusses general observations that were made about the performance of the boundary management approaches.

Even though *cl* completely outperformed the other boundary management approaches with regards to the overall wins and losses, there were a few DMOOPs where *cl* did not perform that well, namely FDA2, dMOP3, HE6 and HE9.

The wins and losses for FDA2 are presented in Table 10.13. *cl* and *re* obtained more losses than wins and *re* performed the worst. For the various performance measures, *cl* obtained the second lowest rank for *acc* and *stab* and the second best rank for NS. However, the difference between the overall wins and losses for FDA2 caused *cl* to obtain the lowest overall rank. For the various n_t-τ_t combinations, *cl* performed poorly for $n_t = 10$ and $\tau_t = 25$, obtaining the worst rank. In addition, *cl* obtained the second lowest rank for all other environments. A similar trend was observed for dMOP3. However, when solving dMOP3, *cl* obtained the worst rank for *acc* and *stab*, but the best rank for NS. Furthermore, *cl* performed the best in the slowly changing environments and the second best for $n_t = 1$ and $\tau_t = 10$. However, for $n_t = 10$ and $\tau_t = 10$, *cl* obtained the second lowest rank and for $n_t = 20$ and $\tau_t = 10$, *cl* performed the worst.

Table 10.13: Wins and Losses of FDA2 for various boundary management strategies

$\mathbf{n_t}$	$\boldsymbol{\tau_t}$	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
10	10	*acc*	Wins	0	2	3	0
10	10	*acc*	Losses	2	1	0	2
10	10	*acc*	Diff	-2	1	3	-2
10	10	*acc*	Rank	3	2	**1**	3
10	25	*acc*	Wins	0	2	3	0
10	25	*acc*	Losses	2	1	0	2
10	25	*acc*	Diff	-2	1	3	-2
10	25	*acc*	Rank	3	2	**1**	3
10	50	*acc*	Wins	0	2	3	0
10	50	*acc*	Losses	2	1	0	2
10	50	*acc*	Diff	-2	1	3	-2
10	50	*acc*	Rank	3	2	**1**	3
1	10	*acc*	Wins	0	2	3	0
1	10	*acc*	Losses	2	1	0	2
1	10	*acc*	Diff	-2	1	3	-2
1	10	*acc*	Rank	3	2	**1**	3

Continued on next page

n_t	τ_t	PM	Results	Boundary management strategies cl	pe	ra	re
20	10	*acc*	Wins	0	2	3	0
20	10	*acc*	Losses	2	1	0	2
20	10	*acc*	Diff	-2	1	3	-2
20	10	*acc*	Rank	3	2	**1**	3
all	all	*acc*	Wins	0	10	15	0
all	all	*acc*	Losses	10	5	0	10
all	all	*acc*	Diff	-10	5	15	-10
all	all	*acc*	Rank	3	2	**1**	3
10	10	*stab*	Wins	0	1	0	1
10	10	*stab*	Losses	0	0	2	0
10	10	*stab*	Diff	0	1	-2	1
10	10	*stab*	Rank	3	**1**	4	**1**
10	25	*stab*	Wins	0	1	1	1
10	25	*stab*	Losses	1	0	2	0
10	25	*stab*	Diff	-1	1	-1	1
10	25	*stab*	Rank	3	**1**	3	**1**
10	50	*stab*	Wins	0	1	0	0
10	50	*stab*	Losses	0	0	1	0
10	50	*stab*	Diff	0	1	-1	0
10	50	*stab*	Rank	2	**1**	4	2
1	10	*stab*	Wins	0	1	0	0
1	10	*stab*	Losses	0	0	1	0
1	10	*stab*	Diff	0	1	-1	0
1	10	*stab*	Rank	2	**1**	4	2
all	all	*stab*	Wins	0	4	1	2
all	all	*stab*	Losses	1	0	6	0
all	all	*stab*	Diff	-1	4	-5	2
all	all	*stab*	Rank	3	**1**	4	2
10	10	NS	Wins	1	3	0	2
10	10	NS	Losses	2	0	3	1
10	10	NS	Diff	-1	3	-3	1
10	10	NS	Rank	3	**1**	4	2
10	25	NS	Wins	1	1	0	1
10	25	NS	Losses	0	0	3	0
10	25	NS	Diff	1	1	-3	1
10	25	NS	Rank	**1**	**1**	4	**1**
10	50	NS	Wins	1	1	0	1
10	50	NS	Losses	0	0	3	0
10	50	NS	Diff	1	1	-3	1
10	50	NS	Rank	**1**	**1**	4	**1**
1	10	NS	Wins	1	1	0	1
1	10	NS	Losses	0	0	3	0
1	10	NS	Diff	1	1	-3	1
1	10	NS	Rank	**1**	**1**	4	**1**
20	10	NS	Wins	1	1	0	2

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$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
20	10	NS	Losses	1	0	3	0
20	10	NS	Diff	0	1	-3	2
20	10	NS	Rank	3	2	4	**1**
all	all	NS	Wins	3	0	15	1
all	all	NS	Losses	5	7	0	7
all	all	NS	Diff	-2	-7	15	-6
all	all	NS	Rank	2	4	**1**	3
10	10	all	Wins	2	3	6	2
10	10	all	Losses	3	4	2	4
10	10	all	Diff	-1	-1	4	-2
10	10	all	Rank	2	2	**1**	4
10	25	all	Wins	0	3	7	1
10	25	all	Losses	4	2	2	3
10	25	all	Diff	-4	1	5	-2
10	25	all	Rank	4	2	**1**	3
10	50	all	Wins	0	3	6	0
10	50	all	Losses	3	2	1	3
10	50	all	Diff	-3	1	5	-3
10	50	all	Rank	3	2	**1**	3
1	10	all	Wins	0	3	6	0
1	10	all	Losses	3	2	1	3
1	10	all	Diff	-3	1	5	-3
1	10	all	Rank	3	2	**1**	3
20	10	all	Wins	1	2	6	0
20	10	all	Losses	3	2	0	4
20	10	all	Diff	-2	0	6	-4
20	10	all	Rank	3	2	**1**	4
all	all	all	Wins	3	14	31	3
all	all	all	Losses	16	12	6	17
all	all	all	Diff	-13	2	25	-14
all	all	all	Rank	3	2	**1**	4

The *cl* approach obtained a mixed performance with two of the HE functions where the decision variables each have their own POS and the POSs are defined by non-linear functions. Table 10.14 presents the wins and losses for HE6. For *acc*, there was no statistical significant difference between the performance measure values of the various approaches for all n_t-τ_t combinations, except $n_t = 1$ and $\tau_t = 10$. Furthermore, for *stab* there was no statistical significant difference for all n_t-τ_t combinations. However, for NS there was a statistical significant difference, and *cl* performed the best with regards to NS. Therefore, *cl* obtained the best overall rank for HE6. The second best rank was obtained by *re*. However, *cl* was the only approach that obtained more wins than losses.

Table 10.14: Wins and Losses of HE6 for various boundary management strategies

n_t	τ_t	PM	Results	Boundary management strategies cl	pe	ra	re
1	10	*acc*	Wins	0	1	1	1
1	10	*acc*	Losses	3	0	0	0
1	10	*acc*	Diff	-3	1	1	1
1	10	*acc*	Rank	4	**1**	**1**	**1**
all	all	*acc*	Wins	0	1	1	1
all	all	*acc*	Losses	3	0	0	0
all	all	*acc*	Diff	-3	1	1	1
all	all	*acc*	Rank	4	**1**	**1**	**1**
10	10	*NS*	Wins	3	0	0	1
10	10	*NS*	Losses	0	1	2	1
10	10	*NS*	Diff	3	-1	-2	0
10	10	*NS*	Rank	**1**	3	4	2
10	25	*NS*	Wins	3	0	0	0
10	25	*NS*	Losses	0	1	1	1
10	25	*NS*	Diff	3	-1	-1	-1
10	25	*NS*	Rank	**1**	2	2	2
10	50	*NS*	Wins	3	0	0	0
10	50	*NS*	Losses	0	1	1	1
10	50	*NS*	Diff	3	-1	-1	-1
10	50	*NS*	Rank	**1**	2	2	2
1	10	*NS*	Wins	3	0	0	1
1	10	*NS*	Losses	0	1	2	1
1	10	*NS*	Diff	3	-1	-2	0
1	10	*NS*	Rank	**1**	3	4	2
20	10	*NS*	Wins	3	0	0	0
20	10	*NS*	Losses	0	1	1	1
20	10	*NS*	Diff	3	-1	-1	-1
20	10	*NS*	Rank	**1**	2	2	2
all	all	*NS*	Wins	15	0	0	2
all	all	*NS*	Losses	0	5	7	5
all	all	*NS*	Diff	15	-5	-7	-3
all	all	*NS*	Rank	**1**	3	4	2
10	10	all	Wins	3	0	0	1
10	10	all	Losses	0	1	2	1
10	10	all	Diff	3	-1	-2	0
10	10	all	Rank	**1**	3	4	2
10	25	all	Wins	3	0	0	0
10	25	all	Losses	0	1	1	1
10	25	all	Diff	3	-1	-1	-1
10	25	all	Rank	**1**	2	2	2
10	50	all	Wins	3	0	0	0
10	50	all	Losses	0	1	1	1
10	50	all	Diff	3	-1	-1	-1

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n_t	τ_t	PM	Results	Boundary management strategies			
				cl	pe	ra	re
10	50	all	Rank	**1**	2	2	2
1	10	all	Wins	3	1	1	2
1	10	all	Losses	3	1	2	1
1	10	all	Diff	0	0	-1	1
1	10	all	Rank	2	2	4	**1**
20	10	all	Wins	3	0	0	0
20	10	all	Losses	0	1	1	1
20	10	all	Diff	3	-1	-1	-1
20	10	all	Rank	**1**	2	2	2
all	all	all	Wins	15	1	1	3
all	all	all	Losses	3	5	7	5
all	all	all	Diff	12	-4	-6	-2
all	all	all	Rank	**1**	3	4	2

The wins and losses for HE9 are presented in Table 10.15. The poorest performance with regards to *acc* was obtained by *cl*. However, for *stab* and *NS*, *cl* performed the best. However, for the various environment types, *cl* performed the best for all n_t-τ_t combinations.

Table 10.15: Wins and Losses of HE9 for various boundary management strategies

n_t	τ_t	PM	Results	Boundary management strategies			
				cl	pe	ra	re
10	10	*acc*	Wins	0	1	1	1
10	10	*acc*	Losses	3	0	0	0
10	10	*acc*	Diff	-3	1	1	1
10	10	*acc*	Rank	4	**1**	**1**	**1**
10	25	*acc*	Wins	0	1	1	1
10	25	*acc*	Losses	3	0	0	0
10	25	*acc*	Diff	-3	1	1	1
10	25	*acc*	Rank	4	**1**	**1**	**1**
10	50	*acc*	Wins	0	1	1	1
10	50	*acc*	Losses	3	0	0	0
10	50	*acc*	Diff	-3	1	1	1
10	50	*acc*	Rank	4	**1**	**1**	**1**
1	10	*acc*	Wins	0	1	1	1
1	10	*acc*	Losses	3	0	0	0
1	10	*acc*	Diff	-3	1	1	1
1	10	*acc*	Rank	4	**1**	**1**	**1**
20	10	*acc*	Wins	0	1	1	1
20	10	*acc*	Losses	3	0	0	0
20	10	*acc*	Diff	-3	1	1	1
20	10	*acc*	Rank	4	**1**	**1**	**1**
all	all	*acc*	Wins	0	5	5	5

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$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
all	all	*acc*	Losses	15	0	0	0
all	all	*acc*	Diff	-15	5	5	5
all	all	*acc*	Rank	4	**1**	**1**	**1**
10	10	*stab*	Wins	3	0	0	0
10	10	*stab*	Losses	0	1	1	1
10	10	*stab*	Diff	3	-1	-1	-1
10	10	*stab*	Rank	**1**	2	2	2
10	25	*stab*	Wins	3	0	0	0
10	25	*stab*	Losses	0	1	1	1
10	25	*stab*	Diff	3	-1	-1	-1
10	25	*stab*	Rank	**1**	2	2	2
10	50	*stab*	Wins	3	0	0	0
10	50	*stab*	Losses	0	1	1	1
10	50	*stab*	Diff	3	-1	-1	-1
10	50	*stab*	Rank	**1**	2	2	2
1	10	*stab*	Wins	3	0	0	0
1	10	*stab*	Losses	0	1	1	1
1	10	*stab*	Diff	3	-1	-1	-1
1	10	*stab*	Rank	**1**	2	2	2
20	10	*stab*	Wins	3	0	0	0
20	10	*stab*	Losses	0	1	1	1
20	10	*stab*	Diff	3	-1	-1	-1
20	10	*stab*	Rank	**1**	2	2	2
all	all	*stab*	Wins	15	0	0	0
all	all	*stab*	Losses	0	5	5	5
all	all	*stab*	Diff	15	-5	-5	-5
all	all	*stab*	Rank	**1**	2	2	2
10	10	NS	Wins	3	1	1	0
10	10	NS	Losses	0	1	1	3
10	10	NS	Diff	3	0	0	-3
10	10	NS	Rank	**1**	2	2	4
10	25	NS	Wins	3	0	0	0
10	25	NS	Losses	0	1	1	1
10	25	NS	Diff	3	-1	-1	-1
10	25	NS	Rank	**1**	2	2	2
10	50	NS	Wins	3	0	1	0
10	50	NS	Losses	0	1	1	2
10	50	NS	Diff	3	-1	0	-2
10	50	NS	Rank	**1**	3	2	4
1	10	NS	Wins	3	0	0	0
1	10	NS	Losses	0	1	1	1
1	10	NS	Diff	3	-1	-1	-1
1	10	NS	Rank	**1**	2	2	2
20	10	NS	Wins	3	0	1	1
20	10	NS	Losses	0	3	1	1

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$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Boundary management strategies**			
				cl	**pe**	**ra**	**re**
20	10	*NS*	Diff	3	-3	0	0
20	10	*NS*	Rank	**1**	4	2	2
all	all	*NS*	Wins	15	1	3	1
all	all	*NS*	Losses	0	7	5	8
all	all	*NS*	Diff	15	-6	-2	-7
all	all	*NS*	Rank	**1**	3	2	4
10	10	all	Wins	6	2	2	1
10	10	all	Losses	3	2	2	4
10	10	all	Diff	3	0	0	-3
10	10	all	Rank	**1**	2	2	4
10	25	all	Wins	6	1	1	1
10	25	all	Losses	3	2	2	2
10	25	all	Diff	3	-1	-1	-1
10	25	all	Rank	**1**	2	2	2
10	50	all	Wins	6	1	2	1
10	50	all	Losses	3	2	2	3
10	50	all	Diff	3	-1	0	-2
10	50	all	Rank	**1**	3	2	4
1	10	all	Wins	6	1	1	1
1	10	all	Losses	3	2	2	2
1	10	all	Diff	3	-1	-1	-1
1	10	all	Rank	**1**	2	2	2
20	10	all	Wins	6	1	2	2
20	10	all	Losses	3	4	2	2
20	10	all	Diff	3	-3	0	0
20	10	all	Rank	**1**	4	2	2
all	all	all	Wins	30	6	8	6
all	all	all	Losses	15	12	10	13
all	all	all	Diff	15	-6	-2	-7
all	all	all	Rank	**1**	3	2	4

The results discussed above indicate that possible future work should include the development of a hyper-heuristic approach that selects the best boundary mechanism on the fly.

The next section discusses results obtained by various knowledge sharing approaches.

10.2.2 Knowledge Sharing Swarm Topologies

This section discusses results obtained by various approaches used to share knowledge between the sub-swarms of DVEPSO. The results are discussed with regards to each performance measure and each n_t-τ_t combination. Results obtained for DMOOPs of

Type I, II and III are also presented. The performance of each knowledge sharing approach measured over all performance measures, n_t-τ_t combinations and DMOOPs are discussed to determine the approach's overall performance. Furthermore, general observations are also highlighted. The wins and losses of the various knowledge sharing strategies are presented in Tables 10.16 to 10.27. In Tables 10.16 to 10.27, ra and ri indicates either a random or ring topology, and g or t indicates whether the guide is selected as the gbest of the selected sub-swarm or through tournament selection applied to the selected sub-swarm's particles' positions.

Results with regards to Performance Measures

This section discusses the results with regards to the performance measures obtained by the various approaches used to share knowledge between the sub-swarms. The wins and losses obtained by the approaches over all n_t-τ_t combinations for the various performance measures are presented in Table 10.16.

Table 10.16: Overall wins and losses for various performance measures obtained by various knowledge sharing strategies

n_t	τ_t	PM	Results	knowledge sharing strategies			
				ra-g	ra-t	ri-g	ri-t
all	all	*acc*	Wins	64	93	23	87
all	all	*acc*	Losses	70	31	132	34
all	all	*acc*	Diff	-6	62	-109	53
all	all	*acc*	Rank	3	**1**	4	2
all	all	*stab*	Wins	60	70	7	76
all	all	*stab*	Losses	43	19	127	24
all	all	*stab*	Diff	17	51	-120	52
all	all	*stab*	Rank	3	2	4	**1**
all	all	*NS*	Wins	62	56	42	62
all	all	*NS*	Losses	38	45	102	37
all	all	*NS*	Diff	24	11	-60	25
all	all	*NS*	Rank	2	3	4	**1**

From the obtained results, the following observations are made:

- The best performance for *acc* was achieved by *ra-t*, while *ri-g* performed the worst. Both approaches that use tournament selection outperformed the approaches using the gbest values.

- For *stab*, *ri-t* performed the best and *ri-g* performed significantly the worst. Once again, both tournament selection approaches outperformed the gbest approaches. A really poor performance was achieved by *ri-g*, obtaining 120 more losses than wins.
- Measured against NS, *ri-t* performed the best and *ri-g* performed significantly the worst. All approaches, except *ri-g*, were awarded more wins than losses.

Results with regards to Various Frequencies and Severities of Change

This section discusses the results obtained with regards to the various n_t-τ_t combinations. The wins and losses obtained by the knowledge sharing approaches are presented in Table 10.17.

Table 10.17: Overall wins and losses for various frequencies and severities of change obtained by various knowledge sharing strategies

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**knowledge sharing strategies**			
				ra-g	**ra-t**	**ri-g**	**ri-t**
10	10	all	Wins	3	4	0	7
10	10	all	Losses	2	2	9	1
10	10	all	Diff	1	2	-9	6
10	10	all	Rank	3	2	4	**1**
10	25	all	Wins	3	4	0	7
10	25	all	Losses	2	2	9	1
10	25	all	Diff	1	2	-9	6
10	25	all	Rank	3	2	4	**1**
10	50	all	Wins	3	4	0	8
10	50	all	Losses	3	2	9	1
10	50	all	Diff	0	2	-9	7
10	50	all	Rank	3	2	4	**1**
1	10	all	Wins	4	3	0	8
1	10	all	Losses	2	4	9	0
1	10	all	Diff	2	-1	-9	8
1	10	all	Rank	2	3	4	**1**
20	10	all	Wins	3	3	0	7
20	10	all	Losses	2	2	9	0
20	10	all	Diff	1	1	-9	7
20	10	all	Rank	2	2	4	**1**

The following observations are made with regards to the various environment types:

- For all environments the best performance was obtained by *ri-t* and the worst by *ri-g*. Furthermore, the tournament selection approaches outperformed the gbest approaches.

Results for Various Dynamic Multi-objective Optimisation Problem Types

The results obtained for DMOOPs of Type I, II and III are discussed in this section.

Type I DMOOPs

This section discusses the results obtained for Type I DMOOPs. The wins and losses obtained by the various knowledge sharing approaches for the performance measures measured over all n_t-τ_t combinations are presented in Table 10.16.

Table 10.18: Overall wins and losses for various performance measures obtained by various knowledge sharing strategies solving Type I DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**knowledge sharing strategies**			
				ra-g	**ra-t**	**ri-g**	**ri-t**
all	all	*acc*	Wins	5	5	0	15
all	all	*acc*	Losses	5	5	15	0
all	all	*acc*	Diff	0	0	-15	15
all	all	*acc*	Rank	2	2	4	**1**
all	all	*stab*	Wins	5	5	0	15
all	all	*stab*	Losses	5	5	15	0
all	all	*stab*	Diff	0	0	-15	15
all	all	*stab*	Rank	2	2	4	**1**
all	all	NS	Wins	6	8	0	7
all	all	NS	Losses	1	2	15	3
all	all	NS	Diff	5	6	-15	4
all	all	NS	Rank	2	**1**	4	3

From the obtained results, the following observations are made:

- For *acc* and *stab* the highest rank was obtained by *ri-t*. The worst rank was obtained by *ri-g*, with zero wins.
- Measured against NS, *ra-t* ranked the best and *ri-g* the worst. Furthermore, *ri-g* was the only approach that obtained more losses than wins.

Table 10.19 presents the wins and losses with regards to the various n_t-τ_t combinations.

Table 10.19: Overall wins and losses for various frequencies and severities of change obtained by various knowledge sharing strategies solving Type I DMOOPs

$\mathbf{n_t}$	τ_t	PM	Results	knowledge sharing strategies			
				ra-g	ra-t	ri-g	ri-t
10	10	all	Wins	3	4	0	7
10	10	all	Losses	2	2	9	1
10	10	all	Diff	1	2	-9	6
10	10	all	Rank	3	2	4	**1**
10	25	all	Wins	3	4	0	7
10	25	all	Losses	2	2	9	1
10	25	all	Diff	1	2	-9	6
10	25	all	Rank	3	2	4	**1**
10	50	all	Wins	3	4	0	8
10	50	all	Losses	3	2	9	1
10	50	all	Diff	0	2	-9	7
10	50	all	Rank	3	2	4	**1**
1	10	all	Wins	4	3	0	8
1	10	all	Losses	2	4	9	0
1	10	all	Diff	2	-1	-9	8
1	10	all	Rank	2	3	4	**1**
20	10	all	Wins	3	3	0	7
20	10	all	Losses	2	2	9	0
20	10	all	Diff	1	1	-9	7
20	10	all	Rank	2	2	4	**1**

With regards to the various n_t-τ_t combinations, the following observations are made:

- For all n_t-τ_t combinations the best performance was achieved by *ri-t* and *ri-g* performed the worst.
- *ri-g* performed poorly, obtaining more losses than wins for all environments.

The wins and losses measured over all performance measures and n_t-τ_t for Type I DMOOPs are presented in Table 10.20.

Table 10.20: Overall wins and losses obtained by various knowledge sharing strategies solving Type I DMOOPs

$\mathbf{n_t}$	τ_t	PM	Results	knowledge sharing strategies			
				ra-g	ra-t	ri-g	ri-t
all	all	all	Wins	16	18	0	37
all	all	all	Losses	11	12	45	3
all	all	all	Diff	5	6	-45	34
all	all	all	Rank	3	2	4	**1**

The best overall rank for Type I DMOOPs was obtained by *ri-t*, with *ri-g* obtaining the worst rank. Only *ri-g* was awarded more losses than wins for Type I DMOOPs.

Type II DMOOPs

This section discusses results obtained for Type II DMOOPs. Table 10.21 presents the wins and losses with regards to the various performance measures over all n_t-τ_t combinations for Type II DMOOPs.

Table 10.21: Overall wins and losses for various performance measures obtained by various knowledge sharing strategies solving Type II DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**knowledge sharing strategies**			
				ra-g	**ra-t**	**ri-g**	**ri-t**
all	all	*acc*	Wins	28	43	12	62
all	all	*acc*	Losses	44	20	79	2
all	all	*acc*	Diff	-16	23	-67	60
all	all	*acc*	Rank	3	2	4	**1**
all	all	*stab*	Wins	37	56	0	49
all	all	*stab*	Losses	29	5	94	14
all	all	*stab*	Diff	8	51	-94	35
all	all	*stab*	Rank	3	**1**	4	2
all	all	NS	Wins	27	31	29	35
all	all	NS	Losses	24	22	59	17
all	all	NS	Diff	3	9	-30	18
all	all	NS	Rank	3	2	4	**1**

The following observations are made:

- The best performance with regards to *acc* was obtained by *ri-t* and the worst performance by *ri-g*. Both gbest approaches performed poorly, obtaining more losses than wins.
- For *stab*, *ra-t* achieved the best rank and the worst rank was obtained by *ri-g*. Once again, the tournament approaches outperformed the gbest approaches.
- Measured against NS, *ri-t* performed the best and *ri-g* performed the worst. Similar to *acc* and *stab*, the tournament approaches outperformed the gbest approaches with regards to NS.

Table 10.22 presents the wins and losses of the knowledge sharing approaches with regards to the various n_t-τ_t combinations for Type II DMOOPs.

Table 10.22: Overall wins and losses for various frequencies and severities of change obtained by various knowledge sharing strategies solving Type II DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**knowledge sharing strategies**			
				ra-g	**ra-t**	**ri-g**	**ri-t**
10	10	all	Wins	22	35	9	33
10	10	all	Losses	25	9	56	9
10	10	all	Diff	-3	26	-47	24
10	10	all	Rank	3	**1**	4	2
10	25	all	Wins	17	24	8	28
10	25	all	Losses	17	7	47	6
10	25	all	Diff	0	17	-39	22
10	25	all	Rank	3	2	4	**1**
10	50	all	Wins	17	26	10	32
10	50	all	Losses	25	12	42	6
10	50	all	Diff	-8	14	-32	26
10	50	all	Rank	3	2	4	**1**
1	10	all	Wins	19	25	5	32
1	10	all	Losses	18	10	48	5
1	10	all	Diff	1	15	-43	27
1	10	all	Rank	3	2	4	**1**
20	10	all	Wins	17	20	9	21
20	10	all	Losses	12	9	39	7
20	10	all	Diff	5	11	-30	14
20	10	all	Rank	3	2	4	**1**

With regards to the various n_t-τ_t combinations, the following observations are made:

- The approach that performed the worst for all n_t-τ_t combinations was *ri-g*.
- *ri-t* performed the best for all environments.
- For $n_t = 10$ and $\tau_t = 10$, the tournament approaches outperformed the gbest approaches. *ri-g* performed poorly, obtaining more losses than wins.
- In a slow changing environment ($\tau_t = 25$ and $\tau_t = 50$)the gbest approaches was completely outperformed by the tournament approaches.

The wins and losses of the knowledge sharing approaches for Type II DMOOPs calculated over all performance measures and n_t-τ_t combinations are presented in Table 10.23. For Type II DMOOPs, *ri-t* obtained the best overall rank and *ri-g* performed the worst. Both tournament approaches performed really well, outperforming the gbest approaches.

Table 10.23: Overall wins and losses btained by various knowledge sharing strategies solving Type II DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**knowledge sharing strategies**			
				ra-g	**ra-t**	**ri-g**	**ri-t**
all	all	all	Wins	92	130	41	146
all	all	all	Losses	97	47	232	33
all	all	all	Diff	-5	83	-191	113
all	all	all	Rank	3	2	4	**1**

Type III DMOOPs

Table 10.24 presents the wins and losses with regards to the various performance measures for the knowledge sharing approaches solving Type III DMOOPs.

Table 10.24: Overall wins and losses for various performance measures obtained by various knowledge sharing strategies solving Type III DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**knowledge sharing strategies**			
				ra-g	**ra-t**	**ri-g**	**ri-t**
all	all	*acc*	Wins	31	45	11	10
all	all	*acc*	Losses	21	6	38	32
all	all	*acc*	Diff	10	39	-27	-22
all	all	*acc*	Rank	2	**1**	4	3
all	all	*stab*	Wins	18	9	7	12
all	all	*stab*	Losses	9	9	18	10
all	all	*stab*	Diff	9	0	-11	2
all	all	*stab*	Rank	**1**	3	4	2
all	all	NS	Wins	29	17	13	20
all	all	NS	Losses	13	21	28	17
all	all	NS	Diff	16	-4	-15	3
all	all	NS	Rank	**1**	3	4	2

The following observations are made:

- For *acc*, the best performance was obtained by *ra-t* and *ri-g* performed the worst. Both ring approaches performed poorly, with more losses than wins.
- The best performance with regards to *stab* was achieved by *ra-g* and the worst performance was achieved by *ri-g*.
- With regards to NS, *ra-g* ranked the best and *ri-g* ranked the worst.

The wins and losses for Type III DMOOPs with regards to the various environment types are presented in Table 10.25.

Table 10.25: Overall wins and losses for various frequencies and severities of change obtained by various knowledge sharing strategies solving Type III DMOOPs

n_t	τ_t	PM	Results	knowledge sharing strategies			
				ra-g	**ra-t**	**ri-g**	**ri-t**
10	10	all	Wins	19	19	8	9
10	10	all	Losses	9	10	21	15
10	10	all	Diff	10	9	-13	-6
10	10	all	Rank	**1**	2	4	3
10	25	all	Wins	13	14	4	6
10	25	all	Losses	7	4	17	9
10	25	all	Diff	6	10	-13	-3
10	25	all	Rank	2	**1**	4	3
10	50	all	Wins	14	7	4	8
10	50	all	Losses	6	4	15	8
10	50	all	Diff	8	3	-11	0
10	50	all	Rank	**1**	2	4	3
1	10	all	Wins	18	16	11	8
1	10	all	Losses	11	10	14	18
1	10	all	Diff	7	6	-3	-10
1	10	all	Rank	**1**	2	3	4
20	10	all	Wins	14	15	4	11
20	10	all	Losses	10	8	17	9
20	10	all	Diff	4	7	-13	2
20	10	all	Rank	2	**1**	4	3

With regards to the various n_t-τ_t combinations the following observations are made:

- For all n_t-τ_t combinations, except $n_t = 10$ and $\tau_t = 25$, and $n_t = 20$ and $\tau_t = 10$, the best performance was obtained by *ra-g*.
- For $n_t = 10$ and $\tau_t = 25$, and $n_t = 20$ and $\tau_t = 10$, *ra-t* performed the best.
- The worst rank was achieved by *ri-g* for all n_t-τ_t combinations, except $n_t = 1$ and $\tau_t = 10$.
- The worst performing approach for $n_t = 1$ and $\tau_t = 10$ was *ri-t*.

Table 10.26 presents the wins and losses for Type III DMOOPs measured over all performance measures and all n_t-τ_t combinations. When solving Type III DMOOPs, the best overall performance was obtained by both *ra-g* and *ra-t*, with *ri-g* performing the worst. The random approaches outperformed the ring approaches on the Type III DMOOPs. Both ring approaches performed poorly, being awarded more losses than wins.

Table 10.26: Overall wins and losses obtained by various knowledge sharing strategies solving Type III DMOOPs

n_t	τ_t	PM	Results	knowledge sharing strategies			
				ra-g	ra-t	ri-g	ri-t
all	all	all	Wins	78	71	31	42
all	all	all	Losses	43	36	84	59
all	all	all	Diff	35	35	-53	-17
all	all	all	Rank	**1**	**1**	4	3

Overall Performance

This section discusses the overall performance of the knowledge sharing approaches. The overall wins and losses over all DMOOPs, n_t-τ_t combinations and performance measures are presented in Table 10.27.

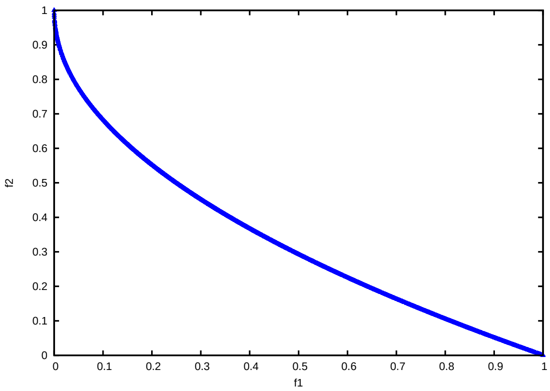
Table 10.27: Overall wins and losses obtained by various knowledge sharing strategies

n_t	τ_t	PM	Results	knowledge sharing strategies			
				ra-g	ra-t	ri-g	ri-t
all	all	all	Wins	186	219	72	225
all	all	all	Losses	151	95	361	95
all	all	all	Diff	35	124	-289	130
all	all	all	Rank	3	2	4	**1**

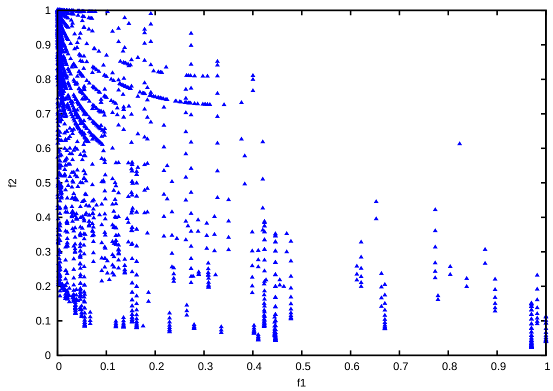
The following are observed:

- The best overall rank was achieved by *ri-t* and the worst performance was obtained by *ri-g*. Both *ri-t* and *ra-t* performed well, obtaining 130 and 124 more wins than losses respectively.
- The tournament approaches completely outperformed the gbest approaches.
- *ra-g* obtained only more wins than losses. In contrast, *ri-g* was awarded 289 more losses than wins.

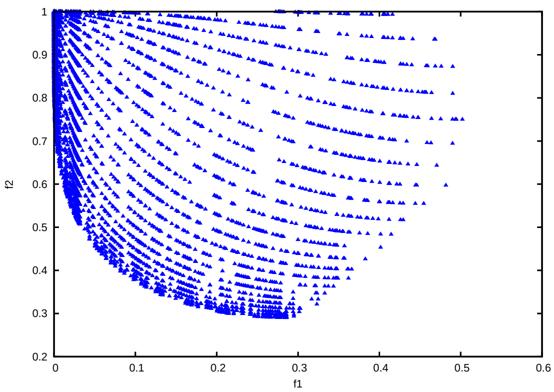
Figures 10.4 to 10.6 illustrate the POF^*s for $n_t = 10$ and $\tau_t = 10$ found by *ra-t*. The POF^*s found by *ra-t* followed the same trend as the POF^*s found by *cl* discussed in Section 10.2.1. However, *ra-t* found a better spread of solutions for FDA3 and FDA3$_{Camara}$. Furthermore, *ra-t* found solutions closer to the true POF of dMOP3.



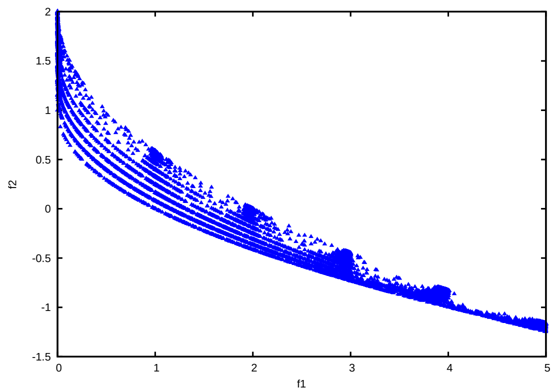
(a) POF^{*} for FDA1$_{Zhou}$



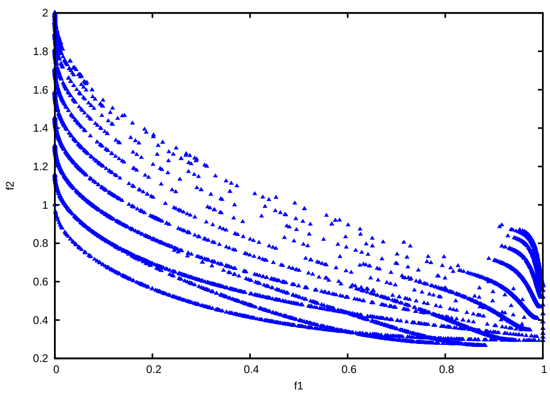
(b) POF^{*} for FDA2



(c) POF^{*} for FDA2$_{Camara}$

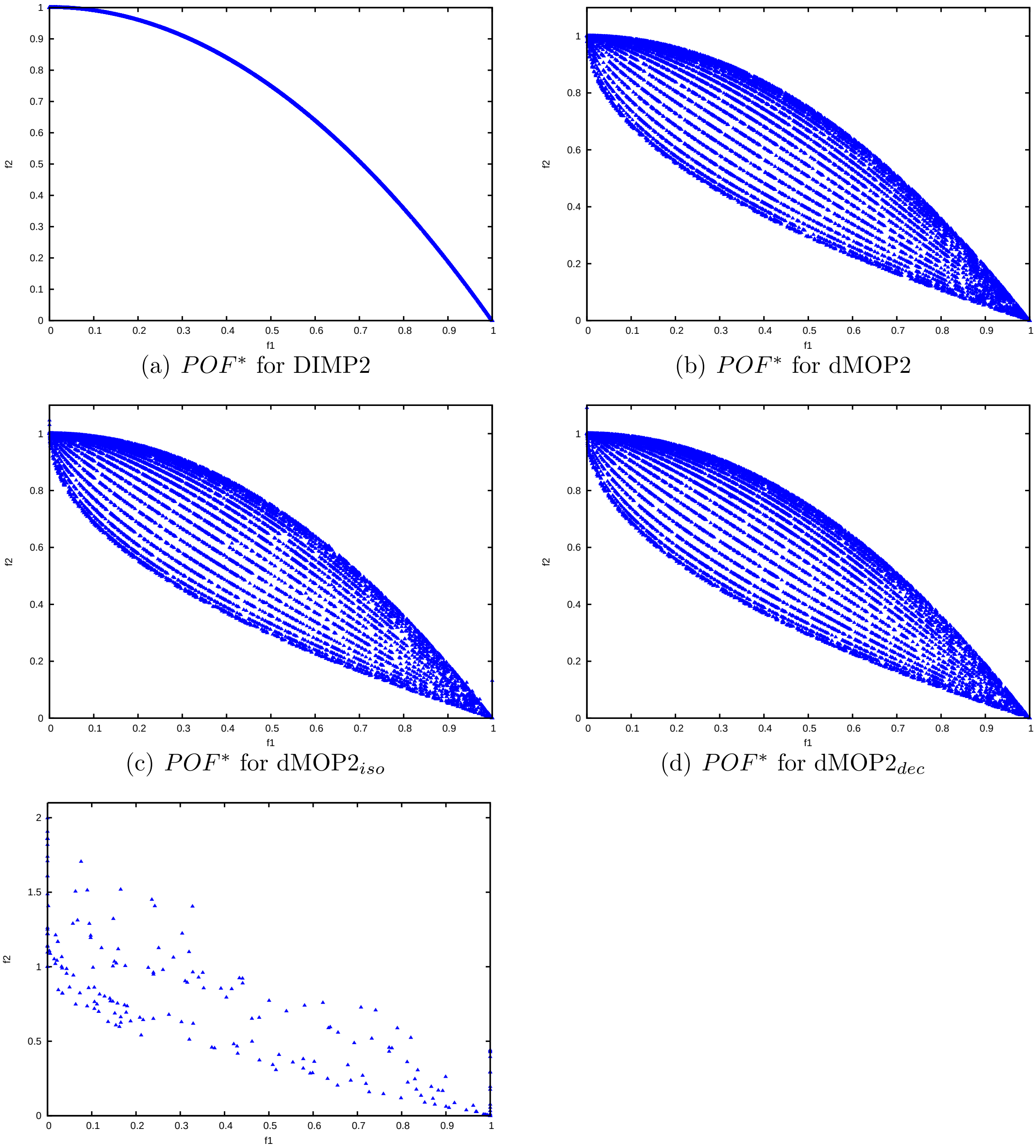


(d) POF^{*} for FDA3



(e) POF^{*} for FDA3$_{Camara}$

Figure 10.4: POF^{*} for FDA functions of DVEPSO using *ra-t* for $n_t = 10$ and $\tau_t = 10$



(a) POF^* for DIMP2

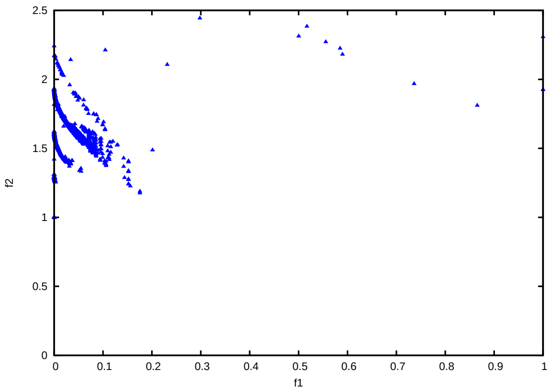
(b) POF^* for dMOP2

(c) POF^* for dMOP2$_{iso}$

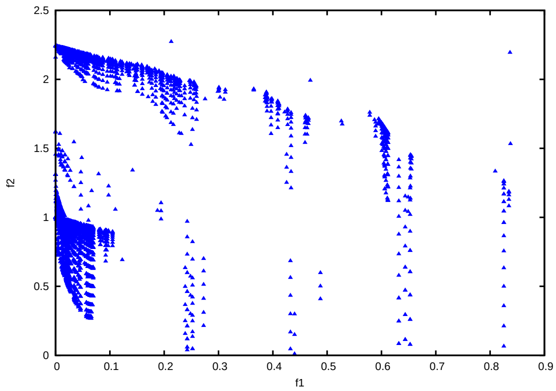
(d) POF^* for dMOP2$_{dec}$

(e) POF^* for dMOP3

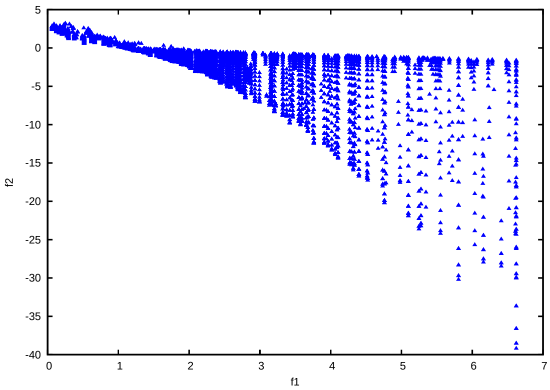
Figure 10.5: POF^* for DIMP2 and dMOP2 functions of DVEPSO using ra-t for $n_t = 10$ and $\tau_t = 10$



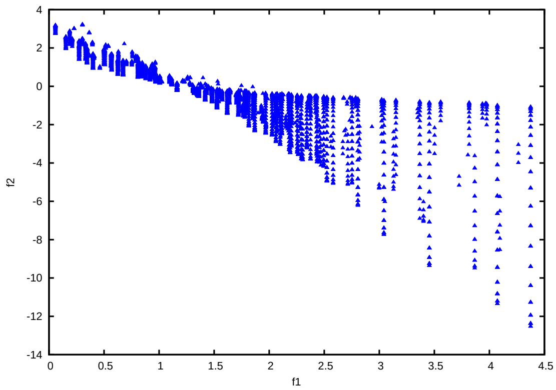
(a) POF^* for HE1



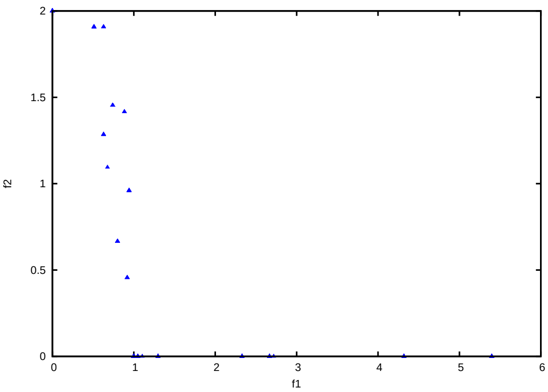
(b) POF^* for HE2



(c) POF^* for HE6



(d) POF^* for HE7



(e) POF^* for HE9

Figure 10.6: POF^* for HE functions of DVEPSO using ra-t for $n_t = 10$ and $\tau_t = 10$

General Observations

This section discusses general observations that were made with regards to the performance of the knowledge sharing approaches.

The second best overall performing knowledge sharing approach, *ra-t*, generally performed well for all DMOOPs. However, it did struggle with DIMP2 and FDA3. The wins and losses for DIMP2 are presented in Table 10.28. When solving DIMP2, *ra-t* performed well with regards to *acc* and *stab*. However, for *NS* it obtained the lowest rank. The number of losses awarded for *NS* to *ra-t*, caused *ra-t* to obtain the second lowest overall rank for DIMP2.

Table 10.28: Wins and Losses of DIMP2 for various knowledge sharing strategies

n_t	τ_t	PM	Results	knowledge sharing strategies			
				ra-g	**ra-t**	**ri-g**	**ri-t**
10	10	*acc*	Wins	1	1	0	3
10	10	*acc*	Losses	1	1	3	0
10	10	*acc*	Diff	0	0	-3	3
10	10	*acc*	Rank	2	2	4	**1**
10	25	*acc*	Wins	1	1	0	3
10	25	*acc*	Losses	1	1	3	0
10	25	*acc*	Diff	0	0	-3	3
10	25	*acc*	Rank	2	2	4	**1**
10	50	*acc*	Wins	1	1	0	3
10	50	*acc*	Losses	1	1	3	0
10	50	*acc*	Diff	0	0	-3	3
10	50	*acc*	Rank	2	2	4	**1**
1	10	*acc*	Wins	1	1	0	3
1	10	*acc*	Losses	1	1	3	0
1	10	*acc*	Diff	0	0	-3	3
1	10	*acc*	Rank	2	2	4	**1**
20	10	*acc*	Wins	1	1	0	3
20	10	*acc*	Losses	1	1	3	0
20	10	*acc*	Diff	0	0	-3	3
20	10	*acc*	Rank	2	2	4	**1**
all	all	*acc*	Wins	5	5	0	15
all	all	*acc*	Losses	5	5	15	0
all	all	*acc*	Diff	0	0	-15	15
all	all	*acc*	Rank	2	2	4	**1**
10	10	*stab*	Wins	1	1	0	3
10	10	*stab*	Losses	1	1	3	0
10	10	*stab*	Diff	0	0	-3	3
10	10	*stab*	Rank	2	2	4	**1**
10	25	*stab*	Wins	1	1	0	3

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n_t	τ_t	PM	Results	knowledge sharing strategy			
				ra-g	ra-t	ri-g	ri-t
10	25	*stab*	Losses	1	1	3	0
10	25	*stab*	Diff	0	0	-3	3
10	25	*stab*	Rank	2	2	4	**1**
10	50	*stab*	Wins	1	1	0	3
10	50	*stab*	Losses	1	1	3	0
10	50	*stab*	Diff	0	0	-3	3
10	50	*stab*	Rank	2	2	4	**1**
1	10	*stab*	Wins	1	1	0	3
1	10	*stab*	Losses	1	1	3	0
1	10	*stab*	Diff	0	0	-3	3
1	10	*stab*	Rank	2	2	4	**1**
20	10	*stab*	Wins	1	1	0	3
20	10	*stab*	Losses	1	1	3	0
20	10	*stab*	Diff	0	0	-3	3
20	10	*stab*	Rank	2	2	4	**1**
all	all	*stab*	Wins	5	5	0	15
all	all	*stab*	Losses	5	5	15	0
all	all	*stab*	Diff	0	0	-15	15
all	all	*stab*	Rank	2	2	4	**1**
10	10	NS	Wins	1	2	0	1
10	10	NS	Losses	0	0	3	1
10	10	NS	Diff	1	2	-3	0
10	10	NS	Rank	2	**1**	4	3
10	25	NS	Wins	1	2	0	1
10	25	NS	Losses	0	0	3	1
10	25	NS	Diff	1	2	-3	0
10	25	NS	Rank	2	**1**	4	3
10	50	NS	Wins	1	2	0	1
10	50	NS	Losses	0	0	3	1
10	50	NS	Diff	1	2	-3	0
10	50	NS	Rank	2	**1**	4	3
1	10	NS	Wins	1	1	0	1
1	10	NS	Losses	0	0	3	0
1	10	NS	Diff	1	1	-3	1
1	10	NS	Rank	**1**	**1**	4	**1**
20	10	NS	Wins	1	1	0	1
20	10	NS	Losses	0	0	3	0
20	10	NS	Diff	1	1	-3	1
20	10	NS	Rank	**1**	**1**	4	**1**
all	all	NS	Wins	5	8	0	5
all	all	NS	Losses	0	0	15	3
all	all	NS	Diff	5	8	-15	2
all	all	NS	Rank	2	**1**	4	3
10	10	all	Wins	3	4	0	7
10	10	all	Losses	2	2	9	1

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$\mathbf{n_t}$	τ_t	PM	Results	knowledge sharing strategy			
				ra-g	ra-t	ri-g	ri-t
10	10	all	Diff	1	2	-9	6
10	10	all	Rank	3	2	4	**1**
10	25	all	Wins	3	4	0	7
10	25	all	Losses	2	2	9	1
10	25	all	Diff	1	2	-9	6
10	25	all	Rank	3	2	4	**1**
10	50	all	Wins	3	4	0	7
10	50	all	Losses	2	2	9	1
10	50	all	Diff	1	2	-9	6
10	50	all	Rank	3	2	4	**1**
1	10	all	Wins	3	3	0	7
1	10	all	Losses	2	2	9	0
1	10	all	Diff	1	1	-9	7
1	10	all	Rank	2	2	4	**1**
20	10	all	Wins	3	3	0	7
20	10	all	Losses	2	2	9	0
20	10	all	Diff	1	1	-9	7
20	10	all	Rank	2	2	4	**1**
all	all	all	Wins	15	18	0	35
all	all	all	Losses	10	10	45	3
all	all	all	Diff	5	8	-45	32
all	all	all	Rank	3	2	4	**1**

Another DMOOP that *ra-t* struggled with, is FDA3. The wins and losses obtained by the various knowledge sharing strategies for FDA3 are presented in Table 10.29. The worst rank was obtained by *ra-g* and *ra-t* for *acc*. For *stab*, *ra-t* was awarded the second lowest rank. With regards to NS, both *ra-g* and *ra-t* performed the best. However, the overall wins and losses for FDA3 measured over all performance measures and all n_t-τ_t combinations, lead to *ra-t* obtaining the best rank for FDA3.

Table 10.29: Wins and Losses of FDA3 for various knowledge sharing strategies

$\mathbf{n_t}$	τ_t	PM	Results	knowledge sharing strategies			
				ra-g	ra-t	ri-g	ri-t
10	10	*acc*	Wins	0	1	0	2
10	10	*acc*	Losses	2	1	0	0
10	10	*acc*	Diff	-2	0	0	2
10	10	*acc*	Rank	4	2	2	**1**
10	25	*acc*	Wins	0	0	0	2
10	25	*acc*	Losses	1	1	0	0
10	25	*acc*	Diff	-1	-1	0	2
10	25	*acc*	Rank	3	3	2	**1**

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n_t	τ_t	PM	Results	knowledge sharing strategy			
				ra-g	ra-t	ri-g	ri-t
10	50	*acc*	Wins	1	0	2	2
10	50	*acc*	Losses	2	3	0	0
10	50	*acc*	Diff	-1	-3	2	2
10	50	*acc*	Rank	3	4	**1**	**1**
1	10	*acc*	Wins	1	1	0	1
1	10	*acc*	Losses	0	0	3	0
1	10	*acc*	Diff	1	1	-3	1
1	10	*acc*	Rank	**1**	**1**	4	**1**
all	all	*acc*	Wins	2	2	2	7
all	all	*acc*	Losses	5	5	3	0
all	all	*acc*	Diff	-3	-3	-1	7
all	all	*acc*	Rank	3	3	2	**1**
10	10	*stab*	Wins	1	1	0	1
10	10	*stab*	Losses	0	0	3	0
10	10	*stab*	Diff	1	1	-3	1
10	10	*stab*	Rank	**1**	**1**	4	**1**
10	25	*stab*	Wins	1	1	0	1
10	25	*stab*	Losses	0	0	3	0
10	25	*stab*	Diff	1	1	-3	1
10	25	*stab*	Rank	**1**	**1**	4	**1**
10	50	*stab*	Wins	1	2	0	2
10	50	*stab*	Losses	2	0	3	0
10	50	*stab*	Diff	-1	2	-3	2
10	50	*stab*	Rank	3	**1**	4	**1**
20	10	*stab*	Wins	1	1	0	1
20	10	*stab*	Losses	0	0	3	0
20	10	*stab*	Diff	1	1	-3	1
20	10	*stab*	Rank	**1**	**1**	4	**1**
all	all	*stab*	Wins	4	5	0	5
all	all	*stab*	Losses	2	0	12	0
all	all	*stab*	Diff	2	5	-12	5
all	all	*stab*	Rank	3	**1**	4	**1**
10	10	NS	Wins	0	0	3	0
10	10	NS	Losses	1	1	0	1
10	10	NS	Diff	-1	-1	3	-1
10	10	NS	Rank	2	2	**1**	2
10	25	NS	Wins	0	0	3	0
10	25	NS	Losses	1	1	0	1
10	25	NS	Diff	-1	-1	3	-1
10	25	NS	Rank	2	2	**1**	2
10	50	NS	Wins	0	0	3	0
10	50	NS	Losses	1	1	0	1
10	50	NS	Diff	-1	-1	3	-1
10	50	NS	Rank	2	2	**1**	2
1	10	NS	Wins	0	0	3	0

Continued on next page

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**knowledge sharing strategy**			
				ra-g	**ra-t**	**ri-g**	**ri-t**
1	10	NS	Losses	1	1	0	1
1	10	NS	Diff	-1	-1	3	-1
1	10	NS	Rank	2	2	**1**	2
20	10	NS	Wins	0	0	3	0
20	10	NS	Losses	1	1	0	1
20	10	NS	Diff	-1	-1	3	-1
20	10	NS	Rank	2	2	**1**	2
all	all	NS	Wins	10	10	0	0
all	all	NS	Losses	0	0	10	10
all	all	NS	Diff	10	10	-10	-10
all	all	NS	Rank	**1**	**1**	3	3
10	10	all	Wins	3	4	0	3
10	10	all	Losses	2	1	5	2
10	10	all	Diff	1	3	-5	1
10	10	all	Rank	2	**1**	4	2
10	25	all	Wins	3	3	0	3
10	25	all	Losses	1	1	5	2
10	25	all	Diff	2	2	-5	1
10	25	all	Rank	**1**	**1**	4	3
10	50	all	Wins	4	4	2	4
10	50	all	Losses	4	3	5	2
10	50	all	Diff	0	1	-3	2
10	50	all	Rank	3	2	4	**1**
1	10	all	Wins	3	3	0	1
1	10	all	Losses	0	0	5	2
1	10	all	Diff	3	3	-5	-1
1	10	all	Rank	**1**	**1**	4	3
20	10	all	Wins	3	3	0	1
20	10	all	Losses	0	0	5	2
20	10	all	Diff	3	3	-5	-1
20	10	all	Rank	**1**	**1**	4	3
all	all	all	Wins	16	17	2	12
all	all	all	Losses	7	5	25	10
all	all	all	Diff	9	12	-23	2
all	all	all	Rank	2	**1**	4	3

Table 10.30 presents the wins and losses for HE6. It is interesting to note that for HE6 there was no statistical significant difference in the performance of the various knowledge sharing approaches for *acc* and *stab* for most n_t-τ_t combinations. Furthermore, there was no statistical significant difference for NS for most of the n_t-τ_t combinations. A similar trend was observed for HE7.

Table 10.30: Wins and Losses of HE6 for various knowledge sharing strategies

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**knowledge sharing strategies**			
				ra-g	**ra-t**	**ri-g**	**ri-t**
1	10	NS	Wins	1	0	0	0
1	10	NS	Losses	0	0	0	1
1	10	NS	Diff	1	0	0	-1
1	10	NS	Rank	**1**	2	2	4
all	all	NS	Wins	0	0	0	1
all	all	NS	Losses	1	0	0	0
all	all	NS	Diff	-1	0	0	1
all	all	NS	Rank	4	2	2	**1**
1	10	all	Wins	0	0	0	1
1	10	all	Losses	1	0	0	0
1	10	all	Diff	-1	0	0	1
1	10	all	Rank	4	2	2	**1**
all	all	all	Wins	0	0	0	1
all	all	all	Losses	1	0	0	0
all	all	all	Diff	-1	0	0	1
all	all	all	Rank	4	2	2	**1**

The next section discusses results obtained by various responses to changes in the environment.

10.2.3 Responses to Change

This section investigates the influence of various responses to environmental changes on the performance of DVEPSO. When a change occurrs, both the particles in the sub-swarms and the archive that stores the non-dominated solutions have to respond to the change in an appropriate manner.

Change Response Strategies applied to Particles

This section discusses results obtained by various responses to changes in the environment applied to particles of the sub-swarms. The results are discussed with regards to each performance measure and each n_t-τ_t combination. Results obtained for DMOOPs of Type I, II and III are also presented. Furthermore, the overall performance of each response is discussed and general observations are highlighted. The wins and losses of the various approaches to respond to environment changes are presented in Tables 10.31 to 10.47. In Tables 10.31 to 10.47, ri and re indicate re-initialisation or re-evaluation of particles respectively, c and a indicate whether the response is applied to only the sub-swarm(s) whose objective function changed or all sub-swarms, and 10, 20 and 30 indicate the percentage of particles that is re-initialised.

Results with regards to Performance Measures

This section discusses the results obtained by the responses applied to the particles for the various performance measures. The wins and losses with regards to the performance measures over all n_t-τ_t combinations are presented in Table 10.31.

Table 10.31: Overall wins and losses for various performance measures obtained by various change response strategies applied to the particles

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Particle response stategies**									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
all	all	*acc*	Wins	163	103	115	112	112	112	43	69	42	71
all	all	*acc*	Losses	24	88	71	73	78	74	139	134	150	111
all	all	*acc*	Diff	139	15	44	39	34	38	-96	-65	-108	-40
all	all	*acc*	Rank	**1**	6	2	3	5	4	9	8	10	7
all	all	*stab*	Wins	113	63	39	56	30	53	29	22	35	33
all	all	*stab*	Losses	6	58	51	24	61	35	46	75	53	64
all	all	*stab*	Diff	107	5	-12	32	-31	18	-17	-53	-18	-31
all	all	*stab*	Rank	**1**	4	5	2	8	3	6	10	7	8
all	all	NS	Wins	108	99	90	89	109	68	70	96	85	110
all	all	NS	Losses	68	90	105	98	50	104	128	83	109	89
all	all	NS	Diff	40	9	-15	-9	59	-36	-58	13	-24	21
all	all	NS	Rank	2	5	7	6	**1**	9	10	4	8	3

The following are observed:

- The best performance for *acc* was obtained by *ri-c*-30 and the worst performance by *reu-c*.
- For *stab*, *ri-c*-30 again obtained the best rank and *re-a* obtained the worst rank.
- Measured against NS, *ri-c*-10 performed the best and *re-c* performed the worst.
- Both *re-c* and *reu-c* were awarded more losses than wins for all performance measures. Three other approaches, *ri-c*-20, *re-a* and *reu-a*, obtained more losses than wins for two of the three performance measures.

Results with regards to Various Frequencies and Severities of Change

The results obtained by the responses applied to the particles for the various environment types are discussed in this section. Table 10.32 presents the wins and losses for the various

n_t-τ_t combinations.

Table 10.32: Overall wins and losses for various frequencies and severities of change obtained by various change response strategies applied to the particles

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Particle response stategies**									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
10	10	all	Wins	92	70	47	61	57	49	25	48	41	51
10	10	all	Losses	26	55	59	40	39	62	80	60	71	49
10	10	all	Diff	66	15	-12	21	18	-13	-55	-12	-30	2
10	10	all	Rank	**1**	4	6	2	3	8	10	6	9	5
10	25	all	Wins	72	32	35	43	46	34	22	25	21	23
10	25	all	Losses	5	40	39	18	26	33	53	49	51	39
10	25	all	Diff	67	-8	-4	25	20	1	-31	-24	-30	-16
10	25	all	Rank	**1**	6	5	2	3	4	10	8	9	7
10	50	all	Wins	59	32	47	34	31	30	21	23	21	27
10	50	all	Losses	12	21	21	41	23	32	49	38	44	44
10	50	all	Diff	47	11	26	-7	8	-2	-28	-15	-23	-17
10	50	all	Rank	**1**	3	2	6	4	5	10	7	9	8
1	10	all	Wins	95	73	67	56	84	71	47	58	44	63
1	10	all	Losses	28	80	57	67	51	53	70	85	86	81
1	10	all	Diff	67	-7	10	-11	33	18	-23	-27	-42	-18
1	10	all	Rank	**1**	5	4	6	2	3	8	9	10	7
20	10	all	Wins	66	58	48	63	33	49	27	33	35	50
20	10	all	Losses	27	40	51	29	50	33	61	60	60	51
20	10	all	Diff	39	18	-3	34	-17	16	-34	-27	-25	-1
20	10	all	Rank	**1**	3	6	2	7	4	10	9	8	5

With regards to the various environment types, the following observations are made:

- For all n_t-τ_t combinations*ri-c*-30 performed the best.
- *re-c* performed the worst in all environments, except severely changing environments ($n_t = 1$ and $\tau_t = 10$).
- For severely changing environments, *reu-c* obtained the worst rank.
- Three approaches, *re-c*, *re-a* and *reu-c*, performed poorly, obtaining more losses than wins for all n_t-τ_t combinations.

Results for Various Dynamic Multi-objective Optimisation Problem Types

This section discusses the results obtained by the various response approaches for solving DMOOPs of Type I, II or III respectively.

Type I DMOOPs

This section discusses the results obtained for Type I DMOOPs. The wins and losses obtained by the various change response approaches for the performance measures over all n_t-τ_t combinations are presented in Table 10.33.

Table 10.33: Overall wins and losses for various performance measures obtained by various change response strategies applied to the particles solving Type I DMOOPs

$\mathbf{n_t}$	τ_t	PM	Results	Particle response stategies									
				ri-c-30	ri-a-30	ri-c-20	ri-a-20	ri-c-10	ri-a-10	re-c	re-a	reu-c	reu-a
all	all	*acc*	Wins	10	13	10	14	9	10	3	4	0	1
all	all	*acc*	Losses	0	0	4	0	6	0	19	15	17	13
all	all	*acc*	Diff	10	13	6	14	3	10	-16	-11	-17	-12
all	all	*acc*	Rank	3	2	5	**1**	6	3	9	7	10	8
all	all	*stab*	Wins	13	13	11	12	11	11	0	0	0	0
all	all	*stab*	Losses	0	0	1	0	0	0	18	18	21	13
all	all	*stab*	Diff	13	13	10	12	11	11	-18	-18	-21	-13
all	all	*stab*	Rank	**1**	**1**	6	3	4	4	8	8	10	7
all	all	NS	Wins	1	0	3	0	8	1	24	15	21	16
all	all	NS	Losses	16	19	10	17	7	14	0	6	0	0
all	all	NS	Diff	-15	-19	-7	-17	1	-13	24	9	21	16
all	all	NS	Rank	8	10	6	9	5	7	**1**	4	2	3

The following observations are made:

- The best performing approach for *acc* was *ri-a*-20 and *reu-c* performed the worst.
- The best performance for *stab* was obtained by both *ri-c*-30 and *ri-a*-30. The worst performing approach was *reu-c*.
- With regards to NS, *re-c* obtained the best rank and *ri-a*-30 the worst rank.
- Four approaches, *re-c*, *re-a*, *reu-c* and *reu-a*, obtained more losses than wins for two performance measures. All of these four approaches re-evaluate the particles after a change in the environment occured. Therefore, the re-evaluation approaches performed poorly. On the other hand, all re-initialisation approaches performed well.

Table 10.34 presents the wins and losses for the n_t-τ_t combinations.

With regards to the various environment types, the following are observed:

- The best performance for $n_t = 10$ and $\tau_t = 10$, and $n_t = 10$ and $\tau_t = 50$ was obtained by *ri-c*-10. The worst performance for $n_t = 10$ and $\tau_t = 10$ was achieved by *reu-a*.
- Three approaches, *ri-c*-10, *re-c* and *reu-a*, performed the best for $n_t = 10$ and $\tau_t = 50$. The worst rank was obtained by *ri-c*-30 and *ri-a*-20.

- For $n_t = 10$ and $\tau_t = 25$, and $n_t = 1$ and $\tau_t = 10$, the best performing approach was *ri-c*-30, with *rea-a* and *reau-a* performing the worst respectively.
- In gradually changing environments, there was almost no difference in the performance of the various response approaches. The best performing approach was *ri-a*-20 and the worst performing approaches were *ri-c*-10 and *ri-c*-20.
- More losses than wins were awarded to *re-a* and *reu-c* for three of the five n_t-τ_t combinations. Therefore, similar to the wins and losses with regards to the performance measures, the re-evaluation approaches performed poorly for most of the n_t-τ_t combinations.

Table 10.34: Overall wins and losses for various frequencies and severities of change obtained by various change response strategies applied to the particles solving Type I DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Particle response stategies**									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
10	10	all	Wins	13	16	13	15	13	13	13	10	11	4
10	10	all	Losses	7	7	9	7	3	7	24	19	20	18
10	10	all	Diff	6	9	4	8	10	6	-11	-9	-9	-14
10	10	all	Rank	4	2	6	3	**1**	4	9	7	7	10
10	25	all	Wins	4	3	2	3	2	2	1	0	0	0
10	25	all	Losses	0	0	0	0	0	0	6	7	3	1
10	25	all	Diff	4	3	2	3	2	2	-5	-7	-3	-1
10	25	all	Rank	**1**	2	4	2	4	4	9	10	8	7
10	50	all	Wins	0	1	3	0	5	0	5	0	3	5
10	50	all	Losses	5	5	0	5	0	3	0	4	0	0
10	50	all	Diff	-5	-4	3	-5	5	-3	5	-4	3	5
10	50	all	Rank	9	7	4	9	**1**	6	**1**	7	4	**1**
1	10	all	Wins	7	6	6	6	8	7	8	9	7	8
1	10	all	Losses	4	7	5	5	9	4	7	9	15	7
1	10	all	Diff	3	-1	1	1	-1	3	1	0	-8	1
1	10	all	Rank	**1**	8	3	3	8	**1**	3	7	10	3
20	10	all	Wins	0	0	0	2	0	0	0	0	0	0
20	10	all	Losses	0	0	1	0	1	0	0	0	0	0
20	10	all	Diff	0	0	-1	2	-1	0	0	0	0	0
20	10	all	Rank	2	2	9	**1**	9	2	2	2	2	2

The wins and losses of the response approaches measured over all Type I DMOOPs, all performance measures and all n_t-τ_t combinations are presented in Table 10.35.

Table 10.35: Overall wins and losses obtained by various change response strategies applied to the particles solving Type I DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Particle response stategies**									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
all	all	all	Wins	24	26	24	26	28	22	27	19	21	17
all	all	all	Losses	16	19	15	17	13	14	37	39	38	26
all	all	all	Diff	8	7	9	9	15	8	-10	-20	-17	-9
all	all	all	Rank	4	6	2	2	**1**	4	8	10	9	7

Measuring the approaches' performance over all performance measures and all n_t-τ_t combinations for Type I DMOOPs, the best rank was obtained by *ri-c*-10, with *rea-a* obtaining the worst rank. All re-evaluation approaches performed poorly and were out-performed by the re-initialisation approaches.

Type II DMOOPs

This section discusses the results for Type II DMOOPs that were obtained by the various response approaches applied to the particles. The wins and losses obtained by the various response approaches for the performance measures over all n_t-τ_t combinations are presented in Table 10.36.

Table 10.36: Overall wins and losses for various performance measures obtained by various change response strategies applied to the particles solving Type II DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Particle response stategies**									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
all	all	*acc*	Wins	108	76	60	79	51	73	19	25	7	37
all	all	*acc*	Losses	8	27	43	21	49	38	85	87	106	71
all	all	*acc*	Diff	100	49	17	58	2	35	-66	-62	-99	-34
all	all	*acc*	Rank	**1**	3	5	2	6	4	9	8	10	7
all	all	*stab*	Wins	94	49	18	41	14	36	25	9	25	19
all	all	*stab*	Losses	4	24	47	15	54	30	23	55	29	49
all	all	*stab*	Diff	90	25	-29	26	-40	6	2	-46	-4	-30
all	all	*stab*	Rank	**1**	3	7	2	9	4	5	10	6	8
all	all	*NS*	Wins	53	37	39	50	41	42	5	24	13	38
all	all	*NS*	Losses	18	34	33	13	17	21	70	40	65	31
all	all	*NS*	Diff	35	3	6	37	24	21	-65	-16	-52	7
all	all	*NS*	Rank	2	7	6	**1**	3	4	10	8	9	5

The following observations are made:

- The best performance for *acc* was obtained by *ri-c*-30 and the worst by *reu-c*. The re-evaluation approaches were awarded the four worst ranks.

- For *stab*, *ri-c*-30 performed the best and *re-a* obtained the worst performance.
- With regards to NS, *ri-a*-20 obtained the best performance and *re-c* the worst.
- More losses than wins were awarded to *re-a* for all performance measures.

Table 10.37 presents the wins and losses with regards to the various n_t-τ_t combinations. With regards to the various environment types, the following are observed:

- In all environments *ri-c*-30 performed the best.
- The worst performance for $n_t = 10$ and $\tau_t = 10$ was achieved by *reu-c*.
- For $n_t = 10$ and $\tau_t = 25$, and $n_t = 10$ and $\tau_t = 50$, *re-a* and *reu-c* performed the worst respectively.
- In a severely changing environment *reu-c* obtained the worst rank. However, for $n_t = 20$ and $\tau_t = 10$ causing a gradually changing environment *re-c* performed the worst.
- Three approaches obtained more losses than wins for all n_t-τ_t combinations, namely *re-c*, *re-a* and *reu-c*.

Table 10.37: Overall wins and losses for various frequencies and severities of change obtained by various change response strategies applied to the particles solving Type II DMOOPs

n_t	τ_t	PM	Results	Particle response stategies									
				ri-c-30	ri-a-30	ri-c-20	ri-a-20	ri-c-10	ri-a-10	re-c	re-a	reu-c	reu-a
10	10	all	Wins	60	44	28	39	23	32	10	11	8	33
10	10	all	Losses	10	21	28	13	28	32	43	44	47	22
10	10	all	Diff	50	23	0	26	-5	0	-33	-33	-39	11
10	10	all	Rank	**1**	3	5	2	7	5	8	8	10	4
10	25	all	Wins	51	27	13	31	16	23	14	12	15	16
10	25	all	Losses	1	25	32	6	25	19	29	33	26	22
10	25	all	Diff	50	2	-19	25	-9	4	-15	-21	-11	-6
10	25	all	Rank	**1**	4	9	2	6	3	8	10	7	5
10	50	all	Wins	44	16	15	18	15	23	7	7	3	11
10	50	all	Losses	4	6	14	8	10	11	25	26	36	19
10	50	all	Diff	40	10	1	10	5	12	-18	-19	-33	-8
10	50	all	Rank	**1**	3	6	3	5	2	8	9	10	7
1	10	all	Wins	52	44	35	42	34	40	14	22	9	22
1	10	all	Losses	7	23	27	17	31	19	43	41	53	53
1	10	all	Diff	45	21	8	25	3	21	-29	-19	-44	-31
1	10	all	Rank	**1**	3	5	2	6	3	8	7	10	9
20	10	all	Wins	48	31	26	40	18	33	4	6	10	12
20	10	all	Losses	8	10	22	5	26	8	38	38	38	35
20	10	all	Diff	40	21	4	35	-8	25	-34	-32	-28	-23
20	10	all	Rank	**1**	4	5	2	6	3	10	9	8	7

Table 10.38 presents the wins and losses of the response approaches' performance over all performance measures and all n_t-τ_t combinations solving Type II DMOOPs. From the data, the following are observed:

- The best overall rank was obtained by *ri*-*c*-30 and the worst by *reu*-*c*.
- All re-evaluation approaches performed badly, obtaining more losses than wins.
- An average or poor performance was obtained by all *ri*-*c* approaches, except *ri*-*c*-30 that obtained the best overall rank. In contrast, all *ri*-*a* approaches performed well.

Table 10.38: Overall wins and losses obtained by various change response strategies applied to the particles solving Type II DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Particle response stategies**									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
all	all	all	Wins	255	162	117	170	106	151	49	58	45	94
all	all	all	Losses	30	85	123	49	120	89	178	182	200	151
all	all	all	Diff	225	77	-6	121	-14	62	-129	-124	-155	-57
all	all	all	Rank	**1**	3	5	2	6	4	9	8	10	7

Type III DMOOPs

This section discusses the results that were obtained by the various response approaches applied to the particles for Type III DMOOPs. The wins and losses obtained by the various response approaches for the performance measures measured over all n_t-τ_t combinations are presented in Table 10.39.

Table 10.39: Overall wins and losses for various performance measures obtained by various change response strategies applied to the particles solving Type II DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Particle response stategies**									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
all	all	*acc*	Wins	45	14	45	19	52	29	21	40	35	33
all	all	*acc*	Losses	16	61	24	52	23	36	35	32	27	27
all	all	*acc*	Diff	29	-47	21	-33	29	-7	-14	8	8	6
all	all	*acc*	Rank	**1**	10	3	9	**1**	7	8	4	4	6
all	all	*stab*	Wins	6	1	10	3	5	6	4	13	10	14
all	all	*stab*	Losses	2	34	3	9	7	5	5	2	3	2
all	all	*stab*	Diff	4	-33	7	-6	-2	1	-1	11	7	12
all	all	*stab*	Rank	5	10	3	9	8	6	7	2	3	**1**
all	all	*NS*	Wins	54	62	48	39	60	25	41	57	51	56
all	all	*NS*	Losses	34	37	62	68	26	69	58	37	44	58
all	all	*NS*	Diff	20	25	-14	-29	34	-44	-17	20	7	-2
all	all	*NS*	Rank	3	2	7	9	**1**	10	8	3	5	6

The following are observed with regards to the various performance measures:

- The best performance for *acc* were obtained by *ri-c*-30 and *ri-c*-10. The worst rank was achieved by *ri-a*-30. With the exception of *re-c*, all re-evaluation approaches performed well.
- For *stab* the best performing approach was *reu-a* and *ri-c*-10 performed the worst. All re-evaluation approaches obtained the top ranks, except *re-c* which performed poorly.
- With regards to NS, the best performance was obtained by *ri-c*-10 and the worst by *ri-a*-10.

Table 10.40 presents the wins and losses for the n_t-τ_t combinations.

Table 10.40: Overall wins and losses for various frequencies and severities of change obtained by various change response strategies applied to the particles solving Type III DMOOPs

n_t	τ_t	PM	Results	Particle response stategies									
				ri-c-30	ri-a-30	ri-c-20	ri-a-20	ri-c-10	ri-a-10	re-c	re-a	reu-c	reu-a
10	10	all	Wins	24	16	11	14	26	9	8	32	27	18
10	10	all	Losses	13	31	25	24	9	27	25	4	12	15
10	10	all	Diff	11	-15	-14	-10	17	-18	-17	28	15	3
10	10	all	Rank	4	8	7	6	2	10	9	**1**	3	5
10	25	all	Wins	17	2	20	9	28	9	7	13	6	7
10	25	all	Losses	4	15	7	12	1	14	18	9	22	16
10	25	all	Diff	13	-13	13	-3	27	-5	-11	4	-16	-9
10	25	all	Rank	2	9	2	5	**1**	6	8	4	10	7
10	50	all	Wins	15	15	29	16	11	7	9	16	15	11
10	50	all	Losses	3	10	7	28	13	18	24	8	8	25
10	50	all	Diff	12	5	22	-12	-2	-11	-15	8	7	-14
10	50	all	Rank	2	5	**1**	8	6	7	10	3	4	9
1	10	all	Wins	36	23	26	8	42	24	25	27	28	33
1	10	all	Losses	17	50	25	45	11	30	20	35	18	21
1	10	all	Diff	19	-27	1	-37	31	-6	5	-8	10	12
1	10	all	Rank	2	9	6	10	**1**	7	5	8	4	3
20	10	all	Wins	13	21	17	14	10	11	17	22	20	34
20	10	all	Losses	15	26	25	20	22	21	11	15	14	10
20	10	all	Diff	-2	-5	-8	-6	-12	-10	6	7	6	24
20	10	all	Rank	5	6	8	7	10	9	3	2	3	**1**

With regards to the different environments, the following are observed:

- For $n_t = 10$ and $\tau_t = 10$, *re-a* performed the best and *ri-a*-10 the worst.
- For both $n_t = 10$ and $\tau_t = 25$, and $n_t = 1$ and $\tau_t = 10$, *ri-c*-10 obtained the best

rank, with *reu-c* and *ri-a*-20 performing the worst respectively.

- In slowly changing environments ($\tau_t = 50$), *ri-c*-20 performed the best, and *re-c* the worst.
- In gradually changing environments ($n_t = 20$), *reu-a* obtained the best rank and *ri-c*-20 the worst rank.

The wins and losses for Type III benchmark functions are presented in Table 10.41. For Type III DMOOPs the best performance measured over all performance measures and n_t-τ_t combinations was obtained by *ri-c*-10. The worst overal rank was obtained by *ri-a*-20.

Table 10.41: Overall wins and losses obtained by various change response strategies applied to the particles solving Type III DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Particle response stategies**									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
all	all	all	Wins	105	77	103	61	117	60	66	110	96	103
all	all	all	Losses	52	132	89	129	56	110	98	71	74	87
all	all	all	Diff	53	-55	14	-68	61	-50	-32	39	22	16
all	all	all	Rank	2	9	6	10	**1**	8	7	3	4	5

Overall Performance

This section discusses the overall performance of the responses applied to the particles. The wins and losses over all DMOOPs, n_t-τ_t combinations and performance measures are presented in Table 10.47.

Table 10.42: Overall wins and losses obtained by various change response strategies applied to the particles

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Particle response stategies**									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
all	all	all	Wins	384	265	244	257	251	233	142	187	162	214
all	all	all	Losses	98	236	227	195	189	213	313	292	312	264
all	all	all	Diff	286	29	17	62	62	20	-171	-105	-150	-50
all	all	all	Rank	**1**	4	6	2	2	5	10	8	9	7

The following observations are made:

- The approach that obtained the best overall rank was *ri-c*-30, completely outperforming the other approaches with 286 more wins than losses. The approach that ranked second obtained 62 more wins than losses.

- With the exception of *ri-c*-10, all re-initialisation approaches performed well. On the other hand, all re-evaluation approaches performed poorly.

The POF^*s found by *ri-c*-30 for $n_t = 10$ and $\tau_t = 10$ are illustrated in Figures 10.7 to 10.9. The same trend as the POF^*s found by *cl* was observed.

General Observations

This section discusses general observations that were made with regards to the performance of the responses applied to the particles of DVEPSO's sub-swarms.

In general, the re-initialisation approaches outperformed the re-evaluation approaches. The re-evaluation approaches performed well for DIMP2, FDA2$_{Camara}$, FDA3, FDA1$_{Zhou}$, HE1 and HE2. However, for DIMP2 and the FDA DMOOPs there was no statistical significant difference in the performance measure values of the various approaches for most of the performance measures. The wins and losses for DIMP2 are presented in Table 10.43.

Table 10.43: Wins and Losses of DIMP2 for various change response strategies applied to the particles

$\mathbf{n_t}$	τ_t	PM	Results	Particle response stategies									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
10	10	*acc*	Wins	0	1	0	0	0	0	1	1	0	0
10	10	*acc*	Losses	0	0	3	0	0	0	0	0	0	0
10	10	*acc*	Diff	0	1	-3	0	0	0	1	1	0	0
10	10	*acc*	Rank	4	**1**	10	4	4	4	**1**	**1**	4	4
10	25	*acc*	Wins	0	0	0	0	0	0	1	0	0	0
10	25	*acc*	Losses	0	0	0	0	0	0	0	1	0	0
10	25	*acc*	Diff	0	0	0	0	0	0	1	-1	0	0
10	25	*acc*	Rank	2	2	2	2	2	2	**1**	10	2	2
10	50	*acc*	Wins	0	1	0	0	0	0	0	0	0	0
10	50	*acc*	Losses	0	0	0	0	0	0	0	1	0	0
10	50	*acc*	Diff	0	1	0	0	0	0	0	-1	0	0
10	50	*acc*	Rank	2	**1**	2	2	2	2	2	10	2	2
20	10	*acc*	Wins	0	0	0	2	0	0	0	0	0	0
20	10	*acc*	Losses	0	0	1	0	1	0	0	0	0	0
20	10	*acc*	Diff	0	0	-1	2	-1	0	0	0	0	0
20	10	*acc*	Rank	2	2	9	**1**	9	2	2	2	2	2
all	all	*acc*	Wins	0	2	0	2	0	0	2	1	0	0
all	all	*acc*	Losses	0	0	4	0	1	0	0	2	0	0
all	all	*acc*	Diff	0	2	-4	2	-1	0	2	-1	0	0

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n_t	τ_t	PM	Results	Particle response stategies									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
all	all	*acc*	Rank	4	**1**	10	**1**	8	4	**1**	8	4	4
10	10	*stab*	Wins	0	1	0	0	0	0	0	0	0	0
10	10	*stab*	Losses	0	0	1	0	0	0	0	0	0	0
10	10	*stab*	Diff	0	1	-1	0	0	0	0	0	0	0
10	10	*stab*	Rank	2	**1**	10	2	2	2	2	2	2	2
all	all	*stab*	Wins	0	1	0	0	0	0	0	0	0	0
all	all	*stab*	Losses	0	0	1	0	0	0	0	0	0	0
all	all	*stab*	Diff	0	1	-1	0	0	0	0	0	0	0
all	all	*stab*	Rank	2	**1**	10	2	2	2	2	2	2	2
10	10	all	Wins	0	2	0	0	0	0	1	1	0	0
10	10	all	Losses	0	0	4	0	0	0	0	0	0	0
10	10	all	Diff	0	2	-4	0	0	0	1	1	0	0
10	10	all	Rank	4	**1**	10	4	4	4	2	2	4	4
10	25	all	Wins	0	0	0	0	0	0	1	0	0	0
10	25	all	Losses	0	0	0	0	0	0	0	1	0	0
10	25	all	Diff	0	0	0	0	0	0	1	-1	0	0
10	25	all	Rank	2	2	2	2	2	2	**1**	10	2	2
10	50	all	Wins	0	1	0	0	0	0	0	0	0	0
10	50	all	Losses	0	0	0	0	0	0	0	1	0	0
10	50	all	Diff	0	1	0	0	0	0	0	-1	0	0
10	50	all	Rank	2	**1**	2	2	2	2	2	10	2	2
20	10	all	Wins	0	0	0	2	0	0	0	0	0	0
20	10	all	Losses	0	0	1	0	1	0	0	0	0	0
20	10	all	Diff	0	0	-1	2	-1	0	0	0	0	0
20	10	all	Rank	2	2	9	**1**	9	2	2	2	2	2
all	all	all	Wins	0	3	0	2	0	0	2	1	0	0
all	all	all	Losses	0	0	5	0	1	0	0	2	0	0
all	all	all	Diff	0	3	-5	2	-1	0	2	-1	0	0
all	all	all	Rank	4	**1**	10	2	8	4	2	8	4	4

For the discontinuous functions, HE1 and HE2, the re-evaluation approaches performed well. For HE1, all re-evaluation approaches performed well, except *re-c*. For HE2, all re-evaluation approaches performed well, except *re-a*. The best overall rank for HE2 was obtained by *reu-a*, and three of the re-evaluation approaches obtained a rank in the top six for HE2. Tables 10.44 and 10.45 present the wins and losses for HE1 and HE2 respectively.

Table 10.44: Wins and Losses of HE1 for various change response strategies applied to the particles

n_t	τ_t	PM	Results	Particle response stategies									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
10	10	*acc*	Wins	5	0	5	2	3	5	1	7	7	2
10	10	*acc*	Losses	3	9	1	5	4	2	7	0	0	6

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n_t	τ_t	PM	Results	Particle response stategies									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
10	10	*acc*	Diff	2	-9	4	-3	-1	3	-6	7	7	-4
10	10	*acc*	Rank	5	10	3	7	6	4	9	**1**	**1**	8
10	25	*acc*	Wins	1	1	3	1	8	6	1	3	0	1
10	25	*acc*	Losses	2	2	1	2	0	0	4	1	9	4
10	25	*acc*	Diff	-1	-1	2	-1	8	6	-3	2	-9	-3
10	25	*acc*	Rank	5	5	3	5	**1**	2	8	3	10	8
10	50	*acc*	Wins	5	1	8	0	1	1	2	1	1	1
10	50	*acc*	Losses	0	2	0	9	3	2	1	1	1	2
10	50	*acc*	Diff	5	-1	8	-9	-2	-1	1	0	0	-1
10	50	*acc*	Rank	2	6	**1**	10	9	6	3	4	4	6
1	10	*acc*	Wins	5	0	2	0	9	8	5	3	6	4
1	10	*acc*	Losses	3	8	7	8	0	1	2	6	2	5
1	10	*acc*	Diff	2	-8	-5	-8	9	7	3	-3	4	-1
1	10	*acc*	Rank	5	9	8	9	**1**	2	4	7	3	6
20	10	*acc*	Wins	2	0	7	1	2	4	2	7	2	7
20	10	*acc*	Losses	3	9	0	8	3	3	4	0	4	0
20	10	*acc*	Diff	-1	-9	7	-7	-1	1	-2	7	-2	7
20	10	*acc*	Rank	5	10	**1**	9	5	4	7	**1**	7	**1**
all	all	*acc*	Wins	18	2	25	4	23	24	11	21	16	15
all	all	*acc*	Losses	11	30	9	32	10	8	18	8	16	17
all	all	*acc*	Diff	7	-28	16	-28	13	16	-7	13	0	-2
all	all	*acc*	Rank	5	9	**1**	9	3	**1**	8	3	6	7
10	10	*stab*	Wins	1	1	1	1	1	1	0	1	1	1
10	10	*stab*	Losses	0	8	0	0	0	0	1	0	0	0
10	10	*stab*	Diff	1	-7	1	1	1	1	-1	1	1	1
10	10	*stab*	Rank	**1**	10	**1**	**1**	**1**	**1**	9	**1**	**1**	**1**
1	10	*stab*	Wins	2	0	2	1	1	2	2	7	2	7
1	10	*stab*	Losses	1	9	2	2	6	2	2	0	2	0
1	10	*stab*	Diff	1	-9	0	-1	-5	0	0	7	0	7
1	10	*stab*	Rank	3	10	4	8	9	4	4	**1**	4	**1**
all	all	*stab*	Wins	3	1	3	2	2	3	2	8	3	8
all	all	*stab*	Losses	1	17	2	2	6	2	3	0	2	0
all	all	*stab*	Diff	2	-16	1	0	-4	1	-1	8	1	8
all	all	*stab*	Rank	3	10	4	7	9	4	8	**1**	4	**1**
10	10	*NS*	Wins	6	9	0	3	5	0	1	7	8	3
10	10	*NS*	Losses	3	0	8	5	4	7	7	2	1	5
10	10	*NS*	Diff	3	9	-8	-2	1	-7	-6	5	7	-2
10	10	*NS*	Rank	4	**1**	10	6	5	9	8	3	2	6
10	25	*NS*	Wins	6	1	5	1	8	1	3	8	5	0
10	25	*NS*	Losses	2	5	3	6	0	6	5	0	2	9
10	25	*NS*	Diff	4	-4	2	-5	8	-5	-2	8	3	-9
10	25	*NS*	Rank	3	7	5	8	**1**	8	6	**1**	4	10
10	50	*NS*	Wins	2	2	2	0	1	2	0	2	2	0
10	50	*NS*	Losses	0	0	0	6	0	0	0	0	0	7
10	50	*NS*	Diff	2	2	2	-6	1	2	0	2	2	-7
10	50	*NS*	Rank	**1**	**1**	**1**	9	7	**1**	8	**1**	**1**	10
1	10	*NS*	Wins	7	8	2	0	8	5	6	4	0	3
1	10	*NS*	Losses	2	0	7	8	0	4	3	5	8	6

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$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Particle response stategies**									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
1	10	NS	Diff	5	8	-5	-8	8	1	3	-1	-8	-3
1	10	NS	Rank	3	**1**	8	9	**1**	5	4	6	9	7
20	10	NS	Wins	0	9	5	4	1	1	5	4	1	8
20	10	NS	Losses	9	0	2	2	6	6	2	4	6	1
20	10	NS	Diff	-9	9	3	2	-5	-5	3	0	-5	7
20	10	NS	Rank	10	**1**	3	5	7	7	3	6	7	2
all	all	NS	Wins	21	29	14	8	23	9	15	25	16	14
all	all	NS	Losses	16	5	20	27	10	23	17	11	17	28
all	all	NS	Diff	5	24	-6	-19	13	-14	-2	14	-1	-14
all	all	NS	Rank	4	**1**	7	10	3	8	6	2	5	8
10	10	all	Wins	12	10	6	6	9	6	2	15	16	6
10	10	all	Losses	6	17	9	10	8	9	15	2	1	11
10	10	all	Diff	6	-7	-3	-4	1	-3	-13	13	15	-5
10	10	all	Rank	3	9	5	7	4	5	10	2	**1**	8
10	25	all	Wins	7	2	8	2	16	7	4	11	5	1
10	25	all	Losses	4	7	4	8	0	6	9	1	11	13
10	25	all	Diff	3	-5	4	-6	16	1	-5	10	-6	-12
10	25	all	Rank	4	6	3	8	**1**	5	6	2	8	10
10	50	all	Wins	7	3	10	0	2	3	2	3	3	1
10	50	all	Losses	0	2	0	15	3	2	1	1	1	9
10	50	all	Diff	7	1	10	-15	-1	1	1	2	2	-8
10	50	all	Rank	2	5	**1**	10	8	5	5	3	3	9
1	10	all	Wins	14	8	6	1	18	15	13	14	8	14
1	10	all	Losses	6	17	16	18	6	7	7	11	12	11
1	10	all	Diff	8	-9	-10	-17	12	8	6	3	-4	3
1	10	all	Rank	2	8	9	10	**1**	2	4	5	7	5
20	10	all	Wins	2	9	12	5	3	5	7	11	3	15
20	10	all	Losses	12	9	2	10	9	9	6	4	10	1
20	10	all	Diff	-10	0	10	-5	-6	-4	1	7	-7	14
20	10	all	Rank	10	5	2	7	8	6	4	3	9	**1**
all	all	all	Wins	42	32	42	14	48	36	28	54	35	37
all	all	all	Losses	28	52	31	61	26	33	38	19	35	45
all	all	all	Diff	14	-20	11	-47	22	3	-10	35	0	-8
all	all	all	Rank	3	9	4	10	2	5	8	**1**	6	7

Table 10.45: Wins and Losses of HE2 for various change response strategies applied to the particles

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Particle response stategies**									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
10	10	acc	Wins	6	0	2	1	6	1	1	6	2	5
10	10	acc	Losses	0	9	3	6	0	4	4	0	4	0
10	10	acc	Diff	6	-9	-1	-5	6	-3	-3	6	-2	5
10	10	acc	Rank	**1**	10	5	9	**1**	7	7	**1**	6	4
10	25	acc	Wins	1	0	4	0	4	0	0	0	0	0
10	25	acc	Losses	0	0	0	0	0	3	2	2	2	0
10	25	acc	Diff	1	0	4	0	4	-3	-2	-2	-2	0

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n_t	τ_t	PM	Results	Particle response stategies									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
10	25	*acc*	Rank	3	4	**1**	4	**1**	10	7	7	7	4
10	50	*acc*	Wins	1	6	4	6	0	0	1	0	2	1
10	50	*acc*	Losses	2	0	0	0	7	3	3	4	0	2
10	50	*acc*	Diff	-1	6	4	6	-7	-3	-2	-4	2	-1
10	50	*acc*	Rank	5	**1**	3	**1**	10	8	7	9	4	5
1	10	*acc*	Wins	4	0	5	1	9	3	4	2	4	4
1	10	*acc*	Losses	2	9	1	8	0	6	1	7	1	1
1	10	*acc*	Diff	2	-9	4	-7	9	-3	3	-5	3	3
1	10	*acc*	Rank	6	10	2	9	**1**	7	3	8	3	3
20	10	*acc*	Wins	2	0	0	0	0	0	3	0	6	4
20	10	*acc*	Losses	0	2	4	1	4	1	0	3	0	0
20	10	*acc*	Diff	2	-2	-4	-1	-4	-1	3	-3	6	4
20	10	*acc*	Rank	4	7	9	5	9	5	3	8	**1**	2
all	all	*acc*	Wins	14	6	15	8	19	4	9	8	14	14
all	all	*acc*	Losses	4	20	8	15	11	17	10	16	7	3
all	all	*acc*	Diff	10	-14	7	-7	8	-13	-1	-8	7	11
all	all	*acc*	Rank	2	10	4	7	3	9	6	8	4	**1**
10	50	*stab*	Wins	1	0	0	0	0	0	0	0	0	0
10	50	*stab*	Losses	0	0	0	1	0	0	0	0	0	0
10	50	*stab*	Diff	1	0	0	-1	0	0	0	0	0	0
10	50	*stab*	Rank	**1**	2	2	10	2	2	2	2	2	2
1	10	*stab*	Wins	1	0	1	1	1	1	1	3	1	1
1	10	*stab*	Losses	0	9	1	0	1	0	0	0	0	0
1	10	*stab*	Diff	1	-9	0	1	0	1	1	3	1	1
1	10	*stab*	Rank	2	10	8	2	8	2	2	**1**	2	2
20	10	*stab*	Wins	1	0	2	0	2	2	1	2	1	2
20	10	*stab*	Losses	0	8	0	5	0	0	0	0	0	0
20	10	*stab*	Diff	1	-8	2	-5	2	2	1	2	1	2
20	10	*stab*	Rank	6	10	**1**	9	**1**	**1**	6	**1**	6	**1**
all	all	*stab*	Wins	3	0	3	1	3	3	2	5	2	3
all	all	*stab*	Losses	0	17	1	6	1	0	0	0	0	0
all	all	*stab*	Diff	3	-17	2	-5	2	3	2	5	2	3
all	all	*stab*	Rank	2	10	5	9	5	2	5	**1**	5	2
10	10	*NS*	Wins	4	2	2	0	9	0	2	8	6	5
10	10	*NS*	Losses	4	2	5	8	0	8	5	1	2	3
10	10	*NS*	Diff	0	0	-3	-8	9	-8	-3	7	4	2
10	10	*NS*	Rank	5	5	7	9	**1**	9	7	2	3	4
10	25	*NS*	Wins	9	0	7	5	8	2	3	2	0	6
10	25	*NS*	Losses	0	8	2	4	1	5	5	6	8	3
10	25	*NS*	Diff	9	-8	5	1	7	-3	-2	-4	-8	3
10	25	*NS*	Rank	**1**	9	3	5	2	7	6	8	9	4
10	50	*NS*	Wins	0	4	6	9	2	4	6	1	3	8
10	50	*NS*	Losses	9	4	2	0	7	4	2	8	6	1
10	50	*NS*	Diff	-9	0	4	9	-5	0	4	-7	-3	7
10	50	*NS*	Rank	10	5	3	**1**	8	5	3	9	7	2
1	10	*NS*	Wins	6	9	8	1	7	3	2	0	4	5
1	10	*NS*	Losses	3	0	1	8	2	6	7	9	5	4
1	10	*NS*	Diff	3	9	7	-7	5	-3	-5	-9	-1	1

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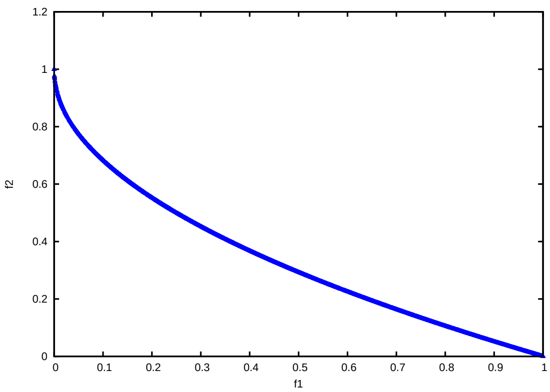
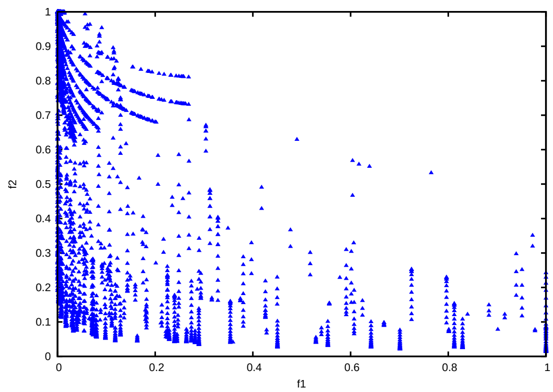
n_t	τ_t	PM	Results	Particle response stategies									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
1	10	NS	Rank	4	**1**	2	9	3	7	8	10	6	5
20	10	NS	Wins	6	9	0	6	1	4	4	2	8	3
20	10	NS	Losses	2	0	9	2	8	4	4	7	1	6
20	10	NS	Diff	4	9	-9	4	-7	0	0	-5	7	-3
20	10	NS	Rank	3	**1**	10	3	9	5	5	8	2	7
all	all	NS	Wins	25	24	23	21	27	13	17	13	21	27
all	all	NS	Losses	18	14	19	22	18	27	23	31	22	17
all	all	NS	Diff	7	10	4	-1	9	-14	-6	-18	-1	10
all	all	NS	Rank	4	**1**	5	6	3	9	8	10	6	**1**
10	10	all	Wins	10	2	4	1	15	1	3	14	8	10
10	10	all	Losses	4	11	8	14	0	12	9	1	6	3
10	10	all	Diff	6	-9	-4	-13	15	-11	-6	13	2	7
10	10	all	Rank	4	8	6	10	**1**	9	7	2	5	3
10	25	all	Wins	10	0	11	5	12	2	3	2	0	6
10	25	all	Losses	0	8	2	4	1	8	7	8	10	3
10	25	all	Diff	10	-8	9	1	11	-6	-4	-6	-10	3
10	25	all	Rank	2	9	3	5	**1**	7	6	7	10	4
10	50	all	Wins	2	10	10	15	2	4	7	1	5	9
10	50	all	Losses	11	4	2	1	14	7	5	12	6	3
10	50	all	Diff	-9	6	8	14	-12	-3	2	-11	-1	6
10	50	all	Rank	8	3	2	**1**	10	7	5	9	6	3
1	10	all	Wins	11	9	14	3	17	7	7	5	9	10
1	10	all	Losses	5	18	3	16	3	12	8	16	6	5
1	10	all	Diff	6	-9	11	-13	14	-5	-1	-11	3	5
1	10	all	Rank	3	8	2	10	**1**	7	6	9	5	4
20	10	all	Wins	9	9	2	6	3	6	8	4	15	9
20	10	all	Losses	2	10	13	8	12	5	4	10	1	6
20	10	all	Diff	7	-1	-11	-2	-9	1	4	-6	14	3
20	10	all	Rank	2	6	10	7	9	5	3	8	**1**	4
all	all	all	Wins	42	30	41	30	49	20	28	26	37	44
all	all	all	Losses	22	51	28	43	30	44	33	47	29	20
all	all	all	Diff	20	-21	13	-13	19	-24	-5	-21	8	24
all	all	all	Rank	2	8	4	7	3	10	6	8	5	**1**

Change Response Strategies applied to the Archive

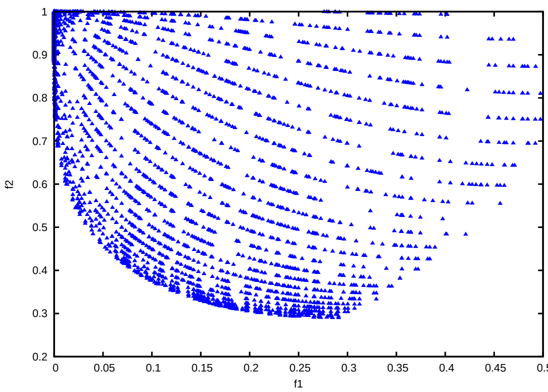
This section discusses results obtained by various responses to changes in the environment applied to the non-dominated solutions in the archive. The wins and losses of the various response approaches are presented in Tables 10.46 to 10.57, where the notation defined in Section 9.2 is used.



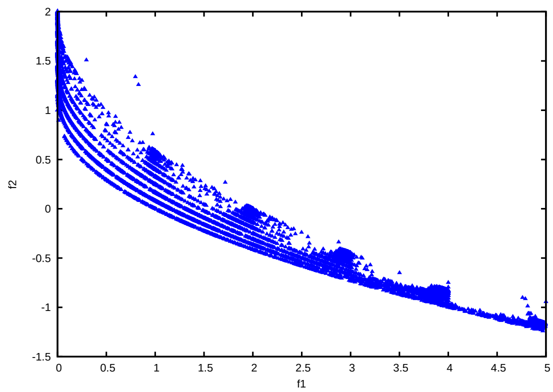
(a) POF^* for FDA1$_{Zhou}$



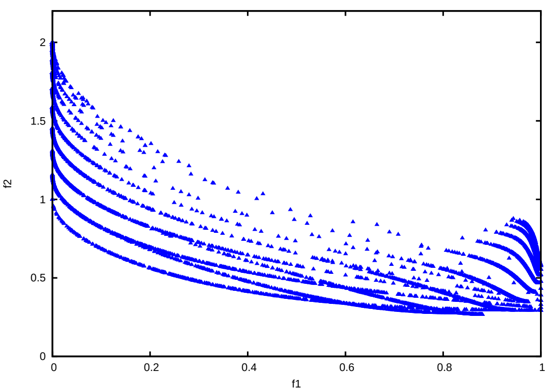
(b) POF^* for FDA2



(c) POF^* for FDA2$_{Camara}$

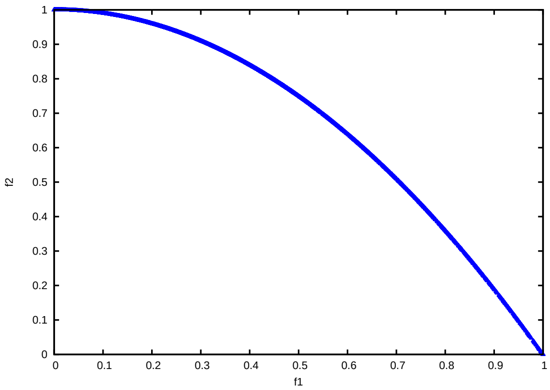


(d) POF^* for FDA3

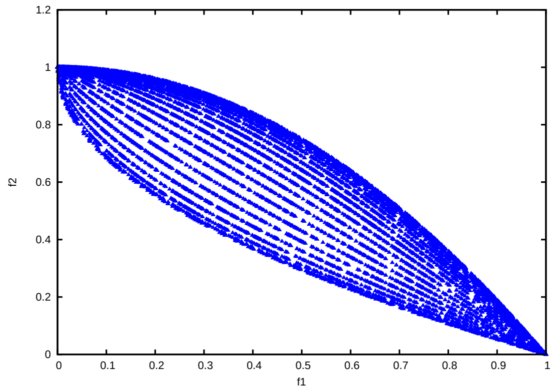


(e) POF^* for FDA3$_{Camara}$

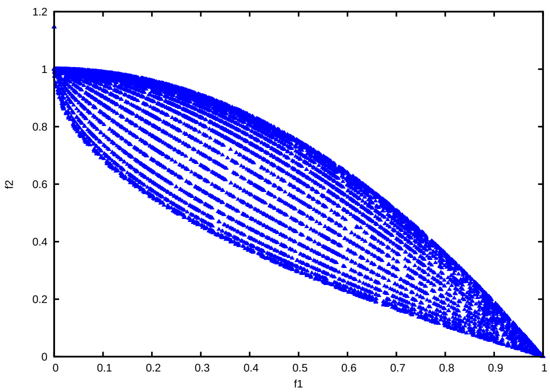
Figure 10.7: POF^* for FDA functions of DVEPSO using ri-c-30 for $n_t = 10$ and $\tau_t = 10$



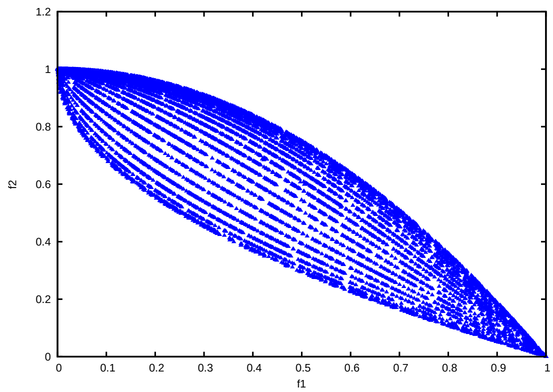
(a) POF^{*} for DIMP2



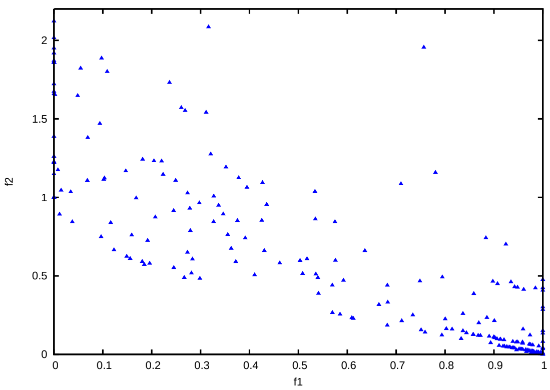
(b) POF^{*} for dMOP2



(c) POF^{*} for dMOP2$_{iso}$

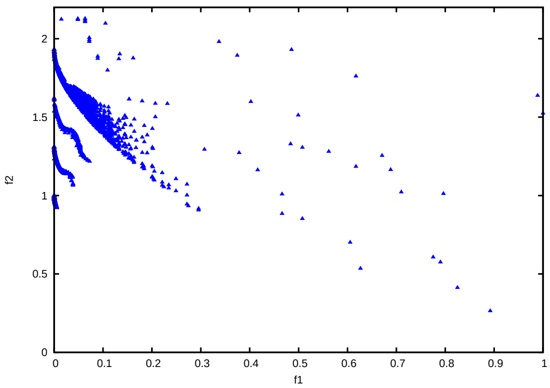


(d) POF^{*} for dMOP2$_{dec}$

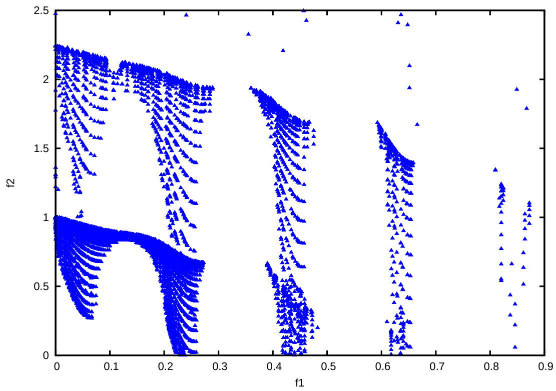


(e) POF^{*} for dMOP3

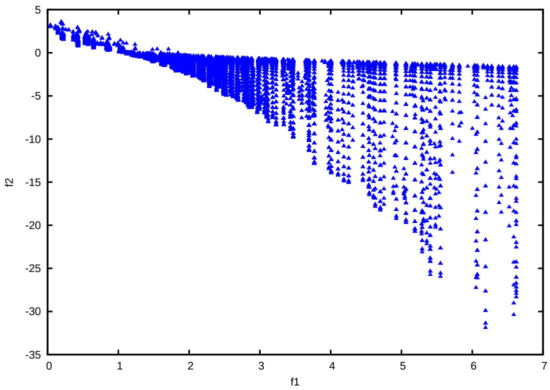
Figure 10.8: POF^{*} for DIMP2 and dMOP functions of DVEPSO using ri-c-30 for $n_t = 10$ and $\tau_t = 10$



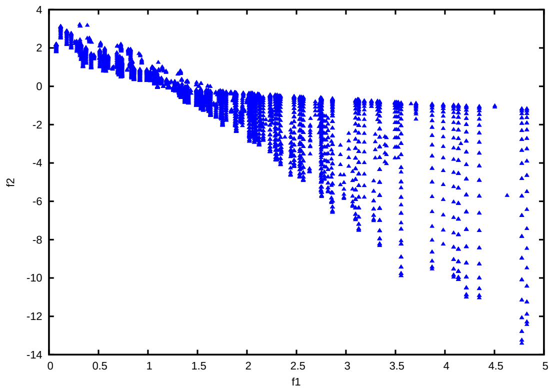
(a) POF^* for HE1



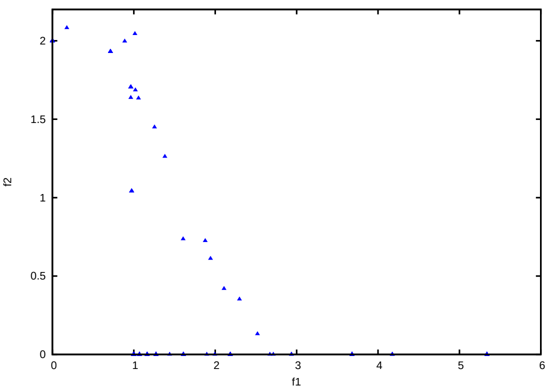
(b) POF^* for HE2



(c) POF^* for HE6



(d) POF^* for HE7



(e) POF^* for HE9

Figure 10.9: POF^* for HE functions of DVEPSO using ri-c-30 for $n_t = 10$ and $\tau_t = 10$

Results with regards to Performance Measures

This section discusses the results obtained for the performance measures by the responses applied to the archive. The wins and losses with regards to the various performance measures over all n_t-τ_t combinations are presented in Table 10.46.

The following are observed:

- The best performance for *acc* was obtained by *ac* and *ra*-0.5 performed the worst.
- For *stab*, *re* obtained the best performance and *ac* the worst.
- Measured against NS, the best rank was obtained by *re* and the worst rank by *ac*.

Table 10.46: Overall wins and losses for various performance measures obtained by various change response strategies applied to the archive

n_t	τ_t	PM	Results	Archive response stategies								
				re	**reh**	**ra-0.5**	**rah-0.5**	**ra-1**	**rah-1**	**ra-1.5**	**rah-1.5**	**ac**
all	all	*acc*	Wins	74	69	48	58	60	65	77	71	158
all	all	*acc*	Losses	88	76	91	73	85	63	75	58	71
all	all	*acc*	Diff	-14	-7	-43	-15	-25	2	2	13	87
all	all	*acc*	Rank	6	5	9	7	8	3	3	2	**1**
all	all	*stab*	Wins	58	48	34	25	35	31	32	28	66
all	all	*stab*	Losses	28	30	30	26	28	31	37	34	113
all	all	*stab*	Diff	30	18	4	-1	7	0	-5	-6	-47
all	all	*stab*	Rank	**1**	2	4	6	3	5	7	8	9
all	all	NS	Wins	175	158	133	120	95	113	74	86	3
all	all	NS	Losses	29	43	49	57	91	72	108	105	403
all	all	NS	Diff	146	115	84	63	4	41	-34	-19	-400
all	all	NS	Rank	**1**	2	3	4	6	5	8	7	9

Results with regards to Various Frequencies and Severities of Change

Table **??** presents the wins and losses for the n_t-τ_t combinations.

For the different environment types the following observations are made:

- The worst performance for all n_t-τ_t combinations was obtained by *ac*.
- For $n_t = 10$ and $\tau_t = 50$, $n_t = 1$ and $\tau_t = 10$, and $n_t = 20$ and $\tau_t = 10$, the best rank was obtained by *re*.
- For $n_t = 10$ and $\tau_t = 10$, *rah*-1 performed the best.
- For $n_t = 10$ and $\tau_t = 25$, the best rank was obtained by *reh*.
- *cl* performed poorly, obtaining more losses than wins for all n_t-τ_t combinations.

Table 10.47: Overall wins and losses obtained by various change response strategies applied to the particles

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Particle response stategies**									
				ri-c-30	**ri-a-30**	**ri-c-20**	**ri-a-20**	**ri-c-10**	**ri-a-10**	**re-c**	**re-a**	**reu-c**	**reu-a**
all	all	all	Wins	384	265	244	257	251	233	142	187	162	214
all	all	all	Losses	98	236	227	195	189	213	313	292	312	264
all	all	all	Diff	286	29	17	62	62	20	-171	-105	-150	-50
all	all	all	Rank	**1**	4	6	2	2	5	10	8	9	7

Results for Various Dynamic Multi-objective Optimisation Problem Types

This section discusses the results obtained by the various response approaches for DMOOPs of Type I, II and III respectively.

Type I DMOOPs

The wins and losses obtained for Type I DMOOPs are presented in Table 10.48.

Table 10.48: Overall wins and losses for various performance measures obtained by various change response strategies applied to the archive solving Type I DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Archive response stategies**								
				re	**reh**	**ra-0.5**	**rah-0.5**	**ra-1**	**rah-1**	**ra-1.5**	**rah-1.5**	**ac**
all	all	*acc*	Wins	2	2	3	3	6	1	1	1	1
all	all	*acc*	Losses	3	1	0	0	0	1	9	1	5
all	all	*acc*	Diff	-1	1	3	3	6	0	-8	0	-4
all	all	*acc*	Rank	7	4	2	2	**1**	5	9	5	8
all	all	*stab*	Wins	1	0	2	1	5	0	1	0	0
all	all	*stab*	Losses	1	1	0	0	0	1	1	1	5
all	all	*stab*	Diff	0	-1	2	1	5	-1	0	-1	-5
all	all	*stab*	Rank	4	6	2	3	**1**	6	4	6	9
all	all	NS	Wins	1	3	1	0	1	0	6	4	0
all	all	NS	Losses	2	0	2	1	2	4	1	2	2
all	all	NS	Diff	-1	3	-1	-1	-1	-4	5	2	-2
all	all	NS	Rank	4	2	4	4	4	9	**1**	3	8

With regards to the performance measures, the following are observed:

- The best performance for *acc* was obtained by *ra*-1 and the worst by *ra*-1.5.
- For *stab*, *ra*-1 also performed the best and *ac* performed the worst.
- Measured against NS, *ra*-1.5 obtained the best rank and *rah*-1 obtained the worst rank.

Table 10.49 presents the wins and losses with regards to the n_t-τ_t combinations.

Table 10.49: Overall wins and losses for various frequencies and severities of change obtained by various change response strategies applied to the archive solving Type I DMOOPs

n_t	τ_t	PM	Results	Archive response stategies								
				re	**reh**	**ra-0.5**	**rah-0.5**	**ra-1**	**rah-1**	**ra-1.5**	**rah-1.5**	**ac**
10	10	all	Wins	0	0	0	0	0	0	0	0	0
10	10	all	Losses	0	0	0	0	0	0	0	0	0
10	10	all	Diff	0	0	0	0	0	0	0	0	0
10	10	all	Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
10	25	all	Wins	0	0	2	1	0	0	1	0	0
10	25	all	Losses	4	0	0	0	0	0	0	0	0
10	25	all	Diff	-4	0	2	1	0	0	1	0	0
10	25	all	Rank	9	4	**1**	2	4	4	2	4	4
10	50	all	Wins	0	0	0	0	0	0	0	4	0
10	50	all	Losses	1	0	1	0	0	0	1	0	1
10	50	all	Diff	-1	0	-1	0	0	0	-1	4	-1
10	50	all	Rank	6	2	6	2	2	2	6	**1**	6
1	10	all	Wins	4	5	4	3	2	1	7	1	1
1	10	all	Losses	1	0	1	1	2	4	8	2	9
1	10	all	Diff	3	5	3	2	0	-3	-1	-1	-8
1	10	all	Rank	2	**1**	2	4	5	8	6	6	9
20	10	all	Wins	0	0	0	0	10	0	0	0	0
20	10	all	Losses	0	2	0	0	0	2	2	2	2
20	10	all	Diff	0	-2	0	0	10	-2	-2	-2	-2
20	10	all	Rank	2	5	2	2	**1**	5	5	5	5

The following observations are made:

- For $n_t = 10$ and $\tau_t = 10$, there was no statistical significant diffference between the performance measure values of the various response approaches.
- The best performance for $n_t = 10$ and $\tau_t = 25$ was obtained by *ra*-0.5, with *re* performing the worst.
- In a slow changing environment ($\tau_t = 50$), *rah*-1.5 performed the best.
- The best rank for a severely changing environment was obtained by *reh*, with *ac* obtaining the worst rank.
- In a gradually changing environment *ra*-1 performed the best. Five approaches performed equally poor obtaining more losses than wins, namely *reh*, *rah*-1, *ra*-1.5, *rah*-1.5 and *ac*.

The wins and losses for Type I DMOO benchmark functions are presented in Table 10.50. For Type I DMOOPs the best rank was obtained by *ra*-1, with *ac* obtaining the worst rank.

Table 10.50: Overall wins and losses obtained by various change response strategies applied to the archive solving Type I DMOOPs

n_t	τ_t	PM	Results	Archive response stategies								
				re	reh	ra-0.5	rah-0.5	ra-1	rah-1	ra-1.5	rah-1.5	ac
all	all	all	Wins	4	5	6	4	12	1	8	5	1
all	all	all	Losses	6	2	2	1	2	6	11	4	12
all	all	all	Diff	-2	3	4	3	10	-5	-3	1	-11
all	all	all	Rank	6	3	2	3	**1**	8	7	5	9

Type II DMOOPs

The wins and losses obtained by the various response approaches for Type II DMOOPs are presented in Table 10.51.

Table 10.51: Overall wins and losses for various performance measures obtained by various change response strategies applied to the archive solving Type II DMOOPs

n_t	τ_t	PM	Results	Archive response stategies								
				re	reh	ra-0.5	rah-0.5	ra-1	rah-1	ra-1.5	rah-1.5	ac
all	all	*acc*	Wins	34	20	19	15	18	22	39	27	89
all	all	*acc*	Losses	33	32	44	35	42	33	25	28	11
all	all	*acc*	Diff	1	-12	-25	-20	-24	-11	14	-1	78
all	all	*acc*	Rank	3	6	9	7	8	5	2	4	**1**
all	all	*stab*	Wins	31	28	16	15	16	23	17	19	54
all	all	*stab*	Losses	25	17	25	18	23	18	31	22	40
all	all	*stab*	Diff	6	11	-9	-3	-7	5	-14	-3	14
all	all	*stab*	Rank	3	2	8	5	7	4	9	5	**1**
all	all	*NS*	Wins	53	53	44	45	39	33	30	30	0
all	all	*NS*	Losses	1	1	9	7	13	17	24	23	232
all	all	*NS*	Diff	52	52	35	38	26	16	6	7	-232
all	all	*NS*	Rank	**1**	**1**	4	3	5	6	8	7	9

The following observations are made:

- The best performance for all performance measures was obtained by *ac*.
- The worst rank for *acc* and *stab* were obtained by *ra*-0.5 and *ra*-1.5 respectively.
- For *NS* the worst performing approach was *ac*.
- Four approaches obtained more losses than wins for two performance measures, namely *ra*-0.5, *rah*-0.5, *ra*-1 and *rah*-1.5.

Table 10.49 presents the wins and losses for the n_t-τ_t combinations. With regards to the various environment types, the following are observed:

- The worst performing approach for all n_t-τ_t combinations was *ac*.

- For $n_t = 10$ and $\tau_t = 10$, *rah*-0.5 performed the best.
- For $n_t = 10$ and $\tau_t = 25$, both *re* and *reh* obtained the best rank.
- In slow changing environments ($\tau_t = 50$), *ra*-1.5 was the best performing approach.
- For severely and gradually changing environments, *re* performed the best.
- More losses than wins were awarded to *ac* for all n_t-τ_t combinations.

Table 10.52: Overall wins and losses for various frequencies and severities of change obtained by various change response strategies applied to the archive solving Type II DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Archive response stategies**								
				re	**reh**	**ra-0.5**	**rah-0.5**	**ra-1**	**rah-1**	**ra-1.5**	**rah-1.5**	**ac**
10	10	all	Wins	28	26	19	26	26	22	17	17	49
10	10	all	Losses	32	13	24	9	22	13	33	18	66
10	10	all	Diff	-4	13	-5	17	4	9	-16	-1	-17
10	10	all	Rank	6	2	7	**1**	4	3	8	5	9
10	25	all	Wins	25	27	19	12	15	17	20	7	9
10	25	all	Losses	4	6	7	10	10	12	12	24	66
10	25	all	Diff	21	21	12	2	5	5	8	-17	-57
10	25	all	Rank	**1**	**1**	3	7	5	5	4	8	9
10	50	all	Wins	16	13	12	13	12	14	23	13	13
10	50	all	Losses	11	11	15	7	10	11	6	10	48
10	50	all	Diff	5	2	-3	6	2	3	17	3	-35
10	50	all	Rank	3	6	8	2	6	4	**1**	4	9
1	10	all	Wins	21	17	8	8	10	14	12	16	46
1	10	all	Losses	4	7	23	22	12	17	12	7	48
1	10	all	Diff	17	10	-15	-14	-2	-3	0	9	-2
1	10	all	Rank	**1**	2	9	8	5	7	4	3	5
20	10	all	Wins	28	18	21	16	10	11	14	23	26
20	10	all	Losses	8	13	9	12	24	15	17	14	55
20	10	all	Diff	20	5	12	4	-14	-4	-3	9	-29
20	10	all	Rank	**1**	4	2	5	8	7	6	3	9

Table 10.53 presents the wins and losses of the response approaches' over all performance measures and all n_t-τ_t combinations solving Type II DMOOPs.

For Type II DMOOPs, *re* performed the best and *ac* the worst. All other approaches were completely outperformed by *ac*, since *ac* obtained 140 more losses than wins. In constrast, all other approaches were awarded more losses than wins. However, *ac* performed the best with regards to *acc* for Type II environments.

Table 10.53: Overall wins and losses obtained by various change response strategies applied to the archive solving Type II DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Archive response stategies**								
				re	**reh**	**ra-0.5**	**rah-0.5**	**ra-1**	**rah-1**	**ra-1.5**	**rah-1.5**	**ac**
all	all	all	Wins	118	101	79	75	73	78	86	76	143
all	all	all	Losses	59	50	78	60	78	68	80	73	283
all	all	all	Diff	59	51	1	15	-5	10	6	3	-140
all	all	all	Rank	**1**	2	7	3	8	4	5	6	9

Type III DMOOPs

The wins and losses obtained for Type III DMOOPs are presented in Table 10.54.

Table 10.54: Overall wins and losses for various performance measures obtained by various change response strategies applied to the archive solving Type III DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Archive response stategies**								
				re	**reh**	**ra-0.5**	**rah-0.5**	**ra-1**	**rah-1**	**ra-1.5**	**rah-1.5**	**ac**
all	all	*acc*	Wins	38	47	26	40	36	42	37	43	68
all	all	*acc*	Losses	52	43	47	38	43	29	41	29	55
all	all	*acc*	Diff	-14	4	-21	2	-7	13	-4	14	13
all	all	*acc*	Rank	8	4	9	5	7	2	6	**1**	2
all	all	*stab*	Wins	26	20	16	9	14	8	14	9	12
all	all	*stab*	Losses	2	12	5	8	5	12	5	11	68
all	all	*stab*	Diff	24	8	11	1	9	-4	9	-2	-56
all	all	*stab*	Rank	**1**	5	2	6	3	8	3	7	9
all	all	NS	Wins	121	102	88	75	55	80	38	52	3
all	all	NS	Losses	26	42	38	49	76	51	83	80	169
all	all	NS	Diff	95	60	50	26	-21	29	-45	-28	-166
all	all	NS	Rank	**1**	2	3	5	6	4	8	7	9

The following observations are made:

- For *acc* the best performance was obtained by *rah*-1.5 and the worst performance by *ra*-0.5.
- The best rank for *stab* was awarded to *re*, while *ac* was awarded the worst rank.
- Measured against NS, *re* performed the best and *ac* the worst.

Table 10.49 presents the wins and losses with regards to the various n_t-τ_t combinations. With regards to the different environments, the following are observed:

- The approach that performed the worst for all n_t-τ_t combinations was *ac*.
- For $n_t = 10$ and $\tau_t = 10$, *ra*-0.5 performed the best.

- The best performing approach for $n_t = 10$ and $\tau_t = 25$, $n_t = 10$ and $\tau_t = 50$, and $n_t = 20$ and $\tau_t = 10$, was re.
- For severely changing environments rah-0.5 performed the best.
- ac performed poorly, obtaining more losses than wins for all n_t-τ_t combinations.

Table 10.55: Overall wins and losses for various frequencies and severities of change obtained by various change response strategies applied to the archive solving Type III DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Archive response stategies**								
				re	**reh**	**ra-0.5**	**rah-0.5**	**ra-1**	**rah-1**	**ra-1.5**	**rah-1.5**	**ac**
10	10	all	Wins	39	32	36	25	15	34	22	15	17
10	10	all	Losses	18	18	7	19	31	13	18	31	80
10	10	all	Diff	21	14	29	6	-16	21	4	-16	-63
10	10	all	Rank	2	4	**1**	5	7	2	6	7	9
10	25	all	Wins	27	28	26	7	9	16	8	10	7
10	25	all	Losses	7	9	7	20	15	9	24	15	32
10	25	all	Diff	20	19	19	-13	-6	7	-16	-5	-25
10	25	all	Rank	**1**	2	2	7	6	4	8	5	9
10	50	all	Wins	30	20	5	15	13	7	10	16	12
10	50	all	Losses	11	10	11	10	16	21	8	20	21
10	50	all	Diff	19	10	-6	5	-3	-14	2	-4	-9
10	50	all	Rank	**1**	2	7	3	5	9	4	6	8
1	10	all	Wins	44	38	37	48	38	41	26	44	24
1	10	all	Losses	29	37	36	26	37	28	49	25	73
1	10	all	Diff	15	1	1	22	1	13	-23	19	-49
1	10	all	Rank	3	5	5	**1**	5	4	8	2	9
20	10	all	Wins	45	51	26	29	30	32	23	19	23
20	10	all	Losses	15	23	29	20	25	21	30	29	86
20	10	all	Diff	30	28	-3	9	5	11	-7	-10	-63
20	10	all	Rank	**1**	2	6	4	5	3	7	8	9

The overall wins and losses for Type III benchmark functions are presented in Table 10.56.

Table 10.56: Overall wins and losses obtained by various change response strategies applied to the archive solving Type III DMOOPs

$\mathbf{n_t}$	τ_t	**PM**	**Results**	**Archive response stategies**								
				re	**reh**	**ra-0.5**	**rah-0.5**	**ra-1**	**rah-1**	**ra-1.5**	**rah-1.5**	**ac**
all	all	all	Wins	185	169	130	124	105	130	89	104	83
all	all	all	Losses	80	97	90	95	124	92	129	120	292
all	all	all	Diff	105	72	40	29	-19	38	-40	-16	-209
all	all	all	Rank	**1**	2	3	5	7	4	8	6	9

The best performing approach for Type III DMOOPs was *re* and the worst performing approach was *ac*. For Type III DMOOPs the response *re* completely outperformed the other responses, obtaining 105 more wins than losses. The approach that was awarded the second highest rank obtained 72 more wins than losses.

Overall Performance

This section discusses the overall performance of the responses applied to the archive. The wins and losses over all DMOOPs, n_t-τ_t combinations and performance measures are presented in Table 10.57. The following observations are made:

- The best overall rank was obtained by *re*, obtaining 162 more wins than losses. Therefore, *re* completely outperformed the other archive response approaches.
- The worst performance was obtained by *ac*, that was awarded 360 more losses than wins.

Table 10.57: Overall wins and losses obtained by various change response strategies applied to the archive

n_t	τ_t	PM	Results	Archive response stategies								
				re	**reh**	**ra-0.5**	**rah-0.5**	**ra-1**	**rah-1**	**ra-1.5**	**rah-1.5**	**ac**
all	all	all	Wins	307	275	215	203	190	209	183	185	227
all	all	all	Losses	145	149	170	156	204	166	220	197	587
all	all	all	Diff	162	126	45	47	-14	43	-37	-12	-360
all	all	all	Rank	**1**	2	4	3	7	5	8	6	9

However, *ac* obtained a lot of losses for NS. Table 10.58 presents the wins and losses without taking NS into account. The effect of NS can clearly be seen. *ac* obtains the best overall rank when taking only *acc* and *stab* into account.

Table 10.58: Overall wins and losses over *acc* and *stab* obtained by various change response strategies applied to the archive

n_t	τ_t	PM	Results	Archive response stategies								
				re	**reh**	**ra-0.5**	**rah-0.5**	**ra-1**	**rah-1**	**ra-1.5**	**rah-1.5**	**ac**
all	all	all	Wins	132	117	82	83	95	96	109	99	224
all	all	all	Losses	116	106	121	99	113	94	112	92	184
all	all	all	Diff	16	11	-39	-16	-18	2	-3	7	40
all	all	all	Rank	2	3	9	7	8	5	6	4	**1**

The POF^{*}s found by *ac* for $n_t = 10$ and $\tau_t = 10$ are illustrated in Figures 10.10 to 10.12. A similar trend was observed than the POF^{*}s found by *ra-t*. However, for FDA3 *ac* obtained a better spread of solutions and for FDA2 *ac* found more solutions that were close to the true POF. It should be noted that for dMOP3, *ac* found a good spread of solutions on the true POF, with some outlier solutions further away from the true POF. Therefore, *ac* found the best POF^{*} for dMOP3 from all the winning approaches illustrated in Figures 10.2, 10.5, 10.8 and 10.12. Furthermore, *ac* obtained a good spread of solutions for HEF6 and HEF7.

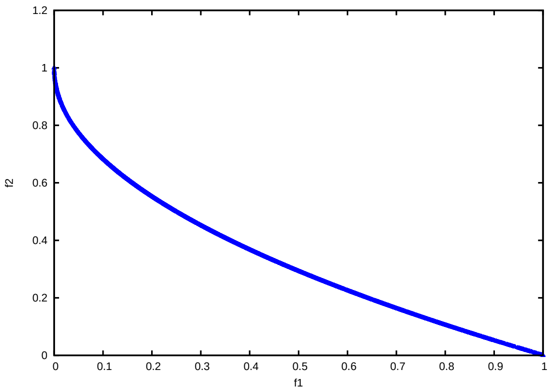
General Observations

This section discusses general observations that were made with regards to the performance of the responses applied to the archive.

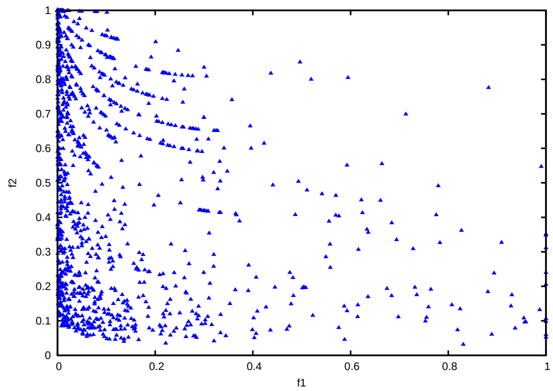
The best performing approach for Type I DMOOPs was *ra*-1. Surprisingly, *ac* performed the best for Type II and second best for Type III DMOOPs with regards to *acc*, but obtained the second lowest rank for Type I DMOOPs. Therefore, *ac* performs well when the POF changes over time. However, when the POF remains static, the other responses perform better, since they re-use previously obtained knowledge.

Similar to the response strategies applied to the particles, with DIMP2 and the FDA functions there was no statistical significant difference between the performance measure values of the various response strategies applied to the archive for many n_t-τ_t combinations.

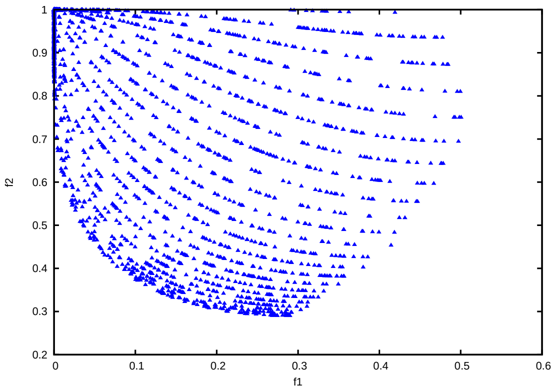
Re-evaluating the solutions in the archive re-uses previously obtained solutions. On the other hand, removing all solutions from the archive also remove optimal solutions found in previous time steps that were either still non-dominant after changes in the environment, or that would have become non-dominant again after applying hill climbing. The solutions found in the time steps that the environment was static (solutions found by *ac*), did not produce as good accuracy values as the combination of previously found solutions and newly found solutions (solutions by the re-evaluation approaches). Therefore, the re-evaluation approaches performed better than *ac* with HE9. It should be noted that HE9 is a difficult DMOOP to solve, since each decision variable has its own POS, the POSs have non-linear functions, and a transformation function is used for



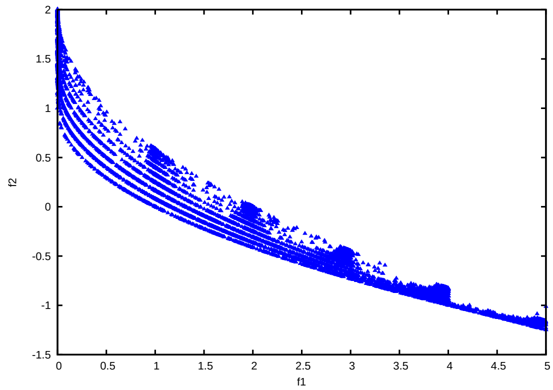
(a) POF^* for FDA1$_{Zhou}$



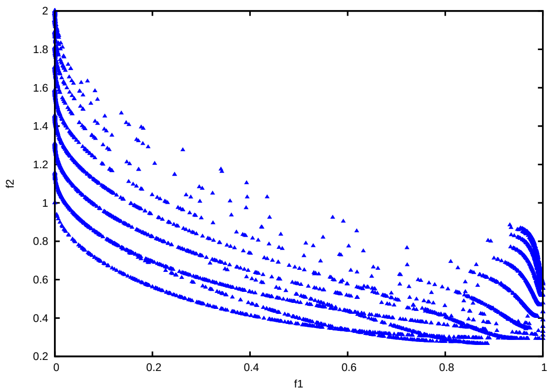
(b) POF^* for FDA2



(c) POF^* for FDA2$_{Camara}$

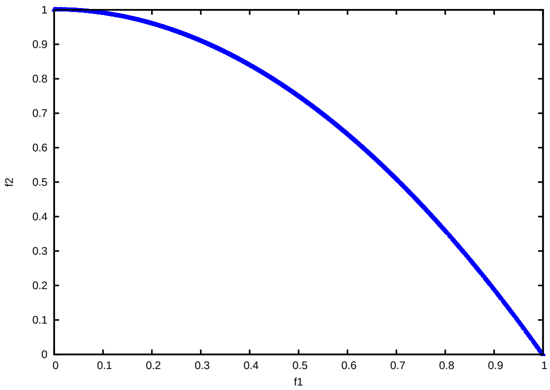


(d) POF^* for FDA3

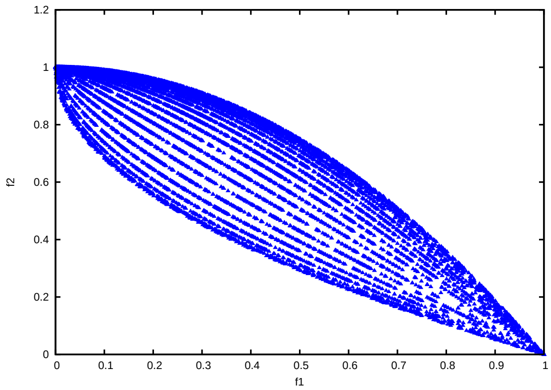


(e) POF^* for FDA3$_{Camara}$

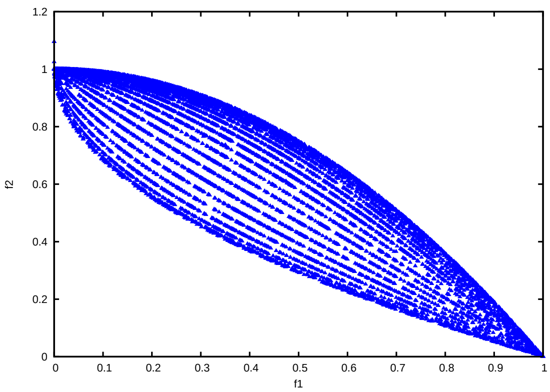
Figure 10.10: POF^* for FDA functions of DVEPSO using ac for $n_t = 10$ and $\tau_t = 10$



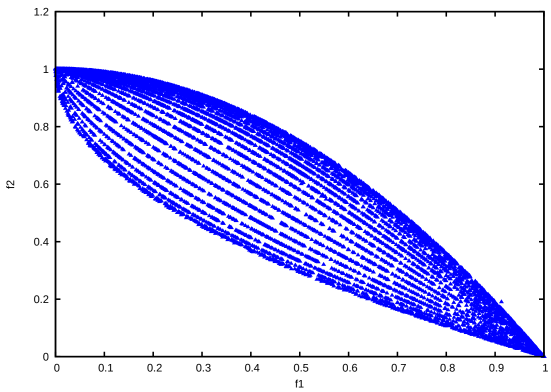
(a) POF^{*} for DIMP2



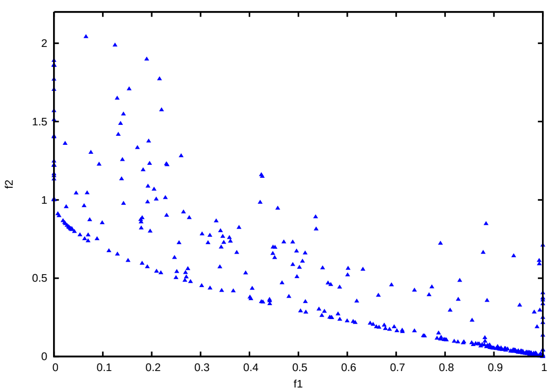
(b) POF^{*} for dMOP2



(c) POF^{*} for dMOP2$_{iso}$

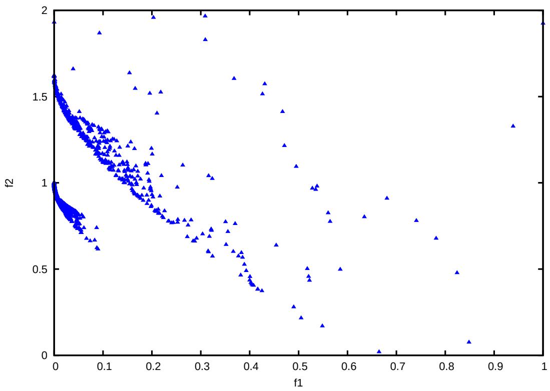


(d) POF^{*} for dMOP2$_{dec}$

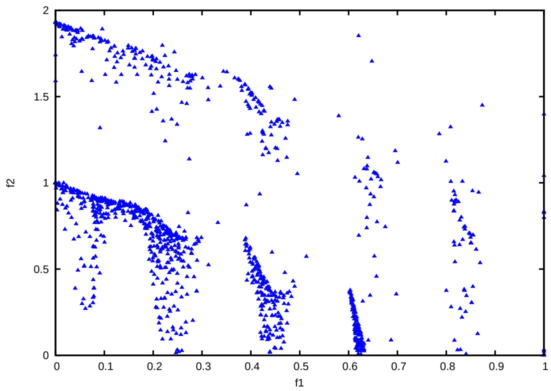


(e) POF^{*} for dMOP3

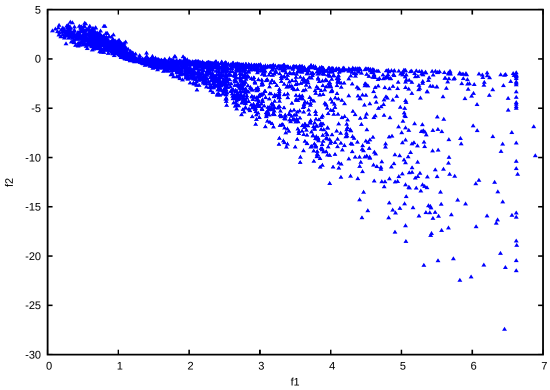
Figure 10.11: POF^{*} for DIMP2 and dMOP2 functions of DVEPSO using ac for $n_t = 10$ and $\tau_t = 10$



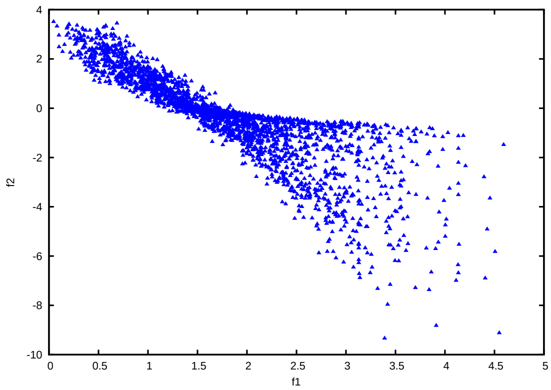
(a) POF^* for HE1



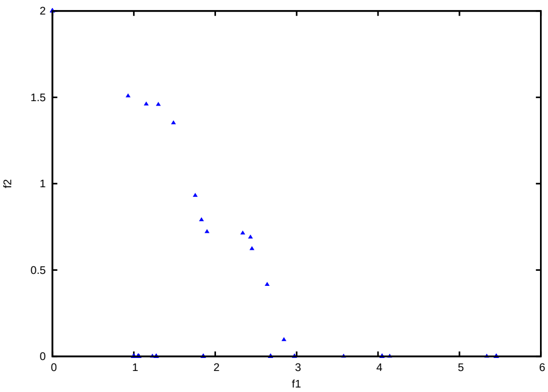
(b) POF^* for HE2



(c) POF^* for HE6



(d) POF^* for HE7



(e) POF^* for HE9

Figure 10.12: POF^* for HE functions of DVEPSO using ac for $n_t = 10$ and $\tau_t = 10$

the decision variables.

The wins and losses for HE9 are presented in Table 10.59. For HE9, *ac* obtained the lowest rank for *acc*, being awarded no wins and 10 losses. With regards to stability, there was not a huge difference in performance between the various approaches. Measured against NS, *ac* performed reasonably well. With regards to the overall wins and losses for HE9, *ac* obtained the second lowest rank. For the re-evaluation approaches, *reh* performed the best for *acc* and *re* the third best. Measured against *stab*, *re* performed the best and *reh* the worst. However, the best performing approaches obtained only one win, and *reh* was awarded no wins and only two losses. With regards to NS, both *re* and *reh* obtained rank six. With regards to the overall wins and losses for HE9, the re-evaluation approaches performed really well, obtaining the best and fourth best ranks.

Table 10.59: Wins and Losses of HE9 for various change response strategies applied to the archive

n_t	τ_t	PM	Results	Archive response stategies								
				re	reh	ra-0.5	rah-0.5	ra-1	rah-1	ra-1.5	rah-1.5	ac
10	10	*acc*	Wins	0	2	0	0	0	0	0	0	0
10	10	*acc*	Losses	1	0	0	0	1	0	0	0	0
10	10	*acc*	Diff	-1	2	0	0	-1	0	0	0	0
10	10	*acc*	Rank	8	**1**	2	2	8	2	2	2	2
10	25	*acc*	Wins	0	0	4	0	0	0	0	0	0
10	25	*acc*	Losses	1	0	0	1	0	1	0	1	0
10	25	*acc*	Diff	-1	0	4	-1	0	-1	0	-1	0
10	25	*acc*	Rank	6	2	**1**	6	2	6	2	6	2
10	50	*acc*	Wins	6	0	0	0	0	0	2	0	0
10	50	*acc*	Losses	0	1	1	0	2	1	0	1	2
10	50	*acc*	Diff	6	-1	-1	0	-2	-1	2	-1	-2
10	50	*acc*	Rank	**1**	4	4	3	8	4	2	4	8
1	10	*acc*	Wins	3	7	4	2	2	0	2	1	0
1	10	*acc*	Losses	1	0	0	3	2	5	2	1	7
1	10	*acc*	Diff	2	7	4	-1	0	-5	0	0	-7
1	10	*acc*	Rank	3	**1**	2	7	4	8	4	4	9
20	10	*acc*	Wins	0	7	0	0	0	1	0	0	0
20	10	*acc*	Losses	1	0	1	1	1	0	2	1	1
20	10	*acc*	Diff	-1	7	-1	-1	-1	1	-2	-1	-1
20	10	*acc*	Rank	3	**1**	3	3	3	2	9	3	3
all	all	*acc*	Wins	9	16	8	2	2	1	4	1	0
all	all	*acc*	Losses	4	1	2	5	6	7	4	4	10
all	all	*acc*	Diff	5	15	6	-3	-4	-6	0	-3	-10
all	all	*acc*	Rank	3	**1**	2	5	7	8	4	5	9
10	25	*stab*	Wins	1	0	0	0	0	0	0	0	0
10	25	*stab*	Losses	0	0	1	0	0	0	0	0	0
10	25	*stab*	Diff	1	0	-1	0	0	0	0	0	0

Continued on next page

n_t	τ_t	PM	Results	Archive response stategies								
				re	reh	ra-0.5	rah-0.5	ra-1	rah-1	ra-1.5	rah-1.5	ac
10	25	*stab*	Rank	**1**	2	9	2	2	2	2	2	2
20	10	*stab*	Wins	0	0	1	1	0	0	0	0	0
20	10	*stab*	Losses	0	2	0	0	0	0	0	0	0
20	10	*stab*	Diff	0	-2	1	1	0	0	0	0	0
20	10	*stab*	Rank	3	9	**1**	**1**	3	3	3	3	3
all	all	*stab*	Wins	1	0	1	1	0	0	0	0	0
all	all	*stab*	Losses	0	2	1	0	0	0	0	0	0
all	all	*stab*	Diff	1	-2	0	1	0	0	0	0	0
all	all	*stab*	Rank	**1**	9	3	**1**	3	3	3	3	3
10	10	*NS*	Wins	0	0	0	0	0	0	0	0	0
10	10	*NS*	Losses	0	0	0	0	0	0	0	0	0
10	10	*NS*	Diff	0	0	0	0	0	0	0	0	0
10	10	*NS*	Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
10	25	*NS*	Wins	0	0	0	0	0	1	0	0	0
10	25	*NS*	Losses	0	0	0	0	0	0	1	0	0
10	25	*NS*	Diff	0	0	0	0	0	1	-1	0	0
10	25	*NS*	Rank	2	2	2	2	2	**1**	9	2	2
10	50	*NS*	Wins	0	0	0	0	0	0	0	0	0
10	50	*NS*	Losses	0	0	0	0	0	0	0	0	0
10	50	*NS*	Diff	0	0	0	0	0	0	0	0	0
10	50	*NS*	Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1	10	*NS*	Wins	2	2	4	5	0	4	1	4	2
1	10	*NS*	Losses	4	4	0	0	8	0	5	0	3
1	10	*NS*	Diff	-2	-2	4	5	-8	4	-4	4	-1
1	10	*NS*	Rank	6	6	2	**1**	9	2	8	2	5
20	10	*NS*	Wins	0	0	0	1	0	4	0	0	0
20	10	*NS*	Losses	1	1	0	0	1	1	0	0	1
20	10	*NS*	Diff	-1	-1	0	1	-1	3	0	0	-1
20	10	*NS*	Rank	6	6	3	2	6	**1**	3	3	6
all	all	*NS*	Wins	2	2	4	6	0	9	1	4	2
all	all	*NS*	Losses	5	5	0	0	9	1	6	0	4
all	all	*NS*	Diff	-3	-3	4	6	-9	8	-5	4	-2
all	all	*NS*	Rank	6	6	3	2	9	**1**	8	3	5
10	10	all	Wins	0	2	0	0	0	0	0	0	0
10	10	all	Losses	1	0	0	0	1	0	0	0	0
10	10	all	Diff	-1	2	0	0	-1	0	0	0	0
10	10	all	Rank	8	**1**	2	2	8	2	2	2	2
10	25	all	Wins	1	0	4	0	0	1	0	0	0
10	25	all	Losses	1	0	1	1	0	1	1	1	0
10	25	all	Diff	0	0	3	-1	0	0	-1	-1	0
10	25	all	Rank	2	2	**1**	7	2	2	7	7	2
10	50	all	Wins	6	0	0	0	0	0	2	0	0
10	50	all	Losses	0	1	1	0	2	1	0	1	2
10	50	all	Diff	6	-1	-1	0	-2	-1	2	-1	-2
10	50	all	Rank	**1**	4	4	3	8	4	2	4	8
1	10	all	Wins	5	9	8	7	2	4	3	5	2
1	10	all	Losses	5	4	0	3	10	5	7	1	10
1	10	all	Diff	0	5	8	4	-8	-1	-4	4	-8

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n_t	τ_t	PM	Results	Archive response stategies								
				re	reh	ra-0.5	rah-0.5	ra-1	rah-1	ra-1.5	rah-1.5	ac
1	10	all	Rank	5	2	**1**	3	8	6	7	3	8
20	10	all	Wins	0	7	1	2	0	5	0	0	0
20	10	all	Losses	2	3	1	1	2	1	2	1	2
20	10	all	Diff	-2	4	0	1	-2	4	-2	-1	-2
20	10	all	Rank	6	**1**	4	3	6	**1**	6	5	6
all	all	all	Wins	12	18	13	9	2	10	5	5	2
all	all	all	Losses	9	8	3	5	15	8	10	4	14
all	all	all	Diff	3	10	10	4	-13	2	-5	1	-12
all	all	all	Rank	4	**1**	**1**	3	9	5	7	6	8

10.3 Summary

This chapter investigated the effect of various parameters on the performance of DVEPSO. These parameters were approaches to manage boundary constraint violations, approaches to share knowledge between the various sub-swarms and responses to a change in the environment applied to either the particles of the sub-swarms or the non-dominated solutions in the archive.

The boundary constraint violation management approach that performed the best was the clamping (*cl*) approach. The clamping approach places any particle that violates a specific boundary of the search space on or close to the violated boundary.

The best and second best performing approaches to share knowledge between the sub-swarms of DVEPSO was the ring-tournament (*ri-t*) and random-tournament approach (*ra-t*) respectively. However, the random-tournament approach (*ra-t*) performed the best with regards to *acc*. With the random-tournament approach, the sub-swarm from which the global guide is selected for a specific swarm's particles' velocity update is randomly selected. Tournament selection is performed to select the global guide from the selected sub-swarm.

When a change in the environment occurs, a response should be applied to both the particles of the sub-swarms and the archive. The best performing response applied to the particles was *ri-c*-30. When a change was detected, 30% of the particles of the swarm(s) whose objective function changed were re-initialised. Re-initialised particles' positions were re-set to new random positions in the search space. After re-initialisation, all particles' pbest was re-set to their current positions and a new gbest was chosen according to the new environment.

The response applied to the archive that performed the best with regards to *acc* and *stab* was *ac*. When a change in the environment was detected, all solutions were removed from the archive.

For each of these four parameters, the best performing approach are used in the experiments of the next chapter. The next chapter compares the performance of the best DVEPSO configuration against four state-of-the-art DMOO algorithms.