Selective Feature Preserved Elastic Surface Registration in Complex Geometric Morphology

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Selective Feature Preserved Elastic Surface Registration in Complex Geometric Morphology

by

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Deforming a complex generic shape into a representation of another complex shape is investigated. An initial study is done on the effect of cranial shape variation on masticatory induced stress. A finite element analysis is performed on two different skull geometries. One skull geometry has a prognathic shape, characterised by jaws protruding forward, while the other has a non-prognathic form.

Comparing the results of the initial finite element analyses, the effect of an undesired variation in shape and topology on the resulting stress field is observed. This variation in shape and topology can not be attributed to the cranial shape variation that is investigated. This means that the variation in the masticatory induced stress field that is due to the relative degree in prognathism can not be quantified effectively.

To best compare results, it would be beneficial to have a computational domain for the different skull geometries that have one-to-one correspondence. An approach to obtain a computational domain that represents various geometries with the exact same mesh size and connectivity between them does exist. This approach involves deforming a generic mesh to represent different target shapes.
This report covers an introductory study to register and deform a generic mesh to approximately represent a complex target geometry. Various procedures are investigated, implemented and combined to specifically accommodate complex geometries like that of the human skull.

A surface registration procedure is implemented and combined with a feature registration procedure. Feature lines are extracted from the surface representation of each skull as well as the generic shape. These features are compared and an initial deformation is applied to the generic shape to better represent the corresponding features on the target.

Selective feature preserved elastic surface registration is performed after the initial feature based registration. Only the registration to surfaces of featureless areas and matched feature areas are allowed along with user selected areas during surface registration.

The implemented procedures have various aspects that still require improvement before the desired study regarding prognathism’s effect on masticatory induced stress could truly be approached pragmatically. Focus is only given to the use of existing procedures while the additional required improvements could be addressed in future work. It is however required that the resulting discretised domain obtained in this initial study be of sufficient quality to be used in a finite element analysis (FEA).

The implemented procedure is illustrated using the two original skull geometries. Symmetric versions of these geometries are generated with a one-to-one correspondence map between them. The skull representations are then used in a finite element analysis to illustrate the appeal of having computational domains with a consistent mapping between them. The variation in the masticatory induced stress field due to the variation in cranial shape is illustrated using the consistent mapping between the geometries as part of this example.
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- The University of Pretoria for the use of skulls in their collection.
- Labuschagne and Partners at the Little Company of Mary Hospital for producing the CT scans used in reconstructing digital surface representations of the skulls.
# Contents

## 1 Introduction

1.1 Background ................................. 1

1.2 Outline of Thesis ......................... 3

1.2.1 Chapter 2: Background Work and Problem Statement 3

1.2.2 Chapter 3: Elastic Surface Registration ................. 4

1.2.3 Chapter 4: Geometric Features ................. 4

1.2.4 Chapter 5: Feature Registration .................. 4

1.2.5 Landmark Nodes ................................ 5

1.2.6 Feature lines ................................ 5

1.2.7 Chapter 6: Proposed Registration Procedure ........ 6

1.2.8 Chapter 7: Conclusions ....................... 8

## 2 Background Work and Problem Statement

2.1 Introduction ................................ 9

2.2 Prognathism in the human skull .................. 10

2.3 Mastication ................................ 11

2.3.1 Teeth .................................. 11

2.3.2 Bone .................................. 11

2.3.3 Muscles ................................ 12

2.4 Finite Element Model ......................... 13

2.4.1 Geometries ................................ 14

2.4.2 Material Properties ....................... 14

2.4.3 Boundary Conditions ....................... 14

2.5 Results .................................. 16

2.6 Problem Statement ........................... 22
3 Surface Registration

3.1 Point Set Registration .................................................. 26
   3.1.1 The Iterative Closest Point Algorithm ...................... 26
           Application .................................................. 28

3.2 Elastic Surface Registration ......................................... 31
   3.2.1 Registration Procedure ....................................... 31
   3.2.2 Application .................................................. 35
           Registration on Femur Geometries ......................... 35
           Registration on Skull Geometries ......................... 40

3.3 Remarks and Conclusions ............................................ 43

4 Geometric Features ...................................................... 44

4.1 Introduction .......................................................... 44

4.2 Local Structure Tensor .............................................. 45
   4.2.1 Feature Classification ....................................... 46
   4.2.2 Spatial Search Speed-up ..................................... 48

4.3 Differential Geometry Surface Information ..................... 49
   4.3.1 Application to a Discretised Surface ...................... 50
   4.3.2 Enhanced Moving Least Squares Approximation ............ 51
           Generating a smooth local surface approximation ..... 51
           Estimating curvatures and their derivatives ............. 53
   4.3.3 Shape Index Feature Points ................................ 54
   4.3.4 Ridges and Valleys .......................................... 56
           Determining ridge nodes: .................................... 56
           Determining valley nodes: ................................... 58
           Connecting nodes into lines ................................ 62
           Thresholding .................................................. 63

4.4 The Selection of Matching Features ............................... 65

5 Feature Registration ..................................................... 68

5.1 Feature Point Registration .......................................... 69
   5.1.1 Radial Basis Function Interpolation ....................... 69

5.2 Feature Line Registration .......................................... 74
   5.2.1 Registration Procedure ..................................... 74
           Point Matching ................................................ 75
Line Matching ........................................ 76
Transformation Computation ....................... 77
Implementation ..................................... 80
Application ......................................... 80
5.3 Surface registration .............................. 81
5.3.1 Orthognathic Representation .................. 83
6 Proposed Registration Procedure ................. 91
   6.1 Step 1: Rotate, Scale and Translate .......... 93
   6.2 Step 2: Use Lines of Curvature .............. 94
      6.2.1 Feature Surfaces ......................... 94
   6.3 Step 3: Register Allowable Surface .......... 99
   6.4 Step 5: Mesh Quality ......................... 102
      6.4.1 Quality Metric .......................... 104
      6.4.2 Usable Skull Mesh Generation .......... 106
   6.5 Analysis on Registered Skull Geometries .... 109
      6.5.1 The effect of non-unique registration on FEA result .... 111
      6.5.2 Linearity and constructing an approximate result from principal shape components .... 118
7 Conclusions .................................... 122
   7.1 Remarks and Possible Future Work .......... 122
A FEA on Skull Geometries ....................... 133
   A.1 Introduction ................................ 133
   A.2 Geometries .................................. 134
      A.2.1 Model Creation ......................... 134
   A.3 Material Properties ........................ 135
   A.4 Boundary Conditions ........................ 135
      A.4.1 Muscle Forces .......................... 136
      A.4.2 Reaction Forces ......................... 138
   A.5 Analysis ................................... 141
   A.6 Results .................................... 149
B Affine Iterative Closest Point Problem .......... 155
   B.1 Reformulating the ICP problem .............. 155
B.2 Lie group and lie algebra ........................................... 156
B.3 Performing an Affine ICP transformation ....................... 157

C Shape Context Correspondence ..................................... 160
C.1 Shape Context ....................................................... 160
C.2 Point Matching ..................................................... 163

D Dolphin Feature Example ............................................. 164
Nomenclature

$a$ - Lower scale bound in the reformulated ICP.

$b$ - Upper scale bound in the reformulated ICP.

$c$ - Counter, indicating the $d^{th}$ triangle patch on the target shape surface mesh.

$c_k$ - Correspondence between a generic and data shape.

$c$ - Vector containing the rotation, reflection and scale variables used in the reformulated ICP.

$d$ - Counter, indicating the $d^{th}$ triangle patch on the generic shape surface mesh.

$d$ - Distance from a point to its registered location.

$D$ - Distance.

$D_j$ - Set of linear bases of a diagonal matrix. The only non-zero entry is $D_{jj} = 1$.

$E$ - Young’s modulus.

$E_j$ - Linearised bases of the special orthogonal group representation of an invertible matrix.

$f$ - Smoothing parameter in the elastic surface registration procedure.

$f_i$ - Function evaluation at location $i$.

$F$ - Force.

$F$ - Implicit surface $F(x) = 0$.

$h$ - Positive increasing function for determining element quality.

$H$ - Reference plane.

$i$ - Counter. $i = 1, 2, ..., k_{max}$ where $k_{max}$ is the maximum number of iterations for example.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Identity matrix.</td>
</tr>
<tr>
<td>J</td>
<td>Jacobian matrix.</td>
</tr>
<tr>
<td>k</td>
<td>Iteration.</td>
</tr>
<tr>
<td>L</td>
<td>A line.</td>
</tr>
<tr>
<td>m</td>
<td>The points on the generic shape.</td>
</tr>
<tr>
<td>m_t</td>
<td>Point correspondence of the target on the generic shape translated so its centroid is at the origin of the Cartesian coordinate axis.</td>
</tr>
<tr>
<td>M</td>
<td>Moment.</td>
</tr>
<tr>
<td>M_{b,b}</td>
<td>Matrix containing evaluations of a radial basis function.</td>
</tr>
<tr>
<td>M</td>
<td>Model or generic shape.</td>
</tr>
<tr>
<td>n</td>
<td>Number of neighbors used in the elastic registration procedure.</td>
</tr>
<tr>
<td>n</td>
<td>Unit normal.</td>
</tr>
<tr>
<td>N</td>
<td>Indicates size. ( N_p ) is the number of points in the target shape and ( N_m ) the number of points in the model shape for example.</td>
</tr>
<tr>
<td>p</td>
<td>Linear polynomial.</td>
</tr>
<tr>
<td>p^j_i</td>
<td>Portion of points on one line ( L_i ) registered to line ( L'_j ).</td>
</tr>
<tr>
<td>p</td>
<td>The points on the target shape.</td>
</tr>
<tr>
<td>p_t</td>
<td>Target shape ( p ) after it is translated so the centroid is at the origin of the Cartesian coordinate axis.</td>
</tr>
<tr>
<td>P_b</td>
<td>Matrix containing boundary coordinates.</td>
</tr>
<tr>
<td>P</td>
<td>Data or target shape.</td>
</tr>
<tr>
<td>q_m</td>
<td>Element quality of tetrahedron ( m ).</td>
</tr>
<tr>
<td>Q</td>
<td>Matrix. ( Q = JW^{-1} ) when determining element quality.</td>
</tr>
<tr>
<td>r</td>
<td>Radius.</td>
</tr>
<tr>
<td>r_j</td>
<td>Rotation variables in the reformulated ICP.</td>
</tr>
<tr>
<td>r</td>
<td>The registration location. ( r_w_j ) is the possible registered location of point ( w_j ) onto a target shape for example.</td>
</tr>
<tr>
<td>R</td>
<td>Rotation Matrix.</td>
</tr>
<tr>
<td>( \mathbb{R} )</td>
<td>Real number indicator. ( \mathbb{R}^3 ) indicates a tensor consisting of three real numbers.</td>
</tr>
<tr>
<td>s_j</td>
<td>Scale variables in the reformulated ICP.</td>
</tr>
</tbody>
</table>
\( S^{k-1} \) - Deformation applied to \( W^{k-1} \) to better approximate the target.

\( S_i \) - Shape index.

\( S \) - Scale Matrix.

\( t \) - Translation vector.

\( T \) - Transformation.

\( T_h \) - Threshold, used when pruning false lines.

\( T_m \) - Indicates size. \( T_m \) is the number of triangles in the model shape.

\( T_p \) - Indicates size. \( T_p \) is the number of triangles in the target shape.

\( \mathcal{T} \) - A tetrahedron.

\( u_j \) - Reflection variables in the reformulated ICP.

\( U \) - Reflection matrix.

\( w \) - The points on the deformable surface.

\( W \) - Jacobian matrix that maps the tetrahedron \( \mathcal{T}_R \) to tetrahedron \( \mathcal{T}_I \).

\( \mathcal{W} \) - Deformable surface in the elastic registration procedure.

\( x \) - Nodal coordinates
Greek Symbols

\( \alpha \) - Coefficients used in radial basis function interpolation
\( \beta \) - Coefficients of the linear polynomial when using RBF interpolation.
\( \gamma \) - Smoothing parameter in the elastic registration procedure
\( \delta \) - Shift variable used in positive increasing function.
\( \varepsilon \) - The average distance of point set correspondence in the ICP procedure.
OR The average total deformation applied to the deformable surface during elastic registration.
\( \varepsilon_T \) - Tolerance.
\( \zeta \) - Machine epsilon or tolerance \((0 < \zeta \ll 1)\).
\( \kappa \) - Principal curvature.
\( \lambda \) - Eigenvalue.
\( \mu \) - Average shape index.
\( \nu \) - Poisson’s ratio.
\( \xi \) - Compact radial basis function scaling factor.
\( \varphi \) - Principal curvature direction.
\( \sigma_0 \) - Smoothing parameter in the elastic registration procedure
\( \sigma_m \) - Determinant of the matrix \( S_m \).
\( \tau \) - Curvature derivative or extremality coefficient.
\( \phi \) - Radial basis function.
Superscripts

$-1$ - Inversion.
$k$ - Iteration counter. $k - 1$ indicates the previous iteration.
$T$ - Tensor transpose.

Subscripts

$0, 1, 2, ...$ - Used where the number represents the index within a list or set.
$i$ - Quantity in list defining the target shape. $i \in \{1, 2, ..., N_p\}$
$i, j, k, ...$ - Used where indicial notation is used and summation is implied.
$j$ - Quantity in list defining the generic shape. $j \in \{1, 2, ..., N_m\}$
$k$ - Iteration counter. $k - 1$ indicates the previous iteration.
$m$ - Indicates a value related to the generic shape $\mathcal{M}$.
$p$ - Indicates a value related to the target shape $\mathcal{P}$.
$x, y, z$ - Indicates coordinates in the $x$, $y$- and $z$- axis.

Mathematical Symbols and Operators

$\in$ - Indicates membership of a set.
$\Sigma$ - Summation.
$|$ - Frobenius norm.
$\|\|$ - Euclidean distance.
$\text{det}(\cdot)$ - Determinant of a matrix.
$\text{tr}(\cdot)$ - Trace of a matrix.
$\partial$ - Partial derivative.
$\nabla$ - Gradient.
## Acronyms and Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>ba</td>
<td>Basion - Landmark position on the human skull.</td>
</tr>
<tr>
<td>CT</td>
<td>Computed Tomography.</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis.</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Model.</td>
</tr>
<tr>
<td>FSI</td>
<td>Fluid Structure Interaction.</td>
</tr>
<tr>
<td>GI</td>
<td>Gnathic index - The distance ratio of the lines connecting the basion landmark to the prostation and nasion landmarks on the human skull. This ratio is expressed as a percentage quantity.</td>
</tr>
<tr>
<td>ICP</td>
<td>Iterative Closest Point - Procedure used in rigid registration.</td>
</tr>
<tr>
<td>LST</td>
<td>Local Structure Tensor.</td>
</tr>
<tr>
<td>MLS</td>
<td>Moving Least Squares.</td>
</tr>
<tr>
<td>MRI</td>
<td>Magnetic Resonance Imaging.</td>
</tr>
<tr>
<td>n</td>
<td>Nasion - Landmark position on the human skull.</td>
</tr>
<tr>
<td>OC</td>
<td>Occipital condyles - Condyles at the foramen magnum where the skull articulates with the spinal column.</td>
</tr>
<tr>
<td>PCA</td>
<td>Principal Component Analysis - Statistical analysis to determine the principal modes of variation within sample data.</td>
</tr>
<tr>
<td>pr</td>
<td>Prostation - Landmark position on the human skull.</td>
</tr>
<tr>
<td>RBF</td>
<td>Radial Basis Function - Interpolation function used to interpolate a scalar quantity known at select positions within spatial data.</td>
</tr>
<tr>
<td>TMJ</td>
<td>Temporomandibular joint - Joint connecting the mandible to the skull.</td>
</tr>
<tr>
<td>TPS</td>
<td>Thin Plate Spline - A type of radial basis function.</td>
</tr>
</tbody>
</table>
List of Figures

1.1 Landmarks on the (a) front, (b) side and (c) bottom of the human skull [43]. .................................................. 2
1.2 Flow diagram illustrating the basic idea of combining feature and surface registration. In the implemented procedure, the result of the feature based registration is used as input and also dictates allowable surfaces used during surface registration. ................................. 7

2.1 Masticatory muscles [12]. (a) Masseter, (b) Temporalis and (c) the lateral and medial Pterygoid. ......................... 13
2.2 Boundary condition positions on the skull. (a) Lateral view of the prognathic skull indicating the Temporal fossa and zygomatic arch (cheek bone). The jaw hinge or temporomandibular joint is indicated by TMJ and the seven sections of the temporalis muscle attachment positions are indicated by numbers 1 through 7. (b) Inferior view of the prognathic skull. The location of boundary constraints are the occipital condyles (OC) on the foramen magnum. Reaction forces are determined at the left and right temporomandibular joints (TMJL and TMJR) and either the first molar (M1) or first incisor (I1) for the applied muscle forces when balancing the system. ................. 15
2.3 Muscles and reaction forces on (a) the prognathic skull form and (b) orthognathic skull form for a vertical molar bite. .... 17
2.4 Lateral view of the working side Von Mises stress for a molar bite on the (a) prognathic and (b) orthognathic skull form as well as for an incisor bite on the (c) prognathic and (d) orthognathic FEA. The units of the stress contours are in N/cm$^2$. ...................... 18
2.5 Semi-transparent view of the (a) prognathic and (b) orthognathic skull form. .................................................. 19
2.6 Lateral cut views through the two skulls to illustrate the difference between their boundary surfaces. (a) Location of the cut planes in (b) and (c). (d) Location of the cut planes (e) and (f). ................ 20

2.7 Frontal cut views through the two skulls to illustrate the difference between their boundary surfaces. (a) Location of the cut planes in (b) and (c). (d) Location of the cut planes (e) and (f). ................ 21

2.8 First three modes of a principal component analysis done on the human femur varied between ±3 standard deviations. This is done after 46 different femur geometries are represented using the same mesh with only updated nodal coordinates [19]. (a) Frontal and (b) lateral view. ................................. 23

3.1 Lateral view of rigid registration results on two femur geometries obtained from the INRIA shape repository [4]. (a) Initial relative position and alignment of the two femurs. From this starting position, the rigid registration result is obtained by ICP for (b) rotation and translation only and (c) allowing isotropic scaling in addition to rotation and translation. ................................. 29

3.2 Isometric view of rigid registration results on two femur geometries obtained from the INRIA shape repository [4]. (a) Initial relative position and alignment of the two femurs. From this starting position, the rigid registration result is obtained by ICP for (b) rotation and translation only and (c) allowing isotropic scaling in addition to rotation and translation. ................................. 30

3.3 Convergence rate of the ICP. The blue line is obtained when only allowing rotation and translation while the red line is the convergence rate when rotation, translation and isotropic scale is allowed. At each iteration, $\varepsilon$ is the function value after performing the least squares optimisation in Equation (3.5). ................................. 30

3.4 Determining the registration of a point $w_j$ onto one of the $n$ closest triangles with (a) the registered point inside and (b) outside of the triangle boundary. ................................. 33
3.5 Elastic registration on the isotropic scale ICP result. (a) Lateral view of the rigid registration result on the two femur geometries. (b) Lateral view of the elastic surface registration result to deform the black wire-frame mesh into the target geometry at iteration 100.

3.6 Elastic registration on the isotropic scale ICP result. (a) Isometric view of the rigid registration result on the two femur geometries. (b) Isometric view of the elastic surface registration result to deform the black wire-frame mesh into the target geometry at iteration 100.

3.7 Improvement on the average deformation similarity criteria in Equation (3.11) subject to varying user controlled parameters. (a) $\sigma_0 = 10$ and $f = 1.0715$ while varying $\gamma$. (b) $f = 1.0715$ and $\gamma = 2$ while varying $\sigma_0$. (c) $\sigma_0 = 10$ and $\gamma = 2$ while varying $f$.

3.8 Various deformed generic surface target representations obtained with different user controlled parameters. Mesh A is the registration result with the parameters set to $\gamma = 2$, $\sigma_0 = 0.5$ and $f = 1.0715$ at 10 registration iterations. Mesh B is the registration result with the parameters set to $\gamma = 2$, $\sigma_0 = 50$ and $f = 1.0715$ at 75 registration iterations. Mesh C is the registration result with the parameters set to $\gamma = 2$, $\sigma_0 = 10$ and $f = 1.3$ at 20 registration iterations. Mesh D is the registration result with the parameters set to $\gamma = 2$, $\sigma_0 = 10$ and $f = 1.1$ at 40 registration iterations. The average total deformation plots in Figure 3.7 also indicate where meshes A - D are chosen.

3.9 Initial position of the smoothed skull mesh and it’s reflection.

3.10 Initial position of the smoothed skull mesh and it’s reflection.

3.11 Problems with the sinus in elastically registering the smooth skull mesh into its reflection. (a) Position of cut plane (c) and (e). (b) Position of cut plane (d) and (f). (c), (d) Initial position of deformable mesh in relation to the target (The blue mesh in Figure 3.10 is set as the deformable mesh and is registered onto it’s reflection). (e), (f) Elastic registration result at iteration 100. The red line represents the position of the target surface in that plane and the black line the deformable mesh surface.
4.1 Points on skull geometry where $\lambda_1 < 5 \times \lambda_2$. This condition represents spheres, saddles, ridges and valleys within a certain degree. (a) Frontal, (b) side and (c) bottom view. ........................................... 46

4.2 Points on skull geometry where $\lambda_1 < 100 \times \lambda_2$. This condition represents spheres, saddles, ridges and valleys within a certain degree. (a) Frontal, (b) side and (c) bottom view. ........................................... 46

4.3 Points on skull geometry where $\lambda_1 > 2'000 \times \lambda_2$. This condition represents planes to a certain degree. (a) Frontal, (b) side and (c) bottom view. .......................................................... 47

4.4 Points on skull geometry where $\lambda_1 > 20'000 \times \lambda_2$. This condition represents planes to a certain degree. (a) Frontal, (b) side and (c) bottom view. .......................................................... 47

4.5 Points for $\lambda_1 < 50 \times \lambda_2$ on a refined dolphin model using (a) 3-ring neighbourhood (b) 15 nearest neighbours .......................................................... 49

4.6 (a) The MLS projection. Given a point $p_j$, the local reference plane $H$ and a point $q$ on $H$ is first determined. An MLS surface is then defined implicitly by the MLS projection as the set of points that project on to themselves [37]. (b) The estimation of a modified MLS approximation to the mesh [37]. .......................................................... 51

4.7 Points on skull geometry where $\kappa_{\text{max}} > |\kappa_{\text{min}}|$. (a) Frontal, (b) side and (c) bottom view. .......................................................... 54

4.8 Points on skull geometry where $\kappa_{\text{min}} < -|\kappa_{\text{max}}|$. (a) Frontal, (b) side and (c) bottom view. .......................................................... 54

4.9 Frontal view highlighting points on skull geometry where $\kappa_{\text{max}} > 5 \times \text{average}(\kappa_{\text{max}})$ in blue and $\kappa_{\text{min}} < 5 \times \text{average}(\kappa_{\text{min}})$ in red. (a) Ridges and valleys split up in (b) to show only ridges and (c) only valleys. .......................................................... 55

4.10 Side view highlighting points on skull geometry where $\kappa_{\text{max}} > 5 \times \text{average}(\kappa_{\text{max}})$ in blue and $\kappa_{\text{min}} < 5 \times \text{average}(\kappa_{\text{min}})$ in red. (a) Ridges and valleys split up in (b) to show only ridges and (c) only valleys. .......................................................... 55

4.11 Lower view highlighting points on skull geometry where $\kappa_{\text{max}} > 5 \times \text{average}(\kappa_{\text{max}})$ in blue and $\kappa_{\text{min}} < 5 \times \text{average}(\kappa_{\text{min}})$ in red. (a) Ridges and valleys split up in (b) to show only ridges and (c) only valleys. .......................................................... 55

4.12 Shape index of a dolphin geometry after using a 5-ring neighbourhood for MLS surface fitting. (a) Isometric (b) top and (c) side view. .... 57
4.13 Shape index of the skull geometry after using a 3-ring neighbourhood for MLS surface fitting. (a) Frontal, (b) side and (c) bottom view.

4.14 Feature points automatically extracted from skull geometry for radius $r_i = 10$ and $\alpha = 0.1, \beta = 0.1$. (a) Frontal, (b) side and (c) bottom view.

4.15 Crest nodes and lines on a hand geometry. (a) Possible ridge (blue) and valley (red) points obtained by using only principal curvatures and derivatives along with (b) the lines after connecting points in the relevant principal direction. (c) The equivalent ridge and valley lines obtained by using curvature derivative zero crossing procedure. For visual clarity only lines with more than 8 segments are displayed.

4.16 Crest lines on a refined and smoothed bishop geometry. (a) Ridge (blue) and valley (red) lines obtained by using only principal curvatures and directions. (b) The equivalent ridge and valley lines obtained by using curvature derivative zero crossing procedure.

4.17 Crest nodes on the refined version of a dolphin geometry. (a) and (c) Ridge (blue) and valley (red) nodes obtained by using only principal curvatures and directions. (b) and (d) The equivalent ridge and valley nodes obtained by using curvature derivative zero crossing procedure.

4.18 Crest lines on the refined version of a dolphin geometry. (a) Ridge (blue) and valley (red) lines obtained by using only principal curvatures and directions. (b) The equivalent ridge and valley lines obtained by using curvature derivative zero crossing procedure. Lines have been pruned and line width scaled according to threshold calculated with Equation (4.11) and a minimum threshold value of $T_h = 10$. 

57

58

60

60

61

61
4.19 Extracted ridge (blue) and valley (red) lines on the refined trim-star geometry. (a) Extracted crest nodes with presence of false edges already evident on the smoother areas of the geometry. (b) Allowable crest nodes after filtering out those that don’t satisfy the local structure tensor condition $\lambda_1 < 10 \times \lambda_2$. (c) The extracted crest nodes that satisfy the local structure tensor filtering condition in (b). (d) The ridge and valley lines constructed using only the filtered crest nodes. In this figure some spurious or false lines are still present indicating that thresholding might still be required. These false lines are likely picked up due to the local discretisation.

4.20 Filtering crest nodes on the refined version of a dolphin geometry. (a) Ridge (blue) and valley (red) nodes extracted using 100 nearest neighbour search with extremality zero crossing procedure. (b) Points for feature areas satisfying $\lambda_1 < 100 \times \lambda_2$ resulting from 3-ring neighbourhood local structure evaluation. (c) Ridge and valley nodes remaining after the filter procedure.

4.21 Frontal view of the skull geometry with extracted ridge nodes in blue and valley nodes in red. (a) and (b) show all of the crest nodes on the geometry and (c) contains only nodes that satisfy the local structure tensor condition $\lambda_1 < 50 \times \lambda_2$.

4.22 Side view of the skull geometry with extracted ridge nodes in blue and valley nodes in red. (a) and (b) show all of the crest nodes on the geometry and (c) contains only nodes that satisfy the local structure tensor condition $\lambda_1 < 50 \times \lambda_2$.

4.23 Lower view of the skull geometry with extracted ridge nodes in blue and valley nodes in red. (a) and (b) show all of the crest nodes on the geometry and (c) contains only nodes that satisfy the local structure tensor condition $\lambda_1 < 50 \times \lambda_2$.

4.24 Thresholded ridge lines on the skull geometry after first applying the filter $\lambda_1 < 50 \times \lambda_2$ and then thresholding lines to $T_h = 200$. (a) Frontal and (b) lateral view. Only lines with more than 4 line segments are displayed.
4.25 Thresholded valley lines on the skull geometry after first applying the filter $\lambda_1 < 50 \times \lambda_2$ and then thresholding lines to $T_h = 200$. (a) Frontal and (b) lateral view. Only lines with more than 4 line segments are displayed.

5.1 Geometric dissimilarity illustrating the average shape of the scapula of male ($n_m = 52$, open symbols) and female ($n_f = 42$, closed symbols) western African lowland Gorillas. (a) Recorded coordinates of homologous points on each specimen. (b) The varying coordinates due to difference in shape as well as location and orientation with respect to axes during landmark digitisation. (c) Superimposed landmark coordinates after applying the Procrustes method. The common coordinate system allows for further statistical analysis. (d) Visualising statistical results, the average male-female variation is shown using both difference vectors and a thin plate spline deformation grid magnified by a scale factor of two [54].

5.2 Radial Basis Function Performance. (a) Original Configuration (b) MQB, (c) IMQB, (d) Gauss, (e) Linear, (f) Cubic, (g) TPS. The deformable mesh is displayed as a black wire-frame and the target as the semi-opaque pink surface. The blue dots indicate deformable landmark positions and the red dots the target positions. In (b) through (g) these landmark coordinates coincide exactly.

5.3 Two sets of lines to be registered [59]. (a) The target skull \( P \) on the left is composed of 591 lines and 19’302 points. (b) Reference skull \( M \) on the right is composed of 583 lines and 19’368 points. These subjects have a variation in shape as well as differences in the number and topology of the lines.

5.4 Registering two lines [59]. (a) Illustration revealing that computing registration parameters is not obvious due to the non-bijectivity of matched points. (b) After discarding non-consistent matched points, line registration parameters are computed consistently.

5.5 Registration of \( M \) towards \( P \) [59]. (a) The deformed set \( M \) with \( P \). Matched points are linked with the two sets reasonably superimposed. In (b) \( M \) is in its original position, allowing an estimated extent of the deformation between the two sets.
5.6 Building and using a topological registration map [59]. (a) The registration graph: each node is a line of a set and an oriented link represents the relation "is registered with". (b) Extracted subsets of corresponding lines of different data sets. If a sub-graph contains at least one line of each data set, it defines a subset of common lines found on all geometries in the sample.  

5.7 Common lines to all six skulls used by Subsol et al. [59]. The thin lines show the lines of the different geometries used and the thicker lines the average common lines constituting the atlas.  

5.8 Feature line registration on dolphin geometries. (a) Original position of a target and base dolphin geometry. (b) Updated position of the target dolphin geometry relative to the base shape after isotropic scale ICP registration. (c) Feature registration of the base dolphin to the aligned target configuration at iteration 100. The target geometry is illustrated in its aligned position with the target features in red and the deformed base geometry features in blue.  

5.9 Frontal view of feature registration on the smooth skull and its reflection. (a) Feature lines of the smoothed skull and its reflection. (b) Feature registration result and (c) the average of the initial and registered positions to create a symmetric model. Blue lines indicate the features of the deformable surface with red lines indicating the target features.  

5.10 Lower view of feature registration on the smooth skull and its reflection. (a) Feature lines of the smoothed skull and its reflection. (b) Feature registration result and (c) the average of the initial and registered positions to create a symmetric model. Blue lines indicate the features of the deformable surface with red lines indicating the target features.
5.11 Elastic surface registration of the smooth skull onto its reflection. The blue mesh in Figure 3.10 is set as the deformable mesh and is registered onto its reflection. (a) Reflected smooth skull geometry. (b) Reflected smooth skull geometry set as the target with the original smooth skull shown as the black wire-frame. (c) Elastic surface registration results after first applying the feature registration of Figures 5.9 and 5.10. (d) The average of the smooth skull and registered nodal coordinates resulting in a symmetric skull surface.

5.12 Asymmetry in the original smooth skull geometry. (a) Displacement from the symmetric skull mesh coordinates back to the original scaled by a factor of 3. (b) The absolute distance (norm of the distance vector) from the original to symmetric nodal coordinates illustrated as scalars on the symmetric skull representation. The color bar values are in millimeters.

5.13 Reflected registration incorporating an initial feature match. Simply registering the smooth skull geometry onto its reflection in Chapter 3 created problems with especially the sinuses. The same cut planes of Figure 3.11 are presented here compared to the registration result after an initial feature match. (a), (c) The initial registration and (b), (d) result after initial feature registration at iteration 100. Recall that the red line indicates the target geometry in the plane with black the surface deformed during registration.

5.14 Elastic registration on the orthognathic skull. (a) The initial deformable mesh. (b) The rigid registration result to align the orthognathic skull to the deformable mesh with (c) the registration result at iteration 60. (d) The smoothed deformed mesh at iteration 60.
5.15 Frontal view of elastic registration on the orthognathic skull. (a) The cut plane illustrating the position of the subsequent figures taken for the registration result. (b) The target and deformable geometry after isotropic scale ICP registration. (c) The result of an initial surface feature registration. Elastic surface registration is performed after an initial feature registration resulting in base mesh deformation at iteration (d) 10, (e) 20, (f) 30, (g) 40, (h) 50 and (i) 60. The red line represents the position of the target surface in that plane and the black line the deformable mesh surface. Note that the topology doesn’t change although it might appear that way. This appearance is due to the registered feature coming in and out of the plane where these figures are generated.

5.16 Lateral view of elastic registration on the orthognathic skull. (a) The cut plane illustrating the position of the subsequent figures taken for the registration result. (b) The target and deformable geometry after isotropic scale ICP registration. (c) The result of an initial surface feature registration. Elastic surface registration is performed after an initial feature registration resulting in base mesh deformation at iteration (d) 10, (e) 20, (f) 30, (g) 40, (h) 50 and (i) 60. The red line represents the position of the target surface in that plane and the black line the deformable mesh surface.

5.17 Difference between the original and smoothed registration result at iteration 60. (a) Result of the elastic surface registration at iteration 60. This is the same cut as visible in Figure 5.15 (i). (b) The result showed in (a) after 10 Taubin [61] smoothing iterations. The red line represents the position of the target surface in the cut plane and the black line the deformable mesh surface.

6.1 Flow diagram illustrating the various components of the registration procedure proposed and implemented.

6.2 (a) Original position of the orthognathic skull geometry relative to the smoothed base skull. (b) Frontal and (c) lateral view of the orthognathic skull and base skull represented by the black wire-frame mesh.
6.3 (a) Rigid registration position of the orthognathic skull geometry relative to the smoothed base skull after an isotropic ICP registration. (b) Frontal and (c) lateral view of the orthognathic skull and base skull represented by the black wire-frame mesh. 93
6.4 (a) User selected allowable features on the symmetric base skull geometry. (b) Frontal, (c) lateral and (d) lower view. 95
6.5 Registration of allowable base geometry features to the orthognathic skull. (a) Frontal, (b) lateral and (c) lower view of the base geometry features relative to the orthognathic skull. (d) Frontal, (e) lateral and (f) lower view of the base geometry features registered and deformed to the corresponding features on the orthognathic skull. 96
6.6 Registration of allowable base geometry features to the orthognathic skull. (a) The registration result on an opaque target skull and (b) semi-transparent target surface. Blue lines indicate the features of the deformable surface with red lines indicating the target features. 96
6.7 All feature points on the symmetric base skull for $\kappa_{\text{max}} > 0.2$ and $\kappa_{\text{min}} < -0.2$. (a) Frontal, (b) lateral and (c) lower view. 98
6.8 All feature points on the orthognathic target skull for $\kappa_{\text{max}} > 0.18$ and $\kappa_{\text{min}} < -0.18$. (a) Frontal, (b) lateral and (c) lower view. 98
6.9 Feature points on the symmetric base skull for $\kappa_{\text{max}} > 0.2$ and $\kappa_{\text{min}} < -0.2$ corresponding to the user specified allowable feature lines in Figure 6.4. (a) Frontal, (b) lateral and (c) lower view. 98
6.10 Feature points on the orthognathic target skull for $\kappa_{\text{max}} > 0.18$ and $\kappa_{\text{min}} < -0.18$ corresponding to the user specified allowable feature lines in Figure 6.6. (a) Frontal, (b) lateral and (c) lower view. 99
6.11 Elastic registration on the orthognathic skull. (a) The rigid registration result to align the orthognathic skull to the deformable mesh with (b) the registration result at iteration 60. This registration result is obtained after an initial allowable feature registration and filtering for allowable surfaces. The compared result of Figure 5.14 employed full feature and subsequent full elastic surface registration. 100
6.12 Frontal view of elastic registration on the orthognathic skull for automatically selected allowable features. (a) The target and deformable geometry after isotropic scale ICP registration. After the initial registration of selected features in Figure 6.6, elastic surface registration is performed and smoothed resulting in (b) the smoothed registration result at iteration 60. The gray and red line sections represent the target surface. Grey represents the automatically discarded areas while the red lines represent the allowable and featureless target surface in the same plane as Figure 5.15. The black line represents the deformable mesh surface.

6.13 Lateral view of elastic registration on the orthognathic skull for automatically selected allowable features. (a) The target and deformable geometry after isotropic scale ICP registration. After the initial registration of selected features in Figure 6.6, elastic surface registration is performed and smoothed resulting in (b) the registration result at iteration 60. The gray and red line sections represent the target surface. Grey represents the automatically discarded areas while the red lines represent the allowable and featureless target surface in the same plane as Figure 5.16. The black line represents the deformable mesh surface.

6.14 (a) Orthognathic target skull geometry with (b) the registration result and (c) the symmetric version on the registration result. (d) Prognathic target skull geometry with (e) the registration result and (f) the symmetric version on the registration result.

6.15 Inverted elements retained after mesh improvement in the orthognathic skull representation. (a) Global position of inverted elements. (b) Detail showing the four inverted surface elements.

6.16 Mesh quality evaluated using Equation (6.4). (a) Symmetric prognathic skull representation. (b) Original mesh generated on the average surface using TetGen [9]. (c) Symmetric orthognathic skull representation.

6.17 Histogram illustrating the element quality of the optimised prognathic and orthognathic mesh representations as well as the element quality of the original mesh generated on the average skull surface.
6.18 Von Mises stress contours for a molar bite scaled to show a maximum of 8 MPa. (a) Prognathic, (b) Average and (c) Orthognathic skull shape. ................................................................. 110

6.19 (a) The Von Mises stress in the prognathic skull shape plotted on the mesh representing the average shape. (b) The Von Mises stress in the orthognathic skull shape plotted on the mesh representing the average shape. (c) The difference in Von Mises stress between the prognathic and orthognathic finite element results $\sigma_{\text{prognathic}}^{\text{vM}} - \sigma_{\text{orthognathic}}^{\text{vM}}$ shown for the range $[-8, 8]$ MPa. All of the contours are plotted on the mesh representing the average skull shape. (a)-(b)=(c) 111

6.20 (a) Three different meshes representing the orthognathic skull shape. (b) Detail of the meshes in (a) illustrating a difference in nodal coordinate positions. This is done for both prognathic and orthognathic skull shape. Three meshes representing each shape is used to quantify the influence the uniqueness of a registration result obtained by this method has on the final FEA result. ......................... 112

6.21 The difference in Von Mises stress between the results obtained using different prognathic and orthognathic skull shape mesh representation. Three mesh versions of the prognathic and of the orthognathic skull shape are used. An FEA is done on the molar bite load case using all six meshes. The FEA result on the prognathic meshes is compared to the result on the orthognathic meshes in the same way as Figure 6.19 (c). In each row the same prognathic mesh is compared to a different orthognathic mesh while each column shows the result of the same orthognathic mesh compared to a different prognathic mesh. Contours are given for the range $[-2, 2]$ MPa. .................. 113

6.22 Histogram illustrating the distribution of stress variation. The results given in Figure 6.21 is categorised to show the small percentage of elements where a significant variation occur. The absolute value of these results are used and normalised to illustrate them on the same histogram. The majority of elements are seen to fall below 5% of the maximum absolute difference in Von Mises stress. ............... 114
6.23 The variation of the difference in Von Mises stress using the original results compared to the difference in Von Mises stress when one of the original results is compared with the result on a new mesh representation. (a) Figure 6.21 (a) - Figure 6.21 (b). (b) Figure 6.21 (a) - Figure 6.21 (c). (c) Figure 6.21 (a) - Figure 6.21 (d). (d) Figure 6.21 (a) - Figure 6.21 (g). 116

6.24 The Von Mises stress result for a molar bite analysis using the same nodes to apply boundary conditions on the three different orthognathic skull shape mesh representations. 117

6.25 (a) The difference in Von Mises stress for the original prognathic and orthognathic skull analyses also displayed in Figure 6.19 (c). (b) The variation noticed when comparing the Von Mises stress for the original prognathic and second orthognathic skull analyses to the original also displayed in Figure 6.23 (a). Contours are given for the range $[-2, 2]$ MPa. 118

6.26 (a) The FEA resulting Von Mises stress on the average skull shape. (b) The average Von Mises result of the prognathic and orthognathic stresses plotted on the average skull mesh. (c) The difference in Von Mises stress $\sigma_{\text{average}}^\text{vM} - (\sigma_{\text{prognathic}}^\text{vM} + \sigma_{\text{orthognathic}}^\text{vM}) / 2$. This falls in the range $[-2.699, 3.247]$ MPa. 120

6.27 Histogram illustrating the distribution of stress variation. The absolute value of the results given in Figure 6.26 (c) is categorised to show the small percentage of elements where a significant variation occur. The majority of elements are seen to fall below 0.162 MPa, which is 5% of the maximum difference in Von Mises stress 3.247 MPa. 121

A.1 Free body diagram of the skull in the $yz$-plane. Muscle force components are visible in their approximate locations (red) as well the reaction forces (blue) for the working side of both the crania and mandible. 139
A.2 Results of sensitivity analysis done on the orthognathic skull for a molar bite. This analysis was chosen because $F_{LRx}$ is the largest in comparison to other resultant forces when an incisor bite or prognathic skull shape is considered. Working: balancing ratio of $F_{LRx}$ left $1:3$ and right $7:3$ for Von Mises stress set to a maximum of (a), (b) $300 \text{ N/cm}^2$ and (c), (d) $50 \text{ N/cm}^2$. Slight variation in stress field is only visible for stresses far below the range of stresses used in drawing conclusions from FEA results. 

A.3 Muscle contribution and reaction forces on the prognathic skull for a vertical molar bite. 

A.4 Muscle contribution and reaction forces on the orthognathic skull for a vertical molar bite. 

A.5 Muscle contribution and reaction forces on the prognathic skull for a vertical incisor bite. 

A.6 Muscle contribution and reaction forces on the orthognathic skull for a vertical incisor bite. 

A.7 Lateral view of the working side stresses for a molar bite on full prognathic and orthognathic FEA results in $\text{N/cm}^2$. (a), (b) 1st principal stress (c), (d) 2nd principal stress (e), (f) 3rd principal stress and (g), (h) Von Mises stress. 

A.8 Lateral view of the working side stresses for incisor bite on full prognathic and orthognathic FEA results in $\text{N/cm}^2$. (a), (b) 1st principal stress (c), (d) 2nd principal stress (e), (f) 3rd principal stress and (g), (h) Von Mises stress. 

A.9 Lateral view of the working side muscle contribution to Von Mises stress. The molar bite for prognathic and orthognathic FEA results are given in $\text{N/cm}^2$. (a), (b) Temporalis (c), (d) superficial masseter (e), (f) deep head masseter and (g), (h) medial pterygoid contributions. 

A.10 Von Mises stress concentrations. (a) Lower view of the incisal bite analysis on the orthognathic skull geometry with detail in (c). (b) Lower view of the molar bite analysis on the prognathic skull geometry with detail in (d). Stress concentrations in these two analyses are shown with reference to Table A.6. Maximum Von Mises stress occurs at stress concentrations and can not be compared.
C.1 a) 2D log-polar histogram bins for 2D shape context. b) 3D spherical coordinates for use in setting up 3D shape context histogram. [68] 162

C.2 Shape context after first creating skeletal lines. This is done here by Xie et al. [69] for two different elephant outlines. The images illustrate the image position for setting up a shape context histogram for points A in (a) and C in (b). 162

C.3 Shape context histograms of the four points a) A, b) B, c) C and d) D marked in Figure C.2 [69]. Here the corresponding points are seen to have similar histograms if (a) and (c) are compared for example. 162

D.1 Original position of a target and base dolphin geometry. The target geometry is illustrated in its original position with the target features in red and the base geometry features in blue. (a) Isometric, (b) lateral and (c) lower view. 165

D.2 Position of a target dolphin geometry relative to the base shape after isotropic scale ICP registration. The target geometry is illustrated in its registered position with the target features in red and the base geometry features in blue. (a) Isometric, (b) lateral and (c) lower view. 166

D.3 Feature registration of a base dolphin to the aligned target configuration. The target geometry is illustrated in its aligned position with the target features in red and the deformed base geometry features in blue. Only the registered feature lines on both geometries are shown. (a) Isometric, (b) lateral and (c) lower view. 167
List of Tables

5.1 Some radial basis functions with compact support $f(\xi)$ and global support $f(x)$ ........................................... 71

6.1 Mesh Quality compared to original mesh generated from the symmetric skull surface. The tetrahedral mesh representations representing the prognathic and orthognathic skull shapes are then improved using MESQUITTE [5] with boundary nodes constrained .................. 107

6.2 Range values for difference in Von Mises Stress. In each case the Von Mises stress result of the analysis of a molar bite on an orthognathic skull representation is subtracted from the same analysis done on a prognathic skull representation per element. .......................... 115

6.3 Distances from the nodal coordinates on each mesh representation to (a) the position on the generic surface mesh for surface points and (b) the tetrahedral mesh representing the average skull shape. ... 117

A.1 Initial muscle forces. (a) Working side force values used and (b) balancing side as a percentage of working side force derived from [57] with equivalent force value. ......................... 136

A.2 Temporalis muscle section force distribution. (a) Activity of muscle sections approximated from [15] for a vertical bite force. (b) Nodes in each section and (c) weighted resultant force value. ............. 136

A.3 Prognathic force values and directions. .............................. 143

A.4 Orthognathic force values and directions. ............................ 144

A.5 Minimum and maximum displacements obtained from finite element analysis for all mastication forces. ............................. 153

A.6 Maximum Von Mises stress obtained from finite element analysis for individual mastication forces and full analysis. ............... 153