

Solving dynamic multi-objective optimisation problems using vector evaluated particle swarm optimisation

by

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Abstract

Most optimisation problems in everyday life are not static in nature, have multiple objectives and at least two of the objectives are in conflict with one another. However, most research focusses on either static multi-objective optimisation (MOO) or dynamic singleobjective optimisation (DSOO). Furthermore, most research on dynamic multi-objective optimisation (DMOO) focusses on evolutionary algorithms (EAs) and only a few particle swarm optimisation (PSO) algorithms exist. This thesis proposes a multi-swarm PSO algorithm, dynamic Vector Evaluated Particle Swarm Optimisation (DVEPSO), to solve dynamic multi-objective optimisation problems (DMOOPs). In order to determine whether an algorithm solves DMOO efficiently, functions are required that resembles real world DMOOPs, called benchmark functions, as well as functions that quantify the performance of the algorithm, called performance measures. However, one major problem in the field of DMOO is a lack of standard benchmark functions and performance measures. To address this problem, an overview is provided from the current literature and shortcomings of current DMOO benchmark functions and performance measures are discussed. In addition, new DMOOPs are introduced to address the identified shortcomings of current benchmark functions. Guides guide the optimisation process of DVEPSO. Therefore, various guide update approaches are investigated. Furthermore, a sensitivity analysis of DVEPSO is conducted to determine the influence of various parameters on the performance of DVEPSO. The investigated parameters include approaches to manage boundary constraint violations, approaches to share knowledge between the sub-swarms and responses to changes in the environment that are applied to either the particles



of the sub-swarms or the non-dominated solutions stored in the archive. From these experiments the best DVEPSO configuration is determined and compared against four state-of-the-art DMOO algorithms.

Keywords: dynamic multi-objective optimisation, particle swarm optimisation, dynamic vector evaluated particle swarm optimisation algorithm, benchmark functions, performance measures, guide updates, management of boundary constraint violations, response strategies, knowledge sharing

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"You have to be fast on your feet and adaptive or else a strategy is useless."

– Charles de Gaulle



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