CHAPTER FIVE

BLIND MULTIUSER DETECTION USING THE CONSTANT MODULUS CRITERION

In this chapter, the focus is narrowed to the field of blind linear multiuser detection employing two different modified versions of the constant modulus criterion. The first section is an introduction, presenting the (rather short) history of blind multiuser detection for the CDMA channel. In the second section, the linearly constrained constant modulus criterion is thoroughly analyzed along with the convexity of the cost function. For the *first time*, a global condition is presented with proof for the convexity of the cost function. The derivation of the linearly constrained constant modulus algorithm is also presented in this section. The third section contains the analysis of the linearly constrained *differential* constant modulus criterion as presented by [60]. The convexity of this cost function is analyzed, and is shown to be globally convex. The linearly constrained differential constant modulus algorithm is also derived and presented in this section. In the final section of this chapter, the performance of the aforementioned criteria is discussed at the hand of the results obtained from this chapter.

5.1 INTRODUCTION

Blind multiuser detection was first conceptualized by Honig et. al. [3], and has been based on the principle of the linearly constrained minimum variance (LCMV), which was originally developed for adaptive array antennas [61]. In [3], Honig describes the blind LCMV detector in terms of a *canonical representation* for the linear detector in the signal space. The principle of the LCMV detector is to minimize the receiver output variance, without cancelling the desired signal component. When a stochastic gradient algorithm is used, the solution of the mean weight vector is equivalent to that of the MMSE solution. The stochastic gradient algorithm in the case of the LCMV receiver is termed the linearly constrained minimum variance algorithm (LCMVA). The LCMV detector has the disadvantage that it may cancel out the desired signal component at the receiver output if there are inaccuracies in the desired signal signature vector. An accurate signature vector estimation is needed for the linear constraint. Another disadvantage of the LCMV detector, is that the weight vector adjusted by the LCMVA fluctuates around the optimum point [3], so that the BEP performance degrades.

Another blind approach which is often used in multipath equalization is the constant modulus algorithm (CMA) [62], [63]. The CMA cannot be directly applied to the CDMA channel, as the weight vector might converge to one of the interfering user signature vectors rather than the desired user signature vector [64]. To overcome this problem, the linearly constrained CMA (LCCMA) was proposed by Miguez and Castedo in [20]. Corrections to the aforementioned paper was introduced in [21] and an incorrect closed form analysis of the LCCMA was done in [22], which was later corrected in [23]. The principle of the linearly constrained constant modulus (LCCM) detector is to minimize the deviation of the receiver output from a constant modulus without cancelling the desired signal component. This means that the desired signal component can be protected from being significantly cancelled even if there are inaccuracies in the estimate of the desired signal vector [20]. Moreover, when the receiver output approaches the target constant modulus, the variance of the weight vector as adjusted by the LCCMA can be expected to be relatively small. These qualities make the LCCMA superior to the LCMVA; however, it has been shown that the LCCMA cannot converge to the optimal point if the desired user amplitude is less than a critical value [60]. To overcome this problem, Miyajima /citeMiyajima00 proposed the linearly constrained differential constant modulus (LCDCM) detector to negate the limitation on the desired user amplitude. The stochastic gradient algorithm employing the LCDCM criterion, is subsequently called the linearly constrained differential CMA (LCDCMA). In this dissertation it will be shown that the LCDCM detector achieves comparable performance to the LCCM detector, while there is no limitation on the desired user amplitude.

Both the LCMVA and the LCCMA have the disadvantage that in a frequency selective channel, multiple propagation paths are suppressed rather than combined [47]. The author in [47] proposes a multichannel LCCMA (MLCCMA) to perform the task of joint blind multiuser detection and equalization or multipath diversity combination. In [60] it is implied that the multipath channel impulse response can be estimated using a subspace method, and used as the linear constraint for the LCDCMA. However, this approach requires singular value decomposition (SVD) which makes this method computationally expensive.

In this chapter we will thoroughly analyze the LCCM and LCDCM detectors. We will investigate the cost functions of each of the detectors and then derive the stochastic gradient algorithms associated with each cost function.

5.2 THE LINEARLY CONSTRAINED CONSTANT MODULUS CRITERION

The linearly constrained constant modulus (LCCM) cost function is given by

$$\underset{\mathbf{v}^H\mathbf{s}_1=1}{J(\mathbf{v})} = \frac{1}{2}E\left[\left(|y_t|^2-\alpha\right)^2\right], \tag{5.1}$$

subject to the linear constraint $\mathbf{v}^H\mathbf{s}_1 = 1$, where y_t is the transformed received signal and α is an arbitrary real scalar. Since, $y_t = \mathbf{v}^H\mathbf{r}$, the LCCM cost can now be written as

$$\underset{\mathbf{v}^H\mathbf{s}_1=1}{J(\mathbf{v})} = \frac{1}{2}E\left[\mathbf{v}^H\mathbf{r}\mathbf{r}^H\mathbf{v}\mathbf{v}^H\mathbf{r}\mathbf{r}^H\mathbf{v}\right] - \alpha E\left[\mathbf{v}^H\mathbf{r}\mathbf{r}^H\mathbf{v}\right] + \frac{1}{2}\alpha^2. \tag{5.2}$$

At this point we make a few assumptions concerning our model. First we assume a synchronous channel. Furthermore, we assume that the Gaussian noise component $\sigma \to 0$, and that the signature waveforms are spanned by $\{\psi_1, \ldots, \psi_L\}$. This leaves us with a K dimensional cost function $J(\mathbf{v})$. In the noise free case, $\mathbf{v}^H\mathbf{r} = \mathbf{v}^H\mathbf{SAb}$. If we let $u_k = A_k(\mathbf{v}^H\mathbf{s}_k)$ and $\mathbf{u} = [u_1, u_2, \ldots, u_K]^T$, then we can write the cost function $J(\mathbf{v})$ as

$$\underset{u_1=A_1}{J(\mathbf{u})} = \frac{1}{2}E\left[\mathbf{u}^H\mathbf{b}\mathbf{b}^H\mathbf{u}\mathbf{u}^H\mathbf{b}\mathbf{b}^H\mathbf{u}\right] - \alpha E\left[\mathbf{u}^H\mathbf{b}\mathbf{b}^H\mathbf{u}\right] + \frac{1}{2}\alpha^2, \tag{5.3}$$

where the linear constraint $\mathbf{v}^H\mathbf{s}_1 = 1$ implies that $u_1 = A_1$.

Since $b_k \in \{\pm 1 \pm j\}$ and for different k, b_k are independent random variables, we have the two expected value terms in (5.3) respectively equal to[1]

$$E\left[\mathbf{u}^H\mathbf{b}\mathbf{b}^H\mathbf{u}\mathbf{u}^H\mathbf{b}\mathbf{b}^H\mathbf{u}\right] = 8\left(\mathbf{u}^H\mathbf{u}\right)^2 - 4\sum_{k=1}^{K}|u_k|^4 \tag{5.4}$$

and

$$E\left[\mathbf{u}^H\mathbf{b}\mathbf{b}^H\mathbf{u}\right] = 2\mathbf{u}^H\mathbf{u}. \tag{5.5}$$

Having removed the influence of the expected values on (5.3), we can now write this equation as

$$\underset{u_1=A_1}{J(\mathbf{u})} = 4\left(\mathbf{u}^H\mathbf{u}\right)^2 - 2\sum_{k=1}^{K}|u_k|^4 - 2\alpha\mathbf{u}^H\mathbf{u} + \frac{1}{2}\alpha^2. \tag{5.6}$$

If we write (5.6) in terms of u_k and u_k^*, we have

[1] The two expected value terms are evaluated in Appendix D

$$\underset{u_1=A_1}{J(\mathbf{u})} = 4\left(\sum_{k=1}^{K} u_k^* u_k\right)^2 - 2\sum_{k=1}^{K}(u_k^* u_k)^2 - 2\alpha\sum_{k=1}^{K} u_k^* u_k + \frac{1}{2}\alpha^2. \tag{5.7}$$

Expanding the terms in (5.7) we obtain

$$J(\mathbf{u}) = 4\left(A_1^* A_1 + \sum_{k=2}^{K} u_k^* u_k\right)^2 - 2(A_1^* A_1)^2 - 2\sum_{k=2}^{K}(u_k^* u_k)^2 - 2\alpha A_1^* A_1 - 2\alpha\sum_{k=2}^{K} u_k^* u_k + \frac{1}{2}\alpha^2. \tag{5.8}$$

5.2.1 THE CONVEXITY OF THE LCCM COST FUNCTION

To investigate the convexity of the cost function $J(u_k)$, a property of a continuous convex function in [65] (Theorem 10.2) will be applied. This theorem states that if

$$J\left(\frac{\mathbf{u}_1 + \mathbf{u}_2}{2}\right) \leq \frac{J(\mathbf{u}_1) + J(\mathbf{u}_2)}{2} \tag{5.9}$$

for any points $\mathbf{u}_1$ and $\mathbf{u}_2$, then the function J is *convex*. The function J is *strictly convex* if the above inequality is true as a strict inequality.

Let the projection of $\mathbf{u}$ with $u_1 = 0$ be denoted by $\bar{\mathbf{u}}$. If we write (5.8) in terms of vector norms of $\bar{\mathbf{u}}$, we have

$$J(\mathbf{u}) = 4\left(A_1^* A_1 + \|\bar{\mathbf{u}}\|^2\right)^2 - 2(A_1^* A_1)^2 - 2\|\bar{\mathbf{u}}\|^4 - 2\alpha A_1^* A_1 - 2\alpha\|\bar{\mathbf{u}}\|^2 + \frac{1}{2}\alpha^2. \tag{5.10}$$

Let us start with the RHS of the inequality in equation (5.9),

$$\begin{aligned}
\frac{J(\mathbf{u}_1)}{2} &= 2\left(A_1^* A_1 + \|\bar{\mathbf{u}}_1\|^2\right)^2 - (A_1^* A_1)^2 - \|\bar{\mathbf{u}}_1\|^4 - \alpha A_1^* A_1 - \alpha\|\bar{\mathbf{u}}_1\|^2 + \frac{1}{4}\alpha^2 \\
&= 2(A_1^* A_1)^2 + 2\|\bar{\mathbf{u}}_1\|^4 + 4A_1^* A_1\|\bar{\mathbf{u}}_1\|^2 - (A_1^* A_1)^2 - \|\bar{\mathbf{u}}_1\|^4 - \alpha A_1^* A_1 - \alpha\|\bar{\mathbf{u}}_1\|^2 + \frac{1}{4}\alpha^2 \\
&= \underbrace{(A_1^* A_1)((A_1^* A_1) - \alpha) + \frac{\alpha^2}{4}}_{A} + \underbrace{(4A_1^* A_1 - \alpha)}_{B}\|\bar{\mathbf{u}}_1\|^2 + \|\bar{\mathbf{u}}_1\|^4 \\
&= A + B\|\bar{\mathbf{u}}_1\|^2 + \|\bar{\mathbf{u}}_1\|^4
\end{aligned} \tag{5.11}$$

and equivalently

$$\frac{J(\mathbf{u}_2)}{2} = A + B\|\bar{\mathbf{u}}_2\|^2 + \|\bar{\mathbf{u}}_2\|^4. \tag{5.12}$$

Thus we have the RHS of (5.9) equal to

$$\frac{J(\mathbf{u}_1)+J(\mathbf{u}_2)}{2} = 2A + B\left(\|\bar{\mathbf{u}}_1\|^2+\|\bar{\mathbf{u}}_2\|^2\right)+\|\bar{\mathbf{u}}_1\|^4+\|\bar{\mathbf{u}}_2\|^4. \tag{5.13}$$

The LHS of (5.9) is given by

$$\begin{aligned}
J\left(\frac{\mathbf{u}_1+\mathbf{u}_2}{2}\right) &= 4\left(A_1^*A_1+\frac{\|\bar{\mathbf{u}}_1+\bar{\mathbf{u}}_2\|^2}{4}\right)^2-2\left(A_1^*A_1\right)^2-2\frac{\|\bar{\mathbf{u}}_1+\bar{\mathbf{u}}_2\|^4}{16}-2\alpha A_1^*A_1\\
&\quad -2\alpha\frac{\|\bar{\mathbf{u}}_1+\bar{\mathbf{u}}_2\|^2}{4}+\frac{1}{2}\alpha^2\\
&= 4\left(A_1^*A_1\right)^2+\frac{\|\bar{\mathbf{u}}_1+\bar{\mathbf{u}}_2\|^4}{4}+2A_1^*A_1\|\bar{\mathbf{u}}_1+\bar{\mathbf{u}}_2\|^2-2\left(A_1^*A_1\right)^2\\
&\quad -\frac{\|\bar{\mathbf{u}}_1+\bar{\mathbf{u}}_2\|^4}{8}-2\alpha A_1^*A_1-\alpha\frac{\|\bar{\mathbf{u}}_1+\bar{\mathbf{u}}_2\|^2}{2}+\frac{1}{2}\alpha^2\\
&= 2\left(A_1^*A_1\right)\left(\left(A_1^*A_1\right)-\alpha\right)+\frac{\alpha^2}{2}+\left(2A_1^*A_1-\frac{\alpha}{2}\right)\|\bar{\mathbf{u}}_1+\bar{\mathbf{u}}_2\|^2\\
&\quad +\frac{\|\bar{\mathbf{u}}_1+\bar{\mathbf{u}}_2\|^4}{8}\\
&= 2\left(A_1^*A_1\right)\left(\left(A_1^*A_1\right)-\alpha\right)+\frac{\alpha^2}{2}+4\left(2A_1^*A_1-\frac{\alpha}{2}\right)\left(\frac{\|\bar{\mathbf{u}}_1+\bar{\mathbf{u}}_2\|}{2}\right)^2\\
&\quad +2\left(\frac{\|\bar{\mathbf{u}}_1+\bar{\mathbf{u}}_2\|}{2}\right)^4
\end{aligned} \tag{5.14}$$

If we use the triangle inequality $\|\bar{\mathbf{u}}_1+\bar{\mathbf{u}}_2\|\leq\|\bar{\mathbf{u}}_1\|+\|\bar{\mathbf{u}}_2\|$ we have

$$\begin{aligned}
J\left(\frac{\mathbf{u}_1+\mathbf{u}_2}{2}\right) &\leq 2\left(A_1^*A_1\right)\left(\left(A_1^*A_1\right)-\alpha\right)+\frac{\alpha^2}{2}\\
&\quad +4\left(2A_1^*A_1-\frac{\alpha}{2}\right)\left(\frac{\|\bar{\mathbf{u}}_1\|+\|\bar{\mathbf{u}}_2\|}{2}\right)^2\\
&\quad +2\left(\frac{\|\bar{\mathbf{u}}_1\|+\|\bar{\mathbf{u}}_2\|}{2}\right)^4
\end{aligned} \tag{5.15}$$

Also using the fact that the functions $(\cdot)^2$ and $(\cdot)^4$ are *strictly convex*, we have by (5.9)

$$\begin{aligned}
J\left(\frac{\mathbf{u}_1+\mathbf{u}_2}{2}\right) &< 2\left(A_1^*A_1\right)\left(\left(A_1^*A_1\right)-\alpha\right)+\frac{\alpha^2}{2}\\
&\quad +2\left(2A_1^*A_1-\frac{\alpha}{2}\right)\left(\|\bar{\mathbf{u}}_1\|^2+\|\bar{\mathbf{u}}_2\|^2\right)\\
&\quad +\|\bar{\mathbf{u}}_1\|^4+\|\bar{\mathbf{u}}_2\|^4
\end{aligned} \tag{5.16}$$

If we write (5.16) in terms of A and B, we have

$$\begin{aligned}
J\left(\frac{\mathbf{u}_1+\mathbf{u}_2}{2}\right) &< 2A+B\left(\|\bar{\mathbf{u}}_1\|^2+\|\bar{\mathbf{u}}_2\|^2\right)+\|\bar{\mathbf{u}}_1\|^4+\|\bar{\mathbf{u}}_2\|^4 \qquad (5.17)\\
&= \frac{J(\mathbf{u}_1)+J(\mathbf{u}_2)}{2}
\end{aligned}$$

Because of the above inequality, and by (5.9), we have proved the *strict convexity* of the LCCM cost function subject to

$$A \geq 0 \tag{5.18}$$

and

$$B \geq 0. \tag{5.19}$$

Since (5.19) is a stricter condition than (5.18), we can discard (5.18). We have thus that the LCCM cost function is convex, subject only to

$$B \geq 0 \text{ or equivalently } A_1^* A_1 \geq \frac{\alpha}{4}. \tag{5.20}$$

Note that the inequality $B > 0$ is a *global condition* on A_1 and α, insuring convexity of the LCCM cost function.

5.2.2 THE STATIONARY POINTS OF THE LCCM COST FUNCTION

Considering equation (5.8) again, if we let $u_k = x_k + jy_k$ for $2 \leq k \leq K$, then the cost function $J(\mathbf{u})$ becomes

$$\begin{aligned} J(\mathbf{u}) &= 4\left(A_1^* A_1 + \sum_{k=2}^{K}\left(x_k^2 + y_k^2\right)\right)^2 - 2\left(A_1^* A_1\right)^2 - 2\sum_{k=2}^{K}\left(x_k^2 + y_k^2\right)^2 \\ &\quad -2\alpha A_1^* A_1 - 2\alpha\sum_{k=2}^{K}\left(x_k^2 + y_k^2\right) + \frac{1}{2}\alpha^2 \end{aligned} \tag{5.21}$$

To solve for the stationary points, we find the gradient (directional derivative) of $J(x_k, y_k)$, and equate it to zero. In this way we can attempt to solve for the points at which the cost function is a minimum. In this case it is more informative to differentiate with respect to the real and imaginary parts of $J(x_k, y_k)$, rather than differentiate with respect to a complex vector:

$$\begin{aligned} \nabla_{x_l} J = \frac{\partial J(\mathbf{u})}{\partial x_l} &= 16\left(A_1^* A_1 + \sum_{k=2}^{K}\left(x_k^2 + y_k^2\right)\right) x_l - 8\left(x_l^2 + y_l^2\right) x_l - 4\alpha x_l \\ &= 4x_l\left(4A_1^* A_1 + 4\sum_{k=2}^{K}\left(x_k^2 + y_k^2\right) - 2\left(x_l^2 + y_l^2\right) - \alpha\right) \\ &= 4x_l\left(4A_1^* A_1 + 4\sum_{\substack{k=2 \\ k \neq l}}^{K}\left(x_k^2 + y_k^2\right) + 2x_l^2 + 2y_l^2 - \alpha\right) \end{aligned} \tag{5.22}$$

and equivalently

$$\nabla_{y_l} J = \frac{\partial J(\mathbf{u})}{\partial y_l} = 4y_l \left(4A_1^* A_1 + 4 \sum_{\substack{k=2 \\ k \neq l}}^{K} \left(x_k^2 + y_k^2\right) + 2x_l^2 + 2y_l^2 - \alpha \right) \tag{5.23}$$

with symmetry evident between (5.20) and (5.21). Letting

$$X = \left(4A_1^* A_1 - \alpha + 4 \sum_{\substack{k=2 \\ k \neq l}}^{K} \left(x_k^2 + y_k^2\right) + 2y_l^2 \right) \tag{5.24}$$

and

$$Y = \left(4A_1^* A_1 - \alpha + 4 \sum_{\substack{k=2 \\ k \neq l}}^{K} \left(x_k^2 + y_k^2\right) + 2x_l^2 \right), \tag{5.25}$$

we have

$$x_l \left(2x_l^2 + X\right) = 0 \tag{5.26}$$

and

$$y_l \left(2y_l^2 + Y\right) = 0. \tag{5.27}$$

At this stage two cases can be identified:

1. $4A_1^* A_1 - \alpha \geq 0$;

2. $4A_1^* A_1 - \alpha < 0$.

In the case of $4A_1^* A_1 - \alpha \geq 0$, it is evident that a unique solution exists at $x_l = 0$ and $y_l = 0$, since all the other terms in (5.24) and (5.25) can only be greater than or equal to zero. In the case of $4A_1^* A_1 - \alpha < 0$, solutions may exist at $x_l = 0$ or $x_l = \pm\sqrt{\frac{-X}{2}}$ and $y_l = 0$ or $y_l = \pm\sqrt{\frac{-Y}{2}}$.

We have already ascertained that the LCCM cost function is strictly convex for certain values of α and A_1. Let us consider the trivial solution of $x_l = 0, y_l = 0$. This implies that $u_k = 0 + j0$ with $2 \leq k \leq K$ or $\bar{\mathbf{u}} = \mathbf{0}$, where $\bar{\mathbf{u}} = [0 \; u_2 \; u_3 \; \ldots \; u_K]^T$. Coincidentally, this is also the solution that cancels out all multiuser interference. To prove that the point $\bar{\mathbf{u}} = \mathbf{0}$ is a global minimum of the cost function J conditioned on A_1 and α, we will have to look at the Hessian matrix $\mathbf{H}(J)$ of the cost function J at the point $\bar{\mathbf{u}} = \mathbf{0}$.

To evaluate the nature of the stationary point $\bar{\mathbf{u}} = \mathbf{0}$ of the LCCM cost function, we use the Hessian matrix as defined below

$$\mathbf{H}(J) = \begin{bmatrix} \mathbf{H}_a & \mathbf{H}_b \\ \mathbf{H}_c & \mathbf{H}_d \end{bmatrix}, \tag{5.28}$$

with

$$(\mathbf{H}_a)_{ml} = \frac{\partial}{\partial x_m}\left(\frac{\partial J}{\partial x_l}\right), \tag{5.29}$$

$$(\mathbf{H}_b)_{ml} = \frac{\partial}{\partial y_m}\left(\frac{\partial J}{\partial x_l}\right), \tag{5.30}$$

$$(\mathbf{H}_c)_{ml} = \frac{\partial}{\partial x_m}\left(\frac{\partial J}{\partial y_l}\right), \tag{5.31}$$

and

$$(\mathbf{H}_d)_{ml} = \frac{\partial}{\partial y_m}\left(\frac{\partial J}{\partial y_l}\right). \tag{5.32}$$

The entries of the Hessian matrix are

$$(\mathbf{H}_a)_{ml} = \begin{cases} 16A_1^*A_1 + 24x_l^2 + 8y_l^2 + 16\sum\limits_{\substack{k=2 \\ k\neq l}}^{K}\left(x_k^2 + y_k^2\right) - \alpha & \text{if } l = m \\ 32x_m x_l & \text{if } l \neq m \end{cases}, \tag{5.33}$$

$$(\mathbf{H}_b)_{ml} = 32y_m x_l, \tag{5.34}$$

$$(\mathbf{H}_c)_{ml} = 32x_m y_l, \tag{5.35}$$

and

$$(\mathbf{H}_d)_{ml} = \begin{cases} 16A_1^*A_1 + 24y_l^2 + 8x_l^2 + 16\sum\limits_{\substack{k=2 \\ k\neq l}}^{K}\left(x_k^2 + y_k^2\right) - \alpha & \text{if } l = m \\ 32x_m x_l & \text{if } l \neq m \end{cases}. \tag{5.36}$$

Normally, for the Hessian matrix to be positive definite (i.e., the cost function convex with a global minimum), the determinants of all the principle submatrices of the Hessian matrix must be zero. This is to say that:

$$\begin{aligned}
\det\left((\mathbf{H}_a)_{11}\right) = (\mathbf{H}_a)_{11} &\geq 0, \\
(\mathbf{H}_a)_{11}(\mathbf{H}_a)_{22} - (\mathbf{H}_a)_{12}(\mathbf{H}_a)_{21} &\geq 0, \\
&\vdots \\
\det \mathbf{H}_a &\geq 0, \\
&\vdots \\
\det \mathbf{H}(J) &\geq 0,
\end{aligned} \tag{5.37}$$

Since we have already proved that the LCCM cost function is strictly convex subject to (5.20), we can now show that the point $\bar{\mathbf{u}} = \mathbf{0}$ is a unique global minimum subject to the same conditions. Since it is possible that a strictly convex function may have only one minimum, we can prove the point $\bar{\mathbf{u}} = \mathbf{0}$ a global minimum, by proving it a local minimum [66].

Let us now prove that a *local minimum* exists only at the point $\bar{\mathbf{u}} = \mathbf{0}$ for certain values of A_1 and α. Implementing this, we have the diagonal Hessian

$$(\mathbf{H}(J_0))_{ij} = \begin{cases} 16A_1^*A_1 - 4\alpha & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \tag{5.38}$$

For each of the diagonal elements to be ≥ 0, and the matrix *positive semi-definite*, the following must be true:

$$A_1^*A_1 \geq \alpha/4. \tag{5.39}$$

We have thus proved the local minimum of J at $\bar{\mathbf{u}} = \mathbf{0}$ subject to the condition (5.39). Note that the condition in (5.39) is a *local condition* on A_1 and α, and insures only a local minimum. Coincidentally, this condition corresponds to the condition on global convexity in (5.20).

In the preceding text we have proved the global convexity of the LCCM cost function subject to (5.20) by using a definition of a continuous convex function in (5.9). We have also seen that the single stationary point (also subject to (5.20)), is a global minimum due to the convexity of the LCCM cost function. The conditions of $A_1^*A_1 > \alpha/4$, $A_1^*A_1 = \alpha/4$ and $A_1^*A_1 < \alpha/4$ are depicted in Figures 5.1, 5.2 and 5.3 respectively. This clearly supports the notion we have developed regarding the convexity of the LCCM cost function. For the case of $A_1^*A_1 \geq \alpha/4$, the cost function is strictly convex. If $A_1^*A_1 < \alpha/4$, convexity cannot be guaranteed anymore, as can be seen in Figure 5.3. From this figure it is also evident that the point $x = 0$, $y = 0$ is also not the only stationary point. This corresponds to the solutions of equations (5.26) and (5.27).

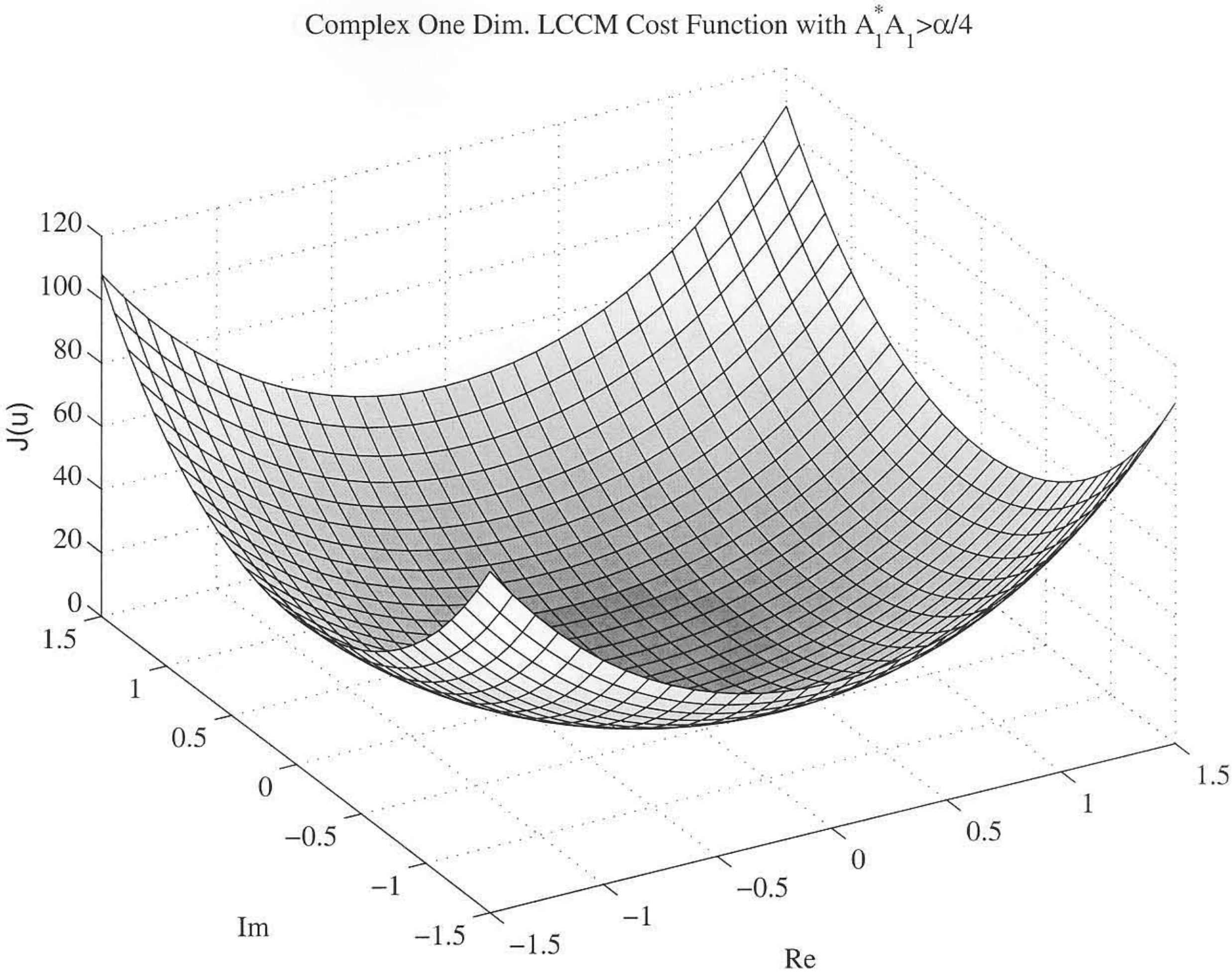

Figure 5.1: Complex LCCM cost function surface with $A_1^* A_1 > \alpha/4$.

5.2.3 LINEARLY CONSTRAINED CONSTANT MODULUS ALGORITHM

The linearly constrained constant modulus algorithm was originally inspired by its application to the field of adaptive arrays [61, 67]. It is based on the generalized sidelobe canceller, which incorporates *a priori* information about the signal. The linear constraint is implemented to capture the user of interest instead of any of the interference signals.

Recall that the LCCM cost function is given by

$$\underset{\mathbf{v}^H \mathbf{s}_1 = 1}{J(\mathbf{v})} = \frac{1}{2} E\left[\left(|y_t|^2 - \alpha\right)^2\right]. \tag{5.40}$$

Let us first consider the unconstrained cost function

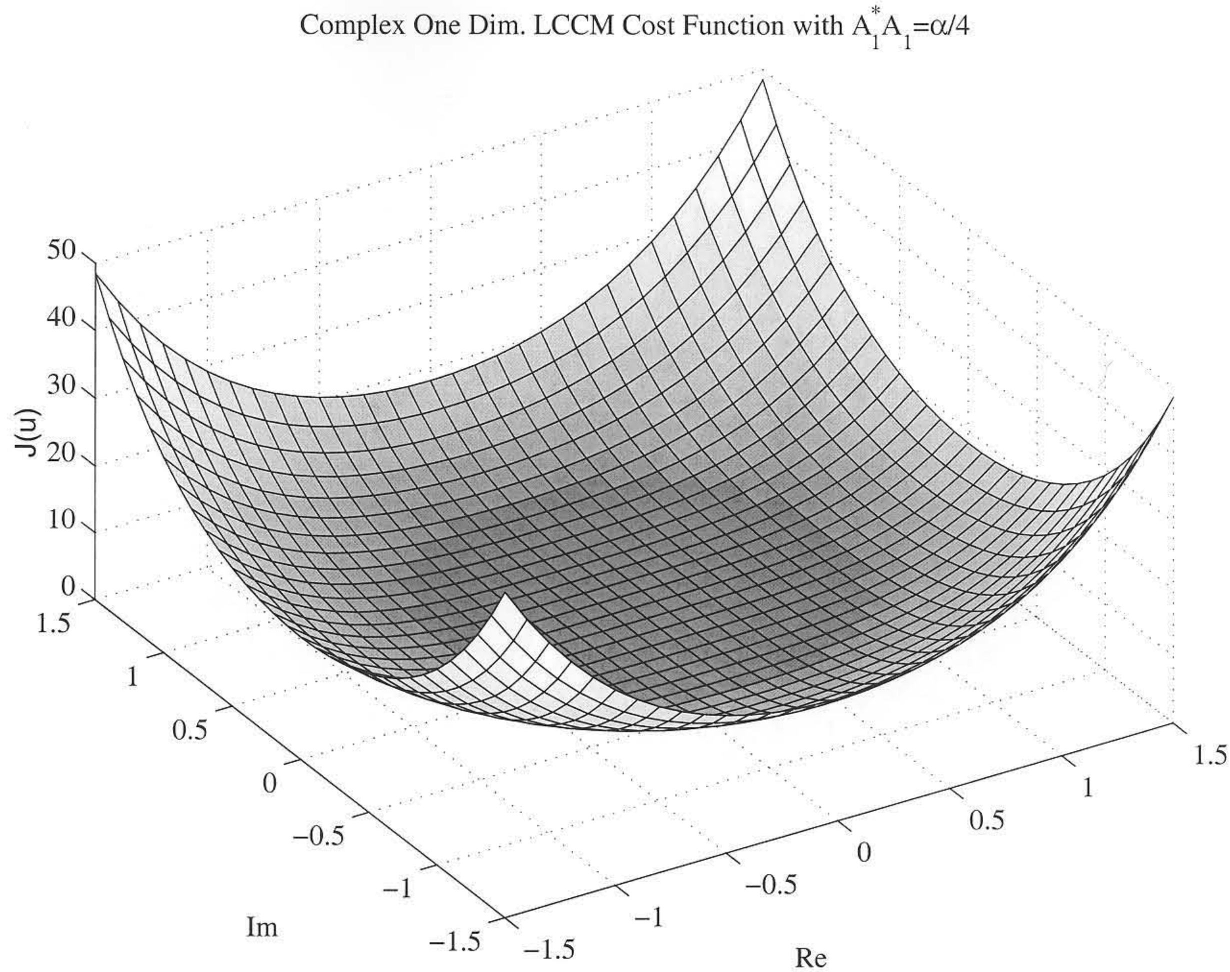

Figure 5.2: Complex LCCM cost function surface with $A_1^* A_1 = \alpha/4$.

$$J = \frac{1}{2} E\left[\left(|y_t|^2 - \alpha\right)^2\right], \tag{5.41}$$

which, if we consider it in terms of inner products, becomes

$$J = \frac{1}{2} E\left[\left(\langle y, c_1^* \rangle^2 - \alpha\right)^2\right], \tag{5.42}$$

where c_1 is (as in the case of the LMS algorithm) the multidimensional parameter which operates on y in the form of a linear transform.

We may consider a *canonical representation* of the linear transform c_1 in terms of the signature waveform of user 1, viz. s_1, and a component orthogonal to s_1, denoted by x_1:

$$c_1 = s_1 + x_1, \tag{5.43}$$

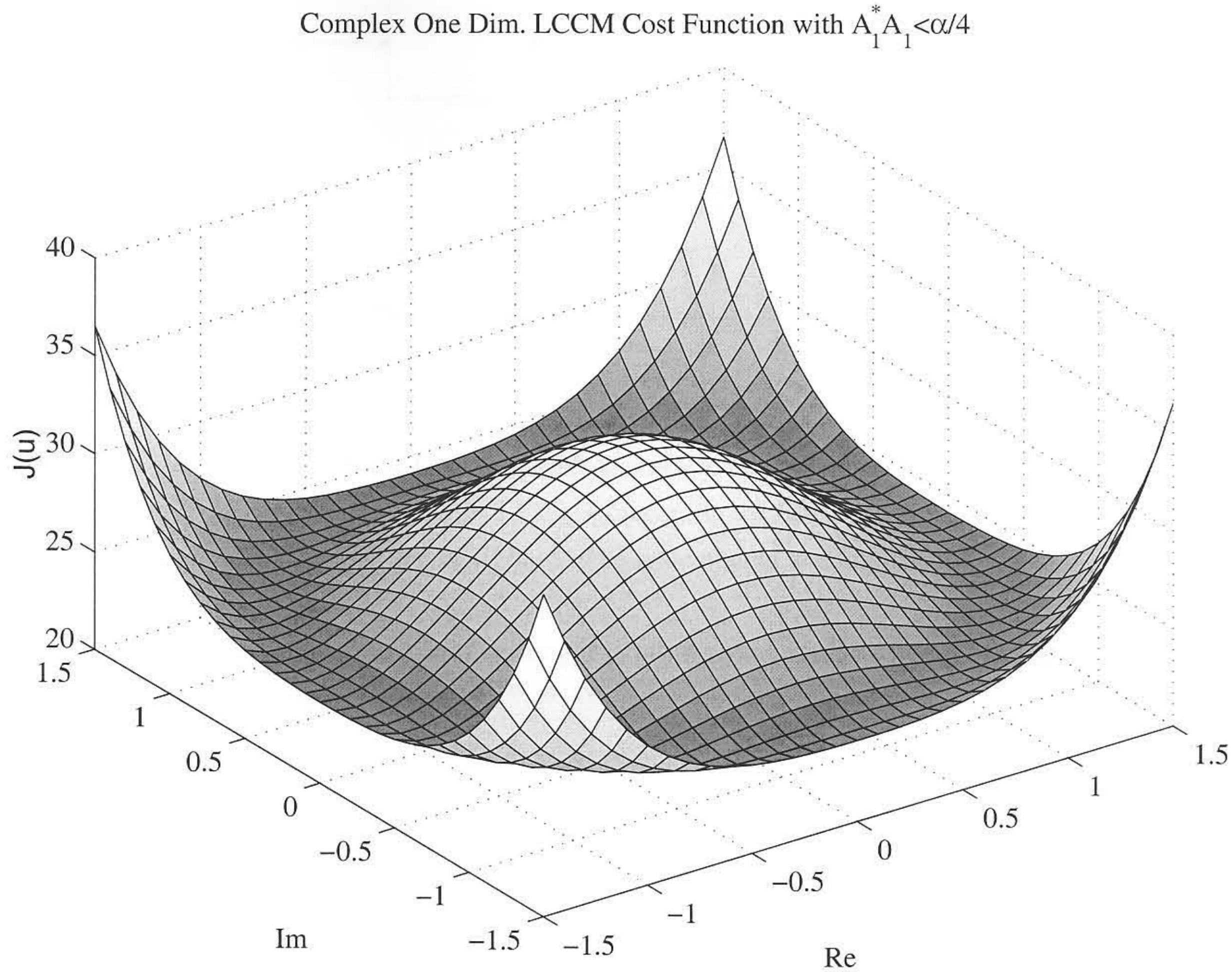

Figure 5.3: Complex LCCM cost function surface with $A_1^* A_1 < \alpha/4$.

where x_1 is such that

$$\langle s_1, x_1^* \rangle = 0 \tag{5.44}$$

This representation is canonical in that every linear multiuser detector of user 1 can be expressed in that form. The set of signals c_1 that can be written as (5.43) and (5.44) are those that satisfy

$$\langle s_1, c_1^* \rangle = \|s_1\|^2 = 1, \tag{5.45}$$

and the decision of $\hat{b}_1 = \mathrm{sgn}(\langle y, c_1^* \rangle)$ is invariant to positive scaling. This means that the only linear transformations that are ruled out by (5.45), are the set of signals c_1 orthogonal to s_1. These signals may be omitted, since they result in an error probability of $1/2$.

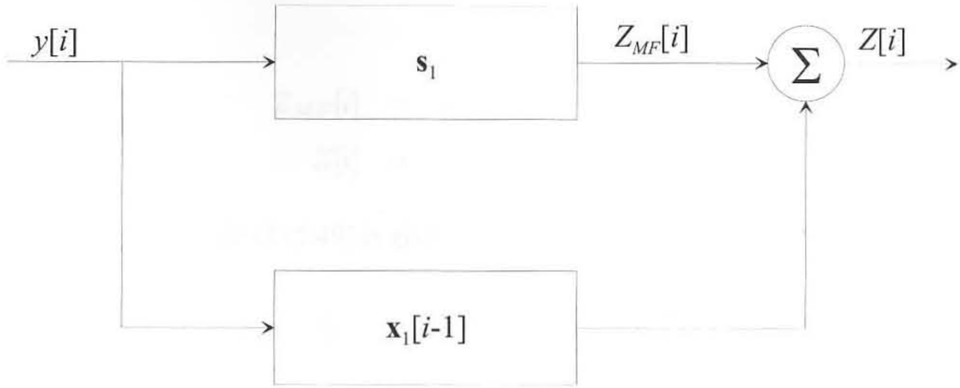

Figure 5.4: Generalized Sidelobe Canceller with $\mathbf{x}[i]$ governed by (5.53) in the case of the LCCM algorithm and (5.79) in the case of the LCDCM algorithm. In the case of the LCDCM algorithm, the previous values $Z_D[i]$, $Z_{MF_D}[i]$ and $\mathbf{r}_D[i]$ need to be remembered to compute $\mathbf{x}[i]$.

Returning to the cost function in (5.43), writing it in terms of the decomposition of c_1 in (5.44) and ignoring the expected value[2], we have

$$J(x_1) = 2\left(\langle y, s_1^* + x_1^* \rangle^2 - \alpha\right)^2. \tag{5.46}$$

If we then find the multidimensional gradient of (5.46), we have

$$\nabla J = 2\left(\langle y, s_1^* + x_1^* \rangle\right)\left(\langle y, s_1^* + x_1^* \rangle^2 - \alpha\right) y^*. \tag{5.47}$$

Note that we are still working with the gradient of the unconstrained cost function. The linear constraint allows the detector to tune out the interference orthogonal to the signature waveform, i.e. restricting the detector from tuning out the desired component. In terms of the gradient, we are looking for the projection or subspace for which the gradient stays orthogonal to s_1. Since the inner product in (5.47) is a complex scaling factor, and y^* is the only multidimensional parameter in the equation, we can restrict y^* (and thus the gradient) to be orthogonal to s_1 by replacing y^* with

$$y - \langle y, s_1^* \rangle s_1. \tag{5.48}$$

Therefore the projection of the gradient in the direction orthogonal to s_1 is

$$\nabla J = 2\left(\langle y, s_1^* + x_1^* \rangle\right)\left(\langle y, s_1^* + x_1^* \rangle^2 - \alpha\right)\left[y - \langle y, s_1^* \rangle s_1\right]. \tag{5.49}$$

Let us denote the matched filter responses for s_1 and $s_1 + x_1[i-1]$ respectively by

[2] As in the case of the LMS algorithm, we may do this. The reason for this is that in the execution of several iterations, the trajectory will be, on average, in the direction of the steepest descent.

$$Z_{MF}[i] = \langle y[i], s_1^* \rangle, \tag{5.50}$$
$$Z[i] = \langle y[i], s_1^* + x_1^*[i-1] \rangle. \tag{5.51}$$

The stochastic adaptation rule of (5.49) is given by

$$x_1[i] = x_1[i-1] - \mu Z[i] \left(Z^2[i] - \alpha \right) \left(y[i] - Z_{MF}[i] s_1 \right), \tag{5.52}$$

which corresponds to the block diagram of the generalized sidelobe canceller in Figure 5.4. As in the case of the LMS algorithm, we may do the following modifications to our system:

- Implementation with finite dimensional vectors rather than continuous time signals.
- Improved convergence speed with more complex recursive algorithms, such as recursive least squares (RLS).
- Implementation in asynchronous channels.

The finite dimensional vector implementation of our LCCM algorithm is given by

$$\mathbf{x}_1[i] = \mathbf{x}_1[i-1] - \mu Z[i] \left(Z^2[i] - \alpha \right) \left(\mathbf{r}[i] - Z_{MF}[i] \mathbf{s}_1 \right), \tag{5.53}$$

with

$$Z_{MF}[i] = \mathbf{s}_1^H \mathbf{r}[i], \tag{5.54}$$
$$Z[i] = \left(\mathbf{s}_1 + \mathbf{x}_1[i-1] \right)^H \mathbf{r}[i]. \tag{5.55}$$

5.3 THE LINEARLY CONSTRAINED DIFFERENTIAL CONSTANT MODULUS CRITERION

The linearly constrained differential constant modulus (LCDCM) cost function is given by

$$\underset{\mathbf{v}^H \mathbf{s}_1 = 1}{J(\mathbf{v})} = \frac{1}{2} E\left[\left(|y_t|^2 - |y_{t_D}|^2 \right)^2 \right], \tag{5.56}$$

subject to the linear constraint $\mathbf{v}^H \mathbf{s}_1 = 1$, where y_t is the transformed received signal and y_{t_D} is a delayed version of the transformed received signal. The LCDCM criterion attempts to keep the modulus of the received signal constant from time t to time $t + D$. Following the same reasoning as in the case of the LCCM detector, we will show that the LCDCM cost function has a *global* *minimum*. Since, $y_t = \mathbf{v}^H\mathbf{r}$ and assuming a quasi stationary CDMA channel, the LCDCM cost can now be written as

$$\begin{aligned} \underset{\mathbf{v}^H\mathbf{s}_1=1}{J(\mathbf{v})} &= \tfrac{1}{2}E\left[\mathbf{v}^H\mathbf{r}\mathbf{r}^H\mathbf{v}\mathbf{v}^H\mathbf{r}\mathbf{r}^H\mathbf{v}\right] - E\left[\mathbf{v}^H\mathbf{r}\mathbf{r}^H\mathbf{v}\mathbf{v}^H\mathbf{r}_D\mathbf{r}_D^H\mathbf{v}\right] \\ &+\tfrac{1}{2}E\left[\mathbf{v}^H\mathbf{r}_D\mathbf{r}_D^H\mathbf{v}\mathbf{v}^H\mathbf{r}_D\mathbf{r}_D^H\mathbf{v}\right], \end{aligned} \tag{5.57}$$

where r_D is the delayed received vector. Again, we assume that the Gaussian noise component $\sigma \to 0$, and that the signature waveforms are spanned by $\{\psi_1, \ldots, \psi_L\}$. This leaves us with a K dimensional cost function $J(\mathbf{v})$. In the noise free case, $\mathbf{v}^H\mathbf{r} = \mathbf{v}^H\mathbf{SAb}$. If we let $u_k = A_k(\mathbf{v}^H\mathbf{s}_k)$ and $\mathbf{u} = [u_1, u_2, \ldots, u_K]^H$, then we can write the cost function $J(\mathbf{v})$ as

$$\begin{aligned} \underset{u_1=A_1}{J(\mathbf{u})} &= \tfrac{1}{2}E\left[\mathbf{u}^H\mathbf{b}[i]\mathbf{b}^H[i]\mathbf{u}\mathbf{u}^H\mathbf{b}[i]\mathbf{b}^H[i]\mathbf{u}\right] - E\left[\mathbf{u}^H\mathbf{b}[i]\mathbf{b}^H[i]\mathbf{u}\mathbf{u}^H\mathbf{b}[i-D]\mathbf{b}^H[i-D]\mathbf{u}\right] \\ &+\tfrac{1}{2}E\left[\mathbf{u}^H\mathbf{b}[i-D]\mathbf{b}^H[i-D]\mathbf{u}\mathbf{u}^H\mathbf{b}[i-D]\mathbf{b}^H[i-D]\mathbf{u}\right]. \end{aligned} \tag{5.58}$$

Let us again assume that the bits of different users are independent and that $b[i], b[i-D] \in \{\pm 1 \pm j\}$. Furthermore, assuming that the delay D is greater than any partial response signalling inherent in the system, we have that bits separated by D seconds are independent, and hence the expectation value terms of (5.58) can be written as

$$E\left[\mathbf{u}^H\mathbf{b}[i]\mathbf{b}^H[i]\mathbf{u}\mathbf{u}^H\mathbf{b}[i]\mathbf{b}^H[i]\mathbf{u}\right] = 8\left(\mathbf{u}^H\mathbf{u}\right)^2 - 4\sum_{k=1}^{K}|u_k|^4, \tag{5.59}$$

$$E\left[\mathbf{u}^H\mathbf{b}[i-D]\mathbf{b}^H[i-D]\mathbf{u}\mathbf{u}^H\mathbf{b}[i-D]\mathbf{b}^H[i-D]\mathbf{u}\right] = 8\left(\mathbf{u}^H\mathbf{u}\right)^2 - 4\sum_{k=1}^{K}|u_k|^4 \tag{5.60}$$

and

$$E\left[\mathbf{u}^H\mathbf{b}[i]\mathbf{b}^H[i]\mathbf{u}\mathbf{u}^H\mathbf{b}[i-D]\mathbf{b}^H[i-D]\mathbf{u}\right] = 4\left(\mathbf{u}^H\mathbf{u}\right)^2. \tag{5.61}$$

This greatly simplifies the LCDCM cost function to

$$\underset{u_1=A_1}{J(\mathbf{u})} = 4\left(\mathbf{u}^H\mathbf{u}\right)^2 - 4\sum_{k=1}^{K}|u_k|^4. \tag{5.62}$$

Writing (5.62) in terms of a summation of u_k and exercising the linear constraint $u_1 = A_1$, we have

$$J(\mathbf{u}) = 4\left(A_1^*A_1 + \sum_{k=2}^{K}u_k^*u_k\right)^2 - 4\sum_{k=2}^{K}\left(u_k^*u_k\right)^2 - 4\left(A_1^*A_1\right)^2. \tag{5.63}$$

Letting $u_k = x_k + jy_k$ for $2 \leq k \leq K$, the cost function $J(\mathbf{u})$ becomes

$$J(\mathbf{u}) = 4\left(A_1^* A_1 + \sum_{k=2}^{K}\left(x_k^2 + y_k^2\right)\right)^2 - 4\sum_{k=2}^{K}\left(x_k^2 + y_k^2\right)^2 - 4\left(A_1^* A_1\right)^2. \qquad (5.64)$$

Once again, the gradient of the cost function ∇J with respect to the lth real and imaginary elements of $\mathbf{u}$ is found, and equated it to zero, yields:

$$\begin{aligned}
\nabla_{x_l} J &= 16x_l\left(A_1^* A_1 + \sum_{k=2}^{K}\left(x_k^2 + y_k^2\right)\right) - 16x_l\left(x_l^2 + y_l^2\right) \\
&= 16x_l\left(A_1^* A_1 + x_l^2 + y_l^2 + \sum_{\substack{k=2 \\ k\neq l}}^{K}\left(x_k^2 + y_k^2\right) - x_l^2 - y_l^2\right) \\
&= 16x_l\left(A_1^* A_1 + \sum_{\substack{k=2 \\ k\neq l}}^{K}\left(x_k^2 + y_k^2\right)\right) = 0 \qquad (5.65)
\end{aligned}$$

and equivalently

$$\nabla_{y_l} J = 16y_l\left(A_1^* A_1 + \sum_{\substack{k=2 \\ k\neq l}}^{K}\left(x_k^2 + y_k^2\right)\right) = 0. \qquad (5.66)$$

This is highly encouraging, since there exists a unique trivial solution of $x_l = 0$ and $y_l = 0$ for the gradient equations (5.65) and (5.66). This corresponds to $\bar{\mathbf{u}} = \mathbf{0}$, once again with $\bar{\mathbf{u}} = [u_2\ u_3\ \ldots\ u_K]$, which is also the solution of the MMSE detector with no multipath or code mismatch.

We will now examine if $\bar{\mathbf{u}} = \mathbf{0}$ is a *global minimum*. As will be seen in the following section, we need not even consider the convexity of the function $J(\bar{\mathbf{u}})$ to determine if $\bar{\mathbf{u}} = \mathbf{0}$ is a global minimum.

5.3.1 GLOBAL MINIMUM OF THE LCDCM COST FUNCTION

In the case of the linearly constrained differential CMA, it is simple to show that the cost function has a global minimum, without even having to consider the convexity of the LCDCM cost function. Since the gradient functions (5.65) and (5.66) only has a trivial solution at $\bar{\mathbf{u}} = \mathbf{0}$, we need only to examine the nature of the stationary point $\bar{\mathbf{u}} = \mathbf{0}$. Again we use the Hessian as defined in (5.28). The entries of the Hessian are given by

$$(\mathbf{H}_a)_{ml} = \begin{cases} 16\left(A_1^* A_1 + \sum\limits_{\substack{k=2 \\ k\neq l}}^{K}\left(x_k^2 + y_k^2\right)\right) & \text{if } l = m \\ 32x_m x_l & \text{if } l \neq m \end{cases}, \qquad (5.67)$$

$$(\mathbf{H}_b)_{ml} = 32 y_m x_l, \tag{5.68}$$

$$(\mathbf{H}_c)_{ml} = 32 x_m y_l, \tag{5.69}$$

and

$$(\mathbf{H}_d)_{ml} = \begin{cases} 16\left(A_1^* A_1 + \sum\limits_{\substack{k=2\\k\neq l}}^{K} \left(x_k^2 + y_k^2\right)\right) & \text{if } l = m \\ 32 x_m x_l & \text{if } l \neq m \end{cases} . \tag{5.70}$$

The Hessian at the point $x_l = y_l = 0$ is given by

$$\mathbf{H}(J_0) = \begin{bmatrix} 16A_1^*A_1 & 0 & \cdots & 0 \\ 0 & 16A_1^*A_1 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & 16A_1^*A_1 \end{bmatrix}, \tag{5.71}$$

which is positive definite. This means that the point $x_l = y_l = 0$ or $\bar{\mathbf{u}} = \mathbf{0}$ is a *unique global minimum*.

We have seen in this section that the LCDCM cost criterion exhibits a global minimum. Unlike the LCCM criterion, this point remains a minimum irrespective of desired user amplitude. Figure 5.5 shows the one dimensional complex surface of the LCDCMA cost function for any value of A_1. It is clearly convex with a global minimum.

5.3.2 LINEARLY CONSTRAINED DIFFERENTIAL CONSTANT MODULUS ALGORITHM

The LCDCMA can be derived by also using the stochastic gradient approach as in the case of the LMS and LCCM algorithms. Again, recall the LCDCM cost function:

$$\underset{\mathbf{v}^H\mathbf{s}_1=1}{J(\mathbf{v})} = \frac{1}{2} E\left[\left(|y_t|^2 - |y_{t_D}|^2\right)^2\right]. \tag{5.72}$$

Let us first consider the unconstrained cost (as with the LCCMA), which is given by

$$J = \frac{1}{2} E\left[\left(|y_t|^2 - |y_{t_D}|^2\right)^2\right]. \tag{5.73}$$

The cost function in terms of inner products representing the transformed received and delayed transformed received signals is given by

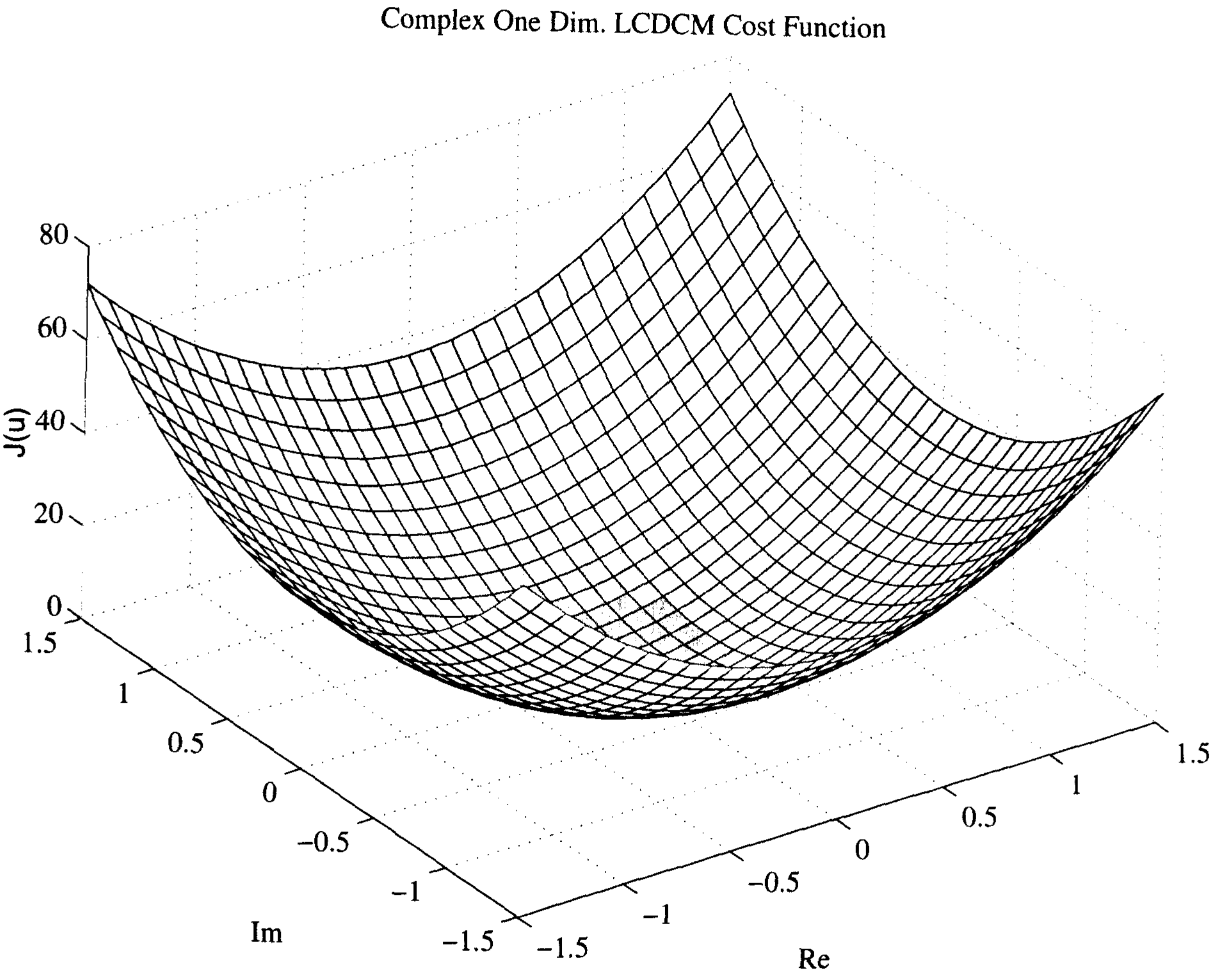

Figure 5.5: One Dimensional Complex LCDCM cost function surface.

$$J = \frac{1}{2}E\left[\left(\langle y, c_1^*\rangle^2 - \langle y_D, c_1^*\rangle^2\right)^2\right], \tag{5.74}$$

where c_1 is the multidimensional parameter which operates on y in the form of a linear transform.

We may now write the cost function in terms of the canonical representation of (5.43) and (5.44), ignoring the expected value[3]:

$$J = \frac{1}{2}\left(\langle y, s_1^* + x_1^*\rangle^2 - \langle y_D, s_1^* + x_1^*\rangle^2\right)^2. \tag{5.75}$$

Finding the multidimensional gradient of (5.75) we get

[3]As in the case of the LMS algorithm, we may do this. The reason for this is that in the execution of several iterations, the trajectory will be, on average, in the direction of the steepest descent.

$$\nabla J = 4\left(y\left\langle y, s_1^* + x_1^*\right\rangle - y_D\left\langle y_D, s_1^* + x_1^*\right\rangle\right)\left(\left\langle y, s_1^* + x_1^*\right\rangle^2 - \left\langle y_D, s_1^* + x_1^*\right\rangle^2\right) \tag{5.76}$$

Let us follow a similar approach whereby (5.49) is established. If we implement the linear constraint, the projection of the gradient in the direction orthogonal to s_1 is

$$\begin{aligned}\nabla J \;\; = \;\; & 4\left(\left[y - \left\langle y, s_1^*\right\rangle s_1\right]\left\langle y, s_1^* + x_1^*\right\rangle - \left[y_D - \left\langle y_D, s_1^*\right\rangle s_1\right]\left\langle y_D, s_1^* + x_1^*\right\rangle\right) \\ & \left(\left\langle y, s_1^* + x_1^*\right\rangle^2 - \left\langle y_D, s_1^* + x_1^*\right\rangle^2\right)\end{aligned} \tag{5.77}$$

where the terms $[y - \langle y, s_1^*\rangle s_1]$ and $[y_D - \langle y_D, s_1^*\rangle s_1]$ are the projections or subspaces of y and y_D respectively for which the gradient stays orthogonal to s_1.

Denoting the matched filter responses for s_1 and $s_1 + x_1[i-1]$ as in (5.50) and (5.51), and the responses of the delayed signals with the subscript letter D, the adaptation rule of (5.77) is given by

$$x_1[i] = x_1[i-1] - \mu\left(\left[y[i] - Z_{MF}[i]s_1\right]Z[i] - \left[y_D[i] - Z_{MF_D}[i]s_1\right]Z[i]_D\right)\left(Z[i]^2 - Z[i]_D^2\right). \tag{5.78}$$

The finite dimensional vector implementation of the LCDCM algorithm is given by

$$\mathbf{x}_1[i] = \mathbf{x}_1[i-1] - \mu\left(\left[\mathbf{r}[i] - Z_{MF}[i]\mathbf{s}_1\right]Z[i] - \left[\mathbf{r}_D[i] - Z_{MF_D}[i]\mathbf{s}_1\right]Z[i]_D\right)\left(Z[i]^2 - Z[i]_D^2\right). \tag{5.79}$$

where the matched filter responses for $\mathbf{s}_1$ and $\mathbf{s}_1 + \mathbf{x}_1[i-1]$ are given by (5.54) and (5.55) and the responses of the delayed signals are again denoted by the subscript letter D.

5.4 PERFORMANCE OF THE LCCM AND LCDCM ALGORITHMS IN MULTIPATH FADING CHANNELS

The MMSE detector optimally combines multiple propagation paths, making it a very suitable receiver structure, given sufficient filter length to span all correlated paths. As we have seen, in single path environment, the LCCM and LCDCM detectors have the same vector weight solutions as the MMSE detector (assuming $\alpha > 1/4$ in the case of the LCCM detector). The single path vector weight solution of the MMSE, LCCM and LCDCM detector is given by (4.47) and [47] as

$$\bar{\mathbf{v}} = \mathbf{C}^{-1}\mathbf{s}_1 \tag{5.80}$$

The question now arises: How will the blind LCCM and LCDCM detectors fare in a multipath environment? Unfortunately, all multiple paths (except one) are suppressed as interference. The reason for this is that in a multipath environment, $\mathbf{p}$ in (4.46) is no longer equal to $\mathbf{s}_1$. The vector $\mathbf{p}$ will now contain the contributions from the correlated parts of the delayed multipath components. Regardless of the multipath value of $\mathbf{p}$, the blind LCCM and LCDCM detectors will continue to extract only one path which correlates with $\mathbf{s}_1$.

Two ways have been proposed to allow the blind LCCM and LCDCM detectors to effectively combine the multiple paths:

1. The multipath channel can be estimated and used as the linear constraint for either the LCCM and LCDCM algorithms [60]. In this way all paths can be effectively combined.

2. A multi-channel LCCM (or LCDCM) algorithm as proposed by Mangalvedhe [47] can be used.

The former method is complex in that is requires singular value decomposition to estimate the multipath channel. The latter method uses several full detectors (channels) to extract each of the multiple paths. Adaptive weights are then used to optimally combine the outputs of the detectors. The fact that one needs a full detector to extract a single path also makes the multi-channel LCCM (or LCDCM) computationally expensive for a large number of paths. Although not discussed in this dissertation, it would be informative to compare the above mentioned two methods for multipath combination, both in terms of complexity, computational cost and performance.

Areas of possible further study could either be a search for a suitable cost function that will optimally combine multiple paths, or other methods to modify the LCCM or LCDCM algorithms which require less complexity than the above mentioned methods.

5.5 SUMMARY

This chapter contains much of the novel theoretical work attempted in this dissertation. The problem of blind multiuser detection utilizing the constant modulus algorithm is explored. An introductory section familiarizes the reader with the relatively recent history associated with blind multiuser detection, as well as all the research that has been attempted in this field.

The second section concerns itself with the thorough analysis of the LCCM cost function. For the first time, through rigorous analysis, a global condition for the convexity of LCCM cost function is derived. The nature of the stationary points are also examined. Subsequently, the LCCM algorithm is derived and presented.

The following section shows that the LCDCM criterion is the solution to non-convergence problems that, under certain circumstances, may plague the LCCMA. This is done by proving that a global minimum exist on the LCDCM cost function. Following the analysis of the LCDCM criterion, the LCDCM algorithm is derived.

In the final section, a qualitative analysis of the LCCM and LCDCM detector performance in a fading multipath channel is conducted. Methods to remedy shortcomings of these detectors in a multipath environment, are proposed.